OTA–Based Non–linear Function Approximations

Edgar Sánchez–Sinencio[1], *Jaime Ramírez–Angulo*[1], *Bernabé Linares–Barranco*[1,2], *Angel Rodríguez–Vázquez*[2]

[1]Department of Electrical Engineering, Texas A&M University
College Station, Texas 77843–3128, U.S.A.

[2]Departamento de Electrónica y Electromagnetismo
Universidad de Sevilla, 41012–Sevilla, Spain

Abstract – **The suitability of operational transconductance amplifiers (OTAs) as the main active element to obtain basic building blocks for the design of programmable non–linear continuous–time networks is presented. The main purpose is to show that the OTA, as the active element in basic building blocks can be efficiently used for non–linear continuous–time functions synthesis. Two efficient non–linear function syntheses approaches are presented. The first approach is a rational approximation and the second is a piecewise–linear approach. Test circuits have been integrated using a 3μ p–well CMOS process. The flexibility of the designed and tested circuits is confirmed.**

I. Introduction

Lately, several authors [1] – [5] have been successfully using the Operational Transconductance Amplifier (OTA) as the main active element in continuous–time active filters. The OTA's programmability nature[1] and the fact that OTAs have only a single high impedance node, in contrast to conventional op amps make the OTA an excellent device candidate for high frequency and voltage (or current) programmable analog basic building blocks. The applicability of OTAs as components to the design of linear networks has been extensively discussed elsewhere [1], [6] and not repeated here. The objective of this paper is to examine the applicability of OTAs as the basic elements to the design of *non–linear* networks. There is not much reported in the literature on the use of OTA for designing non–linear block components [7] –[8]. There are reported excellent contributions [9] – [11], [16] of non–linear circuits dealing with particular important non–linear problems. In our proposed approach rather than try to tackle a specific problem, we focus our attention in a general approach dealing with non–linear basic building blocks using OTAs as the main active elements. At this point no emphasis was done to optimize the circuit performance but to explore the potential and applicability of the OTA–based non–linear system approach.

[1] The output current I_0 of an OTA due to a differential input v_{id} is $I_0 = g_m v_{id}$ and g_m is a voltage (current) controllable parameter[1], [6], [7].

II. Basic Building Blocks

Multiplier Block. A two input four quadrant multiplier has an output(current) given by

$$I_0 = K_M V_1 V_2 \tag{1}$$

where the multiplier constant K_M has units of A/V^2. If V_1 and V_2 can take any positive or negative sign, the multiplier is called a four–quadrant multiplier. This multiplier is represented in Fig. 1(a). The corresponding OTA–based implementations are shown in Fig. 1(b). The block "a" represents a signal attenuator, its function is such that the maximum voltage swing of V_1 and V_2 are equalized, and $-V_{bb}$ is the usual bias control of the OTA. An active attenuator can be implemented in CMOS technology [15]. Although not indicated in Fig. 1, assume the power supplies of the OTAs are V_{DD} and $-V_{SS}$. For the circuit of Fig: 1(b) we obtain

$$I_1 = g_{m1} V_1 = K(V_{I_1} + V_{SST}) V_1 \tag{2a}$$

and

$$I_2 = -g_{m2} V_1 = -K(V_{I_2} + V_{SST}) V_1 \tag{2b}$$

where K is a process– and geometry–dependent constant, $V_{SST} = V_{SS} - V_t$, and V_t is a transistor threshold voltage[2]. The output current becomes $I_0 = -aKV_1V_2 = -|K_M|V_1V_2$ where $|K_M| = aK$. We have the flexibility of making the sign of K_M positive or negative by injecting V_2 to OTA1 instead of to OTA2.

Divider Block. A two input divider has an output which is the ratio of the two inputs, multiplied by a dimensional (in volts) constant K_R, i.e., $V_0 = K_R \frac{V_1}{V_2}$. A symbol of the divider is shown in Fig. 2(a), where the notation n and d stands for numerator and denominator, respectively. The corresponding OTA–based circuit implementation using the multiplier symbol is shown in Fig. 2(b).

Squaring and High Powers (Exponentiation) Blocks. A one input squarer has an output proportional to the square of

[2] We have assumed equal K's and threshold voltages V_t's for the OTAs.

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the input, $I_0 = K_M V_i^2$. The implementation of the squarer is obtained by simply using a multiplier with equal inputs. To obtain an *exponentiation* (raising to a power) block operator with an input V_i and an output to be proportional to V_i^p where $p > 2$, it is required $(p+1)/2$ multipliers for p odd and $p/2$ multipliers for p even. Furthermore, since the proposed multipliers are of the transconductance type, the outputs must be converted into voltages to be able to use them as the inputs of following multipliers. This can be easily obtained by connecting an equivalent resistor at the output. An equivalent resistor using an OTA [5] - [6] is implemented by connecting the output to the negative OTA input and grounding the positive OTA input.

Square-Rooter Block. A one input square-rooter has an output with the negative or positive square root of an input voltage multiplied by a constant of a proper polarity, e.g., $V_0 = \pm\left|\sqrt{K_R V_i}\right|$, $V_i > 0$ or $V_0 = \pm\left|\sqrt{-K_R V_i}\right|$, $V_i < 0$. Fig. 3(a) shows the implementation of the square-rooter, where the output V_0 is given by $V_0 = K_R \frac{V_i}{V_0}$ which yields $V_0 = \left|\sqrt{K_R V_i}\right|$. A more detailed description of the implementation is shown in Fig. 3(b).

Piecewise-Linear Function Generators. Diodes interconnected with OTAs can simulate ideal diodes, thus allowing a piecewise-linear approximation to any desired non-linear function. The accuracy, naturally, improves with the number of line segments involved. The ideal basic building block for piecewise-linear function approximation is shown in Fig. 4. Note that $I_D = 0$ until the breaking point (voltage reference V_r) is reached. The slopes of the linear segments are proportional to the g_m's. The diodes can be implemented with MOS transistors with their gate and drain tied together. If a step type input-output characteristic is needed to implement discontinuities in the function approximation, the linear OTA can be substituted by an OTA comparator which ideally simulates a large g_m and a saturation (output) current of $\pm I_{bias}$.

III. Nonlinear Function Syntheses

A *rational approximation* that has the general form of a polynomial function or of a ratio of polynomials,i.e.,

$$\frac{y_O}{y_{in}} = \frac{\sum_{i=0}^{M} A_i x^i}{\sum_{i=0}^{N} B_i x^i} \tag{16}$$

where i is a positive integer number. In fact, the exponent i can be a fractional exponent of the form p/q, where p and q are negative or positive integers. The exponentation blocks are of the type of Fig. 5. If a negative $-p/q$ is needed, an additional divider has to be used.

A piecewise linear approximation can be obtained by using the basic building block of Fig. 4. Changing the polarity of diodes and input terminals of OTA's allow the obtention of negative and positive slopes. Arbitrary functions with variable positive and negative slopes can be approximated. Furthermore, the *slopes* are *voltage programmable*[3] which gives an additional flexibility in the function approximation design problem. One example of an arbitrary function approximation containing negative and positive slopes is discussed in the next section. Details on the practical considerations of the OTA-based piecewise-linear circuits are under consideration.

IV. Experimental Results

Several test-circuits containing OTAs and transistors connected as diodes were fabricated using a 3μm p-well CMOS process by MOSIS. The linearized OTA used to synthesize the different non-linear analog functions is reported in [3]. The die area of each OTA is $220 \times 700 \mu m^2$ and it consumes 10 mW of power with ± 5V supply voltages. A chip photomicrograph showing two complete OTAs of the test circuit is depicted in Fig 6.

A. *Transconductance Multiplier.* The structure used is shown in Fig. 6. The measured value of K_M is $2.4\mu A/V^2$. In all measurements described here a $100K\Omega$ load resistor was used. The large-signal characteristics of the multiplier are shown in Fig 7. V_1 was held constant (at $0.0V$, $\pm 0.33V$, $\pm 0.66V$, $\pm 1.00V$), while the input V_2 varied between $\pm 1V$. The non-linearity error is shown in Fig 8. For V_2, a triangular 2 volts peak-to-peak signal was applied, while keeping $V_1 = 1V$. The output current produced a triangular voltage signal of 660 mV peak-to-peak. Substracting this signal from an ideal triangular wave, the resulting peak-to-peak error signal was 17 mV which yields a non-linearity error of nearly 2%. Repeating the measurement but interchanging V_1 and V_2 and being V_1 a triangular signal of 2 volts peak-to-peak. A peak-to-peak error signal with an amplitude of 23 mV corresponding to a 3.5% non-linearity error was measured. Fig. 9 shows the multiplier being used as a modulator for the case where both input signals are sinusoidal.

B. *Piecewise Linear Approximation.* The intended transfer characteristic to be considered is shown in Fig. 10(a) and consists of three linear segments. The individual slopes due to each OTA are indicated in the lower part of Fig. 10(a), and the composed resulting transfer characteristics are shown in the upper part of Fig. 10(a). The actual OTA circuit implementation is shown in Fig. 10(b) where an optional diode and voltage sources have been added at the OTA (2 and 3) and shown with broken lines to improve high frequency performance of the circuit. Note that the slopes of the transfer characteristics can be easily modified by changing the OTA voltage-dependent transconductances. The experimental results are shown in Fig. 10(c).

V. Conclusions

The suitability of OTAs as the main active element to obtain basic building blocks for the design of non-linear networks was established. Methods to implement prac-

[3] Additionally, if a resistive load simulated with an OTA is used, the slopes become ratios of transconductances which provides a very good temperature compensation and accuracy improvement.

tical non-linear circuits in a systematic design approach were developed. Two practical synthesis approaches were introduced. The programmability and flexibility of the OTA provides the potential to design adaptive non-linear circuits. Implementations of other non-linear synthesis approaches are feasible using the basic blocks here introduced. The proposed OTA-based building blocks can be incorporated in a CAD software [12] to fully exploit their functionality and versatility. The test-integrated circuit experimental results verified the theoretical predictions.

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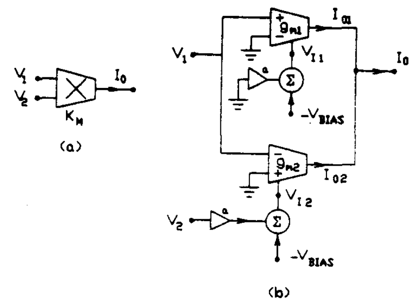


Fig. 1 Multiplier (a) Symbol, (b) OTA Implementation 1.

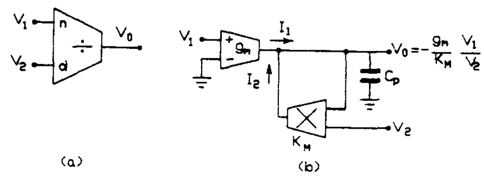


Fig. 2 Divider (a) Symbol, (b) OTA Implementation.

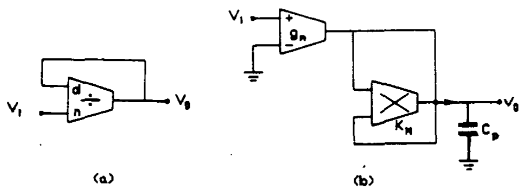


Fig. 3 Square-Rooter, (a) Implementation, (b) OTA Implementation.

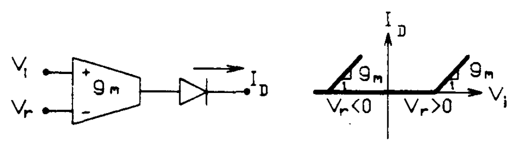


Fig. 4 Piecewise-Linear (PL) Function Generator Building Block .

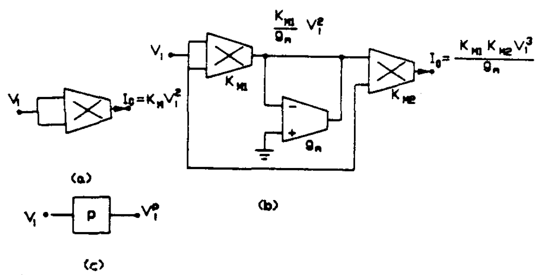
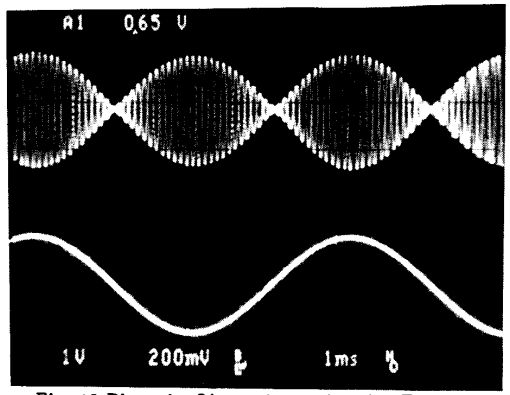


Fig. 5 Exponentiation (Raising to a power) Operation, (a) Squarer, (b) Cubit, (c) p-th.

Fig. 9 Modulation for Two Input Sinusoidal Signals. ⟹



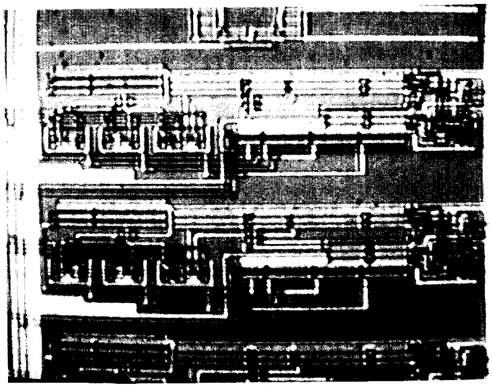


Fig. 6 A Chip Photo Micrograph of Two Complete OTAs.

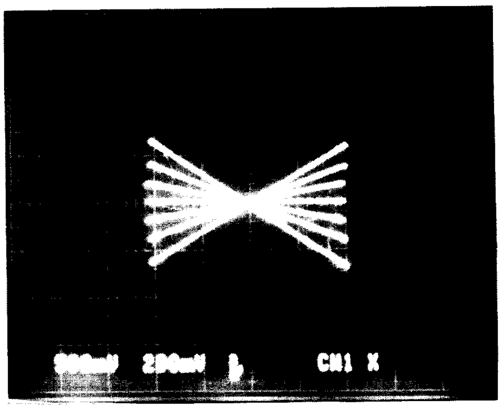


Fig. 7 Large-Signal Characteristics of Multiplier. $V_1 = \pm\{1.00, 0.66, 0.33, 0.0\}V$.

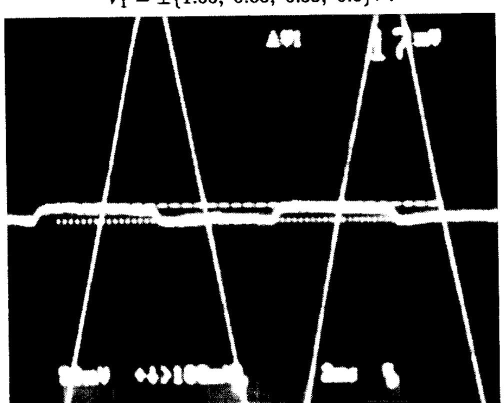
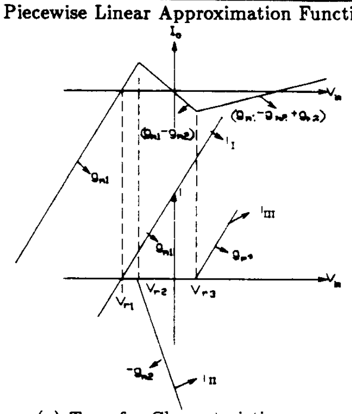
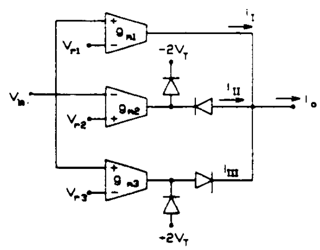


Fig. 8 Non-Linearity Multiplier Error, Fixed $V_1 = 1V$ and Variable Triangular Wave for V_2.

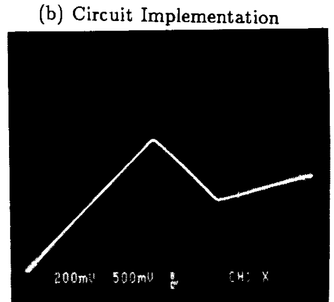
Fig. 10 Piecewise Linear Approximation Function.



(a) Transfer Characteristic



(b) Circuit Implementation



(c) Experimental Results.