View metadata, citation and similar papers at core.ac.uk
brought to you by CORE
provided by idUS. Depósito de Investigación Universidad de Sevilla

respect to the principal (cardinal) axes, the contour graphs are shown for one quadrant in θ-ϕ space.

The array of the double square loops set in a square lattice forms an FSS which is stationary at any 90 degree rotation, and has principal axes at 45 degree intervals. Therefore, the incidence in the plane at $\phi = 45$ degrees does not produce any depolarisation (Fig. 3), and peak values of the orthogonal polarised component appear at nearly $\phi = 22{\cdot}5$ degrees from this plane.

Conclusions: The proposed method overcomes two basic difficulties in FSS applications: it reduces the computation time to calculate the FSS parameters and also requires less FSS data storage space. This is achieved by relating the values of the scattering coefficients for any polarisation and direction of the incident wave to the principal-plane values. An obvious advantage of this method is that only the principal-plane values of the scattering coefficients need be known before the antenna analysis program is invoked. When the FSS information is supplied in the form of measured data, the storage requirement is considerably reduced, since the storage space is only required for the principal planes. For the FSS with a square lattice of elements which are stationary under a 90 degree rotation, such as rings or square loops, the values of the scattering coefficients need be specified only on one principal plane. When the FSS is to be represented by a mathematical model, the model can be simplified considerably for the case of principal-plane incidence, and hence save computation time. For a type of FSS with elements composed of straight-line segments, the proposed method offers an additional advantage. This type of FSS includes rectangular (square) loops, double square loops, a rectangular (square) patch array, a looped patch array, their gridded versions and their complements. They can be represented by their transmission-line equivalent-circuit models using strip grating formulas. The computation time by the equivalent-circuit method is much less than required by a moment method solution of comparable accuracy.

M. J. ARCHER

British Aerospace
Space & Communications Division
Stevenage SG1 2AS, United Kingdom

4th March 1987

References

1 ANDERSON, I.: 'On the theory of self-resonant grids', *Bell Syst. Tech. J.*, 1975, **54**, pp. 1725–1731
2 MARCUVITZ, N.: 'Waveguide handbook' (McGraw-Hill, 1951), pp. 280–284
3 ARCHER, M. J.: 'Periodic multielement strip gratings', *Electron. Lett.*, 1982, **18**, pp. 958–959
4 ARCHER, M. J.: 'Inductive twin-strip gratings in incident TE waves', *ibid.*, 1983, **19**, pp. 118–120
5 ARCHER, M. J.: 'Wave reactance of thin planar strip gratings', *Int. J. Electron.*, 1985, **58**, pp. 187–230

CHAOS VIA A PIECEWISE-LINEAR SWITCHED-CAPACITOR CIRCUIT

Indexing terms: Oscillators, Switched-capacitor circuits

A nonlinear switched-capacitor circuit that generates chaotic signals is reported. The circuit is described by a first-order piecewise-linear discrete equation that exhibits a chaotic dynamics. Experimental results illustrating the circuit performance and its use as a noise generator are included.

Introduction: The study of chaotic behaviour appearing in nonlinear circuits has attracted great attention in the past few years.[1–8,11] In particular, several circuit structures have been reported that can be used as electronic chaos generators. Together with their circuit-theoretic properties, these circuits are also interesting because they provide simple practical tools for experimenting on chaotic phenomena.

To date, most of the proposed chaos generators are continuous-time circuits described by systems of ordinary differential equations that exhibit chaotic solutions.[3–7] However, chaos has also been observed in numerous dynamical systems modelled by nonlinear finite-difference equations, henceforth called discrete maps. Moreover, it has been shown elsewhere that discrete maps are the simplest mathematical models for explaining chaos, the most extended way to deal with this phenomenon being to simulate discrete maps using a digital computer.[9,10] Nevertheless, in Reference 8 a switched-capacitor circuit described by a parabolic discrete map is given. It provides us with an empirical tool that can substitute digital computers for the study of chaotic phenomena arising in different areas of science and technology.

In this letter we report a novel switched-capacitor circuit which is modelled by a piecewise-linear discrete map. The new circuit is suitable for generating chaos while using fewer components than the circuit in Reference 8; in particular, the inconvenience of using analogue multipliers is avoided. In this sense, we can say the circuit to be described below is the simplest reported circuit modelling a discrete map.

Mathematical model and circuit structure: The circuit we will propose herein is intended to implement the following discrete map:

$$x_{n+1} = \begin{cases} A + Bx_n & x_n < 0 \\ A - Bx_n & x_n > 0 \end{cases} \tag{1}$$

The chaotic dynamics of this map have been extensively studied.[9–10] Its asymptotic behaviour comes from the values of both A and B parameters. For values of B less than 1 a stable point is reached for any value of A. For values of B greater than 2 all solutions are attracted to $-\infty$. On the other hand, for values of $B \in (1, 2)$ all solutions are aperiodic. This is called the chaotic region. Inside this region, the values of subsequent iterates of x are randomly distributed on one or more subintervals of the real axis, the number of subintervals and their corresponding length depending on the value of A.

Let us now consider the switched-capacitor circuit shown in Fig. 1*a*. Switches labelled '*e*' (respectively '*o*') turn ON in synchronisation with the even (respectively odd) phase of the clock signal shown in Fig. 1*b*. On the other hand, switches

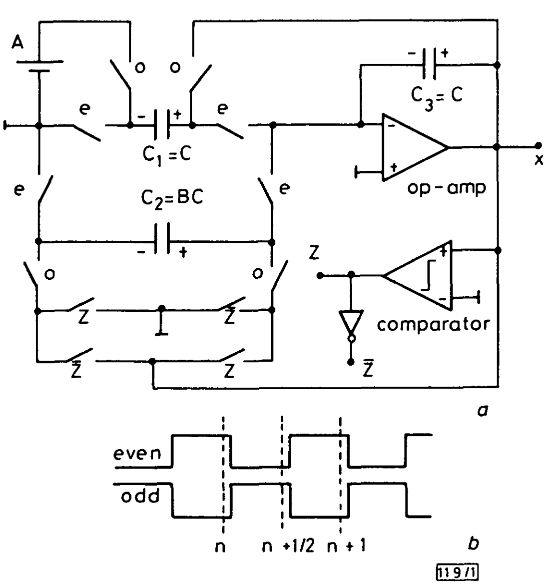

Fig. 1
a Piecewise-linear switched-capacitor circuit
b Timing diagram for switches

labelled Z (respectively $\bar{Z}$) are controlled by the binary output signal (respectively, by the logical complement) of the comparator.

The circuit operates as follows: during the $(n + 1/2)$th odd phase capacitor C_1 charges at

$$V_{1_{n+1/2}} = x_{n+1/2} - A \tag{2}$$

Authorized licensed use limited to: Universidad de Sevilla. Downloaded on March 31,2020 at 15:47:04 UTC from IEEE Xplore. Restrictions apply.

while the charge in capacitor C_2 will depend on the sign of $x_{n+1/2}$ as below,

$$V_{2_{n+1/2}} = \begin{cases} -x_{n+1/2} & x_{n+1/2} < 0 \\ x_{n+1/2} & x_{n+1/2} > 0 \end{cases} \tag{3}$$

Moreover, the charge in C_3 does not change during the odd phase, which means

$$x_{n+1/2} = x_n \tag{4}$$

During the next $(n+1)$th even phase both capacitors C_1 and C_2 deliver all the previously stored charge to C_3, thus from the charge conservation principle,

$$C_3(x_{n+1} - x_n) = -C_1 V_{1_{n+1/2}} - C_2 V_{2_{n+1/2}} \tag{5}$$

Combining eqns. 2 to 4 with eqn. 5 we obtain eqn. 1, A being the value of the external battery shown in Fig. 1*a* and B the capacitor ratio C_2/C_3.

Experimental results: We have built the circuit in Fig. 1*a* using off-the-shelf components (a μA741 opamp, a μA709 comparator, MC14066 analogue switches, and 20% tolerance capacitors ranging between 1 nF and 10 nF). We have used a clock frequency $f_c = 20$ kHz and made many experiments by varying both A and B parameters. Fig. 2 shows a set of measured frequency spectra for different values of B comprised in the chaotic region, namely for $B = 1{\cdot}01$ (Fig. 2*a*), $B = 1{\cdot}2$ (Fig. 2*b*), $B = 1{\cdot}33$ (Fig. 2*c*) and $B = 1{\cdot}83$ (Fig. 2*d*). For each case

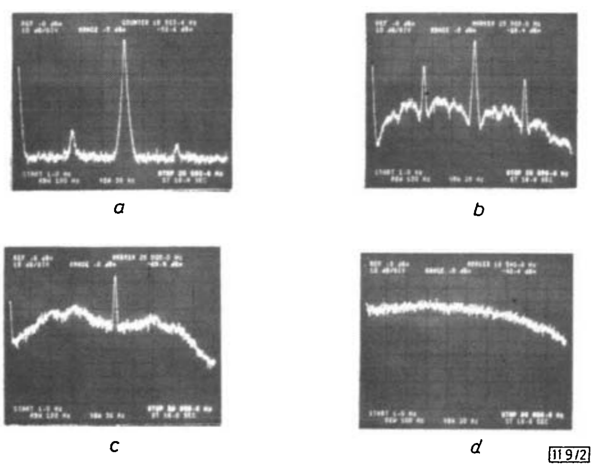

Fig. 2 *Frequency spectra for some aperiodic output signals for circuit of Fig. 1a*

a $A = 3, B = 1{\cdot}01$
b $A = 3, B = 1{\cdot}2$
c $A = 3, B = 1{\cdot}33$
d $A = 3, B = 1{\cdot}83$

we used $A = 3$. The peakings appearing at $f_c/4$, $f_c/2$ and $3f_c/4$ in Figs. 2*a*–*b* are a consequence of the accumulation of iterates inside separate subintervals of the real axis, either 4 (Figs. 2*a*, *b*) or 2 (Fig. 2*c*). Notice that, as B increases, the spectrum becomes approximately flat for most of the frequencies, being flat along the whole frequency range going from DC to f_c for $B = 1{\cdot}83$ (Fig. 2*d*). Since white noise is known to be characterised by a flat frequency spectra, we can appreciate the potential use of the new circuit as a white-noise source.

The time-domain evolution of the chaotic signal provided by the circuit is illustrated in Figs. 3*a* and *b*. There we show

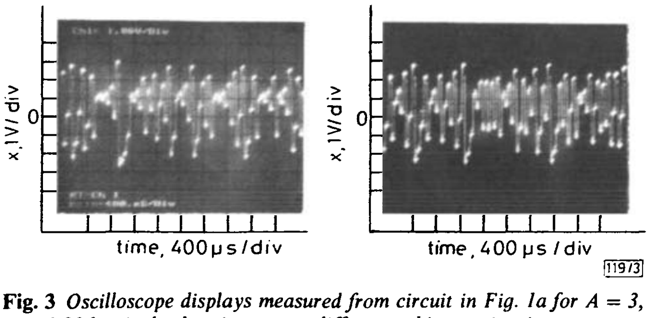

Fig. 3 *Oscilloscope displays measured from circuit in Fig. 1a for $A = 3$, $B = 1{\cdot}83$ by single-shooting at two different arbitrary time instances*

two displays measured from the circuit for $A = 3$, $B = 1{\cdot}83$ and obtained by single shooting the oscilloscope at two different time instances. As corresponds to a chaotic regime, no periodicity can be observed. Moreover, a careful observation shows us that apparently coincident values of x_n result in quite different values after a few iterations, this kind of behaviour relying on the very foundations of any chaotic regime.[9,10]

Discussion of results: We have reported a new circuit and illustrated its performance as a generator of chaos. It can be useful either for the simulation of chaotic phenomena or as a source of white noise. The circuit is also of circuit-theoretic interest because its dynamic equation is equivalent to a well known discrete map. Going beyond the purpose of this letter, the obtained results reveal the existence of a close connection between nonlinear switched-capacitor circuits and discrete maps: chaos can occur in a one-integrator loop switched-capacitor circuit owing to the presence of nonlinearities. This connection is worth considering in future research because switched-capacitor circuits are important in VLSI technology.[11]

Acknowledgment: This work was supported by CAICYT Research Contract 3467-83.

A. RODRIGUEZ-VAZQUEZ *20th March 1987*
A. RUEDA
B. PEREZ-VERDU
J. L. HUERTAS

Dto. de Electricidad y Electrónica
Facultad de Fisica
Universidad de Sevilla, 41012 Sevilla, Spain

References

1 CHUA, L. O., KOMURO, M., and MATSUMOTO, T.: 'The double scroll family', *IEEE Trans.*, 1986, **CAS-33,** pp. 1073–1118
2 TANG, Y. S., MEES, A. I., and CHUA, L. O.: 'Synchronization and chaos', *ibid.*, 1983, **CAS-30,** pp. 620–626
3 MATSUMOTO, T., CHUA, L. O., and TANAKA, S.: 'Simplest chaotic non-autonomous circuit', *Phys. Rev. A*, 1984, **30,** pp. 1155–1157
4 FREIRE, E., GARCIA-FRANQUELO, L., and ARACIL, J.: 'Periodicity and chaos in an autonomous electronic system', *IEEE Trans.*, 1984, **CAS-31,** pp. 237–247
5 AZZOUZ, A., DUHR, R., and HASLER, M.: 'Transition to chaos in a simple nonlinear circuit driven by a sinusoidal voltage source', *ibid.*, 1983, **CAS-30,** pp. 913–914
6 NEWCOMB, R. W., and SATHYAN, S.: 'An RC op amp chaos generator', *ibid.*, 1983, **CAS-30,** pp. 54–56
7 MATSUMOTO, T.: 'A chaotic attractor from Chua's circuits', *ibid.*, 1984, **CAS-31,** pp. 1055–1058
8 RODRÍGUEZ-VÁZQUEZ, A., HUERTAS, J. L., and CHUA, L. O.: 'Chaos in a switched-capacitor circuit', *ibid.*, 1985, **CAS-32,** pp. 1083–1085
9 MAY, R. M.: 'Simple mathematical models with very complicated dynamics', *Nature*, 1976, **261,** pp. 459–467
10 SCHUSTER, H. G.: 'Deterministic chaos: an introduction' (Physik-Verlag, Weinheim, W. Germany, 1984)
11 RODRÍGUEZ-VÁZQUEZ, A., HUERTAS, J. L., RUEDA, A., PÉREZ-VERDU, B., and CHUA, L. O.: 'Chaos from switched-capacitors circuits: discrete maps', *Proc. IEEE*, to be published

ERRATUM

FORSBERG, G. S.: 'Optical receiver with optical feedback', *Electron. Lett.*, 1987, **23,** (9), pp. 478–480

In the seventh line on p. 479, T should be defined as the 'time delay due to delay in optical source'

Eqn. 1 should read

$$I_i(t) = RP_i(t)$$

Eqn. 3 should read

$$A(s)e^{-sT} = P_o(s)/U_i(s)$$

Authorized licensed use limited to: Universidad de Sevilla. Downloaded on March 31,2020 at 15:47:04 UTC from IEEE Xplore. Restrictions apply.