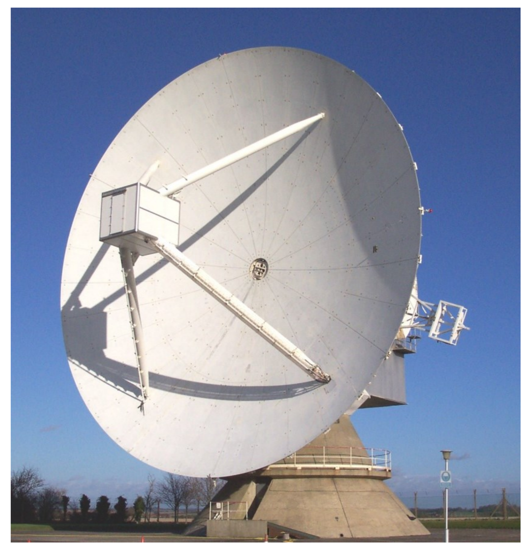
Fractal Modelling Of Rain Fields: From Event-On-Demand To Annual Statistics

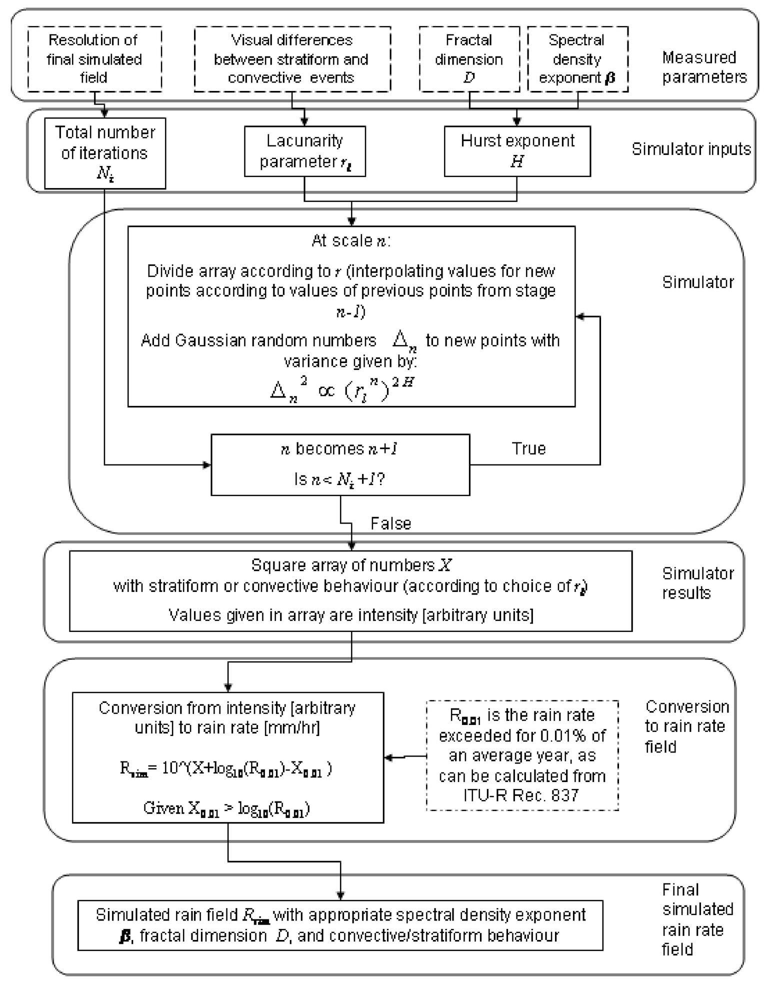
Sarah Callaghan

CCLRC Rutherford Appleton Laboratory, Chilton, Didcot, OXON, OX11 0QX, UK Email:S.A.Callaghan@rl.ac.uk



Chilbolton Advanced Meteorological Radar (CAMRa)

Radio systems operating at frequencies of above 10 GHz are adversely affected by rain, clouds and atmospheric gases. As the radio spectrum below 10 GHz becomes increasingly congested, there is significant pressure to open up these higher frequencies to commercial exploitation. Rain fade mitigation techniques (FMT) such as adaptive transmit power control are often suggested as methods of improving spectrum efficiency, either through allowing exploitation of higher frequency bands, or through improving the packing density of terrestrial links at lower bands. However, in order to accurately implement these FMT, detailed knowledge of the spatio-temporal variation of rain fields is required, on scales of hundreds of metres and timescales of the order of seconds. As yet, radar rain maps at these resolutions are not available, hence the need for accurate simulated rain fields.



Flowchart to generate single simulated rain field scaled to $R_{0.01}$

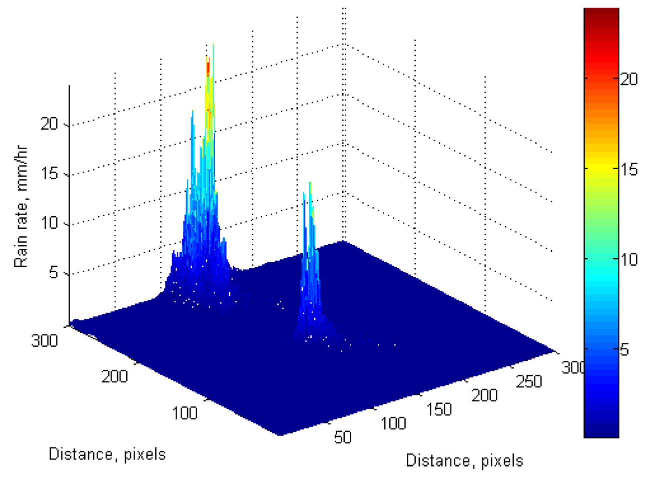
Rain field simulator

The rain field simulator is based on the Voss [1985] successive random additions algorithm for simulating fractional Brownian motion. It is an additive discrete cascade process and produces a monofractal field in the logarithmic domain.

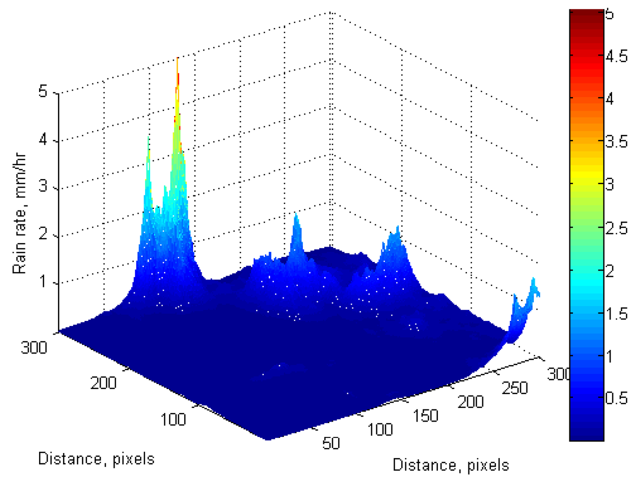
The key parameters are the Hurst exponent H, which determines the fractal dimension of the simulated field and the lacunarity r_l (equivalent to the cascade branching number) which determines whether the simulated fields are stratiform-like or convective-like.

Each simulated field is independent of the others. To create a simulated dataset, a set of stratiform and convective arrays must be assembled, (with proportions given according to M_c and M_s [ITU-R Rec 837-4]) and the whole dataset scaled according to $R_{0.01}$ [ITU-R Rec 837-4].

The resulting simulated rain fields have appropriate spectral density exponent, fractal dimension, and behaviour that is visually consistent with experimentally observed convective or stratiform type of events (according to what is desired).



Example of a simulated **convective** rain event, scaled to R0.01 = 26mm/hr



Example of a simulated **stratiform** rain event, scaled to R0.01 = 26mm/hr

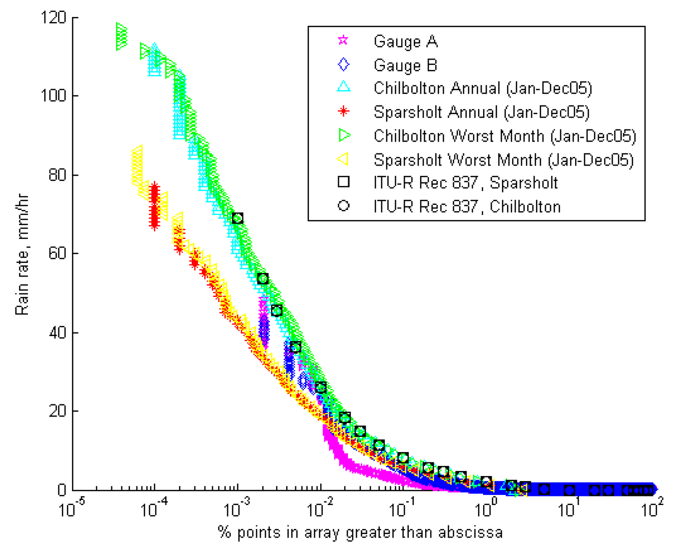
Application of fractal rain fields to radio systems

Fractal analysis of rain fields also leads us to the use of fractal methods to simulate visually and statistically realistic rain fields. These then can be used by systems designers in place of the expensive and difficult to obtain radar measurements of rain fields in order to test their proposed systems before deployment. The simulated rain fields presented here have been used in this context, in a case study of a switching algorithm for a two site Earth-space system using site diversity as a FMT, and in a project investigating the implementation of Adaptive Transmit Power Control (ATPC) on terrestrial links for bands above 18GHz.

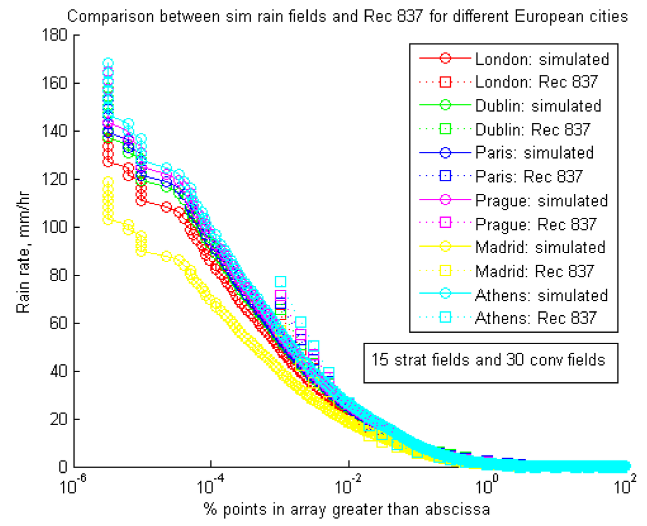
In order to validate the simulator, we compare cumulative distributions of rain fall exceedance as measured by a drop counting rain gauge located at Sparsholt and Chilbolton, both located in Hampshire, UK, with the rainfall as produced by the simulator at two simulated gauges, and the ITU model Rec 837-4.

In order to test the validity of the simulator to other climactic regions, we assume that a spatio-temporal equivalence exists (i.e. that each spatial point in the array can be considered to be equivalent of a temporal point as measured by a rain gauge). We therefore can show the cdfs of the simulated datasets in comparison with the ITU-R Rec. 837-2 curves for different European cities. In general there is a good agreement between the ITU-R curve and the simulated curve for the lower rain rates. It is only at the high rain rates where the curves diverge. This is due the method of combining the simulated rain fields into a simulated “annual” dataset (a 50-50 stratiform-convective mix), which is not physically realistic. These simulations will be repeated using a stratiform-convective mix given according to M_c and M_s (also from ITU-R Rec. 837-4).

CDFs of measured rain events in comparison with simulated events



Cumulative distribution functions for two simulated gauges A and B (7.5km apart), along with the Rec. 837-4 curves calculated for the sites of Sparsholt and Chilbolton (~7.5km apart) and annual and worst month cdfs as calculated from measured rain data recorded at Sparsholt and Chilbolton.



Cumulative distribution functions of rain rate exceedance derived for simulated “annual” data (15 stratiform fields and 30 convective fields) in comparison with the curves produces using ITU-R Rec. 837-4 for different European cities.

Conclusions and Further Work

Presented here is a method for simulating two-dimensional rain fields through the use of a discrete additive cascade procedure in the logarithmic domain. These simulated rain fields have the same fractal dimension, spectral density exponent and can be customised to different European regions through the use of the parameter $R_{0.01}$. They can also produce stratiform-like and convective-like behaviour, according to the type of event desired by the operator.

The procedure used produces simulated rain fields which are mono-fractal fields. This is justified by multifractal analysis of meteorological radar data recorded in the south of England reported elsewhere, which shows that log rain rate fields may be accurately characterised as monofractal fields, as their K(q) functions are straight lines. The transformation of the variables from rain rate to log rain rate allows us to linearise the problem, showing that rain rate fields can be characterised as “meta-Gaussian”.

The rain field simulator can be used in a variety of system planning cases, for instance in an Earth-space system using site diversity, in a terrestrial multi-point to multi-point (mesh) network, or a system of multiple point-to-point links. As the simulator can effectively produce rain fields over whatever area the operator desires, it has potential benefits in the design of satellite systems using adaptive antennas, as well as other micro and macro-scale applications.

Results presented here suggest that even though simulating a complete annual database will be computationally and memory intensive, the resulting statistics will compare favourably with current models. Further work is planned to create a database that will accurately recreate the tail of the annual rain cumulative distribution curve (in the range of 0.1 to 0.0001% exceedence) which is of particular interest to system planners.

Voss, R. F., “Random fractal forgeries”, in *Fundamental Algorithms for Computer Graphics*, editor R. A. Earnshaw, NATO ASI Series F, Computer and System Sciences, Vol 17, 1985.

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