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Synthesis of Two Dimensional Rain Fields for Systems Using Spatial Diversity

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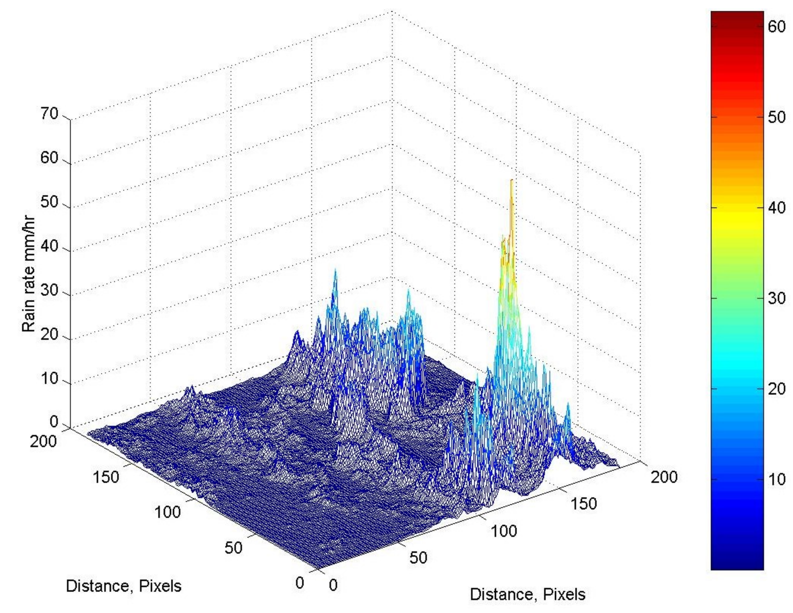
Chilbolton Advanced Meteorological Radar (CAMRa)

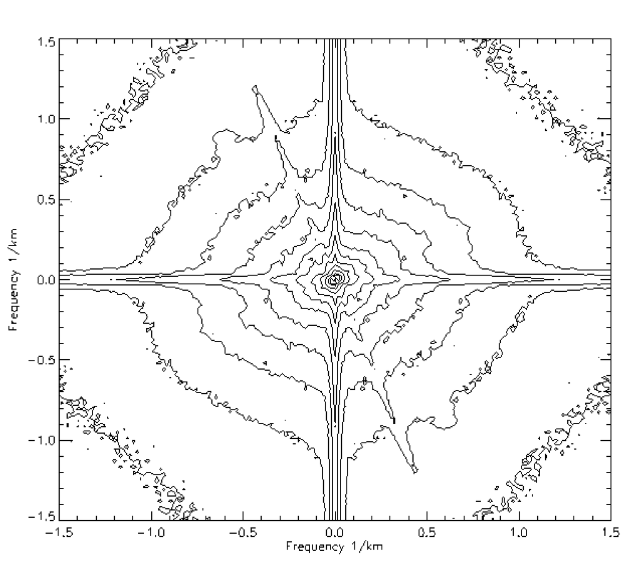
Radio communications systems operating at 10 GHz and above suffer significant impairment due to rain, clouds and atmospheric gases. This attenuation is unlikely to be compensated for by fade margin alone. Fade Mitigation Techniques such as site and route diversity rely on the temporal and spatial inhomogeneity of rain to improve the availability of a system. To correctly configure such a system to optimise the availability and minimize the cost requires detailed knowledge of typical rain fields.

In some cases it is sometimes more convenient to use simulated data for the testing and development of a system. Cascade models have been proposed as a computationally effective method of performing such simulations, and have also been shown to produce the same statistics as real rain fields.

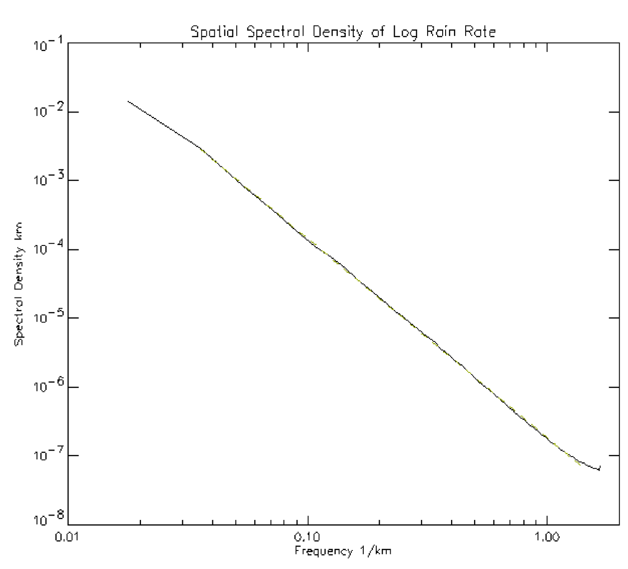
Example of a typical rain field as recorded by the radar (right). If we assume that the rain field can be approximated to a fractal surface, then techniques from fractal geometry can be used to analyse and simulate it.



Spatial Spectral Density Function



2D spatial spectral density of rain rate for event on 7/12/00, averaged over 260 scans. The near circular contours are consistent with a rotationally symmetric, and hence quadrant symmetric, spectral density and spatial autocorrelation.



Averaged radial spectral density of log rain rate for event on 7/12/00. Exponent = -2.8951

If each near-horizontal radar scan is treated as an instantaneous snapshot of the rain rate field then the spatial spectral density may be calculated via 2-D Fourier transform. The spectral density function for an isotropic random field is given by:

$$S(\omega) \propto \omega^{-2H-2}$$

where H is the Hurst exponent, and is equal to 1/3. For surfaces, the fractal dimension D_S is related to the Hurst exponent by $D_S = 3 - H$. The fractal dimension of the contour lines is given by $D_L = D_S - 1$.

Discrete cascades

These types of cascade exploit self-affinity and self-similarity relationships to produce rain rate fields through an iterative random cascade procedure. These cascades are able to incorporate non-rainy regions, and have concepts and ideas in common with the disaggregation and downscaling studies done in hydrology. Most discrete cascades do not include a method for estimating the temporal evolution of the produced fields, though this can be developed.

Further Work

The algorithm presented provides a convincing method for the generation of fractal surfaces that can be used to simulate rain fields. It can be made more realistic by fine-tuning certain parameters, to better suit regions with different climatological behaviours. The average rain rate across the entire simulated rain field is determined by the random process that generates the field, but can be influenced by the choice of value given to the four outer corners during the first iteration of the cascade process. This will not adversely affect the statistics and spectral density function of the field. The fractal dimension D of the generated field is determined only by H; varying the value of r changes the lacunarity, or proportion of holes in the field.

The algorithm produces rain fields that are statistically independent of each other, i.e., creating a series of rain fields by this method will not result in a convincing synthetic event. However, as the fields can be generated over far wider areas than measurements can be taken, it is possible to use Taylor's frozen storm hypothesis to give some indication of temporal variation.

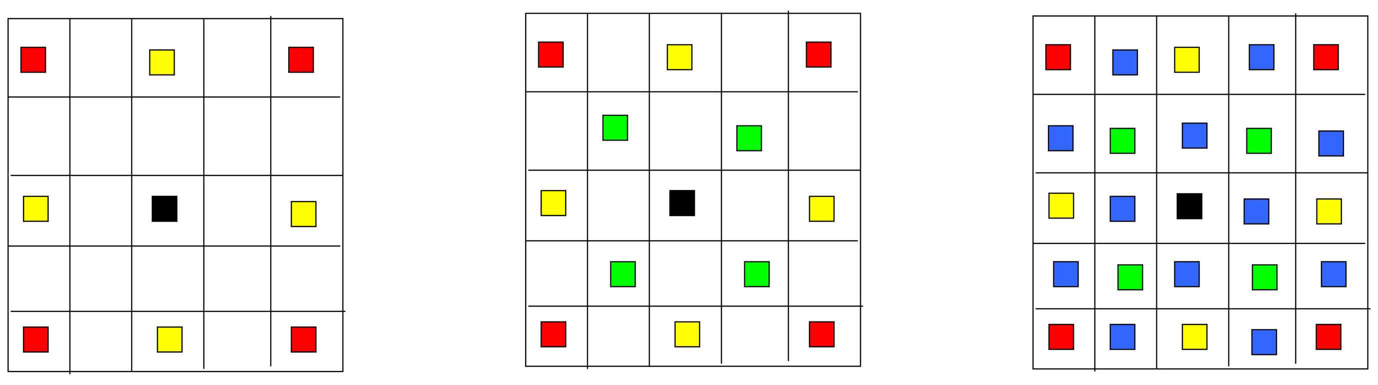
What is a fractal?

A fractal is as an object that appears self-similar under varying degrees of magnification, with each small part of the object replicating the structure of the whole. Some natural fractals, e.g. the boundary of clouds, cracks in a wall and a hillside silhouette possess statistical self-similarity, i.e. they possess the same statistical properties (the same degree of ruggedness) as we zoom in.

Mathematically speaking, the fractal dimension D characterizes any self-similar system; if the linear dimension of a fractal observable is changed by a scale factor f, then, for any value of f the values of the fractal observable will be changed by the factor f^D. For surfaces, the value of the surface dimension lies in the range $1 \le D_L \le 2$ A smooth surface has $D_S = 2$. Similarly, for a contour line , and $D_L = 1$ for smooth lines. The more twisted and "wriggly" the contour line is, the higher the value of D . If pathological cases are disregarded a planar section of a fractal surface has $D_L = D_S - 1$

It has been previously shown, on the assumption that rain fields can accurately be described as a fractal surface, that the fractal dimension of rain rate contours calculated by the area-perimeter and box counting methods has a nearly constant value of ~1.2, independent of the rain rate threshold.

Cascade Algorithm



The cascade algorithm described was first developed by Voss (1985) and is a discrete additive cascade operating in the log domain.

1. (black) Independent Gaussian variable ξ with zero mean and unit variance

2. (red) Value = set to a constant determined by the value of the spatial rain rate average required

3. (yellow) Value = average of two endpoints and middle point

4. (green) Value = average of their diagonal neighbours

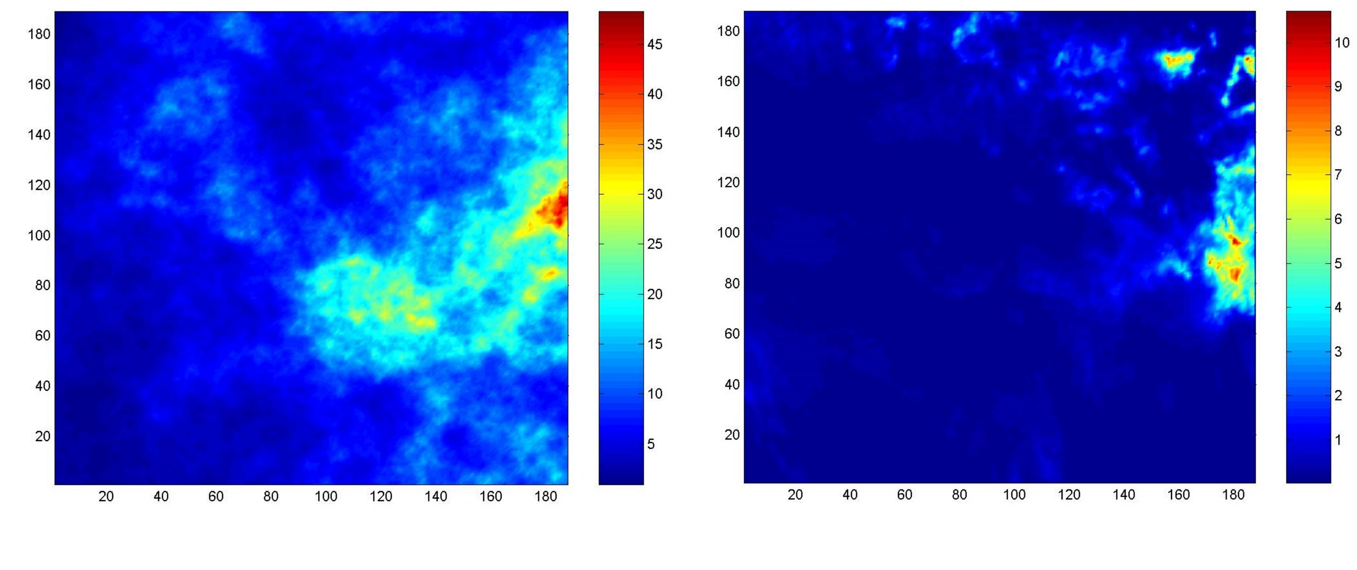
5. All the points plotted then have independent values of $\xi_{n=1}$ added to them, where the Gaussian random variable now has the variance given by:

$$\langle \xi_n^2 \rangle = \sigma_n^2 = r^{2nH} \quad \text{with} \quad r = 1/\sqrt{2} \quad \text{and } n=1.$$

6. (blue) Average as described in 3 and 4

7. All the points plotted then have independent values of $\xi_{n=2}$ added to them, with variance given by equation above.

Repeat steps 6 and 7 to the required level of resolution, with n increasing by 1 for each level.



Example simulated rain field (left) in comparison with a measured rain field (right)