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A Mathematical Model for Predicting the Performance of the Renewable River Current Turbine with Consideration of Rotation

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Abstrak

Turbin yang memanfaatkan aliran sungai merupakan salah satu energi alternatif yang dipercaya tidak terlalu banyak berpengaruh terhadap kerusakan lingkungan dan tidak membutuhkan pekerjaan sipil yang komplek. Studi ini melanjutkan studi sebelumya yang bertujuan untuk mengembangkan model matematika untuk memprediksi perhitungan prestatsi turbin yang memanfaatkan aliran sungai. Dalam studi sekarang ini, pengaruh putaran turbin dimasukan ke dalam model yang baru. Hal ini dilakukan untuk mengetahui lebih dalam tentang prilaku dari turbin aliran sungai. Dalam mengembangkan model, studi ini mengadopsi model " *momentum theory* " yang biasanya digunakan untuk turbin angin. Dari hasil dari pengembangan ini, untuk menghitung prestasi turbin, paramerter yang dimasukan kedalam model berupa kecepatan aliran air, jari- jari turbin, kecepatan putaran air sungai dan kecepatan putaran turbin. Dengan menggunakan model, studi ini melakukan aktivitas investigasi parameter terhadap prestasi turbin. Studi ini juga akan mendiskusikan keterbatasan dari model yang dikembagkan.

Kata kunci: *turbin aliran sungai, putaran turbin, model matematika, teori momentum, prestasi*

Abstract

River current turbine is one of decentralized alternative energy technologies which is believed to have less environmental impacts and to offer no complex civil work. This study is the continuous work of previous study on developing a mathematical model for predicting the performance of river current turbine. In present model, the effect of turbine rotation is included, providing a widening understanding on the behavior of the turbine. To develop the model, the momentum theory which is generally applied for wind turbine is adopted. The result of the model requires inputting parameters of the water velocity, the turbine radius, the wind rotational velocity and the blade rotational velocity to calculate the performance. Verification and parametric study using the model are demonstrated and the limitation of the model is discussed.

Keyword : *river current turbine, turbine rotation , mathematical model, momentum theory, performance*

1. Introduction

A river current turbine is one of the clean decentralized renewable energy technologies. This is an electromechanical energy converter that harnesses the kinetic energy of river water. It operates at almost zero head that requires little or no civil work, causing less environmental impact (Khan, 2009).

Some references show the application of this technology; the Garman Turbine for irrigation & power generation (Anyi et al, 2010), the wheel turbine built by The Renewable Energy Centre of Tacna (Peru) for irrigation purposes (Ponta et al 1999), the Darrieus turbine by the Nova Energy System (St Lawrence River, Canada) and the Intermediate Technology Development Group (ITDG) for irrigating a 0.4 ha plantation in Juba (Sudan, Africa), the axial flow turbine by Lowe in 2007 for power generation (Anyi 2010), the axial flow turbine by The University of Brasilia for power generation (Anyi 2010), the 'Tyson Turbine' by Alternative Way Australia for power generation (Ponta et al, 1999).

Problem Definition

To design the technology, it urgently requires understanding the behavior of the turbine. A mathematical model is one of the methods to recognize the behavior of a system (Science Daily, 2004).

This study is the continuous work of the development of mathematical model for river current turbine in Rachman, et al (2011).

In the previous model which is based on the disk theory, the turbine is only represented by a single non–rotating constant disk. One of the limitations of this representation is that it resists the understanding the effect of a turbine rotation.

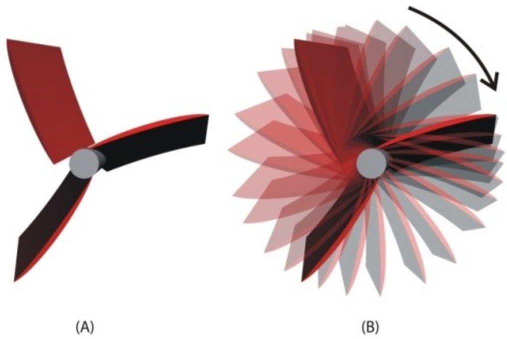

Figure1. (A) non rotating rotor (B) rotating rotor

Actually, as the water energy conversion exists, it is not only thrust which is generated, but also torque does. This is because that the force created rotates the blade which is about an axis of the turbine.

In the present work, the effect of a turbine rotation is included in a developed mathematical model for river current turbine in order to derive a better understanding of flow behavior within the turbine.

Aims of the study

The aim of this study is to develop a mathematical model for river current turbine with consideration of the turbine rotation.

Problem Limitation

This study limits its problem to area of mechanical conversion of the turbine. No transmission or electrical consideration is discussed. Other boundary is put on the concentration of the study on the bare (non channeling device) water current turbine.

2. Literature Reviews

The development in wind engineering design has significantly contributed in the development of water current turbine (Khan, 2009). One of the reasons is that it is believed that the valid equation of the water current turbine is analogous to that of the wind turbine (Clark, 2007).

The disk theory equation, which is generally applied for wind power, is based on the Bernoulli equation and the momentum balance equation which can be applied for not only air, but also water (Rachman, et al 2011).

The momentum theory is the extension of the disk theory with consideration of the turbine rotation (Duran, 2005).

Thus, to develop the river current turbine model which includes the effects of this rotational motion, in this study, based on the analogous equation of the wind and water, the momentum theory for wind turbine will be adopted.

3. Methodology

Mathematical Model

The following is a brief discussion of the previous model based on the disk theory. The Figure2 is the system analyzed in the model.

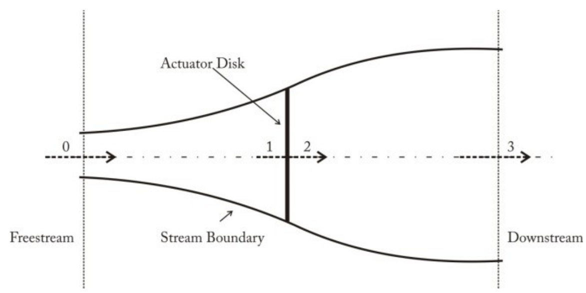

Figure2. Actuator Disk Model (Previous Model)

By applying Bernoulli equation and momentum balance equation with assumptions ; the continuous velocity through the disk, homogenous and incompressible fluid, steady state fluid flow, no frictional drag, constant pressure increment per unit area over the disk, no rotational component of the velocity in the slipstream, an infinite number of blades and equal static pressure at freesteram and downstream (Matthew JCS,2009) (Hartwanger D & Horvat A , 2008) and (Duran, 2005) , and introduction the dimensionless parameter of loading coefficient (K), it results in following equations for the velocity at rotor, the power and the coefficient of performance respectively,

$$V_1 = V_0 \frac{4}{K+4} \tag{1}$$

$$P = 32 \rho V_0^3 A \frac{K}{[K+4]^3} \tag{2}$$

$$C_p = 64 \frac{K}{[K+4]^3} \tag{3}$$

Where V_0 is the freestream wind velocity (m/s), ρ is specific mass of fluid (Kg/m^3) and K is resistance coefficient, which is defined as

$$K = \frac{P_1 - P_2}{\frac{1}{2}\rho V_1^{\,2}} \qquad (4)$$

To develop the momentum model, it requires modifying the disk model with consideration of a rotational rotor which imparts the wind behind the rotor to swirl. In this model, the axial velocities are assumed to remain unchanged (Duran, 2005). It is also assumed that the velocity has only axial and the rotational direction with no radial orientation.

Referring to Figure3, r is the radial distance of the rotor plane, V_2 is the axial velocity immediately after the rotor plane and has exactly same value with V_1, ω is wind angular velocity immediately after the rotor plane and Ω is blade angular velocity. In downstream, r_3, is the radial distance from the axis of the downstream, V_3 is the axial velocity and ω_3 is the angular velocity. In this model it is assumed that the velocity in radial direction is neglected.

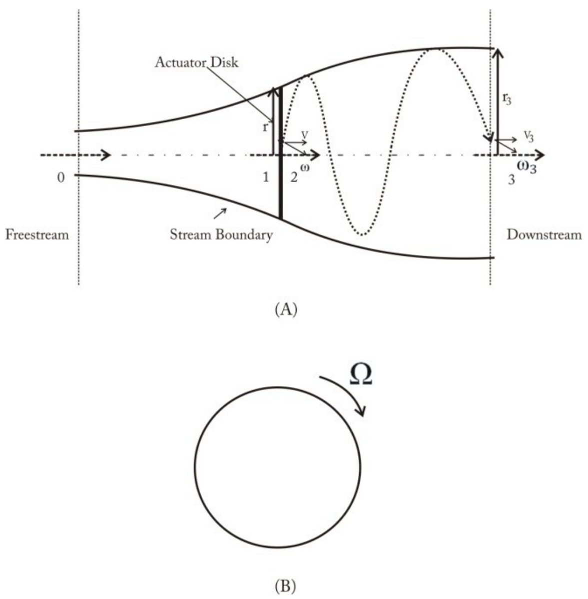

Figure3. Momentum Model Considering the Rotation Effect (Extension Model)

In Duran (2005), the thrust is actually created by not only the differential pressure across the rotor but also the differential pressure due to the rotational motion $(\Delta P = \frac{1}{2}\rho(\omega_3^{\,2} r_3^2 - \omega^2 r^2))$. For the approximate solution, the study assumed that, the angular velocity (ω) imparted to the slipstream is very small compared to the angular velocity (Ω) of the blades. Therefore, terms involving ω^2 are dropped, which leads to the effect of the sum force caused by rotational flow being neglected.

As the force created by the rotational motion is neglected, the thrust can be assumed as in the disk model, defined as

$$T = 8\,\rho\, V_0^{\,2}\ \frac{K}{[K+4]^2}\ A \qquad (5)$$

The angular momentum is defined as

$$L = \omega\, r^2 \qquad (6)$$

A dimensionless parameter of rotational induction factor (a′) is introduced, formulated by

$$a' = \omega / 2\,\Omega \qquad (7)$$

As the element of torque of the radial blade element is defined as the angular momentum extracted in unit time to the corresponding annular element of the slipstream, the elemental torque defined as

$$dQ = \omega\, r^2\ V_1\, \rho\, dA \qquad (8)$$

In this new model, as it is assumed that a single disk with radius of r rotates, thus the imparted a rotational wind velocity (ω) must be for the whole disk area. Thus, the equation of the torque becomes

$$Q = \omega\, r^2\ V_1\, \rho\, A$$

By inserting the definition of the velocity at rotor (Equation (1), and the rotational wind velocity(ω) as function of the rotational induction factor (a’) into Equation(7), the torque becomes,

$$Q = \frac{8}{K+4}\ a'\, r^2\, \Omega\, V_0\, \rho\ A \qquad (9)$$

The power from can be is defined as

$$P = \Omega\ Q \qquad (10)$$

By inserting the definition of torque in Equation (9) into Equation (10), the power can be defined as

$$P = \frac{8}{K+4}\ a'\, r^2\, \Omega^2\, V_0\, \rho\ A \qquad (11)$$

Coefficient of performance is defined as the ratio of the power generated by the turbine and the power available on the river with regard to the turbine area($0.5\,\rho\, V_0^3\, A$) . Using the definition of power in Equation (11), the coefficient of performance can be determined as

$$CP = \frac{16\, r^2\, \Omega^2 a'}{V_0^2\,(K+4)} \qquad (12)$$

A new equation can be obtained to relate the loading coefficient and the parameters of the rotational motion, by equating the power in both the disk theory (Equation (2)) and the momentum theory (Equation (11), it results in following relation,

$$\frac{\Omega^2 r^2}{V_0^2} = \frac{4K}{a'[K+4]^2} = \frac{\Omega 8K}{\omega [K+4]^2} = \lambda_T^2 \qquad (13)$$

Using algebraic manipulation in Equation (14), the loading coefficient (K) can be written as

$$K = \frac{1-\frac{\omega\Omega r^2}{V_0^2} - \sqrt{1-2\frac{\omega\Omega r^2}{V_0^2}}}{\frac{\omega\Omega r^2}{4V_0^2}} \qquad (14)$$

Thus, the coefficient of performance equation becomes

$$CP = \frac{2\left(\frac{r^2\omega\Omega}{V_0^2}\right)^2}{1-\sqrt{1-2\frac{r^2\omega\Omega}{V_0^2}}} \qquad (15)$$

Verification of the Betz limits

The maximum efficiency turbine is 0.59, known as Betz limits. This is the theoretical limit to the percentage of kinetic energy that can be removed from the flowing fluid to the kinetic energy maximum available in fluid. It became a common practice to use this limit for estimating the maximum efficiency of water current turbines (Guney, 2010).

The following activity is to verify the developed model to the Betz limits constraints.

In this verification, the Equation (15) is utilized with assumption of the rotor radius of 1 m at water current velocity of 1m/s. The parameter of a rotational factor (R_F) is introduced, formulated by $R_F = \Omega\omega$.

Figure4 shows the relation of the coefficient of performance and the rotational factor in the verification process. It is shown that the trend on the graph has a maximum coefficient of performance of 0.59, which is equal to the Betz limit. Thus, the model verifies the Betz limit constraint.

The Effect of The Rotation On The Performance

The following activity is to investigate the effect of the blade rotation on the performance of the turbine.

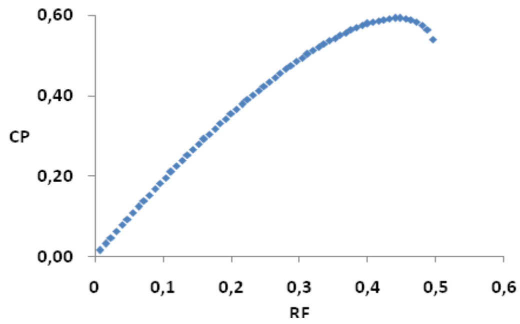

Figure4. Coefficient of Performance as function of the rotational factor

As defined in previous sub chapter, rotational factor (R_F) is the combination of both rotational turbine (Ω) and the rotational flow (ω). The low value of this factor means the low blade rotation as well as the wind rotation, verse versa. The analogous of this condition is explained as the turbine operates at high rotation; the wind imparted will be high also. Thus, the factor will be high. This condition can be applied for the reserved situation.

Thus, based on the Figure4, it can say that as initially the factor increases; the blade rotation increases, the performance increases as well. This trend peaks at the point of 0.59. But, from this point onward, as the factor is increased further; the rotation is increasing, the performance starts decreasing.

4. Discussion

Based on the parametric activity, it is shown that the rotational wind velocity and the blade rotation do affect the performance of the turbine.

Physically, it can possibly be explained that when rotor is operating at low rotation, the incoming water flow sees a rotor as a sparsely resistance. Thus, it makes easy for wind to go through the turbine, enhancing the mass flow rate. However, the non-densely resistance also means less differential pressure across the turbine is. Thus, the thrust created will be low. In turn, the high mass flow is not well supported by the thrust generated, resulting in low turbine performance.

As the rotation is increased further, more resistance is seen by the incoming flow; the water may encounter the turbine blade more frequently. It results in increasing the thrust generated. Thus, the performance is increasing. However, as the rotation is continuously increased, more resists the turbine is. It result in more blockading are the turbine. Thus it resists the flow through the turbine, decreasing the performance.

Limitation of the Model

This model is developed under assumption of a rotating disk. The amount of angular momentum is based on one full area of a rotating disk. Actually each radial element on the disk has its own angular momentum. Thus, the model still requires to be modified to consider the angular momentum for each angular element to derive more precisely calculation.

Even the radius of the rotor is included in the model, the effect of the other rotor geometry; number of blades or blade properties still cannot be explained in this model.

5. Conclusion & Recommendation

A mathematical model has been developed to predict the performance of the river current turbine. This new equation is slightly different with the model in the previous study in term of considering effect the blade rotation (see Equation (3) and Equation (14).

The present model requires inputting parameters of the water velocity, the turbine radius, the wind rotational velocity and the blade rotational velocity to calculate the performance.

This model verifies with Betz limit constraint.

In the parametric study activity, as the blade rotation increases, the coefficient of performance increases initially. But it has a maximum point at 0.59. On this point onward, a further increasing in the rotation reduces the performance. Physically, this can be explained, that the rotation is very important also to create a force on the turbine as the rotation can create a resistance. However as it put on more rotation, the more resist the turbine are. This condition blockades the flow thus reducing the mass flow rate.

Thus, to derive an optimum design of the turbine when the rotation is considered, it requires designing a turbine as such that the rotation is enough to create the force at maintained water flow rate.

The limitation of this model is the restriction in understanding the effect of the other rotor geometry; blade number and blade properties. Indeed, as the model calculates the angular momentum equation based on the single rotating disk, the contribution of the elemental angular area of the turbine may be not well represented. Thus, it reduces the precision of the calculation.

A future work is required to develop a mathematical model for river current turbine which considers the effect of rotor geometry and the contribution of the each elemental area of the turbine on the angular momentum calculation.

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Notation

A Area of turbine (m^2)
a′ Rotational induction factor
CP Coefficient of performance, $C_p = \frac{P}{\frac{1}{2}\rho (V_0{}^3) A}$
dQ Elemental torque ($kg.m^2/s^2$)
K Resistance coefficient, $K = \frac{P_1 - P_2}{\frac{1}{2}\rho V_1{}^2}$
L Angular Momentum
P Power output(Watt)
P_0 Wind static freestream pressure (kg/m $s^{2`}$)
P_1 Wind static pressure at region immediately before the turbine (kg/m $s^{2`}$)
P_2 Wind static pressure at region immediately after the turbine (kg/m $s^{2`}$)
P_3 Wind static pressure at downstream (kg/m $s^{2`}$)
T Thrust (kg. m/s^2)
V_0 Freestream wind velocity (m/s)
V_1 Wind velocity at region immediately before the turbine (m/s)
V_2 Wind velocity at region immediately after the turbine (m/s)
V_3 Wind velocity at downstream (m/s)
R_F Rotational Factor
r Radius of the rotor rotor plane (m)

Symbols

$\dot{m}$ Mass flow rate of fluid (Kg/s^{-1})
ρ Specific mass of fluid (Kg/m^3)
ω The angular velocity immediately behind the rotor plane (rad/s)
ω_3 The angular velocity at a radial distance r_3 from the axis of the slipstream (rad/s)
Ω The angular velocity of the blades (rad/s)