THE CALCULATION OF S2 TIDAL ENERGY IN THE MALACCA STRAIT WITH A THREE-DIMENSIONAL NUMERICAL MODEL

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ABSTRACT

In this paper, the S2-tidal energy of the Malacca Strait is computed using a three-dimensional model based upon a semi-implicit numerical scheme. The impact of using this scheme is that the time of calculation can be saved 4-5 times compared to time used by the explicit scheme, since the semi-implicit scheme does not respect the Courant-Friedrichs-Lewy (CFL) criterion. As results the kinetic and potential energy, the dissipation of energy by bottom friction and horizontal eddy viscosity, the energy input and energy balance caused by the S2-tide are determined.

INTRODUCTION

The Malacca Strait, a passage between the Malay Peninsula and Sumatra, connects the Indian Ocean with the South China Sea. It is approximately 980 km long, varies in width from 52 km in the south to 445 km in the north and has a complex topography (Fig. 1)

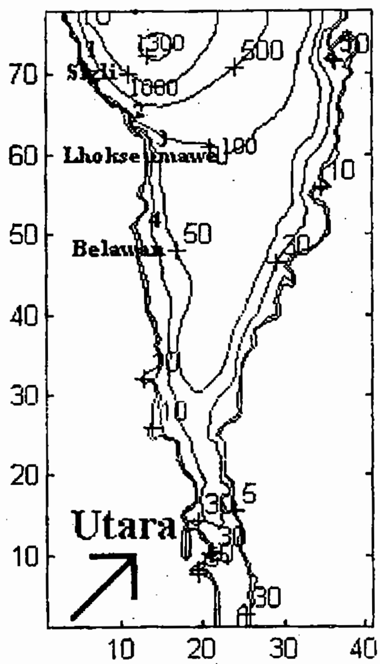

Fig. 1 The Depth of the Malacca Strait in meters, the open boundaries values are given in the top and bottom of the figure

The climate of the region is typically equatorial-hot and humid. Monsoonal effects are not revere in the strait as in the more open surrounding seas because of the sheltering effect of the Malay Peninsula and the island of Sumatra. Indirectly, however, the monsoon seasons greatly influence the circulation in the strait. As a result of the monsoons, two rainy seasons of unequal magnitude occur without any really dry period (Keller and Richards, 1967).

But in the Malacca Strait, the transports are of minor importance for general circulation. Tidal currents predominate and a northwest current is superimposed during the whole year (Wyrtki, 1961).

From the Andaman Sea a part of tidal wave enters the Malacca Strait, where it advances slowly. Because of the contraction of the strait the amplitude of $M_2 + S_2$ rise from 80 cm at the entrance to more than 250 cm in the narrowest part (Wyrtki, 1961).

In this paper, tidal energy and energy balance of the S_2-tide are computed using a three-dimensional model based upon a semi-implicit numerical scheme of the Malacca Strait. The analysis is done using numerical data from the last period of 60 periods of calculation.

FUNDAMENTAL EQUATIONS

The equations of motion can be written as (Sündermann, 1971) :

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}-fv=-g\frac{\partial \zeta}{\partial x}+A_h\nabla_H^2 u+\left[\frac{A_v}{H_{x_k}}\frac{\partial u}{\partial z}\right]_{H_{x_k}}^{H_{x_{k-1}}} \quad 1)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}+fu=-g\frac{\partial \zeta}{\partial y}+A_h\nabla_H^2 v+\left[\frac{A_v}{H_{y_k}}\frac{\partial u}{\partial z}\right]_{H_{y_k}}^{H_{y_{k-1}}} \quad 2)$$

The equation of continuity (3) reads :

$$\frac{\partial \zeta}{\partial t}+\frac{\partial}{\partial x}\int_{-h}^{\zeta}udz+\frac{\partial}{\partial y}\int_{-h}^{\zeta}vdz=0, \quad 3)$$

where u (x,y,z,t), v(x,y,z,t) and w(x,y,z,t) are the current velocity in the x, y and in the z directions, respectively, $f = 2\omega \sin \varphi$ is the Coriolis parameter, ω the angular speed of the Earth's rotation and φ the geographical latitude, $\zeta(x,y,t)$ is the water surface elevation measured from the undisturbed water surface,

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h(x,y) is the water depth also measured from the undisturbed water surface, g is the constant gravitational acceleration, H_{x_k} and H_{y_k} are the layer thicknesses in the u and v points in the k-th layer, respectively, ∇_H is the horizontal gradient operator. A_h is the horizontal turbulent exchange coefficient and A_v is the coefficient of vertical eddy viscosity. At the bottom, the conditions

$$\left.\frac{A_v}{H_b}\frac{\partial u}{\partial z}\right|_{H_b} = \gamma\, u_b \qquad 4)$$

and

$$\left.\frac{A_v}{H_b}\frac{\partial v}{\partial z}\right|_{H_b} = \gamma\, v_b \qquad 5)$$

are assumed, with

$$\gamma = \frac{g\sqrt{u_b^2 + v_b^2}}{C^2 H_b} \qquad 6)$$

where H_b is the bottom layer thickness and C is the Chézy bottom friction coefficient.

The solution of equation 1) to 3) can be seen in Rizal and Sündermann (1994), Rizal (1994), Rizal et. al. (1997) and Rizal (1997). This solution is based on the work from Backhaus (1985).

THE ENERGY EQUATION

The energy equation can be derived from the equation of motion, 1) and 2), and from the continuity equation 3) by multiplication with $\rho H u$ in 1), $\rho H v$ in 2), and $\rho g \zeta$ in 3) (von Trepka, 1967; Zahel, 1976; Mihardja, 1991):

$$\underbrace{\frac{\partial}{\partial t}(E_{kin} + E_{pot})}_{A_1} + \underbrace{\rho g\left(\frac{\partial u\zeta H}{\partial x} + \frac{\partial v\zeta H}{\partial y}\right)}_{A_2} =$$

$$\underbrace{-\frac{\rho g}{C^2}(u^2 + v^2)\sqrt{u^2 + v^2}}_{A_3} + \underbrace{\rho H A_h (u\nabla_H^2 u + v\nabla_H^2 v)}_{A_4} \qquad 7)$$

A_1 denotes the rates of change of kinetic and potential energy per unit area; the expression averaged over one tidal period is zero. A_2 denotes the rate of divergence of energy flux. The right hand side of 7) is composed of the rate of energy dissipation per unit area by bottom friction (A_3), and by the horizontal eddy viscosity (A_4), respectively.

The kinetic energy integrated over the entire investigated area for one period is given by 8) :

$$\langle E_{kin}\rangle = \frac{\rho}{2T}\int_T \iint_{Surface} H(U^2 + V^2)\,dx\,dy\,dt \qquad 8)$$

where

$$U = \sum_{k=1}^{b} u_k H_{x_k} \Big/ \sum_{k=1}^{b} H_{x_k} \qquad 9)$$

and

$$V = \sum_{k=1}^{b} v_k H_{y_k} \Big/ \sum_{k=1}^{b} H_{y_k} \qquad 10)$$

Analogously, for the potential energy is defined by equation 11):

$$\langle E_{pot}\rangle = \frac{\rho g}{2T}\int_T \iint_{Surface} \zeta^2\,dx\,dy\,dt. \qquad 11)$$

The change in energy input/output is integrated along the open boundaries of the Malacca Strait over one period of the partial-tide

$$\langle E_{i/o}\rangle = \frac{\rho g}{2T}\int_T \int_L H V_n \zeta\, dL\, dt \qquad 12)$$

where V_n is the vertically integrated velocity normal to the line open boundaries L. $\langle\ \rangle$ denoted the time derivative.

The change in energy dissipation by bottom friction and horizontal eddy viscosity, integrated over the entire investigated area and over one period of the partial-tide, is defined by equation 13).

$$\langle E_{diss}\rangle = \frac{\rho}{2}\int_T \iint_{Surface} \left[(g/C^2)(U^2 + V^2)^{3/2} - A_h(UV\nabla_H^2 U + V\nabla_H^2 V)H\right] dx\,dy\,dt. \qquad 13)$$

THE PARAMETERS USED IN MODEL

Applying the model described above, the Malacca Strait is discretized with a spatial horizontal grid size of $\Delta x = \Delta y = 13104.5$ meters. 8 levels are taken vertically : 0-10, 10-20, 20-30, 30-50, 50-100, 100-400, 400-800 and 1331 m.

The remaining flow parameters are $g = 9.81\ m/s^2$, $\rho = 1000\ kg/m^3$,

$$A_h = 100\, H_{i,j,k} \qquad 14)$$

and coefficient of vertical eddy viscosity :

$$A_v = (0.05 H_{i,j,1})^2 \sqrt{\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2} \qquad 15)$$

Both A_h and A_v are in m^2/s.

The Chézy-coefficient C $\left[m^{1/2}s^{-1}\right]$ approximated as a depth dependent function. This approach has been successfully applied by Verboom et al. (1992) for the

North Sea. For the Malacca Strait, the following values were achieved by testing and show good results:

$$
\begin{aligned}
C &= 62.64 && \textit{for} && H_b \le 40\text{m} \\
&= 62.64 + (H_b - 40) && \textit{for} && 40\text{ m} \le H_b \le 65\text{ m} \\
&= 87.64 && \textit{for} && H_b \le 65\text{ m}
\end{aligned} \qquad 16)
$$

The time step is Δt = 349.3 s. For the explicit scheme, since the stability restriction imposed by Courant-Friedrichs-Lewy, Δt would have required 81.1 s or about a factor 4.3 smaller. However, the good semi-implicit method in solving ζ^{n+1} should be chosen. The Successive Over Relaxation (SOR) method is an economic way to solve ζ^{n+1}, see Backhaus (1983). The time subdivision, the parameter introduced by Casulli (1990), is taken as $\tau = \Delta t/4$. An implicitness parameter $\alpha = 0.5$ is chosen.

THE RESULT OF THE MATHEMATICAL MODEL

The kinetic and potential energies are calculated according to 8) and 11), respectively. In Fig.2 the time-dependent behavior of the kinetic, potential and total energies in the calculation for the 60th period are shown. The mean value of the kinetic energy is equal to

$$\langle E_{kin} \rangle = 0.82 \times 10^{14} J.$$

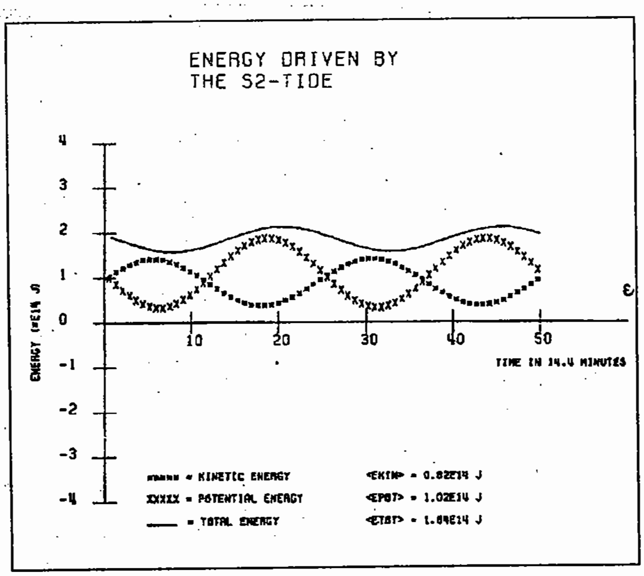

Fig. 2 Kinetic, potential and total energy in the 60th S_2-tide period

For the potential energy it is

$$\langle E_{pot} \rangle = 1.02 \times 10^{14} J.$$

The total energy thus is

$$\langle E_{tot} \rangle = 1.84 \times 10^{14} J.$$

This total energy is about 27.4% of that for the M_2-tide.

The Eq. 12) is used to calculated the net energy input from the Andaman Sea to the Malacca Strait and net energy output from the Malacca Strait to the South China Sea.

The rate of net energy input has the value of 0.34 x 10^{10} *J/s*. This value about 24% of the value for the M_2-tide. The rate of net energy output has the value of -0.06 x 10^{10} *J/s*, or about 27% of that for the M_2-tide. Thus, the rate of total energy input in the Malacca Strait is 0.28 x 10^{10} *J/s*.

In Fig.3 the time-dependent behavior of the rate of net energy input, net energy output and total energy input in the calculation for the 60th period are shown.

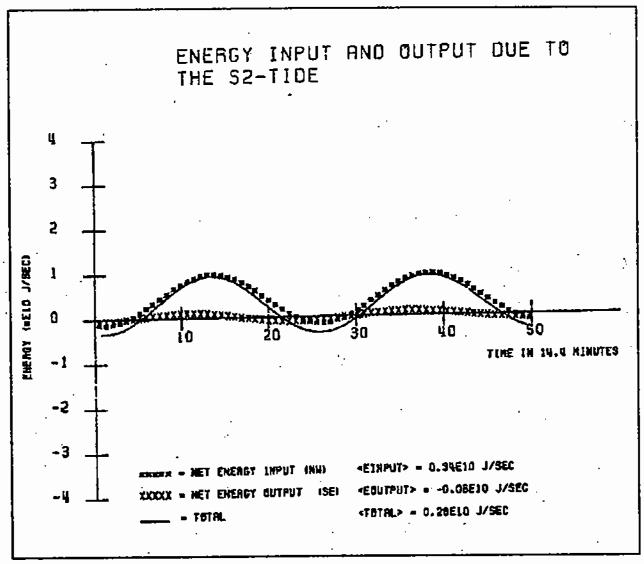

Fig.3 Rate of change in net input (northwestern boundary) and net output (southeastern boundary) energy and total of input energy in the 60th S_2-tide

According to 13), the energy dissipation by bottom friction amounts for the Malacca Strait to

$$\langle \dot{E}_b \rangle = 0.23 \times 10^{10} J/s,$$

and the energy dissipation by eddy horizontal viscosity

$$\langle \dot{E}_h \rangle = 0.05 \times 10^{10} J/s,$$

The total energy dissipation is then

$$\langle \dot{E}_{diss} \rangle = 0.28 \times 10^{10} J/s.$$

In Fig.4 the time-dependent behavior of the rate of energy dissipation caused by bottom friction, horizontal eddy viscosity and total in the calculation for 60th period are shown.

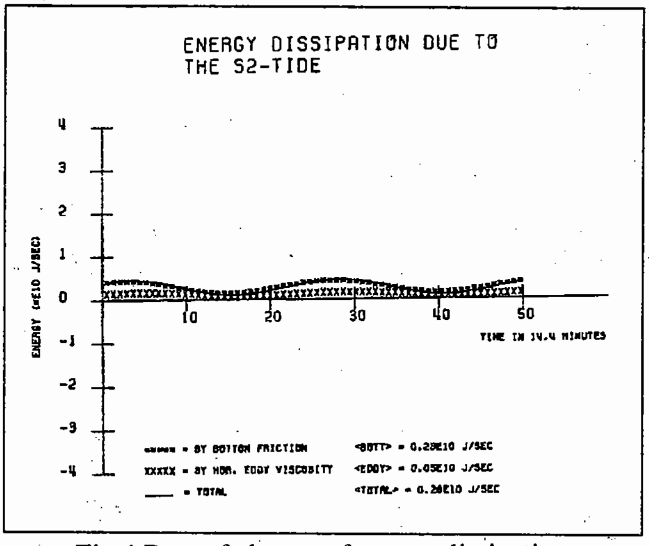

Fig.4 Rate of change of energy dissipation of the S_2 in the 60th S_2-tide period

Then, if this value of total energy dissipation and total energy input are introduced in Eq.7) one obtains

$$\underset{A_2}{0.28 \times 10^{10}\ J/s} = \underset{(A_3 + A_4)}{0.28 \times 10^{10}\ J/s} + \underset{remainder}{0.00 \times 10^{10}\ J/s}$$

It is seen that in the 60th period of calculation, there is no remainder is obtained.

In Fig.5 the time-dependent behavior of the rate of total energy input, total energy dissipation and energy balance in the calculation for 60th period are shown.

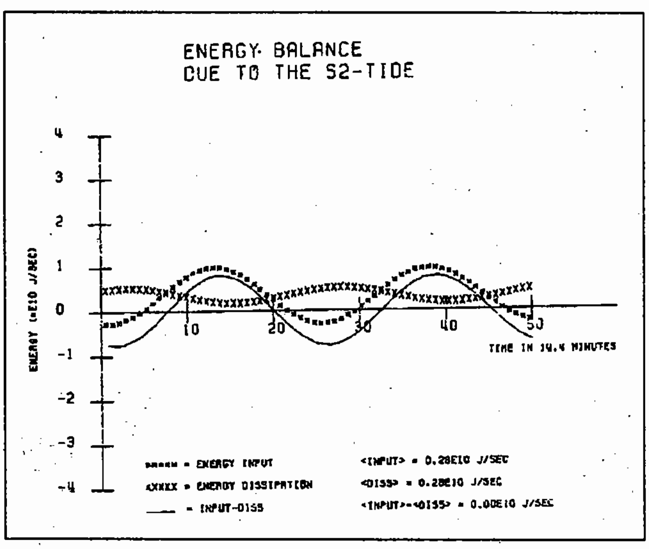

Fig.5 Rate of change in energy balance of the S_2-tide in the 60th S_2-tide period

CONCLUDING AND REMARKS

The S_2 tidal energy balance has been calculated by means a semi-implicit numerical three-dimensional model with a very good balance. In the numerical model sight, it can be concluded that the model works very wheel. The values of mean kinetic and potential energy reflect the dynamics of the Malacca Strait. The value of total energy $\langle E_{tot} \rangle = 1.84 \times 10^{14} J$, or 27.4% of that for M_2-tide is very reasonable since the type of tide in the Malacca Strait is semi-diurnal.

As pointed out by Brosche and Sündermann (1971), the tidal friction quits as the main cause for the deceleration of the earth since the tidal energy transmitted to the world ocean -- according to Jeffreys and Heiskanen as quoted by Sündermann (1977) -- is dissipated by bottom friction. For example, Jeffreys and Heiskanen estimated that 50 to 75% of the whole tidal energy transmitted to the world ocean by the work of the moon and the sun is dissipated within the Bering Sea. Later computations of Munk and Macdonald in 1960 and Miller in year 1966 as also quoted by Sündermann (1977) yielded smaller, but different values. All these calculations are based on a relatively poor data material. Sündermann (1977) found that the energy dissipation by bottom friction yields for the whole Bering Sea, $\langle \dot{E}_b \rangle = 0.29 \times 10^{11} J/s$; this value is smaller by a factor 25 than Jeffrey's value of $7.5 \times 10^{11} J/s$ and by factor 8 than Miller's value of $2.4 \times 10^{11} J/s$. It is in a close agreement with the estimate of Munk and Macdonald ($0.24 \times 10^{11} J/s$). If we compare of our calculation for S2-tide in the Malacca Strait, $\langle E_b \rangle = 0.23 \times 10^{11} J/s$, to the value of Munk and Macdonald for M_2-tide in the Bering Sea, it is approximately 10%.

ACKNOWLEDGMENT

This work was carried out during my Ph.D. program in *Institut für Meereskunde der Universität Hamburg* as a DAAD scholar. Therefore, I would like to thank Prof. Dr. J. Sündermann, Prof. Dr. W. Zahel, Dr. N. Carbajal, Dr. T. Pohlmann and Prof. Dr. J.O. Backhaus from *Institut für Meereskunde der Universität Hamburg* for valuable discussions. I would like also to thank Dr. Radianta Triatmadja as a reviewer who gave a good comments on improving this paper. The topography data was prepared kindly by M. Metzner.

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