PLASTIC COLLAPSE LOAD ANALYSIS OF STRUCTURES CONSIDERING MOMENTS AND AXIAL FORCES

By :

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Abstract

A procedure for the analysis of plastic collapse load of structures considering moments and axial forces is presented. The method is developed based on the static or lower bound theorem. Essentially, this approach involves the determination of the largest load factor for which a statically admissible and safe distribution of internal forces exists. In linear programming form, it is equivalent to maximizing the load factor subject to the conditions of equilibrium and yield.

In this formulation, the displacement method is employed to obtain the internal forces influence coefficients. Those coefficients are used to obtain the sets of statically admissible internal forces. By substituting those forces into the yield surface, which is approximated by piece wise linear functions, the problem of plastic collapse load analysis can be cast into linear programming problem. A linear programming solution algorithm so called Phase 1-Phase 2 procedure is used to solve the problem.

A computer program was prepared for the implementation of the method and a number of numerical examples involving multistory frames and tied arch bridge structures was also considered. Numerical results indicated that the method developed in the present study is found to be accurate, much simpler than the available methods, and is capable of analysing structures with irregular geometry.

1. Introduction

It is well known that plastic collapse load analysis of structures can be cast into linear programming problem to make the solution process suitable for computer programs [1]**). Two methods so called "static" and "kinematic" approaches are usually employed to solve the problems. In static approach, one should maximize the load factor subject to the conditions of equilibrium and yield, while in kinematic approach minimizing the load factor subject to the conditions of equilibrium and mechanism should be carried out [2].

The first approach is related to the force method of structural analysis, while the second is related to the combination of independent elementary mechanisms.

From the micro computer programming point of view, the first method is not popular because of the following reasons : a) the selection of redundant forces by *topological method* may lead to somewhat *ill-conditioned* flexibility matrices [1], and b) the well conditioned matrices could always be obtained by employing the Gauss-Jordan method, however, this procedure tends to fill in the problem data, making the method unattractive or even impractical for large size, sparse problems [3].

In the second method, an essential feature is the fact that every possible mechanism can be constructed from some combination of independent elementary mechanisms [2]. For relatively simple geometry, such as rectangular frames, evaluation of the independent elementary mechanisms is not difficult. However, when the geometry of the structure is not simple and the axial forces effect (in addition to the bending moments) should be taken into account in the analysis, the independent elementary mechanisms are not easy or even impossible to evaluate.

The objective of the present study is to develop a procedure for analysing plastic collapse load of structures considering moments and axial forces, which is

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**)Numbers in parantheses refer to entries in the list of references.

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applicable to both regular (simple) and irregular structural geometry. An example of the irregular geometry is a tied arch bridge structures which is shown in Figure 1.

2. Formulation of The Problem

2.1. General

As mentioned previously, the method discussed in this paper was originally developed for solving plastic collapse load problem of structures having relatively complex geometry, such as tied arch bridge structure. However, the formulation discussed in this paper is applicable to general structures made up of straight beam elements.

The method is developed based on the static or lower bound theorem. Essentially, this approach involves the determination of the largest load factor for which a statically admissible and safe distribution of internal forces exists. In linear programming form, it is equivalent to maximizing the load factor subject to the conditions of equilibrium and yield.

In this formulation the structure is represented by straight beam elements, and the loads are assumed to be applied only at the nodes. The displacement method is employed to obtain the internal forces influence coefficients. Those coefficients are used to obtain the sets of statically admissible internal forces. By substituting those forces into the yield surface, which is approximated by piece wise linear functions, the problem of plastic collapse load analysis can be cast into linear programming problem. A linear programming solution algorithm, so called Phase 1-Phase 2 procedure, discussed in reference [4] is used to solve the problem.

In this study only two dimensional problem is considered. The extension of the method to three dimensional case is quite straightforward.

A computer program was prepared for the implementation of the method, and a number of numerical examples involving multistory frames, and tied arch bridge was also considered.

2.2. Statically Admissible Stress Distribution

Consider a typical tied arch bridge structure illustrated in figure 1. The structure consists of three main components, i.e. : arch rib, hangers, and tie beam. As mentioned previously, the arch rib is approximated by straight beam elements. Nodes are chosen to be located at the intersection points between any two of those three structural components. Loads are applied only at the nodes.

In this method a number of cuts is introduced in the structure to obtain a statically determinate or released structure. For any tied arch bridge structure, a convenient choice of cuts is found to be that illustrated in figure 2. At each hanger cut there is a redundant (axial force), while at the rib cut there are three redundants (axial, shear, and moment). The number of variables in the structures, n, is turned out to be ($3 + \bar{n}$), where $\bar{n}$ is the total number of hangers. This step is similar to that normally done in flexibility method of structural analysis.

Lets consider now the released structure illustrated in figure -2. The external loads, which will be increased proportionally by multiplying them with a load factor, are applied first to the released structure. By means of stiffness method the corresponding internal forces, i,e.: moment (M), axial force (P), and shear at all member ends of the released structure are calculated. The next steps of the analysis involve the determination of the internal forces in the released structure corresponding to unit redundant forces applied separately to the structure. These steps are illustrated in figure -3. Thus for structure having n redundants, there are $K = (n + 1)$ systems that have to be considered.

Let i = member (i = 1,2, ..., s; where s is the total number of members in the structure), and j = end (j = 1,2). For each member end, write expression of moment and axial force. The "internal forces influence coefficient" for moment, m_{ijk}, is defined to be moment at (i, j) due to $X_k = 1$, in which $k = 1,2,. . . . . K$. X_k are basically scalar multiplier and clearly shown in figure -3. Similarly, the "internal forces influence coefficient" for axial force, p_{ijk}, is defined to be axial force at (i,j) due to $X_k = 1$.

Lets consider the original structure shown in figure -1. If the prescribed loading is increased proportionally, the moment and axial force at (i,j), m_{ij} and p_{ij} respectively, can be expressed as a linear combination of the internal forces influence coefficients as follows :

$$m_{ij} = \sum_{k=1}^{(n+1)} X_k \, m_{ijk} \qquad (1)$$

The examples are divided into two categories, i.e.: simple plastic collapse load analysis (only moment is considered), and plastic collapse load analysis under combined stresses (considering both moment and axial force).

In the former category, two multistory frames discussed in references (5) and (6) are considered. To show the effects of combined stresses, those two frames are reanalyzed by means of the latter category. Next, a tied arch bridge structure subject to symmetrical and unsymmetrical loadings are also considered. These tied arch problems have not been solved elsewhere.

4.1. Simple Plastic Collapse Load Analysis

4.1.1. Cohn's Three Story Frame

The geometry, physical properties, and loading condition for this problem are shown in figure-5. This problem has been solved by Cohn and Rafay (5) using kinematic approach based on the elementary independent mechanisms.

To apply the method discussed in section 2 to frame structures, the number of hangers, n is taken to be zero. A convenient cut for this type of problem is shown in figure 6. Since there are three cuts in the structure and each cut has three redundants, the total number of redundants, n, turns out to be 3x3 = 9. Therefore the number of system that have to be considered, K, is (n + 1) = 10. Those 10 systems are illustrated in figure 7.

By applying the procedure given in section 3 and taking $b_\ell = 0$ (ℓ = 1, 2, ..., r), the load factor X_1 was obtained to be 2.211. As expected, the result agrees very well with that obtained by Cohn and Rafay (X_1 = 2.238).

The resulting plastic hinges location is given in figure 8. These plastic hinges are associated with the complete collapse condition [1].

4.1.2. Grierson's Four Story Frame

The problem is described in figure 10. This problem was solved by Grierson [6] using kinematic approach. The prosedure used herein is similar to that discussed in the previous section. Figure 11 shows the convenient cuts for this problem. The total number of redundants, n, is 12. Thus, there are (n + 1) = 13 systems that have to be considered. Those systems are shown in figure 12.

The resulting load factor, X_1, is 2.24, which agrees very well with that obtained by Grierson of 2.236.

The plastic hinges location associated with the complete mechanism is given in figure 13.

4.2. Plastic Collapse Load Analysis Under Combined Stresses

Four problems are considered herein. Two of them are the multistory frames discussed in section 4.1, and the rest of them are tied arch bridge problems. In these analyses, both moment and axial force are taken into account.

4.2.1. Cohn's Three Story Frame

The procedure used here is exactly the same as that discussed in section 4.1.1, except the coefficient b_ℓ defining yield surface should be specified. In solving this problem the yield condition is approximated by simple square (r = 4), as shown in figure 14.

The resulting load factor is X_1 = 1.943, which is very closed to 1.952 obtained by Cohn and Rafay.

As expected, the load factor for combined stresses is lower than that for simple analysis obtained in section 4.1.1. The plastic hinges location is given in figure 9.

4.2.2. Grierson's Four Story Frame

The procedure used to solve this problem is the same as that discussed in section 4.1.2., except the coefficient b_ℓ should be specified. As in the previous case, the yield function is approximated by simple square (r = 4).

The load factor X_1 is found to be 2.11, which agrees well with Grierson's result of 2.08. Figure 15 shows the plastic hinge locations associated with the complete collapse condition.

4.2.3. Tied Arch Bridge

The geometry, physical properties, and loading condition for this problem are illustrated in figure 16. This problem has not been solved elsewhere.

As discussed in section 2, a convenient cut is chosen to be that shown in figure 17. The number of hangers, $\bar{n}$, in the structure is 4. Therefore the total number of redundants, n, turns out to be $(\bar{n} + 3) = 7$. The systems that have to be considered is $K = (n + 1) = 8$. They are shown in figure 17.

Assuming that the yield condition can be approximated by eight linear functions as shown in figure 4 ($a_1 = 1.$, $a_2 = 0.69$, $a_3 = -0.69$, $a_4 = -1.$, $a_5 = -1.$, $a_6 = -0.69$, $a_7 = 0.69$, $a_8 = 1.$, $b_1 = 0.45$, $b_2 = 1.$, $b_3 = 1.$, $b_4 = 0.45$, $b_5 = -0.45$, $b_6 = -1.$, $b_7 = -1.$, $b_8 = -0.45$), the load factor is found to be 1393.15. The plastic hinges location associated with the complete mechanism is given in figure 18.

If the external loads cover only one half of the arch span (unsymmetrical loading), which is illustrated in figure 19, the load factor is found to be 1834.129.

5. Discussion and Conclusion

5.1. Discussion

In the preceding chapter, a procedure for the computation of plastic collapse load of structures was presented. The method was developed based on the static or lower bound theorem, which essentially involved the determination of the largest load factor subject to the conditions of equilibrium and yield.

In the first step of the formulation, as described in section 2.2., a number of cuts was introduced to obtain a statically determinate or released structure. The step is similar to that normally done in flexibility method of structural of analysis. In the next steps, however, instead of using the rest of flexibility method procedure, the stiffness method was employed to obtain the internal forces influence coefficients. The use of stiffness method is desirable because of the following reasons : (a) if we use the flexibility method, arbitrarily selected redundant forces may lead to somewhat "ill-conditioned" flexibility matrices [1], and (b) it has been observed that the force (flexibility) method carried out by the conventional Gauss-Jordan method/process tends to "fill-in" the problem data, making the method unatractive or even impractical for large size, sparse problems (3). By employing the stiffness method, those two disadvantages are avoided.

As mentioned in section I, the method was originally developed for solving plastic collapse load problem of tied arch bridge structures. Since the geometry of the structure is not simple and the axial forces effect should be taken into account in the analysis, the independent elementary mechanisms associated with the kinematic approach are not easy or even impossible to evaluate by hand. Although a procedure for generating the independent elementary mechanisms for arbitrary structures is readily available [7], and another method developed by Ronca and Cohn [8] could also be used to solve the tied arch bridge problem, it is believed that the method described in the previous chapter is much simpler and easy to be implemented by practician engineers. Furthermore, in the preliminary design stage where the changing some members in the structure always happens and thus in need of reanalysis, such simpler method is always desirable.

5.2. Conclusion

In the preceding sections, the procedure for the computation of plastic collapse load of structures has been presented. In the formulation, a number of cuts is introduced to obtain the released structure. By means of stiffness method the so called internal forces influence coefficients are evaluated. These coefficients are used to obtain the sets of statically admissible internal forces. By substituting those forces into the yield surface, which is approximated by piece wise linear functions, the problem of plastic collapse load analysis of structures can be cast into linear programming problem. A linear programming solution algorithm, so called Phase 1-Phase 2 procedure, is used to solve the problem. Since the procedure has no restriction in the sign of variables, the direction of the unit redundant forces applied separately to the released structure can be taken in either way (+ or −) with no effect on the solution.

A computer program has been prepared for the implementation of the method, and a number of numerical examples involving multistory frames and tied arch bridge structure was considered.

The method, which is developed based on the static or lower bound theorem, is found to be very simple to apply to any structures made up of straight beam (and truss) elements.

6. References

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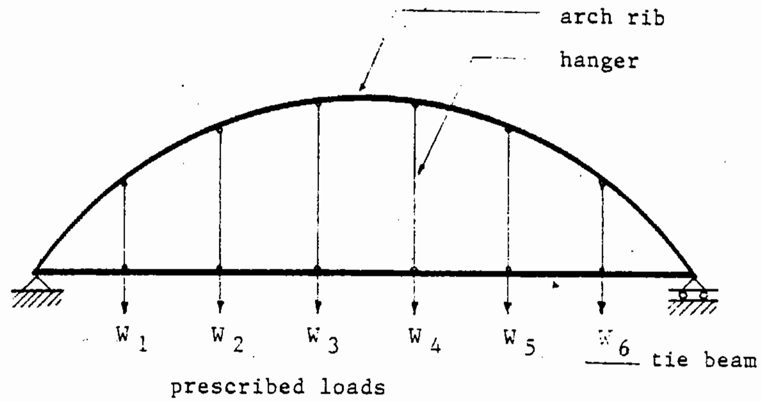


Figure 1. **A Typical Tied Arch Bridge Structure**

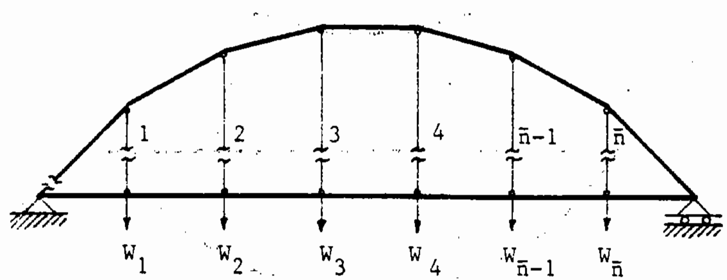


Figure 2. **Released Structure**

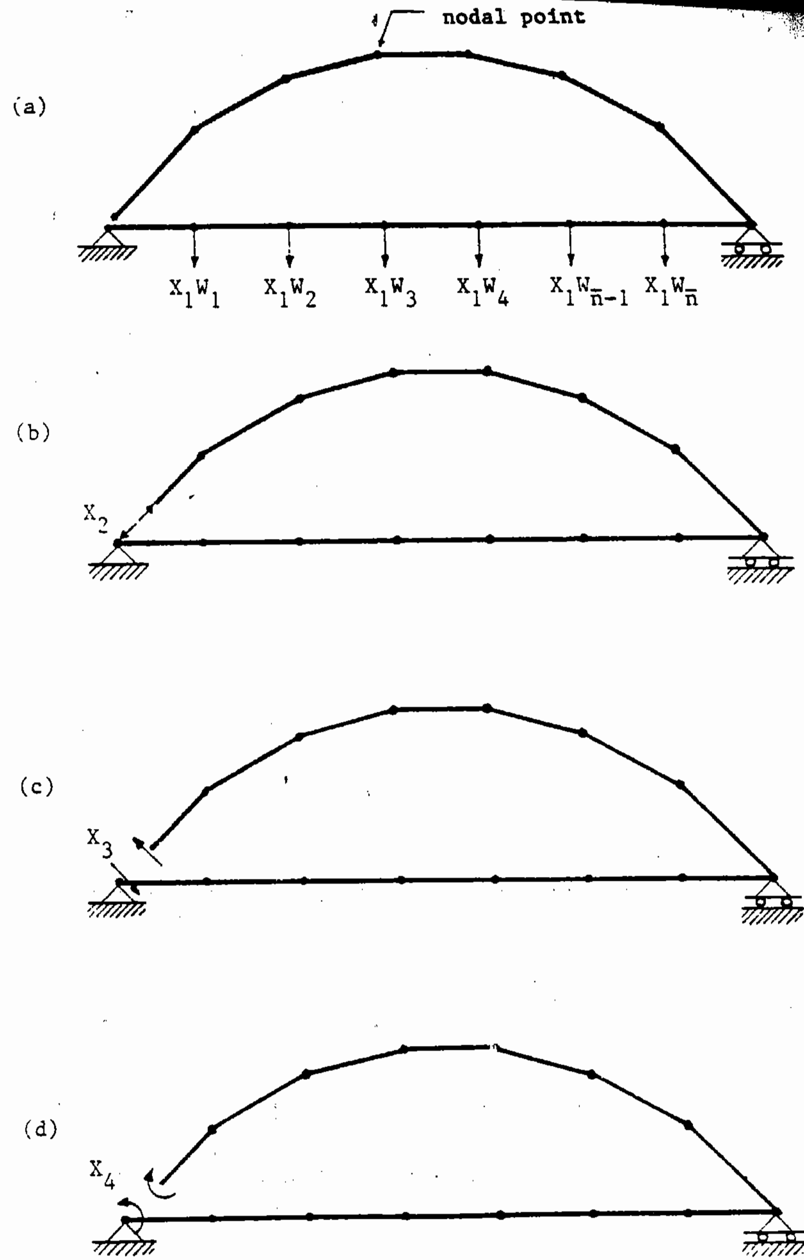


Figure 3. **External Loads and Redundant Forces Applied Separately to the Released Structure.**
(continued)

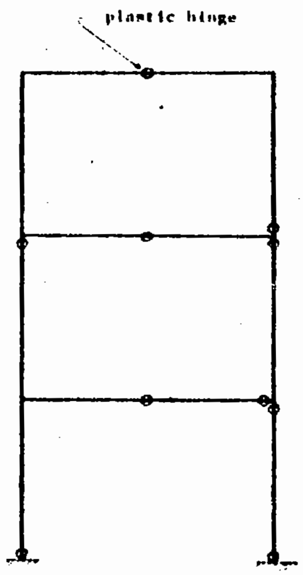


Figure 8. **Plastic Hinge Locations**
(simple analysis)

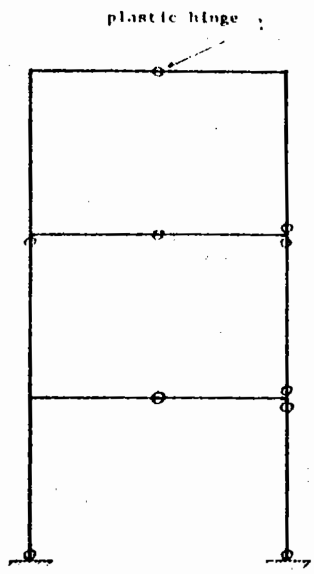


Figure 9. **Plastic Hinge Locations**
(combined stresses)

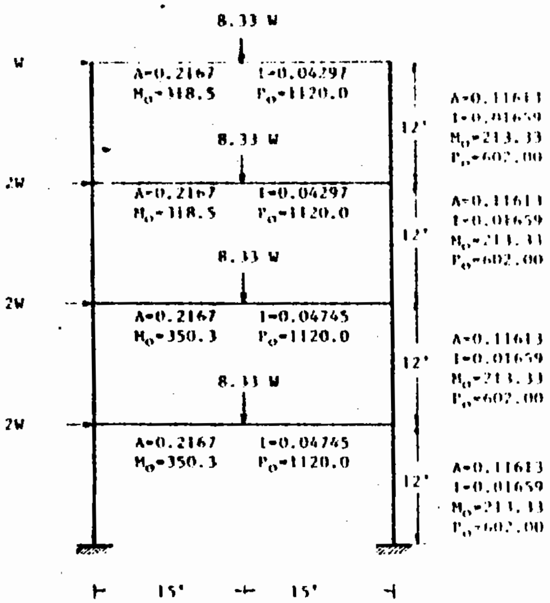


Figure 10. Grierson's Four Story Frame

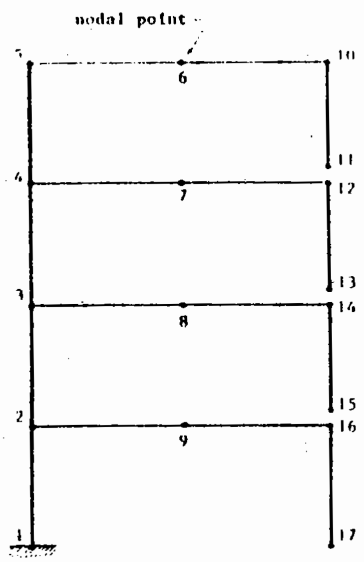


Figure 11. Released Frame Structure

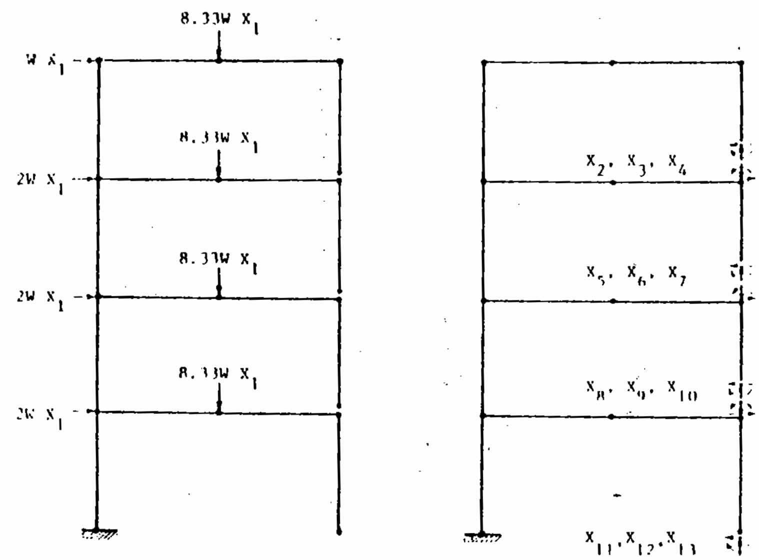


Figure 12. **External Loads and Redundant Forces Applied Separately to the Released Structure**

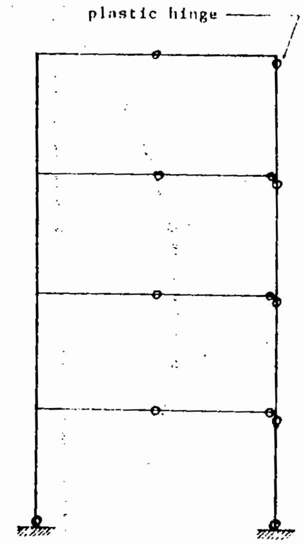


Figure 13. **Plastic Hinge Locations**
(simple analysis)

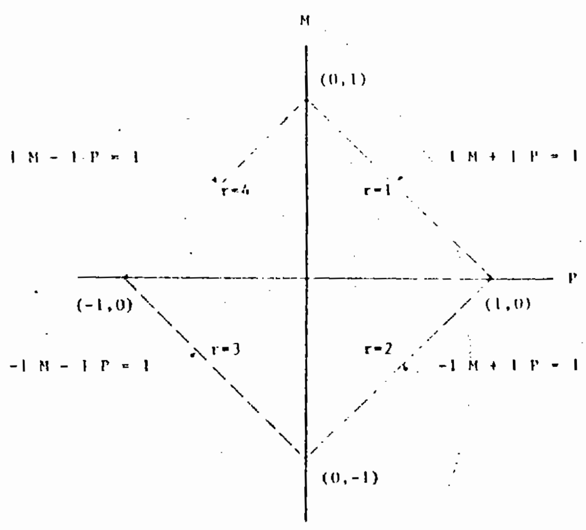


Figure 14. **Yield Function Approximation**

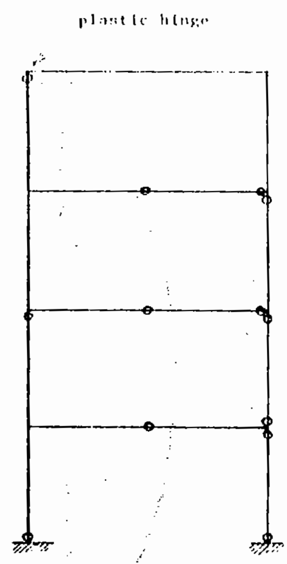
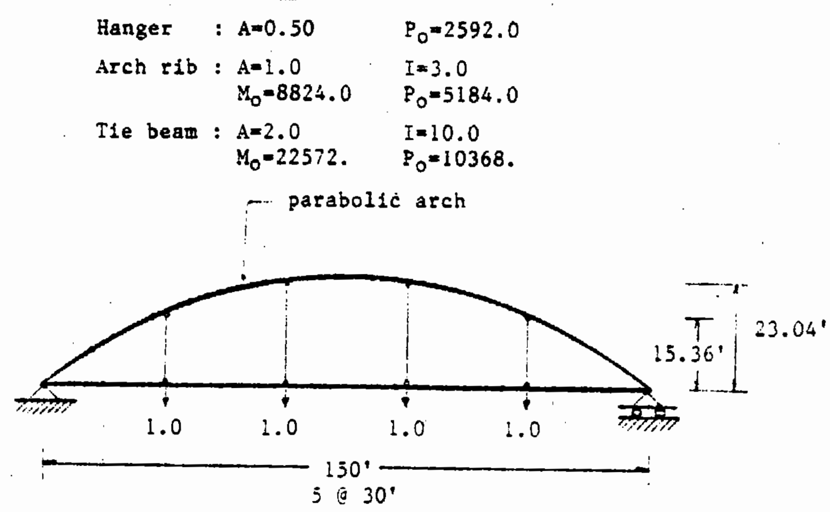


Figure 15. **Plastic Hinge Locations**
(combined stresses)



Fugure 16. **Tied Arch Bridge**

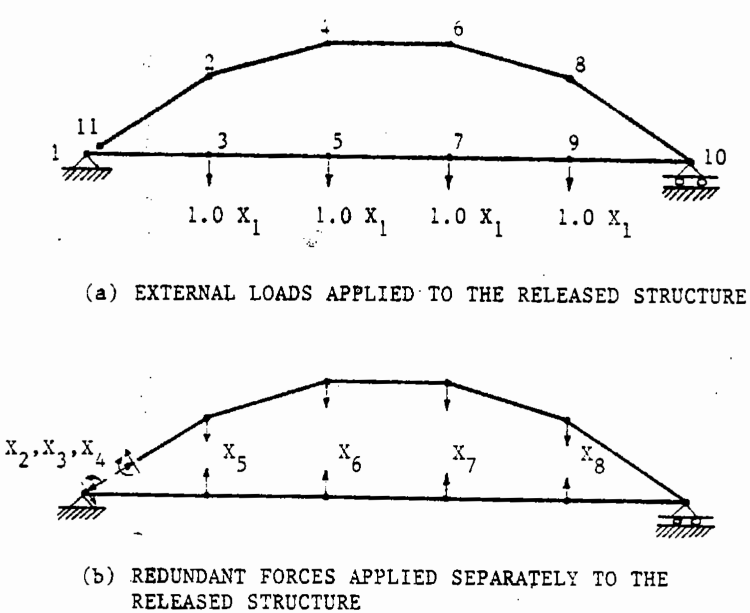


Figure 17. **Released Structure**

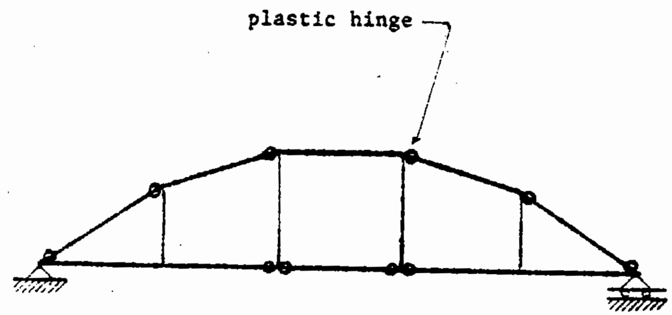


Figure 18. **Plastic Hinge Locations** (symmetrical loading)

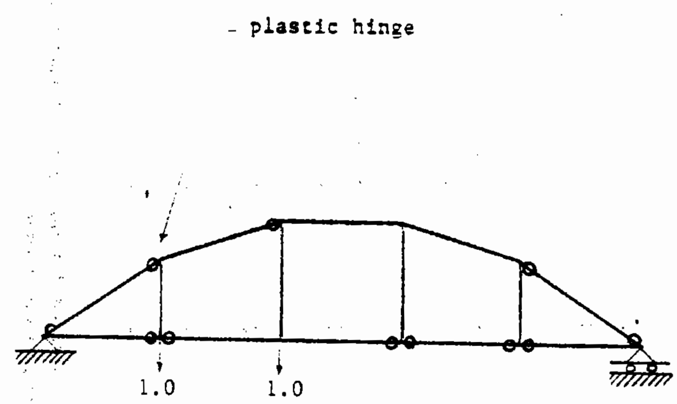


Figure 19. **Plastic Hinge Locations** (unsymmetrical loading)

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