THE HUMAN VISUAL SYSTEM'S MODULATION TRANSFER FUNCTION IN OTHER TRANSFORM DOMAINS

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ABSTRACT

The Human Visual System's (HVS's) Modulation Transfer Function (MTF) has been utilized well in many image processing activities. However, its utilization is limited specifically in the Fourier Transform domain due to the nature of its parameters. On the other hand, many class of images are best represented in other Transform domains, such as the Cosine's, the Sine's, the Haar's, the Slant-Haar's. Here, we show how the MTFs in those other transform domains were constructed and how they look like in the 1-D version.

INTRODUCTION

The HVS's MTF or simply the MTF represents the spatial frequency response of the average person's perception in viewing black-and-white images (Jain, 1989). It is expressed as

$$H(\rho) = [0{,}05 + 0{,}3\rho]e^{-(0{,}114\rho)^{1{,}1}} \qquad (1)$$

where

$$\rho = \sqrt{\xi_1^2 + \xi_2^2}$$

ξ_1 and ξ_2 are the horizontal and vertical spatial frequencies, respectively. The peak value has been normalized to unity and the peak frequency is 8 cycles/degree. Therefore, at lower as well as higher spatial frequencies the HVS's sensitivity to contrast is less. This property has been utilized in some image coding and data compression schemes (Susanto, 1986). The compression gain obtained in these schemes was from 8 to about 0.79 bits/pixel.

In any image data compression scheme, the possibility of bit reduction in the representation of images without lowering their visual qualities is based on the elimination of pixel's gray level redundancies and irrelevancies. Image transforms are the means to annul these redundancies and coarse quantizations are the manner to abolish the irrelevancies. The resulting compression gains are determined by the choice of the transform types and the quantization schemes [Pratt, 1991].

The choice of transforms, on the other hands, is led by the class of images at hand, while the quantization schemes are directed to spread the resulting overall distortion such that it is perceived least by the observers. In this respect, the application of MTF had shown positive results in association with the Fourier Transform. For some classes of images, which contains sharp and regularly structured objects, the Haar and Slant-Haar Transforms are best in reducing the inherent redundancies. Therefore, MTF for these transforms domains should be found accordingly.

PROCEDURE TO OBTAIN THE MTFs IN OTHER TRANSFORM DOMAINS

The basic idea for obtaining the MTFs in domains other than Fourier Transform's is illustrated in Figure 1. In this figure, $H(\rho)$ is the original MTF and $H_T(\tau)$ is the resulting MTF associated with any transform, T, other than the Fourier, F. Analytically, the overall procedure is described as follows [Susanto, 1986].

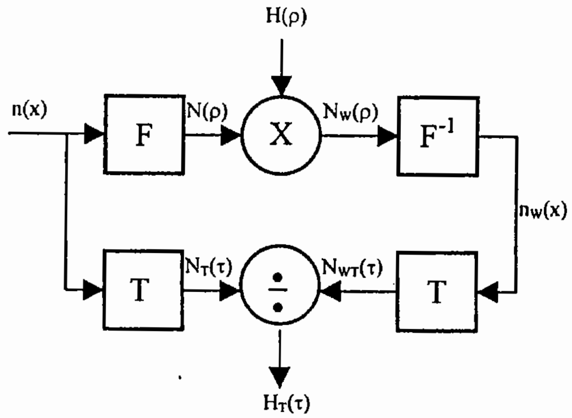

Figure 1. The block diagram which shows the idea for obtaining HT(τ) from H(ρ), where τ is the sequency in the T transform domain.

To excite all possible frequency components, a white noise in spatial domain, n(x), is used. In the Fourier Transform domain this white noise becomes

$$N(\rho) = F\{n(x)\} \qquad (2)$$

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which, theoretically, is

$$\int_{-\infty}^{\infty} n(x)e^{-j2\pi\rho x}dx \quad (3)$$

The white noise spectrum, N(ρ), is then weighted with the (original) MTF, and the result is

$$N_w(\rho) = H(\rho)N(\rho) \quad (4)$$

This MTF-weighted white noise spectrum is Inverse-Fourier transformed to

$$n_w(x) = T^{-1}\{N_w(\rho)\} \quad (5)$$

which, again theoretically, is

$$\int_{-\infty}^{\infty} N_w(\rho)e^{+j2\pi\rho x}d\rho \quad (6)$$

If T represents any transform other than the Fourier, the MTF-weighted white noise in this transform domain is

$$N_{WT}(\tau) = T\{n(x)\} \quad (7)$$

and the transformed original white noise in T domain is

$$N_\tau(\tau) = T\{n(x)\} \quad (8)$$

Now, supposed we had $H_T(\tau)$, i.e. the MTF in the T-transform domain, then

$$H_T(\tau) \times N_T(\tau) = N_{WT}(\tau) \quad (9)$$

therefore

$$H_T(\tau) = \frac{N_{WT}(\tau)}{H_T(\tau)} \quad (10)$$

as indicated in Figure 1; and this basically the procedure to find the MTF in any other transform domain than the Fourier one. However, in practice, since we use a white noise to excite the system, the random nature of the involved functions of x as well as the functions of ρ and τ introduced some problems with regard to the associated computation process and to the accuracy of the results. These problems will be addressed in the following section.

SOME PREDICTION ON THE EXPERIMENTAL ERRORS

It is simpler to express any error, ε, in terms of its variance, say σ_ε^2. If we have a white noise input of variance σ_n^2, then the result of a transform T has the same variance, i.e.

$$\sigma_N^2 = \sigma_n^2 \quad (11)$$

following the Parseval's Theorem. The error in the reconstruction of the MTF in the transform domain is derived from equation (1), so that it is determined by the error Δ_{NT} of $N_{NT}(\tau)$ and Δ_T of $N_T(\tau)$ which are assumed independent of the sequency τ. Using the first degree approximation, the error [Russ, 1995, Neuhoff and Pappas, 1994] is

$$\Delta H_T = \frac{\Delta_{NT}}{N_T(\tau)} - \frac{N_{NT}(\tau)}{N_T^2(\tau)}\Delta_T$$

$$= \frac{1}{N_T(\tau)}\left[\Delta_{NT} - \frac{N_{NT}(\tau)}{N_T(\tau)}\Delta_T\right]$$

$$= \frac{1}{N_T(\tau)}[\Delta_{NT} - H_T(\tau)\Delta_T] \quad (12)$$

Since Δ_{NT} and Δ_T can also be assumed independent of each other, the respective variances in equation (12) are related according to the equation :

$$\sigma_T^2 = \left[\frac{1}{N_T(\tau)}\right]^2 [\sigma_{NT}^2 + H_T^2(\tau)\sigma_T^2] \quad (13)$$

Equation (13) shows clearly the variance of the error of $H_T(\tau)$ is, first of all, inversely proportional to the square of the noise spectrum in the transform domain. Secondly, it is proportional to the square of $H_T(\tau)$. Therefore, we expect some large errors wherever the values of $N_T(\tau)$ are extremely small due to the nature of the dynamics random noise *n(x)* used. The results of our experiments show a few of these cases, see Figure 3(a). In this figure, the MTF is retained to its original scale, deliberately to show the possibility that some of the errors swing out of scale.

RESULTS OF THE EXPERIMENTS

To excite all frequency components of the MTF, a White Gaussian Noise (WGN) was created by running a routine to generate random numbers which are uniformly distributed between 0 and 1. To obtain the WGN, we subtracted 0,5 off these numbers to eliminate the dc component and summed every 12 of them, or

$$WGN = n(i) = \sum_{j=1}^{12}[RANDOM](1.) - 0.5], 0 \le i \le 511 \quad (14)$$

Then, *n(i)* was Discrete Fourier Transformed to get *N(k)*. After weighted by the original MTF, the result was inversely Fourier Transformed to $n_w(i)$. For Fourier Transform the symmetric weighting MTF is shown in Figure 2. In this figure, the maximum frequency is located at the center of the abscissa, i.e. on the 255th sequency, while the peaks occur on the 60th and 452nd which correspond to the frequency 8 cycles/degree.

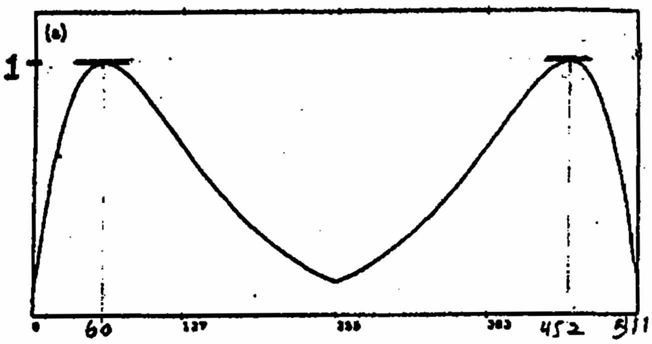

Figure 2. The MTF in the Fourier Transform domain.

The WGN was also Discrete Cosine Transformed to obtain $N_\varepsilon(k)$. The MTF in DCT domain was obtained by carrying out division :

$$H_\varepsilon(k) = \frac{N_{W\varepsilon}(k)}{N_\varepsilon(k)} \tag{15}$$

where $N_{W\varepsilon}(k)$ is the DCT domain MTF weighted WGN $n_w(i)$. The results is shown in Figure 3(a), where it exhibits clearly the experimental errors, especially some extreme or spiking errors. After assemble averaging 512 outputs of similar experiments with different WGNs, the result is shown in Figure 3(b).

For Discrete Sine Transform (DST), the ensemble averaged result is shown in Figure 4(a), while Figure 4(b) shows "low-pass filtered" MTF of (a). For Haar and Slant-Haar Transforms, the results are shown in Figure 5 and Figure 6, respectively, which exhibit significantly higher values at higher sequencies, due to the nature of their basis functions. As we are already aware, each of the nonsinusoidal basis functions in fact consists of a band of spatial frequencies; therefore the nonsinusoidal transforms, in general, can also be called subband transforms.

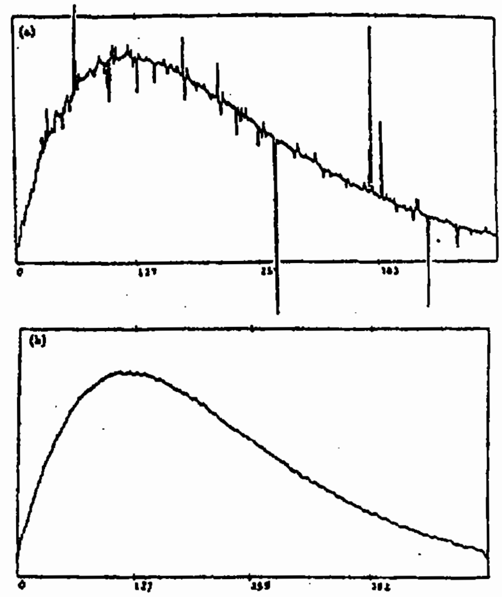

Figure 3. (a) One result of obtained MTF in DCT domain
(b) Assemble averaged MTF of 512 results in (a).

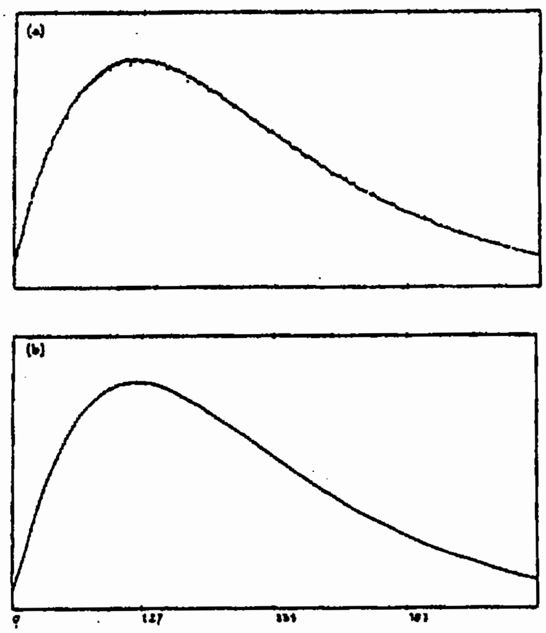

Figure 4(a) Assemble averaged MTF For DST domain
(b) Low-pass Filtered of (a).

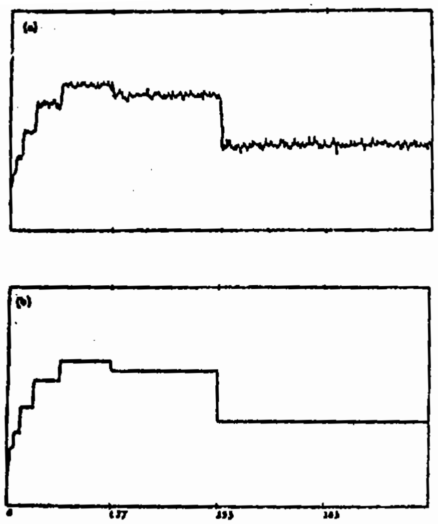

Figure 5. (a) The ensemble averaged MTF in Haar transform domain
(b) The low-pass filtered MTF of (a).

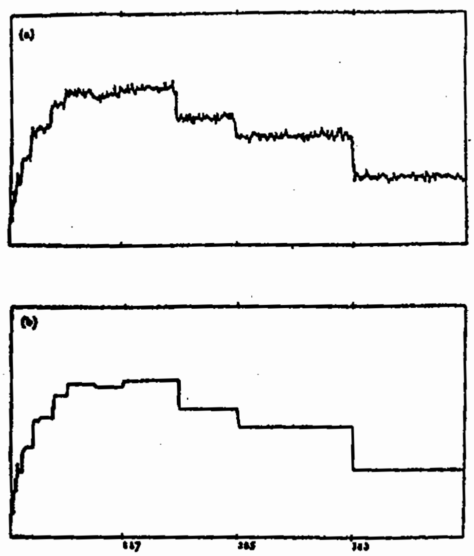

Figure 6. (a) The ensemble averaged MTF in Slant-Haar transform domain
(b) The low-pass filtered MTF of (a).

CONCLUSION

It has been shown that using this technique, the MTF, i.e. the human visual system's frequency response can be acquired for any transform other than the Fourier. For Cosine and Sine Transforms the resulted MTFs are apparently similar to the original. For Haar and Slant-Haar Transforms, the obtained MTFs exhibit piecewise-constant properties.

These generated MTFs open possibilities for image processings which exploit human visual system characteristics, e.g. for image data compression, in any transform domain other than the Fourier one.

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