An integrable deformation of the $AdS_5 \times S^5$ superstring action

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An integrable deformation of the type IIB $AdS_5 \times S^5$ superstring action is presented. The deformed field equations, Lax connection, and κ-symmetry transformations are given. The original $\mathfrak{psu}(2,2|4)$ symmetry is expected to become q-deformed.

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INTRODUCTION

Integrability plays a central role in the study of the AdS/CFT correspondence [1] between type IIB superstring theory on the $AdS_5 \times S^5$ background [2] and the maximally supersymmetric Yang-Mills gauge theory in four dimensions (see [3] for a review). On the Anti-de Sitter side of this correspondence, integrability entered the scene with the discovery that the lagrangian field equations of the $AdS_5 \times S^5$ theory can be recast in the zero curvature form [4]. This implies the existence of an infinite number of conserved quantities.

It is quite natural to seek deformations of the $AdS_5 \times S^5$ superstring which preserve this integrable structure. An important example of such an integrable deformation is the so called β-deformation associated with strings on the Lunin-Maldacena background [5]. The integrability of this model was shown in [6, 7] (see also the review [8] and references therein). Here we shall take a more systematic approach to the construction of integrable deformations by demanding the deformed theory to be integrable from the very outset. This requires approaching the problem from the hamiltonian perspective.

Let us recall that in order to prove integrability in the hamiltonian formalism, one must show the existence of an infinite number of conserved quantities in involution. More precisely, this follows at once if the Poisson bracket of the hamiltonian Lax matrix can be shown to take the specific form in [9, 10]. This was achieved in the case of the $AdS_5 \times S^5$ superstring in [11].

The algebraic structure underpinning this property of the $AdS_5 \times S^5$ superstring was identified in [12]. By utilising this structure, an alternative Poisson bracket with the same property was subsequently constructed in [13]. Moreover, this second Poisson bracket is compatible with the original one, giving rise to a one-parameter family of Poisson brackets sharing the same property [9, 10] which ensures integrability.

These features of the superstring theory are in fact shared with bosonic integrable σ-models [14]. In this latter context, the two compatible Poisson brackets were used very recently in [15] as a building block for constructing integrable q-deformations of the principal chiral model associated with a compact Lie group and of the σ-model on a symmetric space F/G with F compact. In the case of the principal chiral model, the deformation coincides with the Yang-Baxter σ-model introduced by Klimčík in [16]. A key characteristic of this procedure is that the integrability of the deformed theories is automatic since it is used as an input in the construction. Moreover, an interesting output is that the symmetry associated with left multiplication in the original models is deformed into a classical q-deformed Poisson-Hopf algebra.

It is possible to generalize the method developed in [15] to deform the $AdS_5 \times S^5$ superstring theory. The whole construction is carried out at the hamiltonian level and will be presented in detail elsewhere. The purpose of this letter is to present the deformed action and indicate its properties.

SETTING

We begin by recalling the necessary ingredients for defining the $AdS_5 \times S^5$ superstring action (see [17] for more details). Define the projectors $P_{\pm}^{\alpha\beta} = \frac{1}{2}(\gamma^{\alpha\beta} \pm \epsilon^{\alpha\beta})$ where $\gamma^{\alpha\beta}$ is the worldsheet metric with $\det\gamma = -1$ and $\epsilon^{01} = 1$. Worldsheet indices are lowered and raised with the two-dimensional metric. Let $\mathfrak{f}$ denote the Grassmann envelope of the superalgebra $\mathfrak{su}(2,2|4)$, namely the Lie algebra

$$\mathfrak{f} = \mathcal{G}r^{[0]} \otimes \mathfrak{su}(2,2|4)^{[0]} \oplus \mathcal{G}r^{[1]} \otimes \mathfrak{su}(2,2|4)^{[1]},$$

where $\mathcal{G}r$ is a real Grassmann algebra. Introduce the two-dimensional field $g(\sigma,\tau)$ taking value in the Lie group F with Lie algebra $\mathfrak{f}$. The corresponding vector current $A_\alpha = g^{-1}\partial_\alpha g$ belongs to $\mathfrak{f}$. The integrability of the $AdS_5 \times S^5$ superstring action relies heavily on the existence of an order 4 automorphism which induces a $\mathbb{Z}_4$-grading of the superalgebra $\mathfrak{su}(2,2|4)$, and thus of $\mathfrak{f}$. We denote by $\mathfrak{f}^{(i)}$ the subspace of $\mathfrak{f}$ with grade $i = 0, \cdots, 3$. The projector on $\mathfrak{f}^{(i)}$ shall be denoted by P_i and we also write $M^{(i)} = P_i M$ for the projection of $M \in \mathfrak{f}$ on $\mathfrak{f}^{(i)}$. The invariant part $\mathfrak{f}^{(0)}$ is the Lie algebra $\mathfrak{so}(4,1) \oplus \mathfrak{so}(5)$, and the corresponding Lie group is $G = SO(4,1) \times SO(5)$. The supertrace is compatible with the $\mathbb{Z}_4$-grading, which means that $\mathrm{Str}(M^{(m)} N^{(n)}) = 0$ for $m + n \neq 0$ mod 4.

The extra ingredient needed to specify the deformation is a skew-symmetric (non-split) solution of the modified classical Yang-Baxter equation on $\mathfrak{f}$. Specifically, this is an $\mathbb{R}$-linear operator R such that, for $M, N \in \mathfrak{f}$,

$$[RM, RN] - R\big([RM, N] + [M, RN]\big) = [M, N] \qquad (1)$$

and $\mathrm{Str}\big(MRN\big) = -\mathrm{Str}\big(RMN\big)$. We shall take R to be the restriction to $\mathfrak{su}(2,2|4)$ of the operator acting on the complexified algebra by $-i$ on generators associated with positive roots, $+i$ on generators associated with negative roots, and 0 on Cartan generators. We will make use of the operator $R_g = \mathrm{Ad}_g^{-1} \circ R \circ \mathrm{Ad}_g$ which is also a skew-symmetric solution of (1). Finally, we define the following linear combinations of the projectors,

$$d = P_1 + \frac{2}{1-\eta^2} P_2 - P_3, \quad \widetilde{d} = -P_1 + \frac{2}{1-\eta^2} P_2 + P_3.$$

The operator $\widetilde{d}$ is the transpose operator of d and thus satisfies $\mathrm{Str}\big(M\, d(N)\big) = \mathrm{Str}\big(\widetilde{d}(M)N\big)$. The real variable $\eta \in [0, 1[$ will play the role of the deformation parameter.

DEFORMED ACTION

As pointed out in the introduction, we will restrict ourselves in this letter to presenting the deformed action and summarising its most important properties. In this section we shall write down this action and indicate the properties it shares with the undeformed action. Properties which depend on the deformation parameter η are presented in the next section.

Action

The action, which can be obtained by generalising the method developed in [15] to the case at hand, reads $S[g] = \int d\sigma d\tau L$ with

$$L = -\frac{(1+\eta^2)^2}{2(1-\eta^2)} P_-^{\alpha\beta}\, \mathrm{Str}\Big(A_\alpha\, d \circ \frac{1}{1 - \eta R_g \circ d}(A_\beta)\Big). \quad (2)$$

The operator $1 - \eta R_g \circ d$ is invertible on $\mathfrak{f}$ for all values of the deformation parameter $\eta \in [0, 1[$. As in the undeformed case, there is an abelian gauge invariance $g(\sigma,\tau) \to g(\sigma,\tau) e^{i\theta(\sigma,\tau)}$ under which the vector field A_α transforms as $A_\alpha \to A_\alpha + \partial_\alpha \theta \,.\, 1$. Indeed, this leaves the action associated with (2) invariant because $\mathrm{Str}(1.M) = 0$ for any M in $\mathfrak{su}(2,2|4)$. This invariance means that physical degrees of freedom do not belong to the whole group F but rather to the projective group PF. From now on the commutators that will appear should be considered as commutators of the projective algebra $\mathfrak{pf}$, and the adjoint action of g, Ad_g, is that of the projective group PF. This peculiarity already appears in the undeformed case and the reader is referred for instance to the review [17] for more details.

Original Metsaev-Tseytlin action

The undeformed action corresponds to $\eta = 0$. Indeed, when η vanishes, the Lagrangian (2) simply becomes

$$\begin{aligned} L|_{\eta=0} &= -\tfrac{1}{2} P_-^{\alpha\beta}\, \mathrm{Str}\Big(A_\alpha d|_{\eta=0}(A_\beta)\Big), \\ &= -\tfrac{1}{2}\, \mathrm{Str}\Big(\gamma^{\alpha\beta} A_\alpha^{(2)} A_\beta^{(2)} + \epsilon^{\alpha\beta} A_\alpha^{(1)} A_\beta^{(3)}\Big). \end{aligned}$$

One therefore recovers at $\eta = 0$ the type IIB superstring action on the $AdS_5 \times S^5$ background. This celebrated Metsaev-Tseytlin action [2] is that of a σ-model on the semi-symmetric space $PSU(2,2|4)/G$ with Wess-Zumino term (see for instance the reviews [17, 18]) [19].

$SO(4,1) \times SO(5)$ gauge invariance

The action corresponding to (2) has a gauge invariance $g(\sigma,\tau) \to g(\sigma,\tau)h(\sigma,\tau)$ where the function $h(\sigma,\tau)$ takes values in the subgroup G. This can be easily shown using the corresponding transformations

$$\begin{aligned} A_\alpha &\to h^{-1}\partial_\alpha h + \mathrm{Ad}_h^{-1}(A_\alpha), \\ d(A_\alpha) &\to \mathrm{Ad}_h^{-1} \circ d(A_\alpha), \\ R_g &\to \mathrm{Ad}_h^{-1} \circ R_g \circ \mathrm{Ad}_h. \end{aligned}$$

This gauge transformation does not depend on the deformation parameter η.

PROPERTIES OF THE DEFORMED ACTION

To present the properties of the action (2), we will follow the approach presented in the review [17] for the undeformed case.

Equations of motion

The equations of motion are most conveniently written in terms of the vectors

$$\begin{aligned} J_\alpha &= \frac{1}{1 - \eta R_g \circ d}(A_\alpha), \\ \widetilde{J}_\alpha &= \frac{1}{1 + \eta R_g \circ \widetilde{d}}(A_\alpha) \end{aligned}$$

and their projections, $J_-^\alpha = P_-^{\alpha\beta} J_\beta$ and $\widetilde{J}_+^\alpha = P_+^{\alpha\beta} \widetilde{J}_\beta$. In the following, we shall often use the fact that the components J_-^0 and J_-^1 are proportional to each other. One has in particular $[J_-^\alpha, J_-^\beta] = 0$ (and similarly for $\widetilde{J}_+^\alpha$). The equations of motion arising from the Lagrangian (2) are given by $\mathcal{E} = 0$ where

$$\mathcal{E} := d(\partial_\alpha J_-^\alpha) + \widetilde{d}(\partial_\alpha \widetilde{J}_+^\alpha) + [\widetilde{J}_{+\alpha}, d(J_-^\alpha)] + [J_{-\alpha}, \widetilde{d}(\widetilde{J}_+^\alpha)].$$

It is easy to check that the projection $\mathcal{E}^{(0)}$ of $\mathcal{E}$ onto $\mathfrak{f}^{(0)}$ vanishes, in accordance with the gauge invariance of the action described above.

Rewriting the Maurer-Cartan equation

We now wish to address the question of integrability of the theory defined by (2). Recall that in the undeformed case, in deriving the Lax connection one makes use of the Maurer-Cartan equation $\mathcal{Z}=0$ satisfied by A_α, where

$$\mathcal{Z} := \tfrac{1}{2}\epsilon^{\alpha\beta}(\partial_\alpha A_\beta - \partial_\beta A_\alpha + [A_\alpha, A_\beta]).$$

To find a Lax connection we therefore start by rewriting $\mathcal{Z}$ in terms of J^α_- and $\widetilde{J}^\alpha_+$. The resulting expression is a quadratic polynomial in η. Using equation (1) for the operator R_g, one can rewrite the coefficient of η^2 of this polynomial to obtain

$$\mathcal{Z} = \partial_\alpha \widetilde{J}^\alpha_+ - \partial_\alpha J^\alpha_- + [J_{-\alpha}, \widetilde{J}^\alpha_+] + \eta^2[d(J_{-\alpha}), \widetilde{d}(\widetilde{J}^\alpha_+)] + \eta R_g(\mathcal{E}).$$

Anticipating the result, let us note here that choosing R to be a non-split solution of the modified classical Yang-Baxter equation is essential in order to preserve integrability as we deform the theory. Before constructing the Lax connection, let us remark that the field equations in the odd sector $P_{1,3}(\mathcal{E})=0$ may be greatly simplified by considering the combinations

$$P_1 \circ (1-\eta R_g)(\mathcal{E}) + P_1(\mathcal{Z}) = -4[\widetilde{J}^{(2)}_{+\alpha}, J^{\alpha(3)}_-], \tag{3a}$$

$$P_3 \circ (1+\eta R_g)(\mathcal{E}) - P_3(\mathcal{Z}) = -4[J^{(2)}_{-\alpha}, \widetilde{J}^{\alpha(1)}_+]. \tag{3b}$$

As a consequence, one can take as field equations in the odd sector

$$[\widetilde{J}^{(2)}_{+\alpha}, J^{\alpha(3)}_-] = 0, \qquad [J^{(2)}_{-\alpha}, \widetilde{J}^{\alpha(1)}_+] = 0,$$

which have the same form as those of the undeformed model written in terms of ordinary currents.

Lax connection

We define the two vectors

$$L^\alpha_+ = \widetilde{J}^{\alpha(0)}_+ + \lambda\sqrt{1+\eta^2}\,\widetilde{J}^{\alpha(1)}_+ + \lambda^{-2}\frac{1+\eta^2}{1-\eta^2}\widetilde{J}^{\alpha(2)}_+ + \lambda^{-1}\sqrt{1+\eta^2}\,\widetilde{J}^{\alpha(3)}_+,$$

$$M^\alpha_- = J^{\alpha(0)}_- + \lambda\sqrt{1+\eta^2}\,J^{\alpha(1)}_- + \lambda^{2}\frac{1+\eta^2}{1-\eta^2}J^{\alpha(2)}_- + \lambda^{-1}\sqrt{1+\eta^2}\,J^{\alpha(3)}_-,$$

where λ is the spectral parameter. Then, the whole set of equations of motion $\mathcal{E}=0$ and zero curvature equations $\mathcal{Z}=0$ are equivalent to

$$\partial_\alpha L^\alpha_+ - \partial_\alpha M^\alpha_- + [M_{-\alpha}, L^\alpha_+] = 0. \tag{4}$$

One may define an unconstrained vector

$$\mathcal{L}_\alpha = L_{+\alpha} + M_{-\alpha},$$

in terms of which the equation (4) becomes an ordinary zero curvature equation

$$\partial_\alpha \mathcal{L}_\beta - \partial_\beta \mathcal{L}_\alpha + [\mathcal{L}_\alpha, \mathcal{L}_\beta] = 0.$$

The existence of this Lax connection shows that the dynamics of the deformed action admits an infinite number of conserved quantities.

Virasoro constraints

It is clear that each term in the Lagrangian (2) is proportional either to the metric $\gamma^{\alpha\beta}$ or to $\epsilon^{\alpha\beta}$. The part of the action proportional to the metric takes the form

$$S_\gamma = -\tfrac{1}{2}\left(\frac{1+\eta^2}{1-\eta^2}\right)^2 \int d\sigma d\tau \gamma^{\alpha\beta}\,\mathrm{Str}\big(J^{(2)}_\alpha J^{(2)}_\beta\big), \tag{5a}$$

$$= -\tfrac{1}{2}\left(\frac{1+\eta^2}{1-\eta^2}\right)^2 \int d\sigma d\tau \gamma^{\alpha\beta}\,\mathrm{Str}\big(\widetilde{J}^{(2)}_\alpha \widetilde{J}^{(2)}_\beta\big). \tag{5b}$$

To obtain this result, the skew-symmetry of R_g has been used. The Virasoro constraints are then found to be

$$\mathrm{Str}\big(\widetilde{J}^{\alpha(2)}_+ \widetilde{J}^{\beta(2)}_+\big) \approx 0, \qquad \mathrm{Str}\big(J^{\alpha(2)}_- J^{\beta(2)}_-\big) \approx 0.$$

Kappa symmetry

The invariance under κ-symmetry is a characteristic of the Green-Schwarz formulation. We now want to show that the kappa invariance is essentially unchanged after deformation. To do this, consider an infinitesimal right translation of the field, $\delta g = g\epsilon$, where the parameter ϵ takes the form

$$\epsilon = (1-\eta R_g)\rho^{(1)} + (1+\eta R_g)\rho^{(3)}.$$

The fields $\rho^{(1)}$ and $\rho^{(3)}$, whose expressions will be determined shortly, respectively take values in $\mathfrak{f}^{(1)}$ and $\mathfrak{f}^{(3)}$. Then the variation of the action with respect to g reads

$$\delta_g S = \tfrac{(1+\eta^2)^2}{2(1-\eta^2)} \int d\sigma d\tau\,\mathrm{Str}\Big(\rho^{(1)} P_3 \circ (1+\eta R_g)(\mathcal{E}) + \rho^{(3)} P_1 \circ (1-\eta R_g)(\mathcal{E})\Big).$$

We may then use equations (3) to write this variation as

$$\delta_g S = -2\tfrac{(1+\eta^2)^2}{(1-\eta^2)} \int d\sigma d\tau\,\mathrm{Str}\Big(\rho^{(1)}[J^{(2)}_{-\alpha}, \widetilde{J}^{\alpha(1)}_+] + \rho^{(3)}[\widetilde{J}^{(2)}_{+\alpha}, J^{\alpha(3)}_-]\Big).$$

In full analogy with the undeformed case (see [17]), we take the following ansatz for $\rho^{(1)}$ and $\rho^{(3)}$:

$$\rho^{(1)} = i\kappa^{(1)}_{+\alpha} J^{\alpha(2)}_{-} + J^{\alpha(2)}_{-} i\kappa^{(1)}_{+\alpha},$$
$$\rho^{(3)} = i\kappa^{(3)}_{-\alpha} \widetilde{J}^{\alpha(2)}_{+} + \widetilde{J}^{\alpha(2)}_{+} i\kappa^{(3)}_{-\alpha},$$

where $\kappa^{(1)}_{+}$ and $\kappa^{(3)}_{-}$ are constrained vectors of respective gradings 1 and 3. Note that we are using the standard convention for the real form $\mathfrak{su}(2,2|4)$ (see for instance appendix C of [20]). Then, a short calculation leads to

$$\mathrm{Str}\big(\rho^{(1)}[J^{(2)}_{-\alpha}, \widetilde{J}^{\alpha(1)}_{+}]\big) = \mathrm{Str}\big(J^{\alpha(2)}_{-} J^{\beta(2)}_{-} [\widetilde{J}^{(1)}_{+\alpha}, i\kappa^{(1)}_{+\beta}]\big),$$
$$\mathrm{Str}\big(\rho^{(3)}[\widetilde{J}^{(2)}_{+\alpha}, J^{\alpha(3)}_{-}]\big) = \mathrm{Str}\big(\widetilde{J}^{\alpha(2)}_{+} \widetilde{J}^{\beta(2)}_{+} [J^{(3)}_{-\alpha}, i\kappa^{(3)}_{-\beta}]\big).$$

At this point, we use the standard property (see [17]) that the square of an element of grade 2 only contains a term proportional to $W = \mathrm{diag}(1_4, -1_4)$ and a term proportional to the identity which does not play a role in the case at hand. We finally obtain

$$\delta_g S = -\tfrac{(1+\eta^2)^2}{4(1-\eta^2)} \int d\sigma d\tau \Big(\mathrm{Str}(J^{\alpha(2)}_{-} J^{\beta(2)}_{-}) \times$$
$$\times \mathrm{Str}\big(W[\widetilde{J}^{(1)}_{+\alpha}, i\kappa^{(1)}_{+\beta}]\big) + \mathrm{Str}(\widetilde{J}^{\alpha(2)}_{+} \widetilde{J}^{\beta(2)}_{+})\, \mathrm{Str}\big(W[J^{(3)}_{-\alpha}, i\kappa^{(3)}_{-\beta}]\big)\Big).$$

This expression comes from the variation of the field g in the action. It may be compensated by another term coming from the variation of the metric γ. To determine this variation we use the result (5). We are then led to choose

$$\delta\gamma^{\alpha\beta} = \frac{1-\eta^2}{2}\, \mathrm{Str}\Big(W[i\kappa^{\alpha(1)}_{+}, \widetilde{J}^{\beta(1)}_{+}] + W[i\kappa^{\alpha(3)}_{-}, J^{\beta(3)}_{-}]\Big)$$

for the transformation of the metric in order to ensure κ-symmetry.

CONCLUSION

The Lagrangian (2) is a semi-symmetric space generalisation of the one obtained in [15] by deforming the symmetric space σ-model on F/G. In the latter case, it was shown that the original F_L symmetry is deformed to a Poisson-Hopf algebra analogue of $U_q(\mathfrak{f})$. The same fate is confidently expected for the $\mathfrak{psu}(2,2|4)$ symmetry of the $AdS_5 \times S^5$ superstring. Hence, the q-deformation proposed here generalizes the situation which holds for the squashed sphere σ-model [21, 22].

As mentioned in the introduction, the construction of the deformed theory relies on the existence of a second compatible Poisson bracket. The latter is known to be related [13] to the Pohlmeyer reduction of the $AdS_5 \times S^5$ superstring [20, 23]. In fact, one motivation for deforming the superstring action comes from the q-deformed S-matrix appearing in this context [24–26], built from the q-deformed R-matrix of [27]. It would therefore be very interesting to make contact between these two deformations.

Let us end on a more conjectural note by commenting on the limit $\eta \to 1$ of the deformed model. The analogous limit in the case of the deformed $SU(2)/U(1)$ σ-model corresponds to a $SU(1,1)/U(1)$ σ-model [15]. If such a property were to generalise to the case at hand, we expect that the cosets $AdS_5 \simeq SO(4,2)/SO(4,1)$ and $S^5 \simeq SO(6)/SO(5)$ would respectively be replaced in this limit by $SO(5,1)/SO(4,1) \simeq dS_5$ and $SO(5,1)/SO(5) \simeq H^5$. Such cosets have already been considered in [28]. This point certainly requires closer investigation and we will come back to it from the hamiltonian point of view elsewhere.

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