Title	An Integrated Inventory Model for Supply Chain Management
Name	Xiaoliang Zhang

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AN INTEGRATED INVENTORY MODEL FOR SUPPLY CHAIN MANAGEMENT

by

XIAOLIANG ZHANG

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ABSTRACT

Improved integration of logistics processes across multiple companies of a supply chain is of increasing interest and importance. With modern information technologies, more and more companies intend to implement a logistics alliance strategy for co-operations in the supply chain. However, the implementation of the strategy highly depends on the integrated logistics models available. To this end, extensions of existing models may be required to facilitate the entire supply chain rather than individuals.

Inventory management is one of the most important parts of logistics management. In this project, an integrated inventory model is built for a supply chain with a manufacturer, multiple upstream factories and multiple downstream vendors. Based on some assumptions, all the individual inventory behaviours are considered together to suggest an overall optimised plan to minimise the total inventory cost of the supply chain. Then, extensions are made to the integrated inventory model for practical considerations. A numerical analysis is conducted to compare the optimised results of the integrated model with the results of some existing models. Finally, conclusions and future perspectives are drawn.

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Chapter One

Introduction

For many years, industrial researches have focused on improving technologies to reduce the manufacturing cost. Over the last decades, fundamental changes have taken place in the relationships between manufacturers of primary products and retailers. For greater efficiency and lower costs, there has been growing importance given to managing the manufacturing processes effectively for the benefit of all parts together. This has led to the development of new technologies in industry, such as Computer Integrated Manufacturing Systems, Concurrent Engineering and Supply Chain Management.

Supply chain management is a relatively new term. It is about concepts of integrated business planning through different divisions of an enterprise and different enterprises that are related by business activities. A supply chain comprises dispersed facilities where raw materials, intermediate products, or finished products are acquired, transformed, stored and sold. For a manufacturing enterprise, efficient material flow management through daily production action is considered as an important part of the supply chain management.

This research is motivated by the increasing desire of manufacturers to re-engineer their supply chains in order to better manage their material flows. The re-engineering is mainly due to the awareness of supply chain competitive advantage achieved through co-operation at different stages of a supply chain. The co-operation has led to the development of new analytical methods for material control.

The research attempts to present an integrated inventory model based on existing integrated supply chain management methods. The aim is to reduce the total inventory cost of all the companies in the supply chain. It is expected that the integrated inventory model could provide co-operated enterprises with some useful guidelines.

This thesis is divided into six chapters. This chapter, Chapter 1 gives an introduction to the research and the thesis. Chapter 2 presents a brief introduction to supply chain management and a review of inventory control technologies. Chapter 3 describes the supply chain and builds an integrated inventory model with some assumptions. Chapter 4 extends the integrated model by changing the assumptions to make the model more flexible for practical applications. Chapter 5 presents a data generating system and compares the integrated model with an existing model using the supply chain data. Finally, Chapter 6 draws conclusions and discusses further work.

Chapter Two

An Overview of Inventory Management

2.1 Introduction

Over the past ten years supply chain management has become an important focus of competitive advantage for firms and organisations. The impact of supply chain management has increased steadily, drawing on developments in data processing, management science, logistics, operation management and other fields [Harrison, 2001]. Supply chain management is about integrated planning. First, it is concerned with functional integration of purchasing, manufacturing, transportation, and warehousing activities. Second, it refers to the integration of these activities over strategic, tactical, and operational planning. Finally, it also refers to spatial integration of these activities across different individuals of vendors, factories and markets.

For effective supply chain management, matching supply with demand is critical. However, in order for a manufacturer firm to match its supply with the market demand, it has to deal with the issue well beyond its own organisation boundary [Kim *et al*, 2002]. For instance, the firm has to take into account its suppliers' capability as well. For a successful supply chain, it is essential that the information of other companies is consistent with the manufacturer.

So improved integration of activities across multiple companies sharing components of a supply chain is a concern of increasing interest and importance [Shapiro, 2001]. Such integration is obviously relevant to the efficient operation of two companies (in the simplest case) that wish to tighten their working arrangements, e.g. a manufacturer of consumer products wants to arrange a regular plan of replenishment for its buyer. In such case, the integration is complicated because both companies have other suppliers and buyers. That is, their supply chains overlap significantly but are far from integrated. Moreover, enhanced integration implies greater sharing of confidential information about costs and capacities as well as business processes. It leads to more and more companies beginning to share their detailed confidential information to make an overall plan for their supply chain. In many practical instances, integrated control shows considerable room for improvement.

With the increased interest in supply chain management, several authors have discussed the relationship between this term and logistics. Supply chain management is not just another name of logistics. It included elements that are not typically included in a definition of logistics, such as information systems integration and coordination of planning and control activities. But it is no doubt that logistics is the core of supply chain.

Logistics, which has a considerable impact on enterprise costs and competitive advantage, is the process of planning, implementing and controlling the flow and storage of goods and services and related information from the point of origin to the point of consumption. The function in a small company may include all of these activities while in a large corporation it may involve only one or a few of these areas [Ding and Zhang, 2000]. Global 2000 manufacturing and distribution companies spend on the average 11 percent of their revenues on logistics, i.e. transportation, import/export and inventory management, amounting to hundreds of millions of dollars annually [Arzoon, 2000].

Recently, inventory control through supply chain is concerned as a more important logistics action. Reducing inventory levels, work-in-progress, and finished items simultaneously in different stages has now become the major focus for supply chain management. A company may hold inventories of raw materials, parts, intermediate products, or finished products for a variety of reasons. Inventories can serve to hedge against the uncertainties of supply and demand or to take advantage of economies associated with manufacturing or acquiring products in large batches. Inventory problems are characterised by holding cost, shortage cost, ordering cost for buyer, setup cost for vendors, etc. Incorporating inventory decisions in a supply chain optimisation model is difficult because it involves many parameters and relationships, such as variance of market demands and delivery times and their impact on stock outages, which are not easily represented in optimisation models. Nevertheless, depending on the scope of analysis, acceptable approximations of integrated inventory cost can be developed. Improving these approximations is an important area of applied research.

A review of research on inventory management is presented in the following sections while an overview of research in the general area of logistic management has been conducted [Zhang *et al*, 2002].

2.2 Related studies

2.2.1 Supply chain management (SCM)

A supply chain is a set of value-adding activities that connect the suppliers and customers of a firm. A supplier means an external vendor or an upstream process within the firm. Similarly, a customer may be a final customer or a downstream operation [Harrison, 2001].

Over the past decades, SCM has become an important element of competitive advantage for enterprises. More and more enterprises are realising the impact of SCM on their business. However, some crucial parts are invisible in this chain (Figure 2-1).

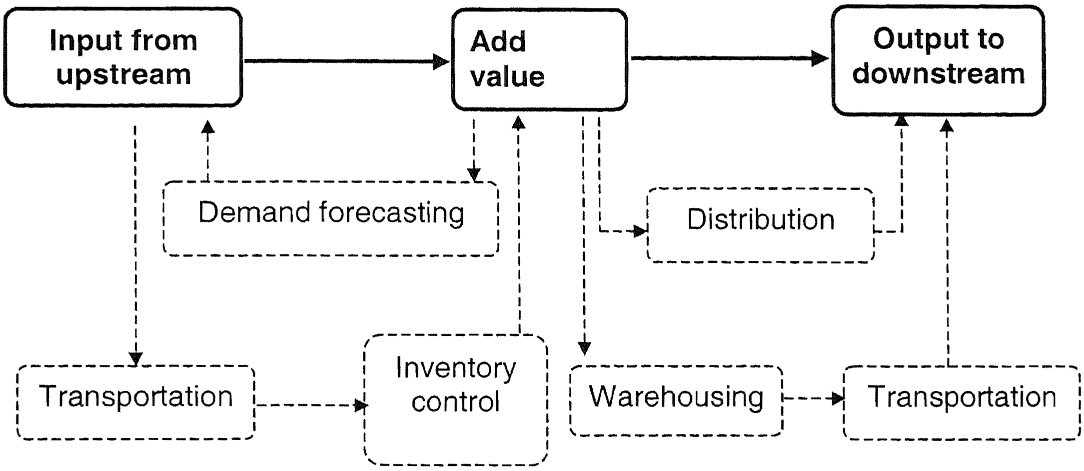


Figure 2-1. Invisible operations in supply chain

From upstream suppliers to the enterprise, demand forecasting and inventory control are needed to ensure the amount of materials for manufacturing requirements. Between the enterprise and downstream, distribution and warehousing (inventory action) are needed to manage the production. Each movement of material flow is implemented by transportation. It is obvious that inventory management plays an important part in SCM.

2.2.2 Global logistics management (GLM)

Large enterprises, especially multinational corporations (MNCs) can no longer restrict themselves to local businesses. They need to accomplish real-time reflection of global market changes. In response to the developing global marketing, GLM is developed. The term GLM refers to the overall management system of a corporation's undertaking of worldwide market distribution, product design, customer satisfaction, production, procurement, logistics, suppliers and inventory [Huang *et al*, 2001]. To implement a GLM system, some concepts are useful. Firstly, Supply Chain Management is directly related, not only to the local suppliers, but also to the global supply chain management. Secondly, standard operational procedures are important to implement logistics globally. Finally, just-in-time reflection to the changes of global market is a most important concept to keep the system working efficiently. So a rough, but descriptive definition of a GLM system is a globally faced, standard, real-time logistics system.

2.2.3 Enterprise resource planning (ERP)

Enterprise resource planning, which is an expanded function from manufacturing resource planning (MRPII), has presented a new efficient solution to enterprise management problems. All the facilities of ERP, such as seamless integration of all divisions, integrated management information, close connection with the supply chain management, seem to be the suitable backbone of a logistics management system. () Core ERP is nothing more than a balance between supply and demand. Demand refers to forecasting, customer orders and safe stock. On the supply side, there are transfer orders, manufacturing orders and purchase orders [Kilian, 2002], and most activities are logistics management operations.

2.3 Research review

The number of published papers on inventory management is vast, with numerous books dedicated to the subject. Most of them are focused on the related fields of inventory information systems improvement, inventory management uncertainties, variable elements in inventory management, logistics activities integration, or co-operation inventory management through supply chain.

2.3.1 Improvement of inventory information systems

Global companies are depending more and more on their IT infrastructure for strategic advantages [Mandal and Gunasekaran, 2002]. But the main challenge in building a credible global IT infrastructure is to manage IT changes at individual locations and integrate those technologies with the enterprise network. Without a proper implementation and integration strategy the enormous benefit of the global IT infrastructure cannot be realized. With the increase of international business, cross-nation inventory actions become more familiar. This leads to the emerging of cross-nation integrated inventory systems. Mandal and Gunasekaran [2002] described how SAP R/3 software came in rescue of a potentially devastating inventory management situation in a large corporation with worldwide operations. They detailed the assessment of the system requirements at various locations, configuration of SAP R/3 as well as the difficulties in implementation and presented an integrated solution for multi-nation enterprises.

Production systems can be categorized as push and pull systems according to the planning strategies. "Push" systems, MRP (Material Requirement Planning) for instance, often have many uncertainties that are not flexible enough in quickly responding to markets changes. It makes more and more researchers focus their further works on "Pull" inventory systems, such as CONWIP (Constant Work-In-Process). Zhang and Chen [2001] presented an integer nonlinear mathematical programming model to determine an optimal production sequence and lot size in a CONWIP production line, which is a good guide for the manufacturing companies that want to implement CONWIP strategy for inventory control.

2.3.2 Inventory management uncertainties

Uncertainties of lead-time, demand rate, delivery time, etc. always exist in inventory control systems. How to estimate them and avoid shortage of inventory is one of the most important focuses in applied research.

In many inventory problems there is an interval of time between the decision to place an order for more stock and the availability of the stock from that order to meet customer demand - this is called the 'lead time'. Many of the inventory models are concerned with how much stock to hold in order to meet a high satisfaction rate of the demand which occurs during this lead-time. Pan and Yang [2002] presented an integrated inventory model with controllable lead-time in their paper for a Just-In-Time production system. Wai [1998] built an inventory model with delivery time guarantees for manufacturing systems while Wu [2001] proposed a mixed inventory model with variable lead-time.

Even in a steady market, the consumer's requirement is not a fixed constant. The demand rates are always uncertain and dynamic. How to keep the supply inventory continuously available under a dynamic market is crucial for the manufacturing companies. Kamath and Pakkala [1999] established a Bayesian approach to dynamic inventory modelling to forecast the unknown demand. Gurler and Parlar [1997] presented a solution by building an inventory system with two random suppliers. Similarly, Janssen and Kok [1999] also built an inventory model with two suppliers, one being comparatively cheap and rigid and the other flexible but expensive. Both of the research works were aimed to guarantee the supply inventory by adding an additional supplier for emergency use.

Other uncertainties of inventory management were also concerned in many published papers. For instance, Downs, *et al* [2001] developed an inventory system to handle with multiple items, resource constraints, lags in delivery and lost sales while Yossi and Awi [2001] presented an inventory model to solve seasonally requirement of multi-items.

2.3.3 Integration of logistics activities

For most enterprises, logistics actions (inventory management, distribution management, transportation management, etc.) are always regarded as a whole integrated system rather than separated actions. In practice, inventory and distribution decisions are often influenced by each other so that they are often considered together. Many researchers have dedicated their work on the integration of inventory management and transportation management. For example, Ahn, *et al* [1994] presented an optimisation mathematical model to minimise the inventory and transportation cost at the parts manufacturer in a JIT production system. Banaszak, *et al* [2000] built an integrated model in a flexible manufacturing system to manage different material flows efficiently.

2.3.4 Co-operation inventory management through supply chain

With more and more companies realising the competitive advantage of co-operation, many spatial integration inventory systems were also developed. Gavirneni [2001] discussed the benefits of co-operation in inventory management for the companies within a supply chain and quantified the benefit due to the co-operation in a typical production distribution environment. Kim, *et al* [2002] built an integrated inventory system for a manufacturing company with multiple suppliers while Woo, *et al* [2001] built an inventory model for single vendor and multi-buyers to optimise the joint total cost for both the vendor and buyers. These pioneer research efforts have extended the overall inventory management from the single vendor and buyer supply chains to more complex supply chains that involve multiple companies for either upstream or downstream.

2.4 Discussions

This chapter has presented an overview of inventory management systems in supply chain management and key research done on various parts in order to find important elements for implementing inventory management systems:

- Inventory information systems: A manufacturing company's competitive advantage depends on the effectiveness of the information flow and subsequent material flow. Better inventory management information systems guide the material flow being transferred more efficiently and accurately.

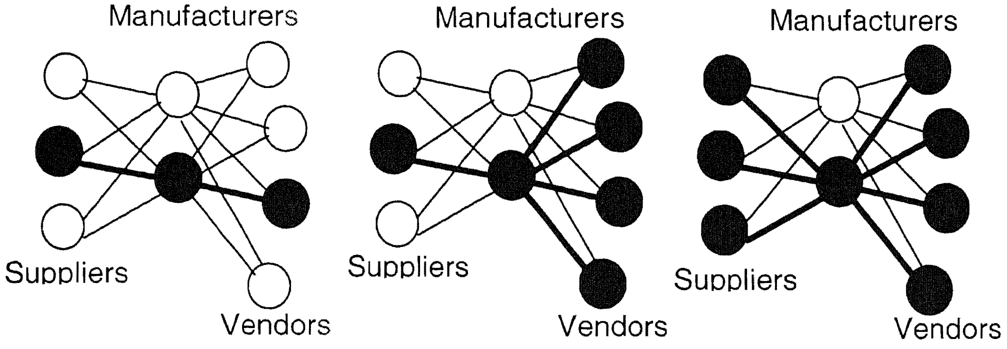
- Uncertainty estimation: With the increasing complexity of supply chain network, uncertainty estimation becomes more important to inventory management. Without uncertainty analysis systems, shortage maybe occur and large extra cost will be brought to the supply chain.

- Strategy integration: The foundation of the integrated logistics management concept is the total cost analysis, which have been defined as optimising the cost of transportation, warehousing, inventory, order processing and information systems. Furthermore, overall enterprise management requires seamless integration of logistics management with other divisions of the enterprise.

- Spatial integration: Supply chain refers to not only functional integration within the company, but also spatial integration within the supply chain. Total inventory planning of the supply chain reduces total inventory costs and gains competitive advantage of logistics.

Although significant progress has been made in the integration of inventory management through different stages of the supply chain, more work is yet to be done. As it is known, the supply chain is often represented as a network similar to the one display in Figure 2-2. The nodes in the network represent companies, which are connected by links that represent direct transportation connections.

The supply chain in the figure has three levels, Suppliers, Manufacturers and Vendors.



(a) Single supplier and single buyer
(b) Single supplier and multi-buyers
(c) Multi- suppliers and multi-buyers

Figure 2-2. Supply chain

The solid nodes and links in Figure 2-2a present a basic integrated inventory system, in which a manufacturer only integrates its inventory with a single supplier and vendor. Woo, *et al* [2001] extended this to a single supplier and multiple vendors, illustrated in Figure 2-2b.

This project extends the model to multiple suppliers and multiple vendors, as shown in Figure 2-2c. It can be seen that integrated multiple suppliers and multiple vendors inventory systems involve almost every individual company in the supply chain, resulting in significant increase in the spatial co-operation between different companies within the supply chain.

Chapter Three

An Integrated Inventory Model for a Manufacturer with Multiple-suppliers and Multiple-buyers

This chapter discusses the inventory behaviours of the supply chain for a single manufacturer with multiple upstream suppliers (factories) and multiple downstream buyers (vendors). An integrated inventory model is built based on a number of assumptions to optimise the total inventory costs of all companies in the supply chain. Then, an optimised total inventory control plan is presented covering the replenishment cycle time, manufacturer inventory level, upstream factories inventory level and replenishment batch quantities for each downstream vendor.

3.1 Introduction

This chapter proposes an integrated inventory system of a manufacturer with multiple suppliers and multiple downstream vendors. The upstream factories make a supply agreement with the manufacturer to deliver intermediate products at a fixed demand rate. The manufacturer produces finished items and delivers them to multiple vendors based on the market information transferred from downstream vendors. The structure of this supply chain can be illustrated by Figure 3-1.

The other companies and their relations, which are not included in this supply chain, are all illustrated by dash lines. The inventory model presents an integrated inventory solution to the supply chain in solid lines without considering of other chains in dash lines.

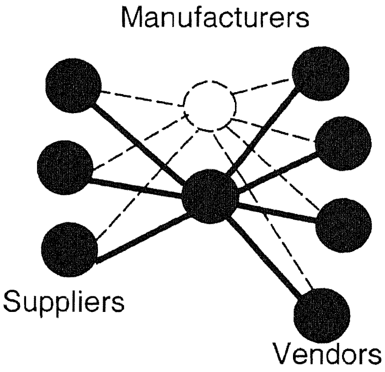


Figure 3-1. The structure of the supply chain

It can be seen that the upstream factories could have other buyers apart from the manufacturer, and downstream buyers maybe have other suppliers. Although, in practice, the relationships among companies are so complex that it is not enough to just consider one single chain; and usually, an optimised solution to one chain does not mean an optimised solution to other chains, which overlap with it for business relations, this integrated inventory model will make allied companies significantly integrate their inventory management and extend the existing model of vendor-buyers to factories-manufacturer-vendors. These kinds of supply chains are ubiquitous in manufacturing industry. For instance, an assembling manufacturer has many part-suppliers, and its finished products are sent to buyers in different locations. Therefore, the integrated inventory model presents a new method, or at least some guide for integrated inventory control in supply chain management.

Two basic inventory models are employed in this research, namely Product Lot Size (PLS) and Economic Order Quantity (EOQ) models. These are briefly described below.

EOQ model: The simplest deterministic inventory model is Economic Order Quantity (EOQ) model. Most of inventory models are developed and extended from EOQ model [Waters, 1992]. Its assumptions include:

- Demand is known with certainty and occurs at a constant rate D unit per unit time.
- Whenever a replenishment order of any quantity (say Q) is placed, a fixed ordering or setup cost S is incurred.
- Each order is delivered immediately; that is, the lead-time is zero.
- Shortages are not allowed.
- The holding cost is a fixed constant h (Pounds per unit per unit time).

Based on such assumptions, the EOQ inventory model presents an economic order quantity $Q^* = (\frac{2SD}{h})^{\frac{1}{2}}$

and optimises the ordering cycle time $C^* = (2SDh)^{\frac{1}{2}}$.

PLS model: A number of simple models have been proposed and studied that extend the EOQ model by combining manufacturing with inventory decisions. One such model is the Product Lot Size (PLS) model [Ding and Zhang, 2000]. Its assumptions include:

- Demand is known as D unit per unit time. Setup cost S is fixed whatever Q is (same with the EOQ assumptions).
- The supplier has a constant production rate P, and $P>D$.
- The optimised ordering size is $Q^* = (\frac{2SD}{h})^{\frac{1}{2}}(\frac{P}{P-D})$.

In this model, it is supposed that the upstream factories have a supply agreement with the manufacturer based on the PLS policy, and that the manufacturer also has a replenishment agreement with downstream vendors based on the EOQ policy. The same assumptions with EQO and PLS will also be made correspondingly when each of the models are related to this model.

3.2 Description of the model

It is assumed that the manufacturer purchases intermediate products (which could be raw materials) from the upstream (factories) and produces finished items output to the downstream (vendors). The procurement action from the upstream is modelled based on the PLS inventory policy because the manufacturing is a continuous action. The replenishment action to the downstream is modelled based on the EOQ inventory policy, which is more economic for vendor-buyer systems. The manufacturer makes all replenishment decisions for intermediate products from upstream and finished items to downstream aims to optimise the joint total inventory cost. All decisions are made based the sharing information from the upstream and downstream, which is interchanged with the EDI (Electronic Data Interchange) order systems. It is expected that the system will result in a lower joint total cost. Other assumptions for the model include the following.

1. Shortages are not allowed for both the manufacturer and its downstream vendors.
2. All the parameters are standardised for the whole supply chain.
3. The delivery cost for the manufacturer and the factories depends on the expenditure incurred per batch to set up replenishment items.
4. The vendors' requirement is roughly fixed, which makes it possible for the manufacturer to make long-term replenishment plan.
5. The transportation time for both upstream and downstream can be neglected comparing with the replenishment cycle time.
6. The capacity of warehouses is sufficient.

The problem for the manufacturer is to determine replenishment cycle time (T) of finished product for vendors, total producing cycle time (C), replenishment batch quantity for vendor *i* (Q_i). All the decisions should be made to minimise joint inventory cost for all the companies throughout the supply chain.

3.2.1 Notations

P_i: Production rates of factory *i* (unit per unit time), which is a known constant for each factory *i*.

D_i: Demand rate of the manufacturer to factory *i* (unit per unit time), which is a known constant to the manufacturer.

t_i: Producing time per replenishment batch of intermediate products of factory *i* for the manufacturer, which is a decision variable and

$$t_i = \frac{D_i C}{P_i} \qquad (3.2.1\text{-}1).$$

H_i: Inventory holding cost (Pound per unit per unit time) of intermediate product of factory *i*, which is a known constant for each factory *i*.

S_i: Setup cost for factory *i*, which is a known constant for each factory *i*.

P: Producing rate of finished item of the manufacturer, which is a known constant.

C: Delivery cycle time of the manufacturer for all vendors, which is also the total producing time of the manufacturer.

D_m: Demand rates of intermediate products of the manufacturer (unit per unit time), which is a known constant.

N: Number of factories, which is a known constant.

M: Number of vendors, which is a known constant.

d_i: Demand rate of finished item of vendor i (per unit per unit time), which is a known constant to each vendor i.

H_{vi}: Inventory holding cost of finished item (Pound per unit per unit time) of vendor i, which is a known constant to each vendor i.

T: Replenishment cycle time of finished products for all the vendors, which is a decision variable.

o_i: Manufacturing time of the manufacturer for vendor i, which is a decision variable and depends on the order quantity.

Q_i: Optimised order quantity of vendor i (replenishment batch quantity for each cycle time), which is a decision variable and related to T.

H_m: Inventory holding cost of finished item of the manufacturer (Pound per unit per unit time), which is a known constant.

S_m: Setup cost of the manufacturer (Pound each time), which is a known constant.

C_T: The integrated inventory cost (Pound per cycle time T) of the supply chain, which is a decision variable.

δ_i: An integer that shows the relationship between the manufacturer's producing rate and demand rate of intermediate product from factory i, which is a known constant and $D_i = \delta_i P$ (3.2.1-2);

C_a: Average integrated inventory cost of the whole supply chain (Pound per unit time), which is a decision variable.

T^*: Optimised replenishment cycle time of finished item, which could make C_a get its minimum value when $T=T^*$.

Q_i^*: Optimised replenishment batch quantity of vendor *i*, which is a decision variable.

C_a^*: Optimised average total inventory cost, which is a decision variable.

The manufacturer orders various kinds of intermediate products from the upstream factories while producing the finished item to satisfy the demands of downstream vendors. The requirement of intermediate products depends on the demands of the finished products from the downstream vendors. The manufacturer should make the replenishment decision for the downstream vendors. To the manufacturer, the downstream vendors are buyers while the upstream factories are suppliers.

3.2.2 Inventory behaviours of the upstream factories

Base on the PLS inventory policy, upstream factories deliver intermediate products continuously to the manufacturer, and their inventory behaviours are all similar to factory *N*.

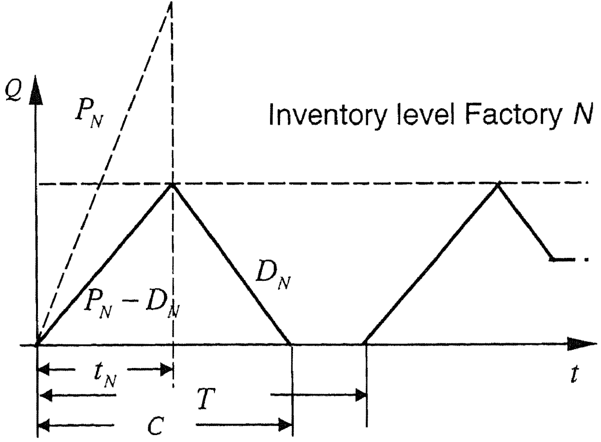


Figure 3-2. Upstream factories inventory behaviours

Factory *N* receives an order from the manufacturer, and begins to produce the intermediate product at time zero. Its inventory behaviour can be illustrated in Figure 3-2.

Factory *N* produces intermediate product at the rate of P_N and continuously delivers it to the manufacturer at the rate of D_N, which is the demand rate of the manufacturer ($P_N > D_N$). At time t_N, Factory *N* finishes the quantity ordered by the manufacturer and at the same time reaches its maximum inventory level.

Its max inventory level should be $(P_N - D_N)t_N$ (3.2.2-1).

Obviously, its average inventory level is $\frac{1}{2}(P_N - D_N)t_N$ (3.2.2-2).

Then its total inventory cost per cycle time should be the average cost multiplies the cycle time *C* and adds its setup cost, i.e.

$$\frac{1}{2}(P_N - D_N)t_N H_N C + S_N \qquad (3.2.2\text{-}3).$$

The inventory behaviours of other factories are similar with factory *N*, and the total inventory cost for all the factories per cycle time should be the sum of these factories' costs, which can be described as

$$\sum_{i=1}^{N}[\frac{1}{2}(P_i - D_i)t_i H_i C + S_i] \qquad (3.2.2\text{-}4).$$

The inventory model of the upstream factories above is based on the assumption that the orders for intermediate products to all the factories are made at the same time. This is due to the situation that the manufacturer requires various kinds of intermediate products from different factories in order to guarantee its production behaviour. The inventory holding costs of these intermediate products are about the same because usually, all the intermediate products are similar for a single type of manufacturer. For instance, a cutter manufacturer which produces various types of milling cutters and lathe cutters maybe require various kinds of steel (e.g. high speed steel and alloy steel), but the inventory holding costs for them are not much different. It is because that the holding cost mostly depends on item size and holding time.

3.2.3 Inventory behaviour from the manufacturer to its vendors

With the market information shared by vendors, the manufacturer decides the downstream vendors' replenishment plan of finished item. Due to it Manufacturing ability limitation, the manufacturer works for only one of the vendors at a time. Thus the time when the replenishment action occurs is different for each vendor.

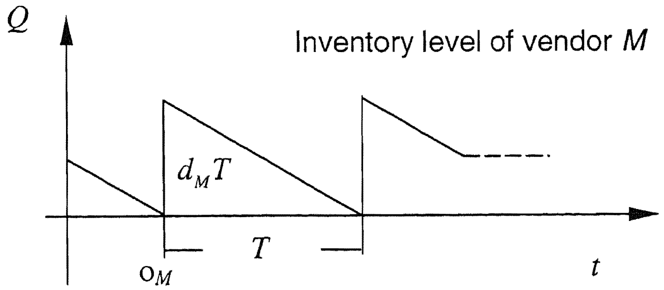


Figure 3-3. Downstream vendors inventory behaviours

But for all vendors, their inventory behaviours within a cycle time *T* are similar. Taking vendor *M*'s inventory behaviour for instance, which is illustrated in Figure 3-3. The manufacturer replenishes the inventory of finish item for Vendor *M* per cycle time T. At time $O_{M,}$ the manufacturer finishes producing the quantity required by vendor *M* for the next cycle time *T* and made a replenishment action. At the same time, vendor *M* reaches its maximum inventory level (assuming a zero lead-time), and the quantity of this maximum inventory level is also its total consumption during the cycle time *T*. Vendor *M*'s demand rate is d_M, and thus the maximum inventory level of vendor *M* should be

$d_M T$ (3.2.3-1).

Its average inventory level should be $\frac{1}{2} d_M T$ (3.2.3-2).

For the holding cost of finished item for vendor *M* per unit per unit time is H_{vM}, the total inventory cost of vendor *M* per cycle time is

$$\frac{1}{2} d_M T^2 H_{vM} \qquad (3.2.3\text{-}3).$$

All the other vendors' inventory behaviours within the cycle time *T* are similar. Therefore, the total inventory cost for all the vendors per cycle time can be described as

$$\sum_{i=1}^{M} \frac{1}{2} d_i T^2 H_{vi} \qquad (3.2.3\text{-}4).$$

It can be seen that all the vendors' total demand for the finished item in the replenishment cycle time *T* is $\sum_{i=1}^{M} d_i T$ (3.2.3-5),

and the manufacturer's producing ability within time *T* is *PT* (3.2.3-6).

If the manufacturer can finish the production for every vendor within time *T*, there is a condition of $P \geq \sum_{i=1}^{M} d_i$ (3.2.3-6).

The distributions of downstream vendors are different from the upstream factories. They are characterised as sporadic and dispersed. Some of them maybe far away from their supplier (the manufacturer) and the transportation cost cannot be ignored when compared with inventory cost. It is not convenient to implement with the PLS policy for transportation difficulties, nor it is economical. That is the reason that most of manufacturer-vendor inventories systems are based on the EOQ policy, with which enterprises can take the advantage of economies of large batch and seldom replenishing actions.

3.2.4 Inventory behaviours of the manufacturer

- *Intermediate product inventory cost from upstream factories*

Based the PLS policy, the upstream factories deliver intermediate products to the manufacturer symmetrically at exactly the demand rate of the manufacturer. This means the manufacturer actually does not hold stock of intermediate items; the upstream factories hold the stock instead of the manufacturer. This is equivalent to the manufacturer distributing its inventory to many sub-inventories of factories. This kind of parts-assemblage structure is vast in the manufacturing industry. Usually the assembling manufacturer chooses nearby suppliers for economical logistics cost. Therefore it is economical and convenient to implement the PLS policy between upstream factories and the manufacturer.

- *Finished item inventory behaviour*

The manufacturer's inventory level of finished product

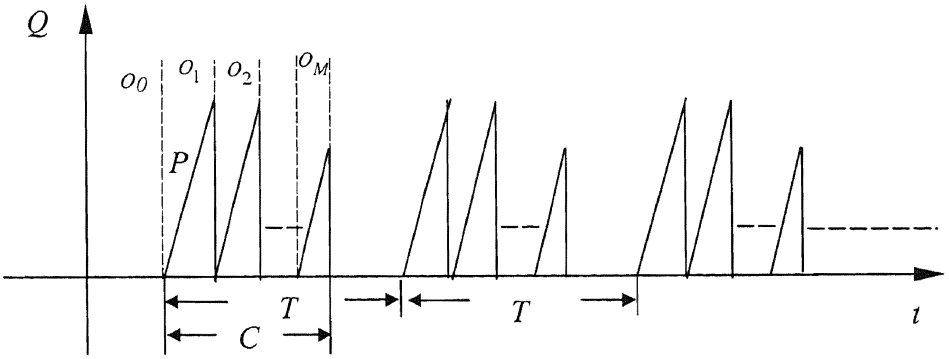


Figure 3-4. Manufacturer's finished item inventory behaviour

As shown in Figure 3-4, the manufacturer begins its production cycle time at o_0. During time period o_0-o_1, the manufacture produces the item required by vendor 1 for replenishment cycle time *T*. At time o_1, replenishment action occurred for vendor 1. The quantity of finished item required by vendor 1 is $Q_1 = d_1 T$ (3.2.4-1), which should also be the finished item inventory level of the manufacturer at time o_1. After time o_1, the manufacturer inventory level becomes zero because all the finished items are delivered to vendor 1.

The producing time of the finished item for vendor 1 should be $o_1 - o_0 = \frac{d_1 T}{P}$ (3.2.4-2).

During time period o_2-o_1, the manufacturer works for vendor 2, the producing time should be $o_2 - o_1 = \frac{d_2 T}{P}$ (3.2.4-3),

and similarly $o_M - o_{M-1} = \frac{d_M T}{P}$ (3.2.4-4).

The total producing time of the manufacturer in cycle time T is obtained by adding producing times for all the vendors together,

$$C = \sum_{i=1}^{M} (o_i - o_{i-1}) = \sum_{i=1}^{M} \frac{d_i T}{P} \qquad (3.2.4\text{-}5).$$

Here the previous equation of $P \geq \sum_{i=1}^{M} d_i$ (3.2.3-6) is used, which means the producing rate is faster than / equal to the total demand rate of all vendors. Form equation (3.2.4-5) and equation (3.2.3-6), it is concluded that

$C \leq T$ (3.2.4-6),

which means the total producing time, is shorter than the replenishment cycle time.

- *The total inventory cost of the manufacturer*

The manufacturer's inventory behaviour has been divided into *M* parts by the producing time periods for different vendors in order to calculate the inventory cost of each part and add them together. The inventory behaviour within time (o_1-o_0) are analysed below as an instance, and others are similar.

The maximum inventory level during o_1-o_0 is

$$(o_1 - o_0)P = \frac{d_1T}{P}P = d_1T \qquad (3.2.4\text{-}7).$$

The average inventory level during o_1-o_0 is $\frac{1}{2}d_1T$ (3.2.4-8).

The inventory cost during o_1-o_0 is

$$\frac{1}{2}d_1TH_m(o_1 - o_0) + S_m = \frac{d_1^2T^2H_m}{2P} + S_m \qquad (3.2.4\text{-}9).$$

Similarly, the inventory cost during $o_i - o_{i-1}$ should be

$$\frac{d_i^2T^2H_m}{2P} + S_m \qquad (3.2.4\text{-}10).$$

Therefore, the total inventory cost of finished product for the manufacturer per cycle time T should be $\sum_{i=1}^{M}\frac{d_i^2T^2H_m}{2P} + MS_m$ (3.2.4-11).

3.2.5 Total inventory cost of the supply chain

After the analysis, three parts of the inventory cost throughout the supply chain have been modelled, which are listed below.

Upstream factories inventory costs of intermediate products in replenishment cycle time *T*, which is

$\sum_{i=1}^{N}[\frac{1}{2}(P_i - D_i)t_i H_i C + S_i]$ (3.2.2-4).

Downstream vendors inventory costs of finished items in replenishment cycle time *T*, which is $\sum_{i=1}^{M}\frac{1}{2}d_i T^2 H_{vi}$ (3.2.3-4).

The manufacturer inventory cost for finished item in replenishment cycle time *T*, which is $\sum_{i=1}^{M}\frac{d_i^2 T^2 H_m}{2P} + MS_m$ (3.2.4-11).

The total inventory cost of the supply chain in cycle time *T* should be the sum of inventory costs from all parts, that is

$$C_T = \sum_{i=1}^{N}[\frac{1}{2}(P_i - D_i)t_i H_i C + S_i] + \sum_{i=1}^{M}\frac{1}{2}d_i T^2 H_{vi} + \sum_{i=1}^{M}\frac{d_i^2 T^2 H_m}{2P} + MS_m \qquad (3.2.5\text{-}1).$$

From previous equations, t_i can be expressed as $t_i = \frac{D_i C}{P_i}$ (3.2.1-1); the relationship between intermediate products demand rate D_i and manufacturer producing rate *P* can be expressed as $D_i = \delta_i P$ (3.2.1-2); and the intermediate products replenishing time *C*, which is also the manufacturer's producing time, can be expressed by $C = \sum_{i=1}^{M}\frac{d_i T}{P}$ (3.2.4-5).

Then if t_i, D_i and *C* in (3.2.5-1) are substituted with (3.2.1-1), (3.2.1-2) and (3.2.4-5), the total cost can be described as:

$$C_T = \sum_{i=1}^{N}[\frac{1}{2}(P_i - \delta_i P)\frac{\delta_i P H_i}{P_i}(\sum_{i=1}^{M}\frac{d_i T}{P})^2 + S_i] + \sum_{i=1}^{M}\frac{1}{2}d_i T^2 H_{vi} + \sum_{i=1}^{M}\frac{d_i^2 T^2 H_m}{2P} + MS_m$$

(3.2.5-2).

In function (3.2.5-2), if *T* is considered as a variable and other notations as parameters, it can be transformed into:

$$C_T(T)=\frac{1}{2}[\frac{\sum_{i=1}^{N}(\delta_i H_i-\frac{\delta_i^2 PH_i}{P_i})(\sum_{i=1}^{M}d_i)^2}{P}+\sum_{i=1}^{M}d_i H_{vi}+\frac{H_m\sum_{i=1}^{M}d_i^2}{P}]T^2+(\sum_{i=1}^{N}S_i+MS_m)$$

(3.2.5-3).

The average total inventory cost of the supply chain per unit time in cycle time *T* should be $C_a=\frac{C_T}{T}$ (3.2.5-4).

Then if C_T in (3.2.5-4) is substituted with equation (3.2.5-3), (3.2.5-4) can easily transformed into:

$$C_a(T)=\frac{1}{2}[\frac{\sum_{i=1}^{N}(\delta_i H_i-\frac{\delta_i^2 PH_i}{P_i})(\sum_{i=1}^{M}d_i)^2}{P}+\sum_{i=1}^{M}d_i H_{vi}+\frac{H_m\sum_{i=1}^{M}d_i^2}{P}]T+\frac{(\sum_{i=1}^{N}S_i+MS_m)}{T}$$

(3.2.5-5).

It can seen that the total inventory cost consists of two parts; one is made of the holding cost and the other is made of the setup cost. The total holding cost is defined as

$$f(T)=\frac{1}{2}[\frac{\sum_{i=1}^{N}(\delta_i H_i-\frac{\delta_i^2 PH_i}{P_i})(\sum_{i=1}^{M}d_i)^2}{P}+\sum_{i=1}^{M}d_i H_{vi}+\frac{H_m\sum_{i=1}^{M}d_i^2}{P}]T$$

(3.2.5-6)

and the total setup cost is defined as

$$g(T) = \frac{(\sum_{i=1}^{N} S_i + MS_m)}{T} \qquad (3.2.5\text{-}7).$$

With a longer replenishment cycle time *T*, which means a longer holding time, the total holding cost is increased. So $f(T)$ is a direct ratio function of *T*. While a longer cycle time *T* means a lower frequency of replenishment action, i.e. a lower frequency of replenishment and set-up, the total set-up cost of the supply chain is decreased. So $g(T)$ is an inverse ratio function of *T*. The relationship between $f(T)$, $g(T)$ and $C_a(T)$ can be illustrated by Figure 3-5.

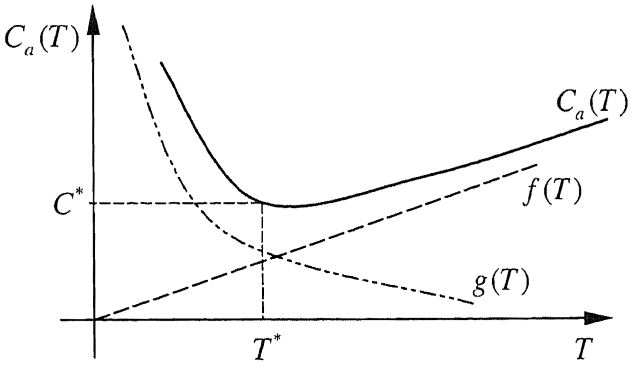


Figure 3-5. $f(T)$, $g(T)$ and $C_a(T)$

As shown in Figure 3-5 $f(T)$ is a straight line, which increases with *T*, while $g(T)$ is a half hyperbola, which decreases with *T*. And they are two approach curves of $C_a(T)$. $C_a(T)$ is a parabola above $f(T)$ and $g(T)$ which decreases in area $(0,T^*)$ and increases in area $(T^*,+\infty)$. Therefore, there must be an optimised replenishment cycle time T^*, which could makes $C_a(T)$ reach its minimum value C_a^*. To get this optimised replenishment cycle time T^*, let the differential of $C_a(T)$ to T be zero, which is

$$\frac{\partial C_a}{\partial T}=\frac{1}{2}\left[\frac{\sum_{i=1}^{N}(\delta_i H_i-\frac{\delta_i^2 PH_i}{P_i})(\sum_{i=1}^{M}d_i)^2}{P}+\sum_{i=1}^{M}d_i H_{vi}+\frac{H_m\sum_{i=1}^{M}d_i^2}{P}\right]-\frac{(\sum_{i=1}^{N}S_i+MS_m)}{T^2}=0$$

(3.2.5-8).

There are two solutions of T to equation (3.2.5-8); one is a plus value and the other is minus. As in practice replenishment cycle time could not be minus, the plus one should be the only optimised replenishment cycle time T^*. The plus solution of equation (3.2.5-9) is

$$T=T^*=\sqrt{\frac{2P(\sum_{i=1}^{N}S_i+MS_m)}{\sum_{i=1}^{N}(\delta_i H_i-\frac{\delta_i^2 PH_i}{P_i})(\sum_{i=1}^{M}d_i)^2+P\sum_{i=1}^{M}d_i H_{vi}+H_m\sum_{i=1}^{M}d_i^2}}$$

(3.2.5-9).

However it cannot be taken as the optimised replenishment cycle time yet. Because, mathematically, T^* is the solution only if it can be proved that there is a minimum solution to equation (3.2.5-5). To prove this, the second differential of $C_a(T)$ to T $\frac{\partial^2 C_a}{\partial T^2}=\frac{2(\sum_{i=1}^{N}S_i+MS_m)}{T^3}$ (3.2.5-10) must have a plus value or zero. Because in (3.2.5-10), set-up cost S_l, S_m and replenishment cycle time T are all positive, (3.2.5-10) must have a plus value.

Thus, it can be concluded that when $T=T^*$, total inventory cost of supply chain $C_a(T)$ reaches its minimum value C_a^* and

$C_a^* = f(T^*) + g(T^*)$ (3.2.5-11).

Correspondingly, the replenishment quantity for downstream vendor i per time should be $Q_i = d_i T^*$ (3.2.5-12).

3.3 Summary

In this chapter a description model for a supply chain with multiple upstream suppliers and multiple downstream buyers has been built to calculate the optimised replenishment cycle time and batch quantity.

Although the integrated inventory control method has been based on some assumptions, many other factors could affect the inventory behaviour of the supply chain in practice. For instance, the delay of transportation will probably cause shortage, so the lead-time is usually taken into consideration. In order to avoid shortage in case of emergency, many inventory modes have safe stocks with a low stock alarm point.

Also, the uncertainty of the demand is an important factor that could influence the accuracy of the optimised replenishment cycle time.

Therefore, the following chapter will discuss the possibility to take these factors into account to make the model more realistic for practical use.

Chapter Four

Extension of the Integrated Inventory Model

4.1 Introduction

The previous chapter has presented an integrated inventory model based on a number of assumptions. Some of them are assumptions of the basic EOQ or PLS models, and others are unique to the integrated model. A few of these assumptions seem a bit unrealistic and could probably make users doubt the practicability of the model. In practice, however, two points should be remembered.

Firstly, all models are simplifications of reality. The main purpose to build these models is to give useful results rather than an exact representation of actual circumstances. The results obtained from these analyses are very widely used, and it can be inferred that the model is accurate enough for many purposes. The results may not be optimal in the strict inventory management sense, but they are good approximations. At least, they give useful guidelines for inventory control management.

Secondly, this is a basic model that can be extended in many ways. In the analysis below attempts will be made to remove or change some of the assumptions to develop more complex models. The aim is to find out how a range of realistic factors can be considered to extend the model. The extensions including:

- Practical considerations for the optimised replenishment quantity.
- Consideration of the lead-time.
- Solution to the uncertainty of demand rate.
- Inclusion of safety stocks.

4.2 The extension

4.2.1 Practical considerations for the optimised order quantity

In the last chapter, the optimised order quantity of finished item for vendor *i* was described as $Q_i = d_i T^*$ (3.2.5-12). This result might be the most economical order quantity mathematically, but there are still several possible reasons that it will not be used in practice sometimes.

- The integrated inventory control model might suggest fractional value for items that come in unitary units, for instance, an order for 3.5 computer systems. It obviously makes no sense and either three or four systems would be bought in practice in spite of the result presented by the integrated inventory model.

- It might not be so convenient for the supplier (the manufacturer) to split standard package size. For instance, an order of 375 cutter tools would be rounded to the nearest 50 (i.e. 350 or 400) because the supplier's standard package size is 50 units per pack.

- Deliveries are made by vehicles with fixed capacities so that 500 cutter tools might fit on to one lorry but the optimised result could be 600 cutter tools. This means that two lorries would be needed and hence the delivery cost would be doubled. This might be economical in terms of inventory, but not economical if the delivery cost is taken into account.

Thus sometimes it is simply more convenient to round order size to a convenient number. An individual company could possibly make a change for its order plan for various kinds of reasons. Although these changes do not always have much difference from the optimised order quantity, it is still important to know how much the rounding will affect the integrated inventory cost. The following analysis will find out the effect of a small change of replenishment batch quantity, which should be around the optimised replenishment batch quantity, on the change of total average inventory cost of the supply chain. It will be also inspected if the effect is small enough to be neglected.

Additional notations are:

c: The change of average total inventory cost corresponding to replenishment batch quantity change.

q: The change of replenishment batch quantity for a single downstream vendor.

α: Change rate of replenishment batch (percent).

k: Change rate of average total inventory cost.

Suppose one of the manufacturer's downstream vendors, which wants to make a small change on its replenishment plan for transportation reasons, has a demand rate of d_e and an optimised order quantity of Q_e (the subscription of e is used to denote the error around the optimised replenishment batch quantity). The objective is to find out how much extra cost the individual change will bring to the whole supply chain.

For the replenishment cycle time $T = \frac{Q_e}{d_e}$ (from 3.2.5-12), and average inventory cost for the whole supply chain is $C_a = f(T) + g(T)$ (3.2.5-11), then it can be easily inferred that $C_a = \frac{1}{d_e}[f(Q_e) + g(Q_e)]$ (4.2.1-1).

As this is an equation between the average total supply chain inventory cost C_a and optimised replenishment batch quantity Q_e, the relationship for the vendor can be illustrated by Figure 4-1.

As shown in Figure, vendor e has optimised replenishment batch quantity of Q_e^* and average total inventory cost of C_a^*.

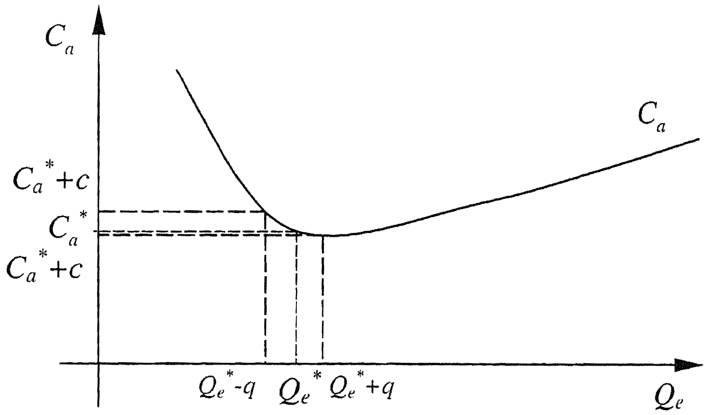


Figure 4-1. Relationship between C_a and Q_e

But for convenient transportation, it wants to slightly increase its replenishment batch quantity around Q_e^*, which is q (q<<Q) where q is the displacement between Q_e^* and Q_e^*+q. Correspondingly, the impact of change of q on the average total supply chain inventory cost could be described as c, which is a displacement between C_a^*+c and C_a^*. The objective is to find the relationship between c and q around Q_e^*.

If c is compared with q, a coefficient (called impact coefficient) will be obtained, which shows the impact rate of replenishment batch quantity on the average total inventory cost to be near the critical point. The coefficient is defined as k and $\frac{c}{q}=\frac{C_a^* k}{Q_e^* \alpha}$ (4.2.1-2).

Here α is a percentage indicate how much percent q takes in Q_e^*, k has the same meaning to C_a^*, which can be described mathematically as $q=Q_e^*\alpha$ and $c=C_a^*k$,

and $k=\frac{Q_e^* \alpha c}{C_a^* q}$ (4.2.1-3).

Obviously, *k* indicates the impact rate of individual change of replenishment batch quantity on the average total inventory cost. The increase of the average total inventory cost can be estimated easily if *k* is known. The average total inventory cost will be increased with the increase of *k*. But if *k* is small enough, which means kC_a^* is also small enough ($kC_a^* << C_a^*$), it can be inferred that the change of individual replenishment batch quantity will not influence the optimised average total inventory cost using this integrated model. With the definition for *c* and *q* above, $\frac{q}{c} = \frac{\alpha Q_e^*}{C_a(Q_e^* + \alpha Q_e^*) - C_a(Q_e^*)} = \frac{\Delta C_a}{\Delta Q_e}$ (4.2.1-4).

For (4.2.1-3) and (4.2.1-4) it can be inferred that

$$k = \frac{C_a(Q_e^* + \alpha Q_e^*) - C_a(Q_e^*)}{C_a^*} \qquad (4.2.1\text{-}5).$$

From equations (4.2.1-1), (3.2.5-6) and (3.2.5-7), the function between C_a and Q_e can be inferred, which is

$$C_a(Q_e) = \frac{1}{2d_e}\left[\frac{\sum_{i=1}^{N}(\delta_i H_i - \frac{\delta_i^2 P H_i}{P_i})(\sum_{i=1}^{M} d_i)^2}{P} + \sum_{i=1}^{M} d_i H_{vi} + \frac{H_m \sum_{i=1}^{M} d_i^2}{P}\right] Q_e + \frac{(\sum_{i=1}^{N} S_i + M S_m) d_e}{Q_e}$$

(4.2.1-6).

Then, the impact coefficient *k* can be estimated from equations (4.2.1-5) and (4.2.1-6).

The impact coefficient *k* shows the increasing rate of the average total inventory cost with the change of vendor *e*'s replenishment batch quantity. But because *k* is complicated relating to so many variables that are not easily measured theoretically, a numerical analysis is carried out below in order to find out how large k could be in practice.

Supposing a manufacturer with five upstream factories and five downstream vendors, the information of the supply chain is given in Tables 4-1 to 4-4.

Table 4-1. Downstream vendors' constants

Vendor	Demand rate for finished item (d_i), unit per month	Holding cost (H_{vi}), Pounds per unit per month
1	867	4
2	580	5
3	813	4
4	662	5
5	996	5

Table 4-2. Upstream factories' constants

Factory	Production rate (P_i), unit per month	Demand rate for intermediate product (D_i), unit per month	Manufacturing parameter (δ_i)	Holding cost (H_i), Pounds per unit per month	Setup cost (S_i), Pounds per time
1	12840	12460	3	2	2379
2	27040	24920	6	2	2219
3	41240	37380	9	2	2188
4	23890	20760	5	2	2338
5	34350	33220	8	2	2182

Table 4-3. Manufacture's constants

Production rate (P_m), unit per month	Holding cost (H_m), Pounds per unit per month	Setup cost (S_m), Pounds per time
4153	4	4000

Table 4-4. Optimised results

Cycle time (T) month	Average total cost (Ca), Pounds per month	Optimised order quantity of vender i, per cycle time (Qi)				
		1	2	3	4	5
1.282	48840	1112	743.57	1042	848.67	1277

The optimised results are calculated from the general constants of a supply chain with five upstream factories and five downstream vendors listed above. But the optimised replenishment batch quantities are not integers for all the five vendors.

For vendor 2, it is absolutely meaningless for an order quantity of 743.57, and it has to be adjusted. The influence of the adjustment can be ignored if *k* is small enough.

Analysing the relationship function between *k* and α,

$$k = \frac{C_a(Q_e^* + \alpha Q_e^*) - C_a(Q_e^*)}{C_a^*} \qquad (4.2.1\text{-}5)$$

using Mathcad with the supply chain data, the following result and graph are obtained.

As it should be, the coefficient *k* is still very small (below 8 percent) even when α (illustrated as *x* in Figure 4-2) is as large as 50 percent. It is because only one single vendor replenishment batch quantity is adjusted in the integrated inventory model.

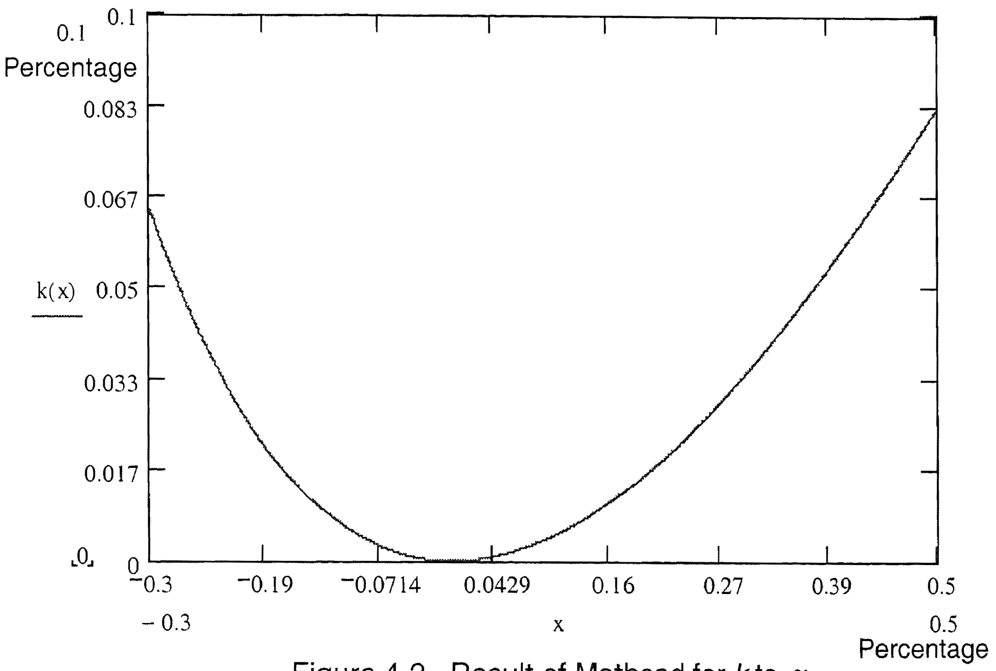


Figure 4-2. Result of Mathcad for *k* to α
(Variable x in the figure has the same meaning with α in the main text)

The case also indicates that making individual change while keeping the integrated inventory plan will lead to similar optimised result and the model will still suggest an economical plan for replenishment.

Figure 4-3 illustrates the increase of average total inventory cost *c* (shown as *g(x)* in Pounds in the figure) and the adjustment rate α (*x* in the figure). Even when an individual's replenishment batch quantity change is as much as 20 percent, the average total cost is increased by no more than 700 Pounds. It can be neglected comparing with the average total cost (i.e. 700 Pounds << 48840 Pounds).

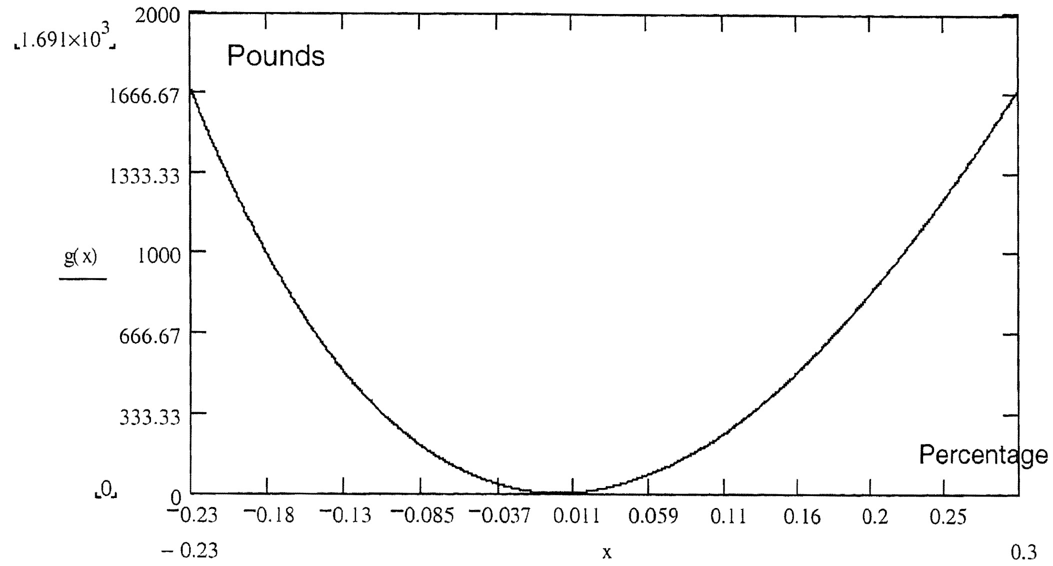


Figure 4-3. Result of Mathcad for total inventory cost
(Variable x in the figure has the same meaning with α in the main text)

The analysis in this section has proved that the integrated inventory model could ignore the impact of slight individual order change. That means vendors' replenishment batch can be easily rounded to an integer number or adjusted to fit transportation convenience without the worry about a large extra cost for the whole supply chain. When the adjustment rate of replenishment batch quantity is below 10 percent, the extra cost in the case study is below 200 Pounds. Although just one vendor's order quantity is adjusted in the case study for simplicity, it can be estimated that the coefficient would not be large enough to be considered even if all the five vendor's replenishment batches are adjusted.

4.2.2 Consideration of lead-time

In many inventory problems there is an interval of time between the decision to place an order for more stock and the availability of the stock from that order to meet customer demand - this is called the “lead-time”. Many of inventory models are concerned with how much stock to hold in order to be able to meet a high satisfaction rate of the demand which occurs during the lead-time. In most of the early literature dealing with inventory problems, lead-time is viewed as a prescribed constant or stochastic variable [Ouyang and Yao, 2002]. This is an assumption of simplification that generally makes the subsequent analysis of the inventory model easier.

So far, the lead-time is assumed zero. It means as soon as a replenishment batch is sent the delivery arrives and is immediately available. In practice, this almost never happens and there are always delays between placing an order and receiving the goods in stock. The lead-time maybe anywhere from a few minutes to several years, and it is typically between a few days and several weeks. If the lead-time is assumed to be a constant, an extension can be made to the previous analysis. The following analysis will add a finite lead-time to the previous model and extend it by taking the lead-time into consideration.

Additional notations are:

LT_i: the replenishment lead-time for vendor i, which is a known constant to each downstream vendor.

Q_{LT}: the inventory quantity that is demanded during lead-time LT_i, which is a known constant and $Q_{LT} = LT_i d_i$.

Figure 4-4 considers the effects of a fix lead-time (LT_M) on the normal inventory behaviours of vendor *M*. Because vendor *M* has a constant demand from the manufacturer and there is no benefit in carrying the stock from one cycle time to the next, each replenishment batch should be timed to arrive just as existing stock is exhausted. To achieve this, the replenishment batch must be sent by a time offset, LT_M before it is needed, as shown in the figure. The optimised order quantity for vendor *M*, which is Q_M^* remains unchanged, but the time when the replenishment batch is sent is brought forward by LT_M.

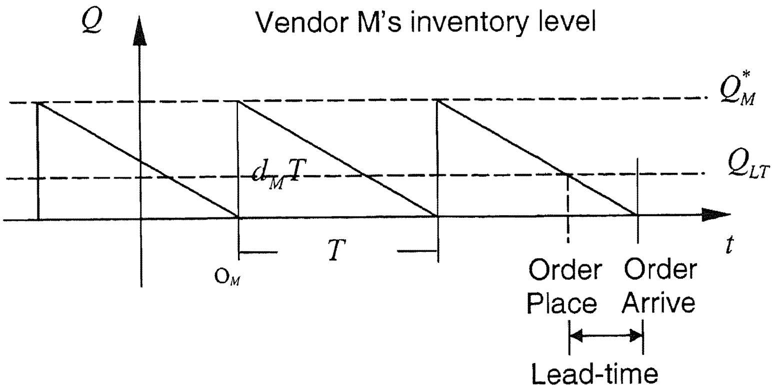


Figure 4-4. Consideration of a finite lead-time to vendor *M*

Due to the assumptions of the model, the stock on hand of a downstream vendor is just enough to cover its demand until the replenishment batch arrives. Because both vendor *M*'s demand and lead-time are assumed constant, the amount of stock needed to cover the lead-time should also be constant, and it could be calculated as Vendor *M*'s lead-time multiplied by Vendor *M*'s demand rate (per unit per unit time).

The manufacturer is the core in the supply chain. It is the manufacturer who makes the integrated inventory decision for the whole supply chain. In fact considering the lead-time consideration for vendor *M* means that the manufacturer must make a replenishment action for vendor *M* LT_M days before its inventory is exhausted. So at the beginning of LT_M days, the manufacturer should start producing the quantity that is required by vendor *M* for the next cycle time *T*.

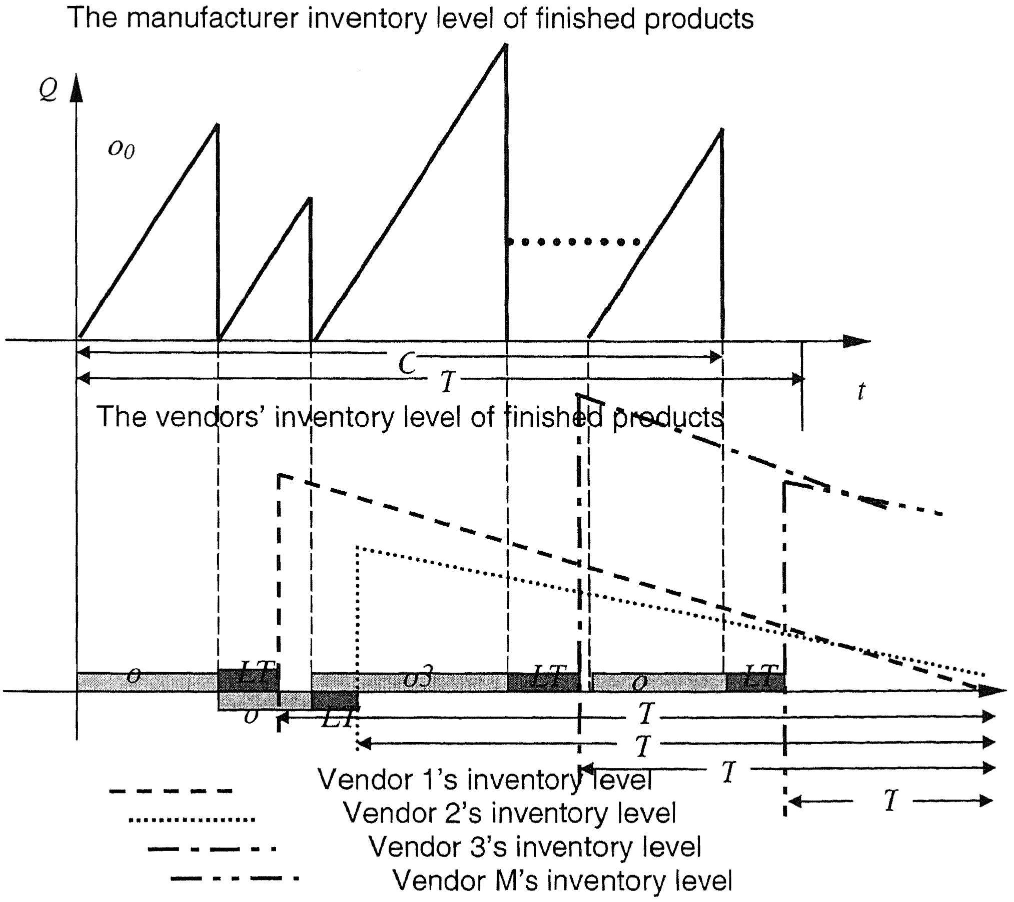


Figure 4-5. Impact of lead-time on the vendors and manufacturer

Figure 4-5 illustrates the regular manufacturing plan is in solid lines (the upper part of the figure). This plan is based on the assumption of zero lead-time as in the previous model. If the manufacturer still keeps the manufacturing plan after the supply chain management takes the lead-time into consideration, the vendors will have to change their inventory plan; otherwise the manufacturer will not be able to finish the replenishment batches for the vendors by the lead-time. In other words, either the manufacturer or the vendor will take the impact of the lead-time. The impact on the manufacturer is to produce ahead of schedule while the impact on the vendors is to delay replenishment batch.

Usually different vendors are located in different places and their lead-times vary because the lead-times are influenced by the delivery time mostly. If the manufacturer's producing plan is changed, multiple lead-times have to be considered within one single production cycle time (the manufacturer production cycle time), which means the manufacturer's production activity may not remain continuous. Furthermore, it will make the model more complex and result in mathematical difficulties. Therefore, adjustments are made to the vendor's inventory plans so that multiple lead-times can be considered in multiple cycle times (vendors cycle times of inventory replenishment).

It can be seen from Figure 4-5 that the time when the manufacturer sends the replenishment batch to vendor 1 is O_1. But the replenishment product will not be available for vendor 1 until $O_1 + LT_1$. For vendor *M* (the last vendor in the list), the time when replenishment batches are ready should be $\sum_{i=1}^{M} o_i + LT_M$ (4.2.2-1),

where *O*

where O_i is the manufacturing time for the replenishment batch of vendor *i*,

$o_i = \frac{d_i T}{P}$ (4.2.2-2).

So equation (4.2.2-1) can be transformed to $\sum_{i=1}^{M} \frac{d_i T}{P} + LT_M$ (4.2.2-3).

By adjusting the downstream vendors inventory plans, the consideration of lead-time in the analysis simply delays the replenishment batches available time for vendors. The inventory behaviours of vendors within a single cycle time *T* remain the same. This is the main reason to adjust the vendor's inventory plan instead of the manufacturer's manufacturing plan. A fixed lead-time is used without changing the mathematical representation of the average total inventory cost of the supply chain. To implement the total inventory management plan, the replenishment batch available time for vendor *i* is a crucial factor. The following numerical analysis will use the information of a supply chain with four upstream suppliers and four downstream vendors. The time when the manufacturer starts production is taken as time zero and the replenishment batch available time for each downstream vendor is calculated.

General information of the supply chain is given in Tables 4-5 to 4-8.

Table 4-5. Downstream vendors' data

Vendor	Demand rate for finished item (d_i), unit per month	Holding cost (H_{vi}), Pounds per unit per month
1	366	12
2	385	10
3	285	12
4	306	12

Table 4-6. Upstream factories' data

Factory	Production rate (P_i), unit per month	Demand rate for intermediate product (D_i), unit per month	Manufacturing parameter (δ_i)	Holding cost (H_i), Pounds per unit per month	Setup cost (S_i), Pounds per time
1	12320	11460	8	6	1091
2	1627	1433	1	7	1148
3	1435	1433	1	6	1119
4	7560	7165	5	6	1147

Table 4-7. Manufacture's data

Production rate (P_m), unit per month	Holding cost (H_m), Pounds per unit per month	Setup cost (S_m), Pounds per time
1433	10	2000

Table 4-8. Optimised results

Cycle time (T), month	Average total cost (C_a), Pounds per month	Optimised order quantity of vender i, per cycle time (Q_i)			
		1	2	3	4
0.985	25390	360.6	379.3	280.8	301.5

Based the above information, fixed lead-times for vendors are taken into account and their replenishment batch ready times are calculated using equation (4.2.2-3) (see Table 4-9).

Table 4-9. Replenishment batch ready times

Vendor	1	1	3	4
Lead-time (*LT*), month	1.5	0.34	0.98	1.4
Replenishment batch ready time, month	1.76	0.85	1.70	2.32

It can be seen that the replenishment batch available time varies from 0.85 to 1.76 (as far as in this case). This is because that the time depends on both producing time and delivery time. The manufacturer has to finish the products for the vendors one by one. Furthermore, the vendors have different locations (different cities and maybe different countries), which means the delivery time for each vendor can be very different. So the replenishment batch ready time for each vendor varies in such a large range of 200 percent.

4.2.3 Uncertainty in downstream vendors demand

One of the assumptions in the previous model is that all demand rates of downstream vendors' are fixed. This assumption is a bit unrealistic because consumption is dynamic in real-life. Even in a steady market, the consumer's requirement is not a fixed constant. But in practice the manufacturers usually can forecast the demand for finished products under the assumption that the demand is related to price, advertising, promotion, and other factors. Usually analysts build forecast models for a supply chain to provide necessary information instead of taking the demand as a fixed constant.

Based on a combination of historical data, managerial judgement, and modelling practitioner expertise, forecast models create demand data for multiple companies in different locations over multiple periods of time [Kamath and Pakkala, 2002]. Although the demand rate of downstream vendors may not be fixed constants in the case discussed in the previous chapter, it can be still assumed that the manufacturer can forecast the demand by implementing a forecast model in the supply chain. However, in constructing and employing forecasting models, it must be kept in mind that good forecast models will still have significant errors. The future will always remain uncertain and the further into the future the model looks, the larger the errors. Better forecasts will, of course, have small errors, but the forecast will never be perfect. Appropriate frequent analysis with demand uncertainty and optimisation in the model will allow management to take actions to correct the mistake due to forecasting errors.

Additional notations are:

E: Proportional error of the forecast model, which can be estimated and assumed to be a constant by the manufacturer.

C_{real}: Optimised average total inventory cost obtained from the previous model, which is not influenced by the demand forecast model error.

C_{error}: Optimised average total inventory cost obtained from the model with demand uncertainty, which is influenced by the demand forecast model error.

α: A coefficient indicating the change of optimised total inventory cost.

Instead of the unrealistic assumption of fixed downstream demand rates, it is now assumed that the manufacturer uses the information provided by its forecasting model and adjusts the replenishment plan for vendors timely according to the forecast demand change. But in fact this is an approximation with some error. Suppose, for example, the demand rates are forecasted and contain proportional error, *E*, so that an actual demand of *d_i* is forecasted to be $d_i * (1+E)$ (4.2.3-1).

Then actually the optimised integrated inventory model should be

$$C_a(T)=\frac{1}{2}\left[\frac{\sum_{i=1}^{N}(\delta_i H_i-\frac{\delta_i^2 PH_i}{P_i})(\sum_{i=1}^{M}d_i(E+1))^2}{P}+\sum_{i=1}^{M}d_i(E+1)H_{vi}+\frac{H_m\sum_{i=1}^{M}d_i(E+1)^2}{P}\right]T+\frac{(\sum_{i=1}^{N}S_i+MS_m)}{T}$$

(4.2.3-2).

Because a different model is used that contains a demand-forecasting error, the errors caused by demand forecasting also bring an error to the optimised result of the average total inventory cost. If the manufacturer can get the real demand rate information of each downstream vendor before making the replenishment plan (which is absolutely impossible), no errors need to be considered in the mathematical model and the model should remain as

$$C_a(T)=\frac{1}{2}\left[\frac{\sum_{i=1}^{N}(\delta_i H_i-\frac{\delta_i^2 PH_i}{P_i})(\sum_{i=1}^{M}d_i)^2}{P}+\sum_{i=1}^{M}d_i H_{vi}+\frac{H_m\sum_{i=1}^{M}d_i^2}{P}\right]T+\frac{(\sum_{i=1}^{N}S_i+MS_m)}{T}$$

(3.2.5-5).

If the optimised result obtained by equation (3.2.5-5) is noted as C_{real} and the optimised result obtained by equation (4.2.3-2) as C_{error}, the resulting error in the average total inventory cost can be described as

$$\frac{C_{error}}{C_{real}} \qquad (4.2.3\text{-}3).$$

Because (4.2.3-3) is complicated relating to many variables, to calculate methodically a case study will be used to find out the result error for the sake of simplicity.

Supposing a supply chain with a manufacturer, four upstream factories and four downstream vendors, the information is given in Tables 4-10 to 4-13.

Table 4-10. Downstream vendors data

Vendor	Demand rate for finished item (d_i), unit per month	Holding cost (H_{vi}), Pounds per unit per month
1	500	11
2	452	14
3	354	11
4	413	13

Table 4-11. Upstream factories data

Factory	Production rate (P_i), unit per month	Demand rate for intermediate product (D_i), unit per month	Manufacturing parameter (δ_i)	Holding cost (H_i), Pounds per unit per month	Setup cost (S_i), Pounds per time
1	1929	1770	1	7	1172
2	3580	3540	2	7	1155
3	16380	14160	8	6	1199
4	1954	1770	1	6	1122

Table 4-12. Manufacturer data

Production rate (P_m), unit per month	Holding cost (H_m), Pounds per unit per month	Setup cost (S_m), Pounds per time
1770	10	2000

Table 4-13. Optimised results

Cycle time (T), month	Average total cost (C_a), Pounds per month	Optimised order quantity of vender i per cycle time (Q_i)			
		1	2	3	4
0.812	31150	406	367	287	335

Supposing the manufacturer in this supply chain uses a forecasted demand rate instead of the real demand rate to make the replenishment plan using the integrated model, in order to find out the impact of the forecasting error on the average total inventory cost, the total inventory costs should be calculated using different models (i.e. (3.2.5-5) without error and (4.2.3-2) with error) with the same information given above and the results should be compared.

Similarly, an impact coefficient is defined as $\alpha = \frac{C_{error} - C_{real}}{C_{real}}$ (4.2.3-4).

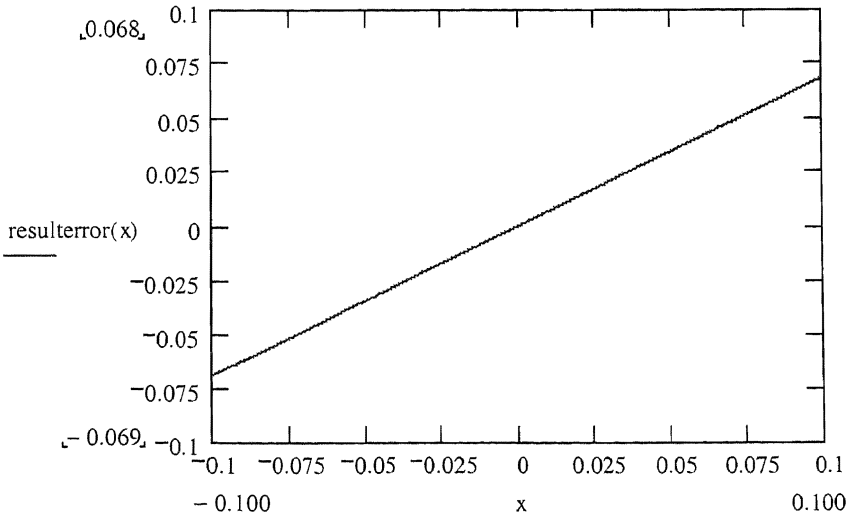


Figure 4-6. Relationship between E and α

The impact coefficient α is related to forecast error E. In this case, the forecast errors for different vendors are assumed to be the same value E for the analysis of the impact of E on α. After analysis with Mathcad, a graph of α versus E is obtained (Figure 4-6).

In this case, E has been limited to the range of (-0.1,0.1) because the forecasting error should not be larger than 10 percent, e.g. if the demand rate is 500 units per month for a vendor, the forecast demand rate should be between 450 and 550. Actually, in most cases the errors of demand forecasting system are around 5 percent, or otherwise it is not efficient and improvement is needed.

Figure 4-6 shows the change of total cost for various errors in forecast. Again it is clear that large errors in forecast demand rate (10 percent) leads to relatively small movements away from the minimum value of the optimised result (less than 7 percent). But it is still larger compared with the impact of change in replenishment batch quantity. Thus the accuracy of the supply chain data becomes more important. That is also the reason why many researchers are focusing on electronic data interchange systems and demand forecasting systems.

4.2.4 Inclusion of safety stocks

If the future demand was known precisely, far enough ahead to cover all the lead-time concerned, or if the forecasting technique were perfect, and if all suppliers could meet their lead-times consistently, there would be no need for safety stock at all. An appropriate system could be selected from among those described above, and there would be no problem of stock control.

But these assumptions are unrealistic, and safety stock is still needed in most cases. In reality, the future demand is not known, forecast is far from perfect, and suppliers (including the company's own machine shop) sometimes fail to deliver the goods on time. There must be some protection against the two special circumstances (as far as stock control is concerned). One is higher rate of usage than forecasted and another is late delivery. Either of these, in the absence of a safety stock of some kind, can lead to stock out, with more or less serious consequences depending on the particular item concerned. This is the function of safety stocks - a form of insurance premium against the danger of a stock out.

With almost any systems, if the safety stock is zero, there is just about a 50 percent chance of a stock out before each replenishment batch reaches. That is because each replenishment action has been carefully calculated that a new batch should be received just as the stock exhausted, just as what is done in the previous analysis. But this only happens if the usage and lead-time are exactly correct.

A comparatively small amount of money invested in safety stock, say 10 percent of the quantity of the average working inventory, will reduce the danger of a stock out significantly, perhaps half of the risk. It requires considerably more safety stock to reduce this to 5 percent or lower. But of course more safety stock will cost more money. Actually there is a balance between safety stock levels and shortage risk.

This section will devote to a discussion of how to add a safety stock consideration to assess risks of shortage, and to decide how much money to invest in safety stocks, for various combination of forecasting technique, inventory control system, and other circumstances.

As indicated above, shortage is often caused by higher usage and late delivery. Higher usage happens because there is an error in the demand forecast while late delivery means a variation in lead-time. Thus, the most two crucial factors, which influence the safety stock level, are the error of forecast and error of lead-time. The impact of each factor will be analysed separately and the two factors will be combined to decide safety stock costs.

Additional notations include:

SS_f : Safety stock for forecast errors, which is a decision variable.

SS_l : Safety stock for lead-time variation, which is a decision variable.

l: Variation of lead-time for vendor i, which can be estimated as a constant.

SS : Combine safety stock for both forecast errors and lead-time variation.

CS: Cost of combined safety stock.

The last section has described demand forecasting errors. By tracking the demand forecasting errors, it is possible to determine the error in replenishment batch quantity, either overstock or shortage. But only the shortage is discussed here, which means *E* has a negative value in the demand-forecast model in Section 4.2.3.

Under this circumstance, the vendor has a risk of shortage because the replenishment batch delivered by the manufacturer is not enough to cover the demand of the vendor during the next cycle time *T*. This can be described mathematically as $d_i(E+1)T < d_iT$ (4.2.4-1).

Obviously, without safety stock of some kind, the shortage will be $|E\ d_i|$ at the end of the next replenishment cycle time. So in order to avoid shortage due to errors in demand forecast, a safety stock needs to be added, which is $SS_f = |E \cdot d_i|T$ (4.2.4-2).

This can be illustrated by Figure 4-7.

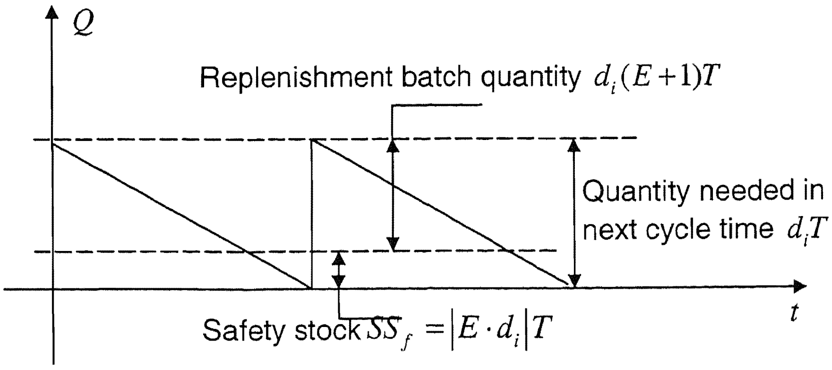


Figure 4-7. Safety stock for forecast error

If every forecast system has an error and cannot guarantee a one hundred percent accuracy, no delivery system can be so accurate that lead-time errors can be ignored.

Normally, the lead-time is more like a time period such as 2 to 3 weeks rather than a fixed constant such as 2.5 week. Furthermore, the lead-time can also be delayed by various kinds of reasons. But in most inventory models, the lead-time is assumed as a fixed constant for simplicity. This assumption will probably cause shortage in practice.

Supposing one of the downstream vendors has a lead-time of LT_i and a variable uncertain time around the lead-time, l_i, the real replenishment batch available time should be $LT_i + l_i$ instead of LT_i. But in the previous analysis, the lead-time is described as a known constant LT_i, which will cause shortage without safety stock guarantee when $l_i > 0$.

Another safety stock can be added for the vendors to avoid shortage caused by the lead-time variation. This safety stock should cover the demand during time l_i. Thus it should be $SS_l = l_i \cdot d_i$ (4.2.4-3).

The safety stock for the lead-time uncertainty can be illustrated by Figure 4-8.

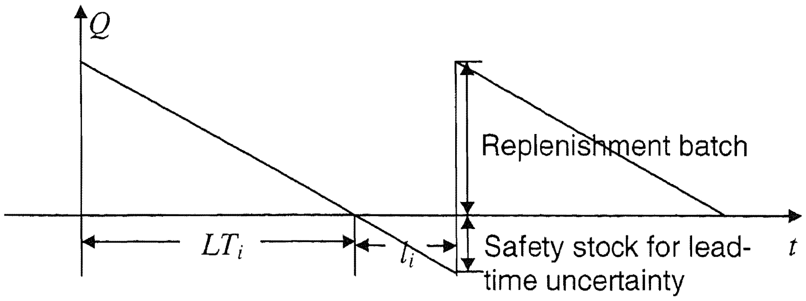


Figure 4-8. Safety stock for lead-time uncertainty

In real supply chain management, there often exists both delay of lead-time and errors in demand forecasting at the same time. To avoid shortage caused by both of them, the safety stocks for forecast error and lead-time variation needs to be combined as

$$SS = SS_f + SS_l = d_i l_i + Ed_i T (E < 0) \qquad (4.2.4\text{-}4).$$

Then the cost of safety stock for vendor *i* is

$$CS_i = (d_i l_i - Ed_i T) H_{vi} T \qquad (4.2.4\text{-}5).$$

The total safety stock cost for all the downstream vendors is

$$CS = \sum_{i=1}^{M} (d_i l_i + Ed_i T) H_{vi} T \qquad (4.2.4\text{-}6).$$

The average total safety stock cost for the whole supply chain is

$$CS_a = \sum_{i=1}^{M} (d_i l_i + Ed_i T) H_{vi} \qquad (4.2.4\text{-}7).$$

Therefore, the average total inventory cost should be the regular inventory cost (3.2.5-5) plus safety stock cost (4.2.4-7). The precious average total inventory cost model (3.2.5-5) can be extended as

$$C_a(T)=\frac{1}{2}[\frac{\sum_{i=1}^{N}(\delta_i H_i-\frac{\delta_i^2 PH_i}{P_i})(\sum_{i=1}^{M}d_i)^2}{P}+\sum_{i=1}^{M}d_i H_{vi}+\frac{H_m\sum_{i=1}^{M}d_i^2}{P}]T+\frac{(\sum_{i=1}^{N}S_i+MS_m)}{T}+\sum_{i=1}^{M}(d_i l_i+Ed_i T)H_{vi}$$

(4.2.4-8).

And it can be transformed as

$$C_a(T)=\frac{1}{2}[\frac{\sum_{i=1}^{N}(\delta_i H_i-\frac{\delta_i^2 PH_i}{P_i})(\sum_{i=1}^{M}d_i)^2}{P}+\sum_{i=1}^{M}d_i H_{vi}(1+E)+\frac{H_m\sum_{i=1}^{M}d_i^2}{P}]T+\frac{(\sum_{i=1}^{N}S_i+MS_m)}{T}+\sum_{i=1}^{M}d_i l_i H_{vi}$$

(4.2.4-9).

Similar to the analysis in Section 3.2.5, the optimised replenishment cycle time T^*, can be calculated, which can make $C_a(T)$ reach its minimum value, by letting the differential of $C_a(T)$ to T be zero.

$$T^*=\sqrt{\frac{2P(\sum_{i=1}^{N}S_i+MS_m)}{\sum_{i=1}^{N}(\delta_i H_i-\frac{\delta_i^2 PH_i}{P_i})(\sum_{i=1}^{M}d_i)^2+P\sum_{i=1}^{M}d_i H_{vi}(1+E)+H_m\sum_{i=1}^{M}d_i^2}}$$

(4.2.4-10).

The optimised average total inventory cost should be

$C_a^*=C_a(T^*)$ (4.2.4-11).

In the analysis below, an investigation on the information of a supply chain, which has three upstream factories, a manufacturer and five downstream vendors, will be implemented, as an instance to find out how much extra cost the safety stock will bring to the supply chain in practice.

The general information of the supply chain is given in Tables 4-14 to 4-17.

Table 4-14. Downstream vendors' data

Vendor	Demand rate for finished item (d_i), unit per month	Holding cost (H_{vi}), Pounds per unit per month
1	577	14
2	772	13
3	509	11
4	631	14
5	902	11

Table 4-15. Upstream factories' data

Factory	Production rate (P_i), unit per month	Demand rate for intermediate product (D_i), unit per month	Manufacturing parameter (δ_i)	Holding cost (H_i), Pounds per unit per month	Setup cost (S_i), Pounds per time
1	15260	14140	4	5	2250
2	15810	14140	4	5	2019
3	12620	10600	3	6	2104

Table 4-16. Manufacture's data

Production rate (P_m), unit per month	Holding cost (H_m), Pounds per unit per month	Setup cost (S_m), Pounds per time
3534	10	4000

Table 4-17. Optimised results

Cycle time (T), month	Average total cost (C_a), Pounds per month	Optimised order quantity of vender i per cycle time (Q_i)				
		1	2	3	4	5
0.886	60880	500	669	441	547	781

The error of the manufacturer demand forecasting system can be estimated as $E=-5\%$ and the lead-time variation for each vendor is list in Table 4-18.

Table 4-18. Optimised results

Vendor	1	2	3	4	5
Lead-time variation	0.175	0.398	0.035	0.295	0.099

If model (4.2.4-9) is used, the average total inventory cost with the consideration of safety stock can be calculated, which is $C_a^* = 69980$ Pounds. Comparing to the original average total inventory cost (60880 Pounds), it is increased by almost 15%. But in practice, it is necessary to spend the extra 15% of the total inventory cost to avoid shortage, especially when shortage cost is much higher than the inventory cost.

4.3 Summary

In this chapter, the integrated inventory model has been extended in several ways by changing some unrealistic assumptions made in the last chapter.

- Moving away from the optimised replenishment batch quantity. This has made the model more flexible for integrated inventory planning. The analysis indicates that a slight individual adjustment will not impact the optimised cost to cause a concern, which means downstream vendors can easily adjust the replenishment batch quantity for convenient delivery or other special requirements without worrying about a large extra cost.

- Consideration of finite lead-times. It is impossible for a replenishment batch to be immediately available as soon as the batch is sent. Therefore a lead-time has been added to the previous model, which is assumed as a constant. An inventory plan with the lead-time considered has been presented for downstream vendors.

- Demand uncertainty. Instead of the unrealistic assumption of fixed demand rates, the model assumes the demand information is forecasted with an error. Extra cost of forecast error is estimated and added to the previous model. Analysis indicates that large errors in forecast demand rate will lead to relatively small movements away from the minimum cost.

- Safety stock. Either finite lead-times or fixed demand rates are simplifications of the real circumstance. There are always changes and uncertainties, and therefore safety stock has been considered for both demand uncertainty and lead-time variation. Although numerical analysis indicates that the cost of safety stock can be almost as large as 10 percent of the total inventory cost, it is still necessary for most companies because it avoids larger cost of shortage.

Chapter Five

Comparison and Numerical Analysis

In this chapter, a random data generating system will be presented to provide random supply chain information. Then, three case studies will be carried out using the integrated inventory model proposed in the research. Finally, analysis will be focused on the relationship between the optimised results and the complexity of the supply chain (the number of individuals in the supply chain).

5.1 Random Data Generating System (RDG)

5.1.1 Introduction

In order to analyse the property and the accuracy of a mathematical model, a large amount of data is usually needed to do numerical analysis. Lack of data could possibly lead to inaccurate and less comprehensive results. Supply chain models are characterised by multiple companies, complex data structure and large number of variables, which means the models are not only complex of business structure, but also complicated mathematically. Therefore, to analyse and prove the integrated inventory model, data collection is a crucial part. However, there are some difficulties to collect sufficient data suitable for this research work.

- Variety of supply chain structure. There are many different structures of supply chain in practice, and even the same supply chain may be looked from different points of views for various reasons. The integrated model intends to present a general solution to a kind of supply chain rather than a case study. Therefore, the analysis should be comprehensive enough, which means a large quantity of data is needed from a wide range of industry. The data collection is both difficult and time consuming.

- Complexity of the supply chain information. As known, the standard of data representation is still a new technique that needs improving in supply chain management [Ballou, 2001]. Different enterprises in industry are still describing their systems using different parameters. This mathematical model should investigate the companies that describe their inventory behaviours using the same parameters; otherwise, the data is meaningless for the integrated inventory model.

Since it is hardly possible to collect sufficient data suitable for the integrated model, a random data generating system (RDG) is built. The random data generating system should have the following functions.

- Automatic generation of supply chain inventory information for upstream factories, the manufacturer and downstream vendors.

- Adjustable generating scale. The upstream factories in the same industry field should have similar production abilities. Different downstream vendors have different demand rates, but the demand rates should be in the same scale. In practice, it is hardly possible that one vendor has a demand rate of 100 units per month while the other has a demand rate of 10000. The RDG system should allow users to set basic values, and generates all the data around the basic values.

- Changeable quantities of upstream factories and downstream vendors. The integrated model presents an approach to a supply chain with multiple upstream factories and downstream vendors. In order to analyse the model numerically; the numbers of factories and vendors should be changeable.

- Integration with the analysis system. For convenient use, the RDG system should be integrated with the numerical analysis systems. Optimised results can be obtained once the supply chain data is generated.

- Extendibility. The analysis needs not only basic data used in the basic mathematical model described in Chapter 3, but also data for the extended model such as lead-time and safety stock. Furthermore, the integrated inventory model could possibly be extended for further research in various ways. It should be possible to extend the RDG system for more complex analysis correspondingly.

5.1.2 Implementation

Basic values which are set by users include

N: Number of upstream factories, indicating for how many factories, the data should be generated.

M: Number of downstream vendors, indicating for how many vendors, the data should be generated.

d_0: Basic demand rate; other demand rates will be generated from d_0.

H_0: Basic intermediate holding cost; other holding costs will be generated from H_0.

HV_0: Basic finished item holding cost. Usually the holding cost of finished item is higher than intermediate products. Therefore, in the RDG system, the basic holding cost of finished items is defined as βH_0, where β is an adjustable constant for different circumstances.

S_0: Basic set-up cost; other set-up costs will be generated from S_0.

Random generation of extension data:

D_j: Demand rate of downstream vendors; each demand rate is generated as the basic demand rate plus a random value.

δ_i: Manufacturing coefficient, which is a integer generated by a random function.

PM: Manufacturer producing rate, generated as the sum of all the demand rates plus a random value. This is because in real work, the producing rate should be larger than / equal to the total demand rate.

P_i: Producing rates of upstream factories. In practice, the manufacturer's demand rates for intermediate products depend on its producing rate and manufacturing coefficient. For instance, if a car manufacturer has a producing rate of 1000 cars per month, then its demand rate for wheels should be 4*1000=4000 per month, where the manufacturing coefficient for wheels is 4. In the RDG system, each vendor's producing rate is generated as the manufacturer producing rate multiplied by its manufacturing coefficient plus a random value because the producing rate should be larger than / equal to the demand rate.

D_i: Demand rate of intermediate products from vendor i, generated as the manufacturer producing rate multiplied by its manufacturing coefficient.

H_i: Intermediate product holding cost of upstream factory i, generated as the basic holding cost plus a random value.

S_i: Set-up cost of upstream factory i, generated as the basic set-up cost plus a random value.

HV

HV_j: Finished item holding cost of downstream vendor *i*, generated as the basic finish item holding cost plus a random value.

HM: Finished item holding cost of the manufacturer, generated as the basic finished item holding cost plus a random value.

SM: Manufacturer setup cost, generated as the basic set-up cost plus a random value.

The structure of the RDG system is illustrated in Figure 5-1.

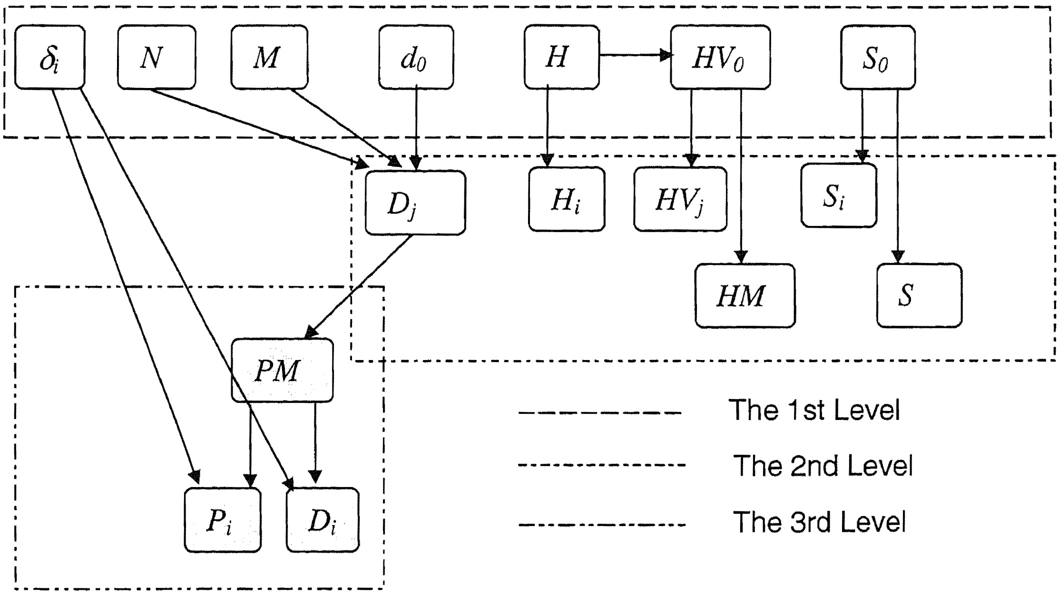


Figure 5-1. Structure of the RDG system

The parameters of the first level in Figure 5-1 are basic parameters, which should be pre-set by the user. The second level parameters are generated based on the first level parameters, and the third level parameters are generated based on the first level and the second level parameters.

5.2 Comparison

Although, in practice, much improvement has been achieved to optimise inventory cost, there is still room for further improvement in the supply chain inventory planning. One of the most important motivations to build an integrated inventory model is to integrate multiple companies' inventory management. The benefit of a lower total cost is expected so that the supply chain gains the competitive advantage in logistics management.

The possibilities of integrating inventory activities through supply chain management have been discussed using various methods in published papers. Complex mathematical models have been presented instead of classic inventory models. Most of them are dedicated to more complex or accurate approximation of real business circumstances. The integrated model in this research, which is built based on classic inventory models, has been focused on the improvement of overall inventory management. It is beyond the concern of optimised replenishment batch quantity and cycle time for single inventory behaviour.

However, the classic inventory models have been used and proved as an economical solution to inventory management for many years. They are still efficient in some simple circumstances, e.g. when there is only one supplier and buyer in the supply chain. But as the complexity of the supply chain structure increases, the analysis for simple chains is far from enough. Therefore, this integrated inventory model has been built for complicated supply chains.

This section investigates three different inventory models: classic EOQ model, PLS model and mixed model. The optimised results of these models will be compared with the integrated model proposed in this research. All the results are optimised inventory cost of a same supply chain. The improvement of the integrated model will be demonstrated for inventory control of complex supply chains.

5.2.1 The average inventory cost of basic models

Single EOQ model. In the single EOQ model analysis, it is assumed that the EOQ policy is employed for inventory planning for both intermediate products and finished items through the supply chain. The manufacturer orders intermediate products from every individual upstream factory separately according to the EOQ policy. Each downstream vendor orders finished items from the manufacturer separately. Every individual company in the supply chain plans its inventory activity without the consideration of other related companies.

According to the EOQ inventory model, the economic order quantity of intermediate products for the manufacturer from upstream factory i should be

$$Qemf_i^* = \left(\frac{2S_iD_i}{H_i}\right)^{\frac{1}{2}} \qquad (5.2.1\text{-}1).$$

The manufacturer's average inventory cost for intermediate products from factory *i* should be $Cemf_i^* = \left(\frac{S_iD_iH_i}{2}\right)^{\frac{1}{2}}$ (5.2.1-2).

The average inventory level of factory *i* should be the same as the average inventory level of the manufacturer because they have the same maximum inventory level. Therefore, the average inventory cost of factory *i* should also be

$$Cef_i^* = \left(\frac{S_iD_iH_i}{2}\right)^{\frac{1}{2}} \qquad (5.2.1\text{-}3)$$

So, the total average inventory cost of factories-manufacturer level is

$$Ceu = Cef_i^* + Cemf_i^* = \sum_{i=1}^{N} (2S_iD_iH_i)^{\frac{1}{2}} \qquad (5.2.1\text{-}4).$$

The finished item inventory control is also based on the EOQ policy. Similarly, for downstream vendor *i*, the economic order quantity should be

$$Qev_i^* = \left(\frac{2S_md_i}{H_{vi}}\right)^{\frac{1}{2}} \qquad (5.2.1\text{-}5).$$

The average inventory cost of vendor *i* should be

$$Cev_i^* = \left(\frac{S_md_iH_{vi}}{2}\right)^{\frac{1}{2}} \qquad (5.2.1\text{-}6).$$

The average inventory cost of the manufacturer should be

$$Cemv_i^* = \left(\frac{S_md_iH_m^2}{2H_{vi}}\right)^{\frac{1}{2}} \qquad (5.2.1\text{-}7)$$

The total average inventory cost for manufacturer-vendors level can be inferred as $Ced = \sum_{i=1}^{M} ((\frac{S_m d_i H_{vi}}{2})^{\frac{1}{2}} + (\frac{S_m d_i H_m^2}{2H_{vi}})^{\frac{1}{2}})$ (5.2.1-8).

The total average inventory cost should be the summation of all inventory costs, which is

$$Ce = \sum_{i=1}^{M} ((\frac{S_m d_i H_{vi}}{2})^{\frac{1}{2}} + (\frac{S_m d_i H_m^2}{2H_{vi}})^{\frac{1}{2}}) + \sum_{i=1}^{N} (2S_i D_i H_i)^{\frac{1}{2}} \quad (5.2.1\text{-}9).$$

Single PLS model. In the single PLS model analysis, it is assumed that the PLS policy is employed for inventory planning for both intermediate products and finished items through the supply chain. The manufacturer orders intermediate products from every individual upstream factory separately according to the PLS policy. Each downstream vendor orders finished items from the manufacturer separately. Every individual company in the supply chain plans its inventory activity without the consideration of other related companies.

According to the PLS policy, the optimised replenishment cycle time from facotry *i* should be $Tpu^* = (\frac{2S_i}{H_i D_i})^{\frac{1}{2}} (\frac{P_i}{P_i - D_i})^{\frac{1}{2}}$ (5.2.1-10).

Thus, the manufacturer's maximum inventory level of intermediate products from factory *i* is $Qpu^* = (\frac{2S_i D_i}{H_i})^{\frac{1}{2}} (\frac{P_i}{P_i - D_i})^{\frac{1}{2}}$ (5.2.1-11).

Then the manufacturer's average inventory cost for the item from upstream factory *i* can be inferred as

$$Cpm_i^* = (\frac{S_i H_i D_i}{2})^{\frac{1}{2}} (\frac{P_i}{P_i - D_i})^{\frac{1}{2}} \qquad (5.2.1\text{-}12).$$

Because factory *i* transports the intermediate products continuously with a production lot-size to the manufacturer, factory *i* has no inventory cost. The total average inventory cost of factories-manufacturer level should be

$$Cpu = \sum_{i=1}^{N} (\frac{S_i H_i D_i}{2})^{\frac{1}{2}} (\frac{P_i}{P_i - D_i})^{\frac{1}{2}} \qquad (5.2.1\text{-}13).$$

The inventory control from the manufacturer to downstream vendors is also based on the PLS inventory policy.

According to the PLS policy, the optimised replenishment cycle time of vendor *i* should be $Tpd^* = (\frac{2S_m}{H_{vi} d_i})^{\frac{1}{2}} (\frac{P}{P - d_i})^{\frac{1}{2}}$ (5.2.1-14).

Thus, vendor *i*'s maximum inventory level of finished products is

$$Qpd^* = (\frac{2S_m d_i}{H_{vi}})^{\frac{1}{2}} (\frac{P}{P - d_i})^{\frac{1}{2}} \qquad (5.2.1\text{-}15).$$

Then vendor *i*'s average inventory cost for the finished products can be inferred as

$$Cpv_i^* = (\frac{S_m H_{vi} d_i}{2})^{\frac{1}{2}} (\frac{P}{P - d_i})^{\frac{1}{2}} \qquad (5.2.1\text{-}16).$$

Because the manufacturer transports the finished products continuously with a production lot-size to the manufacturer, the manufacturer has no inventory cost. The total average inventory cost of manufacturer-vendors level should be

$$Cpd=\sum_{i=1}^{M}\left(\frac{S_m H_{vi} d_i}{2}\right)^{\frac{1}{2}}\left(\frac{P}{P-d_i}\right)^{\frac{1}{2}} \qquad (5.2.1\text{-}17).$$

The total average inventory cost should be the summation of all inventory costs, which is

$$Cp=\sum_{i=1}^{N}\left(\frac{S_i H_i D_i}{2}\right)^{\frac{1}{2}}\left(\frac{P_i}{P_i-D_i}\right)^{\frac{1}{2}}+\sum_{i=1}^{M}\left(\frac{S_m H_{vi} d_i}{2}\right)^{\frac{1}{2}}\left(\frac{P}{P-d_i}\right)^{\frac{1}{2}} \qquad (5.2.1\text{-}18).$$

EOQ and PLS mixed model. In the EOQ and PLS mixed model analysis, it is assumed that the PLS policy is employed for inventory planning for intermediate products and the EOQ policy is employed for finished items. The manufacturer orders intermediate products from every individual upstream factory separately according to the PLS policy. Each downstream vendor orders finished items from the manufacturer based on the EOQ policy separately. Every individual company in the supply chain plans its inventory activity without the consideration of other related companies. In fact, the assumptions of EOQ and PLS mixed model are similar to those for the integrated inventory model proposed in this research. But the most important difference is, in the mixed model, each individual is optimised instead of the supply chain.

From the upstream factories to the manufacturer, the intermediate product inventory is controlled with the PLS policy. The total cost should be the same with the analysis in the single PLS model, which is

$$Cxu = \sum_{i=1}^{N} \left(\frac{S_i H_i D_i}{2}\right)^{\frac{1}{2}} \left(\frac{P_i}{P_i - D_i}\right)^{\frac{1}{2}} \qquad (5.2.1\text{-}19).$$

From the manufacturer to the downstream vendors, the finished item inventory is controlled based on the EOQ policy. The total cost should be the same with the analysis in the single EOQ model, which is

$$Cxd = \sum_{i=1}^{M} \left(\left(\frac{S_m d_i H_{vi}}{2}\right)^{\frac{1}{2}} + \left(\frac{S_m d_i H_m^2}{2H_{vi}}\right)^{\frac{1}{2}}\right) \qquad (5.2.1\text{-}20).$$

Therefore, the total average inventory cost of the EOQ and PLS mixed model should be

$$Cx = \sum_{i=1}^{N} \left(\frac{S_i H_i D_i}{2}\right)^{\frac{1}{2}} \left(\frac{P_i}{P_i - D_i}\right)^{\frac{1}{2}} + \sum_{i=1}^{M} \left(\left(\frac{S_m d_i H_{vi}}{2}\right)^{\frac{1}{2}} + \left(\frac{S_m d_i H_m^2}{2H_{vi}}\right)^{\frac{1}{2}}\right) \qquad (5.2.1\text{-}21).$$

5.2.2 Case studies

The following numerical analysis will concentrate on some cases using the single EOQ model, the single PLS model and the mixed model as well as the integrated model presented in this research. The results (optimised total average inventory cost) of different model for the supply chain will be compared. The analysis will begin with the simplest supply chain and then more complex supply chains are investigated to see the advantages of the integrated model for supply chains with different structures. All the data in these cases is generated by the RDG system presented in this research.

1. Suppose the simplest supply chain, in which a manufacturer has one upstream factory and one downstream vendor, has the data listed in Table 5-1. The optimised total inventory costs will be calculated using different models and compared.

Table 5-1. Data for the simplest supply chain

<table>
<tr><th colspan="3">Downstream Vendors</th></tr>
<tr><td>Vendor</td><td>Demand rate for finished item (d_i), unit per month</td><td>Holding cost (H_{vi}), Pounds per unit per month</td></tr>
<tr><td>1</td><td>291</td><td>13</td></tr>
<tr><th colspan="3">Upstream Factories</th></tr>
<tr><td colspan="2">Factory</td><td>1</td></tr>
<tr><td colspan="2">Production rate (P_i), unit per month</td><td>340</td></tr>
<tr><td colspan="2">Demand rate for intermediate product (D_i), unit per month</td><td>312</td></tr>
<tr><td colspan="2">Manufacturing parameter (δ_i)</td><td>1</td></tr>
<tr><td colspan="2">Holding cost (H_i), Pounds per unit per month</td><td>6</td></tr>
<tr><td colspan="2">Setup cost (S_i), Pounds per time</td><td>559</td></tr>
<tr><th colspan="3">The Manufacturer</th></tr>
<tr><td>Production rate (P_m), unit per month</td><td>Holding cost (H_m), Pounds per unit per month</td><td>Setup cost (S_m), Pounds per time</td></tr>
<tr><td>312</td><td>10</td><td>1000</td></tr>
<tr><th colspan="3">Optimised Results using the Integrated Model</th></tr>
<tr><td>Cycle time (T), month</td><td>Average total cost (C_a), Pounds per month</td><td>Optimised order quantity of vendor i per cycle time (Q_i)</td></tr>
<tr><td>0.686</td><td>4547</td><td>200</td></tr>
<tr><td>Optimised Result using the Single EOQ Model</td><td>Optimised Result using the Single PLS Model</td><td>Optimised Result using the Mixed Model</td></tr>
<tr><td>3880</td><td>7822</td><td>6748</td></tr>
</table>

In this case, the integrated model is not the best solution for the supply chain. The optimised total inventory cost of the single EOQ model, which is 3880 Pounds, is the most economical cost. The relationship of results of different models is illustrated in Figure 5-2.

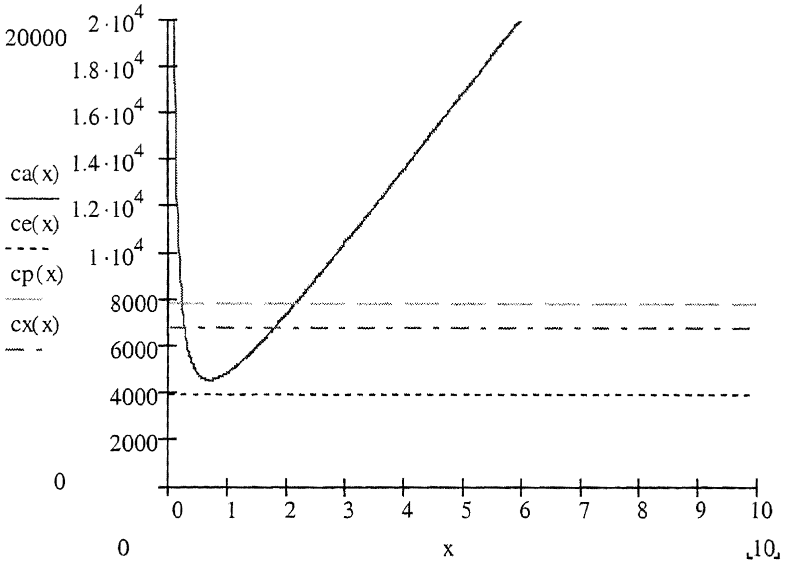


Figure 5-2. Relationship between models for the simplest supply chain

In Figure 5-2, *ca*(*x*) indicates the total average inventory cost of the integrated model, *ce*(*x*) indicates the total cost of the single EOQ model, *cp*(*x*) indicates the total cost of the single PLS model, and *cx*(*x*) indicates the total cost of the mixed model. It can be seen that the integrated model is the second economical solution for this supply chain. The optimised result of the single EOQ model saves about 14.7% of total inventory cost over the integrated model.

2. A more complex supply chain is considered, in which a manufacturer has four upstream factories and three downstream vendors. The basic information is listed. The optimised results are compared in Table 5-2.

Table 5-2. Data for the second supply chain

Downstream Vendors

Vendor	Demand rate for finished item (d_i), unit per month	Holding cost (H_{vi}), Pounds per unit per month
1	4241	13
2	4772	14
3	4632	13

Upstream Factories

Factory	1	2	3	4
Production rate (P_i), unit per month	46880	110800	29190	92380
Demand rate for intermediate product (D_i), unit per month	42370	98870	28250	84740
Manufacturing parameter (δ_i)	3	7	2	6
Holding cost (H_i), Pounds per unit per month	5	6	6	7
Setup cost (S_i), Pounds per time	2382	2215	2184	2344

The Manufacturer

Production rate (P_m), unit per month	Holding cost (H_m), Pounds per unit per month	Setup cost (S_m), Pounds per time
14120	10	4000

Optimised Results using the Integrated Model

Cycle time (T) month	Average total cost (C_a), Pounds per month	Optimised order quantity of vendor *i* per cycle time (Q_i)		
		1	2	3
0.345	122600	1462	1645	1595

Optimised Result using the Single EOQ Model	Optimised Result using the Single PLS Model	Optimised Result using the Mixed Model
220800	337100	203100

In this case, the integrated model is absolutely the best solution for the supply chain. Its optimised total inventory cost, which is 12260 Pounds, is the most economical total inventory cost for the supply chain. The relationship of results of different models is illustrated in Figure 5-3.

The functions *ca*(*x*), *ce*(*x*), *cp*(*x*) and *cx*(*x*) in Figure 5-3 has the same meaning with Figure 5-2.

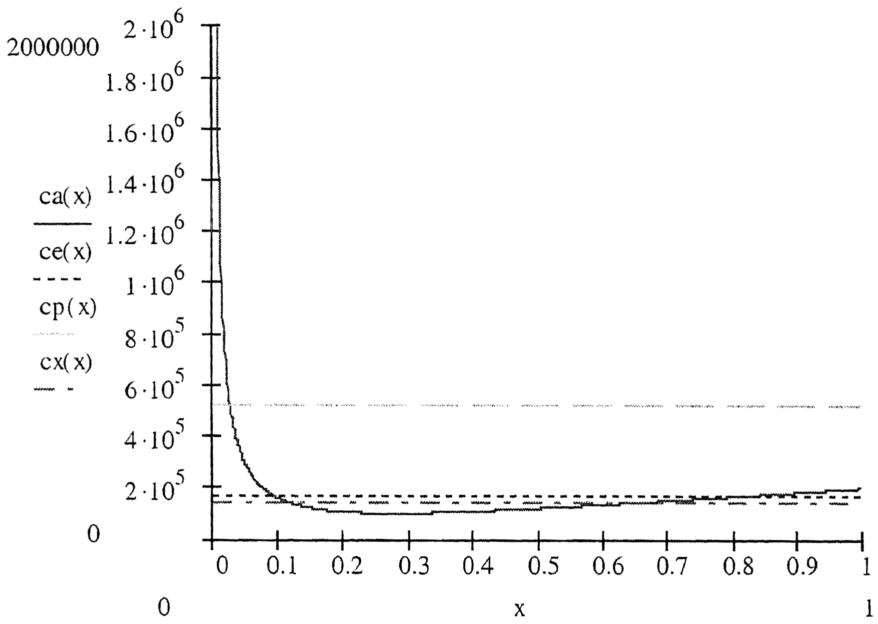


Figure 5-3. Relationship between models for the second supply chain

It can be seen that the optimised inventory cost of the integrated model saves about 44.5% of total inventory cost over the single EOQ model, 63.6% over the single PLS model and 39.6% over the mixed model.

3. A very complex model is considered, in which a manufacturer has ten upstream factories and ten downstream vendors. The basic information is listed and optimised results are compared in Table 5-3.

Table 5-3. Data for a complex supply chain

Downstream Vendors

Vendor	Demand rate for finished item (d_i), unit per month	Holding cost (H_{vi}), Pounds per unit per month
1	2404	20
2	1584	23
3	1963	23
4	2808	22
5	1807	24
6	2334	23
7	3071	25
8	2222	23
9	2287	26
10	1801	24

Upstream Factories

Factory	Production rate (P_i) unit per month	Demand rate for intermediate product (D_i), unit per month	Manufacturing parameter (δ_i)	Holding cost (H_i), Pounds per unit per month	Setup cost (S_i), Pounds per time
1	377300	324900	14	11	1286
2	217700	208900	9	11	1332
3	515600	464200	20	12	1385
4	94950	92840	4	9	1342
5	453900	394600	17	9	1378
6	102900	92840	4	11	1441
7	429000	394600	17	9	1384
8	105800	92840	4	9	1370
9	378600	348200	15	13	1276
10	155400	139300	6	13	1367

The Manufacturer

Production rate (P_m), unit per month	Holding cost (H_m), Pounds per unit per month	Setup cost (S_m), Pounds per time
23210	18	2462

Optimised Results using the Integrated Model

Cycle time (T), month	Average total cost (Ca), Pounds per month
0.16	479900

Optimised Result using the Single EOQ Model	Optimised Result using the Single PLS Model	Optimised Result using the Mixed Model
968400	1528000	907400

In this case, the integrated model is also the best solution for the supply chain. Its optimised total inventory cost, which is 479900 Pounds, is the most economical total inventory cost for the supply chain. The relationship of results of different models is illustrated in Figure 5-4.

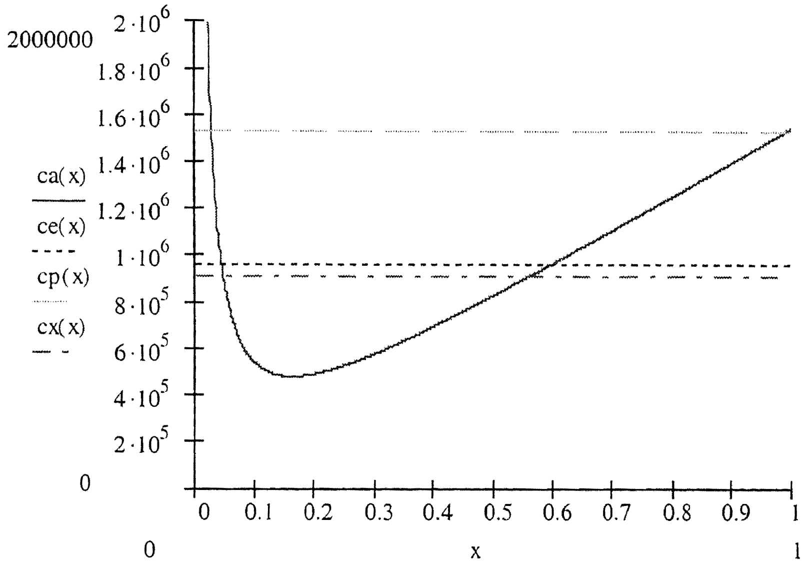


Figure 5-4. Relationship between models for the complex supply chain

The functions *ca*(*x*), *ce*(*x*), *cp*(*x*) and *cx*(*x*) in Figure 5-4 has the same meaning with Figure 5-2. It can be seen that the optimised inventory cost of the integrated model saves about 50.4% of total inventory cost over the single EOQ model, 68.6% over the single PLS model and 47.1% over the mixed model.

The numerical analysis indicates a trend that the integrated inventory model works better as the number of the companies increases in the supply chain. The following analysis will focus on the change of the optimised results with the number of the supply chain individuals. The information in Table 5-4 is generated by the RDG system with the same basic parameters.

Table 5-4. Change of optimised results with the number of supply chain individuals

Basic information of the RDG system					
Demand rate d_0		Holding cost H_0		Setup cost S_0	
3113		9.00		1231	
Supply chain information		**The optimised results of different models**			
Number of factories	Number of vendor	Single EOQ model	Single PLS model	Mixed model	Integrated model
1	1	26810	60430	50820	31150
3	3	123200	207500	108800	85720
5	5	308500	533900	280400	149800
7	7	550000	1090000	508600	228100
9	9	673300	1056000	620700	327500
11	11	1015000	2900000	947400	479200
13	13	1608000	3310000	1528000	708300
15	15	2301000	3698000	2205000	1129000
17	17	2698000	4857000	2592000	1364000
19	19	3539000	5765000	3416000	1773000
21	21	3685000	8754000	3548000	1604000
23	23	4620000	14390000	4476000	2326000
25	25	6058000	14150000	5899000	2546000
27	27	6175000	14420000	6002000	2882000
29	29	7486000	18180000	7301000	3611000

The table above can be illustrated by Figure 5-5.

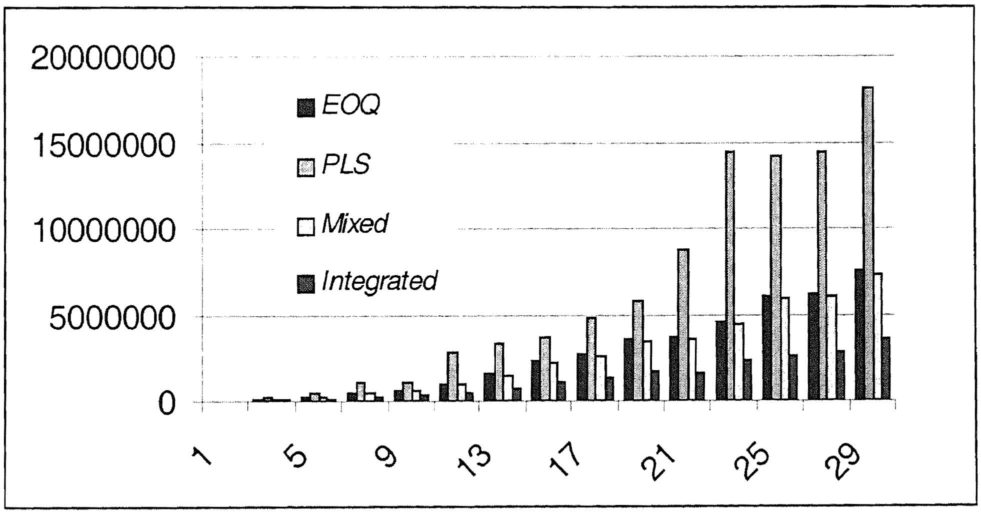


Figure 5-5. Graphic illustration of Table 5-4

Both the table and the figure present an overview of the relationships between the optimised results and the number of the supply chain individuals. It can be seen that the integrated inventory model shows its significant strength in inventory control of complex supply chains. The analysis has also proved that the optimised result of the integrated inventory model becomes better as the number of the companies increases in the supply chain.

Chapter Six

Conclusions and Further Work

6.1 Conclusions

Reducing inventory levels of raw materials, intermediate products and finished items simultaneously in different stages has become the major focus for supply chain management. In recent years, there has been a growing trend in both research and practical applications of integrated inventory control through supply chain management. An important ingredient for a successful integrated inventory control policy is closer co-ordination and co-operation among the trading parties. Therefore, the models that can control inventory levels integrally have been inevitable for firms to improve their inventory management.

This research has considered an inventory policy for a manufacturer with multiple upstream suppliers and multiple downstream vendors. A general replenishment agreement is made between the manufacturer and upstream factories based on the EOQ policy, and the EOQ policy has been implemented between the manufacturer and downstream vendors. An integrated model has been built to optimise the total inventory cost of the supply chain. Through some extensions and comparisons with numerical analysis, it can be concluded that

- The integrated model is not suitable for the simplest supply chains (a manufacturer with only one supplier and one buyer). The numerical results for this kind of supply chain show an increase of 14.7% in the total inventory cost over the single EOQ model.

- With the increasing complexity of the supply chain, the integrated model shows its advantage in total inventory control. The optimised results of the most complex supply chain in this work (a manufacturer with ten suppliers and ten vendors) indicates that the integrated model saved about 50.4% of the total inventory cost over the single EOQ model, 68.6% over the single PLS model and 47.1% over the mixed model. The percentage of saving increases with the complexity of the supply chain.

- Individual replenishment batch quantity changes will not influence the optimised result for the total inventory cost. In the case study, the total inventory cost increases by less than 0.05% when the individual replenishment batch quantity changes by more than 50%.

- The optimised result of the integrated model depends on the accuracy of the demand forecasting system when downstream vendors' demand information cannot be acquired through electronic data interchange systems. The total inventory cost increases by 7% when the demand forecasting system has an error of 10% in the case study.

6.2 Further Work

Although the numerical analysis indicates that the integrated inventory model significantly decreases the total inventory cost in complex supply chains, several extensions are still worthwhile to be considered.

- The integrated model presents an economic solution to a manufacturer who just produces a single kind of finished item. In practice, many manufacturers produce various kinds of finished item at the same time. Therefore, the integrated model should be extended to support multiple items inventory.

- The integrated model optimises a supply chain's total inventory cost without the consideration of other chains. Sometimes, two or more supply chains overlap heavily so that the optimised plan for a single chain may increase the inventory cost of other chains. Therefore, consideration for overlapped chains is an important focus in further extension.

- Transportation is also a crucial part of logistics management. This research presents an economical solution to supply chain inventory management without the consideration of transportation cost. Frequent replenishment may be economical for inventory management, but it may also increase transportation cost, and thus the total logistics cost of the supply chain. Therefore, further work should concentrate on strategy integration of logistics actions.

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