THE DEVELOPMENT AND ASSESSMENT OF AN ANALYTICAL STRUCTURAL MAINTENANCE DESIGN SYSTEM FOR ROADS.

by

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This thesis is submitted to the Council for National Academic Awards in partial fulfilment of the requirements for the Degree of Doctor of Philosophy.

This work was undertaken within the Department of Civil Engineering at Plymouth Polytechnic, Plymouth, in collaboration with the Transport and Road Research Laboratory (TRRL), U.K..

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#### ABSTRACT.

Previous work at Plymouth has shown the deflected shape as measured by the Deflectograph can be used to estimate the thickness of surfacing. This thesis extends the earlier work to develop relations between deflected shape and the stiffness and thickness of pavement layers.

A literature study has been carried out to identify the factors causing the deterioration of flexible pavements. The literature study also assesses the various pavement evaluation equipment that is available and describes various methods of analysis and the interpretation of the pavement surface deflection shape, that have been proposed. The properties of the materials of the various layers of the flexible pavement have been reviewed. Various structural models of a pavement are considered and the study indicates that the finite element method provides a most accurate prediction of actual pavement response to a moving wheel load.

A 3D finite element model of a flexible pavement has been produced and partially validated with data obtained from the TRRL. A Fortran Programe has been developed to convert absolute deflections predicted by the 3D model into equivalent Deflectograph deflections. The model has been used to carry out parametric study to establish appropriate relationships between the deflected shape and material properties of the pavement layers. Relationships have been established to determine the thickness, modular ratio and modulus of the pavement layers and its support subgrade from measurement of the deflected shape. An Analytical Pavement Evaluation and Design System has been set up based on the relationships. The system has been validated by comparing the material properties obtained from the laboratory testing with those predicted by the design model using the Deflectograph measurements obtained from local roads with measurement of layer thickness and subgrade strength measured in-situ.

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## DECLARATION.

While registered as a candidate for the degree for which this submission is made, the author has not been a registered candidate for another award of the C.N.A.A. or of a University during the research programme.

All work described in this thesis is wholly original and was carried out by the author except where specifically noted by reference. Assistance received in the research work and perparation of this thesis is acknowledged.

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## STATEMENT OF ADVANCED STUDIES.

The author has, during his period of research, attended two residential courses: 'Symposium on Highway Maintenance and Data Collection', held at Nottingham University in July 1983, and 'Analytical Design of Bituminous Pavements', held at Nottingham University in April 1984.

The author has written a paper to the International Symposium on 'The Bearing Capacity of Roads and Airfields' to be held in Sept 1986 at Plymouth.

The author has attended the course entitled 'The Deflectograph Use and Analysis of Results', organised by the Department of Civil Engineering at Plymouth Polytechnic for practising enginners.

The author attended the 'European Flexible Pavement Study Group Meeting' held at Plymouth Polytechnic in July 1983. TABLE OF CONTENTS.

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# NOTATIONS.

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F	= Axle Load Factor.
L	= Axle Load.
L <sub>s</sub>	= Standard Axle Load.
R	= Radius of Curvature.
a	= Radius of Circular Loaded Area.
đ	= Deflection.
d <b>o</b>	= Deflection at Distance 0 m or Deflection at Centre of
	Loaded Area.
μ	= Poisson's Ratio.
Е	= Elastic Modulus.
D <sub>d</sub>	= Differential Deflection.
d <sub>r</sub>	= Deflection at Distance 'r' from the Load.
Q <sub>r</sub>	= Ratio of the Deflection, $d_r$ , at a Distance 'r' from
	the Load to the Deflection Under the Centre of the
	Load, d <sub>0</sub> .
<sup>H</sup> in	= Layer Thickness Layers $i = 1$ to n.
E <sub>in</sub>	= Elastic Modulus of Layers $i = 1$ to n.
$\mu_{in}$	= Poisson's Ratio of Layers $i = 1$ to n.
W <sub>1</sub> or d <sub>1</sub>	= Deflection at Sensor No 1.
W <sub>2</sub> or d <sub>2</sub>	= Deflection at Sensor No 2.
W <sub>3</sub> or d <sub>3</sub>	= Deflection at Sensor No 3.
W <sub>4</sub> or d <sub>4</sub>	= Deflection at Sensor No 4.
W <sub>5</sub> or d <sub>5</sub>	= Deflection at Sensor No 5.
DMD	= Dynaflect Maximum Deflection $(W_1)$ .
SCI	= Surface Curvature Index $(W_1 - W_2)$ .
BCI	= Base Curvature Index $(W_4 - W_5)$ .

.

SP8	= Spreadibility $[100(W_1 + W_2 + W_3 + W_4 + W_5)]$
	5w <sub>1</sub>
HRA	= Hot Rolled Asphalt.
DBM	= Dense Bitumen Macadam.
S <sub>D</sub>	= Bitumen Stiffness.
s <sub>m</sub>	= Mix Stiffness.
t	= Loading Time, sec.
f	= Frequency, Hz.
SP(or SP <sub>r</sub> )	= Softening Point(or Recovered Softening Point), $^{\circ}C$ .
PI(or PI <sub>r</sub> )	= Penetration Index(or Recovered Penetration Index),
	°c.
Т	= Temperature, ${}^{0}C$ .
P	= Penetration at 25 $^{\circ}C$
v	= Vehicle Speed in Km/hr.
c <sub>v</sub> -	= Volume Concentration of the Aggregate.
vv	= Volume of Voids.
VMA	= Voids in the Mixed Aggregate.
$\sigma_1$	= Major Principal Stress.
$\sigma_{3}$	= Minor Principal Stress = Cell Pressure = Applied
	Confining Pressure.
€ <sub>vr</sub>	= Axial Strain.
€ <sub>rr</sub>	= Radial Strain.
<sup>є</sup> р	= Permanent Strain.
M <sub>r</sub> or E <sub>r</sub>	= Resilient Modulus (Stiffness).
E  <sup>*</sup>	= Complex Modulus (Stiffness).
$\sigma_{\!\!\!\phi}$	= Amplitude of the Sinusoidal Vertical Stress.
$\epsilon_{oldsymbol{\phi}}$	= Amplitude of Resultant Sinusoidal Steady State Vertical
	Strain.

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$\phi$	= Phase Lag, $0_{C}$ .
t <sub>i</sub>	= Time Lag Between a Cycle of Sinusoidal Stress and the
	Resultant Cycle of Sinusoidal Strain, sec.
t p	= Time for a Cycle of Sinusoidal Stress, sec.
N <sub>s</sub>	= Service Life.
€ <sub>s</sub>	= Initial Strain Corresponding to a Given Number of
	Cycle to Failure.
ε	= Maximum Value of Applied Tensile Strain.
n <sub>1</sub>	= Number of Cycles of Strain Applied.
Nl	= Number of Cycles to Produce Failure Under Constant
	Strain Amplitude.
$\sigma_{3}'$	= Initial Effective Confining Stress.
$\sigma_{3}$	= Applied Confining Stress.
U	= Initial Pore Pressure.
S	= Soil Suction.
I p	= Plasticity Index, %.
CBR	= California Bearing Ratio.
р	= Mean Normal Stress.
<sup>E</sup> subgrade	= Elastic Modulus of Subgrade.
q	= Deviator Stress.
q <sub>r</sub>	= Cyclic Deviator Stress.
θ	= Sum of Principle Stress = $\sigma_1$ + $2\sigma_2$ = q + $3\sigma_3$
x	= 'x' Direction.
У	= 'y' Direction.
Z	= 'z' Direction.
Line xx	= Line Through the Centre of the Twin Rear Wheels
	Near the Kerb.
Line yy	= Line Through the Centre of the Vehicle.

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Line	ZZ	=	Line	Through	the	Centre	of	the	Twin	Rear	Wheels
			Away	from the	e Kei	rb.					

NS	=	Near	Side	Twin	Rear	Wheels	of	the	Vehicle.
----	---	------	------	------	------	--------	----	-----	----------

OS = Off Side Twin Rear Wheels of the Vehicle.

- P = Tyre Inflation Pressure.
- A = Actual Tyre Contact Area.
- $A_{p}$  = Computed Tyre Area.
- W = Static Wheel Load.
- 2D = Two Dimensional.

\_

- 3D = Three Dimensional.
- H = Equivalent Thickness;

$$f \times H_{i} \sqrt{\frac{\Xi_{i}(1-\mu_{n})}{\Xi_{n}(1-\mu_{i})}}$$

- $D_x D_y$  = Differential Deflection; Difference in Magnitude of Deflection at Distances x and y away from the Point of Maximum Deflection.
- D = Deflection at Distance x away from the Point of Maximum Deflection.
- D<sub>0</sub> = Deflection at 0 mm away from the Point of Maximum Deflection.
- D<sub>1</sub> = Deflection at 100 mm away from the Point of Maximum Deflection.
- D<sub>2</sub> = Deflection at 200 mm away from the Point of Maximum Deflection.
- D<sub>3</sub> = Deflection at 300 mm away from the Point of Maximum Deflection.
- D<sub>4</sub> = Deflection at 400 mm away from the Point of Maximum Deflection.

- D<sub>8</sub> = Deflection at 800 mm away from the Point of Maximum Deflection.
- $H_{G}$  = Thickness of the Granular Material.
  - = Thickness of the Pavement.

Н<sub>р</sub>

Deflection.

#### 1.1 General.

Structural deterioration under traffic takes place to some extent in all road pavements although in well designed ones its development is a very slow and seasonal process. An increasing proportion of the network of major roads built in the last two decades will require strengthening in the years ahead; this is additional to the periodic strengthening required to improve the older and largely undesigned road network, to maintain it at structural standards appropriate to today's heavier traffic. It is therefore important that the considerable expenditure "on strengthening of roads should be made as effective as possible by the use of a suitable method for designing strengthening measures.

A system for the design of strengthening measures should be capable of two functions:

- (a) predicting the remaining life of a pavement under traffic so that strengthening by overlaying can be timed to coincide with the onset of critical conditions;
- (b) designing the thickness of overlay required to extend the life of a road to carry any given traffic and to indicate length of roads which have deteriorated sufficiently to require partial or total reconstruction.

Essential to a design method is some form of measurement of the structural condition of the road. The procedure must be sufficiently rapid and convenient to enable closely spaced measurements to be made over long lengths of road in a realistic period of time.

The measurement of the surface deflection basin provides valuable information for the structural evaluation of flexible pavements. The surface deflection of a pavement system and its curvature under a load are influenced by the stiffness of its component layers, thickness, load intensity and its overall structural integrity. A mechanistic method of pavement design generally starts with a component analysis of the different materials (laboratory testing of material specimens). The different components are then incorporated into a system (layered model), and the behaviour of the whole system under load is analysed (stresses, strains, deflections). In a mechanistic pavement evaluation method, the system response is measured (surface deflections), the response is analysed with the use of a layered model, and the material properties are back calculated.

# 1.2 Factors Influencing the Deterioration of Road Pavements. 1.2.1 Traffic Loading.

Both the magnitudes and the numbers of commercial traffic loads contribute to the damage of flexible pavements. For pavement design purposes, it is usual to consider vehicle loadings in terms of axle loads. Tyre pressure and wheel or axle configuration also influence the pavement performance. Defects caused by traffic include fatigue cracking, deformation, wear by loss or polishing of aggregate and excessive embedment of chippings.

Tyre pressure primarily affects the wearing course and has little influence on the loading of the lower layers. Wheel and axle configurations influence mainly the upper layers of the pavement. The AASHO (American Association of State Highway Officials) Road Test (1) has shown that an 80 KN single axle load has the same structural

damaging effect as a 142 KN tandem axle load, i.e. equivalent pavement performance achieved on similar pavement structure. A complete system of classifying commercial traffic in terms of vehicle types and axle configurations is given in Ref (2).

Traffic on a road is mixed in composition and therefore it is very important for design purposes to express cumulative traffic in terms of an equivalent number of standard axles. Investigations were carried out at the AASHO Road Test (1) to determine the relative damaging effects of loads of different sizes and the following relationship was developed:

$$F = \left(\frac{L}{L_s}\right)^a$$
(1.1)

The concept of equivalent load means that one application of a load, L, is equivalent in terms of pavement damage to, F, applications of a standard load,  $L_s$ , is generally taken as equal to 80 KN. The value of 'a' was found to be dependant upon the thickness of the pavement and the strength of the subgrade but the average value from the AASHO Road Test (1) was found to be approximately 4.0. The relationship shown in Eqn 1.1 is also known as the Fourth Power Law. Recent work carried out in S.Africa (3) with a heavy vehicle simulator has shown that the value of 'a' can vary depending upon the pavement type and the criterion of distress. The approximate equivalences obtained for different pavement types is given in Table 1.1. A value of 4.2 was recommended for use in S.Africa if an average value for all pavement types was required.

The average equivalence factor suitable for use for flexible pavements in the U.K. is given in Road Note 29 (4). If the composition of traffic is expressed in terms of the number of axle loads in each

of a range of categories, the damaging power of the traffic flow may be assessed. The proportion of pavement damage caused by each load category is obtained by multiplying the number of axle loads in that category by the appropriate equivalence factor. The equivalence factor is used in conjuction with measured load spectra obtained from weighbridges installed in roads to assess the damaging power of traffic of different compositions (5). The damaging effect of small axle loads is insignificant despite their large number. Consequently all but commercial vehicles over 15 cwt (762 Kg) unladen weight can be ignored for the purpose of pavement design.

Pavement Base Type	Value of 'a'
Natural Gravel	2-3
Crushed Stone	3-4
Asphalt	4
Cement Treated	6

Table 1.1 Recommended Standard Axle Factor for use in S.Africa (after Freeme et al, Ref 3).

Where weighbridge data are not available, average numbers of standard axles per commercial vehicle has to be used. Because this average is a function of both commercial vehicle loading and the percentage of commercial vehicles in a traffic flow, its value depends on the type of road. Typical values for design purposes in the U.K. are given Ref (6).

The speed of traffic has an influence on the design of flexible pavement because it affects the mechanical properties of bituminous materials. The critical factor is the time for which a load pulse

due to a moving vehicle affects the bituminous layer. This is a function of both vehicle speed and the thickness of bituminous layer.

## 1.2.2 Environmental Effects.

The environmental effects which are important in pavement design are moisture, influenced by the position of the water table, and temperature. These factors influence the subgrade performance and the properties of bituminous materials respectively.

#### 1.2.2.1 Moisture Content and Water Table.

Croney and Bulman (7) have reviewed the influence of climate on subgrade soil and have given attention to the relationship between elastic modulus and moisture content. The amount of water present is defined by the position of the water table which varies from season to season. In the U.K. the water table is generally at its highest during winter and spring, and at its lowest during summer. When the water table is at its highest, it weakens the subgrade and the pavement is then most susceptible to deformation.

Crony and Bulman (7) stated that unless the water table in a road structure was very high, there was no evidence of long term moisture exchange within the subgrade sufficent to affect the strength of sub-base/base layers.

## 1.2.2.2 Temperature.

The stiffness of the bituminous material decreases with increasing temperature. This in turn increases the traffic stress imposed on the material below the bituminous layer. Therefore permanent deformation is most likely to occur in the summer when the temperature is

high, especially for flexible pavements containing rolled asphalt and crushed stone as opposed to lean concrete road bases (8). Fig 1.1 shows the development of deformation in a rolled asphalt pavement and its subgrade related to monthly temperature durations within the pavement.



Fig 1.1 Development of Deformation in a Rolled Asphalt Pavement and its Subgrade Related to Monthly Temperature Durations within the Pavement (after Lister Ref 8).

At low temperatures, the stiffness of bituminous material is relatively high and under these conditions it is least able to resist the horizontal tensile strain developed at the bottom of these layers caused by repeated load application of traffic. Under these conditions, fatigue cracking is most likely to occur. Fatigue failure may take place as a result of low temperature and a great number of repeated loads. This also depends on the pavement structure, the layer stiffness at the low temperature and the strain level generated. The time for the cracks to propagate can also be shorter under these conditions than at higher temperatures.

It is also possible for thermal cracking of the bituminous layer due to tensile failure to take place, especially under extreme cold conditions (7). Decrease in temperature causes the bituminous material to become stiffer and to contract, and since the bituminous layer is restrained by the underlying layers, the contraction induces stresses which often exceed the tensile strength causing fracture (9). The fracture is generally initiated at the surface.

Very low pavement temperature can result in pavement damage due to heave in the subgrade or sub-base. This heave arises not from the expansion of water on freezing, but from a continuous migration of moisture into the freezing zone from the unfrozen material below. On certain types of foundation, heave as great as 80 mm may occur, followed by a temporary but considerable loss in strength during the thaw.

#### 1.2.3 Design and Construction.

Underdesigned pavements deteriorate rapidly which results in premature failure. Underdesign can be due to an incorrect design, but it is more likely to be as a consequence of an error in either the estimate of traffic or the strength of the subgrade and/or pavement layers. This error can arise due to unexpected changes in traffic

loading, similar to the increase recorded in the U.K. in 1972/73 or to inadequte soil surveys (10).

The use of poor quality materials and/or construction techniques will have the same effect as underdesign, since the pavement will be weaker in those areas.

#### 1.3 Maintenance.

The serviceability of a pavement is a measure of its state of fitness to carry traffic comfortably, safely and economically. The level of serviceability of a pavement declines gradually and continually as shown diagrammatically in Figs 1.2 and 1.3 (11). If no remedial action is taken the pavement will ultimately reach a state in which it can no longer carry traffic safely. Pavement maintenance consists of remedying and/or preventing road surface deterioration using techniques such as surface dressing, overlaying or reconstruction at intervals throughout its life so that the level of serviceability is always maintained above the acceptable miniumum as shown in Fig 1.3 (11). The structural condition of the pavement may be assessed by measuring surface cracking, permanent deformation, transient surface deflection and the shape (curvature) of the transient deflection bowl.

Deformation and cracking induced by traffic loading are the criteria most commonly used for assessing the state of structural serviceability of a pavement and hence for deciding when maintenance is necessary. Transient deflection produced under a moving wheel load is commonly used to assess serviceability in connection with structural maintenance programmes. Measurements of curvature are also used for this purpose.





Fig 1.2 Serviceability History (after Peattie, Ref 11).



## 1.4 Statement of the Problem.

The failure of a flexible pavement, generally results from a loss in strength of one or more layers in the pavement structure caused by some of the factors discussed in Section 1.2. There are various techniques of rehabilitating a failed pavement (see Section 1.3), but the selection of an appropriate technique requires a knowledge of where the failure originates. One method of identifying the weakened layer would be to analyse the stresses and strains generated in each layer. Pavement response to applied load, in which these stresses and strains are calculated, may be analysed using the finite element method, elastic layer analysis based on Burmister's theory, visco-elastic layer analysis and other methods all of which are discussed in Chapter 4.0. All this analysis requires the input of the pavement layer material properties.

# 1.5 Research Hypothesis.

The shape of surface deflection basin of a pavement system under load is not constant, but varies with many factors. The most significant influencing factors are pavement temperature, subgrade moisture, cracking and elastic modulus of the layers.

The current structural maint enance design method used in the U.K. is based on the empirically derived relations between the deflection of a road's surface produced by the passage of a rolling wheel load and the road's performance. The measurement of deflection is made with a Deflectograph, which can also provide information on the deflected surface shape under load. The maximum deflection of a pavement measured with a Deflectograph is used to assess the overall strength of a pavement and to determine the thickness of overlay required to strengthen a pavement (12). However, the method is not capable of identifying which layers of the pavement are contributing to the weakness.

Previous work (13) has shown that the deflected shape measured by the Deflectograph can be used to estimate the thickness of the surfacing, the highest strength layer in the pavement. The research described in this thesis is a continuation of previous work (13), and is intended to produce an analytical model that makes use of the measurement of the deflected shape in order to define the properties and the thickness of all or some of the layers in the pavement. In this way layers of weakness will be identified and alternative strategies including the use of overlays or partial reconstruction to restore the pavement strength can be evaluated. In the case of reconstruction or partial reconstruction, the prediction of the maximum deflection can be used as an input to a pavement design method in which the performance model is the LR 833 Charts (12).

# 2.1 Introduction.

The structural analysis of pavement systems, the prediction of their life expectancy and the design of necessary strengthening measures are among the most significant aspects of pavement management. The need for the development of a rapid method of pavement condition evaluation arises from an ever increasing demand for rational design and rehabilitation of pavement systems. There is general agreement among pavement engineers and researchers that the measurement of the surface deflection basin provides valuable information for the structural evaluation of flexible pavement. The maximum transient surface deflection of a pavement system and its transient deflected shape or curvature under load are influenced by the stiffness of its component layers together with their thickness, the applied load intensity and its overall structural integrity.

Structural evaluation is, to an extent, an inverted design process. If the cross section and properties of the paving material and support system are known, it is possible to compute the pavement response (stresses, strains and displacements) for a given loading condition. In the evaluation process, the response of the pavement is observed and the material properties are back calculated.

Among the different responses to load, exhibited by a pavement, only surface deflection is easily measurable. The deflection at various points away from the point of maximum deflection must be measured together with the maximum deflection since the shape of the deflection bowl can be different for the same maximum deflection, as
illustrated in Fig 2.1. The shape depends on the layer thickness and material properties, while maximum deflection is sensitive primarily to variations in subgrade support (15).



maximum deflection

Fig 2.1 Different Deflection Bowl for Same Maximum Deflection.

The Deflection Beam (16), Curvature Meter (16), Dynaflect (16), Deflectograph (16), Falling Weight Deflectometer (16) and Road Rater (16) have been used either to measure the maximum deflection only or to measure both the maximum deflection and the deflection at various other points away from the point of maximum deflection. A number of different methods have been suggested by various authors for defining and interpreting the curvature measured, each related to the design method adopted and equipment used to measure the deflection bowl.

# 2.2 Characterization of Measuring Device.

The maximum transient surface deflection and curvature generated at the road surface depend to a large extent on the method of testing. This dependency is caused mainly by non-linear response of the materials in a pavement to load magnitude, together with time of loading or rate of loading effects. Measuring equipment can be classified into two categories:

- (a) Rolling wheel techniques (measure displacement of the road's surface under the action of a rolling wheel load);
- (b) Stationary loading techniques (measure dispacement under a single or repeat pulse load.

The structural condition of the pavement may also be assessed by the Wave Propagation techique (16) which normally involves the application of very small loads and uses, rather than the deflection level, the velocity of propagation and the wavelength of the waves developed by the applied constant dynamic load to determine material properties.

# 2.3 Rolling Wheel Techniques.

These techniques involve the measurement of vertical transient deflection of the road surface under the action of a rolling wheel load, travelling at a creep speed. The Benkelman Beam (16), Travelling Deflectometers (17) and Deflectograph (16) are all classified under this heading. Rolling wheel techniques are often criticised on the grounds that normal traffic does not move at the creep speed and that the dynamic response of bituminous materials, whose stiffness is frequency dependent, is not characterised correctly. Much of the damage in the U.K. is due to the deformation of the pavement layers and the subgrade, which takes place during warm or hot weather. The stiffness of the bituminous material, which is also temperature dependent, is low under such conditions. It can be shown that the stiffness of the bituminous material under creep speed and moderate temperature at which the deflections are measured, are similar to those obtained under normal commercial vehicle speeds at high temperature.

# 2.3.1 Deflection or Benkelman Beam.

The Deflection Beam also known as the Benkelman Beam was originally designed by Benkelman (17) for the use in the WASHO (Western Association of State Highway Officials) Road Test in the U.S.A.. Since then it has been modified by the TRRL (Transport and Road Research Laboratory) for use in the U.K.. The transient deflection of the road surface as it is loaded by the passage of a wheel is measured by the rotation of a long pivoted beam in contact with the road at the point where deflection is to be observed. The aluminium alloy beam, of length 3.66 m passes between the dual rear wheels of a loaded lorry. It is pivoted at a point 2.44 m from the tip giving a 2:1 length ratio on the either side of the pivot. The pivot is carried on a frame which also carries a dial gauge arranged to measure the movement of the free end of the beam. The datum frame is supported by three adjustable legs. The deflection measured is not the absolute deflection. On most road pavements the bowl of the deflection surrounding the load wheel extends to a radius of greater than about 1.5 m. The beam tip and the forward feet of the beam frame are thus within the bowl. Details of the Deflection Beam and its operating procedures are presented in Ref (18, 19). Making 2 measurements at a point, an experienced team can complete about 250 measurements of a maximum deflection in a working day. Measurements of deflected shape can also be obtained by connecting the Deflection Beam to a Chart Recorder but at the expense of a reduction in the rate of testing.

### 2.3.1.1 Pavement Evaluation.

The transient deflection is used extensively as an indicator of the overall pavement performance both in the U.K. (16) and elsewhere. Maximum deflection provides a measure only of the overall pavement condition. To evaluate the condition of individual layers of the pavement requires a knowledge of both maximum deflection and deflected shape.

Kung (20) reviewed some of the theories of the curvature measurement, proposed by several authors and also suggested a new method based on 'slope of deflection' for assessing pavement performance. The term 'slope of deflection' is defined as the tangent of the angle made by the orginal pavement surface and the extension of the straight line connecting the point of inflection of the deflected curve and the point of maximum deflection, i.e.  $\tan \theta = cd/bc$ , see Fig 2.2. Carey et al (20) suggested the 'Bending Index' relationship for the correlation study of pavement deflection and performance:

Bending Index, b = d/a (2.1)

where d = deflection in inches, and

a = one half the deflected length.

A similar suggestion was made by Ford (20) and Bisselt (20) except instead of 'one half the deflected length', 'Radius of influence'was used which was assumed to be from the point of maximum deflection back to where the curve becomes tangential to the horizontal, i.e. cd/ac, see Fig 2.2. According to Kung (20) 'one half deflected length' and 'Radius of influence' are too long in length for use in this type of study because of the fact that deflection increases slowly until the wheel is within a certain distance from the probe of the Benkelman Beam and then increases rapidly to the maximum point.

Kung (20) stated that the ratio of 'cd' to 'bc' in the 'slope of deflection' method, would be more justifiable than that of 'cd' to 'ac', see Fig 2.2, because the line 'bd' passes through the point of inflection 'e', which is one of the sharpest portions of the curve. The unit elongation are greatest on the sharpest portion of the curve, i.e. point of maximum tensile stress. Kung (20) verified his suggest-ion, 'slope of deflection', by carring out 40 field tests on the Virginia Test Highways. A high degree of correlation was found between slope of deflection and flexible pavement cracking and it was suggested that the maximum allowable slope of deflection of about  $0.75 \times 10^{-3}$  was appropriate.



Fig 2.2 Defining the 'Slope of Deflection'.

The Benkelman Beam has also been used in S.Africa (21) to measure the deflection recorded as a truck stopped briefly every 6 inches (153 mm) of its travel for distance of 4 ft (1220 mm) on either side of the point of measurement. The resulting deflection was plotted and the radius of curvature determined graphically by fitting a curve to the points of measured maximum deflection. This method proved to be quite accurate but in practice it proved difficult because of the need to stop the truck at exactly 6 inches (153 mm) intervals. Dehlen (22) used both maximum deflection and the radius of curvature at the point of maximum deflection to investigate the problem of 'chicken net' cracking, and concluded that a correlation existed between the condition of the surfacing and both the maximum deflection and the radius of curvature. These relationships indicate that 'chicken net' cracking was due to excessive flexure of the bituminous surfacing.

Leger et al (23) recorded the deflected shape and calculated the radius of curvature by using a Benkelman Beam linked to a chart recorder. Measurement of the radius of curvature would involve the difference between two points very close to each other and consequen tly will result in large errors in measurement. Therefore it was necessary to obtain the radius of curvature by means of analytical methods. The deflected shape was analyzed by fitting the recorded curve to an analytical curve. The two peaks are fitted together and the radius of curvature was calculated from the analytical curve. The curve adopted was of the form (23):

$$a(x) = \frac{d_0}{1 + ax^2}$$
 (2.2)

The author reported that the matching obtained with this curve was in general, excellent up to 400 to 500 mm from the peak of the deformation curve. The matching was generally performed numerically by a regression calculation in the axis Y = 1/d and X = x, in which the matching curve is a straight line. The radius of curvature can be obtained from the following expression once 'a' and 'd\_' are known:

$$2Rd_{o} = 1/a$$
 (2.3)

Huang (24) stated that curvature was definitely related to the tensile strain in the asphalt layer. The curvature-tensile strain

ratio depends primarily on the thickness of the surfacing and was independent of the road base thickness. The use of curvature, instead of deflection, as a criterion for controlling fatigue, was highly desirable because the curvature-tensile strain ratio was not affected by modular ratios.

The major difficulty with the use of all these methods is that speed of operation; while suitable for short failure investigation the Deflection Beam does not provide sufficient route capacity for routine testing.

#### 2.3.2 Deflectograph.

The Deflectograph, originally designed by the Laboratoire des Ponts et Chaussees in France (25), operates within the wheel base of a standard rigid wheel base lorry. The principle of the pivoted beams supported by a datum frame is similar to that of the Deflection Beam. Two beams, one in each wheel path, are mounted on a T-shaped datum frame, which rests on the road surface at its extremities. The two beams, see Plate 2.1, are attached through bearings to the recording heads. During the measuring cycle the beams and the datum frame are stationary on the road surface. The tips of the measuring arms are then at a position approximately 1100 mm ahead of the centreline of the rear axle, see Fig 2.3. At this point a switch is activated that energises a solenoid within the recording heads, which in turn causes two anvils to grip a vertical spring attached to the core or armature of a displacement transducer, see Plate 2.2 (18). The operation of these anvils links the measurement arm and the transducer through the beam arm extension. The tips of the measurement arm are, by then, approximately 990 mm in front of the centreline of the rear axle and

it is from this point, B in Fig 2.3, that the measurement of deflection begins.

The downward movement is detected by the measuring arms and is transferred to the displacement transducer via the extension arm and the clamping solenoid. The output from the transducer is fed to the analogue and digital recording equipment housed in the rear cabin on the vehicle. When the centreline of the twin rear wheels has reached a point, D in Fig 2.3, 230 mm in front of the tip of the measurement arms, the clamping solenoids are de-energised; this allows the transducer's armature and vertical spring to fall back to their rest or zero position. After the maximum deflection has been recorded, the datum frame and deflection beams are pulled forward on steel skids at twice the speed of the lorry by a cable system, operated through an electromechanical clutch, to the next measuring point. The measurements are taken at approximately 4 m intervals.

The vehicle speed is about 2 to 2.5 Km/hr giving a maximum surveyed length of about 10 to 16 Km, providing about 2,500 to 4,000 readings per day in each wheel path. The deflection measured by the Deflectograph is recorded relative to the datum provided by the T-frame. The Deflectograph assembly, shown in Figs 2.3, 2.4 and 2.5, indicates that all the supports of the T-frame will be affected by the loaded wheels during the measuring cycle and that the front wheel will play a greater part in influencing the measured values than in the case of the operation of the Deflection Beam.

The complex relation between absolute and measured deflection resulting from the datum frame rotation is a major drawback of the Deflectograph system. A further difficulty with this approach is the slow loading speed and the difficulty this produces in selecting

appropriate layer moduli which complicates any comparison between measured deflections and those predicted by theory although in principle, this is possible if suitable layer moduli are selected (26).



Plate 2.1 Deflectograph-Beam Assembly (after Kennedy et al Ref 18).

The Deflectograph has been modified by the TRRL to record the deflected shape of the pavement's surface as a maximum deflection and ten ordinate deflections, i.e. a multivalue curvature measurement, see Figs 2.6 and 2.7. The curvature of the pavement's surface is expressed as a differential deflection, which in turn gives an indication of the condition of the pavement structure. Differential deflection is the difference between the maximum deflection and an ordinate deflection and is specified in terms of the horizontal distance between the two points of measurement, i.e. the differential deflection 50 mm from the maximum deflection relates to the shape of the deflection dish over that distance.

#### 2.3.2.1 Pavement Evaluation.

Butler et al (13) carried out theoretical studies, using a two dimensional finite element approach, and practical studies, using in-situ measurements of maximum deflection and deflected shape obtained with a Deflectograph on a wide range of pavement types. The finite element model was used to determine the relationship between the deflected shape of the pavement surface and the thickness of the bituminous layer. Charts (13) were produced which enabled the position of the differential deflection most influenced by a change in bituminous layer thickness to be identified and it was found to correspond to an offset 200 mm from the point of maximum deflection. It was found that the stiffness of the bituminous layer has a greater influence on the deflected shape (differential deflection) than on the maximum deflection, whereas the opposite is true for the modulus of the subgrade. The differential deflection was less influenced by the stiffness of the subgrade and the stiffness and thickness of the granular layers.

The main draw back with the use of the Deflectograph for analysing the response of a pavement is the limitation of its current accuracies of measurements; 1 to 2 x  $10^{-2}$  mm. To be effective accuracies of 1 to 2 x  $10^{-3}$  mm are required.



Fig 2.3 Cycle of Operation: Diagrammatic

(after Kennedy et al, Ref 18).



Fig 2.4 Diagrammatic Representation of Deflectograph

(after Kennedy et al, Ref 18).





Detail	Satisfactory range	
Dimension A	290-380mm	
Dimension B	130-190mm	
Dimension C	1830-1875mm	
Dimension D	1980-2013mm	
Dimension E	4445-4510mm	
Front axle load	4500kg ± 5%	
Rear axle load	6350kg ± 10%	
Twin rear wheel load	3175kg ± 10%	
Tyres	12.00 X 20	
Tyre pressure	690kN/m2 (100psi)	

Fig 2.5 Essential Chassis Details for Deflectograph Vehicles (after Kennedy et al Ref 18).



Fig 2.6 Recording the Deflected Shape as a Maximum Deflection and Ten Ordinate Deflection.



Fig 2.7 The Relationship Between the Maximum, Ordinate and Differential Deflections.

### 2.3.3 Travelling Deflectometer.

The Deflectometer (16) consists of automated versions of the Deflection Beam with the beams mounted in front of each pair of dual rear wheels of a 15 m long articulated lorry. It is longer and less manoeuvrable than the Deflectograph (16). The beams are carried on a large datum frame which stands stationary on the road, independent of the continuously moving lorry while the deflection generated by the approaching wheel is detected and recorded. The datum frame and beams are then carried rapidly forward relative to the lorry, ready to begin the next cycle of measurement. The lorries operate at 1 to 1.5 Km/hr and taking measurements at 6 and 11 m intervals respectively. The capacity of this system is about 2,000 measurements per day. This equipment is in use in California and Denmark and is capable of recording both the maximum deflection and deflected shape of the road surface (16).

#### 2.3.3.1 Pavement Evaluation.

The radius of curvature (R) of the road surface at the point of maximum deflection (d) or the product of 'Rd' can sometimes be determined from the influence line, i.e. the deflection recorded as the wheel approaches (27). Since the radius of curvature is difficult to measure, an influence length is sometime calculated as a measure of the shape of the influence line. In Denmark, the length of deflection curve is determined as the distance between two points at which deflection is approximately 95% and 5% respectively of the maximum (27).

# 2.3.4 Curvature Meter.

The Curvature Meter (21, 28) was used in S.Africa to measure the deflected shape and is based on the principle that the differential deflection occurring over a short distance is related to the curvature of the surface over that distance. The instrument, Fig 2.8, consists essentially of a short bar resting on the road surface at both ends with a dial gauge mounted centrally between the feet with its spindle in contact with the surface. The instrument is placed 5 ft (1525 mm) ahead of the dual rear wheels of a loaded truck, in an approximately level position. The truck is driven forward slowly, without stopping at any point, and the maximum reading is recorded as the wheels pass directly opposite the instrument (point B in Fig 2.8). Two negative maxima will also be observed, corresponding to pointsA and C. A final reading is taken when the wheels have passed 10 ft (3050 mm) beyond the instrument.



Fig 2.8 The Curvature Meter (after Dehlen, Ref 21).

The relation between curvature and differential deflection may be deduced by simple geometry, by fitting an approximate curve to the three points of measurement: the maximum and two points of inflection. Previous work (21) undertaken in S.Africa with a Benkelman Beam has indicated that in the vicinity of the point of maximum deflection, the deflected shape is typically a sine form. It has also been noted that points of inflection (points P in Fig 2.8, where the curve changes from concave to convex) occur fairly consistently at distances of 6 inches (153 mm) on either side of the point of maximum deflection. The relationship between the curvature and differential deflection in the case of a sine curve is (21):

$$R = L/D_a f$$
(2.4)

where L = distance between the dial gauge and each support,

D\_= differential deflection,

f = factor which varies between 2.0 and 2.47 as the ratio L/S
varies between 0 and 1.



Fig 2.9 Recording Curvature with a Curvature Meter (after Dehlen, Ref 21).

The Curvature Meter is unable to measure the total deflection occurring beneath the wheels and a telescope or a pair of binoculars is needed to read the dial gauge, while sitting on the road behind the vehicle . Use of this equipment in the U.K. has indicated that  $'D_{d}'$  has a range of 0.001 to 0.002 inches on motorways, and up to 0.015 inches on roads of light construction.

# 2.3.4.1 Pavement Evaluation.

This instrument was developed to carry out the same task as that of the Benkelman Beam. The evaluation process is the same as that described in Section 2.2.1.1.

### 2.4 Stationary Loading Techinques.

Stationary Loading techniques cover a wide range of loading conditions including static loading of the Plate Bearing Test and two forms of dynamic loading which apply either steady state sinusoidally varying force of a single impulse transient force, to the pavement surface. The pavement is loaded by a pulse of constant duration irrespective of depth and the planes of principle stress do not rotate. Dynamic deflection is represented by a curve, indicating pavement reaction, with the amplitude of the deflection expressed in terms of the frequency of application of the load. A particular deflection value can be determined for a given frequency from the curve. Stressstrain conditions generated by these tests are not representative of conditions in the pavement structure under normal loading conditions and the deflection response of frequency and stress dependent materials will be affected.

#### 2.4.1 Plate Bearing Test.

The Plate Bearing test consists of an almost static load applied to a circular loading plate. The pavement is usually subjected to a number of load cycles before the rebound deflection is measured. There are several test methods and the results obtained depend on the equipment used, load, duration of the load application, stiffness and diameter of the plate and the location of the measuring devices.

Repeated static Plate Bearing Tests to determine pavement layers properties (estimate of layer moduli) and deflection are commonly used in Denmark and Finland (27). The test is slow and in general is not accurate. The speed of operation limits the number of test results that can be obtained, usually about 2 to 10 measurements per day, and these are usually insufficient to assess the homogeneity of the pavement with distance.

#### 2.4.1.1 Pavement Evaluation.

#### It is possible to:

- (a) measure the deflection under a given load;
- (b) measure the load corresponding to a given deflection;
- (c) determine the elastic deformation modulus corresponding to a pressure/deformation curve;
- (d) determine the modulus of each layer of a 2 layer system by using several loads with plates of two or three different diameters.

The deflection is measured through the centre of the plate. The repeated rebound deflection is used for calculation of E-values.

# 2.4.2 The Falling Weight Deflectometer.

The Falling Weight Deflectometer equipment (27) applies a pulse load by dropping a weight of 150 Kg onto a set spring system which in turn transmits a load pulse of about 28 ms duration to the road surface by means of a circular plate of 300 mm diameter. The maximum force developed is 60 KN when dropped from 400 mm high. The deflection of the pavement is measured by means of velocity transducers, one in the centre of the loaded area and up to 5 transducers at a fixed distance from the load. This enables the shape of the deflection bowl to be characterized. The equipment is carried on a single axle trailer towed by a vehicle carrying the power supply and recording equipment. The towing vehicle also houses the controls for the loading cycle and for the operation of the hydraulic jack, used to raise the trailer when a test is being carried out. Maximum output is of the order of 200 measurements per day.

The equipment does not measure the absolute deflections, because of the seating load, but a datum is fixed. Repeat measurements are required because of the single load pulse method of testing. The pattern of load pulse developed within the structure differs from that developed under traffic in that its duration is independent of depth and principal stresses do not rotate.

A parameter,  $Q_r'$ , is used to define the ratio of the deflection  $(d_r)$  at a distance 'r' from the load to the deflection under the centre of the test load  $(d_0)$ . Claessan et al (29) stated that "the parameter  $Q_r'$  was choosen instead of the radius of curvature, because  $Q_r'$  can be measured more easily and provides equivalent information". For typical structures, 'r' has been fixed at a distance of 600 mm and the curvature expressed as the  $Q_{600}$  ratio.

# 2.4.2.1 Pavement Evatuation.

Using the BISAR computer program (30), graphs such as that reproduced as Fig 2.10 (29) have been prepared giving the relationship between  $E_1$  (modulus of the asphalt layer),  $d_0$ ,  $Q_r$  and  $H_1$ (thickness of the asphalt layer) for predetermined values of  $E_2$ (modulus of unbound or cemented base layer),  $H_2$  (thickness of base layers),  $E_3$  (modulus of the subgrade) and the distance r, for a given test load. From such graphs, with  $d_0$  and  $Q_r$  measured, two unknown structural parameters can be determined if the other variables are known or can be estimated.

In the studies carried out by Van der Poel (31), the deflection is calculated at three points of the pavement surface, one at the centre of the loaded area ( $d_0$ ), one at a distance of 600 mm of the centre ( $d_{600}$ ) and one at a distance of 2000 mm ( $d_{2000}$ ). The shape of the deflection bowl is characterised by the surface curvature index (SCI), calculated by subtracting  $d_{600}$  from  $d_0$ . In relation to the maximum deflection ( $d_0$ ), the SCI value gives an indication of the pavement properties, while  $d_{2000}$  can be related to the bearing capacity of the subgrade. As described in Ref (31) the modulus of the subgrade can be calculated from the deflection,  $d_r$ , measured at a distance, r, from a loaded area with a contact stress, q, and a radius of 'a':

$$E_{r} = \frac{qa^{2}(1-\mu^{2})}{d_{r} \times r}$$
 (2.5)

where  $\mu$  = Poisson's ratio.

The major restriction to the use of this form of equipment for pavement evaluation is the strong dependence of predicted pavement material properties on the measurement of maximum deflection and deflected shape; information on the repeat ibility of the measurements has not been published.



Fig 2.10 Deflection Interpretation Chart (after Claessen et al, Ref 29).

2.4.3 Dynaflect.

The Dynaflect (32) is an electromechanical system for measuring the dynamic deflection on the road's surface caused by oscillatory load. Measurements are independent of a fixed surface reference. It consists of a small, two wheeled, trailer, carrying a dynamic sinusoidal force generator in the form of two contro-rotating masses. The dynamic force varies at a fixed frequency of 8 Hz with a maximum value of 227 N, which is superimposed on a static load of 725 Kg and is transmitted to the road surface through the two small rigid wheels located 0.5 m apart. The maximum deflection is recorded midway between the wheels together with the deflection at four other points along the centre line, between the wheels, spaced in the longitudinal direction at 305 mm intervals. Deflections are measured with velocity sensitive transducers and are recorded by equipment carried in the towing vehicle. The deflection basin parameters associated with the Dynaflect measurement are shown in Fig 2.11. The five parameters associated with each Dynaflect measurement are described as follows (32):

(a) Dynaflect Maximum Deflection (DMD)  $= W_1$  (1st sensor deflection); measure of pavement structural characteristics and support condition. (b) Surface Curvature Index (SCI)  $= W_1 - W_2$  (differential deflection between the 1st and 2nd sensors); an indicator of the structural condition of the surface layer. (c) Base Curvature Index (BCI)  $= W_4 - W_5$  (differential

> deflection between the 4th and 5th sensor); measure the base support conditions.

(d) Spreadability (SP%), 
$$\frac{[100(W_1 + W_2 + W_3 + W_4 + W_5)]}{5W_1}$$

= (average deflection as a percentage of the maximum); measure the load carrying

capacity and stiffness ratio of the pavement structure. = W<sub>5</sub>; indirect measure of subgrade modulus.

(e) Fifth Sensor Deflection  $(W_5)$ 



Fig 2.11 Dynaflect Deflection Basin Parameters

(after Bandyopadhyay, Ref 32).

#### 2.4.3.1 Pavement Evaluation.

Studies of pavement condition evaluation from Dynaflect deflection basin parameters have been carried out by several authors (15, 32, 33) and several conclusions have been deduced from their work. DMD is a measure of pavement structure characteristic and support conditions.

SCI is predominantly an indicator of the structural condition of the surface layer. SCI is inversely proportional to the radius of

curvature and is therefore a measure of tensile strains in the pavement. Majidzadeh (15) showed that SCI is dependent on pavement thick-E1/E2 (pavement modulus/support ness and the modular ratio modulus) using a two layer elastic model. For a relatively thick pavement, SCI decreases with an increase in the modular ratio  $E_1/E_2$ , whereas for a thin pavement, the  $SCI-E_1/E_2$  relation depends greatly on the magnitude of thickness, H (pavement thickness), an increase in thickness reduces the SCI parameters. It was shown (15) that the tensile strain at the bottom of the pavement layer and the vertical strains on the subgrade are proportional to the SCI. It has been reported (33) that the SCI parameters could be used for detection of problem areas in the pavement layer. The adequacy of load transfer at joints, effect of transverse crack planes and roads containing concrete quality have been evaluated in numerous pavements in Ohio (U.S.A.) using the SCI parameter. This parameter can be used to detail areas of deficiency at the pavement surface layer.

The BCI has been widely used as a parameter suitable for detection of problems in the subgrade and the base layers. BCI values ranging from 0.05 to 0.11 are generally representative of satisfactory performance, whereas poorly performing pavements show BCI values greater than 0.15 to 0.2.

 $W_5$  has been shown (33) to be an indirect measure of subgrade modulus and is relatively insensitive to pavement thickness and modulus of surface layer.

The SP% describes the slab action of the pavement and its ability to distribute load. Pavements with high SP% values distribute loads more effectively and the resulting stresses and strains on the subgrade are smaller. The ability of the pavement to absorb the

induced stresses is reflected in the spreadibility parameter, which, in turn, is a function of pavement thickness (H), modular ratio,  $E_1/E_2$  (pavement modulus/support modulus), and other geometrical variables. It was shown (33) that a higher SP% is not always a guarantee for satisfactory pavement performance, especially for thin pavement where the effectiveness of load distribution decreases with a decrease in pavement thickness. A structural diagnosis chart based on Dynaflect deflection parameters, prepared by Bandyopadhyay (32), is shown in Table 2.1.

The main drawback of this system is that the low force level does not represent lorry traffic and a fixed frequency of 8 Hz sometimes produces resonance. The equipment can therefore be used to identify possible weak areas but the estimation of material properties from the results must be questionable.

DMD	SCI	BCI	CONDITION OF PAVEMENT STRUCTURE
(	CHECK EACH	EVALUATION	WITH SP% AND WS CRITERIA
G.T. I.40	G.T. 0.16	G.T. 0.12	PAVEMENT AND SUBGRADE WEAK
		L.E. 0.12	SUBGRADE STRONG, PAVEMENT WEAK
	L.E. 0.16	G.T. 0.12	SUBGRADE WEAK, PAVEMENT MARGINAL
		L.E. 0.12	DMD HIGH, STRUCTURE OK
L.E. I.40	G.T. 0.16	G.T. 0.12	STRUCTURE MARGINAL, DMD OK
		L.E. 0.12	PAVEMENT WEAK, DMD OK
	L.E. 0.16	G.T. 0.12	SUBGRADE WEAK, DND OK
		L.E. 0.12	PAVEMENT AND SUBGRADE STRONG

Table 2.1 Structural Diagnosis Chart Based on Dynaflect Deflections (after Bandyopadhyay, Ref 32).

#### 2.4.4 Road Rater.

The Road Rater (34) is powered from an electro-hydraulic power unit attached to a van or truck. It serves as a means of measuring a road's strength by the application of a dynamic load of 445 N, operating at frequencies between 10 and 50 Hz superimposed on a static load of 500 Kg. The road's deflection profile, measured by four velocity sensors housed on a boom and spaced 0.35 m apart, is displayed visually on instrumentation located inside the vehicle. About 300 measurements at a single frequency or alternatively 100 at 3 frequency are possible in a working day. The actual number is dependent on the spacing of the points of measurement. This equipment is used in the U.S.A. and in Italy (16).

Kilareki et al (35), used the Road Rater deflection basin to calculate the Surface Curvature Index (SCI), the Base Curvature Index (BCI) and the Base Damage Index (BDI). These criteria were correlated with the maximum tensile strain, the fatigue cracking of the pavement sections and traffic loading; a relationship was established between the SCI and the traffic loading resulting in the prediction of remaining pavement life.

The main drawback is similar to that of the Dynaflect in that unrealistically small loads are applied in relation to those applied by lorry traffic.

# 2.4.4.1 Pavement Evaluation.

Using the BISAR computer program (30), the deflection values corresponding to the assumed modulus values are computed (34). The calculated deflections are then compared with the measured deflections and the assumed modulus values are corrected. The correction begins

from the subgrade modulus and proceeds successively to sub-base, base and surface moduli. This iteration process is repeated until the difference between the calculated and the measured deflections for the four sensors are within the specified tolerance. In the theoretical development of the modulus evaluation procedure, it was shown that the deflection at the fourth sensor is essentially dependent on the subgrade modulus only, whereas, the deflection at the third sensor can be approximated with sufficient degree of accuracy as a function of only the moduli of sub-base and subgrade. The following relationship were suggested (34):

$$E_4 = 67.88 \times S_4^{-1.04}$$
(2.6)  

$$E_3 = 1492.85 \times S_3^{-7.26} \times S_4^{-6.24}$$
(2.7)  
where  $E_4 =$  Subgrade modulus in 10<sup>5</sup> lbf/in<sup>2</sup>,  

$$E_3 =$$
 Sub-base modulus in 10<sup>5</sup> lbf/in<sup>2</sup>,  

$$S_4 =$$
 Deflection at fourth sensor in 10<sup>-6</sup> in,  

$$S_3 =$$
 Deflection at third sensor in 10<sup>-6</sup> in.

It was reported (34) that the modulus values of surface, base and subgrade will be greater and those of the sub-base will be lower when evaluated from Road Rater deflections compared to those determined from repeated load and plate load tests. The discrepancy in the modulus values could be due to the effects of dynamic loading and stress (or strain) level effects. Plate load test is a static test and the repeated load test can also be considered as a static load test because of its loading frequency (20 cycles/min) when compared with the Road Rater loading with a frequency of 25 Hz or 1500 cycles/min. The stress level in the pavement induced by the Road Rater is considerably lower than those generated by the plate load. Therefore, under the low confining pressure, the sub-base material will exhibit a smaller modulus. At a deeper level, the low deviator stress level induced by the Road Rater results in a significantly higher subgrade modulus compared to those determined from plate load and repeated load test. Therefore the combined effects of dynamic loading and low stress intensity yield higher modulus values for the subgrade and lower modulus values for the sub-base material when compared with the modulus values determined from the laboratory repeated load tests and field plate load tests.

Kilareski et al (35) reported that the surface deflection at geophone No.1, d, (located at the centre between the loading plate), is more sensitive to the variation of the surface modulus,  $E_1$ , than  $d_2$ . Both  $d_3$  and  $d_4$  are insensitive to the variation of the modulus  $E_1$ . A similar effect was observed with the variation of the base modulus ,  $E_2$  as that of  $E_1$ . The sub-base modulus  $E_3$ , significantly affects, all deflection values except for  $d_{\underline{A}}$  (geophone deflection No 4). All four deflection values decrease considerably with an increase in the subgrade modulus  $\mathbf{E}_{\mathtt{A}}$  . The BISAR computer program (30), was used to analyse a four layer elastic system, and the deflection values  $d_1$ ,  $d_2$ ,  $d_3$  and  $d_4$  corresponding to the assumed values of moduli were calculated (34). These calculated deflections are compared with deflections measured at the four geophones. New assumed modulus values based on the previous values were calculated using the equation stated in Ref (35). The iteration process is repeated until the difference between the calculated and measured deflection for all sensors are within a specified tolerance.

# 2.5 Wave Propogation Technique.

The structural condition of the pavements can also be assessed ed by measuring the velocity of propagation and length of waves propogating from an applied dynamic load (36). Steady state loading is normally used. High frequency produces a short wave length surface or Rayleigh Waves, which travel at speeds determined by the surface layer. At lower frequency, wavelengths several times that of the structure are produced and these characterise the lower layers of the pavement and the subgrade.

### 2.5.1 Pavement Evaluation.

The stiffness of the surfacing and subgrade layers can be determined by extrapolating parts of the dispersion curve (plot showing the relation between phase velocity = frequency x wavelength, and wavelength) to zero wavelength. Interpretation of the remainder of the dispersion curve can only be achieved by the comparison with the theortical curves based on the theory of elasticity. Using the Wave Propagation techniques it is only possible to cover two sites per day. The main drawback of the method is therefore that it is very slow. In addition, the stress level imposed by these techniques is small and the pavement stiffness obtained will therefore, differ from that generated under moving traffic loads.

# 2.6 Factors Affecting the Selection of Pavement Evaluation Equipment.

The following factors must be considered before undertaking any structural strength measuring equipment:

- (a) General Requirement.
  - (i) the total distance and range of environmental conditions of

the site;

- (ii) the purpose of the investigation failure or routine structural strength survey;
- (iii) the range of construction and material type to be tested;
  - (iv) availability of suitable skilled staff to operate the equipment and to analyse and interpret the data.
- (b) Equipment Requirements.
  - (i) suitability for daily and yearly route capacity;
  - (ii) ability to test a wide range of construction and material types;
  - (iii) provision of a rapid system for processing and assesing the collected data;
    - (iv) should be linked to an authoritative design method.

For the output of the method to be effective in designing strengthening measures, it is important that the measurement input to any design method is sufficient.

# 2.7 Justification for the Selection of Deflectograph.

Research carried out over a number of years at the TRRL, has established that a significant relation exists between the magnitude of the deflection of a road pavement under a rolling wheel moving at creep speed and the structural performance of that road. Under traffic, systematic measurements of deflection obtained on both orginal and overlaid pavements has provided the basis for prediction of the unexpired lives of pavements and for the design of bituminous overlays to extend their lives.

In selecting a method of measurement, the TRRL opted for the realism of the rolling wheel and given the empirical approach orginally planned, accepted the Deflection Beam for development into a standard measurement technique. However, extended surveys over long lengths of motorway and trunk road, demonstrated the need for a measurement techniques of greater capacity than that provided by the Deflection Beam, which testing at 10 m intervals, can cover only 1 Km of road in a working day.

The TRRL therefore purchased the Deflectograph which carries out measurement of deflection at closely spaced intervals and it is possible to survey 8 to 10 Km in a working day. Despite the drawbacks given in Section 2.3.2, its use for measuring curvature has been investigated because:

- (a) It is used extensively for monitoring highway networks;
- (b) It has the greatest route capacity and the ability to take measurement at short intervals compared to the other equipment;
- (c) The performance aspect of existing design method is accepted;
- (d) If the curvature information could also be obtained and interpreted, it would give a very powerful evaluation system.

#### 3.1 Introduction.

In the design of a road pavement there are two aspects of material properties which have to be considered. Firstly, the loaddeformation or the stress-strain characteristics of the material used in the various layers of the structure need to be known so that analysis of the stresses and strains at the critical points can be performed. Secondly, information is required on the likely mode of failure of the various materials under repeated application of load so that the design criteria in the form of maximum allowable stresses or strains can be incorporated into a design procedure. The material used in the flexible pavement construction fall essentially into four categories, bituminous bound, cement bound, unbound aggregate and cohesive subgrade soil. Non-cohesive material can be classified as unbound aggregate. All these materials are non-homogenous, anisotropic, nonlinear and generally non-elastic.

Pavement performance does not depend solely on the charactistic of the materials in the individual layers but rather on the interaction of the various layers. This interaction of the layers will itself depend on the thickness of the layers as well as the material characteristics.

# 3.2 Bituminous Materials.

Bituminous material is mineral aggregate mixed with either tar or bitumen or with one of the low viscosity cut back bitumen or bitumen emulsion binders. Traditionally in the U.K., bituminous road

materials can be classified under three basic mixes types which are: (a) Hot Rolled Asphalt (BS594, Ref 37);

(b) Dense Bitumen (or tar) Macadam (BS4987, Ref 38);

(c) Open Textured Macadam (BS4987, Ref 38).

# 3.2.1 Hot Rolled Asphalt (HRA, Ref 37).

This is a gap graded material including filler and a relatively hard grade of bitumen. The bitumen content is high and the mechanical properties of the material are influenced primarily by those of the bitumen sand-filler mortar. The coarse aggregate bulks the mixture and contributes to its stability. Fine aggregates forms the major proportion of the mortar while the filler stiffens and strengthens the binder. Bitumen binds the whole mixture together and provides water proofing. Compacted rolled asphalt mixes are dense and basically impervious from the time of laying and possess good load spreading properties and durability. HRA is extensively used for the wearing course, on heavily trafficked roads, but can also be used for the base course and the roadbase. It is easy to compact and therefore relatively insensitive to site conditions and quality of technical supervision.

#### 3.2.2 Dense Bitumen macadam (DBM, Ref 38).

This is a uniformly graded material often using a softer binder than rolled asphalt. Its properties are partly influenced by interparticle friction and interlock, which is the characteristic of a macadam, and partly by the viscosity of the binder. The coarse aggregate provides the main skeleton of the interlocked aggregate and provides the distribution of the traffic loads. Fine aggregate fills, or

partially fills the voids in the coarse aggregate skeleton and it has a significant bearing on the finished surface texture. Filler increases the viscosity of the binder, thus reducing the risk of binder draining from the aggregate. Combined with the binder, it helps to fill small voids. Binder acts as a lubricant between the aggregate particles particularly during compaction and also as a water proofing and bonding agent.

The bitumen content is relatively low and this makes it a cheaper material than rolled asphalt. These materials tend to be used for the roadbase and the base course layers, though they may also be used for surfacing (39).

# 3.2.3 Open Textured Macadam (38).

This is a uniformly graded material with low binder content. It has a high void content when compacted and therefore it is not used as a major structural layer. The main function of its ingredients are the same as those of DBM. It is used for surfacing minor roads and has the advantage of providing a relatively dry surface under wet conditions. The ability of very open textured mixes to drain surface water has been exploited for surfacing runways when the material is underlaid by an impermeable layer.

# 3.3 Stiffness Characteristics.

Bituminous materials are viscoelastic and therefore their stress-strains characteristics are dependent on both time of loading (t) and temperature (T). Van der Poel (40) defined the stress-strain relationship as the stiffness (S):

S(t, T) = Stress/Total Strain (3.1)

The variation of stiffness with time and temperature is shown in Fig 3.1 for a typical bitumen stiffness. The mix stiffness depends on the mix variables such as aggregate type, and grading, bitumen type and content and degree of mix compaction and the resulting air voids content.



Fig 3.1 Variation of Bitumen Stiffness With Time and Temperature (after Brown, Ref 40)

# 3.3.1 Estimation of Bitumen and Mix Stiffness.

# 3.3.1.1 Bitumen Stiffness.

Bitumen stiffness varies with loading time and temperature exhibiting elastic behaviour at low temperature and short loading time, viscous behaviour at high temperature and long loading time and viscoelastic response in between. This is illustrated clearly in Fig 3.1. The stiffness of bitumen, S<sub>b</sub>, can be estimated with the aid of a nomograph, see Fig 3.2, developed by Van der Poel (41) and which requires the following data:
- (a) Loading time, t(sec) or frequency, f(Hz);
- (b) Temperature,  $T(^{0}C)$ ;
- (c) Softening Point (recovered,  $SP_r$ ) from the Ring and Ball Test,  $SP_r^{(0}C)$ ;
- (d) Penetration Index, PI, which is a measure of temperature suscept ibity.

The penetration index (recovered) is calculated using the following equation (42):

$$PI_{r} = \frac{1951 - 500 \log P_{r} - 20 SP_{r}}{50 \log P_{r} - SP_{r} - 120.1}$$
(3.2)

If recovered binder properties are not available, then they can be estimated from the initial properties using the following equations (41):

$$P_r = 0.65 P_i$$
 (3.3)  
 $SP_r = 98.4 - 26.4 \log P_r$  (3.4)

The loading time may be determined from Fig 3.3, which requires an estimate of the asphalt layer thichness (H mm) or alternatively the following equation (42) may be used:

$$\log t = 5 \times 10^{-4} \text{ H} - 0.2 - 0.94 \log V \quad (3.5)$$

As an alternative to using the nonograph, the following equation (42) may be used to calculate the binder stiffness:

$$S_{b} = 1.157 \times 10^{-7} \times t^{-0.368} \times 2.718^{-PI} r \times (SP_{r} - T)^{5}$$
(3.6)

The ranges of applicability of this equation are as follows:

$$PI_r = -1 \text{ to } 1,$$
  $(SP_r - T) = 20 \text{ to } 60 {}^{0}C,$   
t = 0.01 to 0.1 sec,  $S_b = MPa$ 



Fig 3.2 Nonograph for Determining the Stiffness Modulus of Bitumens

(after Van der Poel, Ref 41).





# 3.3.1.2 Mix Stiffness.

Van der Poel (40) has shown that the stiffness of a mix  $(S_m)$  containing dense graded aggregate and bitumen is primarily dependent on the aggregate. Heukelom and Klomp (43) extended the work of Van der Poel and deduced the following relationship for the mix stiffness:

$$\frac{S_{m}}{S_{b}} = \begin{bmatrix} 1 + \frac{2.5}{n} \times \frac{C_{v}}{1 - C_{v}} \end{bmatrix}^{n}$$
(3.7)
where n = 0.83 log  $\begin{bmatrix} 4 \times 10^{4} \\ S_{b} \end{bmatrix}$ ,  $C_{v} = \frac{\text{Vol of Agg}}{\text{Vol of Agg + Vol of bitumen}}$ 

 $S_{\rm b}$  = stiffness of bitumen from nonograph in MN/m<sup>2</sup>.

This expression in only valid when the bitumen stiffness is greater than 10 MN/m<sup>2</sup>, for  $C_v$  between 0.7 to 0.9 and with compacted mixes having about 3% air voids. Van Drat and Sommer (40) have proposed a correction to  $C_v$  for mixes having air voids greater than 3% :

$$C_{v}' = \frac{C_{v}}{1 + (v_{v} - 0.03)}$$
(3.8)

where  $V_v = Volume$  of voids.

Typical creep test results are shown in Fig 3.4 for a dense mix having a  $C_v$  value of 0.85. A single relationship between  $S_m$ and  $S_b$  exists under high bitumen stiffness, but at low bitumen stiffness the location and shape of the curve depends on the composition of the mix, properties and grading of the aggregate and the state and method of compaction of the mix.



Fig 3.4 Relationship Between Mix Stiffness and Binder Stiffness (after Brown, Ref 40).

Researchers from the Shell Organization have presented (44) a new nonograph, see Fig 3.5, from which the mix stiffness can be derived for a given volume concentration of aggregate and bitumen and a given bitumen stiffness. Van der Poel's (40) nonograph formed the basis for this new nomograph.



Fig 3.5 Nomograph for Mix Stiffness (after Claessen et al, Ref 44).

For typical U.K. dense bitumen macadam (DBM) and rolled asphalt (RA) mixes, Brown (45) modified the Shell nomograph and developed an equation to determine the Young modulus (stiffness) as a function of the temperature, vehicle speed and the layer thickness:  $Log_{10}(E) = Log(aT^2-bT+c) - 10^{-4}(dT^2+eT+f) \times (0.5H-0.2-0.94LogV)$ where: E = Young Modulus (stiffness),  $MN/m^2$ , (3.9)

- V = Vehicle Speed, Km/hr,
- T = Temperature, C,
- H = Layer Thickness, m,

	a	b	C	a	Ą	£
ואת	10.2	557	8120	1.8	36.0	1470
ΒV	20.2	793	10 <b>3</b> 60	0	51.5	1200

a, b, c, d, e, f = Material Constants as given below:

### 3.3.2 Measured Stiffness.

The nomograph of Van der Poel (40) and the latest Shell nomograph may be used to estimate the value of stiffness over the range of temperature and loading times associated with pavement design requirements to within a factor of about 1.5 to 2.0 times the indicated values. However, if a more accurate value of stiffness of a particular mix is required, and also preferably for the use of the analytical design method, it is necessary to measure the actual material response under loading conditions, including stress levels, to be used in practice. The types of test which have been used to measure stiffness as a function of time are as follows:

- (a) Creep under constant stress;
- (b) Stress relaxation under constant strains;
- (c) Constant rate of strain;

(d) Dynamic tests under sinusoidal stress or strain;

(e) Dynamic tests under repeat step function pulse loading. More details of the testing procedure and the necessary testing equipment are presented in Ref (46, 47).

The work of Van der Poel (40) and others mentioned above is based on the assumption that bituminous mixes behave as linear viscoelastic material but careful measurement of dynamic stiffness has indicated that two conditions of behaviour occur, linear (stiffness independent of stress level) below a certain stress, and non-linear at higher stress values. The non-linear behaviour appears to depend on the composition and void content of the mix as well as the temperature and loading conditions. The effect of the composition and void content of the mix on stiffness is small when compared with the effect of temperature and speed of loading and may safely be ignored in most cases.

### 3.3.2.1 Dynamic or Repeated Load Triaxial Test.

When a rolling wheel load passes over a point in a road structure, the various layers are subjected to stress variations like those shown in Fig 3.6. These variations will differ between layers and between points in the same layer, but the basic pattern is similar everywhere. The repeated load triaxial test is the best practical method for the testing of the pavement material as it allows the determination of both resilient and permanent strains. The major limitation of the triaxial test is that only principal stresses can be applied to a test specimen. Furthermore, as shown in Fig 3.7, two of the principal stresses are necessarily equal because of the axial symmetry of the arrangment.

The cylindrical sample is subjected to an equal all-round pressure, the confined stress, on which an additional axial stress, the deviator stress, is added. Fig 3.8 shows the the applied stress and the form of the measured strain response. Sinusoidal loading is usually applied. For the measurement of resilient behaviour, a short interval between pulses would seem advisable to allow for the delayed elastic recovery.



Fig 3.6 Stresses on a Soil Element due to a Passing Wheel load (after Pell, Ref 46).

Barksdale (48) has presented a chart, see Fig 3.9, with which to determine the length of the pulse corresponding to various vehicle speeds and depths in the pavement. Resilient modulus (stiffness),  $M_r$ , has been defined, which is analogous to Young's modulus and is calculated from (see Fig 3.8, Ref 46):

$$M_{r} = \frac{\text{Applied deviator stress}}{\text{Vertical resilient strain}} = \frac{\sigma_{1} - \sigma_{3}}{\epsilon_{vr}} (3.10)$$

Kallas (49) investigated the dynamic modulus of the bituminous material in tension, tension-compression and compression. He found little difference between the results except under the condition of low frequency (1 Hz) combined with high temperature (21 to 38  $^{\circ}$ C) when the dynamic tension or tension-compression modulus could be as low as one half of the dynamic compression modulus. The Deflectograph (18) moves at about 2 Km/hr, i.e. loading of low frequency, and it is usually operated at an average temperature of 25  $^{\circ}$ C. Therefore, under these loading conditions the dynamic modulus (stiffness) of the bituminous material should be determined from compression testing.



σ<sub>1</sub> = Major principal stress σ<sub>3</sub> = Minor principal stress = Cell pressure

(01-03) = Deviator stress

Fig 3.7 Stress System in the Triaxial Test

(after Pell, Ref 45).

Bituminous core samples obtained from some of the AASHO Road Test pavement sections were tested under repeated load conditions to determine dynamic complex modulus (stiffness) values (50). Each of the core samples were subjected to various loading conditions and temperatures. The change in the vertical stress did not have an appreciable influence in the dynamic modulus; however, the dynamic modulus significantly varied with a change in temperature (50). The resilient modulus (stiffness),  $M_r$ , obtained from the repeated load triaxial test is defined as the deviator (or repeated axial) stress divided by the recoverable strain (or resilient strain), associated with the bounce of the specimen (51). By definition, a secant modulus is obtained that corresponds to the minimum value occurring during the loading portion of the test.



Fig 3.8 Measurements in Repeated Load Triaxial Test with Constant Cell Pressure (after Pell, Ref 46).

Kallas and Riley (52) carried out repeated load tests on unconfined cylindrical samples of asphalt paving mixtures at several temperatures. The resultant axial and radial strains were measured and the dynamic complex moduli and the Poisson's ratio were determined. The vertical strains were measured with strain gauges cemented to the specimen at mid-height with epoxy cement. The horizontal strains were measured with strain gauges cemented at mid-height to specimens, at right angles to the direction of loading.





Absolute values of the complex modulus (stiffness),  $|\mathbf{E}|^*$ , and the phase lag,  $\theta$ , were calculated as follows (52):

$$\left|\mathbf{E}\right|^{\star} = \frac{\sigma_{\varrho}}{\epsilon_{\varrho}} \tag{3.11}$$

where |E| = absolute value of the complex modulus;

 $\sigma_{0}$  = amplitude of the sinusoidal vertical stress;

 $\epsilon_{\varphi}$  = amplitude of resultant sinusoidal steady state vertical strain.

and 
$$\theta = \frac{t_i (360^0)}{t_p}$$
 (3.12)

where  $\theta$  = phase lag, degree,

t; = time lag between a cycle of sinusoidal stress

and the resultant cycle of sinusoidal strain, sec,

 $t_{n}$  = time for a cycle of sinusoidal stress, sec.

The complex modulus stiffness is found to decrease with increasing temperature and with decreasing frequency, i.e. complex modulus is dependent on temperature and loading frequency (52). The complex modulus increases with increasing vertical stress. Since the increment is relatively small, the effect of the vertical stress on the modulus is not of practical importance.

Khanna et al (53) determined the stiffness of bituminous material by triaxial tests. In a loaded pavement system, the vertical stresses in the bituminous surface course are always compressive whilst the horizontal stresses may be compressive or tensile in nature. The condition when horizontal stresses are compressive, was stimulated by triaxial tests. To stimulate horizontal tension in the bituminous layer, a test method was developed making use of thick hollow cyclindrical triaxial specimens. The specimens were subjected to internal radial pressure and tested under axial compressive loading. The stiffness was found to be dependent on the axial stress and internal radial pressure. An analytical finite element model was developed to predict load response characteristics of the multi-layer pavement system. The stiffness determined from the above tests were used as inputs into the analytical model to predict the load deformation responses of the semi-full scale pavement test sections. The predictions from the developed model show closer agreement with the experimental result in comparison with classic linear elastic model (53).

Poisson's ratio is defined as equal to the lateral strain divided by the axial strain.

Poisson's ratio may be calculated as:

$$\mu = \frac{\epsilon_{\rm rr}}{\epsilon_{\rm vr}} \tag{3.13}$$

Poisson's ratio depends on temperature and therefore it has a minimum value at low temperature and increases with increasing temperature (46, 52). Poisson's ratio also decreases with increasing frequency. Tests carried out in a constant rate of strain triaxial compression showed that Poisson's ratio was essentially independent of rate of strain, but at 4  $^{0}$ C had a value of approximately 0.35 and at 25  $^{0}$ C and at 60  $^{0}$ C a value approaching 0.5. When deformation is large, it is possible that "dilation" (volume increase) will occur and Poisson's ratio will exceed 0.5.

### 3.3.3 Permanent Deformation.

## 3.3.3.1 Introduction.

A significant cause of pavement failure is the development of excessive permanent deformation, a rut, which at 10 mm depth is regarded as critical and at 20 mm depth constitutes failure (54). The surface deformation is a summation of the vertical deformation developing in each layer of the pavement structure. The viscous response of bituminous materials can contribute substantially to the total deformation recorded at the surface, particularly when operating under high temperature conditions.

## 3.3.3.2 Creep Testing.

The simplest way to investigate the permanent deformation characteristics of bituminous materials is by the use of the static creep test. The Shell Laboratories (40) conducted a large number of creep tests involving simple uniaxial loading in compression in order to study the effect of mix parameters on permanent deformation. The effects of temperature, binder type and applied stress can be ignored

if the results are plotted in terms of a mix stiffness against binder stiffness, see Fig 3.4. On the  $S_m$  against  $S_b$  plot, see Fig 3.10, the creep results move from right to left since increasing time implies decreasing bitumen stiffness (55). Mixes made with crushed sand provide higher mix stiffness and therefore, lower permanent deformation than mixes made with rounded sand. The compaction method and compactive effort influences the density and the packing arrangement which in turn affects the mix stiffness (40).



Fig 3.10 Creep Results (after Hills et al, Ref 55).

Brown et al (51) carried out some dynamic triaxial and static creep tests on dense bitumen macadam and produced the graphs in Fig 3.11. It can be seen that at low stress levels the static and dynamic results are similar. However, at the higher stress levels, a static stress of only about 65% of the dynamic value would be required to produce the same strain at a particular time. The effects of different stress levels and temperature (51) are shown in Figs 3.12 and 3.13.



Fig 3.11 Comparison of results from Dynamic Tests and Creep Test at 20  $^{\circ}$ C (after Brown et al, Ref 51).



Fig 3.12 Creep Tests- Effect of Stress Level



Fig 3.13 Creep Tests-Effect of Temperature

(after Brown et al, Ref 51).

# 3.3.3.3 Repeated Load Testing.

Pell et al (56) have indicated that the repeated load, triaxial test, which has been used extensively for the evaluation of resilient properties of materials, could be used for studying permanent deformation under more realistic conditions than those of the creep test. Brown et al (51) have shown that the permanent strain which gradually accumulates under repeated loading is essentially a creep phenomenon. They studied the effects of vertical stress, temperature, confining stress, frequency of the vertical stress pulse, rest periods and binder content by carrying out triaxial tests on dense bitumen macadam.

Brown et al (57) measured the sample deformation over two diametrically opposed gauge lengths about the mid-height of the sample. The measurements are thus free of specimen end effects and errors which can be introduced by the use of an arbitary datum such as the cell base or the cell lid. They used a set of three "strain collars"; one pair measured the vertical deformation while the third monitored lateral deformation (57). Four locating targets, glued to the specimen, are at each end of the two gauge lengths. The relative deformation of each pair of targets are measured by using a pair of linear variable differential transformers (L.V.D.T). The central collar is used for radial deformation measurement. It is a hinged collar locating onto two diametrically opposed targets at the centre of the sample. As the sample expands laterally, the gap between free end of the collar changes and this change is measured by an L.V.D.T. The samples were tested with lubricated loading platens to minimize lateral end restraint.

Vertical stress has a major influence on permanent strain as indicated in Fig 3.14. The vertical stress applied consisted of a  $30 \text{ KN/m}^2$  constant value onto which the dynamic pulse was superimposed (51). This value of "dead load" was in fact too high in relation to the likely over-burden pressure to be expected in a pavement.





Permanent strain increases with a decrease of application of the vertical stress, i.e. the total loading time rather than the number of load applications controls the permanent strain. Frequency of loading between 1 and 10 Hz did not affect the relationship between permanent strain and time. Permanent deformation is time rather than frequency dependent and the rest periods between stress pulses does not seem to affect the accumulation of strain.

The effect of static confining stress on the accumulation of vertical strain under a repeated load testing is illustrated in Fig 3.15. Similar effects were noted when the confining stress was applied cyclically in phase with the vertical stress. Brown et al (51) have made a simplification to the test techniques to produce essentially the same response from the material by using a static confining stress equal to the mean values of the desirable cyclic stress.



Fig 3.15 Influence of Confining Stress on Vertical Strain (after Brown et al, Ref 51).

Temperature plays an important part in the build up of permanent strain, see Fig 3.16. Resistance to permanent deformation comes from a combination of binder viscosity which is highest in the rolled asphalt, and interparticle action, which is highest in the dense bitumen macadam.



Fig 3.16 Influence of Temperature on Vertical

Strain (after Brown et al, Ref 51).

Based on field studies, Chou (58) has suggested that the most critical conditions for bituminous material occurs in spring when the subgrade normally contains excessive moisture and provides the least amount of support and the bituminous material is relatively cold and cannot withstand large strain repetitions.

#### 3.3.4 Fatigue Cracking.

#### 3.3.4.1 Introduction.

Under traffic loading the layers of a flexible pavement structure are subjected to a continuous flexing with the life of the pavement decreasing with increasing magnitude of stress and strains. The magnitude of the strain is dependent on the overall stiffness and nature of the pavement construction but measurement has indicated tensile strains of the order  $100 \times 10^{-6}$  for a 3 ton (26.7 KN) wheel load.

Service life  $(N_g)$  is the accumulated number of load applications necessary to cause failure in a test specimen. The fatigue behaviour of a specimen subjected to repeated loading depends primarily on the load, environmental and specimen variables. The service life of specimens tested in the laboratory depends greatly on the mode of testing used, either controlled stress, when the loading is in the nature of alternating stress of constant amplitude, or controlled strain, when the loading is in the form of an applied alternating strain or deflection of constant amplitude.

## 3.3.4.2 Effect of Stiffness and Criterion of Failure.

If specimens are tested in controlled stress, then for different stiffnesses, results such as those shown in Fig 3.17a are obtained (46). At a particular stiffness, S, the mean fatigue lives can be represented by a straight line on a log plot of stress, against the number of cycles of load,  $N_s$ , to cause failure. Different levels of stiffness are represented by approximately parallel lines, showing that with this type of testing the fatigue life is highly dependent on stiffness, the stiffer the mix the longer the life. When the results are replotted in terms of strains, as shown in Fig 3.17b, it can be seen that the results for different stiffness coincide, indicating that strain is a major criterion of failure and that the effects of temperature and speed of loading can be accounted for by their effect on the stiffness-strain criterion (46).

When similar specimens are tested in a controlled strain mode results such as those shown in Fig 3.17c are obtained (46). Although the lines coincide at higher stiffness, those at lower stiffness show the reversed effect of stiffness from that found from controlled stress tests. This is due to the different mode of failure developed in the two tests. In controlled stress, the formation of a crack results in an increase in actual stress at the tip of the crack due to the stress concentration effect and this leads to rapid propagation and complete fracture of the specimen. In controlled strain, the formation of a crack results in a decrease in stress and hence a slower rate of propagation (46).

The criterion of fatigue crack initation is one of applied tensile strain, and a general relationship defining the fatigue life is as follows (46):

$$N_s = C \times (1/\epsilon)^m$$
 (3.14)  
where  $N_s =$  number of application of loads to initiate a fatigue  
crack (failure),

Ś

 $\epsilon$  = maximum value of applied tensile strain,

C, m = Constants which depends on the mix Characteristics. For most dense mixes the factor, m, defines the slope of the strainlife line. The slope of the fatigue line appears to depend on the stiffness characteristic of the mix and nature of the binder; mixes having high stiffness and linear viscoelastic behaviour giving a flatter line. The behaviour is characteristic of mixes having a relatively high binder content of a harder bitumen. Mixes with softer grades of binder show considerable non-linearity, particulary at higher stress levels and they have a steeper fatigue line.

Kirk (59) reported that for a given mix, the initial strain  $\epsilon(n_s)$  corresponding to a given number of cycles to failure is an unequivocal function of the stiffness of the bitumen.  $\epsilon(n_s)$  increases with increasing maximum size aggregate and decreases sharply when the filler content is reduced below a certain limit. With a given gradation containing sufficient filler,  $\epsilon(n_s)$  increases with increasing binder content. This increase is greater, when the mix contains a large number of small voids. The above effects are only valid for mixes with a small number of voids (59).

## 3.3.4.3 Effect of Mix Variables.

It has been found that the fatigue life of a mix at a certain strain level may be increased by the use of a more viscous binder and/or an increase in the binder content. All other mix variables are apparently secondary and only influence fatigue performance in so far as they affect the relative binder content. It is possible to increase the stiffness of a mix by changes in aggregate grading, but if the relative binder volume is unaffected, the fatigue performance at a

certain strain will not improve. It is also possible to increase the fatigue life of a mix at a certain strain whilst reducing the stiffness by increasing the binder content beyond the optimum. Therefore, for good fatigue performance in association with maximum stiffness, a dense grading is required in which the voids are filled with a hard grade of binder (40).

#### 3.3.4.4 Prediction of Fatigue Performance.

Fatigue tests are carried out under simple loading conditions that apply continuous cycles of loading of particular magnitudes under controlled conditions of temperature and speed. In practice, the material is subjected to a succession of load pulses of varying size and with rest period between strain. Since stress controlled tests essentially determine the time for crack initation, some allowance should be made for the time it takes for a crack to propagate up through the bituminous layer. The results of simple loading tests may be used to predict the performance under compound loading condition by the use of Miner's Law, namely (45):

$$\sum_{i=1}^{n} \frac{1}{N_{1}} = 1$$
 (3.15)

where  $n_1 = number$  of cycles of strain applied;

N1 = number of cycles to produce failure under constant strain
amplitude from simple loading tests;

r = number of different strain levels involved.

Chou (58) also suggested that when determining the pavement requirements to prevent excessive strain in the subgrade, the stiffness of the bituminous layer should be evaluated at the highest temperature expected in the field, i.e. when the bituminous material contributes the least amount of resistance to deformation.



Controlled Stress Fatigue Tests At Different Stiffnesses S<sub>1</sub>, S<sub>2</sub> Etc.

Log or v Log Ns



Controlled Stress Fatigue Tests Log E v. Log Ns



Controlled Strain Fatigue Tests Log E v Log N.s.



#### 3.4 Cement Bound Materials.

Cement is utilised as a binder in material used for both base and sub-base layers, producing lean concrete, cement bound granular material and soil cement. Its ability to resist cracking largely depends on the relationship between its stiffness and strength. The dynamic modulus depends on the type and content of cement, water content, aggregate type and grading (56). Its high value of stiffness results in generating tensile stresses of considerable magn itude due to both traffic and temperature. Fig 3.18 shows the relationship between the flexural strength and elastic modulus. It also shows the approximate thermal and traffic stresses for some typical thickness of lean concrete base. It seems that the combined stresses exceed the flexural strength except at the high values of modulus (60). It is considered that a lean concrete base will probably be cracked (primary cracking) to some extent due more to thermal effects than to shrinkage before the permanent construction is complete. Further secondary cracking may or may not occur depending on the magnitude of the traffic-induced stresses. A reduction of approximately 30% in strength can be expected under repeated loading conditions (61). It is suggested (56) that a modulus value of 500  $MN/m^2$  and a Poisson's ratio of 0.25 would be appropriate for design calculations.

Research (56) on soil cement under repeated loading and full scale road experiments show that soil cement layers under heavy traffic quickly become extensively cracked. Therefore for design calculations they should be treated as unbound granular materials having a modulus related to that of the underlying subgrade, and a Poisson's ratio of 0.3.



Fig 3.18 Variation of Induced Stresses and Flexural Strength With Elastic Modulus for Various Lean Concrete Bases (after Lister, Ref 60).

# 3.5 Cohesive Soil.

Cohesive soil has an elastic modulus which is non-linear and very much dependent on stress. The non-linearity can be generally represented by the curves in Fig 3.19.



Fig 3.19 The Effect of Stress Condition on the Resilient Modulus of Cohesive Soil (after Pell et al, Ref 56).

#### 3.5.1 Resilient Behaviour.

The resilient modulus of silty clay is of the form (40):

$$M_{r} = \frac{K}{\left(\frac{q}{r}/\sigma_{3}'\right)^{n}}$$
(3.16)

where  $\sigma'_{3}$  = inital effective confining stress,

q\_ = cyclic deviator stress,

K, n = constants which depend on soil type and test conditions. The load frequency of between 1 and 10 Hz has little effect on resilient strain. The resilient modulus will tend to be lower for load tests with rest periods than for those obtained from continously loaded tests. The rest period allows some elastic recovery in the clay. Confining stress does not influence the resilient modulus significantly but the deviator stress does. Fig 3.20 illustrates the dependence of resilient modulus on the deviator stress for a partially saturated compacted silty clay (61). From the tests carried out on both





saturated and partly saturated soil, it was found that it is the inital effective stress that influences resilient characteristics. This stress is the one to which the elements of soil are subjected prior to the application of wheel loading. The magnitude of this stress depends on the position of the water table and the depth of the elements. For deep water tables, high suction will develop and the magnitude of the suction may be taken as equal to the effective stress. Resilient modulus is related to soil suction. For high water tables, it is possible to estimate the negative pore pressure and hence effective stress of the water table present (40).

$$\sigma_3' = \sigma_3 - U$$
 (3.17)

where  $\sigma_3$ = applied confining stress;

U = initial pore pressure;

 $\sigma'_3 \simeq$  S, Suction, for deep water table.

The elastic modulus may be estimated from the relationship between CBR and dynamic modulus (resilient modulus) obtained from Wave Propagation testings (63) which is usually defined as (44, 64, 65):

$$M_r = 10 \times CBR (MNm^{-2})$$
 (3.18)

although alternative models based on power function have also been proposed (66):

$$M_r = 17.6 \times (CBR)^{0.64} (MPa)$$
 (3.19)

The elastic modulus of clay can also be determined from the Plasticity Index  $(I_p)$  based on reseach at TRRL (67, 68):

$$E_{subgrade} = 70 - I_{p}$$
(3.20)

where E is in MPa and  $I_p$  is a Percentage.

Kirwan et al (69) have developed a chart for the prediction of resilient modulus of glacial till soil from moisture and density information for the plasticity index range from 14 to 20, see Fig 3.21.

Poisson's ratio of 0.5 will be applicable to the saturated condition under undrained conditions. Lower values will apply as the degree of saturation decreases and 0.4 is often taken for design purposes.



Fig 3.21 Relationship Between Relative Compaction, Relative Moisture Content and Resilient Modulus (after Kirwan et al, Ref 69).

## 3.5.2 Permanent Strain.

Permanent strain under repeated loading depends on the applied deviator stress and also on the stress history, density and moisture content of a soil. A definite failure point can be established when the rate of permanent deformation shows a sharp increase which can be seen from the relationship between permanent strain and number of stress applications for normally consolidated silty clay, shown in Fig 3.22. The concept of 'threshold stress' has been used to define the boundary between 'stable behaviour', where no failure will occur, and 'unstable behaviour', where, provided sufficient applications of load are applied, failure will eventually occur (56). For heavily over-consolidated clay, a well defined failure condition is not reached, but strain levels are about twice as high as in the normally consolidated case. Hyde et al (70) have shown that reasonable prediction of permanent strain rate under repeated load conditions can be obtained from the creep tests.



Fig 3.22 Relationship Between Permanent Strain and Number of Stress Application for a Normally Consolidated Saturated Silty Clay (after Lashine, Ref 56).

#### 3.6 Granular Material.

Granular materials are successfuly used as load distributing layers in the pavement structure. Well compacted granular materials are effective in reducing the stresses trans mitted to the generally weaker subgrade. Granular materials generally contain water in their pore spaces and extensive repeated load testing has revealed that the resilient modulus of elasticity is very stress dependent. The nonlinearity is represented by the curves in Fig 3.23.

## 3.6.1 Resilient Behaviour.

The resilient modulus of elasticity being a secant modulus, is a function of both deviator and confining stress. In general, the modulus depends on both shear stress (q/2) and mean normal stress, p, which is related to confining stress.

$$E_{r} = f(q, \sigma_{3}), p = 1/3(\sigma_{1} + \sigma_{3}) = 1/2q + \sigma_{3} \quad (3.21)$$
  
where q = Deviator Stress, p = Mean Normal Stress,

 $\sigma_1$  = Axial Stress,  $\sigma_2 = \sigma_3$  = Radial Stress. Normal stress level has the greatest effect in defining the modulus than shear stress. The non-linear behaviour can be expressed in the form:

$$E_{r} = \kappa_{1}(\theta)^{K_{2}}$$
(3.22)

where  $E_r = resilient modulus;$ 

 $\theta$  = sum of principal stress =  $\sigma_1 + 2\sigma_2 = q + 3\sigma_3$ ; K<sub>1</sub>, K<sub>2</sub> = constant which depends on the material and test condition.

More recent research (71) has shown that the resilient behaviour is much more complex than Eqn (3.22), but under conditions well removed from failure, this equation seems to be valid. Frequency has little or no effect on the resilient strain, although only limited experimental evidence of this is available. The stress history does have some effect on the resilient behaviour. Boyce et al (71) found that the resilient stress decreases and reaches a steady value after about 200 to 1000 cycles, without any substantial increase in permanent strain. It was suggested (70) that resilient strain measurement should be taken after a few cycles (about 4) on each stress path.



Fig 3.23 The Effect of Stress Condition on the Resilient Modulus of Granular Soil (after Pell et al, Ref 56).

In-situ investigation using the Wave Propagation technique has indicated that the modulus of elasticity which a granular layer develops is dictated by the subgrade support conditions (63). In practice, the modulus is normally estimated using Eqn (3.18). Field measurements (63), together with theore tical analysis (72), suggest a modular ratio of two (42) whilst other investigations (73, 74) suggest that the modulus of the granular layer,  $E_2$ , is influenced both by its layer thickness,  $H_2$ , and the modulus of the underlying layer,  $E_3$ :

 $E_2 = K_2 E_3$  (3.23) where  $K_2 = 0.2 H_2^{0.45}$ 

Limits on the application of this approach have been proposed for situations in which more than one layer of granular material is laid (44). These limits, based on the application of elastic theory, suggested that multiple layers act as a composite material for which the ratio of the subgrade modulus to combined modulus of the granular layer is unlikely to be greater than about 4 (75). However, results have been reported (76) which show that, on the basis of measured CBR

values, higher ratios up to 5 or 6 can be generated.

Poisson's ratio depends on the principal stress ratio or ratio of deviator stress to mean normal stress, P (=  $1/3\theta$ ), with a better defined relationship resulting from the use of effective stress rather than total stress (40). Its magnitudes increase with increasing q<sub>r</sub> (i.e. cyclic deviator stress)/P, but a value of 0.3 seems most appropriate for design. Resilient modulus and Poisson's ratio also depend on the aggregate grading, density, particle shape, texture and moisture conditions. Higher density produces higher resilient modulus and the dependence on normal stress is greater with rounded aggregate than with angular aggregate.

#### 3.6.2 Permanent Strain.

The relationship between permanent deformation and the number of loading cycles for granular material is shown in Fig 3.24. There is a sharp increase in permanent deformation during the early load cycles followed by very little subsequent increase, so the material reaches an equilibrium strain. The value of this equilibrium strain depends on the ratio  $q_r/\sigma_3$  under drained conditions.

#### 3.7 Summary of Procedure to Determine Material Properties

## for Pavement Design.

The procedures available to determine material properties for the purposes of pavement design are summarised in the following sections. Those procedures used to estimate materials properties in developing the models used in the work described in this thesis are denoted by \*, while those used in validating the model are denoted by \*\*.



Fig 3.24 Permanent Strain in a Granular Material as a Function of Applied Stresses (after Brown, Ref 40).

# 3.7.1 Bituminous Material.

## Stiffness.

(a) Use:

(i) Van der Poel's Nomograph (bitumen stiffness, Ref 41) with \*either (a) The equation derived by Heukelom and Klomp (43),

Eqn (3.7),

- or (b) Mix Stiffness Nomograph (44), \*
- (ii) Equation derived by Brown (45), Eqn (3.9), \*\*
- (b) Measure Stiffness Using:
  - (i) Creep under constant strains;
  - (ii) Stress relaxation under constant strain;
  - (iii) Constant rate of strain;

- (iv) Dynamic tests under sinuosidal stress or strain; \*\*
- (v) Dynamic tests under repeated step function pulse loading.

# Poisson's Ratio.

- (a) Generally assumed (45) as 0.35; \*, \*\*
- (b) Measured from experiments.

#### Permanent Deformation.

- (a) Creep testing;
- (b) Repeated load testing.

#### 3.7.2 Cement Bound Materials.

#### Stiffness.

(a) Measured from experiments.

#### Poisson's Ratio.

(b) Generally Assumed (56) as 0.25.

#### Permanent Deformation.

(a) Measured from experiments.

#### 3.7.3 Cohesive Soil.

Stiffness.

- (a) Wave Propagation test (63, 64, 65), Eqn (3.18); \*, \*\*
- (b) Model based on Power function (54), Eqn (3.19);
- (c) Based on Plasticity Index (67, 68), Eqn (3.20);
- (d) Measured Stiffness.

#### Poisson's Ratio.

- (a) Generally assumed (56) as 0.4 for design purpose; \*, \*\*
- (b) Measured.

#### Permanent Strain.

(a) Measured from experiments.

# 3.7.4 Granular Soil.

# Stiffness.

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(a) Measured from experiments,

(b) Eqn (3.23), (73, 74). \*, \*\*

Poisson's Ratio.

(a) Generally assumed (56) as 0.3; \*, \*\*

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(b) Measured from experiments.

## Permanent Strain.

(a) Measured from experiments.

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#### CHAPTER 4.0 SELECTION OF STRUCTURAL MODEL.

#### 4.1 Introduction.

In recent years, highway engineers have shown considerable interest in developing a mechanistic basis for designing and evaluating pavement structures. Theories based on linear elastic, non-linear elastic or viscoelastic layered systems, together with linear elastic half spaces, determined using Equivalent Layer Thickness concept, for the prediction of pavement response, are all currently either in use or under development (48, 77, 78, 79, 80). The finite element method of analysing the pavement structure is also used, because of its ability to accommodate non-linear materials properties that are stress dependent and because it is capable of analysing structures with more complex loading and boundary conditions. Many of the design systems being developed now include guidelines regarding the selection of appropriate materials properties and the design criteria for use with that system. Some of the theories and computer programs available to analyse the pavement structures are discussed briefly in the following sections.

### 4.2 Linear Elastic Half-Space Theory.

Boussinesq (77) developed a theory for predicting the stress distribution in a perfectly elastic, homogeneous mass which extended infinitely in the horizontal directions and obeyed Hooke's Law. The load is applied orginally as point load and subsequently extended to a circular contact area with uniform pressure. This theory is not capable of analysing a multi-layer elastic structure directly unless

the multi-layer elastic structure is first transformed into an equivalent semi-infinite space and then analysed using Boussinesq's equations (77). This transformation is achieved by using the Equivalent Thickness Theory (80).

## 4.2.1 Equivalent Thickness Theory.

The basic assumption is that the stresses, strains and deflections below a layer will be unchanged as long as the flexural stiffness of the layer remains constant. The flexural stiffness is a function of the cube of the thickness of the layer and its modulus of elasticity and Poisson's ratio. The equivalent thickness,  $H_e$ , of the (n-1) layers above layer n, is defined as follows (80, 81, 82, 83, 84, 85):

$$H_{e} = f \times \sum_{i=1}^{n-1} H_{i} \times \sqrt[3]{E_{i}(1 - \mu_{n})/E_{n}(1 - \mu_{i})}$$
(4.1)

where f = Correction factor.

It can be represented diagrammetrically as shown in Fig 4.1.



Fig 4.1 Diagrammatic Representation of The Equivalent Thickness Theory.

Boussinesq's equations (77) are used to calculate the stress and strains at the underside of the interface. Where Poisson's ratio is not known, the transformation may be simplified by assuming a single value for all the layers. For a multi-layer system, it is assumed that the material below the layer considered, has the same modulus as the layer considered. Results obtained via the Equivalent. Thickness method deviate from the exact layered elastic theory (80), BISTRO (85). For better agreement, a correction factor, 'f', is introduced. The correction factor is 0.8 except for the first interface where it is 1.0. For a two-layer system a factor of 0.9 is often used.

The accuracy of the equivalent layer method reduces as the number of layers increases (83). The main advantage of the Boussinesq's equations (77) is their relative simplicity which provides easy access by practising Engineers. It would be advantageous if a simplified design procedure could be developed, to satisfy the relationships derived from the 3D finite element model (see Chapt. 7.0), using the Equivalent Thickness theory.

### 4.3 Elastic Theory.

. This method is incorporated in linear layer elastic theory and the finite element method. The finite element method can be used either to carry out linear or non-linear elastic analysis.

### 4.3.1 Layered Elastic Theory.

Since Burmister (77) first used elastic theory to formulate equations for the deflection and stress in aircraft runways in 1943, there have been enormous developments in the analytical design method for pavements. The assumptions that Burmister (77) and others have made are as follows:

- (a) Each layer acts as a continuous, isotropic, homogeneous, linearly elastic medium, infinite in the horizontal extent;
- (b) The surface loading can be represented by a uniformly distributed

vertical stress acting over a circular area;

- (c) The interface conditions between layers can be represented as being either perfectly smooth or perfectly rough;
- (d) Each layer is continuously supported by the layer beneath,
- (e) Inertia forces are negligible;
- (f) Deformations throughout the system are small;
- (g) Temperature effects are neglected.

Several different multi-layer elastic theories have been proposed in recent years by different authors who assigned different properties to the uppermost layers. Westergard (87), Hogg (88), Odemark (88), Pickett et al (88) have treated it as an elastic plate. Tables 4.1 and 4.2 (88) gives brief details of the type of analysis carried out by each of the authors. Burmister (89) and Hank et al (90) have treated it as an elastic layer. The differences between the two approaches is that an elastic plate is subjected to bending deformation but suffers no vertical deformation due to direct stresses, whereas in an elastic layer all the stresses are considered and no restrictions are placed on the deflections.

Measurements of the vertical deflection of a flexible pavement under moving wheel loads carried out by the TRRL and WASHO shows that transient compression of the upper layer occurs, indicating that it behaves as an elastic layer rather than as a plate. Burmister's theory (88) treated the road as a system with three elastic layers with rough interface. Jeuffory et al (88) assumed the top layer to be an elastic plate and the middle layer to be an elastic layer with no friction at the upper interface and perfect roughness at the lower interface. The interfaces occurring in practice are unlikely to be ideally rough, but they are also far from smooth.

Author	Assumed conditions of the layers	Assumed condition at the interface	Form of loading	Parameters used	Stresses and deflec-	Comments
Westergaard(2) (1926)	Elastic plate on a subgrade acting as a series of parallel vertical, independent springs.	Discontinuous	Uniform, circular loading,	Modulus of subgrade reaction, (i.e. spring constant)	Tenatle stresses at three points on the surface of the plate.	The subgrade is unrealistic and stresses in the subgrade are not considered.
Hogg <sup>(3)</sup> (1938)	Elastic plate on a remi-infinite elastic subgrade.	-		Young's modulus and Poisson's ratio for the subgrade.	Tensile scresses in the plate,	Extension of Westergaard's theory.
Burmister(4) (1943)	Elastic layer on a semi-infinite elastic subgrade.	a) Continuous b) Discontinuous	Bessel function loading	a, E <sub>1</sub> h E <sub>2</sub>	w. zz, rr, rz in the pavement and the subgrade.	Equations derived but not evaluated for specific cases.
Fox <sup>(5)</sup> (1948)			Uniform, circular loading.	-	All stresses in the pavement and subgrade.	Estension and numerical evaluation of Burmister's
Hank 4 Scrivner(6) (1948)				*	Stresses at both sides of the interface.	-
Pickett & Al <sup>(7)</sup> (1948)	Elastic plate on a semi-infinite elastic subgrade.	a) Discontinuous b) No horizontal displacement al the interface.		a/b and functions of $E_1/E_2$ , $\mu_1$ and $\mu_2$ .	Ail stresses and deflection in the subgrade.	Massiy applicable to concrete pavements. Results were modified to allow for shear in the pavement on deflection, and horizontal shear at the interface, by making them agree with Barmister's results (approxi-
Odemark <sup>(8)</sup> (1949)	-	Discontinuous			Deflections at the surface and radial stress at the base of the upper layer.	

Table 4.1 Elastic Theories of Two Layered System

(after Whiffin et al, Ref 88).

Author	Assumed conditions of the layers	Assumed condition at the interface	Form of loading	Parameters used	Stresses and deflec- tions determined	Comments
Burmister <sup>(9)</sup> (1945)	Two elastic layers on a semi-indefinite	z) Continuous b) Discontinuous	Bessel function loading.	$\frac{a}{h_2}, \frac{h_1}{h_2}, \frac{E_1}{E_2}, \frac{E_2}{E_3}$	Deflection at the second interface.	A method of cal- culating all the stresses is given, but no numerical evaluation of the deflection or stresses.
Hank & Scrivner(6) (1948)	-	Continuous	Uniform circular loading.	-	All the stresses at the first surface.	Extension and numerical evalu- ation of Burmisters equations for the first Interface.
Acum & Fra (1951)	-	a) Continuous			Vertical and radial scresses at both interfaces.	Extension and numerical evalu- ations of Burmister's equa- tions for vertical and radial stresses.
Jeuffroy & <sup>(11)</sup> Bachelez (1957)	Elastic plate on an elastic layer on a semi-infinite elastic subgrade.	Discontinuous at the first interface, continuous at the second interface.		$\frac{h_2}{a}$ , $\frac{h}{a} \frac{i(E_1)}{(6E_2)}^{\frac{1}{2}}$ , $\frac{E_2 - E_3}{E_2 + E_3}$	w1, ff1 and ££2	

Table 4.2 Elastic Theories of Three Layered Systems

(after Whiffin et al, Ref 88).

It is justifiable to treat the flexible pavement as a three layer system, with the whole surfacing (wearing course and base course) as the uppermost layer, the road base as an intermediate layer and the soil subgrade as forming the lower semi-infinite layer. In the paper presented by Acum et al (91), the authors published the interfacial stresses in a three layer system on the vertical axis through the centre of the tyre contact area. The analysis was based on the equations developed by Burmister (77). The loaded area was assumed to be circular and subjected to a uniformly distributed pressure. The equation, first solved by Burmister (77), defined the deflection, D, of a two layer elastic system of pavement in terms of its thickness, H, and its modulus,  $E_1$ , laid on subgrade of modulus,  $E_2$ , and loaded over a circular area of radius, a, with an applied pressure, P. This equation is of the form (77):

 $D = P.f[(a/H), (E_1/E_2), (1/E_2)] \quad (4.2)$ It is possible to solve this equation to find the values of  $E_1$  and  $E_2$  by measuring values of deflection obtained from two sizes of loaded area. Similarly a three layer system is theoretically treatable by using three loaded areas.

The advances in computer technology has overcome the difficulties in solving complex mathematical equations. Shell and Chevron oil companies have considered the pavement as a multi-layered elastic system and have developed computer programmes known as BISAR (Shell, Ref 30, 44) and the CHEVRON (Chevron, Ref 92, 64).

### 4.3.1.1 Elastic Theory Approach.

Tables and charts of influence values for determiningstresses and deflection in a variety of two layer systems have been given by Burmister (77, 89), Hank et al (90) and Fox (93). For three layer systems tables and graphs have been presented by Acum et al (91), Jones (94) and Peattie (95). All these authors assumed a uniformly

distributed, vertical surface load acting over a circular area and a Poisson's ratio of 0.5 for each layer. Influence values were given only for points along the axis of symmetry.

Coffman et al (88) compared the deflection measured at the AASHO Road Test with deflections calculated from results of laboratory tests in order to verify layered theory. A three layered elastic system was solved with tables prepared by Jones (94) and Peattie (95). Bases, sub-base and subgrade materials were characterized by creep tests and the results were transformed to a dynamic modulus. The dynamic modulus of the asphalt concrete surface was determined by a cyclic triaxial test.

Brown et al (96) also verified elastic layered theory in a carefully controlled laboratory experiment, using a program developed by Jones (96). They found that elastic theory predicted vertical and maximum shear stresses and maximum surface deflection satisfactorily. Klomp et al (97) used the Shell BISTRO (86) program to compare predicted applied strain in the asphalt bound surface and base layers with values measured by wire resistance strain gauges. Flexural tests were used to determine the modulus of the asphalt bound layers and Wave Propagation technique was used for the subgrade. They found that strain in the bound layers could be predicted with reasonable accuracy. However, surface strain measurements traditionally have been more difficult to predict because of the effects of the factors including tyre profile, temperature variations and tensile stresses in the asphalt concrete.

Thrower et al (98) have also presented evidence indicating that layered elastic theory can be used to predict stresses and strain in pavements subjected to moving wheel loads. Dynamic moduli of the

asphalt concrete were measured by means of a dynamic flexural test. Sub-base moduli were determined by Wave Propagation technique and the subgrade moduli by the Shell correlation procedure and the measured CBR value (98). There was good agreement between theory and measured response for stiff pavement sections. There were some discrepancies at higher road temperature, partly because of the difficulties in evaluating 'E' values of the asphalt concrete at high temperature.

Hicks (72), using classical elastic theory, predicted surface deflection and strains in the asphalt concrete, strain in the untreated base layer and stresses in the subgrade with reasonable accuracy. However, for the prototype pavement investigated, better comparison was obtained with finite element technique.

## 4.3.2 Finite Element Method.

The continuum mechanics approach described in the previous section satisfies exactly the governing differential equations associated with the pavement design problem but this is true only for the case of homogeneous, isotropic and elastic layers. The finite element method on the other hand, offers the ability of modelling pavements in a considerably more realistic manner. Each element in the system can be given independent anisotropic material properties and the layers need not be infinite in width. Solutions of the finite element formulation for displacements, stresses and strains are obtained for each element grid. In the elastic layered system, those quantites must be calculated individually at each desired point. The finite element method therefore provides an extremely powerful technique for problem solving involving the behaviour of the structure subjected to accelerations, loads, displacement or changes in temperature, but the method

is a numerical approximation, and as a result, the cost of computer time to solve problems may be as much as 2 to 5 times that as for classical elastic layered solution procedures.

Detailed description of the finite element method has been presented by Zienkiewicz (99), Nath (100), Huebner (101) and Naylor (102). For the analysis by this method, the layered flexible pavement is divided into a number of discrete triangular or rectangular finite elements. Each adjacent element is inter-connected by frictionless pins, called nodes. The structural stiffness properties of each element can be determined by the application of energy principles using an approximate displacement continuity inside each element. Inter-element displacement compatibility is also maintained so that gaps will not open up between adjacent elements. When a fine element mesh size is used, the finite element theory will usually give a satisfactory solution. Convergence to the correct solution with decreasing mesh size is not, however, always guaranteed.

### 4.3.2.1 Finite Element Approaches.

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Shifley (48) and Duncan (79) used finite element technique to analyse pavement structures composed of an asphalt concrete surface, granular roadbase and clay subgrade. Shifley (48) conducted a series of rigid plate repeated load tests on a full scale test section to simulate slowly moving traffic. Transient deformation computed from the results of the repeated load laboratory tests for multi-layer pavement structures compared reasonably well with the measured test road deflections. The non-linear behaviour for both aggregate base and subgrade were approximately accounted for by an iterative finite element procedure.

Duncan et al (79) analysed a pavement structure for both summer conditions (low asphalt stiffness) and winter conditions (high asphalt stiffness). The response of in-service pavements was calculated by an iterative finite element program together with material properties determined from repeated load triaxial tests. Predicted deflections were found to be in good agreement with those measured by the California Travelling Deflectometer.

Wolff (103) compared the stresses generated in three layer pavements using a constant modulus of elasticity in the unbound layers, with those generated when variations of modulus within the layers were included. The finite element method, using a load incremental procedure, in which the total load is divided in equal increments and applied step by step, was used to analyse the pavement with non-linear elastic behaviour. Wolf (103) found that the consideration of the non-linear stress-strain behaviour of the unbound sub-base results in completely different stress distribution in the whole pavement compared with constant modulus of elasticity.

Brown et al (104) used a pavement test facility for the validation of the analytical design method. The stress, strain and permanent deformations of asphalt pavement with granular bases were compared with those computed by the finite element computer program called SENOL. Each structure has about 40 mm of asphalt, about 140 mm of crushed limestone base and a silty clay subgrade of about 6% CBR. Resilient strains were determined with a reasonable accuracy using SENOL (104, 105). Transient stresses were less satisfactory but permanent strains and deformation were reasonably good.

### 4.4 Viscoelastic Layered Systems.

The elastic layered theory developed by Burmister (77), assumes that the material which constitutes the individual layers are linearly elastic as characterized by the time independent constants of proportionality between stresses and strains. This assumption is only valid under dynamic loads with a relatively short loading time. Under static loads of long duration, the stress-strain behaviour relationship of these materials will not be constant but vary with time. In order to take this time dependency into account, it was necessary to renew the existing elastic theory and to develop the viscoelastic theory. It has been claimed by Monismith et al (78) that viscoelastic theory would give a better approximation than the elastic theory for predicting the displacement in a pavement system.

The viscoelastic analysis is quite similar to the elastic analysis and based on the same assumption that proportionality exists between stresses and strains. However, the coefficients of proportionality of the former are differential time operators whereas those of the latter, generally referred to as shear and bulk moduli, are time independent constants. This principal of elastic-viscoelastic correspondence, as orginally developed by Lee (106), has been used recently by Huang (106), Ishihara et al (107), Barksdale et al (108) for the analysis of the layered system. By applying the Laplace transformation to the governing equations to remove the time variable, a viscoelastic problem can usually be transformed into an associated elastic problem, with a transformed variable. The solution of the associated elastic problem, when transformed back into the real time variable, will give the desired viscoelastic solution.

Moavenzadeh et al (109) reported on a computer based design procedure which incorporates linear viccoelastic theory as the structural model. The time dependent function evaluated by them in the current formulation of linear viscoelastic theory is the creep compliance function. The following equation was used as a mathematical approximation to this function (109):

$$D_{j} = \sum_{i=1}^{n} j(e)^{-tq} i, \quad j = 1, 2, 3....$$
(4.3)
the creep compliance function:

where  $D_j$  = the creep compliance function;

 $G_{i} = coefficient of the Dirichlet series;$ 

q<sub>i</sub> = exponent for Dirichlet series;

- e = natural base;
- t = time interval.

In order to determine values for  $G_j$  and  $q_i$ , it is necessary to determine the complex modulus and time temperature shift factor of the material being evaluated. A rather difficult and lengthy test procedure is required to determine the complex modulus and time temperature shift factor for asphalt concrete.

### 4.4.1 Viscoelastic Approach.

The stresses and displacements induced by the application of a vertical load to the surface of a semi-infinite viscoelastic two layer system was analysed by Ishihara et al (107). This analysis was based on the assumption that a frictionless horizontal interface separated the upper layer from the lower one. The two layer system consisted of Maxwell type viscoelastic materials and the stresses and displacement changed with the loading time. Later he studied the results obtained from a multi-layer system of the AASHO Test Road and those obtained using an equivalent layer system. The author (107) reported that the agreement between the theory and the test data was quite satisfactory.

Barksdale et al (108) compared measured and computed response by using a linear viscoelastic approach. They calculated both permanent and resilient deflections for three layer systems on the AASHO Test Road. The stress-strain relations were obtained from the result of repeated load triaxial tests. The calculated deflections were in general close enough to suggest that viscoelastic theory can give reasonable estimates of resilient and permanent deformations.

Huang (106) developed two methods of analysis, one based on a direct method of Laplace inversion and the other on an approximate method of collocation. The two methods were used to determine the stresses and displacement in a two layer incompressible system, made up of both simple and complex viscoelastic materials. Both methods agreed closely with a maximum discrepancy of not over 5%. The direct method generally gives more accurate results and is applicable to the case where the stress and displacement equations can be reduced to a form suitable for direct Laplace inversions. When a greater number of layers and more complex materials are involved, such equations are almost impossible to obtain. Therefore the approximate method can be used and it will give fairly good results.

# 4.5 Theory Selection.

Pichumani (110) carried out a comparative study of several computer programs. The AFPAV computer program, which is based on the finite element analysis techniques, was shown to be more efficient and economical than the BISTRO (86) program for analysing layered pavement system, loaded by a multiple wheels of modern aircraft. The stresses

and strains predicted by all the computer programs were similar, but BISTRO (86) predicted slightly more deflection than the finite element programs.

Duncan et al (79) carried out an analysis of a three layered system using elastic half-space theory and finite element method. for analysis of systems with linear material properties, the finite element method offers little or no advantage over layered system analysis. However, because it is feasible to incorporate non-linear material behaviour in finite element analysis, the technique may be used for obtaining solutions to problems which cannot be solved accurately by other available methods.

It is the author's opinion that finite element method should be used in the analysis of flexible pavement. This has the ability to carry out non-linear analysis which enables it to incorporate nonlinear material properties. The computing time required to carry out such analysis is tremendous. Therefore, it would be very beneficial to have some sort of simplified design method based on results of the expensive finite element method. To develop a simplified design procedure, it requires the results from a more sophisticated and accurate model against which the consequence of the simplification can be judged and an estimate of error can be made.

One must not forget that at present electronic technology is improving rapidly and in future even more powerful computers are likely to be developed. The computing time and the running cost will be reduced considerably and every highway agency could be in a position to afford a computer of low cost capable of carrying out finite element calculations. Therefore, the exploitation of the finite element method should not be abandoned.

The finite element method has been adopted as the primary analysis technique in the investigation reported in this thesis. Although the analysis presented is limited to a linear system, the model developed and results obtained provide a basis for further work involving complex and non-linear materials.

### 5.1 Introduction.

This chapter describes the use of finite element modelling to compute the stresses and deformations within a pavement structure subjected to traffic loading. A suitable model is developed and calibrated by reference to the measured deflection profiles obtained under field loading conditions. Finally, a procedure to convert absolute surface deflection profiles predicted from the model into equivalent Deflectograph deflections is presented including a description of the appropriate computer programs.

## 5.2 The Finite Element Method.

The finite element method is a numerical analysis technique for obtaining approximate solutions to a wide variety of engineering problems. The method represents the extension of matrix methods for skeletal structures to the analysis of continuum structures. The analysis of a continuum structure differs from those of the skeletal structure in two basic aspects only, the subdivision into a number of elements which are connected at the nodal points and the division of the element stiffness characteristics. The finite element method is essentially dependent for its success on the skillful use of computers. The method consists of reducing a structure to a skeletal form consisting of individual members (elements) connected at their ends (nodes). Analysis of the skeletal structure is carried out by first considering the behaviour of each individual element independently and then by assembling the elements together is such a way that the following three conditions are satisfied at each ordal point:

- (a) Equilibrium of forces;
- (b) Compatibility of displacements;
- (c) Laws of material behaviour.

The finite element method is used in preference to other numerical methods for the following reasons:

- (a) Owing to flexibility of their sizes and shapes, finite element is .
   able to represent a given body more faithfully, however complex its shape may be;
- (b) Time dependent and non-linear material properties can be dealt with relatively easily;
- (c) Boundary conditions are dealt with easily;
- (d) The versatility and flexibility of the finite element method can be used very effectively to evaluate accurately the cause and effect relationships in a complex continuum.

### 5.2.1 Basic Structural Analysis.

The Principal of Virtual Work (111, 112, 113) is used in deriving the stiffness properties of various elements. The principle is concerned with the relationship which exists between a set of external loads and the corresponding internal forces, and also with sets of joint (node) displacements and the corresponding deformation of members which satisfy the conditions of compatibility. The principle may be stated as follows; 'The virtual work done by the external loads is equal to the internal virtual work absorbed by the structure'. The forces and displacement may be either real or virtual but the conditions of equilibrium and compatibility must be satisfied. It can be expressed as:

$$E \cdot D = \int^{V} I \cdot e \cdot V \tag{5.1}$$

where E = systems of external loads;

D = deflection of the loads;

I = systems of internal forces;

e = internal deformation of the structure;

V = volume of the structure.

The matrix method may be formulated in three different ways:

(a) Stiffness (displacement) method;

(b) Flexibility (force) method;

(c) Mixed method.

In the stiffness method, the displacements compatibility conditions (deformed structure must fit together) are satisfied and the equations of equilibrium set up and solved to yield the unknown nodal displacements. In the flexibility method the conditions of joint equilibrium (internal forces balance the external applied loads) are first satisfied and the equations, arising from the need of compatiblity of nodal displacement, solved to yield the unknown forces in the members. In addition to these two basic approaches, in recent years a mixed formulation involving both approaches has also been used. Further information about these methods is available elsewhere, see Ref (112, 114 and 115).

## 5.2.2 Subdivision of Structure.

In a continuum structure, the continuum has to be artificially divided into a number of elements called finite elements or discrete elements, before the matrix method of analysis can be applied. In reality, the elements are connected together along their common

boundaries, but in order to make the matrix method of analysis possible, it is assumed that the elements are interconnected only at their nodes. This means that continuity requirements are satisfied only at the nodal points and this would make the structure very much more flexible. In the finite element, the individual elements are constrained to deform in specific patterns. The choice of a suitable pattern of deflection for the elements can lead to the satisfaction of some, if not all, of the continuity requirements along the sides of adjacent elements (111).

### 5.3 The Finite Element Program (PAFEC 75).

The finite element program, PAFEC, is readily available on the Plymouth Polytechnic main frame computer. PAFEC 75 (116, 117, 118) is a version of PAFEC which has been designed so that users may input data in a very straightforward manner. This is a very general program, i.e. not specially designed for pavement design like the elastic layer program, BISAR (44). All the information about the models is fed into the program data file and the program creates about ten output files. Each of the output files or phases consists of various information about the model and engineering analysis. The important phases for analysis of the structure are phases 1, 2, 4, 6, 7 and 9. Detailed description of each phase is given in Table 5.1. The program has the facility to restart a particular job and this enables various models of different material properties, pressure and restraint to be analysed without the need to re-start the job from the beginning. This facility reduces significantly the computing time required to analyse most of the models. The program has to be started from the beginning when the co-ordinates of any of the nodes is changed or when the

thickness of layers is changed.

PHASE	Short description	Detailed description
1	Read	Data modules are read in, default values are inserted and the modules are placed onto backing store. The NODES module is expanded so that all mid-side nodes are included.
2	PAFBLOCKS	Any PAFBLOCK data is replaced by the full nodal coordinate and topological description of the complete mesh of elements.
3	IN.DRAW structure	The structure itself is drawn. At this stage it is not possible to show any results such as displacements, stresses or temperatures since these have not yet been evaluated.
4	Pre-solution housekeeping	In this PHASE the constraints on the problem are considered and a numbering system for the degrees of freedom is derived.
5	IN.DRAW constraints	This PHASE is very similar to the PHASE 3 except the constraints which have been applied are shown. Conversley the degrees of freedom can be indicated on a drawing.
6	Elements	The stiffness (or other such as conductivity, mass etc) matrices of all the elements are found and put onto backing store.
7	Solution	The system equations are solved for displacements, temperatures or whatever happens to be the primary unknowns in the problem being tackled.
8	OUT.DRAW displacements	The primary unknowns in the problem (i.e. displacements or temperatures) are drawn.
ý	STRESS .	The stresses are found
10	OUT.DRAW	Stress contour, stress vector plots etc. are produced.

Table 5.1 Brief Description of the Ten Phases of PAFEC 75 (Ref 117).

## 5.3.1 Method of Analysis.

The model of the pavement can be analysed as a plane elasticity problem or as a non-linear analysis. Plane elasticity problems involve continual loaded in their plane and may be separated into two separate classes, namely plane stress problems and plane strain problems. In a plane stress problem, the continuum (such as a plate) is thin relative to other dimensions, and stress normal to the plane is neglected. In a plane strain problem, the strain normal to the plane of loading is assumed to be zero. Neither the assumption of zero stress nor zero strain in the direction normal to the plane of loading is strictly applicable to the flexible pavement problem but all the pavement models were analysed as a plane strain problem since it was considered to be a more realistic approach.

#### 5.3.2 Basic Element Shapes.

Since the fundamental premise of the finite element method is that a continuum of arbitary shape can be accurately modelled by an assemblage of simple shapes, most finite elements are geometrically simple. For analysis with the PAFEC computer program, the structure to be analysed is divided into a series of quadrilaterals and/or triangles. The basic element shapes which can be used for analysis of two dimensional or three dimensional flexible pavement model are illustrated in Figs 5.1 and 5.2. Triangular elements an enable irregular boundary structure to be modelled and they are often used to model a circular structure. A set of functions, depending to the type of elements used, is chosen to define uniquely the state of displacement within each element in terms of its modal displacement.

The displacement function defines uniquely the state of strain within an element in terms of its nodal displacement. These strains will define the state of stress throughout the element and hence also on its boundaries. The chosen displacement functions satisfy the requirement of displacement continuity between adjacent elements and this will ensure that no gaps will develop between the edges

of adjacent deformed elements. The rectangular plane elasticity elements are slightly more accurate than triangular elements because they assume a linear distribution of strain over the element, whereas triangular elements assume the strain is constant within the element.



Fig 5.1 Two Dimensional (2D) Elements.



Fig 5.2 Three Dimensional (3D) Elements.

### 5.3.3 Two and Three Dimension Isoparametric Elements.

Elements which have one midside node are known as 'parabolic elements' and they offer better accuracy. Higher order elements with more than one midside node are available but although they give better accuracy per element it is doubtful if they offer any advantage on a 'per node' basis. The shape of a solid element can be expressed in terms of an interpolation function, also known as shape function. The interpolation function for the various elements are as follow (119):

(a) Linear = No midside node element;

(b) Qualdratic = One midside noded element;

(c) Cubic = Two midside noded element.

When the shape function defining the geometry and the function are of the same order, the element is called isoparametric. The isoparametric element enables curve sided elements to be modelled. Isoparametric elements are used to model the flexible pavement. The sides of the element are considered to be straight when the mid-modes of the elements are not specified in the program.

A two dimensional (2D) element has two degrees of freedom in its own plane at each node and the three dimensional (3D) element has three translatory degrees of freedom at each node. 3D elements are expensive to use and should only be employed when stresses vary in three directions.

## 5.3.4 Geometric Aspect Ratios of the Elements.

The geometric aspect ratios of the different basic element shapes are given in Table 5.2 (117). Great distortion of the elements results in a reduction of accuracy. The program provides a 'warning' message if the element is very slightly distorted and is approaching

the critical condition stated in Table 5.2 (117). An 'error' message is provided if the element is distorted and has already approached these critical conditions stated. The program fails to operate when one error message or a few warning messages are generated. This geometric checking is carried out in phase 2. If there is any 'error' message then the program will come to a halt after phase 3 (See Table 5.1). The phase 3 drawing(s) are a very useful aid for correcting poor element geometries (117).

## 5.3.5 Automatic subdivision of Structure.

The starting point of an analysis is the division of the structure into elements. The program has the facility to generate a graded mesh of rectangular or triangular elements under the 'PAFBLOCK' module. The 'MESH' module specifies the spacing of the elements and its size. These facilites reduce the amount of input data required. The nature of the finite element method means that, in general, the accuracy of the solution increases with the number of elements. However, it must be realised that as the number of elements increase, the computer time required will also increase, with a consequent increase in cost. Fine graded mesh around the zone of high stress concentrations and coarse graded mesh, away from these stress concentrations, can lead to economy in solution time without any loss of accuracy, but the choice of a suitable subdivision for a particular structure must be based on previous experience of similar solutions.

Shape of element	Warning	Error		
Triangle	5 <r<sub>max&lt;15 15&lt;θ<sub>min</sub>&lt;30 150&lt;θ<sub>max</sub>&lt;165</r<sub>	R <sub>max</sub> ≥15 θ <sub>min</sub> ≤15 θ <sub>max</sub> ≥165		
Quadrilateral	5 <r<sub>max&lt;15 25&lt;θ<sub>min</sub>&lt; 45 135&lt;θ<sub>max</sub>&lt;155 0.00001<h<sub>max&lt;0.01</h<sub></r<sub>	R <sub>max</sub> ≥15 <sup>θ</sup> min <sup>≤25</sup> θ <sub>max</sub> ≥155 H <sub>max</sub> >0.01		
Wedge, Brick	For three dimensional elements each face is checked depending upon whether it is a triangle or a quadrilateral using the above criteria except that for a quadrilateral a check on H is omitted.			

Notes: 1.  $R_{max} = \frac{\text{length of longest side}}{\text{length of shortest side}}$ 

- 2.  $\theta_{\min} = \min \text{ minimum angle between chords across}$ any adjacent element sides.
- 3.  $\theta_{max} =$ maximum angle between chords across any adjacent element sides.
- 4. H<sub>max</sub> = distance of fourth node from the plane of the first three maximum side length
- This check (No. 4) is not made in the case of the Semi-Loof shell elements which may be generally curved out of a plane.

Table 5.2 Warning, Error and Element Shape

for 2D and 3D Elements (Ref 117).

### 5.3.6 Material Properties.

The finite element method has the ability to carry out both linear and non-linear analysis, i.e. able to solve problem involving material with linear and non-linear properties. For linear analysis, which is straightforward, only the elastic modulus, Poisson's ratio and the density of the material must be specified (115, 117).

The solution of non-linear problems by the finite element method is usually attempted by one of three basic techinques:

(a) Incremental or step wise procedures;

(b) Iterative or Newton Methods;

(c) Step-iterative of mixed procedure.

## 5.3.6.1 Incremental Procedure.

The basis of the incremental procedure is the subdivision of the load into many small partial loads or increments. The load is applied one increment at a time, and during the application of each increment, the displacement equations are assumed to be linear. In other words, a fixed value of the elastic stiffness matrix is assumed throughout each increment, but the elastic stiffness matrix may take different values during different load increments. The solution of each step of loading is obtained as an increment of the displacements. These displacement increments are accumulated to give the total displacement at any stage of loading, and the incremental process is repeated until the total load has been reached (112, 114).

### 5.3.6.2 Iterative Procedure.

The iterative procedure is a sequence of calculations in which the structure is fully loaded in each iteration. Because some approximate constant value of stiffness is used in each step, equilibrium is not necessarily satisfied. After each iteration, the portion of the total loading that is not balanced is calculated and used in the next step to compute an additional increment of the displacements. The process is repeated until equilibrium is approximated (112, 114).

### 5.3.6.3 Mixed Procedures.

The step-iteration or mixed procedure utilizes a combination of the incremental and iterative schemes. Here the load is applied incrementally, but after each increment successive iterations are performed. The method yields higher accuracy but at the price of more computational effort (114).

#### 5.3.6.4 Comparison of the Basic Procedures.

The principle advantage of the incremental procedure is its complete generality. It is applicable to nearly all types of nonlinear behaviour. It provides a relatively complete description of the load deformation behaviour. Useful results can be obtained at each of the intermediate states corresponding to an incremental of load. It guarantees convergence to the exact solution. The incremental procedure is usually more time consuming than the iterative technique and it is difficult to know in advance what increment of loads are necessary to obtain a good approximation to the exact solution.

The iterative procedure is easier to program than the incremental method and it is faster, provided it need analyse only a few different loadings. The principle disadvantage of the iterative method is that there is no assurance that it will converge to an exact solution. The technique is not applicable to dynamic problems.

The mixed method combines the . advantage of both the incremental and iterative procedures and tends to minimize the disadvantage of each. Step-iteration is being utilized increasingly. The additional computational effort is justified by the fact that the iteration of the procedure permits one to assess the quality of the approximate equilibrium at each stage.

### 5.3.6.5 Non-Linear Analysis -PAFEC.

The PAFEC program uses the incremental procedure to carry out the non-linear analysis. The material is assumed to behave elastically before yield according to Hooke's Law. If the material is loaded beyond yielding, additional plastic strain occurs, which if the load is then removed, leaves a residual deformation. It is assumed that data is available for the yield stress and stress-strain gradient after yield for the case of uniaxial tension. Often the uniaxial stress -strain curve will approximate to the two straight lines of linear elasticity and linear strain hardening after yielding, so that it is defined by the elastic modulus, the yield stress, and the plastic stress-strain gradient.

### 5.3.7 Loadings.

These may consist of point loads applied at the node or pressure loading applied over a plane enclosed by a set of nodes. A constant or varying pressure load can be applied over one surface of a 3D brick element and the program then converts the pressure load into equivalent nodal loads. The magnitude of the nodal forces is determined by interpolating between the nodes that are enclosed within the area of the pressure. Fig 5.3 shows the equivalent nodal forces for a given pressure applied on the surface of the most commonly used elements. Detailed calculation of the equivalent nodal force is illustrated in Appendix I.



Note: (i) Total uniform load applied over the element = 1; (ii) + ve = inwards, and - ve = outwards.

Fig 5.3 Equivalent Nodal Loads due to Constant Pressures on Element Surface (Ref 117).

## 5.4 Three Dimensional Pavement Model.

2D analysis requires less computing time, disc space and a smaller base (computer memory) than a 3D analysis. 2D analysis of the pavement model using the PAFEC program was carried out at the Plymouth Polytechnic by Butler (13). The pavement was modelled to predict the surface deflection in the wheel path under the loading action of a Deflectograph (18). However the stress on each element of material varies in three directions when subjected to a load. In addition, the deflection measured by the Deflectograph is relative to a datum of the T-frame which has one of its supports off the line of the wheel path. The layout of the Deflectograph assembly is given in Plate 2.1, Figs 2.3 and 5.4. The deflection profile along the road through the centre of the twin rear wheels (line XX on Fig 5.4) and the centre line of the vehicle along which the T-frame moves (line YY on Fig 5.4) is illustrated in Fig 5.5.



Fig 5.4 Diagrammatic Representation of Deflectograph (after Kennedy et al, Ref 18).

Fig 5.6 gives the deflection profile across the road through the centres of the twin rear wheels (line AA on Fig 5.4) and through points 1 m away from the centre of the twin rear wheels. Since the datum provided by the T-frame is affected by movement of its support foot lying in the plane of the centre line of the vehicle, a 3D analysis of the pavement model was considered to be necessary. It is necessary to obtain the deflections along the path of the T-frame, from the analysis of the model, in order to transform the absolute deflections into deflections that would have been obtained from the Deflectograph.





Line XX (through the centre of the twin rear wheels).

Fig 5.5 Absolute Deflection Along the Road.

In order to develop a realistic 3D model, a number of initial models were developed and there are detailed in the next few sections. These initial models were intended to provide information on the shape of tyre area, type of elements, number of element, geometric dimension, accuracy, computing time, base, disc space required and on the type of restraint and loading to be used.

Undeflected Surface



- - 1 m away from the centre of the twin rear wheels
 Through the centre of the twin rear wheels (line AA on Fig 5.4).

Fig 5.6 Deflection Profiles Across the Road.

## 5.4.1 Model A.

The aim of this model was to choose the most suitable shape of the tyre contact area with the road. The shape of the tyre contact area is not circular or rectangular but more elliptical as shown in Fig 5.7 (18). Research (26, 120) into the shape of the contact area suggests that it is possible to assume a circular shape of area equal to the load, 'W', carried by the tyre divided by the Pressure (P) of the wheel (see Appendix II), i.e.

Area 
$$(A) = W/P$$
 (5.2)

The radius of the circle could be deduced from from the area. From the details of the Deflectograph given in LR 834 (18), the radius was found to be in the range between 77 to 94 mm, but for convenience the radius was assumed to be 100 mm. There are two methods of incorporating the circular area into the program:

- (a) Divide the circular area into a number of triangles with one side curved. This method involves allocating a mid-node on the curved surface. During the analysis process, the curved surfaces are divided into a number of small triangles as shown in Fig 5.8a. This method is very tedious and requires a great amount of data to be input.
- (b) The process adopted divides the circular area into a number of straight sided triangles as shown in Fig 5.8b. This method is very much simpler and straightforward. It requires a smaller amount of data to be input and both methods offer more or less the same accuracy (116).

The area of circle can easily be approximated by the use of the inscribing and circumscribing polygons. The polygons are divided into triangles each with a vertex at the centre of the circle. It is then a simple matter to evaluate the sum of the area of all the triangles. It is also possible to approximate the circle with triangles, the lengths of which when summed provides a good approximation to the perimeter of the circles. Since pressure would be applied over a circular area, it is better to approximate the circle by triangles of equal area.

The circular area was split into 8 equal isosceles triangles, each subtending  $45^{0}$  at the centre of the circle as shown in Fig 5.8b. It was decided to use a maximum of 8 triangles inscribing the

circle since a greater number will increase the number of elements which in turn will increase the file, base size and the computing time

Model A was constructed and three loading cases were considered:

(a) Pressure load over 8 equal isosceles triangles;

(b) Pressure load over 4 equal squares;

(c) Single Point load at the centre.

The model consist of one layer only. The value of the elastic modulus is irrevelant at this stage, provided that the value used was the same for all the loading cases. At this point, the load case No (a) was assumed to be the correct one and remaining two cases were compared with load case No (a). From the comparison of the results, case No (b) was found to be similar to case No (a). It was concluded that initially eight equal isosceles triangles would be used but if any problems were encountered with the disc space or the computing time required, then the four equal square shape would be used.

## 5.4.2 Model B.

The main purpose of constructing this model was to get an estimation of the necessary dimensions of the model and to study the effects of the nearside front wheel and the offside twin rear wheels, OS, (away from the kerb) on the deflection profile along the centre line of the vehicle and centre line of the nearside twin rear wheels, NS, (close to the kerb). The definition of the centre line of the vehicle (line YY) and centre line through the twin rear wheels, nearside wheel path (line XX) and offside wheel path (line ZZ) are illustrated in Fig 5.4. It would be very difficult to model the front wheels and two sets of twin rear wheels together since this would require an

enormous amount of computing resources. Therefore, it is crucial to know if is possible to exclude the nearside front and/or the offside twin rear wheels, in the final model, see Fig 5.9.







Fig 5.7 Deflectograph (a) Curve Sides (b) Straight Sides Tyre Print (Ref 18). Fig 5.8 Triangles with: (a) Curve Sides and (b) Straight Sides.

Model B1 was constructed and the dimension of the model are as shown in Fig 5.9a. The deflection along the lines XX and YY (see Fig 5.9) were recorded for the two loading cases:

(a) Two sets of twin rear wheels, NS (nearside), OS (offside), see Fig 5.9, loaded together,

(b) Twin Rear wheels, OS, loaded alone.

From these two load case it is possible to determine the effect of the twin rear wheels, OS, on the deflection profile along lines XX and YY. The result indicates that the twin rear wheels, OS, do affect the deflection profile along both the lines XX and YY.



Fig 5.9 Model B1 - B2.
Model B2 was constructed, as shown in Fig 5.9b, with the dimension in the z direction decreased by 750 mm. The deflection along the lines XX and YY was recorded and compared with those obtained from Model B1. There was no significant change in the deflection. Therefore, it is possible to limit the width of the model in the z direction to 3830 mm and restrain it in the z direction.

Model B3 consists of one front wheel and twin rear wheels, NS, as shown in Fig 5.10a. Two loading cases were considered: (a) Front and twin rear wheels, NS, loaded together;

(b) Front wheel loaded alone.

From these load cases it is possible to determine the effects of the front wheel on the deflections along the line XX. The deflections generated along the line XX using this model were compared with those obtained from Model B1. From the results it could be deduced that the front wheel affects the deflection along the lines XX to a distance about 2 m away from the centre of the twin rear wheels, NS. The twin rear wheels, OS, have a greater effect on the deflections along the line XX than the front wheel.

The length of the model, in x direction, was then reduced: first the distance in front of the front wheel, see Model B4 in Fig 5.10b, and secondly the length both in front of the front wheel and to the rear of the twin rear wheels, see Model B5 in Fig 5.10c. The deflection along the line XX from both models were compared with those obtained from Model B3. There were no significant changes in the deflections.



All the above Models, B1 to B5, consist of two layers of total thickness 680 mm. Each model was run twice with different elastic modulus for the upper layer;  $1.5 \times 10^8$  and  $5 \times 10^9$  N/m<sup>2</sup> but the same Poisson's ratio of 0.35. In both cases, the lower layer had the same elastic modulus. In all these models, the front and rear wheels loads were input as a pressure load of  $6.9 \times 10^5$  N/m<sup>2</sup>. The contact area of the tyre was represented by eight equal isosceles triangles as shown in Fig 5.8b. Twenty noded brick elements and fifteen noded wedge elements were used. The boundary conditions of the models are as follows:

Model	Plane	Direction Restrained		
B1 to B5	ABCD	x, y, z		
	AECG	X, Z		
	BFDH	х, у		
	CDGH	X, Z		
	ABEF	X, Z		

Table 5.3 Boundary conditions of Model B.

#### 5.4.3 Conclusions to be Drawn from The Initial Models A-B.

Analysis of the above models leads to the following conclusions:

- (a) The front wheel affects the deflection along the line XX to a distance about 2 m.
- (b) The front wheel could not be excluded from the final model even though it only affects the deflection to a distance about 2 m, since in order to get a Deflectograph deflection over a meter, the

measuring arm should be at least 2.54 m away from the centre of the twin rear wheels (see Fig 5.4, Point A is about 2.54 m away from Point B).

- (c) The twin rear wheels, OS, affects the deflection along the lines XX and YY.
- (d) The twin rear wheels, NS, have a greater influence on the deflection along the line XX than the front wheel. This could be due to the fact that the twin rear wheels loads are twice the magnitude of the front wheel load and also they are closer to the twin rear wheels, NS.
- (e) The distance in front of the front wheel could be reduced to 1 m and in the rear of the twin rear wheels to about 1.5 m.
- (f) If the front and the twin rear wheels, OS, could not be accommodated together in the final model, then the front wheel would have to be excluded. (In the final model only the front wheel and the twin rear wheels, NS, were used, see Section 5.4.7).

#### 5.4.4 Model C.

The main purpose of constructing this model was to find a method of including the effect of the twin rear wheels, OS, on deflection along the lines XX and YY, but without including the wheels in the final model, since this would reduce the size of the model and the computing time and the disc space associated with it. The Model Cl, whose dimension is based on conclusions stated in Section 5.4.3, was cut along the line of symmetry, i.e. line YY, see Fig 5.11a, to a model, Model C2, as shown in Fig 5.11b. In these two models, the tyre contact area was represented by a square (100 mm x 100 mm) instead of 8 equal isosceles triangles, since the 8 isosceles triangles require

greater computer resources with no improvement in accuracy. The effect on the shape of deflection profile by this change was studied. Both forms of contact area gave the same maximum deflection. The deflections at some distance away from the point of maximum deflection were greater for the triangular shape compared to the square shape, by about 3 to 6%. Twenty noded brick elements were only used for the entire construction of Model C1 and C2. The model consists of two layers of total thickness 680 mm with the same elastic modulus of  $1.5 \times 10^9 \text{ N/m}^2$ . The plane of the model was restrained as shown in Fig 5.11. The deflections along the lines XX and YY from Model C1 and C2 were compared and the following conclusions were deduced:

- (a) The deflection along the lines XX and YY for both model were very similar, within engineering limits, about 4%.
- (b) The computing time and disc space required for Model C2 was half of that required for Model C1.
- (c) The number of elements, computing time and disc space required for a square contact area was much less compared to the triangular area.

The boundary conditions of Model Cl and C2 are listed in Table 5.4.

### 5.4.5 Model D.

Using the results from Model C2, it is possible to exclude the twin rear wheels, OS, without excluding its effect in the final model. This made it possible to include the front wheel as shown in Model D1, see Fig 5.12a. The main purpose of constructing Model D was to find a means of dividing the Model D1, into two separate models, Model D2 and D3 (see Fig 5.12), then to analyse the two models separately and finally to superimpose the deflection, along the lines XX and

Plane	Direction Restrained		
ABCD	x,y,z		
ACGE	x		
BFDH	x		
CGDH	Z		
ABEF	Z		

YY, by using the Principle of Superposition.

Table 5.4 Boundary Conditions of Model Cl and C2.

From deflection profiles obtained from TRRL experimental sections of roads loaded with a Deflectograph moving at creep speed, Figs 5.13 and 5.14, it can be seen that the deflection due to front wheel is negligible at a distance of about 2.5 m from the centre of the front wheel (Point P in Figs 5.13 and 5.14). Therefore, the models were superimposed over a distance of 3 m between the front wheel and the twin rear wheels, NS, as shown in Fig 5.12. The deflection along the lines XX and YY from Model D2 and D3 were superimposed and compared with those obtained from Model D1. The comparison was carried out for different combinations of restraint applied to plane BDFE. The deflections were not found to be similar and therefore it is not possible to divide the Model D1 into Model D2 and Model D3. The thickness, number of layers, tyre contact area and elastic modulus of the Models D were similar to Models C.

#### 5.4.6 Model E.

Model E was constructed in order to determine the geometric dimension and the type of elements to be used in the final model. During the operation of the Deflectograph (18), the datum T-frame projects about 600 mm beyond the centre line of the twin rear wheels. The deflection profile illustrated in Figs 5.13 and 5.14 indicates that the absolute deflections along line YY, see Fig 5.12, on either side centre of the wheels are the same for a distance of 600 mm. Therefore, it is possible to model only half of the twin rear wheels as shown in Model E, see Fig 5.15, and use the deflection at 600 mm in front of the twin rear wheels as equal to the deflection at 600 mm behind the wheels.

It is costly to use 20 moded brick elements for a large 3D model and therefore it would be economical to use only 8 moded brick elements. The same accuracy could be achieved to a certain extent by using a finer mesh near the loaded zone and a coarse mesh for the zone away from the loaded zone. Figs 5.15 and 5.16 give the detail description of Model E, which consist of eight layers. Each layer can have different material properties, but all the sub-layers within each main layer must have the same material properties.



Fig 5.11 Model C.



Fig 5.12 Model D.



Fig 5.13 Absolute Deflection of The Granular Roadbase Pavement.



Fig 5.14 Absolute Deflection of The Bituminous Roadbase Pavement.



Fig 5.15 3D View of Model E.



5.16 Plan of Model E.

When the model was analysed using appropriate but approximate elastic moduli values for each layer, the deflection at the centre point between the front and twin rear wheels was quite small. The surface at this point was deflecting upwards (+ ve deflection) rather than deflecting downward (-ve deflection). According to Desai et al (114), if the structure is symmetrical, the thickness V (see Fig 5.17) should be least 10 to 12 times greater than D, the diameter of the wheel. The distance H should be at least 4 to 6 times greater than the diameter D. The distance H and D are the critical distance beyond which the boundary conditions have a negligible effect on the surface deflection profile.



Fig 5.17 Finite Representation of Infinite Bodies (Desai et al, Ref 114).

#### 5.4.7 Model F.

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Model F was constructed in order incorporate Desai et al (113) suggestions. Therefore Model E was reconstructed with 9 layers instead of 8, i.e. the subgrade thickness was increased to 2.244 mm, to give Model F (see Fig 5.18 and 5.19). The distance between the centre of the twin rear wheels and the kerb boundary (plane EFAB in Fig 5.16), was increased to 1.3 m. When the model was analysed with elastic modulus similar to Model E, the deflection between the twin rear wheels increased by 15% but the deflection at the centre point between the front and the twin rear wheels increased by 200%. To reduce the computing time due to the increase in the number of layers, the manner of division of the model into a number of finite elements was changed. The dimension of the tyre contact area was changed from 100 mm square to 85 mm square and the distance between the wall of the twin rear wheels was increased from 90 mm to 150 mm so that the dimensions are more in accordance with those stated in Ref (18). These changes in the dimension of the contact area and the distance between the walls of the twin rear wheels did not produce a significant effect on the deflection profile. This change enables the deflection at centre of the twin rear wheels, point 'a' in Fig 5.18, to be obtained unlike the case with Model E in Fig 5.16.

The plane BDHF and ACEG, see Fig 5.18, was restrained from moving in the x direction, since the deflection for a distance of 1 m on either side of the line HF is symmetrical. The plane CDGH was restrained from moving in the z direction, since the deflections are symmetrical about the line GH. Total restraint (in x, y, z direction) was applied to the lower plane ABCD to represent the bedrock surface. The plane ABEF was not restrained in any direction. The plane ABEF represents the interface between the pavement and the soil. In practice, there will be a very partial restraint on the plane in all three directions and this restraint will increase with depth. Ideally some sort of spring mechanism with increasing stiffness with depth at the interface between the pavement and the soil could be used to model this effect. Since this suggestion could not be implemented easily

with the PAFEC program, the plane (ABEF) was treated as a free plane. The boundary conditions and the number of elements used are summarized in Tables 5.5 and 5.6. It is possible to divide each layer into a number of sub-layers. The thickness of each sub-layer must be within the range specified in Fig 5.19. Each layer can have different material properties but all the sub-layers within each of the main layers must have the same material properties. The PAFEC program for this model is shown in Appendix III.

Plane	Direction Restrained
ABCD	x, y, z
ABEF	FREE
CDGH	Z
ACEG	x
BDHF	x

Table 5.5 Boundary Conditions of Model F.

Total Nodes	2920
Total Elements	2331
Total Degrees of Freedom	7434
Element Types	37100 (8 Noded Brick Element)
	37200 (6 Noded Wedge Element)

Table 5.6 General Information about Model F.



136

Units = mm

5.18 Plan of Model F.



Fig 5.19 Various Layers of Model F.

B

#### 5.5 Validation of The Final Model (Model F).

Before the model could be used to investigate the relationship between the deflected shape and the pavement condition, it was necessary to ensure that its response was similar to that of an actual pavement structure. Absolute deflection recorded by in-situ displacement transducers, inserted into an experimental pavement with a granular road base, have been used to define the pavement response to be modelled. These measurements were carried out by the TRRL under the loading action of a Deflectograph.

#### 5.5.1 Granular Road Base Pavement.

The pavement with a granular road base was represented by the layered system as shown in Fig 5.20. The only information available about the parameters controlling the response of each layer is the thickness of the surfacing, road base and sub-base. The other information available are the loads of the front and the twin rear wheels. The temperature of the environment at the time of measurement of the deflection was not available. In developing the model, the elastic modulus and the Poisson's ratio of each layer of the pavement layers and subgrade were initially deduced from typical published values for each material type. These values were subsequently modified until the response of the theoretical model was similar to that of the actual pavement structure.

Asphalt Bound Layer,  $E_1$ ,  $H_1$ ,  $\mu_1$ 

Intermediate Granular Layer, E<sub>2</sub>, H<sub>2</sub>,  $\mu_2$ 

Subgrade,  $E_3$ ,  $H_3$ ,  $\mu_3$ 

Fig 5.20 Granular Road Base Pavement.

#### 5.5.1.1 Subgrade layer.

The response of the subgrade was controlled by the resilient modulus  $E_3$  ( $E_s$ ), Poisson's ratio  $\mu_3$  and the limiting thickness  $H_3$ . According to Desai et al (114), the depth of the whole structure should be at least 10 or 12 times the diameter (170 mm) of the wheel. Since the pavement thickness was about 550 mm, the thickness of the subgrade was fixed to 2250 mm. This will ensure that the lower boundary condition will not have a significant effect on the surface deflection profile.

It has been reported (56) that the subgrade soil is stress dependent and relationships have been developed between stress, strain and modulus. However, Bleyenberg et al (121), have demonstrated from full scale road experiments, that linear elastic theory may be used to describe the pavement response, provided that the moduli of the material were determined from appropriate loading conditions. Where in-situ measured subgrade modulus is not available, the empirical relationship between dynamic subgrade modulus and the CBR value may be used (115):

$$E_{c} = 10' \times CBR N/m^{2}$$
(5.3)

The subgrade modulus for the granular road base pavement was not

available, but based on the knowledge of the soil type, a CBR 15% was chosen as an appropriate value at the formation level and these give a subgrade modulus of  $E_3 = 15 \times 10^7 \text{ N/m}^2$ . The subgrade modulus was assumed to vary linearly from 15  $\times 10^7 \text{ N/m}^2$  at the formation level to 100  $\times 10^7 \text{ N/m}^2$  at 2.25 m below the formation level. The Poisson's ratio,  $\mu_2$ , was assumed to be 0.35.

#### 5.5.1.2 Intermediate Granular Layer.

The modulus of the unbound base layer is to a large extent stress dependent (122). Theoretical analysis and field measurements have shown that the modulus of the unbound base layer,  $E_2$ , depends on its thickness  $H_2$ , and the modulus of the underlying subgrade,  $E_3$ , according to the following relationship (43):

$$E_2 = K_2 E_3 \tag{5.4}$$

where  $K_2 = 0.2 H_2$  with  $H_2$  in mm, with the limit 2 K<sub>2</sub> 4. An initial value of  $E_2 = 2 \times 10^8 \text{ N/m}^2$  was assumed for the combination of the wet mix macadam base and type 2 sub-base. The Poisson's ratio,  $\mu_2$ , was assumed to be 0.35. The thickness of the granular layer,  $H_2 = 450$  mm, was split into two layers of 300 mm and 150 mm for analysis. This enables different modulus values to be assigned to the wet mix macadam base and the type 2 sub-base layer if required.

#### 5.5.1.3 The Asphalt Bound Layer.

The stiffness modulus of the asphalt mix can vary considerably from around 1 x  $10^7$  to about 5 x  $10^{10}$  N/m<sup>2</sup>. The upper range of stiffness values (say 1 x  $10^9$  to 5 x  $10^9$  N/m<sup>2</sup>) has been determined for a large number of asphalt mixes by means of dynamic and/or semi-static (e.g, constant rate of loading) test at various

temperature and under different loading conditions. The modulus is dependent solely on the bitumen content, the stiffness of the bitumen and the voids in the mix. The stiffness of bitumen which varies with loading time and temperature can be estimated with the aid of Van der Poel's nonograph (41). The loading time was determined using following equation (42):

log t = 5 x  $10^{-4}$ H - 0.2 - 0.94 log V (5.5) For the pavement considered, H = 150 mm, and V = 2 Km/hr and therefore the loading time, t = 0.36 secs.

A 50 pen grade bitumen was assumed. Properties of the binder for typical mixes and the properties of HRA wearing course were obtained from Tables 2 and 3 of Ref (42).

$$P_{i} = 50$$
 (5.6)

$$P_r = 0.65 P_i = 32.5$$
 (5.7)

$$SP_{r} = 98.4 - 26.35 \log P_{r} = 58.6 \text{ °C}$$
(5.8)  
$$PI_{r} = \frac{1951.4 - 500 \log P_{r} - 20SP_{r}}{50 \log P_{r} - SP_{r} - 120.14}$$
$$= -0.2$$
(5.9)

It was reported (123) that the temperature (T) was low when the measurements were taken and therefore 10  $^{\circ}$ C was assumed.

$$SP_r - T = 48.6 °C$$
 (5.10)

Using the Van der Poel's nonograph (41), Fig 3.2,  $S_{b}$  (stiffness of bitumen) is equal to 200 MPa.

$$M_{B} = 7.9\%, V_{V} = 4.0\%, G_{b} = 1.01, G_{a} = 2.65,$$

$$V_{B} = (100 - V_{V}) \left[ \frac{(M_{B}/G_{b})}{(M_{B}/G_{b}) + (M_{A}/G_{a})} \right] = 17.6\% \quad (5.11)$$

$$VMA = V_{B} + V_{V} = 21.6\% \quad (5.12)$$

Using the relationship between mix stiffness  $(S_m)$ , binder stiffness

 $(S_b)$  and VMA in Fig 5.22 (41), the mix stiffness is equal to  $6 \times 10^9 \text{ N/m}^2$ . Therefore, the modulus of the asphalt bound layer was first assumed initially be  $6 \times 10^9 \text{ N/m}^2$ .



Fig 5.21 The Relationship Between Mix Stiffness, Binder Stiffness and VMA (after Brown et al, Ref 42).

## 5.5.2 Matching The Response of The 3D Model and Actual Pavement

#### Structure.

The elastic modulus, Poisson's ratio and the thickness of initial model are given in Fig 5.22. The front wheel load is 22 KN and the rear wheel load is 17 KN. When the model was analyzed using the values, the maximum deflection obtained was less than the actual value. The deflection at a distance 1 m away from the point of maximum deflection was positive instead of negative, i.e. surface deflecting upwards. This shows that the subgrade value assumed is very stiff and therefore the elastic modulus of the subgrade was lowered to CBR 10% at the surface of the subgrade. The model was then analyzed with the subgrade modulus varying linearly from 1 x 10<sup>8</sup> N/m<sup>2</sup> at the surface to 1 x 10<sup>10</sup> N/m<sup>2</sup> at the base of the subgrade layer. This very much increases the deflection, both at the point of maximum deflection and at a distance 1 m away from the point of maximum deflection, towards the actual value.

$E_1 = 6 \times 10^9 \text{ N/m}^2$ , $\mu_1 = 0.35$ , $H_1 = 100 \text{ mm}$				
$E_2 = 2 \times 10^8 \text{ N/m}^2$ , $\mu_2 = 0.35$ , $H_2 = 300 \text{ mm}$				
$E_3 = 2 \times 10^8 \text{ N/m}^2$ , $\mu_3 = 0.35$ , $H_3 = 150 \text{ mm}$				
$E_4 = Varied linearly with depth from 15 \times 10^7$				
to 10 x 10 <sup>8</sup> N/m <sup>2</sup> , $\mu_4 = 0.35$ , $H_4 = 2245$ mm				

Fig 5.22 Elastic Modulus, Poisson's Ratio and The Thickness of the Initial Model.

The modulus of the subgrade at various depths was also altered so that it is no longer follows a linear variation with depth. Change in modulus values at various depths, affects the shape of the deflection profile at some distance, say 500 to 800 mm, away from the point of maximum deflection. The final values of the subgrade modulus at various depths is shown in Figs 5.23 and 5.24. Figs 5.25 and 5.26 shows the deflection profile, along the centre line through the twin rear wheels, Fig 5.25 (line XX of Fig 5.4), and along the centre line of the vehicle, Fig 5.26 (line YY of Fig 5.4), obtained from the 3D finite element model and those obtained by the TRRL. Fig 5.27 shows the Deflectograph deflection profile for the in-situ pavement structure, and the profile a Deflectograph would see if measuring the deflected shape generated by the 3D finite element model. Those figures show that deflection profile, both absolute and Deflectograph, for the actual pavement and the model are very similar for the first 1.5 m away from the point of maximum deflection. Beyond the 1.5 m, the computed deflections are less than the actual deflection and this could be due to the coarse mesh size around the front wheel. Full details of the effect of change of the elastic modulus of the various layers, on the deflection profile is discussed in Chapter 6.0.

#### 5.5.3 Comparison Of Asphalt Tensile Stresses.

The Shell method (124) presents a procedure for calculating the average stress levels in the bituminous layer. The bituminous layer is first divided into three sub-layers: the first two layers should be 40 mm thick and the third layer equal to the total bituminous layer thickness less 80 mm. The bituminous layer is subdivided so that the temperature at different depths can be taken into account. It is based on detailed studies (124), from which it has been concluded that the uppermost layers are subjected to the greatest temperature changes and are usually made of different types of mix from that used

in the lower layers. The lower layers are subjected to smaller temperature change and are usually of similar mix type, so that they are not subdivided to the same extent.

$E_1 = 6 \times 10^9 \text{ N/m}^2$ ,	$\mu_1 = 0.35,$	$H_1 = 100 \text{ mm}$
$E_2 = 2 \times 10^8 \text{ N/m}^2$ ,	$\mu_2 = 0.35,$	$H_2 = 300 \text{ mm}$
$E_3 = 2 \times 10^8 \text{ N/m}^2,$	$\mu_3 = 0.35,$	H <sub>3</sub> = 150 mm
$E_{4.1} = 1.0 \times 10^8 \text{ N/m}^2$ ,	$\mu_{4.1} = 0.35,$	$H_{4.1} = 374 \text{ mm}$
$E_{4.2} = 1.1 \times 10^8 \text{ N/m}^2$ ,	$\mu_{4.2} = 0.35,$	$H_{4.2} = 374 \text{ mm}$
$E_{4.3} = 2.4 \times 10^8 \text{ N/m}^2$ ,	$\mu_{4.3} = 0.35,$	$H_{4.3} = 374 \text{ mm}$
$E_{4.4} = 3.5 \times 10^8 \text{ N/m}^2$ ,	$\mu_{4.4} = 0.35,$	$H_{4.4} = 374 \text{ mm}$
$E_{4.5} = 5.0 \times 10^8 \text{ N/m}^2$ ,	$\mu_{4.5} = 0.35,$	$H_{4.5} = 374 \text{ mm}$
$E_{4.6} = 10.0 \times 10^8 \text{ N/m}^2,$	$\mu_{4.6} = 0.35,$	$H_{4.6} = 374 \text{ mm}$

Fig 5.23 Elastic Modulus, Poisson's Ratio and The Thickness of the Final Model.

The average stress in each sub-layer is dependent on six variables:  $E_3$  (subgrade modulus),  $H_2$  (thickness of unbound layer),  $H_{1-3}$  (thickness of the lower bituminous sub-layer),  $E_{1-3}$  (modulus of the bottom asphalt layer),  $E_{1-2}$  (modulus of the intermediate layer and  $E_{1-1}$  (modulus of the top asphalt layer). The average stress in each of the sub-layers is the product of the contact stress of the standard design wheel,  $6 \times 10^5 \text{ N/m}^2$  (dual wheels, each 20 KN) and the proportionality factor, Z. The Z factor is a function of thickness and the modular ratios between the different asphalt sublayers and the layers in the structure, and is therefore determined for each sub-layer. For the pavement structure with a granular road base:

$$E_3 = 10 \times 10^7 \text{ N/m}^2$$
,  $H_2 = 450 \text{ mm}$ ,  
 $E_{1-1} = E_{1-2} = E_{1-3} = 6 \times 10^9 \text{ N/m}^2$ ,  $H_{1-3} = 20 \text{ mm}$ .

The proportionality factors, Z, for this non-typical structure of 100 mm of bituminous material on top of 450 mm of granular material are not available from the data tables presented in the Shell Method (124); tabulated values corresponding to the structure closest to that investigated were used. Details of these structures and the corresponding Z factors derived from the Table Z8 and Table Z48 in the Shell Manual (124) are given in Table 5.7 from which it can be deduced that the Z factors for the lower bituminous sub-layer lies between 0.8 and 1.1. This gives an average stress level between 4.8 x 10<sup>5</sup> and  $6.6 \times 10^5 \text{ N/m}^2$  in the lower bituminous sub-layer.

Subgrade	Thickness of	Thickness of the mid	Zl	Z2	Z3
Modulus,N/m	unbound layer,mm	asphalt sub-layer,mm			
1×10 <sup>8</sup>	300	0	-0.3	1.1	1
1x10 <sup>8</sup>	300	50	0.0	0.5	0.8
1×10 <sup>8</sup>	600	0	-0.1	1.0	-
1×10 <sup>8</sup>	600	50	0.1	0.5	0.8

Table 5.7 Proportionality Factor, Z, Derived

from the Shell Manual (Ref 124).











Fig 5.27 Comparison of Deflectograph Deflection.

The stress at the bottom of the bituminous layer, on the vertical axis of symmetry between the two loaded areas is greater than those on the vertical axis through the centre of the loaded area for the 3D finite element model. The stress on the vertical axis of symmetry between the two areas is  $1.7 \times 10^6 \text{ N/m}^2$ . This value is slightly greater than that suggested by the charts in the Shell Manual (124) and it may probably be due to the difference in the magnitude of the wheel loads, layer moduli and layer thickness. However, the stress obtained is not unrealistic, which when taken together with the fact that the deflected shape of the theoretical model was very similar to that of the actual pavement structure. This suggests that the response was sufficiently accurate to be used as the basis of an investigation of the relationship between the deflected surface shape and the properties and thickness of pavement structures.

# 5.6 Fortran Program to Change Absolute Deflection to Deflectograph

#### Deflection.

PAFEC program (116, 117, 118) creates an output file, Phase 7, which contains the deflection, in three directions, of all the nodes in the structures. This output file is quite large and the PAFEC program does not have the facility to only print the deflection at certain nodes.

A Fortran Program, called TRANS.F77 (see Appendix IV), has been written to pick out the vertical deflection, y direction, at certain particular nodes from the output file, Phase 7. The nodes along the centre line between the twin rear wheels, line XX of Fig 5.18, and along the centre line of the vehicle (Deflectograph), line CH of Fig 5.18, have been numbered in an ascending order, i.e. 1

to 25. Therefore when the Fortran Program is run, it picks up only the first 25 deflection values in the output file, Phase 7.

Another Fortran Program, called PLOT.F77 (see Appendix V), has been written to convert the absolute deflection into Deflectograph deflection using the following equation (125):

$$D_{D} = (Y_{BF} - Y_{BO}) - (1_{2}/1_{1}) (Y_{CF} - Y_{CO})$$
  
- (1 - 1\_{2}/1\_{1}) (Y\_{AF} - Y\_{AO}) (5.13)

where  $Y_{AO}$  and  $Y_{AF}$  = Initial and final deflection of point A;  $Y_{BO}$  and  $Y_{BF}$  = Initial and final deflection of point B;  $Y_{CO}$  and  $Y_{CF}$  = Initial and final deflection of point C;  $l_2$  = Length of T-Datum frame = 2.13 m;  $l_1$  = Length of Measuring Arm = 1.53 m.



Fig 5.28 Simplified Diagrammatic Representation of Deflectograph.

The program changes all the absolute deflections along the centre line through the front and twin rear wheels, at either 50, 100, 125, 200 or 250 mm intervals, into equivalent Deflectograph recorded deflection.

#### 5.7 Summary.

The specification of the final model used to investigate the relationships between deflected shape and the properties and thickness of the pavements layers is summarised below.

#### (a) Loading.

- (i) A square, 170 x 170 mm, contact area was used in the model;
- (ii) The loads were applied as pressure loads over the contact area;
- (iii) Only one set of twin rear wheels, nearside, and front wheel were modelled;
- (iv) Only half of the nearside twin rear wheels and front wheels were modelled.
- (b) Dimension of Model,
  - (i) The distance of the centre of the nearside twin rear wheels to the kerb is about 1300 mm;
  - (ii) The dimensions of the model are as follow: Length = 4500 mm, Width = 2225 mm, Height = 2040 mm (min);
  - (iii) The model consist of 9 main layers and each layer can be subdivided into a number of sub-layers. All the main layers can have different material properties but all the sublayers within the main the layer must have the same material properties;
    - (iv) The thickness of the main layer and sub-layer within any main layer must be between 75 to 374 mm.
- (c) The Boundary Conditions.
  - (i) The plane near the kerb is free, i.e. not restrained in any direction;
  - (ii) The plane through the cut-off plane of the twin rear wheels and front wheel is restrained from moving in the x direction (horizontal longitudinal);
  - (iii) The plane through the line along which the the T-frame moves is restrained from moving in the z direction (horizontal

transverse);

(iv) The lower plane of the model is restrained in all directions Table 5.5 shows the restraint applied to the various planes of the model.

#### (d) Material Properties.

(i) The material properties required for linear analysis are the elastic modulus, Poisson's ratio and the density. The properties required for non-linear analysis are: yield stress, plastic modulus, gradient of the stess-strain curve before and after yielding and the size of incremental load.

#### (e) Element Types.

- (i) Eight noded brick element and six noded wedge elements are used;
- (ii) Total number of elements used required are 2331;
- (iii) Total number of nodes required are 2920.
- (f) Computing Time.
  - (i) The computing time required for the analysis of the model from the beginning is 24,600 sec but when the job is restarted from phase 6, i.e. when only the material properties ' of the model is changed, the time required is only 15,113 sec.

#### (g) Special Programs.

- (i) Fortran Program, TRANS.F77, picks out the deflection along the line through the centre of the twin rear wheels and the centre line of the vehicle,
- (ii) Fortran Program, PLOT.F77, converts the deflection obtained from the model into deflection that would be recorded by the Deflectograph.

## CHAPTER 6.0 RELATIONSHIPS BETWEEN CURVATURE AND/OR DEFLECTION AND THICKNESS AND ELASTIC MODULUS OF THE VARIOUS LAYERS OF THE PAVEMENT.

#### 6.1 Introduction.

It would be very beneficial to establish relationships between measurements that can be taken on the road surface and the thickness and elastic modulus of the pavement layers, since they could be used as:

- (a) a tool for identifying the weakest layer(s) of a pavement;
- (b) a design tool to determine the thickness and elastic modulus of the pavement layers to produce a given surface deflection.

A parametric study has been carried out to develop such relationships between curvature and/or deflection of a pavement surface under load and the thickness and the elastic modulus of its various layers. The study involved the use of a 3D finite element model of a pavement in which the thickness and elastic modulus of the various layers were varied independently. The model was used to determine the distance over which the surface curvature of the road surface is influenced significantly by changes in the thickness and elastic modulus of the various layers. Separate routines were developed to convert the absolute deflected shape predicted by the model into the equivalent deflection dish that would be measured by a Deflectograph. In all cases the measures of deflected road surface shape or 'curvature', used in developing the relationships are those that would be measured by a Deflectograph.

This chapter attempts to identify some of the factors, i.e. pavement layer thickness and modulus, which influence the deflection and the differential deflection recorded by the Deflectograph. The
results of the analysis are used in developing an Analytical Pavement Evaluation and Design system. Various relationships have been established between Maximum Deflection,  $D_0$ , Deflection at distance x away from point of maximum deflection,  $D_x$ , and Differential Deflections,  $D_x-D_y$  (difference in deflection at distances x and y away from the point of maximum deflection), Equivalent Thickness,  $H_e$ , and the thickness and elastic modulus of the various pavement layers.

#### 6.2 Parametric Study.

In a parametric study, the thickness and elastic modulus of the various layers of a pavement are changed independently, one parameter at a time, and the effects on the resulting deflected shape and deflection of the pavement under load are obtained. To carry out a full parametric study covering the range of all variables for every pavement layer requires a substantial computer resource and, therefore, only two different values of elastic modulus and thickness for each layer were considered. Table 6.1 shows the different values of thickness and elastic modulus of the different layers. The upper and lower value of layer thickness and modulus are considered to represent the limiting pratical values to be found in most pavements, see Fig 6.1. Relationships have been established for different elastic modulus and thickness by interpolating between the two specified values. Only linear elastic analysis was carried out. The Deflectograph deflection derived from the 3D model was used to establish relationships between maximum deflection/differential deflection of the road surface and the thickness and modulus of individual pavement layers.

++	100	/	/		100				150			
100	0 1 0	Tar	/		150	30	0	15	0	3	300	
2 2 10	140	/	/	150	350	150	350	150	350	150	350	
2	2	2	4									
			10						_			
		9	4									
			10									
	9	2	4									
			10									
		9	4									
			10						_			
10	2	2	4								-	
			10						-			
		9	4						_		-	
			10								-	
	9	2	4						-	-		
			10									
		9	4								-	
			10						$\dot{a}$			

Note: All combinations have been investigated. Table 6.1 Different Combinations of the Thickness and Elastic Modulus the Different Layers of Pavement.



Subgrade: CBR 2 - CBR 10;  $E_4 = 2 - 10 \times 10^7 \text{ N/m}^2$ .

Fig 6.1 Range of Thickness and Elastic Modulus of Individual Pavement

Layers.

# 6.3 Selecting the Position of a Suitable Ordinate Differential

## Deflection.

The results of the parametric Study showed that the shape in the vicinity of the maximum deflection can be used as an indicator of the strength of the upper bound layer. The next step was to select the position of the ordinate deflection within this area whose changes in magnitude, as a result of changes in the thickness and modulus of the layer, was large enough to be recorded by a Deflectograph. Deflection parameter,  $D_x$ , decribed in this thesis refers to the vertical surface deflection of the pavement at distance x away from the point of maximum deflection. Differential deflections,  $D_x-D_y$ , refers to the difference in the vertical surface deflection of the pavement at distances x and y away from the point of maximum deflection.

The suitable position and distance over which the differential deflection (recorded by Deflectograph) should be obtained is determined by comparing the magnitude of the differential deflections over various distances throughout the deflected dish. The position of the differential deflection, used to characterise a particular parameter, was selected as the point at which the maximum change in differential deflection is produced for a small change in the parameter considered. Fig 6.2 shows the deflected shape recorded by the Deflectograph and the symbol used to represent the deflection ordinates at various distances away from the point of maximum deflection. Fig 6.3 illustrates the method used to obtain the most suitable position and the distance at which the differential deflection should be measured. The example presented in Fig 6.3 relates the differential deflections  $D_0 - D_8$  to the equivalent thicknesses,  $H_{\rho}$ , for a given value of  $H_{C}$ , see Fig 6.7. From the graph it can be seen that the maximum differential deflection is  $D_0 - D_8$  (i.e. difference in deflection at point 0 mm and 800 mm away from the point of maximum deflection).

This procedure was adopted to obtain the most suitable position and distance of the differential deflection for the change in the thickness and elastic modulus of the various layers.

### 6.4 Method of Statistical Analysis.

In order to develop an analytical pavement evaluation and design system, relationships must be established between the Deflectograph deflection/differential deflection and the thickness and modulus of individual layers of the pavement. Several relationships were investigated but in many casses the variation of the original data demonstrates that estimates of predicted layer properties could

not be made with reasonable levels of statistical confidence.

### 6.4.1 Regression and Fitting of Common Slope.

The statistical analysis was carried out using computer based Statistical Analysis packages called MINITAB (126) and GLIM (127). Several statistical models, i.e. Power, Exponential and Polynomial (k orders), were fitted to investigate the relation between the differential deflection and pavement layers thickness and modulus. The R-Squared, i.e. (Sum of squares explained by the regression/Sum of squares of the independent variables), were compared for all the statistical models and the model which gave the largest value of R-squared was identified. The Power Law model gave the largest value R-squared and was linearised by plotting  $Log_{10}Y$  Vs  $Log_{10}X$ . When investigating the difference in several regression lines for the different pavement layer variables, i.e. modulus and thickness, the following procedures were carried using the Statistical Package GLIM (127):

- (a) Fit separate lines, linearised by plotting  $Log_{10}Y$  Vs  $Log_{10}X$ , (different slopes and intercepts).
- (b) Use the F-test in the analysis of variance to test for significant difference in slopes. If the differences are not significant then a common slope (different intercepts) can be fitted for the different lines.
- (c) Once parallel lines have been established, they are then tested for a common line, i.e. single intercept. The analysis of variance is carried out using F-test to test if there is any significant difference in intercepts. If the differences are significant then the lines remain different and only a common slope is fitted. Details of the statistical analysis are shown in Appendix VI.



Fig 6.2 Nomenclature to Represent the Deflection Ordinates at Various Distances Away from the Point of Maximum Deflection.



the Point of Maximum Deflection.

Fig 6.3 Diagrammatic Representation of the Method to

Obtain the Position and Distance of the

Differential Deflection.

#### 6.4.2 Confidence Limit.

The 90% confidence limits for a future predicted observation have also been calculated using the t-test for the relationships defined in Section 6.4.1 above. The confidence limits are the band width within which 90% of the results of the different types of structure considered fall, i.e scatter of data. The band represent the range of data from the weakest structure, i.e. combination of all the minimum thicknesses and moduli of each layer, to the strongest structure, i.e. combination of all the maximum thicknesses and moduli of each layer. It is also the band width within which 90% of the future predicted analysis will fall. 90% of the pavements with structure within the range specified in Fig 6.1 will fall within this band. Pavement structure for motorways and other modern designed roads might be expected to have somewhat narrower 90% confidence limits if good quality control has been achieved in layer properties and thickness. An example of the regression line and the 90% confidence limits are shown in Figs 6.4 and 6.6.

# 6.5 Relationships Between Deflected Shape and Pavement Condition. 6.5.1 Relationship between $D_0$ and CBR of Subgrade.

Several authors, using various measuring equipment (15, 31, 32), have reported that the deflection at some distance away from the point of maximum deflection is a measure of support strength. The distance from the point of maximum deflection at which the deflection is taken depends on the types of equipment used. It seems that for the Deflectograph, the deflection at a distance 800 mm away from the point of maximum deflection is a good indicator of the support strength.

Analysis of the effect of different CBR of the subgrade on the response of the finite element model has led to the derivation of a relationship between the deflection at distance 800 mm away from the point of maximum deflection,  $D_8$ , and the CBR of the subgrade. Variation of elastic modulus and thickness of each layer above the subgrade for a given CBR has only a very small influence on the  $D_8$ .

Fig 6.4 shows the Power Law relationship between  $D_8$  and CBR with 90% confidence limits. Table 6.2 shows the equation to describe the relationship. From this relationship, it is possible to determine the CBR of the subgrade knowing the deflection,  $D_8$ . For a given  $D_8$  of 0.004 mm the relationship predicts a CBR of 2% with a 90% confidence limits of ±1% (see Fig 6.4). It predicts a CBR of 7% with a 90% confidence limits band narrows at higher CBR since the variation in  $D_8$  reduces as the subgrade increases in strength when other pavement variables, i.e. layer thickness and modulus, have negligible influence on  $D_8$ . To predict the subgrade strength requires that the Deflectograph can measure to  $1 \,\mu$ m.

Equations	No
$\log_{10}(D_8) = 0.776 - 0.572 \log_{10}(CBR)$	6.1

Table 6.2 Equations to Describe The Relationship Between  $D_{R}$  and CBR.



Fig 6.4 Power Law Relationship Between D<sub>8</sub> and CBR.

# 6.5.2 Relationships Between Maximum Deflection, $D_0$ , and Equivalent

Thickness, He.

Equivalent Thickness,  $H_e$ , for a four layer system is defined as follows (80, 81, 82):

$$H_{e} = 0.8 \times \left( H_{1} \sqrt[3]{\frac{E_{1}}{E_{4}}} + H_{2} \sqrt[3]{\frac{E_{2}}{E_{4}}} + H_{3} \sqrt[3]{\frac{E_{3}}{E_{4}}} \right)$$
(6.2)

where  $H_{p} = Equivalent Thickness;$ 

 $H_1$  = Thickness of Surfacing;

 $H_2$  = Thickness of Road Base;

 $H_3 = Thickness of Sub-Base;$ 

 $E_1$ ,  $\mu_1$  = Elastic Modulus and Poisson's ratio of Surfacing;  $E_2$ ,  $\mu_2$  = Elastic Modulus and Poisson's ratio of Road Base;

 ${\rm E}_{3},\;\mu_{3}$  = Elastic Modulus and Poisson's ratio of Sub-Base;

 $E_4$ ,  $\mu_4$  = Elastic Modulus and Poisson's ratio of Subgrade. This equation (Eqn.6.2) allows a four layer system to be transferred into a one layer system of thickness,  $H_e$ . In the parametric study, the Poisson's ratio for all the layers was assumed to be 0.35.

The maximum deflection,  $D_0$ , was plotted against  $H_e$  for various combinations of pavement structures investigated, see Table 6.1, on two different subgrades: CBR 2 and CBR 10. Fig 6.5 shows the scatter of the data for CBR 2 when the  $Log_{10}(D_0)$  is plotted against  $Log_{10}(H_e)$ . The Power Law equations to describe the relation ships are shown in Table 6.3. A common slope could not be fitted for the two CBR levels using the procedures stated in Section 6.4.1. Fig 6.6 illustrates the relationship with the 90% confidence limits. It seems that the maximum deflection,  $D_0$ , is greatly influenced by  $H_e$ . The lines of the two CBRstend to converge at a higher value of  $H_e$  or at lower value of maximum deflection, i.e. when the pavement is very stiff. For the various combinations of structures investigated, the range of  $H_e$  for CBR 2 is 0.98 to 2.8 m and for CBR 10 is 0.58 to 1.64 m. The minimum and maximum value of the  $H_e$  range represents the weakest and strongest structure of the combinations of structures investigated. At the 90% confidence level, the relationship for CBR 2 predicts a value of  $H_e$ :  $1.5 \pm 0.2$  m for  $D_0$  of 0.27 mm. To give some engineering appreciation of this variation, a confidence interval of  $H_e$ :  $\pm$  0.2 m about the mean value would result in the following variations in the thickness and modulus of pavement layers when converted back into individual properties:

 $E_1 = Variation of \pm 4.0 E_1 (mean);$ 

$$E_2 = E_3 = Variation of \pm 4.0E_2(mean);$$

 $H_1 = Var_i \text{ intion of } \pm 1.5H_1 \text{ (mean)};$ 

 $H_2 = H_3 = Variation of \pm 1.4H_2(mean).$ 

For CBR 10, a D<sub>0</sub> of 0.2 mm gives a 90% confidence range of H<sub>e</sub>: 0.82  $\pm$  0.16 m. This confidence interval of H<sub>e</sub>:  $\pm$  0.16 m about the mean value has the same order of variation in the pavement variables as CBR 2. A small variation of H<sub>e</sub> would change any  $\rightarrow$ ? moduli  $\rightarrow$  of the pavement layers, i.e. E<sub>1</sub>, E<sub>2</sub>, E<sub>3</sub>, by 2 to 3 times. H<sub>e</sub> is influenced more by the modulus than the thickness of the pavement layers. This shows that the modulus of the pavement layers should not be determined from any relationship having H<sub>e</sub> as one of the variables.

CBR	Equations	NO		
10	$\log_{10} D_0 = 1.197 - 1.23 \log_{10} H_e$	6.3		
2	$\log_{10} D_0 = 1.731 - 1.73 \log_{10} H_e$	6.4		

Table 6.3 Equations for the Relationships Between  $D_0$  and  $H_e$ .



Fig 6.5 Scatter of Data:  $Log_{10}(D_0)$  Vs  $Log_{10}(H_e)$  for CBR 2.



Fig 6.6 Relationships Between  $D_0$  and  $H_e$  for a Given CBR.

# 6.5.3 Relationships Between $D_0-D_8$ and $H_e$ for a Given Thickness

of Granular Material,  $H_{G}$  (H<sub>2</sub>+H<sub>3</sub>).

In most pavements, containing a granular road base, the road base and the sub-base are constructed with similar material, i.e. having a similar modulus value when compared with the modulus of layers immediately above and below. Relationships between differential deflection and equivalent thickness for a given value of road base,  $H_2$ , and sub-base,  $H_3$ , have been investigated but no practical relationships that could be used to interpret site measurements were established. Therefore,  $H_2$  and  $H_3$  were combined together as  $H_G$ (total thickness of granular material) and relationships were established between  $H_e$  and differential deflection,  $D_0-D_8$ , for a given value of  $H_c$ .

The differential deflection,  $D_0-D_8$ , was plotted against  $H_e$ , for a given thickness of granular material,  $H_G$  ( $H_2$ + $H_3$ ), see Fig 6.1. A common slope was fitted for the different regression lines. The statistical details and the Power Law equations to represent the relationships are shown in Tables A.1 and A.2 of Appendix VI. The variation of the thickness of granular material greatly influences the differential deflection,  $D_0-D_8$ . It is possible to determine  $H_G$  from Fig 6.7; knowing  $H_e$  (see Fig 6.6) and CBR (see Fig 6.4). The following information could be deduced from the relationships: Given: CBR 2;  $H_G = 300$  mm;  $D_0-D_8 = 0.3$  mm; Prediction:  $H_e = 1.21 \pm 0.1$  m at 90% confidence limits; Variation in the thickness and modulus of pavement layers due to the confidence interval of  $H_e = \pm 0.1$  m:

 $E_1: \pm 1.6E_1 \text{ (mean)}; E_G: \pm 1.6E_G \text{ (mean)};$  $H_1: \pm 1.2H_1 \text{ (mean)};$  This clearly indicates that by knowing  $H_G$  and CBR, the variation of  $E_1$  due to variation of  $H_e$ , or vice-versa, is small. The variation for CBR 10 is of same order as for CBR 2.

6.5.4 Relationships Between  $D_0-D_8$  and  $H_e$  for a Given Pavement

Thickness,  $H_p (H_1 + H_G)^{-}$ .

The differential deflection,  $D_0$ - $D_8$ , was plotted against  $H_e$ , for a given pavement thickness,  $H_p$  ( $H_1$ + $H_G$ ), see Fig 6.1. Table A.3 of Appendix VI shows the equations describing the relationships.

It is observed that the differential deflection,  $D_0^{-D_8}$ , is influenced by the variation of the total pavement thickness,  $H_p$ . The relationships could be used to determine  $H_p$  from Fig 6.8; knowing  $H_e$  (see Fig 6.6) and CBR (see Fig 6.4). The following information could be deduced from the relationships: Given: CBR 2;  $H_p = 400 \text{ mm}$ ;  $D_0^{-D_8} = 0.3 \text{ mm}$ ; Prediction:  $H_e = 1.2 \pm 0.1 \text{ m}$  at 90% confidence limits; Variation in the thickness and modulus of pavement layers due to the confidence interval of  $H_e = \pm 0.1 \text{ m}$ :

$$\begin{split} E_{1} &: \pm 1.7E_{1} \text{ (mean)}; \\ E_{G} &: \pm 1.4E_{G} \text{ (mean)}; \\ H_{1} &: \pm 1.2H_{1} \text{ (mean)}; \\ H_{G} &: \pm 1.2H_{G} \text{ (mean)}; \end{split}$$

Given: CBR 10;  $H_p = 600 \text{ mm}; D_0 - D_8 = 0.3 \text{ mm};$ 

Prediction:  $H_{p} = 0.75 \pm 0.05$  m at 90% confidence limits;

The variation in the thickness and modulus of pavement layers due to the confidence interval of  $H_e = \pm 0.05$  m is of the same order as for CBR 2 detailed above. The value of  $H_1$  (thickness of surfacing) could be determined by subtracting  $H_G$  from  $H_P$  for a given CBR,  $H_e$  and  $D_0^{-D}_8$ .







Fig 6.8 Relationships Between  $D_0^{-D_8}$  and  $H_e$  for a Given  $H_p$ .

# 6.5.5 Relationships Between $D_0 - D_8$ and $H_e$ for a Given Value

# $\frac{E_1/E_2}{E_1}$ Ratio.

Relationships between  $D_0-D_8$  and  $H_e$  for a given value of  $E_1/E_2$  (surfacing modulus/road base modulus) ratio and  $E_2$  have been established. It is observed that the variation of  $E_1/E_2$  ratio largely influences the differential deflection,  $D_0-D_8$ . The Power Law equations for the relationships are shown in Table A.4 of Appendix VI. The regression lines, see Figs 6.9, 6.10, 6.11 and 6.12, for the different  $E_1/E_2$  ratios tender to converge at high values of  $H_e$ , i.e. thicker and/or greater modulus of one or more layers of the pavement. To use the relationships;  $H_e$ ,  $D_0-D_8$ , and  $E_2$  have to be known. The relationships could be interpreted as follows: Given: CBR 2;  $D_0-D_8 = 0.1$  mm;  $E_2 = 2 \times 10^8$  N/m<sup>2</sup>;

$$E_1/E_2 = 40/2$$
.

Prediction:  $H_e = 1.6 \pm 0.3$  m at 90% confidence limits. The variations in the thickness and modulus of the pavement layers due

 $H_1: \pm 1.8H_1 \text{ (mean)}; H_G: \pm 1.4H_G \text{ (mean)}.$ Given: CBR 10;  $D_0-D_8 = 0.055 \text{ mm}; E_2 = 2 \times 10^8 \text{ N/m}^2;$ 

the confidence interval  $\pm 0.3$  m are as follows:

$$E_1/E_2 = 40/2$$
.

Prediction:  $H_e = 0.76 \pm 0.4 \text{ m}$  90% confidence limits. Variations due to the confidence interval  $H_e = \pm 0.3 \text{ m}$ :

 $H_1: \pm 1.8H_1 \text{ (mean)}; H_G: \pm 1.8H_G \text{ (mean)}.$ 

For the calculation of variation in  $H_1$  and  $H_G$ , the values of  $E_1$ and  $E_2$  were kept constant since the ratio is known.

From the relationships it is possible to determine  $E_1$  knowing  $E_2$ . In most pavements with a granular road base, the sub-base and road base are of similar material, i.e. having similar modulus values when compared with the modulus of layers immediately above and below. Therefore,  $E_2$  may be obtained from the following equation (73, 74):

 $E_2 = E_3 = K E_4$ (6.5) where K = 0.2 H<sub>3</sub><sup>0.45</sup>; 2<k<4; H<sub>3</sub> = Sub-base Thickness;  $E_3 =$  Sub-base Modulus;  $E_4 =$  Subgrade Modulus. Once E<sub>2</sub> is known, E<sub>1</sub> may be determined from the relationships.

6.5.6 Relationships Between  $D_0-D_2$  and  $H_1$  for a Given Ratio of

 $E_1/E_2$ 

Relationships have been derived between  $D_0-D_2$  and  $H_1$ for a given value of  $E_1/E_2$  ratio and  $E_2$ . Table A.6 of Appendix VI show the equations to describe the relationships. The relationships are independent of the support strength, i.e. CBR. The value of  $H_1$ may either be obtained from subtracting  $H_G$  from  $H_p$  or by coring from the pavement. The relationships may be used to determine either the value of  $H_1$  knowing the ratio  $E_1/E_2$  or the ratio  $E_1/E_2$ knowing  $H_1$ . It could be observed from Figs 6.13 and 6.14 that the slopes of the regression lines are gentle, i.e. the rate of change of  $D_0-D_2$  with  $H_1$  is very low. For a given  $D_0-D_2$  of 0.07 mm and  $E_1/E_2$  ratio of 400/20, the relationship predicts a range of  $H_1 =$ 120 ± 20 mm. The confidence interval for other ratios of  $E_1/E_2$  is about ± 25 mm. This shows that the relationships could predict  $H_1$ to a nearest value of 50 mm. The confidence interval width narrows for the regression lines as  $H_1$  increases.



 $E_1/E_2$  and  $E_2 = 2 \times 10^8 \text{ N/m}^2$ .



Fig 6.10 Relationships Between  $D_0 - D_8$  and  $H_e$  for CBR 2 for Given  $E_1/E_2$  and  $E_2 = 9 \times 10^8 \text{ N/m}^2$ .





Fig 6.12 Relationships Between  $D_0^{-D_8}$  and  $H_e$  for CBR 10 for Given  $E_1/E_2$  and  $E_2 = 9 \times 10^8 \text{ N/m}^2$ .



Fig 6.13 Relationships Between  $D_0^{-D_2}$  and  $H_1$  for Given Ratio of  $E_1^{/E_2}$  and  $E_2^{} = 2 \times 10^8 \text{ N/m}^2$ .



Fig 6.14 Relationships Between  $D_0 - D_2$  and  $H_1$  for Given Ratio of  $E_1/E_2$  and  $E_2 = 9 \times 10^8 \text{ N/m}^2$ .

# 6.5.7 Relationships Between $D_0-D_2$ and $E_1$ for a Given Value of

# $E_2$ and $H_1$ .

Power Law relationships have been established between differential deflection,  $D_0-D_2$  and  $E_1$  for a given value of  $E_2$  and  $H_1$ . The equations to represent the relationships are given in Table A.7 of Appendix VI. It is observed that the differential deflection,  $D_0-D_2$ , is greatly influenced by the variation of  $E_1$  for a given value of  $E_2$  and  $H_1$ . The regression lines tend, see Figs 6.15 and 6.16, to converge at higher values of  $E_1$ . This indicates that as  $E_1$  increases, the pavement becomes stiff and  $D_0-D_2$  is incapable of detecting the different values of  $E_2$ . By comparing the regression lines for  $H_1$  of 100 and 150 mm for a given  $E_2$ , it could be seen that the regression line for  $H_1$ : 150 mm would intercept the x axis at lower value of  $E_1$  than the regression line  $H_1$ : 100 mm.

Given  $D_0 - D_2 = 0.07 \text{ mm}$ ,  $E_2 = 20 \times 10^7 \text{ N/m}^2$  and  $H_1$ = 100 mm, the relationship predicts 90% confidence limits of  $E_1 = \pm 1.25E_1$  (mean  $E_1 = 6.5 \times 10^9 \text{ N/m}^2$ ). The confidence interval is of same order, i.e.  $\pm 1.25E_1$  (mean), for  $H_1 = 150 \text{ mm}$  and  $E_2 = 20 \times 10^7 \text{ N/m}^2$ . The value of  $H_1$  could be obtained by subtracting  $H_G$  from  $H_P$  or by coring from the pavements. The value of  $E_2$ could be obtained from the relationships established in the last few sections or from Eqn.6.5



Fig 6.15 Relationships Between  $D_0^{-}D_2^{-}$  and  $E_1^{-}$  for Given  $E_2^{-}$  and  $H_1^{-} = 100 \text{ mm}$ .



 $E_2$  and  $H_1 = 150$  mm.

## 6.6 Analytical Pavement Evaluation and Design System.

An analytical Pavement and Evaluation and Design System flow chart, Fig 6.17, has been drawn up based on the relationships established in the previous section. The Deflectograph deflection measurements required for input to the system are  $D_0$ ,  $D_2$  and  $D_8$ . The Deflectograph deflections should be adjusted to equivalent values at the standard temperature of 20  $^{O}$ C using the appropriate temperature correction charts in LR 833 (12). The differential deflection between two points should be calculated first from the recorded Deflectograph deflection and then adjusted to equivalent values a 20  $^{O}$ C.

Differential deflections are used in the relationships to obtain the modulus of  $E_1$  or the modular ratio  $E_1/E_2$ . The deflected shape recorded by the Deflectograph is influenced by the thickness and modulus of the pavement layers at the temperature at which measurement are obtained. The maximum deflection is influenced primarily by the modulus of pavement layers. In order to derive appropriate moduli values, therefore, it has been found necessary to use as measured differential deflection and correct these to the standard temperature to give the appropriate values of moduli. Leger et al (23) reported that the curvature (similar to differential deflection) need not be corrected for temperature if it is within the limits specified in Ref (23).

#### 6.7 Limiting Factors.

The investigation of relationships developed shows that it is not possible to determine  $H_2$  and  $H_3$  separately and so  $H_2$  and  $H_3$  have to be combined together as  $H_G$ .  $H_1$  can not be determined independently of other pavement variables. Therefore  $H_1$  can only be

determined from  $H_p-H_G$  or from relationships which involve knowing  $E_1/E_2$  ratio and  $E_2$ . It is not possible to determine  $E_1$ ,  $E_2$  and  $E_3$  separately and independently of other pavement variables. Only the modular ratio  $E_1/E_2$  could be determined from the relationships established.  $E_1$  could be determined from Figs 6.15 and 6.16 if  $H_1$  and  $E_2$  is known. From the investigations, it is observed that  $E_2$  (=  $E_3$ ) have to be known first before any of the relationships could be used.  $E_2$  or  $E_3$  could be determined from Eqn.6.5 knowing CBR of the support from Fig 6.4. To avoid build up of errors, i.e. errors in  $H_p$  and  $H_G$ , it is better to determine  $H_1$  by coring from the pavement.

## 6.7.1 Insignificant Relationships.

Several relationships have been rejected on the grounds that the confidence limits for the variables considered, overlap. An example of this is shown in Fig 6.18 for the relationships between differential deflection  $D_0-D_3$  and equivalent thickness,  $H_e$ , for a given thickness of surfacing,  $H_1$ . For a given  $H_e$  of 1 m and  $D_0-D_3$  of 0.1 mm, the relationship could predict an average value of either 100 or 150 mm. The relationship is not therefore an accurate indicator of  $H_1$  and so it is rejected.



Fig 6.17 Flow Chart: Analytical Pavement Evaluation and Design System.



Fig 6.17 Flow Chart: Analytical Pavement Evaluation and Design System (continued).



Given H1.

#### 6.8 Summary.

- (a) Relationships Between Deflection and/or Differential Deflection and Layer Thickness and Modulus.
  - (i) CBR ( $E_A$ ) mainly influences  $D_R$ ;
  - (ii)  $E_1/E_2$  ratio mainly influences  $D_0-D_8$ ;
  - (iii)  $E_1$  mainly influences  $D_0-D_2$  for a given  $H_1$  and  $E_2$ .
  - (iv)  $H_1$  mainly influences  $D_0-D_3$ ;
  - (v)  $H_1$  mainly influences  $D_0-D_2$  for a given value of  $E_1/E_2$  and  $E_2$ .
  - (vi)  $H_{G}$  mainly influences  $D_{O}-D_{8}$ ;
  - (vi)  $H_p$  mainly influences  $D_0 D_8$ ;
  - (vii)  $H_e$  mainly influences  $D_0$ ;

## (b) Required Deflection Inputs.

In order to use the relationships, the following ordinate Deflectograph deflections are required:

(i)  $D_0$ ; (ii)  $D_2$ ; (iii)  $D_8$ .

## (c) Support Modulus, CBR.

The CBR of the support can be determined from the deflection,  $D_{\rm g}$ , see Fig 6.4.

(d) Equivalent Thickness, 
$$H_e$$
.  
 $H_e = (0.8 \times H_1 \sqrt[3]{E_1/E_4}) + (0.8 \times H_2 \sqrt[3]{E_2/E_4})$   
 $+ (0.8 \times H_3 \sqrt[3]{E_3/E_4})$  (6.2)

The equivalent thickness,  $H_e$ , may be obtained from the relationships between maximum deflection,  $D_0$ , and  $H_e$ , see Fig 6.6.

# (e) Thickness of Surfacing, H<sub>1</sub>.

Relationships between the differential deflection,  $D_0-D_2$ and  $H_e$  may be used to determine  $H_1$  for a given value of  $E_1/E_2$ and  $E_2$ , see Figs 6.13 and 6.14.  $H_1$  may also be obtained by subtracting  $H_G$  (Fig 6.7) from  $H_p$  (Fig 6.8). However, to prevent a build up of errors,  $H_1$  should be determined by coring.

# (f) Thickness of Granular Material, $H_{G}$ .

In most pavements, with a granular road base, the road base and the sub-base are constructed with the similar material, i.e. having a similar modulus value when compared with the modulus of layers immediately above and below. Therefore, the total thickness of granular material,  $H_{\rm G}$ , is determined from the relationships between the differential deflection,  $D_{\rm O}$ - $D_{\rm B}$  and equivalent thickness,  $H_{\rm e}$ , see Fig 6.7.

# (g) Total Thickness of the Pavement Layers, Hp.

Relationships between the differential deflection,  $D_0^{-D_8}$ and equivalent thickness,  $H_e$ , may be used to determine the total thickness of the pavement, i.e  $H_1 + H_G$ , see Fig 6.8.

# (h) Elastic Modulus of Surfacing, E1.

Relationships between the differential deflection,  $D_0 - D_8$ , and  $H_e$  may be used to determine the values of  $E_1$ , if  $E_2$  is known, see Figs 6.9, 6.10, 6.11 and 6.12. Relationships between  $D_0 - D_2$  and  $H_1$  may also be used to determine  $E_1$  if  $H_1$  and  $E_2$ are known.
## (i) Elastic Modulus of Road Base, $E_2$ or Sub-Base $E_3$ .

The similarity of material in most granular road base pavements results in comparable values of  $E_2$  and  $E_3$  modulus, and therefore it may be obtained from the relationships between  $D_0^{-}D_8$ and  $H_e$  if  $E_1$  is known, see Figs 6.9, 6.10, 6.11 and 6.12. It may also be determined from Figs 6.13, 6.14, 6.15 and 6.16 if  $H_1$  and  $E_1$  are known. Alternatively  $E_2 = E_3$  may be calculated from Eqn.6.5.

#### (j) Analytical Pavement Evaluation and Design System.

Based on the established relationships, an Analytical Pavement Evaluation and Design System flow chart has been drawn up, see Fig 6.17. Validation of the system and the effects of errors (confidence interval) on the predicted values from the system are discussed in Chapter 7.0.

# CHAPTER 7.0 VALIDATION OF THE ANALYTICAL PAVEMENT EVALUATION

AND DESIGN SYSTEM.

#### 7.1 Introduction.

An Analytical Pavement Evaluation and Design System has been set up in Chapter 6.0. The design system was calibrated using deflection results obtained from the TRRL. To validate this system, requires Deflectograph deflection measurements together with information about the thickness and modulus of the layers of the pavement with which to compare predicted values of the layers' properties. The Deflectograph deflection used as input was obtained from the Deflectograph surveys conducted previously on local roads (131). Cores of bituminous material were extracted from these roads to determine the thickness, and laboratory triaxial testing was undertaken to determine the modulus of these layers. In-situ bearing capacity tests using a cone penetrometer were carried out previously by Butler. (131) on these roads to determine the subgrades type and strength. Information about the road base construction was also obtained from the previous work (131).

Using the in-situ Deflectograph deflection (131) as an input the thickness and elastic modulus of pavement layers have been predicted using the Analytical Pavement Evaluation and Design System set up in Chapter 6.0. The predicted layer's thickness and modulus have been **compared** with those occurring in the real pavement structure.

7.2 Thickness and Modulus of Pavement Layers.

#### 7.2.1 Bituminous Material.

The laboratory triaxial testing of the bituminous material is described in Appendix VII. The elastic modulus of the bituminous samples was between 2.5 to 5 x  $10^9$  N/m<sup>2</sup>. It is observed that the elastic modulus increased with increasing in confining pressure.

The elastic modulus was also calculated using the following equation derived by Brown (45):  $\log_{10}E = \log_{10}(aT^2 - bT + c) - 10^{-4}(dT^2 + eT + f)(0.5H - 0.2 - 0.94Log_{10}V)$ 

where: E = Young Modulus (stiffness), MN/m<sup>2</sup>; (7.1)

V = Vehicle Speed: 2.5 Km/hr; T = Temperature: 18  $^{\circ}C$ ;

H = Layer Thickness: 0.1 to 0.14 m;

a, b, c, d, e, f = Material Constants as given below:

	a	b	с	Ъ	е	f
DBM	10.2	557	8120	1.8	36.0	1470

The calculated modulus range for the range of thickness given above is  $1.8 \text{ to } 1.9 \times 10^9 \text{ N/m}^2$ . The modulus determined from triaxial testing was about 1.7 to 3.3 times greater than the calculated modulus. The difference could be due to the error in setting up the sample and in the abstracting of values from the graphical outputs but the values obtained from laboratory testing are more likely to be able to characterise the particular mix obtained from the road. The thickness of the bituminous material of cores extracted from the roads was about 90 to 140 mm.

#### 7.2.2 Road Base Thickness and Subgrade Strength.

The test sites are lengths of undesigned road. Neither construction drawings or design specification were available. The road base thickness was estimated by the local divisional surveyor to be between 150 to 200 mm (131).

In-situ bearing capacity tests using a hand held cone penetrometer shows that the subgrade strength is between CBR 10 to CBR 15 (131).

#### 7.3 Deflectograph Deflection (Input Values).

The Deflectograph deflection was obtained from the previous work (131). The Deflectograph deflections were adjusted to equivalent values at the standard temperature of 20  $^{0}$ C using the appropriate temperature correction charts in LR 833 (12). The differential deflection between two points was calculated first from the recorded Deflectograph deflection and then adjusted in a similar manner to equivalent values at 20  $^{0}$ C.

It is observed that the Deflectograph deflections were recorded only to  $1 \times 10^{-2}$  mm. To use the relationships in Chapter 6.0 accurately, especially Fig 6.4, the deflection should be recorded  $1 \times 10^{-3}$  mm and at distances 200 mm and 800 mm away from the point of maximum deflection in addition to the maximum deflection. The recorded deflections should not be interpolated between two values, since the rate of change in deflection between the two points is not linear, especially from 700 mm onwards from the point of maximum deflection.

#### 7.4 Limitation of Validation.

The parametric study (see Chapter 6.0) which forms the basis of the Analytical Pavement Evaluation and Design system, was carried out over a limited range of thickness and modulus of the pavement layers. The in-situ thicknesses and moduli of the pavement layers obtained was beyond this limited range. Values of  $E_2$ , from Eqn.7.2, and  $E_1$ , experimental and calculated value, are well below the range of  $E_2$  and  $E_1$  investigated in the parametric study. The moduli and thicknesses of pavement layers were estimated from the system by interpolating between the upper and lower values investigated. This could result in a build up of large errors. Therefore, the validation of the system is very much limited.

#### 7.5 Estimation of Thickness and Modulus Pavement Layers.

#### 7.5.1 Estimation of Subgrade Strength, CBR.

To obtain the subgrade strength from Fig 6.4 (step 2 of Fig 6.17), the Deflectograph deflection at  $D_8$  was approximated, since the deflection at 800 mm away from point of maximum deflection was not recorded.

$$D_o: 2.0 \times 10^{-2} \text{ mm} \simeq \text{CBR 8 (mean value)};$$

 $D_{o}$ : 1.5 x 10<sup>-2</sup> mm  $\simeq$  CBR 11 (mean value).

This range of CBR is within the range of CBR obtained from the previous work (131) which was between CBR 10 and CBR 15, and given the reduced accuracy of the recording system gives an acceptable prediction.

### 7.5.2 Estimation of Equivalent Thickness, ${\rm H}_{\rm e}$ .

The temperature corrected maximum deflections,  $D_0$ , were used to obtain H<sub>e</sub> from Fig 6.6 (step 3 of Fig 6.17) by interpolating for the different CBR.

CBR 8:  $D_0 = 110 \times 10^{-2} \text{ mm}$ :  $H_e \simeq 0.34 \text{ m}$ ; CBR 11:  $D_0 = 98 \times 10^{-2} \text{ mm}$ :  $H_e \simeq 0.25 \text{ m}$  (for simplicity  $H_e$ 

of CBR 10 was assumed).

The calculated values of  $H_e$  for CBR 8 and CBR 11 using the predicted values of CBR,  $H_1$ ,  $H_G$ ,  $E_1$  and  $E_2$  (detailed in the next few sections) are: 0.38 and 0.36 m. These values are almost within the 90% confidence intervals discussed in Chapter 6.0.

7.5.3 Estimation of Total Pavement Thickness,  $H_{\rm p}$ , and Road Base

Thickness, H <sub>G</sub> .					
Using Fig	s 6.7 and 6.8 (step 4 and 5 of Fig 6.17):				
CBR 8: D <sub>0</sub> -	$D_8 = 112 \times 10^{-2} \text{ mm}; H_e \simeq 0.34 \text{ mm};$				
H <sub>G</sub>	$\simeq$ 100 to 200 mm; H <sub>P</sub> $\simeq$ 200 to 300 mm;				
CBR 11: D <sub>0</sub> -	$D_8 = 98 \times 10^{-2} \text{ mm}; H_e \simeq 0.25 \text{ mm};$				
H <sub>G</sub>	$\simeq$ 100 to 200 mm; H <sub>p</sub> $\simeq$ 200 to 300 mm.				
Therefore, $H_1 = H_p - H_G \simeq 100$ to 200 mm (step 6 of Fig 6.17).					
The range of $H_p$ , $H_g$ and $H_1$ are within the range estimated in the					
previous study and from the cores extracted which are: $H_1 = 90 \text{ mm}$ to					
150 mm; $H_{G} = 150$ to 200 mm.					

E<sub>1</sub>, and E<sub>2</sub>.

Using the predicted  $H_{G}$  and the following equation (73, 74, step 7 of Fig 6.17):

 $E_2 = E_3 = K E_4$ (7.2)where  $K = 0.2 H_{c}^{0.45}$ , 2 < k < 4,  $H_{c} = mm$ ; For an average  $H_G = 150$  mm, <u>CBR 8</u>:  $E_2 = E_3 = 1.5 \times 10^8 \text{ N/m}^2$ ;  $D_0 - D_2 = 34 \times 10^{-2} \text{ mm}$ ; assume  $H_1 = 100 \text{ mm};$ Using Fig 6.15 (step 13 of Fig 6.17):  $E_1 \simeq 4 \times 10^9 \text{ N/m}^2$  (less than the calulated and experimental value of  $E_1 = 2 \times 10^9 \text{ N/m}^2$ ). Using Figs 6.13 and 6.14 (step 10 of Fig 6.17): Given  $H_1 = 100 \text{ mm}$  and  $E_2 = 1.5 \times 10^8 \text{ N/m}^2$ : Therefore:  $E_1 \simeq \langle 4 \times 10^9 \text{ N/m}^2$  (less than the calulated and experimental value of  $E_1 = 2 \times 10^9 \text{ N/m}^2$ ; Given  $E_1 = 2.0 \times 10^9 \text{ N/m}^2$  and  $E_2 = 1.5 \times 10^8 \text{ N/m}^2$ : Therefore (step 10 of Fig 6.17):  $H_1 \simeq <100 \text{ mm}$  (less than the average thickness of the sample extracted, i.e. 90 to 100 mm). Using Figs 6.9, 6.10, 6.11 and 6.12 (step 8 of Fig 6.17) for  $E_2 = 1.5 \times 10^8 \text{ N/m}^2$ ;  $D_0 - D_8 = 112 \times 10^{-2} \text{ mm}$ :  $E_1 \simeq 4.0 \times 10^9 \text{ N/m}^2$  (less than the calulated and experimental value of  $E_1 = 2 \times 10^9 \text{ N/m}^2$ ). <u>CBR 11</u>:  $E_2 = E_3 = 2.1 \times 10^8 \text{ N/m}^2$ ;  $D_0 - D_2 = 18 \times 10^{-2} \text{ mm}$ ; assume  $H_1 = 100$  mm; Using Fig 6.15 (step 13 of Fig 6.17):  $E_{1}\simeq$  2 x  $10^{9}~\text{N/m}^{2}$  (almost same as the calulated and experimental value of  $E_1 = 2 \times 10^9 \text{ N/m}^2$ ).

Using Figs 6.13 and 6.14 (step 10 of Fig 6.17):

Given  $H_1 = 100 \text{ mm}$  and  $E_2 = 2.1 \times 10^8 \text{ N/m}^2$ : Therefore:  $E_1 \approx 4 \times 10^9 \text{ N/m}^2$  (less than the calulated and experimental value of  $E_1 = 2 \times 10^9 \text{ N/m}^2$ ); Given  $E_1 = 2.0 \times 10^9 \text{ N/m}^2$  and  $E_2 = 2.1 \times 10^8 \text{ N/m}^2$ : Therefore (step 10 of Fig 6.17):  $H_1 \approx 400 \text{ mm}$  (less than the average thickness of the sample extracted, i.e. 90 to 100 mm). Using Figs 6.9, 6.10, 6.11 and 6.12 (step 8 of Fig 6.17) for

 $E_2 = 2.1 \times 10^8 \text{ N/m}^2; D_0 - D_8 = 53 \times 10^{-2} \text{ mm};$ Therefore:  $E_1 \approx <4 \times 10^9 \text{ N/m}^2$  (less than the calulated and experimental value of  $E_1 = 2 \times 10^9 \text{ N/m}^2$ );

It seems that the system predicts the moduli and thicknesses of the pavement layers with a reasonable accuracy within the limitation of the validation stated in Section 7.4. Table 7.1 gives the summary of the comparison of thicknesses and moduli of the pavement layers estimated from the system with those obtained from laboratory testing and previous work (131).

	System	In-situ
CBR	8 & 11	10 to 15
<sup>Н</sup> G	100 to 200 mm	150 to 200 mm.
H <sub>1</sub>	Having determined $H_{P} \& H_{G}$ ;	90 to 150 mm
	$H_1$ 100 to 200 mm	
	or H <sub>l</sub> <100 mm	
El	<4 x 10 <sup>9</sup> N/m <sup>2</sup>	$2 \text{ to } 5 \times 10^9 \text{ N/m}^2$
		$(at 18 ^{0} C)$

Table 7.1 Comparison of Thicknesses and Moduli of Pavement Layers.

#### 7.6 Comparison of Deflected Shape.

Comparison has been carried out between the deflected shape recorded by the Deflectograph at site and the equivalent Deflectograph recorded deflected shape produced by the 3D model. Values of thickness and modulus of the pavement layer predicted by the Analytical Pavement Evaluation and Design System were used in the 3D model. The thickness and modulus of the pavement layers used in the 3D model analysis are as follows:  $H_1 = 100 \text{ mm}; H_G = 150 \text{ mm}; E_1 = 2 \times 10^9 \text{ N/m}^2$  and  $E_2$ =  $E_3 = 2 \times 10^8$  N/m<sub>2</sub>. Figs 7.1 and 7.2 show the deflected shape recorded at site and those produced by the 3D model. The deflected shapes are quite similar except for the maximum deflection where there is a slight difference in deflection. This could be due either to the different value of  $E_1$  used in producing the deflected shape or the 3D model not being well refined. It should be noted that a value of  $2 \times 10^9$  ${\rm N/m}^2$  corresponding to the modulus of the asphalt surfacing layer at 18  $^{\circ}$ C has been used in the validation. This value has been used because of the difficulty of predicting accurately the value corresponding to a road temperature of 6  $^{\circ}C$  and also because it is unlikely to influence the value of  $D_0 - D_2$  although the value of  $D_0 - D_8$ , which controis the ratio  $E_1/E_2$  will have been affected.

#### 7.6 Conclusion.

Almost all the relationships in the Analytical Pavement Evaluation and Design System predict the thickness and modulus of the pavement layers with a reasonable accuracy. The comparison of the deflected shapes, Figs7.1 and 7.2, show that the 3D model needs to be further refined to set up a more accurate Analytical Pavement Evaluation and Design system based on measurement of the deflected shape recorded by the Deflectograph.









#### CHAPTER 8.0 SUMMARY, CONCLUSIONS AND RECOMMENDATIONS.

#### 8.1 Summary.

The current structural maintenance design method used in the U.K. is based on empirically derived relations between the deflection of a road's surface produced by the passage of a rolling wheel load and the road's performance. The measurement of deflection is made with a Deflectograph which can also provide information on the deflected surface shape under load.

Previous work at Plymouth has shown that the deflected shape as measured by the Deflectograph can be used to estimate the thickness of the surfacing. The current project has developed from the results of this initial investigation to produce an analysis model that can use the measurements of deflection shape to define the properties and thickness of all layers of a flexible pavement.

A 3D finite element model of a flexible pavement has been produced, based on a commercially available package (PAFEC) and this has been partially validated with data obtained from the TRRL. The 3D model has been used to carry out parametric study to provide an understanding of the link, and to establish appropriate relationships between deflected shape and the thickness and modulus of the pavement layers. The development of such relationships allows back analysis of the deflected shape of a road measured at its surface to identify 'strong' and 'weak' layers within the pavement structure from which maintenance strategies can be developed. The relationships can also be used to design both new and strengthened pavements.

Relationships have been established to determine the modular ratio  $E_1/E_2$ , thickness of granular material and the thickness of the pavement and its support subgrade from measurements of the deflected shape. An Analytical Pavement Evaluation and Design System has been set up based on the established relationships.

Comparison was carried out between the material properties determined from laboratory and previous (131) work with those predicted by the design model using the measured Deflectograph deflections as an input. The comparison shows that the design model can predict the pavement layers properties within the accuracy required for practical engineering analysis.

#### 8.2 Conclusions.

The work reported in this thesis is a study to investigate the relationships between Deflectograph deflection and pavement layer thickness and modulus, and to set up the basis for an Analytical Pavement Evaluation and Design System. In some relationships there are only two variables along the X axis and a straight line was fitted between the two points. In fact it may not be a straight line between the points but a curve or a set of straight lines with different slopes. A common slope was fitted for regression lines of two variables, i.e. H<sub>G</sub>: 300 and 500 mm, of a given CBR but it should not be interpolated or extrapolated for different values, i.e. H<sub>G</sub>: 400, 600 mm. A further parametric study involving a wider range of variables, i.e. E<sub>1</sub> = 2, 4, 6, 8, 10 and 12 x 10<sup>9</sup> N/m<sup>2</sup>, should be carried out to set up a more accurate system. This initial study has indicated the deflection inputs associated with different thickness and modulus of pavement layers.

#### 8.3 Recommendations.

It could be observed from the work reported in this thesis that there are a number of areas where further work is required to set up a more powerful, accurate and simplified Analytical Pavement Evaluation and Design System. The following recommendations are made:

- (i) A further parametric study involving a wider range of variables should be carried out using the 3D finite model that has already been set up.
- (ii) Further validation of the Analytical Pavement Evaluation and Design System using a triaxial test to obtain the modulus of bituminous material should be carried out. Deflectograph deflections should be recorded to 1  $\mu$ m on roads where road base type and thickness and the subgrade strength are known. The Deflectograph deflection should be recorded at following distances: 0, 100 ,200, 300, 400, 500, 600, 700, 800, 900 mm away from the point of maximum deflection. Results from this form of approach could be used as an alternative to (i) if a suitable range of layer thicknesses and moduli could be identified.
- (iii) The Analytical Pavement Evaluation and Design System should be simplified using either the BISAR (30) or TI 59 programmable Calculators (80, 85).

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#### APPENDIX I. EQUIVALENT NODAL FORCES.

The initial step of any finite element analysis is the unique description of the unknown function, U, within each element in terms of n parameter of  $U_i$ . U is the value of the quantity at some point x, y and  $N_i$  is the shape function of x, y for node i; n is the number of nodes in the element. Clearly, if the n values of  $U_i$  are known, then U may be determined at any point inside the element.

$$U = \sum_{i=1}^{n} U_{i}$$
 (AI.1)

Convergence to the exact solution is assured as the number of nodes within each element increases. The order of the polynomial equation of the shape function to describe the unknown quantities within each element increases as the number of nodes in an element increases. High order polynomial equations are difficult to solve and it requires a great deal of computing resources. This problem can be overcome by dividing the region into a finite number of elements and assuming a lower order polynomial approximation within each element. The shape functions must be of the form such that when the x, y (for 2D) values of  $U_i$  corresponding to the same node, i.e. node i, are substituted, the function assumes the value 1. Also it must assume a value of zero for x, y values corresponding to all the other nodes.

The nodal force vector is derived by considering the work done on an element by the external loads or forces. The components of the nodal force vector are known as the equivalent nodal forces and are defined as those forces which, if they acted at the nodes would do the same work on the structure as the actual point or distributed forces. If the forces act over an area or throughout a volume, the work done must be evaluated by summing over each small section of the

element, i.e. by integration.

Force x Displacement. (AI.2)

Derivation of bilinear shape function and the appropriate equivalent nodal force for a four noded elemet (2D) is shown in this section. In order to derive the shape function (any order) and the equivalent nodal forces of an 2D or/and 3D element of any shape with a number of nodes, please refer to one of the following Ref (112, 113, 114, and 119). (1.1) (01) U(x, y) = a + bx + cy + dxy3 at  $(0, 0), U_1 = a, (x = Y = 0)$ at (1, 0),  $U_2 = a + b$ , (x = 1, Y = 0)at  $(0, 1), U_3 = a + c, (x = 0, Y = 1)$ (0,0 at (1,1),  $U_4 = a + b + c + d$ , (x = 1, Y = 1) $a = U_{1}$  $b = U_2 - U_1$  $c = U_3 - U_1$  $d = U_4 - U_1 - (U_2 - U_1) - (U_3 - U_1)$  $= U_1 - U_2 - U_3 + U_4$  $U(x, y) = U_1 + (U_2 - U_1)x + (U_3 - U_1)y$  $+ (U_1 - U_2 - U_3 + U_4)$  $U(x, y) = U_{1}(1 - x + xy - y) + U_{2}(x - xy)$  $+ U_{4}(y - xy) + U_{4}(xy)$  $U(x, y) = \sum_{i=1}^{4} N_i(x, y)$ where  $N_1(x, y) = 1 - x - y + xy$  $N_2(x, y) = x(1 - y)$  $N_3(x, y) = y(1 - x)$  $N_4(x, y) = xy$ 

Note: That each  $N_i(x, y)$  is unity at the ith node and zero at the

other nodes.

Equivalent Nodal Force (E.N.F) =  $\iint_{\Box} f(x, y) N_i(x, y) dxdy$ where force is function f(x, y).

Assume a constant Pressure (=1) over the square element.

E.N.F = 
$$\int_{0}^{1} \int_{0}^{1} N_{1}(x, y) dxdy$$
  
For Node 1:- E.N.F =  $\int_{0}^{1} \int_{0}^{1} (1 - x - y + xy) dxdy$   
E.N.F = P  $\int_{0}^{1} [y - xy - 1/2(y^{2}) + 1/2(xy^{2})] dx$   
E.N.F = P  $\int_{0}^{1} [1 - x - 1/2 + 1/2(x^{2})] dx$   
E.N.F = P  $\int_{0}^{1} [1/2 - x/2] dx$   
E.N.F = P  $[x/2 - 1/2(x^{2})]_{0}^{1}$   
E.N.F = P/4 = 1/4

The same procedure is repeated for node 2, 3 and 4. The above calculation applies to a 8 noded 3D brick element (37100) since the pressure is applied uniformly over a flat surface. When two similar elements are joined together at some common node, the equivalent nodal force distribution is as follows:



Totat Pressure = 1

#### APPENDIX II. CONTACT AREA AND PRESSURE BETWEEN THE TYRE

#### AND THE ROAD.

There is general agreement among investigators that the contact area under the wheels on the flexible pavement is approximately elliptical. The pneumatic tyres on which virtually all road vehicles move do not apply a uniform contact pressure over a circular area of the road surface as is generally assumed in the analysis of multilayer elastic systems. For unloaded conditions, the envelope of the contact area of tyres closely approximates to circles. At full load the tyre walls define the edge of a parallel sided oval of length : breath ratio of about 1.4, while over loading results in further elongation of the envelope between the tyre walls (26).

Lister (26) has reported that an approximately constant contact area is maintained for full load conditions over the range of recommended inflation pressure  $P_p$  but no general relation exists between this actual contact area,  $A_c$ , and the computed area,  $A_p$ , defined by  $W/P_p$ , where W is the static wheel load. He reported that it is reasonable to assume that for load ratios:

(actual wheel load)

(recommended wheel load)

of the order 0.5, the tyre contact area is reasonably circular and the contact pressure across it is parabolic in form. The loaded area is a parallel sided oval under a fairly uniform distribution of pressure, and under overloaded conditions, the envelope is elongated towards a length/breadth ratio of 2:1 with peak contact pressures under the side walls and with flat longitudinal profiles. Fig A.1 shows the change of lateral profile of contact pressure with inflation pressure (26).
These loading conditions differ considerably from the usual assumption of a uniformly loaded circular area defined by  $(W/P_p)$  i.e. static wheel load divided by the tyre inflation pressure. However, Lister (26) found that making the assumption of circular loading, results in errors of less than 2% when used to calculate interfacial pavement stresses and surface deflection under the whole range of tyre load conditions.

Sanborn et al (120) carried out a comparison of stress and displacement produced by a semi-elliptical loads and that produced by uniform circular loads for varying depths and offsets. The semi-ellipsoidal load distribution produced significantly higher stresses at shallow depth than did a uniform circular load of equal magnitude. At some-what greater depths, stresses were the same for both distribution of load. Semi-ellipsoidal loads also indicate larger displacement near the surface than does the equivalent uniform circular loads.





APPENDIX III. PAFEC PROGRAM OF THE 3D MODEL.

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| 7.  | 0 4   | F. O   | 1.2   
   
   
   | 55  
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| 7.  | 0 4   | I. O   | 1.3   
   
   
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| 7.  | 04  | I. O   | 1.4   
   
   
   | 25  
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| 7.  | 04  | I. O   | 1.5   
   
   
   | 75  
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| 7.  | 04  | I. O   | 1.6   
   
   
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| 7.  | 04  | . <b>O</b>   | 1.7   
   
   
   | 45  
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| 7.  | 0 4   | F. O   | 1.8   
   
   
   | 95  
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| 7.  | 0 4   | 1.0  | 2.1   
   
   
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| 6.  | 915   | <b>4</b> .   | 02  
   
   
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| 2.  | 54  | I. O   | 0.2   
   
   
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| 2.  | 5 4   | I. O   | 1.4   
   
   
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| 2.  | 5 4   | I. O   | 1.5   
   
   
   | 85  
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| 2.  | 585   | 5 4.   | 0 0   
   
   
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| 2.  | 585   | 34.  | 0 1   
   
   
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| 2.  | 585   | 54.  | 0 1   
   
   
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| 2.  | 585   | 54.  | 0 1   
   
   
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| 2.  | 66  | 4.0  | 1.  
   
   
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| 6.  | 84  | <b>4</b> 0   | 0   
   
   
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|   | 84<br>84<br>91:<br>97:<br>97:<br>55<br>58:<br>58:   | 4. C<br>4. C<br>5 4. C<br>5 5 4. C<br>5 5 5 5 5 6 C<br>5 5 7 C<br>5 7 C  | 0       1.         0       1         0       1         0       1         0       1         0       1         0       1         0       1         0       1         0       1         1       5         2       4         7       7         1       5  
   
   
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|   | 84<br>84<br>91<br>97<br>97<br>97<br>97<br>55<br>58<br>58<br>58<br>58<br>58<br>58<br>58<br>58<br>58<br>58<br>58<br>58  | 4. CC<br>4. CC<br>5 4. C<br>5 5 7 4. C<br>5 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7  | 0       1.         0       1         0       1         0       1         0       1         0       1         0       1         0       1         0       1         1       5         7       1         7       1         7       1  
   
   
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  | 55<br>4<br>25<br>75<br>5<br>45   |  |   |  |   |   |   |  |  |   |   |   |  |  |  | | | | | | | |
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   |   |   |  |  |                                      |   |  |   |   |
|   | 84<br>84<br>97<br>97<br>97<br>97<br>97<br>55<br>55<br>58<br>58<br>58<br>58<br>58<br>58<br>58<br>58<br>58<br>58<br>58  | 4. CC<br>5. 4. CC<br>5. 4. 4. 5. 5. 4. CC<br>5. 5. 5. 5. 5. 5. 5. 5. 5. 5. 5. 5. 5. 5  | 0       1.         0       1         0       1         0       1         0       1         0       1         0       1         0       1         0       1         1       2         4       7         1       2         4       7         1       2         4       7         1       3         1       5  
   
   
   | 34<br>66<br>0. 2<br>34<br>36<br>36<br>37<br>37<br>37<br>37<br>37<br>37<br>37<br>37<br>37<br>37<br>37<br>37<br>37  
  | 55<br>4<br>25<br>5<br>5<br>4<br>5<br>4<br>5<br>15<br>85  |  |   |  |   |   |   |  |  |   |   |   |  |  |  | | | | | | | |
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34<br>662:34<br>34<br>34<br>2<br>138525<br>35<br>2<br>138552<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br>35<br> | 55<br>4275<br>45<br>5<br>5<br>5<br>5<br>5<br>5<br>5<br>5<br>5<br>5<br>5<br>5<br>5<br>5<br>5<br>5<br>5        |  |   |  |   |   |   |  |  |   |   |   |  |  |  |   
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  | 55<br>4275<br>45<br>15<br>55<br>55<br>55<br>55<br>55<br>55<br>55<br>55<br>55<br>55<br>55<br>55               |  |   |  |   |   |   |  |  |   |   |   |  |  |  | | | | | | | |
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   |   |  |  |                                      |   |  |   |   |
|   | 04         84         915         975   | 4. CC<br>5. 4. C<br>5. 3. 4. C<br>5. 3. 3. 7<br>5. 3. 3. 5<br>5. 3. 7<br>5.   | 0       1.         0       1         0       1         0       1         0       1         1       2.4         1       2.4         1       2.4         1       2.4         1       2.4         1       2.4         1       1.5 <tr td="">       1.5         1<!--</td--><td>24<br/>6224<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>24<br/>2</td><td>55<br/>4<br/>25<br/>75<br/>4<br/>5<br/>15<br/>85<br/>45<br/>25</td></tr> <tr><td></td><td>04       84       91       97       <td< td=""><td>4. CC<br/>5. 4. CC<br/>5. 4. CC<br/>5. 4. CC<br/>5. 4. CC<br/>5. 5. 4. CC<br/>5. 5. 5. 4. CC<br/>5. 5. 5. 5. 5. 5. 5. 5. 5. 5. 5. 5. 5. 5</td><td>1.         0         1.         0         0         1.         0         1.         0         1.         0         1.         0         1.         0         1.         0.1         1.         0.1         1.         1.         1.         1.       
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      3       5       6       2       3       5       6       2</td><td>55       275       45       15       855       5       5       5       5</td></td></tr> <tr><td>66666666666688888888888888888888888888</td><td>889999999996555555555556666688889999</td><td>4.00         5         5         5         5         5         5         5         5         5         5         6         7        &lt;</td><td>1.       0       0       1</td><td>2       3       6       2       3       6       7       1</td><td>55     25       15     55       15     55       5     5       5     5</td></tr> | 24<br>6224<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>24<br>2        | 55<br>4<br>25<br>75<br>4<br>5<br>15<br>85<br>45<br>25  |  | 04       84       91       97 <td< td=""><td>4. CC<br/>5. 4. CC<br/>5. 4. CC<br/>5. 4. CC<br/>5. 4. CC<br/>5. 5. 4. CC<br/>5. 5. 5. 4. CC<br/>5. 5. 5. 5. 5. 5. 5. 5. 5. 5. 5. 5. 5. 5</td><td>1.         0         1.         0         0         1.         0         1.         0         1.         0         1.         0         1.         0         1.         0.1         1.         0.1         1.      &lt;</td><td>2 3 6 2 2 3 4 5 6 7 2 1 3 8 2 2 4 3 5 7 4 5 6 7 2 5 5 5 5 7 4 5 7 5 7</td><td>55<br/>4<br/>25<br/>75<br/>4<br/>5<br/>15<br/>85<br/>45<br/>25</td></td<> | 4. CC<br>5. 4. CC<br>5. 4. CC<br>5. 4. CC<br>5. 4. CC<br>5. 5. 4. CC<br>5. 5. 5. 4. CC<br>5. 5. 5. 5. 5. 5. 5. 5. 5. 5. 5. 5. 5. 5 | 1.         0         1.         0         0         1.         0         1.         0         1.         0         1.         0         1.         0         1.         0.1         1.         0.1         1.      < | 2 3 6 2 2 3 4 5 6 7 2 1 3 8 2 2 4 3 5 7 4 5 6 7 2 5 5 5 5 7 4 5 7 5 7 | 55<br>4<br>25<br>75<br>4<br>5<br>15<br>85<br>45<br>25 |  | 0887777756555555555555555555555555555555 | 4.00         5.4.00         5.4.00         5.5.4.00         5.5.4.00         5.5.5.5.00         5.5.5.00         5.5.5.00         5.5.5.00         5.5.5.00         5.5.5.00         5.5.5.00         5.5.5.00         5.5.5.00         5.5.5.00         5.5.5.00         5.5.5.00         5.5.5.00         5.5.5.00         5.5.5.00  
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   |   |   |  |  |                                      |   |  |   |   |
|   | 04       84       91       97 <td< td=""><td>4. CC<br/>5. 4. CC<br/>5. 4. CC<br/>5. 4. CC<br/>5. 4. CC<br/>5. 5. 4. CC<br/>5. 5. 5. 4. CC<br/>5. 5. 5. 5. 5. 5. 5. 5. 5. 5. 5. 5. 5. 5</td><td>1.         0         1.         0         0         1.         0         1.         0         1.         0         1.         0         1.         0         1.         0.1         1.         0.1         1.      &lt;</td><td>2 3 6 2 2 3 4 5 6 7 2 1 3 8 2 2 4 3 5 7 4 5 6 7 2 5 5 5 5 7 4 5 7 5 7</td><td>55<br/>4<br/>25<br/>75<br/>4<br/>5<br/>15<br/>85<br/>45<br/>25</td></td<> | 4. CC<br>5. 4. CC<br>5. 4. CC<br>5. 4. CC<br>5. 4. CC<br>5. 5. 4. CC<br>5. 5. 5. 4. CC<br>5. 5. 5. 5. 5. 5. 5. 5. 5. 5. 5. 5. 5. 5   | 1.         0         1.         0         0         1.         0         1.         0         1.         0         1.         0         1.         0         1.         0.1         1.         0.1         1.      <   
   
   
   | 2 3 6 2 2 3 4 5 6 7 2 1 3 8 2 2 4 3 5 7 4 5 6 7 2 5 5 5 5 7 4 5
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  |   |   |  |  |  |   |  |   |   |  |                                  |   | | | | | | | | | | |
  |   |   |  |                                |   |  |   |   |  |                                   |  |   |   |  |  |                                    |  |  |   |  |  |   |  
  |   |   |  |  |                                      |   |  |   |   |
|   | 0887777756555555555555555555555555555555  | 4.00         5.4.00         5.4.00         5.5.4.00         5.5.4.00         5.5.5.5.00         5.5.5.00         5.5.5.00         5.5.5.00         5.5.5.00         5.5.5.00         5.5.5.00         5.5.5.00         5.5.5.00         5.5.5.00         5.5.5.00         5.5.5.00         5.5.5.00         5.5.5.00         5.5.5.00         5.5.5.00         5.5.5.00         5.5.5.00         5.5.5.00         5.5.0  | 1.         0         1.         0         1.         0         1.         0         1.         0         1.         0         1.      <   
   
   
   | 2       3       4       5       6       2       1      
1          | 55<br>4<br>25<br>5<br>5<br>4<br>5<br>1<br>5<br>5<br>4<br>5<br>5<br>5<br>5<br>5<br>5<br>5<br>5<br>5<br>5<br>5 |  |   |  |   |   |   |  |  |   |   |   |  |  |  |   
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  |   |   |  |                                |   |  |   |   |  |                                   |  |   |   |  |  |                                    |  |  |   |  |  |   |  
  |   |   |  |  |                                      |   |  |   |   |
|   | 0887777756<br>55555555555555566<br>66666  | 4.00         5         4.00         5         4.00         5         4.00         5         5         6         7         7         7         8         7         8         7         8         7         8         7         8         7         8         7         8         7         8 <t< td=""><td>1.         0         1.         0         0         1.         0         1.         0         1.         0         1.         0         1.         0.1         1.         0.1         1.</td><td>2 3 4 6 2 2 3 4 5 6 7 2 1 3 5 6 7 4 5 6 7 4 5 6 7 4 5 6 4 7 4 7 4 5 6 4 7 4 7 4 7 4 7 4 7 4 7 4 7 6 4 7 4 7</td><td>55<br/>425<br/>75<br/>45<br/>15<br/>55<br/>45<br/>55<br/>5</td></t<> | 1.         0         1.         0         0         1.         0         1.         0         1.         0         1.         0         1.         0.1         1.         0.1         1.   
   
   
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4 7 4 5 6 4 7 4 7 4 5 6 4 7 4 7 4 7 4 7 4 7 4 7 4 7 6 4 7 4 7  | 55<br>425<br>75<br>45<br>15<br>55<br>45<br>55<br>5   |  |   |  |   |   |   |  |  |   |   |   |  |  |  |   
  |   |   |  |  |  |   |  |   |   |  |                                  |   | | | | | | | | | | |
  |   |   |  |                                |   |  |   |   |  |                                   |  |   |   |  |  |                                    |  |  |   |  |  |   |  
  |   |   |  |  |                                      |   |  |   |   |
|   | 084<br>9999999955555555555555555555555555555  | 4.00         5         4.00         5         4.00         5         4.00         5         4.00         5         4.00         5         4.00         5          5         5         5         5         5         5         5         5         5         5         5         5         5         5 <td>1.         0         1.         0         0         1.         0         1.         0         1.         0         1.         0         1.      <t< td=""><td>2     3     6     2     2     3     4     5     6     7     1</td></t<><td>55<br/>25<br/>25<br/>25<br/>45<br/>15<br/>55<br/>55<br/>55<br/>55<br/>55<br/>55<br/>55<br/>55<br/>5</td></td>  | 1.         0         1.         0         0         1.         0         1.         0         1.         0         1.         0         1. <t< td=""><td>2     3     6     2     2     3     4     5     6     7     1</td></t<> <td>55<br/>25<br/>25<br/>25<br/>45<br/>15<br/>55<br/>55<br/>55<br/>55<br/>55<br/>55<br/>55<br/>55<br/>5</td>  
   
   
   | 2     3     6     2     2     3     4     5     6     7     1   
 1      | 55<br>25<br>25<br>25<br>45<br>15<br>55<br>55<br>55<br>55<br>55<br>55<br>55<br>55<br>5                        |  |   |  |   |   |   |  |  |   |   |   |  |  |  |  
   |   |   |  |  |  |   |  |   |   |  |                                  |   | | | | | | | | | | |
   |   |   |  |                                |   |  |   |   |  |                                   |  |   |   |  |  |                                    |  |  |   |  |  |   |   
   |   |   |  |  |                                      |   |  |   |   |
|   | 08879797976555555555555556666688  | 4.00         5         5         5         5         5         6         7         8         7         8         9        <  | 1.         0         1.         0         1.         0         1.         0         1.         0         1.         0         1.         0         1.         0         1.         0.         1.         1.         0.         1.         0.         1.         0.         1.         0.         1.         0.         1.         0.         1.         0.         1.         0.         1.         0.         1.         0.         1.         0.         0.         1.         0.         0.         0.         0.         0.         0.         0.         0.         0.         0.         0.         0. <tr< td=""><td>2 3 6 2 2 3 4 2 5 6 7 2 5 5 5 7 4 2 5 6 4 2 3 5 1 1 2 2 5 6 4 2 3 5 1 1 1 2 2 5 6 4 2 3 5 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1</td><td>55<br/>4<br/>25<br/>75<br/>4<br/>5<br/>4<br/>5<br/>5<br/>5</td></tr<>   
   
   
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5 6 4 2 3 5 1 1 2 2 5 6 4 2 3 5 1 1 1 2 2 5 6 4 2 3 5 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1  | 55<br>4<br>25<br>75<br>4<br>5<br>4<br>5<br>5<br>5  |  |   |  |   |   |   |  |  |   |   |   |  |  |  |   
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  |   |   |  |  |                                      |   |  |   |   |
|   | 889999999965555555555666668888  | 4.00         3         4.4.4.4.4.4.4.4.4.4.4.4.4.4.4.4.4.4.4.  | 1.       0       1         0       1       1         0       1       1         0       1       1         1       2       7         1       3       3          1       3       3          1       3       3  
   
   
   | 2       3       6       2       3       4       5       6       7       5       5       7       4       2       3       6       4       2       3       6       4       2       3       6       4       2       3       6       4       2       3       6      
4       2       3       6       4       2       3       6       4       2       3       6       4       2       3       6       4       2       3       6       4       2       3       6       4  | 55<br>4<br>25<br>75<br>4<br>5<br>15<br>85<br>5<br>5  |  |   |  |   |   |   |  |  |   |   |   |  |  |  |   
  |   |   |  |  |  |   |  |   |   |  |                                  |   | | | | | | | | | | |
  |   |   |  |                                |   |  |   |   |  |                                   |  |   |   |  |  |                                    |  |  |   |  |  |   |  
  |   |   |  |  |                                      |   |  |   |   |
|   | 889999999996555555555556666688888   | 4.00         3         4.00         5         5         5         4.00         5         5         5         4.00         5         5         4.00         5   | 1.       0       1.         0       1       1.         0       1       1.         1       1.       1.         0       1.       1.         1   
   
   
   | 2       36       2       36       2       3     
 3        | 55<br>4<br>25<br>75<br>4<br>5<br>15<br>85<br>5<br>5<br>5   |  |   |  |   |   |   |  |  |   |   |   |  |  |  |  
   |   |   |  |  |  |   |  |   |   |  |                                  |   | | | | | | | | | | |
   |   |   |  |                                |   |  |   |   |  |                                   |  |   |   |  |  |                                    |  |  |   |  |  |   |   
   |   |   |  |  |                                      |   |  |   |   |
|   | 8899999999965555555555556666688888  | 4.00         5         4.00         5         5         4.00         5         5         4.00         5         5         4.00         5         5         5         6         7         7         7         8         7         8         7         8         7         8         7         8   | (1, 0) $(1, 0)$ $($  
   
   
   | 2       36       2       36       2       3     
 3        | 55<br>4<br>275<br>4<br>5<br>1<br>5<br>5<br>5<br>5<br>5   |  |   |  |   |   |   |  |  |   |   |   |  |  |  |  
   |   |   |  |  |  |   |  |   |   |  |                                  |   | | | | | | | | | | |
   |   |   |  |                                |   |  |   |   |  |                                   |  |   |   |  |  |                                    |  |  |   |  |  |   |   
   |   |   |  |  |                                      |   |  |   |   |
|   | 844 11111111111111111111111111111111111   | 4.00         5         5         5         5         5         6         7         4.00         7  | 1.       0       1       1.       0       1       1.       0       1       1. <td>2       36       2       36       2       3       6       2       3       6       2       3       6       2       3       6       2       3       6       2       3       6       2       3       5       6       2</td> <td>55       275       45       15       855       5       5       5       5</td>   
   
   
   | 2       36       2       36       2       3       6       2       3       6       2       3       6       2       3       6       2       3       6       2       3       6       2       3       5       6       2       3       5       6       2       3     
 5       6       2       3       5       6       2  | 55       275       45       15       855       5       5       5       5                                     |  |   |  |   |   |   |  |  |   |   |   |  |  |  |  
   |   |   |  |  |  |   |  |   |   |  |                                  |   | | | | | | | | | | |
   |   |   |  |                                |   |  |   |   |  |                                   |  |   |   |  |  |                                    |  |  |   |  |  |   |   
   |   |   |  |  |                                      |   |  |   |   |
| 66666666666688888888888888888888888888  | 889999999996555555555556666688889999  | 4.00         5         5         5         5         5         5         5         5         5         5         6         7        <  | 1.       0       0       1  
   
   
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54	2	37200	13	1	1	1	1 153 158 159 128 133 134
55	1	37100	13	1	1	o o	1 153 159 154 160 128 134 129 135 5 154 140 155 141 139 135 130 134
57	1	37100	±3 ≈13	1	1	0 : 0	12 156 162 161 163 131 137 136 138
58	1	37100	14	1	1	ŏ	11 164 166 165 171 139 141 140 146
59	1	37100	14	1	1	0	9 166 172 300 323 141 147 297 321
60	1	37100	14	1	1	0	1 300 323 167 301 297 321 142 298
61 47	2	37200	14	1	1	1	1 JU1 167 173 298 142 148 1 167 168 173 143 143 149
63	2	37200	14	1	1	1	1 168 173 169 143 148 144
64	2	37200	14	1	1	1	1 173 169 174 148 144 149
65	1	37100	14	1	1	0	1 169 174 170 324 144 149 145 322
66	1	37100	14	1	1	0	10 170 324 171 175 145 322 146 150
68	1	37100	14	1	1	0	8 176 181 177 302 151 156 152 299

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70 2 37200 14 1 1 1 1 182 178 183 157 183 158	
70 2 37200 14 1 1 1 1 182 178 183 157 153 158	
71 2 37200 14 1 1 1 1 178 183 184 153 158 159	
72 1 37100 14 1 1 0 1 178 184 179 185 153 159 154 140	
73 1 3/100 14 1 1 0 3 1/4 185 180 188 134 180 135 181	
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76 1 37100 15 1 1 0 9 191 107 202 325 144 172 300 232	
77 1 37100 15 1 1 0 1 303 325 192 304 300 323 167 301	
78 2 37200 15 1 1 1 1 304 192 198 301 167 173	
79 2 37200 15 1 1 1 192 193 198 147 148 173	
BU 2 3/200 15 1 1 1 1 143 148 144 188 173 184	
81 2 37200 15 1 1 1 1 198 194 199 173 169 174	
82 1 37100 15 1 1 0 1 194 199 195 326 169 174 170 324	
83 1 37100 15 1 1 0 10 195 324 194 200 170 324 171 175	
84 1 37100 15 6 1 0 7 197 201 200 205 172 176 175 180	
85 1 37100 15 1 1 0 8 201 206 202 305 176 181 177 302	
86 1 37100 15 1 1 0 1 202 305 203 207 177 302 178 182	
B/ 2 3/200 13 1 1 1 20/ 203 208 162 1/8 163	
<b>68 2 37200 15 1 1 1 208 203 209 183 178 184</b>	
87 1 37100 15 1 1 0 1 203 207 204 210 178 184 177 185	
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92 1 37100 16 1 1 0 11 214 216 215 221 189 191 190 196	
93 1 37100 14 1 1 0 9 214 222 304 327 191 197 303 325	
74 1 3/100 18 1 1 0 1 308 32/ 21/ 30/ 303 325 142 304	
95 2 37200 16 1 1 1 307 217 223 304 192 198	
96 2 37200 16 1 1 1 217 218 223 192 193 198	
97 2 37200 14 1 1 1 1 218 222 218 183 188 184	
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102       1       37100       16       1       0       8       226       231       227       308       201       206       202       305         103       1       37100       16       1       1       0       1       227       308       201       206       202       305         103       1       37100       16       1       1       0       1       227       308       228       232       202       305       203       207         104       2       37200       16       1       1       1       232       228       233       207       203       208         105       2       37200       16       1       1       1       228       233       234       203       208       209         106       1       37100       16       1       1       1       228       234       229       235       203       209       204       210         107       1       37100       16       1       1       0       12       231       237       236       238       206       212       211       213 <t< td=""><td>3</td></t<>	3
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102       1       37100       16       1       1       0       8       226       231       227       308       201       206       202       305         103       1       37100       16       1       1       0       1       227       308       201       206       202       305       203       207         104       2       37200       16       1       1       1       232       228       233       207       203       208         105       2       37200       16       1       1       1       232       228       233       207       203       208         105       2       37200       16       1       1       1       228       233       234       203       208       209         106       1       37100       16       1       1       0       1       229       235       203       209       204       210         107       1       37100       16       1       1       0       12       231       237       236       238       206       212       211       213         108	3
102       1       37100       16       1       0       8       226       231       227       308       201       206       202       305         103       1       37100       16       1       1       0       1       227       308       201       206       202       305         103       1       37100       16       1       1       0       1       227       308       228       232       202       305       203       207         104       2       37200       16       1       1       1       232       228       233       207       203       208         105       2       37200       16       1       1       1       228       233       234       203       208       209         106       1       37100       16       1       1       0       1       228       234       229       235       203       209       204       210         107       1       37100       16       1       1       0       12       231       237       236       238       206       212       211       213	3
102       1       37100       16       1       1       0       8       226       231       227       308       201       206       202       305         103       1       37100       16       1       1       0       1       227       308       201       206       202       305         103       1       37100       16       1       1       1       227       308       228       232       202       305       203       207         104       2       37200       16       1       1       1       232       228       233       207       203       208         105       2       37200       16       1       1       1       228       233       234       203       208       209         106       1       37100       16       1       0       1       237       236       238       204       210       205       211         108       1       37100       16       1       0       12       231       237       236       238       204       216       212       212       212       211       213	3
102       1       37100       16       1       1       0       8       226       231       227       308       201       206       202       305         103       1       37100       16       1       1       0       1       227       308       201       206       202       305         103       1       37100       16       1       1       1       227       308       228       232       202       305       203       207         104       2       37200       16       1       1       1       232       228       233       207       203       208         105       2       37200       16       1       1       1       228       233       234       203       208       209         106       1       37100       16       1       1       0       1       237       235       230       236       204       210       205       211         107       1       37100       16       1       0       12       231       237       236       238       206       212       211       212 <t< td=""><td>3</td></t<>	3
102       1       37100       16       1       1       0       8       226       231       227       308       201       206       202       305         103       1       37100       16       1       1       0       1       227       308       201       206       202       305       203       207         104       2       37200       16       1       1       1       232       228       233       207       203       208         105       2       37200       16       1       1       1       232       228       233       207       203       208         105       2       37200       16       1       1       1       228       233       234       203       208       209       204       210         106       1       37100       16       1       1       0       1       228       234       229       235       203       209       204       210       205       211         107       1       37100       16       1       1       0       12       231       237       236       238	3
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102       1       37100       16       1       1       0       8       226       231       227       308       201       206       202       305         103       1       37100       16       1       1       0       1       227       308       228       232       202       305       203       207         104       2       37200       16       1       1       1       232       228       233       207       203       208         105       2       37200       16       1       1       1       232       228       233       207       203       208         105       2       37200       16       1       1       1       228       233       234       203       208       209       204       210         106       1       37100       16       1       1       0       1       228       237       235       230       236       204       210       205       211       213         107       1       37100       17       1       0       11       237       241       240       246       214 <t< td=""><td>3</td></t<>	3
102       1       37100       16       1       1       0       8       226       231       227       308       201       206       202       305         103       1       37100       16       1       1       0       1       227       308       228       232       202       305       203       207         104       2       37200       16       1       1       1       232       228       233       207       203       208         105       2       37200       16       1       1       1       232       228       233       207       203       208         105       2       37200       16       1       1       0       1       228       233       234       203       208       207         106       1       37100       16       1       0       5       227       235       230       236       204       210       205       211         108       1       37100       17       1       0       12       237       241       240       246       214       216       215       222       306 <t< td=""><td>3</td></t<>	3
102       1       37100       16       1       1       0       8       226       231       227       308       201       206       202       305         103       1       37100       16       1       1       0       1       227       308       228       232       202       305       203       207         104       2       37200       16       1       1       1       232       228       233       207       203       208         105       2       37200       16       1       1       1       232       228       233       207       203       208         105       2       37200       16       1       1       0       1       228       234       229       235       203       209       204       210         107       1       37100       16       1       0       1       237       236       238       206       212       211       213         108       1       37100       17       1       0       11       237       241       240       246       214       216       223       306 <t< td=""><td>3</td></t<>	3
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102       1       37100       16       1       1       0       8       226       231       227       308       201       206       202       305         103       1       37100       16       1       1       0       1       227       308       228       232       202       305       203       207         104       2       37200       16       1       1       1       232       228       233       207       203       208         105       2       37200       16       1       1       1       232       238       233       207       203       209       204       210         106       1       37100       16       1       0       1       228       233       236       204       210       205       211         107       1       37100       16       1       0       12       231       237       236       238       204       210       205       211       213         108       1       37100       17       1       0       12       231       237       246       214       216       215	3
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102       1       37100       16       1       1       0       8       226       231       227       308       201       206       202       305         103       1       37100       16       1       1       0       1       227       308       228       232       202       305       203       207         104       2       37200       16       1       1       1       232       228       233       207       203       208         105       2       37200       16       1       1       1       228       233       234       203       208       207         106       1       37100       16       1       0       1       228       233       236       204       210       205       211         107       1       37100       16       1       0       12       237       236       238       206       212       211       213         108       1       37100       17       1       0       12       237       236       238       206       212       217       237         110       1       <	9 1
102       1       37100       16       1       1       0       8       226       231       227       308       201       206       202       305         103       1       37100       16       1       1       0       1       227       308       228       232       202       305       203       207         104       2       37200       16       1       1       1       232       228       233       207       203       208         105       2       37200       16       1       1       1       232       233       234       203       208       207         106       1       37100       16       1       0       1       237       235       230       236       204       210       205       211         108       1       37100       16       1       0       12       231       237       236       238       204       210       205       211       215       231       237       246       214       216       215       212       216       222       306       327       217       307         107	3 1
102       1       37100       16       1       1       0       8       226       231       227       308       201       206       202       305         103       1       37100       16       1       1       0       1       227       308       228       232       202       305       203       207         104       2       37200       16       1       1       1       232       228       233       207       203       208         105       2       37200       16       1       1       1       232       233       207       203       209         106       1       37100       16       1       1       0       1       237       235       230       236       204       210       205       211         107       1       37100       16       1       0       12       231       237       236       238       204       210       205       211       213         108       1       37100       17       1       1       0       12       241       240       246       214       216       223 <td< td=""><td>3</td></td<>	3
102       1       37100       16       1       0       8       226       231       227       308       201       206       202       305         103       1       37100       16       1       1       0       1       227       308       228       232       202       305       203       207         104       2       37200       16       1       1       1       232       228       233       207       203       208         105       2       37200       16       1       1       1       232       233       207       203       208       207         106       1       37100       16       1       0       1       228       234       229       235       203       209       204       210       205       211         107       1       37100       16       1       0       12       231       237       236       236       204       210       205       211       213         108       1       37100       17       1       0       12       241       247       309       327       217       237	3
1021 $37100$ $16$ 1108 $226$ $231$ $227$ $308$ $201$ $206$ $202$ $305$ $103$ 1 $37100$ $16$ 111 $227$ $308$ $228$ $232$ $202$ $305$ $203$ $207$ $104$ 2 $37200$ $16$ 111 $1227$ $308$ $228$ $232$ $202$ $305$ $203$ $207$ $104$ 2 $37200$ $16$ 111 $1228$ $233$ $204$ $203$ $208$ $105$ 2 $37200$ $16$ 110 $1228$ $233$ $234$ $203$ $209$ $204$ $210$ $107$ 1 $37100$ $16$ 110 $5279$ $235$ $230$ $236$ $204$ $210$ $205$ $211$ $108$ 1 $37100$ $16$ 10 $12$ $231$ $237$ $236$ $238$ $206$ $212$ $211$ $215$ $109$ 1 $37100$ $17$ 10 $12$ $237$ $246$ $214$ $216$ $215$ $221$ $110$ 1 $37100$ $17$ 11 $0$ $1237$ $241$ $240$ $246$ $214$ $216$ $215$ $111$ 1 $37100$ $17$ 111 $1242$ $243$ $248$ $217$ $218$ $223$ $111$ 2 $37200$ $17$ 111 $244$ <td>3 1 5 38</td>	3 1 5 38
1021 $37100$ $16$ 1108 $226$ $231$ $227$ $308$ $201$ $206$ $202$ $305$ $103$ 1 $37100$ $16$ 111 $227$ $308$ $228$ $232$ $202$ $305$ $203$ $207$ $104$ 2 $37200$ $16$ 1111 $232$ $228$ $233$ $207$ $203$ $208$ $105$ 2 $37200$ $16$ 111 $1228$ $233$ $207$ $203$ $209$ $106$ 1 $37100$ $16$ 110 $1228$ $234$ $229$ $235$ $203$ $209$ $204$ $210$ $107$ 1 $37100$ $16$ 110 $5279$ $235$ $230$ $236$ $204$ $210$ $205$ $211$ $108$ 1 $37100$ $16$ 10 $12231$ $237$ $236$ $238$ $206$ $212$ $211$ $213$ $109$ 1 $37100$ $17$ 10 $1237$ $241$ $240$ $246$ $214$ $216$ $215$ $221$ $111$ 1 $37100$ $17$ 11 $0$ $1242$ $243$ $248$ $217$ $218$ $223$ $111$ 1 $37100$ $17$ 11 $1242$ $243$ $248$ $217$ $218$ $223$ $113$ 2 $37200$ $17$ 11 $1244$ $249$ $223$ <td< td=""><td>3 1 5 38</td></td<>	3 1 5 38
102       1       37100       16       1       1       0       8       226       231       227       308       201       206       202       305         103       1       37100       16       1       1       1       227       308       228       232       202       305       203       207         104       2       37200       16       1       1       1       232       228       233       207       203       208         105       2       37200       16       1       1       1       232       233       207       203       209       204       210         106       1       37100       16       1       0       1       237       235       230       236       204       210       205       211         108       1       37100       17       1       0       12       237       237       236       238       204       216       227       307       217       223       217       107       11       1       1       307       327       216       222       306       327       217       307       17	3 1 5 38
102       1       37100       16       1       0       8       226       231       227       308       201       206       202       305         103       1       37100       16       1       1       1       227       308       228       232       202       305       203       207         104       2       37200       16       1       1       1       232       228       233       207       203       208         105       2       37200       16       1       1       1       228       233       234       203       208       207         106       1       37100       16       1       0       1       228       234       229       235       203       204       210       205       211         107       1       37100       16       1       0       12       231       237       236       238       206       212       211       213         108       1       37100       17       1       0       12       237       237       216       222       306       327       217       307	3 1 5 38 6
1021 $37100$ $16$ 1106 $226$ $231$ $227$ $308$ $201$ $206$ $202$ $305$ $103$ 1 $37100$ $16$ 111 $1227$ $308$ $228$ $232$ $202$ $305$ $203$ $207$ $104$ 2 $37200$ $16$ 111 $1228$ $233$ $204$ $203$ $208$ $207$ $106$ 1 $37100$ $16$ 110 $1228$ $233$ $234$ $203$ $209$ $204$ $210$ $107$ 1 $37100$ $16$ 110 $1228$ $231$ $237$ $236$ $238$ $206$ $212$ $211$ $211$ $108$ 1 $37100$ $16$ 110 $12231$ $237$ $236$ $238$ $206$ $212$ $211$ $211$ $109$ 1 $37100$ $17$ 10 $12397$ $241$ $240$ $246$ $214$ $216$ $222$ $306$ $327$ $111$ 1 $37100$ $17$ 11 $0$ $12397$ $242$ $310$ $306$ $327$ $217$ $307$ $112$ $37200$ $17$ 11 $1$ $242$ $248$ $307$ $217$ $223$ $113$ $237200$ $17$ 11 $1$ $242$ $248$ $217$ $218$ $223$ $114$ $237200$ $17$ 11 $1$ $244$ $249$	3 1 5 38
1021 $37100$ $16$ 1106 $226$ $231$ $227$ $308$ $201$ $206$ $202$ $305$ $104$ 2 $37200$ $16$ 111 $1227$ $308$ $228$ $232$ $202$ $305$ $203$ $207$ $104$ 2 $37200$ $16$ 111 $1228$ $233$ $204$ $203$ $208$ $209$ $106$ 1 $37100$ $16$ 110 $1228$ $233$ $234$ $203$ $209$ $204$ $210$ $107$ 1 $37100$ $16$ 110 $1228$ $235$ $236$ $204$ $210$ $205$ $211$ $108$ 1 $37100$ $16$ 110 $12231$ $237$ $236$ $238$ $206$ $212$ $211$ $211$ $108$ 1 $37100$ $17$ 10 $11239$ $241$ $240$ $246$ $214$ $216$ $215$ $223$ $110$ 1 $37100$ $17$ 10 $1309$ $329$ $242$ $310$ $306$ $327$ $217$ $307$ $112$ $2$ $37200$ $17$ 11 $1$ $1242$ $248$ $307$ $217$ $223$ $114$ $2$ $37200$ $17$ 11 $1$ $243$ $248$ $217$ $218$ $223$ $219$ $115$ $2$ $37200$ $17$ 11 $1$ $244$ $247$ <	3 1 5 38 6
102137100161106226231227308201206202305103137100161111237308228232203203203104237200161111232228233207203208105237200161111228234203208209106137100161101228234227235203209204210107137100161101228234237236238206212211211108137100161101237236238206212211211109137100171012372412402462142162152211101371001711012372423103063272173071122372001711124224821721822321911323720017111248244249223219224116137100171101<	3 1 5 38
102       1       37100       14       1       0       6       226       231       227       308       201       206       202       305         103       1       37100       16       1       1       1       227       308       228       232       202       305       203       207         104       2       37200       16       1       1       1       232       228       233       207       203       208         105       2       37200       16       1       1       1       228       233       234       207       203       208         106       1       37100       16       1       0       1       228       234       229       235       203       207       204       210         107       1       37100       17       1       0       12       231       237       236       238       206       212       211       213         108       1       37100       17       1       0       1       247       307       327       217       307         112       2       37200       17	3 1 5 38 6
10213710016110622623122730820120620230510423720016111227308228232202305203207105237200161112282332072032082091061371001611012282332342072032092042101071371001611012282332342072032092042101081371001611012282372302362042102052111081371001711012231237236238206212211211109137100171101123724124024621421621522111137100171101242248247217237217307112237200171111242248244218223219115237200171111244247245330219224220328117 </td <td>3 1 5 38 6</td>	3 1 5 38 6
1021 $37100$ $16$ 1106 $226$ $231$ $227$ $308$ $201$ $206$ $202$ $305$ $104$ 2 $37200$ $16$ 111 $227$ $308$ $228$ $232$ $202$ $305$ $203$ $207$ $104$ 2 $37200$ $16$ 111 $228$ $233$ $207$ $203$ $208$ $105$ 2 $37200$ $16$ 111 $228$ $233$ $234$ $207$ $203$ $209$ $106$ 1 $37100$ $16$ 1101 $228$ $234$ $229$ $235$ $203$ $207$ $204$ $210$ $107$ 1 $37100$ $16$ 110 $1228$ $231$ $237$ $236$ $238$ $206$ $212$ $211$ $211$ $108$ 1 $37100$ $17$ 10 $12231$ $237$ $236$ $238$ $206$ $212$ $211$ $211$ $109$ 1 $37100$ $17$ 10 $1227$ $237$ $237$ $216$ $222$ $306$ $327$ $111$ 1 $37100$ $17$ 10 $1247$ $247$ $248$ $217$ $218$ $223$ $114$ 2 $37200$ $17$ 11 $1$ $248$ $244$ $248$ $217$ $218$ $223$ $217$ $115$ 2 $37200$ $17$ 11 $1$ $247$ $251$	3 1 5 38 6
1021371001611062262312273082012062032031042372001611112322282332072032081052372001611112282332042032092042101061371001611012282342292352032092042101071371001611052272352302362042102052111081371001611012231237236238206212211211109137100161101223123723623820621221121110137100171101223123723723623721723711137100171113093292423103063272173071122372001711114424824421822321911523720017111244249245330219224242328117137100	3 1 5 38 6

=

/66	
767	
768	
769	
770	
771	
772	
773	
774	
775	
776	
777	
REST	FRAINTS
NODE	E. NUMBER PLANE DIRECTION
333	2 123
362	2 123
334	2 123
363	2 123
333	1 1
334	1 1
362	1 1
363	1 1
334	3 3
363	3 3
IN. I	DRAW
DRA	TYPE INFO ORIE SIZE
13	0 4 1
	OUT. DRAW
	DRAW PLOT ORIE SIZE
1 1	4 1
	END. OF. DATA

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APPENDIX IV. TRANS.F77.

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С С \* PROGRAM TRANS. F77 46 С \* \_\_\_\_\_ \* С ¢ С \* THIS PROGRAM TRANSFERS THE DATAS FROM THE "PAFEC" DUTPUT FILE CALLED # "OC\_\_\_\_>7", TO ANOTHER SPECIFIED FILE. THIS PROGRAM TRANSFER THE X VALUES С - 44 (THE DISTANCE ALONG THE ROAD), DEFLECTION OF THE NEAR SIDE REAR WHEELS С ¥ С \* PATHS AND THE DEFLECTION OF THE MIDDLE OF THE LANE. С -84 С 4 COLUMN INFORMATION С # с # 1 X VALUES (M) С -11 2 DEFL. OF. WHEEL PATH (mm) С # з DEFL. OF. MID-LANE (mm) С # 4 **\*\* LINE NUMBER** ¢ 4 С \* \*\* THE LINE NUMBER PROVIDES A MEAN TO DETECT THE END OF DATA WHEN USED IN ANOTHER PROGRAM, CALLED "DEFL. F77", WHICH С \* С # CHANGES THE ABSLOUTE DEFLECTION INTO DEFLECTOGRAPH С # DEFLECTION. THE LAST LINE(NO 24) IS GIVEN A VALUE O С # IN ORDER TO CARRY OUT THE ABOVE TASK. С \* С \* TO RUN THE PROGRAM, PLEASE TYPE THE FOLLOWING COMMANDS: -С \* С . .. \$77 TRANS -IN #>PAFEC2>OMDTH7 -OUT PMDTH7 " С ..... С . ASSUMED: - 1) PAFEC OUTPUT FILE =OMDTH7 2) TRANSFERED OUTPUT FILE = PMDTH7 С \* С -3)FILE = OMDTH7 IS IN THE SUB-UFD CALLED PAFEC2 -86 С # С \*\*\*\*\* C PROGRAM TRANS DIMENSION X(30), YAAD(30), YBAD(30), K(30) REAL #8 C, YAAD, YBAD, X INTEGER LK C C THESE STATEMENTS SKIPS THE FIRST 64 LINES. DO 10, I=1,64 READ (5,16) 15 FORMAT (A20) CONTINUE 10 С C THESE STATEMENTS PICKS UP THE MULTIPLICATION FACTOR. READ (5,16) C 16 FORMAT (29%, E6. 0) С С THESE STATEMENTS SKIPS THE NEXT 3 LINES. DO 20, I=1,3 READ (5,15) 20 CONTINUE С C THESE STATEMENTS PICKS UP THE FIRST 24 DEFLECTIOS (WHEEL PATH) AND THE X C VALUES. DO 50, I=1, 24 READ (5,30) YAAD(I), X(I) 30 FORMAT (19X, F9. 3, 45X, F5. 2) С THIS STATEMENT DIVIDES THE DEFLECTIONS BY THE MULTIPLICATION FACTOR AND С С THEN CHANGE THE DEFLECTION FROM M TO mm. YAAD(I) = (YAAD(I)/C) \* 1000.0С 50 CONTINUE С THESE STATEMENTS PICKS UP THE SECOND 24 SETS OF DEFLECTIONS (MID-LANE). С DO 60, I=1, 24

```
.
      READ (5,40) YBAD(I)
  40 FORMAT (19X, F9. 3)
С
C THIS STATEMENT DIVIDES THE DEFLECTIONS BY THE MULTIPLICATION FACTOR AND THEN
C CHANGES THE DEFLECTIONS FROM M TO mm.
     YBAD(I)=(YBAD(I)/C)*1000.0
С
   60 CONTINUE
С
C THESE STATEMENTS NUMBER THE LINES 1 TO 24. LINE 24 IS GIVEN THE NUMBER 0.
      DO 18 I=1,24
      K(I)⊐I
   18 CONTINUE
      K(24)=0
С
C THESE STATEMENTS PRINTS THE RESULTS IN THE FOLLOWING ORDER: X VALUES,
C DEFLECTIONS (WHEEL PATH), DEFLECTIONS (MID-LANE) AND LINE NUMBER
      DO 80, I=1,24
      WRITE (6,70) X(I), YAAD(I), YBAD(I), K(I)
  70 FORMAT (5X, F5. 2, 10X, 2PE12, 4, 10X, 2PE12, 4, 10X, I3)
С
   80 CONTINUE
С
      STOP
      END
```

APPENDIX V. PLOT.F77.

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C *********************
                                        ***
C #
                      PROGRAM PLOT. F77
                                                                               ÷
С
 4
                        _____
                                                                               #
С
                                                                               *
 ÷
 *
С
    THIS PROGRAM CHANGES THE ABSOLUTE DEFLECTION INTO
                                                                               #
С
  *
     DEFLECTOGRAPH DEFLECTION.
                                                                               Ħ
                                                                               #
С
  *
С
 .
    LENGTH OF LONG ARM(T DATUM FRAME) = L1 = 2130MM
                                                                               #
                                                                               ¥
¢
  *
    LENGTH OF SHORT ARM(BEAM)
                                       = L2 = 1530MM.
                                                                               ¥
¢
                                             = 0.7183MM
 *
    RATIO (L2/L1)
    (1-(L2/L1))
                                             = 0.2187MM
                                                                               #
С
         B = TIP OF T-DATUM FRAME
                                                                               ¥
С
 -
         A = HEAD OF BEAM FRAME
С
 *
                                                                               ¥
         C = TIP OF BEAM FRAME
                                                                               ¥
C
  -8
                                                                               #
С
    XADCO = ORGINAL POSITION OF C
 -84
     XADBO = ORGINAL POSITION OF B
С
  #
                                                                               *
С
     XADAD = ORGINAL POSITION OF A
                                                                               #
 *
С
                                                                               *
 -84
С
  *
     XADCT = FINAL POSITION OF C
                                                                               ÷
С
  *
     XADBT = FINAL POSITION OF B
                                                                               ÷
С
 *
     XADAT = FINAL POSITION OF A
                                                                               ŧ
С
  *
                                                                               *
C
     XAD = DISTANCE ALONG THE ROAD FOR ABSOLUTE DEFL.
 *
                                                                               ÷
С
     XAA = DISTANCE ALONG THE ROAD FOR DEFLECTOGRAPH DEFL
 - 44
                                                                               *
С
  *
                                                                               #
     YAAD = DEFLECTION(ABS) OF ROAD ALONG THE BEAM
С
 #
                                                                               *
С
     YBAD = DEFLECTION(ABS) OF ROAD ALONG THE T-DATUM FRAME
 .
                                                                               æ
С
  #
     YAA = DEFLECTION(DEFLIOGRAPH) OF THE ROAD ALONG THE
                                                                               Ħ
С
           BEAM OF THE NEAR SIDE WHEELS
 4
                                                                               #
С
 #
                                                                               th-
С
     XAD = 7.0 POSITION OF THE CENTRE OF REAR WHEELS
 #
                                                                               #
     XAD = 2.5 POSITION OF THE CETREE OF FRONT WHEEL
С
 - 44
                                                                               Ħ
С
  #
                                                                               ×
С
 *
    DEFLECTOGRAPH DEFLECTIONS CAN ONLY BE GIVEN AT FOLLOWING INTERVALS: -
                                                                               ¢
С
 #
                   50, 100, 125, 200 AND 250mm
                                                                               #
С
 *
С
 #
                                                                               #
С
  #
     TO RUN TYPE THE FOLLOWING COMMAND: -
                                                                               *
С
 4
                 $F77 PLOT
С
                         (WHEN YOU ARE ASKED FOR THE NAME OF THE DATA FILE. )
 #
               · PMDTH6
                                                                               *
С
  #
                         (WHEN YOU ARE ASKED FOR THE NAME OF THE OUTPUT
                 SMDTH6
                                                                               *
С
 4
                          FILE .)
                                                                               #
С
 *
                  100
                         (WHEN YOU ARE ASKED FOR THE INTERVAL. )
                                                                               *
С
  *
                                                                               #
С
    ASSUMED: -1) DATA FILE NAME = PMDTH6
 #
                                                                               ¥
С
 *
              2) OUTPUT FILE NAME = SMDTH6
                                                                               *
С
  *
              3) INTERVAL = 100mm
                                                                               *
С
 *
                                                                               *
С
 *****
С
      PROGRAM PLOT
      DIMENSION XAD(400), YAAD(400), YBAD(400)
      LOGICAL #2 QU
      CHARACTER #20 FILONE, FILTWO
C DECLARE VARIABLES AS REAL
      REAL #4 XDD, YDD, XADCT, YADCT, XADCO, YADCO, XADBT, YADBT, XADBO, YADBO,
     *XADAT, YADAT, XADAQ, YADAQ, XAD, YAAD, YBAD, MAX, MIN, INT, INTER
С
C DECLARE SOME INTEGERS.
     INTEGER #4 I, K, M
C
C ENTER THE DATA FILE NAME WHEN YOU ARE ASKED
    1 PRINT *, 'PLEASE ENTER THE NAME OF THE DATA FILE'
      READ (*,2) FILONE
    2 FORMAT (A20)
```

```
C CHECK IF THE NAMED DATA FILE EXISTS
     INQUIRE(FILE=FILONE, EXIST=QU)
      IF(QU) GO TO 3
C IF THE DATA FILE DOES NOT EXISTS THEN
     PRINT *, 'FILE NOT FOUND! PLEASE TRY AGAIN'
     GO TO 1
с
C IF THE NAMED DATA FILE EXISTS THEN
    3 PRINT *, 'PLEASE ENTER THE NAME OF THE OUTPUT FILE '
     READ (+,2) FILTWO
C
C ENTER THE INTERVAL (50, 100, 125 OR 200 mm) OF THE DEFLECTOGRAPH READING
C REGUIRED
     PRINT *, 'INTERVAL(50,100,125,200 DR 250 mm)'
     READ (*,5) INTER
    5 FORMAT (F6.0)
C TO OPEN THE OUTPUT FILE
     OPEN(6, FILE=FILTWD, STATUS= 'UNKNOWN')
С
     WRITE (6,81)
   81 FORMAT (28X, '-----')
     WRITE (6,82)
  82 FORMAT (28%, "IDEFLECTION: ")
     WRITE (6,83)
  83 FORMAT (28X, '----', ///)
     WRITE (6,94)
   94 FORMAT (6X, 'NOTE', /, 6X, '----', /)
     WRITE (6,93)
   93 FORMAT (6%, 'MEA. DEFL = DEFLECTOGRAPH DEFLECTIONS MEASURED. ', /, 6%, '
    *WHL. DEFL = ABSLOUTE DEFLECTIONS THROUGH THE REAR WHEELS. ', /, 6X, 'MI
    *D. DEFL = ABSLOUTE DEFLECTIONS THROUGH THE MID-LANE. ', ///)
     WRITE (6,84)
   WRITE (6,85)
   85 FORMAT (1X, 'DEFLECTOGRAPH DEFLECTION', 13X, 'ABSLOUTE DEFLECTION')
     WRITE (6,86)
   */)
     WRITE (6,87)
   87 FORMAT (3X, '-----', 2X, '-----', 8X, '-----', 8X, '-----', 2X, '------
      --', 3X, '--
                -----/)
     WRITE (6,88)
  88 FORMAT (4X, 1X(M) 1, 4X, 1MEA. DEFL(mm) 1, 10X, 1X(M) 1, 4X, 1WHL, DEFL(mm) 1, 3
    *X, 'MID. DEFL(mm) ')
     WRITE (6,89)
  --', 3X, '----')
С
C TO PRINT THE DEFLECTOGRAPH AND ABSLOUTE DEFLECTIONS TOGETHER IN THE
C SAME OUTPUT FILE (FILTWO), THE SUBROUTINE CALLED "DATASO", IS USED.
C THIS SUBROUTINE OPEN THE ORGINAL DATA FILE(FILONE) AND READS THE DATAS.
     CALL DATASO(XAD, YAAD, YBAD, FILONE)
С
C CHANGE INTERVAL TO METERS.
     INT=(INTER/1000)
С
C NUMBER OF INTERVALS + 1
     N = ((1000/INTER)+1)
C
C SET I=O FOR PRINTING PURPOSE
     I=0
С
C OPEN THE MAIN LOOP FOR VARIOUS POINTS(X VALUES) ALONG THE ROAD, AT
C SPECIFIED INTERVAL
```

```
245
```

```
C STARTION POINT OF INTERPLOTING.
      MAX=7.0
C LAST POINT OF INTERPLOTION
      MIN=6.0
        DO 10 XDD=MIN, MAX, INT
          XADCO=(XDD-0. 99)
          XADAD=(XDD-2.52)
          XADBO=(XDD-0. 39)
          XADCT=(XDD)
          XADAT = (XDD - 1, 53)
          XADBT=(XDD+0.6)
С
C SINCE THE ABSLOUTE DEFLECTIONS ON EITHER SIDE OF THE WHEEL IS ASSUMED
C TO BE THE SAME, THEN THE DEFLECTION AT DISTANCE X BEHIND THE WHEEL
C WOULD BE THE SAME AS THE DEFLECTION AT DISTANCE X INFRONT OF THE
C WHEEL
          IF (XADBT. GT. 7. 0) THEN
          XADBT=(7.0-((XDD+0.6)+7.0))
          END IF
С
C USE THE SUBROUTINE, "WHEEL", TO INTERPLOTE FOR VARIOUS POINTS
C ALONG THE ROAD THROUGH THE CENTRE OF THE WHEEL.
          CALL WHEEL (XADCO, YADCO, FILONE)
          CALL WHEEL (XADCT, YADCT, FILONE)
          CALL WHEEL (XADAO, YADAO, FILONE)
          CALL WHEEL (XADAT, YADAT, FILONE)
C
C USE THE SUBROUTINE, "CENTRE", TO INTERPLOTE FOR VARIOUS POINTS
C ALONG THE ROAD FOR THE CENTRE LINE OF THE CENTRE.
          CALL CENTRE (XADBO, YADBO, FILONE)
          CALL CENTRE (XADBT, YADBT, FILONE)
С
C TO CHANGE ABSLOUTE DEFLECTIONS INTO DEFLECTOGRAPH DEFLECTIONS.
          YDD=(YADCT-YADCD)-(0.7183)*(YADBT-YADBD)-(0.2817)*(YADAT-YADAD)
С
C TO CHANGE REAL NUMBERS(N) TO INTERGERS(L).
          L=N
C TO PRINT THE RESULTS DISTANCE(ALONG THE ROAD), DEFLECTOGRAPH AND C ABSLOUTE DEFLECTIONS.
          I = I + 1
          IF(I.LE.L) THEN
          WRITE (6,15) XDD, YDD, XAD(I), YAAD(I), YBAD(I)
          FORMAT (3X, F6. 3, 3X, 2PE10. 1, 11X, 0PF5. 2, 3X, 2PE12. 4, 3X, 2PE12. 4)
   15
          END IF
С
C TO CLOSE THE MAIN LOOP
   10 CONTINUE
С
C TO PRINT THE REMAINING SETS OF THE ABSLOUTE DEFLECTION: -
      M=L+1
       DO 18, K=M, 24
          WRITE (6,16) XAD(K), YAAD(K), YBAD(K)
          FORMAT (33X, F5. 2, 3X, 2PE12, 4, 3X, 2PE12, 4)
   16
       CONTINUE
   18
C TO CLOSE THE OUTPUT FILE(FILTWO)
      CLOSE(6)
С
      STOP
      END
C
C SUBROUTINE, "WHEEL", LAYOUT.
      SUBROUTINE WHEEL (XE, YE, FILONE)
      DIMENSION XAD(400), YAAD(400)
      REAL #4 XAD, YAAD, XE, YE
      INTEGER I.J.K
      CHARACTER #20 FILONE
```

```
С
с
  TO OPEN THE DATA FILE(FILONE).
       OPEN(5, FILE=FILONE, STATUS= 'UNKNOWN')
       I == 1
С
C ALL THE LINES IN THE DATA FILE WILL HAVE A LINE NUMBER IN THE LAST
C COLUMN EXCEPT FOR THE LAST LINE WHICH WILL HAVE A INTERGER=0. THIS
C ENABLE THE END OF DATA TO BE DETECTED.
   20 READ (5,4) XAD(I), YAAD(I), K
    4 FORMAT (5X, F5, 2, 10X, E12, 4, 32X, 13)
С
       I = I + 1
С
C THIS COMMANDS ENSURES THAT, I HAS NOT EXCEEDED 400(DIMENSION=400).
       IF(I.GT. 400) STOP
С
C THIS COMMAND ENSURES THAT EACH DATA IS READ TILL THE LAST CARD WITH
С
  INTEGER=0 IS DETECTED.
      IF (K. GT. 0) GD TD 20
С
C CHECK FOR X VALUE LESS THAN SMALLEST XAD VALUE ON CURVE.
      IF (XE. LT. XAD(1)) STOP
       J=2
С
C THE FOLLOWING ARITHMETIC IF STATEMENT COMPARES THE GIVEN X VALUE
C WITH SUCCESSIVE VALUES ON THE CURVE, LOOKING FOR AN X VALUE ON THE
C CURVE THAT IS GREATER THAN THE GIVEN VALUE.
   25 IF(XE-XAD(J)) 40,35,30
С
C NOT FOUND YET.
   30 J=J+1
С
C CHECK WHETHER GIVEN X VALUE IS GREATER THAN LARGEST X ON CURVE.
      IF(J.LT. I) GO TO 25
      STOP
С
C EQUAL
   35 YE=YAAD(J)
      GO TO 100
C
C TWO CURVE VALUES BRACKET GIVEN X -- INTERPOLAE.
   40 YE=YAAD(J-1)+(YAAD(J)-YAAD(J-1))/(XAD(J)-XAD(J-1))*(XE-XAD(J-1))
С
  100 CLOSE(5)
      RETURN
      END
C
C SUBROUTINE, "CENTRE", LAYOUT.
      SUBROUTINE CENTRE(XE, YE, FILONE)
      DIMENSION XAD(400), YBAD(400)
      REAL #4 XAD, YBAD, XE, YE
      INTEGER I. J.K
      CHARACTER +20 FILONE
C
      OPEN(5, FILE=FILONE, STATUS= 'UNKNOWN')
      I = 1
С
   20 READ (5,4) XAD(1), YBAD(1), K
    4 FORMAT (5X, F5. 2, 32X, E12. 4, 10X, I3).
С
      I = I + 1
С
      IF(I.GT. 400) STOP
С
      IF (K. GT. 0) GD TD 20
С
```

```
IF (XE. LT. XAD(1)) STOP
      J=2
С
   25 IF(XE-XAD(J)) 40,35,30
С
   30 J=J+1
Ç
      IF(J.LT. I) GO TO 25
     STOP
С
   35 YE=YBAD(J)
     GO TO 100
С
   40 YE=YBAD(J-1)+(YBAD(J)-YBAD(J-1))/(XAD(J)-XAD(J-1))*(XE-XAD(J-1))
С
  100 CLOSE(5)
     RETURN
     END
С
C SUBROUTINE, "DATASO", LAYOUT.
     SUBROUTINE DATASO(XAD, YAAD, YBAD, FILONE)
                                             .
     DIMENSION XAD(400), YAAD(400), YBAD(400)
     REAL #4 XAD, YAAD, YBAD
     INTEGER I
     CHARACTER #20 FILONE
     OPEN(3, FILE=FILONE, STATUS= 'UNKNOWN')
С
      DO 60, I=1, 24
        READ (5,28) XAD(1), YAAD(1), YBAD(1)
  28
        FORMAT (5X, F5. 2, 10X, E12. 4, 10X, E12. 4)
      CONTINUE
   60
С
     CLOSE(5)
С
     RETURN
     END
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APPENDIX VI. TABLES OF STATISTICAL ANALYSIS DETAILS.

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SOURCE OF VARIATION	DEGREE OF FREEDOM (df)	SUMS OF SQUARES (SS)	MEAN SQUARE (MS) (SS/df)	MEAN SQUARE RATIO F-TEST
Fitting of Separate Lines Residual (or Error)	$\frac{2\kappa-1}{\sum_{i=1}^{K} 2\kappa}$	Mp Rp	$M_{p} / (2K-1) = M_{p}$ $S_{p} = \frac{R_{p}}{\Sigma_{n-2K}}$	Test of some difference in the separate lines M <sub>pp</sub> / <sub>S<sup>2</sup>p</sub>
Total	$\sum_{i=1}^{K} n_{i-1}$	Т		
Fitting of Common Slope	К	Mc		Test of whether or not $\beta = 0$
Residual	К 2n <sub>i</sub> -К-1 i=1	Rc	s <sup>2</sup> c	(n <sub>0</sub> :p-0vn <u>i</u> :p v)
Total	κ Σn <sub>i</sub> -1 i=1	т		
Difference due to Separate Slopes and Common Slope	к-1	<sup>R</sup> c <sup>−R</sup> p	S <sup>2</sup> șlope	$F=S^{2}Slope/S^{2}p$ Reject H <sub>0</sub> if $F>F_{\alpha}, K-1, \Sigma n-2K$ at a significance level
H <sub>O</sub> :Parallel Lines VH <sub>1</sub> : Some	Lines not	e		
Fitting a Common Line	. 1	Mo		Test if common line has zero slope or not
Residual	κ Σn <sub>i</sub> -2 1	Ro	s <sup>2</sup> 0	
Total	Σn <sub>i</sub> -1	т		
Given Parallel Lines Difference due to Separate Intercepts and Common Intercepts	К-1	R <sub>o</sub> -R <sub>c</sub>	s <sup>2</sup> <sub>INT</sub>	$F=S^{2}_{INT}/S^{2}_{C}$ Reject H <sub>0</sub> if F>F <sub>a</sub> ,K-1, <u>c</u> -K-1
H <sub>O</sub> : Equal Intercepts Parall	el Lines V	H <sub>1</sub> :Unequa	al Ints. <sup>e</sup> Lin	es
$\hat{y} \pm \bar{t}_{\nu} \frac{\alpha}{2}  s_c \sqrt{1 + \frac{1}{n_1}} + \frac{1}{j}$	$\frac{(x^* - \bar{x}_i)^2}{(x_{ij} - \bar{x}_i)^2}$			

Table A.1 Statistical Analysis: Fitting of Common Slope and Confidence Limit.

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COMPARISON OF 4 REGRESSION LINES.						
SOURCE OF VARIATION	DEGREE OF	SUMS OF	MEAN SQUARE	F-TEST.		
	FREEDOM,	SQUARES	MS,			
	df	SS	(SS/df)			
Fitt. of Separate Lines	7	2.187	0.312			
Residual	65	0.156	0.002	130≃F <sub>7,65</sub>		
				Signi.,		
				Separate Lines		
Total	72	2.343				
Fitt. of Common Slope	4	2.177				
Residual	68	0.166	0.002			
Total	72	2.343				
Diff. Due to Separate						
Slopes and Common Slope	3	0.009	0.003	1.6≃F <sub>3,65</sub>		
				Not Signi.		
				Common Slope		
Fitt. a Common Line	1	0.184				
Residual	71	2.159	0.030			
Total	72	2.343				
Given Parallel Lines	3	1.993	0.664	$332 \simeq F_{3,68}$		
Diff. Due to Separate				Signi.		
Intercepts and Common				Different		
Intercepts				Intercepts		
Note: Fitt. = Fitting,	Diff. = Di	fference	, Signi.=	Significant.		

Table A.2 Statistical Details of the Power Law Relationships Between

 ${\rm D_{0}-D_{8}}$  and  ${\rm H_{e}}$  for a Given Thickness Granular Material,  ${\rm H_{G}}$  .

CBR	Η <sub>G</sub> ππ	Equations	No
	300	$\log_{10}(D_0 - D_8) = 1.679 - 2.346 \log_{10} H_e$	A.1
2	500	$Log_{10}(D_0 - D_8) = 1.839 - 2.346 Log_{10}H_e$	A.2
	300	$\log_{10}(D_0 - D_8) = 1.003 - 2.346 \log_{10}^{H}e$	A.3
10	500	$Log_{10}(D_0 - D_8) = 1.210 - 2.346 Log_{10}H_e$	A.4

Table A.3 Equations to Describe the Relationships Between  $D_0^{-D_3}$ and  $H_e$  for a Given Value of Granular Material Thickness,  $H_G^{-}$ .

CBR	H <sub>P</sub> ma	Equations	No
	400	$\log_{10}(D_0 - D_8) = 1.688 - 2.535 \log_{10}^{H_e}$	A.5
2	600	$Lcg_{10}(D_0 - D_8) = 1.830 - 2.585 Lcg_{10}H_e$	A.6
	400	$Log_{10}(D_0 - D_8) = 0.944 - 2.585 Log_{10}H_e$	A.7
10	600	$Log_{10}(D_0 - D_8) = 1.203 - 2.585 Log_{10}H_e$	A.3

Table A.4 Equations to Describe the Relationships Between  $\rm D_0-D_S$  and  $\rm H_e$  for a Given Pavement Thickness,  $\rm H_P$ .

°CBK	E1/E2	Equations	NO
	40/2	$Log_{10}(D_0 - D_8) = 1.137 - 0.66 Log_{10}H_e$	<u>.</u> \.9
2	100/2	$Log_{10}(D_0 - D_8) = 0.936 - 0.55 Log_{10}H_e$	A.10
	40/2	$\log_{10}(D_0 - D_8) = 0.974 - 0.75 \log_{10} H_e$	A.11
10	100/2	$Log_{10}(D_0 - D_8) = 0.680 - 0.56 Log_{10}^{H_e}$	A.12
	40/9	$Log_{10}(D_0 - D_8) = 0.950 - 0.59 Log_{10}^{H_e}$	A.13
2	100/9	$Log_{10}(D_0 - D_8) = 0.858 - 0.52 Log_{10}H_{e}$	A <b>.1</b> 4
	40/9	$Log_{10}(D_0 - D_8) = 1.201 - 1.44 Log_{10}H_e$	A.15
10	100/9	$Log_{10}(D_0 - D_8) = 0.764 - 0.70 Log_{10}H_e$	A.16
E <sub>1</sub> =	$E_2 = x$	$10^8  ext{ if } /  ext{in}^2$	-

Table A.5 Equations to Describe the Relationships Between  $D_0^{-D_8}$ and  $H_e$  for a Given Value of  $E_1^{/E_2}$  Ratio.

E1/E2	Equations	No		
100/2	$Log_{10}(D_0 - D_2) = 3.145 - 1.221 Log_{10}H_1$	A.17		
40/2	$\log_{10}(D_0 - D_2) = 3.402 - 1.221 \log_{10}H_1$	A.18		
100/9	$Log_{10}(D_0 - D_2) = 2.970 - 1.221 Log_{10}H_1$	A.19		
40/9	$Log_{10}(D_0 - D_2) = 3.200 - 1.221 Log_{10}H_1$	A.20		
$E_2 = E_1 = x 10_8 \text{ N/m}^2$				

Table A.6 Equations to Describe the Relationships Between  $D_0^{-D_2}$ and  $H_1$  for a Given Ratio of  $E_1^{/E_2}$ .

Hı	E <sub>2</sub>	Equations	No		
100	2	$\log_{10}(D_0 - D_2) = 2.504 - 0.597 \log_{10}E_1$	A.21		
	9	$\log_{10}(D_0 - D_2) = 1.746 - 0.399 \log_{10}E_1$	A.22		
150	2	$\log_{10}(D_0 - D_2) = 2.734 - 0.762 \log_{10}E_1$	A.23		
	9	$\log_{10}(D_0 - D_2) = 2.337 - 0.680 \log_{10}E_1$	A.24		
$H_1 = mm$ , $E_2 = \times 10^8 \text{ N/m}^2$ .					

Table A.7 Equations to Describe The Relationships Between  $D_0^{-D_2}$ and  $E_1$  for a given Value of  $E_2$  and  $H_1$ .

#### A.1 Introduction.

To validate the pavement evaluation and design system set up in Chapter 6.0, requires Deflectograph deflection measurements together with information on the thickness and modulus of the layered pavement with which to compare predicted values of the layer properties. Cores of bituminous material were extracted from local roads to determine the thickness and laboratory triaxial testing was undertaken to determine the modulus of these layers.

# A.2 Field Sampling.

Samples of bituminous material were collected from local roads, i.e. Totnes to Halwel (Road No.A381), where Deflectograph surveys have been conducted previously. The thickness of the bituminous material along these roads was not consistent but is usually less than 150 mm. A 100 mm core barrrel was used so that the thickness of the sample is at least greater than the diameter of the sample. The height of the sample should be greater than the diameter in order to obtain stress-strain results free of the influence of end effects during laboratory testing. The cores of bituminous material obtained were of Dense Bituminous Macadam (D.B.M) with a layer thickness in the range of 90 to 140 mm.

## A.3 Sample Preparation and Platen Connection.

All the core samples were trimmed at both ends using a diamond tipped circular saw, so that the ends were smooth and perpendicular to the sample axis. In conventional triaxial testing, where large deformations are involved, it is often sufficiently accurate to use the loading frame as a datum and to take measurements external to the triaxial cell by monitoring the movement of the load plunger. However, for dynamic repeated load testing, it is desirable to obtain measurement directly on the sample. This procedure eliminates extraneous deformation in the system due to the sample end effects and sundry movements occurring externally between the cell base and the plunger (132).

Very small holes were drilled in the samples and six locating targets, for measurement of longitudinal deformation, were secured with glue to the sample at each end of the three gauge length. The locating targets were spaced at 120 degrees of arc around the sample. Two further small metal locating studs, diametrically opposite, were secured with glue at the centre of the sample for measurement of lateral deformation.

A latex membrane was then placed over the sample necessitating the perforation of the membrane. The perforation was then sealed to the locating targets with glue. A metal washer was then placed over the locating targets and these were glued to the latex membrane.

Because of the small resilient deformations which were expected in these experiments, it was considered important that the cylindrical samples remained cylindrical as long as possible during the test (57). This can only be achieved by providing "frictionless" end platens whose diameters are greater than that of the sample. The nonuniformity of the stress distribution within the sample is primarily due to the effects of the friction on the end plates. The radial displacement on the periphery at both ends varies inversely with a

friction factor (137). The platens were made from ground and chromed, hardened steel; these produce a low coefficient of friction. The coefficient of friction was further reduced by using a sandwich of thin rubber discs and silicone grease (138). Lateral slipping of the sample can occur with this form of testing due to slight inaccuracies of setting up. To prevent excessive movement due to this lateral slipping a small retaining lip in the base platen was inserted. The diameter of the retaining lip was slightly larger than the sample and the top platen diameter.

## A.5 Stess Application System.

Vertical stress is applied to the specimens by an hydraulic actuator, see Plate A.1. The actuator is controlled by closed servo loops operating a servo valve on the actuator. A constant static load is applied by setting the D.C level of the reference signal at an appropriate value. This "dead load" is introduced to represent the effects of overburden pressure in an actual road. The dynamic load in either the sinusoidal or triangular mode, is superimposed on this dead load by selecting the required amplitude and frequency of the reference signal from the function generator. This represents the effect of the wheel loads on the pavement known as 'live load'. The triaxial cell is based on the design discussed by Brown et al (57, 132, 133).

# A.6 Measuring System.

#### A.6.1 Applied Vertical Load.

The servo hydraulic actuator has a built in transducer to measure and display the load applied. To determine the effect of friction between the loading ram and the hole in the cell top plate, a

cyclic load was applied to a specimen and the load applied was checked against an internal load cell. The built-in transducers displayed an average load value only 0.8% greater than the actual load applied as monitored by an internal load cell and indicated that only small inaccuracies would be introduced by using this approach. Therefore all the cyclic loads applied were monitored by means of the built-in transducers because inaccuracies in trimming and setting up of the sample could result in permanent damage of the internal load cell.

#### A.6.2 Confining Pressure.

Brown et al (51) studied the effects of confining pressure applied cyclically and statically on bituminous material. Brown et al (51) have made a simplification to the test techniques to produce essentially the same response from the material by using a static confining pressure equal to the mean values of the desirable cyclic stress. Air was used as a confining medium to provide the all-round equal static pressure. The confining pressure was measured using a Pressure Gauge.

# A.6.3 Longitudinal Deformation.

The longitudinal deformation was measured by three LVDTs (Linear Variable Differential Transformers) spaced at 120 degrees of arc around the sample, about the mid-height of the sample. The measurements are thus free of specimen end effects and errors which can be introduced by the use of an arbitrary datum such as the cell base. The LVDTs are of armature assembly (core and push rod). The push rod is held by the support arm and the core is held by the support bracket as shown in Plate A.2. This arrangement ensures that the LVDTs are

vertical, since sometimes it is not possible to align the two mounting studs. The three LVDTs were calibrated using a micrometer.

# A.6.4 Radial Deformation.

The radial collar to measure the radial deformation at the centre of the sample was based on a design procedure proposed by Bishop et al (138). The radial perspex collar is split into two halves; hinged at one end and connected by a tension spring at the other end as shown in Plate A.3. The collar is located onto two diametrically opposed locating studs at the centre of the sample. As the sample expands laterally, the gaps between the free ends of the collar change and this change is measured by an LVDT (core and push rod armature) mounted onto the collar across the tension spring. The action of the tension spring keeps two pointers, one on each half of the collar, located in the locating studs, as shown in Plate A.4. Small pieces of sponge are placed between the collar and the sample to prevent tilting The lateral LVDT is calibrated using Slip Gauges. The radial strain collar is convenient to use but it has the disadvantage that the measurement obtained is a "point value" of the deformation rather than an average for all diameters.

# A.7 Data Recording.

All the signals from the LVDTs were fed into the amplifiers which inturn are connected to a multi-channel recorder. The longitudinal deformation obtained from the three LVDTs are averaged to eliminate any tilting effects and provide a measure of the average deformation of the central portion of the sample. The vertical load applied by the **ser**vo **hydraulic** actuator is **measured** by the servo control transducers and displayed by its digital meter.



Plate A.1 Triaxial Testing System.



# Vertical Displacement Transducer.

Plate A.2



Lateral Displacement Collar and Transducer.

Plate A.3



# Deformation Measuring System.

Plate A.4

# A.8 The Testing Programme.

The vertical stress consisted of a 32  $\text{KN/m}^2$  constant value 'dead load', onto which the dynamic pulse was superimposed. It was appreciated that this value of 'dead load' was high in relation to the likely overburden pressure to be expected in a pavement but it was selected in order to ensure that the top end platen remains in contact with the specimen during testing. The type pressure of the Deflectograph rear wheel is about 690  $\text{KN/m}^2$ . The total vertical pressure of 1380  $\text{KN/m}^2$ , pressure of twin rear wheels, was applied to the sample at different static confining pressures of 100, 200, 300  $\text{KN/m}^2$ . The speed of the Deflectograph is about 2.5 Km/hr. The width of the Deflectograph type is about 230 mm. Assuming, 45 degrees of load spread, the frequency of the vertical stress was calculated as follows:



H = Sample Thickness; 100 to 140 mm, V = 2.5 Km/hr = 694.44 mm/sec, S = (240 + H) mm,

t = S/V, f = 1/(2 t), Range of f = 0.32 to 0.28 Hz.

Therefore a frequency of 0.3 Hz was the vertical stress pulse used for all tests. The testing was carried  $\operatorname{out}_{1}^{48}$  <sup>0</sup>C. The modulus was calculated after about 300 cycles. After about 250 cycles the curve of the plot of deformation against time tended to become flat. This is probably due to the fact that the samples were already compacted by the moving traffic on the roads and the initial changes reflected 'bedding in' of the testing system and particularly 'bedding in' of the sample ends.

The elastic modulus of the bituminous samples was between 2.5 to 5 x  $10^9$  N/m<sup>2</sup>. The elastic modulus increased with the increase in confining pressure.