Multi-Agent Fitness Functions For Evolutionary Architecture

by

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Abstract

The dynamics of crowd movements are self-organising and often involve complex pattern formations. Although computational models have recently been developed, it is unclear how well their underlying methods capture local dynamics and longer-range aspects, such as evacuation. A major part of this thesis is devoted to an investigation of current methods, and where required, the development of alternatives. The main purpose is to utilise realistic models of pedestrian crowds in the design of fitness functions for an evolutionary approach to architectural design.

We critically review the state-of-the-art in pedestrian and evacuation dynamics. The concept of 'Multi-Agent System' embraces a number of approaches, which together encompass important local and longer-range aspects. Early investigations focus on methods—cellular automata and attractor fields—designed to capture these respective levels.

The assumption that pattern formations in crowds result from local processes is reflected in two dimensional cellular automata models, where mathematical rules operate in local neighbourhoods. We investigate an established cellular automata and show that lane-formation patterns are stable only in a low-valued density range. Above this range, such patterns suddenly randomise. By identifying and then constraining the source of this randomness, we are only able to achieve a small degree of improvement. Moreover, when we try to integrate the model with attractor fields, no useful behaviour is achieved, and much of the randomness persists. Investigations indicate that the unwanted randomness is associated with 2-lattice phase transitions, where local dynamics get invaded by giant-component clusters during the onset of lattice percolation. Through this in-depth investigation, the general limits to cellular automata are ascertained—these methods are not designed with lattice percolation properties in mind and resulting models depend, often critically, on arbitrarily chosen neighbourhoods.

We embark on the development of new and more flexible methodologies. Rather than treating local and global dynamics as separate entities, we combine them. Our methods are responsive to percolation, and are designed around the following principles: 1) Inclusive search provides an optimal path between a pedestrian origin and destination. 2) Dynamic boundaries protect search and are based on percolation probabilities, calculated from local density regimes. In this way, more robust dynamics are achieved. Simultaneously, longer-range behaviours are also specified. 3) Network-level dynamics further relax the constraints of lattice percolation and allow a wider range of pedestrian interactions.

Having defined our methods, we demonstrate their usefulness by applying them to laneformation and evacuation scenarios. Results reproduce the general patterns found in real crowds.

We then turn to evolution. This preliminary work is intended to motivate future research in the field of Evolutionary Architecture. We develop a genotype-phenotype mapping, which produces complex architectures, and demonstrate the use of a crowd-flow model in a phenotype-fitness mapping. We discuss results from evolutionary simulations, which suggest that obstacles may have some beneficial effect on crowd evacuation. We conclude with a summary, discussion of methodological limitations, and suggestions for future research.

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Author's Declaration

Declaration:

At no time during the registration for the degree of doctor of Philosophy has the author been registered for any other University award.

This study was financed with the aid of a University of Plymouth studentship.

A programme of advanced study was undertaken and relevant scientific seminars and conferences were regularly attended. Work was presented and published.

Publications:

A preliminary vision of the project was presented as a poster (Holden and Parmee, 2000) based on preliminary work, none of which is contained herein. Since that preliminary work, and with further investigations, the project changed emphasis. These investigations have been reviewed, presented and published (Holden and Cangelosi, 2004b), based on the work, which underpins chapter 5. Another publication, which contains details of our general methodology was reviewed, presented and published (Holden and Cangelosi, 2004a), based on the work which underpins chapter 6. These three publications are contained within the appendices (see Appendix C).

A further publication resulted from work at a summer school (Holden, Sanjana and Seiferle-Valencia, 2002). This is not directly relevant to the thesis, but informed an analysis of those models outlined herein, which attempt to exploit statistical techniques based on *Small World* theory.

Conferences Attended:

ACDM 2000, University of Plymouth, Plymouth. Complex Systems Summer School 2002, Santa Fe Institute, Santa Fe. Inteflam 2004, Heriot-Watt University, Edinburgh.

Industrial Work:

Between 10/2001 and 08/2002, work was undertaken in the *Intelligent Systems Lab*, *BTexact*. This inspired some of the ideas developed in Chapter 5 on pulsed neural networks.

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In loving memory of Elsie Horne.

Chapter 1

Introduction

Egress refers to the process of movement in populations of pedestrians. Each pedestrian must move within a built configuration, from a given origin location to a destination location. There are a number of complexities involved, which incorporate configuration-pedestrian interactions and pedestrian-pedestrian interactions. Also, both kinds of interaction can take place at varying degrees of locality. One problem is to represent such complexities at an appropriate level of abstraction, so that we can build models that capture real pedestrian flows.

Our longer-term aim is to utilise egress models as fitness functions in an evolutionary approach to the design of building architecture. However, an essential first step is, of course, to develop sound models. Most of the thesis is devoted to the latter, but after successfully modelling important aspects of real crowds, we also manage to provide a preliminary evolutionary approach to the former. The main purpose of this chapter is to set the background of the thesis, both in terms of our longer-term aims, and the immediate problem of egress modelling.

We provide an overview, which outlines the thesis structure and contributions (Section 1.1). We introduce a number of relevant fields of enquiry and concepts; Evolutionary Algorithms (Section 1.2), Artificial Life (Section 1.3), Evolutionary Architecture (Section 1.4), methods of architectural decomposition (Section 1.5), and the concepts of function and form in an evolutionary sense (Section 1.6). We introduce the meaning of fitness functions in an

evolutionary-algorithmic sense (Section 1.7) and the field of Crowd Dynamics (Section 1.8). While doing this we hook-on particular beliefs and general aims, which we summarise at the end of the chapter (Section 1.9).

1.1 Overview of Research

1.1.1 Thesis Outline

Chapter 2: We address the broad aspects of scientific modelling in order to identify an appropriate level of abstraction for models of egress behaviour. We introduce the idea of a Multi-Agent System (MAS) and refer to pedestrian crowds as examples of such systems. We critically review some egress and crowd dynamics models from the literature, introduce the concepts on which these models are based and identify issues relevant in an evolutionary context ¹. We organise this review around concepts of discrete and continuous space, and indicate which techniques from both are worth looking at in more detail.

Chapter 3: Having abandoned a number of modelling approaches, we offer a more detailed technical review of the methods, which underpin models that remain relevant to our chosen level of abstraction. The aim is to introduce the details of computational representations used and, therefore, identify in more detail which kinds of pedestrian behaviours are important and at what level model dynamics should be specified. In this way, chapter 3 is a more detailed exposé of the methods previously ring-fenced as potentially useful in Chapter 2. As a result of this more in-depth review, further modelling techniques are deemed inappropriate in accordance with our aims. We are left with the approaches, which utilise Cellular Automata (CA) methods or field attractors, which

¹Throughout the thesis we use the terms *egress* and *crowd dynamics* interchangeably because their meaning is often interwoven. However, it is also useful to think of egress as the general processes and patterns of crowd behaviours that occur when a crowd population vacates a configuration. Crowd dynamics refer to the rules that give rise to these patterns as the process of egress unfolds. The reader may also find the glossary in Appendix A useful for definition of other terms.

capture local and global dynamics, respectively.

Chapter 4: We replicate and thoroughly investigate the properties of an existing CA model. We choose a well-published model from the literature and attempt to analyse its behaviour. The model is designed to produce important dynamic patterns found in populations of real crowds. Based on our evidence, we argue that this CA is inadequate. The problem stems from an underlying attempt to model complex and high-level behaviours from the development of overly simple rules, which operate in limiting local neighbourhoods. We therefore attempt to incorporate the local dynamics of the CA with a global attractor field. The aim is to integrate local with global methods, in order to model the short and long-range behaviours, observed in real pedestrian crowds. However, we fail to meet our aims and this approach is taken no further.

Chapter 5: One property of the CA, observed from the investigation of Chapter 4, is the sudden collapse of intended pattern formation and a phase transition between a relatively stable behavioral regime and a behavioral regime associated with large-scale randomness. Following on from these observations, we take a detailed look at the nature of information interaction in 2-lattice graphs. The 2-lattice is assumed in almost all discrete, MAS models of pedestrian and evacuation dynamics. If we are to model interactive systems by using this kind of lattice, then we need to understand the properties of this spatial domain. We replicate results from the field of computational physics and investigate information percolation on the 2-lattice. We present statistical results, which show phase transition behaviours of the 2-lattice space. We also show how these phase transitions define an upper limit to the success of any rules specified in local neighbourhoods.

This chapter is, therefore, the point where we abandon any attempt to specify rules at a purely local level, and the hope of any higher-level emergence of global dynamics from such localities. We suggest reasons why localised models of egress behaviour might be generally limited, and what is required to represent local interactions between agents

and more global strategies of escape. These requirements lead us to the introduction of the Pulse Coupled Neural Network (PCNN). Most of this chapter is then dedicated to the investigation of a pulsing neuron model and coupled populations. Originally applied in the area of image processing, PCNN's have been shown to produce clear and well-defined patterns of autowave expansion. We argue that in view of 2-lattice properties these expanding waves may offer an alternative and more robust method of search in local neighbourhoods. We enhance autowave features by pruning neuron equations. We demonstrating how this model may be useful in the specification of appropriate dynamics for pedestrian egress behaviours.

Chapter 6: Results from Chapter 5 provide a basis for a new approach to modelling pedestrian dynamics and egress. Instead of using the disparate methods of other approaches, such as locally defined CA and globally defined attractor fields, this new approach integrates the local and global perspective. We provide a complete description of a multi-agent system, capable of modelling adaptive behaviour in a population of escaping individuals. This system is based on a dynamic boundary, which defines *locality*, and is derived from time series information in autowave expansions. Essentially, this method provides a robust region around a pedestrian, which is able to return useful information without suffering the catastrophic effects of lattice space.

We demonstrate the potential use of these tools in modelling pedestrian evacuation scenarios with two models, which reproduce crowd behaviour patterns found in real crowds.

Chapter 7: By producing models of the real emergent patterns found in actual crowds, we are confident enough to then take a step back from the detail of modelling and look forward to our longer-term aims. We provide a preliminary demonstration of how the model might be used in future evolutionary approaches to the design of architectural space. This chapter is preliminary in the sense that we use a restricted density param-

eter, due to 2-lattice constraints. We present two evolutionary simulations. The first depends on a nature-inspired algorithm, which automatically develops valid architectural representations of space. The second is a constrained application of evolution to the optimisation of exit-sign locations. We introduce an evolutionary algorithm, and a genotype \mapsto phenotype \mapsto fitness mapping for both evolutionary simulations. Although we should treat these preliminary results with caution, they demonstrate both useful selection pressure and the efficacy of evolution in the functional optimisation of building design.

Chapter 8: We critically summarise the thesis in the light of original aims. Conclusions are drawn and possible future directions considered.

1.1.2 Original Contributions

Five central contributions are made:

- 1. Investigation into Cellular Automata model (see Chapter 4):
 - (a) We replicate a number of CA models and design statistical techniques, which provide indicators of model breakdown.
 - (b) We attempt to improve the rule sets by introducing methods, which reduce inherent randomness. This improves some of the runaway random effects in the CA.
 - (c) However, further investigation reveals that intended pattern formations fail catastrophically as density parameters increase.
- 2. Fundamental limitations of local methods (see Chapters 4 and 5):
 - (a) By considering the interaction complexities typical in percolation studies of the 2lattice graph we offer reasons as to why 2-D CA models fail catastrophically. This has important implications as to:
 - i. how to represent dynamics and

- ii. how to specify egress models.
- 3. Investigation into pulsed neuron properties in a Multi-Agent System approach to crowd modelling (see Chapter 5):
 - (a) We develop ideas derived from studies of the Pulse Coupled Neural Network (PCNN).
 - (b) We argue that the PCNN provides a more adequate, network-level search and demonstrate how when tuned appropriately, spaces of high complexity are explored.
 - (c) We identify a number of useful properties in these networks and adapt equations in order to enhance these properties.
- 4. Efficient implementation of autowave search (see Chapter 6):
 - (a) We implement a simplified version of the search algorithm because the PCNN is computationally expensive, particularly in multi-agent scenarios.
 - (b) We execute network search through the implementation of a locally executable CA, which has two primary advantages:
 - i. arbitrarily complex networks can be explored.
 - ii. computational cost is reduced.
- 5. Techniques for the development of models for egress scenarios (see Chapter 6):
 - (a) We present general methods for model specification.
 - (b) In order to demonstrate the potential power of our tools we produce two models, which replicate important optimising behaviour found in real crowds. The first model achieves this in simple spatial configurations and the second in more complex spatial domains. These models serve to demonstrate the self-organising potential of our approach and provide a rationale for utilising such techniques as fitness functions for evolutionary architecture.

- 6. Multi-agent fitness functions for evolutionary architecture (see Chapter 7):
 - (a) We preset details of a developmental algorithm, which can be used to generate an arbitrary amount of spatial configurations with various levels of complexity.
 - (b) We present details of Genotype→Phenotypes→Fitness mappings and present evolutionary results, using the fitness function in the context of two different kinds of evolutionary simulations.

1.1.3 A Note on Style

There is no pre-ordained style to every chapter, although we have decided to adhere to some organising principles while writing. Each chapter firstly contains an abstract-styled overview of its contents. This is intended to provide a general feel for the chapter before details appear. At the end of each chapter a summary recaps the main concepts, results and/or arguments and provides a bridge to the following chapter. Where we feel a section inside a chapter has been particularly long or detailed, we provide a similarly styled summary, but local to that section.

We now begin our introduction to the general fields of enquiry and concepts, which motivate and are implicated in the work of this thesis.

1.2 Evolutionary Algorithms

Since Darwin, it has been possible to think of the latest end products of evolution as manifestations of a plodding accumulation of small, adaptive change. Complex phenotypes are selected over millennia and their forms are shaped by relentless environmental pressures. Considering how complex some of these forms are, it has been difficult to imagine how these form have arisen, other than through the skill of a conscious creator. But in spite of our imagination, or perhaps because of a different, scientific one, evidence suggests that Darwins theory is correct—given the right building materials, nature juggles them, with stunning creativity, into

the beautiful configurations that we call 'life'. Yet, despite its appearance and true to form, nature appears to follow a comparatively simple set of rules, which convert matter into life. Although fine-grained details of natural evolution are not fully understood, at another level, general Darwinian process can be viewed as algorithmic; as a program of change executed in the medium of nature (Dennet, 1995). On this view, it is not a necessary condition that selection be implemented only in accordance with natural constraints. The implication is that that fundamental Darwinian mechanisms (Variation, Inheritance, Reproduction, Mutation) can be implemented in other media. In the past thirty years strides have been made in computer-based evolution and today Evolutionary Algorithms are a huge and various industry, the GA (Holland, 1975; Goldberg, 1989) being the most popular and widely used (Bently, 1999).

The general aim of all Evolutionary Computation is to exploit scientific ideas about natural, solution-driven innovations in the face of a ceaseless, selection-driven process of survival. Questions are open regarding the detailed accuracy of Evolutionary Algorithms as appropriate representations of real biological populations. For example, the fundamental Darwinian processes found in natural evolution are not always well represented, but the GA is perhaps the most extensive evolutionary algorithm in this respect (Bently, 1999). This said, we are not directly interested in biological details other than for their adaptive properties. In this sense we do not intend to construct a *model* of evolution, but rather exploit its power in the context of design. We will encounter some of the specific GA mechanisms later in the thesis, but for now two general concepts suffice; 'exploration' and 'exploitation':

Exploration involves the discovery of new solutions. As in biology, GA's operate on genotypes where probabilistic mutations occur at given sites. Mutations introduce the possibility of innovations, which might contribute to the overall fitness of a phenotype. Without mutation, GA's can only accumulate random pieces of information and evolution is likely to stagnate with early convergence to poor solutions. The concepts of innovation and stagnation are combined in a population-dependant concept of *fitness*, where each population member is judged according the the fitness of the entire population. Innovations through mutation, over time, come to dominate the population and this is where the second concept, exploitation, derives its meaning.

Exploitation produces an accumulation of information, which contributes positively to fitness. In biology, the dominant view is that genes are the selected units (Dawkins, 1976) and, although provisional, fit genetic material tends to accumulate in phenotypes. Any fit genes are, by implication, more likely to survive because they contribute to the survival of the phenotype carrier. Because phenotypes reproduce with inheritance, and hence accumulate genetic information, even though mutations explore genotypes, a species' genetic historical identity is stubbornly apparent due to this selective pressure. For example, phenotypes as different as humans and rats share large amounts of genetic material, which in a Darwinian interpretation, is inherited from an origin—points in biological history where a species began to branch into separate reproductive futures. Although evolution clearly innovates through mutation, and diverges from this origin over time, there is always this power to exploit.

By analogy to these biological process, Holland argues that artificial evolution selects building blocks. Just like we combine known strategies in solving a new problem, building blocks on binary strings are combined through a process akin to sexual reproduction (Holland, 1975; Holland, 1995). These building blocks contribute to the success of the entire string and 'good' building blocks accumulate and persist, both over time and across a population, even though mutations keep on exploring. In short, the GA is a multi-tasking algorithm. It harnesses both explorative and exploitative forces. Any conference proceeding on GA's over the last ten years will testify to the great amount of research dedicated to these general properties of the GA. These mechanisms need to be tuned to the particular application domain and thus various techniques based on explorative and exploitative forces have arisen in Evolutionary Design (For example, Parmee, 2000).

1.3 Artificial Life

Whereas EA's have a clear and singular, nature-inspired foundation, A-Life is more generally nature-inspired. According to Bonabeau and Theraulaz, it is a "method consisting in generating at a macroscopic level, from microscopic, generally simple, interacting components, behaviours that are interpretable as lifelike" (p 303, Bonabeau and Theraulaz, 1995). This definition broadly captures many activities in A-Life approaches, but A-Life has many faces and other fields of enquiry often cut across it. For example, in the study of adaptive agents, techniques such as Artificial Neural Networks and GA's are used. Many scientists in the field of A-Life often find that they have to define what is artificially life-like in their work. But there are concepts that define the field, even if they are shared with other domains (Boden, 1996a).

In the definition provided by Bonabeau and Theraulaz, generating is a very useful term for our current purposes. A generative procedure is one where simple rules interact and develop into higher-level patterns. A good example are CA, collections of low-level, locally interactive rules, which develop into global patterns. Generative procedures are part of many 'bottom-up' approaches, but are particularly important in A-Life—interactions between artificial units may produce significantly interesting patterns in comparison with those found in natural systems (Langton, 1996a). Maybe the biological medium is also not a necessary condition for the production of life-like behaviour, just as Dennet (1995) argues in terms of evolution? Some even claim that certain computerised behaviour is life rather than life-like, leading to the internal separation of the so-called 'strong' and 'weak' A-Life brigades (Levy, 1993).

Good examples of generating procedures are found in the new field of computational morphogenesis, where preliminary work investigates complex mappings between the genotype and the phenotype (Kumar and Bentley, 2003; Stanley and Miikkulainen, 2003). Tools in this field include CA, L-Systems and various other self-organising techniques. A general theme in computational morphogenesis is to analyse the nature and complexity of the mechanisms that control the process of development. Prusinkiewicz (1995) suggests that morphogenesis can be used in fields as far from theoretical biology as graphic design, computer art and landscape

architecture. Developmental procedures are therefore useful in domains other than theoretical biology. This is true of the field of Evolutionary Architecture, which we now introduce.

1.4 Evolutionary Architecture

We have so far presented Artificial Evolution and A-Life separately, but nowhere are overlaps more apparent than in this new domain. Evolutionary Architecture was conceived by Frazer (1995) and refers to architecture in the sense of man-made building design. Evolutionary Architecture is interdisciplinary and various techniques from the field of computational intelligence are used. As Frazer states:

"An evolutionary architecture investigates the fundamental form-generating processes in architecture, paralleling a wider scientific search for a theory of morphogenesis in the natural world. It proposes the model of nature as the generating force in architectural form. The profligate prototyping and awesome creative power of natural evolution are emulated by creating virtual architectural models, which respond to changing environments... Architecture is considered as a form of artificial life, subject, like the natural world, to principles of morphogenesis, genetic coding, replication and selection."

(p 9, Frazer, 1995)

Frazer's work on evolutionary architecture is mostly speculative, but he does outline some of the computational techniques, which may be useful for the future of the field. He includes Artificial Neural Networks (ANN's), Classifier Systems, the GA and CA among his list. Since Frazer's (1995) book, a small but growing group of computer scientists are beginning to apply techniques mentioned by Frazer for evolving architectural forms. Architecture per se is primarily concerned with space, structure and form and Evolutionary Architecture is concerned with decomposing these concepts into primitive units, expressing these in generative rules, which can be genetically coded, developed, tested and evolved (O'Reilly and Testa, 2000). Also, CA have been used to try and explore the fundamental rules of form in architecture (Broughton, Tan and Coates, 1997; O'Reilly and Testa, 2000). This work defines a small, but growing trend in the use of nature-inspired computation for evolutionary architectural design.

Unlike the majority of GA applications, Evolutionary Architecture often provides no explicit fitness function. Evolution is used with the emphasis very much on exploration, where fitness is determined with a human-in-the-loop rather than a fixed fitness function. Humans select aesthetically pleasing forms and evolution searches for an inherited variety. This process is repeated until the user is satisfied with the emergent shapes (Broughton et al., 1997), similarly to Dawkins's (1991) popular 'biomorph' program. The assumption behind these explorative techniques is that stronger materials and reduced costs allow form to act as a precursor to function. It had been assumed that function generally determined form and that building shape was also constrained by the strength of available materials. These constraints determined a narrow range of forms with little room for exploration or inventiveness (Broughton et al., 1997). However, with the increasing availability of strong and flexible materials, the general feeling behind Evolutionary Architecture is that Computer-Aided Architectural Design can exploit nature-inspired algorithms for form exploration and that these forms can, more likely than not, be realised.

1.5 Architectural Components

In order to realise the evolution of architecture it is important to identify primitive components of space. Theories of configurational architecture argue that topological features can describe space. The use of networks in architectural theory was first used extensively by Alexander, Ishikoawa and Silverstein (1977) and inspired the development of Space Syntax (Hillier and Hanson, 1984; Hillier, 1996). Essentially, this is a strand of applied Graph Theory where space is discretely represented as a graph so that mathematical techniques can be mobilised and space understood from a more rigorous standpoint. For example, Space Syntax theory has been used to analyse configurational relationships in buildings and street plans (Hillier and Hanson, 1984; Hillier, 1996; Hanson, 1998). An added advantage of this decomposition is that

it provides a framework for thinking about spatial representation in genetic codes.²

Graphs are fundamentally important, not only at the level of architectural theory, but also for pedestrian models. Although many discrete models rely on graph representations, nothing is ever said of their statistical properties. As we will see, these statistical properties are very important and inform some of the decisions that we make in later chapters.

1.6 Function and Form

In the Evolutionary Architecture literature we believe form generation is over emphasised. The exploration of form is an important area of research (see above), but in this thesis we suggest that there are other important features of built environments, which should be considered if the power of evolution in architectural design is to be fully realised. As implied by Space Syntax, important questions revolve around the spatial features of a building. In particular, the following, related questions represent our concerns: What are the effects of a building configurations on inhabitants?, What are the effects of interactions between inhabitants? and Can we sufficiently model the dynamic aspects of these interactions?

In Evolutionary Architecture these kinds of questions have not so far been asked. One intention behind this thesis is to address them. The over emphasis on form generation has so far precluded any attempt to harness the power of nature-inspired techniques for developing building-occupant friendly designs. We agree with the general ideas behind Evolutionary Architecture—Artificial Evolution and other nature-inspired techniques can provide a powerful medium for design. However, we believe that in order to be of real, practical use, Evolutionary Architecture will need to address issues that stem from the above questions. At one level this thesis can be seen as a preliminary investigation into evolving buildings, which cater for

²In the language of Graph Theory, Space Syntax approximates continuous space to a 'graph', a collection of 'vertices' with various internal relationships specified by 'edges'. We use these terms consistently throughout the thesis in order to avoid confusion. However, these methods are no longer confined to abstract mathematics and other fields use graph elements without naming them in the same way. For 'vertex', point, node, junction, or 0-simplex might be found and for 'edge', line, arc, branch or 1-simplex are common (Harary, 1997). Unfortunately, this list is by no means exhaustive, but we have found that graph, vertex and edge are perhaps the most frequent, particularly where statistical techniques are applied (Newman, 2003; Watts, 1999). We therefore follow this convention.

interactions between the building and its inhabitants.

In attempting to persuade people as to the power of selection, Darwin points out how various natural forms reflect human intention. Referring to specific domesticated breeds, Darwin writes: "One of the most remarkable features in our domesticated races is that we see in them adaptation, not indeed to the animal's or the plant's own good, but to man's use or fancy" (p 26 Darwin, 1998). Indeed, with domesticated variation in animals, form mirrors the breeder intentions in the same way that evolutionary architectural form will mirror the aesthetic preferences of the architect. However, the appearance of breeds, unless bred purely for aesthetic pleasure, will hide some underlying function. For example, the beauty and elegance of a greyhound hides the necessity for speed in the economics of greyhound racing. When architects design buildings, they too consider forms, which must fulfil given functions.

One particular context we have in mind for this thesis is the area of building safety. Safety science is a large area of research in its own right. For example, staff training standards are issued regularly as a means to minimise human error factors in evacuation situations and, because such standards are relentlessly updated, these minimum requirements often demand strenuous efforts on behalf of a company in order that their current safety practices comply. Staff training for the development of mental and behavioural competence in real evacuation situations is very important. However, improvements in this area are constrained by the inherent safety of a particular building configuration. Questions relating to spatial layout, exit positioning and hazard warnings, such as alarms and smoke detectors, become significant.

Although evolutionary architecture may be useful in the exploration of form, whether or not it can provide an adequate framework for these functional aspects remains unclear. To contribute towards Evolutionary Architecture in this respect is a further aim of this thesis. We are primarily interested in the use of computationally intelligent techniques and providing an evolutionary tool with which to explore forms of building layout that function well with respect to evacuation efficiency. Specifically, we are interested in the design of fitness functions to this end in order that, in the longer-term, architectural design may benefit from a coupling

of crowd behaviour models and evolution.

1.7 Fitness Functions

It is in this sense that that the title of this thesis comes to the fore; *Multi-Agent Fitness Functions for Evolutionary Architecture*. In the GA literature, simple fitness functions are often used in order to demonstrate evolutionary optimisation capability. Benchmark fitness functions exist (Goldberg, 1989), which are easy to implement, mathematical functions and used in order to test the dynamics of evolution and success of algorithm implementation. However, not all real problems are easy to express as a simple function.

Fitness function variety stretches from simple mathematical function optimisation to the design of complex ecosystem simulations. In the latter, fitness is determined by some period of learning in a virtual environment, perhaps even with the use of ANN's (Mitchell, 1999; Cangelosi, 2001). There are computational constraints on the complexity of a fitness function, but in order to be of any use it needs to model the essential features of a problem.

The complex detail of the evacuation process is important and it is unlikely that simple or overly abstract models will suffice. Human behaviour is itself complex and here specific questions appear, such as, How do we account for this complexity in fitness functions for evolutionary architecture? Is it possible to predict the safety of a given configuration or, if not, what techniques are available to us in order that we can better understand the relationship between human behaviour and building layout? and What techniques are appropriate when designing fitness functions that represent the complexities in interactive human behaviour?

This thesis is broadly concerned with these questions and the techniques available for modelling this complexity. One aim is to develop simulations of crowd evacuation from building configurations of arbitrary complexity. The process of evacuation is known as egress. We need to represent the dynamics of crowding people in egress situations. This would be a step towards providing a useful fitness function for an evolutionary approach to the design of spaces, which implicitly cater for interactive human behaviour. The evolutionary and adaptive design of safe spatial configurations might then be conceivable. The success of our approach will depend on the usefulness of such a fitness function. In short, an egress scenario is interactive, and adaptive—it is a complex system and it is often said that within such systems the devil is in the detail. The aim of this thesis is to ask which details need to be captured and encapsulated inside a fitness function?

1.8 Models of Crowd Dynamics

Current models of crowd dynamics provide a starting point for an investigation into the types of models that may be useful in capturing this complexity. If we can identify some techniques for modelling the crowd dynamics of escaping pedestrians, then we are well on the way to identifying fitness functions for use in the evolution of building design and spatial layout.

There are a variety of techniques already available. Current models vary widely, however, and we need to be very careful, particularly in the case of modelling egress behaviour, that we capture relevant behaviour. The immediate question is Do we have the correct level of detail in our fitness function model, enough to capture the complexity of the egress process? If we are to use existing models as fitness functions, we need to ensure that the details of these models do not represent unusual instances. Evolution is population-based—the fitness function will need to be general and applicable to various configuration scenarios and not contain quirks of behaviour specific to certain crowds or configurations. For this reason, the fitness function will have to be robust and able cope with a large amount of diversity while converging towards a fit set of phenotypes.

General features of real crowds are implicit in many models of egress behaviour and recent conference proceedings testify to the variety of techniques used (for example, Schreckenberg and Sharma, 2002). Some will be more apt than others and we will identify those relevant in the next chapter. Here we provide a small list of generally important *types* of behaviour:

• Pedestrian-pedestrian interaction

- Pedestrian-environment interaction
- Escape behaviour
- Adaptive decision-making

1.9 Summary

In this chapter, after presenting a thesis overview we:

- introduced general background areas of research:
 - We emphasised the importance of Evolutionary Design and briefly described explorative and exploitative evolutionary forces. We introduced A-Life as a sister discipline to Artificial Evolution and Evolutionary Architecture as a child of both. We mentioned that Graph Theory is the formal undercurrent in architectural representations of space. We introduced Evolutionary Architecture and discussed the relative concepts of function and form.
- which led us to the concept of a fitness function and important issues in developing models of crowd behaviour.

As we have mentioned, this latter point represents the most significant obstacle to the design of fitness functions appropriate for egress processes. We begin our investigations by reviewing literature in the field of Pedestrian dynamics and egress modelling.

Chapter 2

Pedestrian and Evacuation Dynamics I

This chapter is our first encounter with some currently employed techniques. As such, it is less focused with particular models than the review that follows in Chapter 3. Overall, we aim to characterise a general modelling style.

We discus the nature of mathematical and simulation models (see Section 2.1)¹. We introduce a conceptual framework (Multi-Agent Systems), which is related to simulation modelling and look at natural and virtual examples, including pedestrian systems (see Section 2.2). We classify and review models based on their underlying spatial representation, i.e., discrete (see Section 2.3), or continuous (see Section 2.4). We provide a chapter summary, which recaps on the reasons why we need to abandon certain approaches, whereas others are ring-fenced for further investigation (see Section 2.5).

2.1 Models

In a general sense, a model is a representation of something in the real world without the inclusion of 'peripheral' detail. For example, children play with model boats because they perhaps have some level of interest in real ones, but they are not necessarily critical of boat

¹We acknowledge the work of Noble (1998) and Bullock (1997) in this discussion.

design or implementation in any detail according to the standards of an engineer. The boat will be designed with enough detail to satisfy a child's interest—'peripheral' detail may include the exact shape of the stern and essential detail might be something like the correct colour. Marine architects have boat interests for different (or additional) reasons and have the knowledge to build models according to oceanic constraints. This filed of knowledge might be so advanced that these interests are satisfied with the use of compressed, equation-based models of real processes. In the design of real boats there might no longer be a need to experiment with real-world models because of the reality captured by virtual boats and virtual water—'peripheral' details may include the colour of the boat whereas the exact shape of the stern might be essential.

Models are, therefore, more or less abstract depending on how they are used and what they are used for. We will find it useful to distinguish between equation models and simulation models in order to help present our own modelling preferences.

2.1.1 Equation Models

In the area of mathematical and physical science abstract models have produced laws—seemingly unbreakable truths about the world. For example, interactions between bodies with mass can be described by $F = G \frac{mm'}{r^2}$. As Noble (1998) points out however, these equations mean nothing unless accompanied by a verbal decoding. In this example, we may say that "the force between a body with a mass m and another body with a mass m' is the product of a gravitational constant and their combined masses. This force diminishes according to the distance between them r, multiplied by itself". For a physicist, there is no more information here than in the equation, which affords a compact and speedy description. Over the years science has accumulated a number of laws, which are sometimes integrated—a wider range of phenomena get described in more simple terms. On this view, laws are therefore not unbreakable or necessarily true, but just the latest entries in a list of provisional descriptions, which are judged according to their predictive power.

The above concept of model seems appropriate for certain aspects of the natural world, but not others because nature "seems to be so designed that the most important things in the real world appear to be an accidental result of a lot of laws" (p12, Feynman, 1992)—not all of the world can be forced into the framework of overly simplistic descriptions. This is why there are a bewildering variety of modelling techniques available in science.

Maynard-Smith (1974) argues that this variety exist on a continuum of detail, beginning with models of simple systems (e.g., $F = G\frac{mm'}{r^2}$) and ending with detailed, real-world simulations. He reserves the word model for representations that contain minimal detail and simulation to mean models, which incorporate maximum detail (Noble, 1998). At the less detailed end of the spectrum, models are often general and may therefore be relevant to a number of fields of enquiry. For example, consider the following discrete time equation (Wilson, 2000):

$$N_{t+\Delta t} = N_t + \alpha \Delta t - \beta \Delta t \tag{2.1}$$

where at time t there is a population of size N_t . Within a time window Δt , individuals enter and leave at rates defined by α and β , respectively. When our clock has swept through its tick Δt the population level is, therefore, described by $N_{t+\Delta t}$. This model can describe a number of open immigration-emigration systems—for example, a population of birds can vary due to birth and death or people can enter or leave buildings. In this way, it is effective equation in the sense that it can be applied to many phenomena. However, it is ineffective in the sense that it will not capture detail in any given one. Therefore, although models should not contain peripheral detail, this does not mean that they should not contain detail per se. Where we require the incorporation of more detail, we slide along towards the simulation end of the continuum.

2.1.2 Simulation Models

We mentioned in Chapter 1 that we require a fitness function to capture the *process* of an evacuation and a useful model will therefore be detailed and specific rather than simple and

abstract. We agree with existing views on egress modelling: "For the evacuation model to function appropriately, it should accurately represent the process involved during the evacuation. This representation should not be based solely upon final occupant actions, but should rely upon constituent factors" (Gwynne, Galea and Owen, 1998). This echoes what Maynard-Smith means by a simulation model and Bullock (1997) provides us with an apt definition:

"A simulation is a model that unfolds over time. Rather than constructing a static representation of the process under examination, such as flow charts or equations, and relying on human interpreters to simulate the passage of time, or determine the state of the system at some arbitrary time analytically, the simulation designer captures the dynamics of the original process by specifying the dynamic mechanisms which govern how the system changes over time. The character of such a simulation's dynamics is determined experimentally, through allowing the simulation to unfold over time."

(p 24)

This definition serves our perspective well. We do not claim that models at the lower-end of the continuum are less useful per se, although it has previously been claimed that simulations are advantageous because they contain more explicit representations (Miller, 1995). Again we align ourselves with Bullock who states: "In general, unqualified claims of the superiority of one style of modelling over another are not compelling. Clarity, ease of design, ease of presentation, etc will vary from model to model to a greater or lesser extent than they vary from modelling style to modelling style" (p25, Bullock, 1997). In short, we regard the usefulness of different styles as a practical concern rather than a philosophical question. It is our aim in the following section to introduce a modelling concept, which is useful for our own practical concerns.

2.2 Multi-Agent Systems

As Feynman (1992) implies (above), when we consider a wider class of phenomena the idea of a law becomes less fruitful. If we reduce the value r in $F = G \frac{mm'}{r^2}$ to sufficiently small values, then the so-called law breaks down. Also, in law-like systems where many objects interact it can be difficult to solve equations—e.g, the three-body problem being the classic

(Stewart, 1997). Pedestrian dynamics result from systems where as well as there being many units these units are also adaptive, autonomous and interactive. It is in this sense that humans can be said to have *agency* (Boden, 1996b). Autonomous agents and MAS have recently been recognised as fields of research in their own right. Agents have the following properties:

"Problem solving systems capable of independent, autonomous action in some environment. Agents must typically take action in the face of partial information and when faced with difficulties need to be able to decide whether its tasks are still appropriate, whether the chosen means of realising them is likely to succeed, or whether a new course of action should be taken instead."

(Jennings, 1998)

Because of the nature of MAS, they are best understood through examples and we now present three different types; natural, virtual, and pedestrian.

2.2.1 Natural MAS

Ant nests are a good examples of MAS. Nest members have a certain amount of autonomy, but a limited set of skills. In particular, they have a very limited capacity in terms of information storage. Single ants are therefore relatively uninteresting, in stark contrast to the staggering level of organisation within an ant nest. The latter show the ability to hunt collectively, make adaptive decisions collectively and solve complex problems while doing so (Holldobler and Wilson, 1990). The behaviour of an individual ant does not map well to the complexity of the nest.

Adaptive behaviour must, therefore, reside in ant interactions. In the laboratory, ants have demonstrated the ability to solve higher-level problems such as the solution of shortest paths between a nests and food source (Deneubourg, Goss, Franks, Sendova-Franks and Detrain, 1990). Explanations rest on the concept of Distributed Intelligence, where the intelligence of the system far outstrips that of its constituent parts—these parts have the ability to distribute and react to chemical 'messages' in the environment. Therefore, it makes more sense to not only study the abilities of individual ants, but also the dynamics that result from complex pheromone networks. An important message from these kinds of systems is that complex behaviour does not always require a correspondingly complex unit. Translated in terms

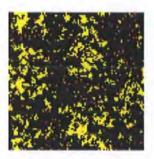






Figure 2.1: A simple example of emergent behaviour. Wood chips are collected over time into larger and larger piles due to the feedback created in the environment rather than an explicit woodchip-collecting rule. Adapted from Resnick (1994).

of a computational model, this sounds like good news for the computer scientist—complex behaviour need not necessarily demand the implementation of a correspondingly complex representation.

2.2.2 Virtual MAS

Indeed, some of the early models in the field of A-Life demonstrate this idea. For example, in natural systems, animals appear to flock for reasons related to survival (e.g., herding prey) or perhaps to minimise energy (e.g., birds slip-streaming in flight). When we observe a flock of birds invisible hands appear to orchestrate cooperative and complex movement. However, for virtual flocking to occur three simple rules suffice: 1) Move within a certain distance of neighbours, 2) Match velocity with neighbours 3) Prevent collision by limiting the effect of rule 1. These local rules, executed in agents called 'boids', produce satisfyingly complex flocking behaviour (Reynolds, 1987).

Another example is Resnick's (1994) virtual termite wood-piling model. Here, termites live on a two dimensional lattice and follow two rules: 1) If encounter wood-chip, then pick-up wood-chip; 2) If carrying wood-chip and encounter wood-chip, then drop wood-chip. With random wood-chip initialisation and random termite movement scattered chips are accumulated into large piles. We present example epochs of termite behaviour (see Figure 2.1).

These kinds of patterns are often referred to as emergent because they are not represented

explicitly in model rules. In the recent past, such algorithms have produced much excitement, but more recently, experts have pointed out reasons for the apparent lack of success of Distributed Intelligence and in particular the lack of application. As Bonabeau, Dorigo and Theraulaz (1999) state:

"It is fair to say that very few applications of swarm intelligence have been developed. Swarm-intelligent systems are hard to "program", because the paths to problem solving are not pre-defined, but emergent in these systems and result from interactions among individuals and between individuals and their environment as much as from the behaviour of the individuals themselves. Therefore, using a swarm-intelligent system to solve a problem requires a thorough knowledge, not only of what individual behaviours must be implemented but also of what interactions are needed to produce such or such global behaviour."

(p 7)

The clear message is that we need to represent the behaviours of single entities well to successfully exploit emergence. Although it might be possible to implement simple rules to produce system-level behaviour, the difficulty is in uncovering the appropriate rules.

2.2.3 Pedestrian MAS

In a similar way to ants in nests and birds in flocks, pedestrians can be thought of as the units or agents in a pedestrian crowd. Most of the time pedestrians co-operate and crowds show unproblematic, well-structured, predictable behaviour. However, there are situations where actions produce unintended consequences because of long chains of interdependence between pedestrian units. For example, it has been known for a long time that decisions made at local points in space and time can have effects that result in blockages, areas of very densely packed pedestrians, which may result in injury or death (Canter, 1980). In a similar way to the natural and virtual systems above, local actions generate global behaviour.

Individuals within the system act interactively with respect to local circumstances and, as Bonabeau et al. (1999) suggest, it is the interaction between pedestrians and between pedestrians and their environment, which presents the main challenge from a modelling point of view. In an extensive review of real human behaviour in egress situations a number of

attributes have been identified (Gwynne et al., 1998). We give a general summary of some of them here, encapsulated by the three broad classes of interactions given in chapter 1.

- Pedestrian-pedestrian interaction: This describes the culmination of interactions between individuals as they escape. Local and global decisions are important—rather than blindly follow a given direction of egress, individuals seem to act adaptively and update decisions according to changing circumstances. Pedestrians also co-operate and demonstrate group behaviour. One widely cited example is the appearance of lane formations, which optimise crowd-flow (Helbing, Farkas, Molnr and Vicsek, 2002). However, pedestrians also show conflicting behaviour, and where flow becomes viscous blockages result in injury.
- Pedestrian-environment interaction: This refers to the direct physical impact of an enclosure upon occupants. Design decisions not only include building structure, but crucial elements are the position of exit signs and the travel distance from a given location in the building to an escape exit. Pedestrian reactions to the environment impinge on how pedestrians interact with each other. Also, pedestrians show a range of navigation ability in dynamic environments.
- Adaptive escape behaviour: Observations indicate that the huge majority of individuals
 react in an adaptive, decision-making manner, rather than by default to some herding
 instinct (a common assumption often made in the past). It is argued that theories based
 on a herding instinct are rather the default reactions of some scientists to behaviours
 that have not been well understood.

There are a wide variety of modelling techniques used, which capture some of these properties to varying degrees. Some approaches can be classified as MAS and others maybe not. We begin with approaches that utilise a discrete representation of space.

2.3 Discrete Models

2.3.1 Discrete Space

With discrete space there cannot be an infinite number of spatial fragments as opposed to continuous space where any given fragment is itself infinitely divisible. Lets consider Zeno's paradox (Huggert, 1999):

"Imagine our runner—her name is Atlanta—bursting over a starting line and hurtling towards the finish at top speed. First, she must travel half the distance, namely 50m, leaving 50m to go. Then half the remaining distance, 50m÷2=25m and so on... There is no end to the number of times we can halve the remaining distance; hence Atlanta must cover an infinite number of finite distances to get to the finish".

(p 39)

With discrete space we choose an a-priori spatial unit and have to stick to it. Then any operation that divides this unit is deemed invalid. Therefore Zeno's paradox cannot occur in discrete space—Atlanta's movement cannot divide a predefined quantity. If we use this quantity as the lowest limit of movement, then Atlanta will pass the finish line at some point in time, rather than never. In the same way, if a pedestrian will move in a given period of time, then it will move at least one unit or an integer multiple of this unit.

There is no rule stating that the unit chosen should be a certain size, but once this is chosen it it is chosen. The following set of discrete models depend on different choices in this respect and these choices introduce specific properties to the model.

2.3.2 Coarse Network Representations

2.3.2.1 Graphs (briefly)

The network approach uses the language of Graph Theory where vertices and edges are the basic units of representation. A vertex represents some abstract object, e.g., a person or an area of land. An edge is an abstract relationship or interaction between one object and another. A Graph represents the interaction network; it consists of the entire set of objects

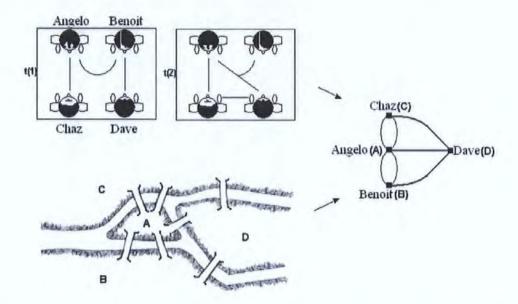


Figure 2.2: Different types of networks. Abstract networks can represent many different objects or events. A single graph can represent social relations as the temporal accumulation of conversations between Angelo, Benoit, Chaz and Dave (top) or spatial relations between areas of land connected by bridges (bottom). These different systems map to the same graph.

within a system and the interactions between them. Graphs are highly abstract and can capture the properties of many different systems, which may map to the same graph. For example, a dynamic network of people, linked via a history of conversations share the same graph with a river and its bridges, which link previously separated areas of land (see Figure 2.2).

We will need to discuss graph theory more in the next chapter, but for now, while discussing all discrete models, it is important to keep in mind that all share this common spatial abstraction.

2.3.2.2 Static, Coarse Networks

We present a configuration with seven rooms of different sizes connected via corridors of varying length (see Figure 2.3). There are explicit network representations for the corridors, but no explicit representation of pedestrians. This as a coarse description of space because the actual space connecting two objects is not regarded as important enough to receive vertex representations. The assumption here, therefore, is that nodes define some functionally

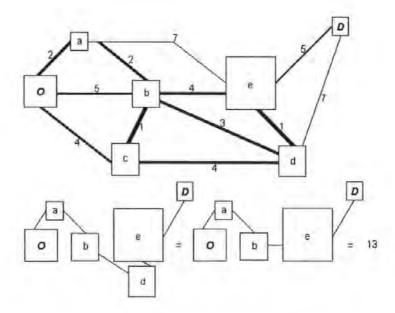


Figure 2.3: Enclosed configurations as coarse networks. Rooms (nodes) have connecting corridors (links). Each corridor has an associated cost (eg distance) and attraction value (eg inverse of the distance, line thickness). With this information, a shortest path can be defined from the origin (O) to the destination (D). There are two optimal routes: 1) D-e-d-b-O and 2) D-e-b-a-O, each measuring an minimal cost of 13 units.

important space.

In the example given, we have shown how distance can be represented on a link. Fahy (2001) uses this method and designs different cost functions, which model differently affected areas. For example, the effects of contra-flows. Calculations of link values are based on information available from observations of a given space. In this way, where people traverse a configuration, it is reasonable to assume that they follow some kind of shortest path or route of least effort. For Fahy (2001), the problem is to traverse space and minimise cost. This can be achieved by searching the attractiveness of possible local decisions and combining minimal-cost decisions through n steps. Fahy applies Hillier and Lieberman's (1990) shortest path algorithm to model pedestrian movement. We present this algorithm (see Table2.1).

We see two main difficulties with this approach. Firstly, it is problematic to define static links because dynamic and potentially important features of a pedestrian scenario are ignored. This problem also cascades—if dynamic features are important, then the 'shortest' path derived from a priori link values may not turn out to be the shortest path at all. The question

Table 2.1: Shortest Path Algorithm (Hillier and Lieberman, 1990)

- 1. Objective of n^{th} iteration: Find n^{th} nearest node to the origin. Iterate for 1,2, ...n, until n^{th} nearest node is the destination.)
- 2. Input for n^{th} iteration: (n-1) nearest nodes to the origin (solved for at previous iterations), including their shortest path and distance from the origin. (These nodes, plus the origin, will be called solved nodes; the other are unsolved nodes.)
- 3. Candidates for n^{th} nearest node: Each solved node that is directly connected by a link to one or more unsolved nodes provides one candidate—the unsolved node with the shortest connecting link (ties provide additional candidates)
- 4. Calculation of n^{th} nearest node: For each such solved node and its candidate, add the distance between them and the distance of the shortest path from the origin to this solved node. The candidate with the smallest such total distance is the n^{th} nearest node (ties provide additional solved nodes), and its shortest path is one generating this distance.

becomes what is shortest; time or distance? It has been stated that "pedestrians normally follow the fastest route to their next destination, but not the shortest one" (p 26, Helbing et al., 2002). Secondly, in some applications important vertices may not be apparent a priori. It might not be possible to identify separable spatial regions that warrant the use of single vertices. The danger is that a coarse structure has to be imposed on spaces, which are then forced into subjective categories. In one application, as Fahy states: "because of the openness of the stores floor-plan, the author divided the large areas into smaller areas to be defined by nodes" (Fahy, 2001).

2.3.2.3 Dynamic, Coarse Networks

Coarse networks have been extended in order to overcome some of the disadvantages of using static representations. The conversation network (see Figure 2.2) is an example of a dynamic network where, at each time step, information is updated. In the case of egress analysis, these kinds of networks represent the state of the art for mathematical modelling (Hamacher and Tjandra, 2002). Initial information (configuration layout, number of evacuees on nodes

etc...) is processed and updated, providing input for the next iteration on the graph. The graph at the next iteration therefore has the same topology, but different, updated vertices and edge values. The resulting network is, therefore, a temporal expansion of the former kind and attempt to capture the dynamic features of escape behaviour.

However, important detail is minimally represented in these models, if at all. Measures such as distances and costs need to be known *a-priori*, based on manual measurements of a given configuration. This is not a disadvantage for a specific application, because time can be spent making these measurements, but when we want to execute a model in any given configuration, this information may not be known. This is particularly true if the configuration is to be generated automatically by some kind of algorithm ².

This thesis is concerned with using a general egress model, which is capable of representing the detailed pedestrian-pedestrian and pedestrian-environment interactions. This detail is not adequately represented in either coarse-static or coarse-dynamic networks. If we are to use a model as a fitness function within an evolutionary context, then we need explicit representation of these fine-grained details.

It is only relatively recently that available computer power has enabled the implementation of such detail. The next section introduces agent-based approaches, which operate on finer-grained networks.

2.3.3 Fine Network Representations

Here we briefly introduce three examples, which utilise a fine-grained network. Not all of them are directly related to egress analysis, but represent general techniques relevant for modelling movement in arbitrarily complex, labyrinthine space.

EXODUS: This is an example of a commercially available software package with a 3-D graphical user-interface, but some of the underlying methods are familiar. The underlying representation of space is two-dimensional. Unlike the approaches outlined

²We will return to the automatic generation of space later on in the thesis (Chapter 7).

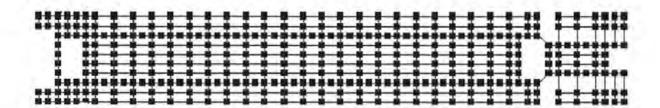


Figure 2.4: Enclosed configurations as fine-grained networks. Detail of spatial representations in EXODUS far exceed that of the earlier coarse grained networks. This example is a network to represent an aeroplane aisle, constructed from a lattice grid.

above, EXODUS looks at pedestrian-fire interactions as well as pedestrian-pedestrian and pedestrian-configuration interactions. A number of sub-models are described. These include Occupant, Movement, Behaviour, Toxicity, Geometry and Hazard sub-models (Owen, Galea, Lawrence and Filippidis, 1998). In each sub-model there may be a number of techniques used to achieve desired behaviour. A 2-lattice is used to map the details of a configuration. Unlike the network-based models above, where each vertex might represent a room, there may be hundreds of vertices, perhaps representing some sub-space. A room represented by thousands of vertices is possible. We present an example from an airline configuration (see Figure 2.4).

PEDFLOW: This is designed with a similar level of modelling in mind as Exodus, but is applied more to the analysis of street layout rather than buildings. PEDFLOW has a number of features intended to model pedestrians in urban situations at a microscopic level. Pedestrians occupy a vertex and move through an urban surface at a given speed. There are algorithms that take into account the range of walking speeds available and pedestrians can stop instantaneously. A journey for an individual is defined by a number of sub-journeys. When compared with evacuation scenarios these kinds of behaviours are not as relevant as EXODUS—behaviour defined in PEDFLOW is a much less urgent kind than the egress type behaviour. However, it is worth including here because different scenarios can be created with relative ease once certain representations of the environment are coded. For example, in PEDFLOW people tend to form following behaviour and do not collide with other pedestrians. Speeds can vary based on the

'urgency' of the journey and pedestrians also avoid obstacles. These behaviours are very similar to the ones defined in *EXODUS*. In addition, *PEDFLOW* incorporates models of traffic flow into the urban environment so that the interaction of 'pedestrian' and 'traffic' models can be analysed (Jones, 1996; Kerridge, 2001).

STREETS: This is developed in *The Centre for Advanced Spatial Analysis* at University College London. It also has the aim of analysing pedestrian behaviour in the built urban environment. Again, although not directly relevant to egress behaviour, the model uses common techniques and a fine-grained representation of space. A particular application is the Notting Hill Carnival (Batty, DeSyllas and Duxbury, 2002).

Like *PEDFLOW*, pedestrians are given a journey and can stop at various positions to complete 'sub-tasks' on their way. Pedestrians follow simple rules to produce behaviours—avoidance of other agents and navigation around representations of buildings. Pedestrians follow street layout on their way to a globally defined destination. There are no generally defined, computational behaviours in such a model. Behaviour will be defined based on observations of real crowds at the application event because the software is developed in the context of real applications. One advantage of this is that the model can be evaluated against data collected from real events. This is important for model calibration (Gwynne et al., 1998). However, the downside is that the algorithms are less generally applicable to other scenarios.

Publications point towards the relevance of Cellular Automata approaches (Torrens, 2000) used to model populations moving through streets. A map of street-layout is developed and the surface are created by the use of a stigmergy-based algorithm³. Successful pedestrians deposit attraction forces that decay over time (Batty et al., 2002). The build up of this dynamic quantity makes some areas more attractive than others and the model then behaves in a way that simulates crowding behaviour as attractors are

³Breifly, *stigmergy* refers to interactive feedback between an agent and its environment. Pheromone interaction in ants (see above) is one example.

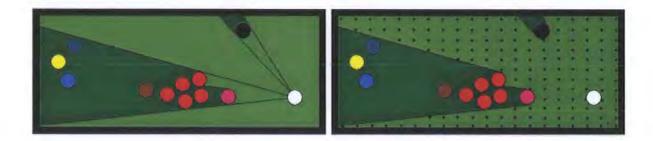


Figure 2.5: Isovist and isovist field. An isovist is a viewable area of space from a given perspective in space. In the example we take the perspective of the white circle. We can create an *isovist* by drawing lines that encompass all directly accessible points in the space, the viewable area. Dark green represents non-viewable areas of a space and light green the area directly accessible (left image). Only one of the red circles, the pink and the black circles are accessible from the white one. We create any isovist field by superimposing values on the picture, represented as small black spots. This is an isovist *field*. Non-viewable areas are not part of the field (right image).

reinforced over time. Rather than creating a specific algorithm, Batty et al. (2002) relies on the development of stigmergy-feedback and self-organisation from agent deposits.

We will analyse the approach by Batty later on in the thesis, when we come to discuss navigation techniques in complex space in more detail. We find that the representation of surfaces has some problems and the rules that pedestrians follow will suffer from large amounts of computational redundancy in the early stages of surface formation.

2.3.4 Space Syntax

Space Syntax theory (Hillier, 1996) informs a particular agent-based model of pedestrian movement (Penn and Turner, 2002). The aim is to model a natural system of movement based on three observations of *browsing* behaviour; 1) pedestrians do not backtrack, 2) pedestrians continue in the same direction of motion 3) pedestrians divert only when a new view opens up to them (Peponis, Zimring and Choi, 1990). Agent navigation is based on the idea of an isovist-based sensor (Penn and Turner, 2002). We illustrate the concepts of isovist and isovist field (see Figure 2.5).

Penn and Turner use clustering coefficients from the field of Social Networks to construct the scalar field values. Later in this thesis, we see in more detail how Penn and Turner (2002) apply these statistics to agent movement on isovist fields. Briefly, agents wander around a configuration by processing information inherent in the topology of the network. It is argued that the use of graph clustering coefficients allow more simple rules of navigation in complex space (Penn and Turner, 2002). The use of clustering information is a foundation for the construction of rule-sets and because this information measures the topology of the space it is plausible that it be manipulated. However, it is an abstraction from these techniques to say that the navigational rules are simplified. The clustering algorithms used have rules themselves and to say that using these techniques simplifies navigation is to take a position relative to the interface of an agent, rather than achieve computational simplicity per se.

In contrast to these implicit rules, the next section introduces an approach based on the execution of explicit rule-sets with local inputs.

2.3.5 Cellular Automata

CA's are usually specified as multiprocessor arrays. They are *cellular* because they exist in a discrete space (usually a lattice) and *automata* because they depend on local input with neighbouring cells, so that rather than following fixed global instructions, each cell reacts to its local environment. The transition rules are usually the same for each cell; given the same input two different cells produce the same output. We present a common CA scheme (see Figure 2.6). In this scheme we can see only two time steps for t = 0 and t = 1 and the T-shape transition rule determines the change of state between these time steps.

We can redraw the current state of the system at each time-step so that time moves down the y-axis. Each separate state of each cell is placed along the x-axis. Although the operations at each cell are simple, over time aggregate output can produce quite complex dynamics. Given small perturbations to transition rules, large changes can occur in the qualitative behaviour of a population over time. We demonstrate this with three CA examples, seeded with identical values at t = 0 (see Figure 2.7).

These kinds of examples have been advocated as potentially applicable for many different

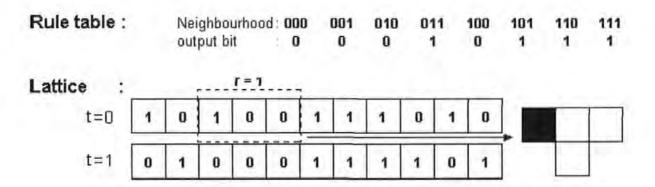


Figure 2.6: CA scheme. Here, a single input is provided by two immediate neighbours and the current vertex itself. This makes three bits in total and, therefore, the rule table consists of $2^3 = 8$ inputs corresponding to eight outputs. A single transition rule can be represented as a T-shape. Adapted from (Mitchell and Crutchfield, 1996).

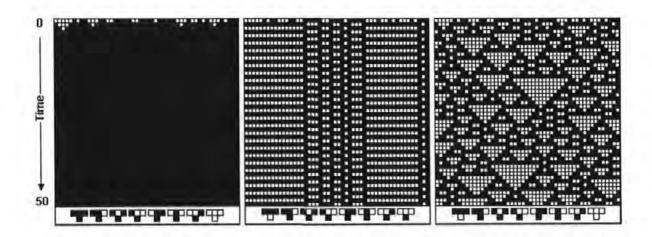


Figure 2.7: CA dynamics. In accordance with the transition rules presented underneath each space-time plot, the behaviour unfolds over time into different patterns. In these examples, we have only changed the right and left hand transition rules, but three very different patterns emerge; fixed-point (left), oscillatory (centre), and chaotic (right).

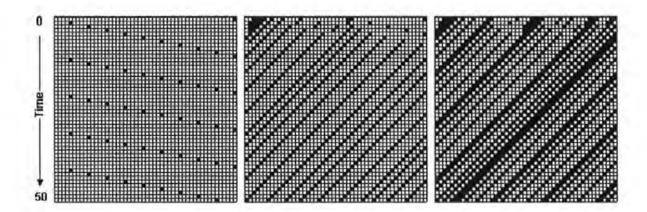


Figure 2.8: Cellular Automata model of simple movement. In the left-hand picture a value one is initialised to the top right hand corner. Over time we see how the rules model the movement of a cell value through space and time. It takes four iterations for blockages to be resolved with ten 'on' cells in the initial condition (centre). With double the amount of on cells, blockages do not disappear and thick lines represent blockages, which travel back in space in a South-West direction.

types of phenomena (Wolfram, 2002). However, if Cellular Automata are to be of any use for modelling objects that move through space as well as time, then we need to change the nature of the neighbourhoods and rule transitions. In the above examples, neighbourhoods are limited to a radius r = 1. It is possible to have neighbourhoods that have a greater radius and also bias the neighbourhood in a single direction. We can also update neighbourhoods over time. We show how very simple movement may modelled in a space-time diagram (see Figure 2.8). In Chapter 3, we will return to the kinds of rules capable of modelling motion, and which form the basis of vehicle CA's (Nagel and Rasmussen, 1994).

Inspired by these kinds of algorithms, some have attempted to model pattern formations in populations of moving pedestrians with the use of two-dimensional CA (Blue and Adler, 2000a). In particular, lane formation is observed. Lane formations are thought to be very important in the crowd modelling literature (Blue and Adler, 2000a; Helbing, Farkas and Vicsek, 2000; Schreckenberg and Sharma, 2002). The rule sets in the Blue and Adler (2000a) model are more complex compared to typical CA, but they are still very simple compared with the potentially complex decision-processes or interactions that might occur in real pedestrian populations.

It might be argued that the simplicity of these programs might urge us to classify them at the lower end of the modelling continuum. However, the important properties of cellular automata are that they are *developmental* and supposedly elicit *self-organisation*. For example, it is shown that from a random, initial configuration, behaviour will not organise until around one hundred time-steps into the simulation, which illustrates the *emergent* nature of this approach (Blue and Adler, 2000a).

In the next chapter we will provide a more technical review of some of the discrete models, which represent individuals, crowds and configurations. Firstly, we will review models based on a continuous concept of space.

2.4 Continuous Models

2.4.1 Continuous Space

With Zeno's paradox, we know there is a problem in the reasoning. It was used to illustrate the concept of discrete space as consisting of an indivisible unit. What is wrong with the paradox? As Feynman states: "...a finite amount of time can be divided into a infinite number of pieces just as the length of a line can be divided into an infinite number of pieces by dividing it by two" (Feynman, 1963). In other words Zeno ignores the time dimension.

Infinitesimal distance must be coupled with infinitesimal time, so that distance coverage occurs in the spatial and temporal domain. We can then find the ratio of this coupling when a change in time approaches zero. This ratio we call velocity v:

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} \tag{2.2}$$

This infinite fragmentation of distance according to time is common to some of the continuous approaches below, which *calculate* a quantity of spatial movement from local forces without having to impose it beforehand.

2.4.2 Fluid Dynamics

It is clear to see how fluid dynamic-based approaches have been used in the study of crowding people. Fluid is a tempting analogy, because it moves through space and avoids obstacles. Applications of this approach have been applied to various problems (Smith and Dick 1993), but are often limited to situations where the density of the crowd is relatively high (Hughes, 2002; Hughes, 2003). The assumption is that the crowd is like a fluid at high density, because the choices that people make are more constrained by the choices of others. As crowd density increases each person has reduced freedom. Reactions become less like a collection of individuals and more like a single, fully connected mass, just like separable particles made of hydrogen and oxygen atoms attract to form the collective we call 'water'.

These kinds of assumptions underpin methods in the fluid dynamics approach. Applications of this approach are therefore limited to situations with large amounts of people (Hughes, 2002). Hughes' equations can model a homogenous population, where speed is assumed to be a function of total density. This is a particularly unrealistic assumption, but the maths gets very complex and tractable analysis requires populations to be handled *en mass* (Hughes, 2002).

The fluid analogy has been criticised and more granular approaches advocated (Still, 2000). Still argues that the underlying assumption, (flow volume = average speed × average density), is simply too unrealistic when compared to complexities in real crowds. The assumptions underneath the fluid analogy are questionable and the failure of these models to produce sensible behaviours at low densities is a severe drawback.

In the current thesis it will be desirable to have a model represent populations of varying densities, both *between* different configurations, but also *within* a configuration. It is well observed that in egress situations some parts of the population can move freely, whereas others form blockages. A real population is heterogeneous, unlike water, and rules of movement need to incorporate density fluctuations. Ideally, a model should remain robust to the density parameter.

2.4.3 Steering Dynamics

Apart from flocking behaviour in 'boids', Reynolds has developed steering behaviours for autonomous characters in an environment (Reynolds, 1999). The motivation for Reynolds is to create characters in games that are autonomous and react to varying environmental conditions. The aim is to produce more natural behaviour and his work has Strong Artificial Life overtones; "real agents in a virtual world" (Reynolds, 1999). Steve Grand shares similar views and warns against attempts to interpret outward behavioural rules by advocating the use of Newtonian mechanics: "...they should be trying to model Newton's basic equations of motion and the curves would turn up all by themselves, complete with swerving, skidding and all the other things that help convince people" (p73, Grand, 2000).

Reynolds attempts to model movement by designing mechanical devices that work at a local level. He defines three levels to the autonomous characters; locomotion, steering and action selection. The locomotion layer is defined as a simple vehicle in the sense meant by (Braitenberg, 1984) and is essentially a point mass with various scalar and vector quantities to describe mass, position and orientation. Steering is simulated by a velocity aligned local space, where incremental adjustments are made on the previous time step to a current velocity vector and a desired velocity vector. We can see that an agent floats in two dimensions (see Figure 2.9), a very different way of modelling motion compared with the lattice-based approaches in Section 2.3.

2.4.4 Social Dynamics

One attempt has been made to apply Newtonian mechanics to moving populations of escaping people, whose vectors represent various desires and constraints in the context of egress behaviour (Helbing et al., 2000; Helbing, Farkas and Vicsek, 2001; Helbing et al., 2002).

To model how a crowd of people move through space over time a mathematical theory using differential calculus must be able to 1) describe the changes in positions of individuals and 2) have some method of recording these changes so that behaviour unfolds dynamically, moment

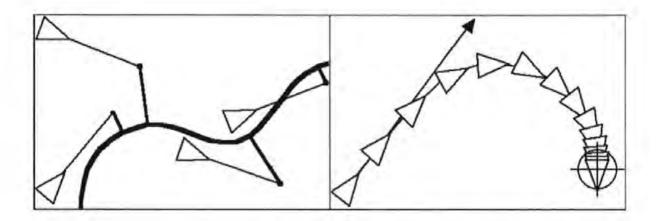


Figure 2.9: Simple, continuous movement. Left: path following. Right: arrival. Adapted from Reynolds (1999).

to moment. Helbing builds on the general equation for motion through space and uses a specific acceleration equation, which describes changes in velocity for a given pedestrian:

$$m_i \frac{d\mathbf{v}_i}{dt} = m_i \frac{v_i^0(t)\mathbf{e}_i^0(t) - \mathbf{v}_i(t)}{\tau_i} + \sum_{j(\neq i)} \mathbf{f}_{ij}(\mathbf{x}_i(t), \mathbf{x}_j(t)) + \sum_W \mathbf{f}_{iW}(\mathbf{x}_i(t))$$
 (2.3)

where a pedestrian with mass m_i moves towards a desired direction \mathbf{e}_i^0 at a desired speed v_i^0 so that velocity \mathbf{v}_i is adapted within a cetain time τ_i . The second right-hand term represents local forces between the current pedestrian and surrounding ones, and the third right-hand term represents forces between the current pedestrian and walls (obstacles). Like many of the discrete models, assumptions are based on the general idea that in an escape scenario pedestrian are motivated to move towards certain destinations. As Helbing et al. (2001) argue, this kind of approach is better than the fluid dynamic approach, because behaviour does not depend critically on density and volume parameters.

Various behaviours are possible with these latter, locally-aligned, continuous velocity-based models. For example, the effects of queuing, blockages at exit points, lane formation, avoidance of approaching pedestrians, fire and panic. We present example behaviours from Helbing's model (see Figure 2.10).

In this thesis, we will need to represent discontinuous concentrations of pedestrians in space and we will need to develop a model, which can incorporate more complex spaces than

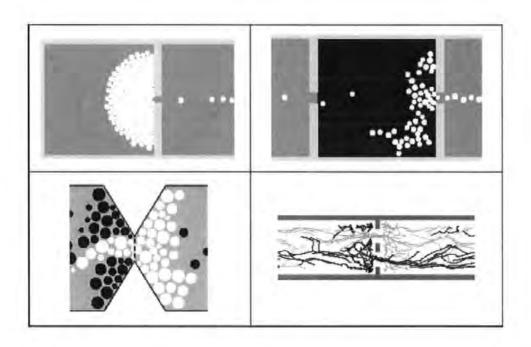


Figure 2.10: Helbing's social force model. Top Left: Panicking individuals are blocked at an exit, or are 'frozen by heating'. Top Right: Crowd-following in a smoky room with panicking individuals; movement is less directed, but paradoxically throughput is enhanced. Bottom Left: bi-directional crowds oscillate at a junction point. Bottom Right: pedestrians demonstrate lane formations behaviour.

those used by Helbing et al. (2002). Although we agree with Helbing et al. (2002) who advocate the use of more local representations (Reynolds, 1999; Helbing et al., 2000; Helbing et al., 2001; Helbing et al., 2002), for the concerns of this thesis, continuous models have not been applied to spaces appropriate to our aims.

The models rely on a desired velocity vector being present and for any complex trajectory this will need to be constructed from collections of way points, which can be seen to represent discrete samples of space. Therefore the problem of manoeuvering a body through a complex field of values can be seen as a kind of discrete dynamical process similar to the approaches presented in Section 2.3. So, although the Helbing et al. (2002) model seem to produce interesting dynamics in simple spaces, we will need to concentrate on a more complex concept of space. For this reason it will be the models with discrete dynamics, which will provide a focus for the rest of the thesis.

2.5 Summary

In this chapter we:

- identified with the idea of a modelling continuum.
 - Models are domain specific and serve different purposes. We found the distinction between simple models and simulation models useful.
- introduced pedestrian crowds as Multi-Agent Systems (MAS).
 - This concept can be thought of as key in the design of simulation models; it can
 encompass the processes of egress behaviour at an appropriate level of abstraction.
- · critically reviewed a number of discrete approaches
 - Coarse network representations do not provide a detailed enough representation of pedestrian crowds, which in light of the current thesis is a severe limitation.
 - Fine network representations are more appropriate methods in the light of the MAS concept. These finer-grained approaches use the concept of an attractor field in order to model long-range movement.
 - Cellular Automata represent local pedestrian decisions and interactions with simple rules. Such rules lead to important emergent properties found in real crowds, such as lane formations.
- · reviewed continuous approaches
 - Although this approach looks like a promising technique, in such models the notion
 of a configuration is rather underdeveloped and inappropriate in the broader context
 of this thesis.

Having narrowed down the potential use of wide-ranging techniques, we now begin to concentrate on 2 kinds of discrete model. We focus on techniques—attractor fields and Cellular Automata—which model *long-range* movement and *local dynamics*, respectively.

Chapter 3

Pedestrian and Evacuation Dynamics

II

Firstly, we outline some technical detail. Because graphs underpin discrete models of pedestrian and evacuation dynamics, we familiarise ourselves with Graph definitions and graph walks (see Section 3.1). We look at two relevant applications of Graph Theory and the statistical techniques used to capture spatial relationships (see Section 3.2).

Secondly, having provided a technical background, we then critique discrete models of pedestrian and evacuation dynamics in the context of thesis aims (see Section 3.3). This critique will lead to a clear view of which methods need more fine-grained investigations. The summary at the end of the chapter clarifies our arguments (see Section 3.3).

3.1 Graph Theory

3.1.1 Graphs

In order that we can properly review discrete models of pedestrian movement, we need to know what their underlying space is made of. The following definitions are important for the rest of the thesis and, in accordance with the following definitions, we present examples (see Figure 3.1).1

Definition 1 A graph G contains a set of vertices V(G) together with a set of vertex pairs called edges E(G). The number of elements |V(G)| and |E(G)| are known as the order and size of G, respectively.

Definition 2 The global structure of G is important—a graph is *connected* if any vertex is reachable from any other, otherwise it is *disconnected*. Connected graphs are therefore referred to as *single-component* whereas disconnected graphs are described as *multi-component*.

Definition 3 A simple graph is an *undirected* graph where edges provide no specific representation for direction. For example, if we consider networks like the examples in Chapter 2 (see Figure 2.2), then undirected means that two people talk to *each other*, or a bridge can be crossed from *either* side. In this way, the objects are connected in a bi-directional relationship.

Definition 4 The number of edges shared by a given vertex v is the degree k_v .

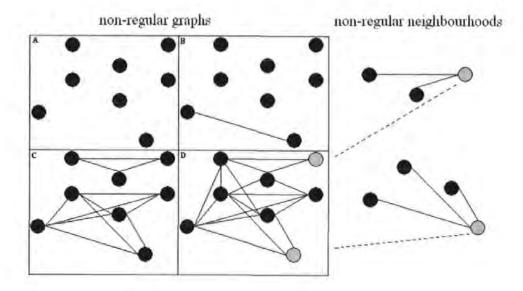
Definition 5 The degree of v is related to its neighbourhood Γ_v , which is a set of vertices not containing v, but sharing an edge with it.

Definition 6 We refer to a *d*-lattice as a case where any vertex v is joined to lattice neighbours throughout the entire graph of dimension d. In models of pedestrian and evacuation dynamics, the d-lattice representation is common, in particular where d = 2. However, d = 1 is also an important kind of lattice².

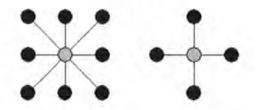
Definition 7 A d-lattice has boundaries, which can either be periodic or non-periodic. 'Periodic' refers to a lattice, which is said to be 'wrapped-around', where the pattern of connectivity applies to the entire graph. A 'non-periodic' lattice is where vertices have heterogeneous neighbourhood structure. For example, boundary vertices may connect with fewer neighbours than is typical of the rest of the lattice and thus periodic structures are broken.

¹These definitions are based on Watts (1999) and Wilson (1985).

²This will become apparent when we consider in more detail the relationship between the Vehicle Cellular Automata (Nagel and Rasmussen, 1994) and the pedestrian Cellular Automata (Blue and Adler, 2000a). See Chapter 4.







d-lattices

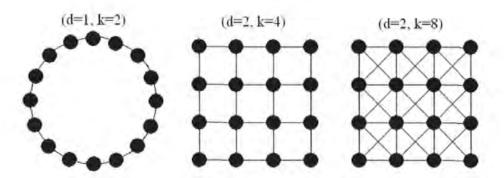


Figure 3.1: Elements and structures of graphs. Top: A, B, C and D are 4 different graphs with the orders and sizes being |V(A)| = 8, |E(A)| = 0, |V(B)| = 8, |E(B)| = 1, |V(C)| = 8, |E(C)| = 12, |V(D)| = 8, |E(D)| = 17. C is two-component and D is single-component. From graph D we show non-regular neighbourhoods from two sampled vertices. Middle: two neighbourhoods typically used in pedestrian and evacuation models. The one on the left is a Moore neighbourhood (k = 8) and the one on the right is known as the von Neumann neighbourhood (k = 4). Bottom: three examples of d-lattices are shown for varying values of d and k. See text for definitions.

3.1.2 Vector Walks on Simple Graphs

In the literature we have been assuming the directional movement of pedestrians, but, as defined above, *simple graphs* are themselves unconstrained in terms of direction and cannot represent movement.

This is appreciated more fully if we consider the associated vector walks of an simple 2-lattice graph. Consider a non-periodic 2-lattice with vertices v_{ij} , where $k_{ij} = \{8, 5, 3\}$ depending on location (see Figure 3.2). This graph has order |V(G)| = 16 and size |E(G)| = 42. If each edge represents two vectors (of opposite directions), then we have $|E(G)| \times 2 = 84$ vectors, which have their origins in a particular vertex in the graph. We draw up a vector table, which contains a row for each vertex v_{ij} and an entry for the number of edges k_{ij} . Moving from the top left vertex to the bottom right vertex, reading the vector table as we would a book, produces a vector walk. After completing this walk we notice that the directional information in the graph results in *nothing*; the vectors span a reletively complex walk and return to the original position (see Figure 3.2).

A simple graph therefore contains no directional information. Models of pedestrian and evacuation dynamics essentially represent ways of cancelling the appropriate vectors from a graph and breaking the edge-defined symmetry in the 2-lattice.

3.1.3 Vector Walks on Constrained Graphs

One method for providing directional information is to store information on the vertices in the form of scalar values. A collection of such scalars defines an attractor field, which can be used to constrain movement. By placing appropriate values on the surrounding neighbourhood vertices, we cancel certain vectors from a simple 2-lattice graph, resulting in a directional move from the origin cell (see Figure 3.3, Top). We can see how canceled vectors constrain movement in the immediate neighbourhood to a single step, represented by a vector. We also present a s-shaped walk where a number of such cancelations in a wider neighbourhood have already been assumed so that the sum of the non-canceled vectors defines a longer-range walk

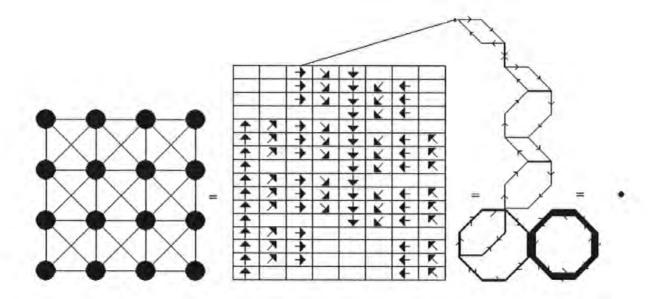


Figure 3.2: Vector walk on a 2-lattice. The graph is a 2-lattice (k = 8) with non periodic boundary conditions. Having walked on all possible moves from each vertex v_{ij} we get a vector walk containing 84 vectors, which returns to the top left-hand origin vertex v_o .

(see Figure 3.3, Bottom).

In this chapter we have so far presented some definitions of graphs and considered the problem of modelling movement as a that of canceling vectors from a simple graph. Before we review in more detail models of pedestrian navigation, we will consider two applications of graphs. This will provide some other important definitions, which are concerned less with the local properties of graphs, such as neighbourhoods, but which use such local properties to derive statistical definitions of a graphs global structure.

3.2 Graph Theory: Applications

3.2.1 Space Syntax

Space Syntax is a social theory of configurational architecture and an attempt to analyse the general properties of *locality* in a space and, in particular, which parts of the space are the most social in nature. Hillier (1996) defines a *j-graph*—a graph, which represents space from a given perspective. All j-graphs for a given space will have the same topology, but arranging

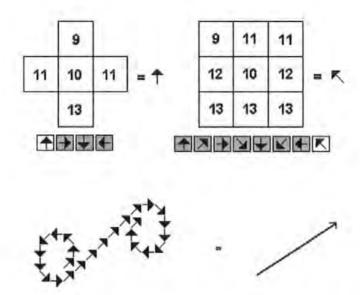


Figure 3.3: Constrained graphs and movement. Top: Constrained von Neumann and Moore neighbours. The neighbourhood represents the connection pattern between nodes, but vertex information defines the allowable moves from a given cell. We show valid (white) and invalid (grey) vectors for the two neighbourhood examples. Movement depends on a minimising option. Bottom: we show a simple s-shaped walk—there can be any number of vector combinations and walks can be more or less complex, but all connect an origin vertex v_o to a destination vertex v_d .

the graph in perspective shows how certain areas of space have specific relationships with other areas in a configuration. With this representation we can see that certain vertices in a graph are in some sense closer than others to the rest (see Figure 3.4). This idea underpins the concept of depth, which is a measure of distance.

Definition 8 Given a vertex origin v_o and destination v_d we calculate the minimum distance $d(v_o, v_d)$ by traversing an edge subset P(E). This returns the shortest path between v_o and v_d , or the depth of v_o from v_d , and vice versa. The total vertex depth tvd of v_o is then the mean shortest path to all other vertices, i.e., we calculate $d(v_o, v_j) \forall j \in V$ and $tvd = \bar{d}$.

This is a measure of the general accessability of v_i from v_j . A simple one-dimensional space serves to illustrate this. We choose a 1-lattice space with non-periodic boundaries, which in the context of a building might represent a corridor. In order to reach some unknown destination in this space, it is best to take a position of minimal depth, represented, in our example, by the centre of the space (see Figure 3.5).

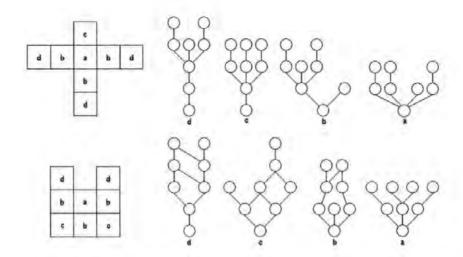


Figure 3.4: Spatial configurations and locality. The j-graphs represent different local features even though the topology of the entire graph is the same. Being in a different area in space offers a different point of access to the rest of the space where some vertices are embedded more deeply than others. Adapted from Hillier (1996).

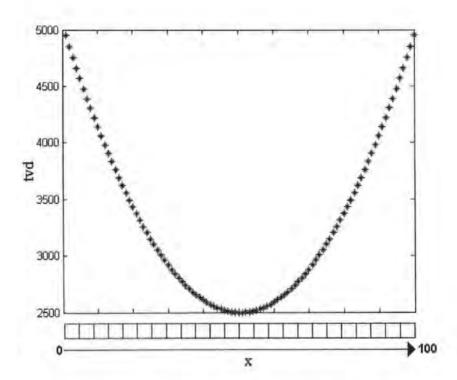


Figure 3.5: Depth values for a one-dimensional space. Each site on a 'corridor' space has a depth. Depths are plotted for each site x in the corresponding space.

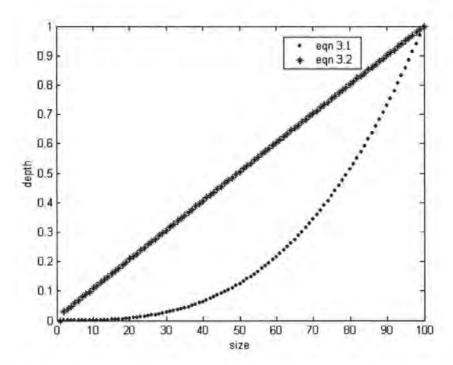


Figure 3.6: Configuration depth against space size. While adding single vertices to a onedimensional space we increase space linearly. We therefore have to normalise Hillier (1996) suggested statistic for *total depth*, which produces an exponential. After applying the correct normalisation a corresponding linear increase is produced.

The total depth of the entire configuration, according to Hillier (1996), is then:

$$td = \frac{\sum_{i=j=1}^{N} d(i,j)}{N}$$
(3.1)

which sums the depth of every vertex to every other vertex. However, we should be cautious of the definition; Hillier does not take account of the increase in N to N-1 relationships while calculating the depth of each cell in a space (see Figure 3.6). We make this simple adjustment:

$$td = \frac{\sum_{i=j=1}^{N} d(i, j)}{N(N-1)}$$
(3.2)

This adjustment produces a linear increase with a linear change in space. Of course, with a non-linear change in space we would still expect a non-linear change in td. We plot the values obtained from both Hillier's equation and our own (see Figure 3.6).³

³Later on in the thesis (see Chapter 7)_τ we use this adjusted definition of depth to help interpret results from our evolutionary simulations.

3.2.2 Small Worlds

In chapter 2 we introduced a dynamic social network between conversing people (see Figure 2.2). These networks represent interactions between people, which are usually characterised by the small world phenomenon; the "...anecdotal notion that you are only ever six 'degrees of separation' away from anybody else on the planet" (Watts, 1999). One of the motivations behind social network theory is to formally define the statistical structure of these networks. As mentioned above, these statistics will rely, in a similar way that that Hillier (1996) relies, on local information. In a social network, who we know defines this locality.

Definition 9 Structure can be derived from friendship ties. An important statistic that captures these ties is the clustering coefficient γ_v of Γ_v , i.e., the propensity of neighbourhood friends to be friends of each other, defined more formally by:

$$\gamma_v = \frac{|E(\Gamma_v)|}{\binom{k_v}{2}} \tag{3.3}$$

where $|E(\Gamma_v)|$ is the number of edges (among *friends*) in the neighbourhood of v and $\binom{k_v}{2}$ is the maximum number of edges that can be constructed in that neighbourhood given k_v . The clustering coefficient of the graph $\gamma(G) = \gamma_v \forall v \in V(G)$.

Another way of thinking about γ is to imagine that if three people form a single component, then they can either form a *triangle* Δ or a simple *triple* Θ and γ can then be expressed as a ratio (Newman, 2003):

$$\gamma = \frac{\sum \Delta}{\sum \Theta} \tag{3.4}$$

We illustrate this idea by presenting an example social network (see Figure 3.7). Notice that a triangle 'is a' triple. Therefore, $\gamma = (0,1)$ and in a fully connected graph $\gamma = 1$ whereas in a fully disconnected graph $\gamma = 0.4$

⁴This global statistic is not much use in creating field information or cancelling vectors, but we will see how local clustering values have been used in models of pedestrian movement to create attractor fields (see Section 3.3.3).

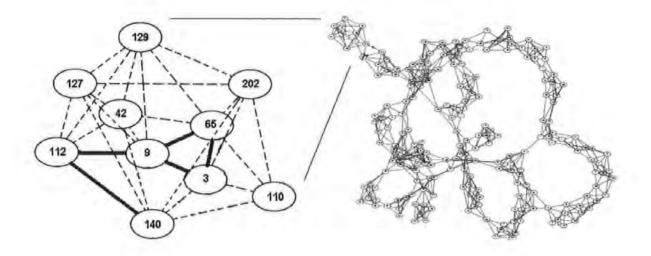


Figure 3.7: Components of communities. A social network contains clusters or communities, which consist of triples and triangles. The clustering coefficient of the entire social network can be seen as the ratio of the number of triangles in the network to the number of connected triples. A sample community (left) is selected from a social network (right). Vertices 112,9 and 140 form a triple. Vertex 9,3 and 65 form a triangle, which is a triple. Adapted from (Jin et al., 2001).

3.3 Attractor Fields

We have established in the previous sections that constructing an appropriate representation of movement through space is akin to solving the problem of crossing out appropriate vectors derived from a simple 2-lattice-graph. We will now review how some of the most recent models of pedestrian and evacuation dynamics have approached this. The first model, STREETS (see Section 3.3.1), and the second model EXODUS (see Section 3.3.2), exemplify the use of scalar values in vertices for the generation of attractor fields. Both approaches attempt to represent labyrinthine structures, but use different procedures for the development of attractor fields. We will see that the STREETS method is particularly weak one. The Space Syntax-based approach (Penn and Turner, 2002) also generates attractor fields, but adopts an alternative method and relies heavily on social network clustering statistics. We critique this approach and demonstrate its weaknesses (see Section 3.3.3). Finally we review Cellular Automata models of movement, which do not rely on surfaces, but on fully explicit, local rules, which model important pedestrian-pedestrian interactions in simple spaces (see Section 3.3.4).

The aim of this section is to identify methods from the current state of the art, which may be useful in developing pedestrian and evacuation models that are appropriate to use as the basis of fitness function design. Where methods are inappropriate, we explain why, but we also summarize our arguments at the end of the chapter (see Section 3.3).

3.3.1 STREETS

This model uses an underlying 2-lattice with von Neumann neighbourhoods. Pedestrians occupy the lattice and an attractor field is generated according to the following global rule:

$$\psi_{xy} = \Phi - \sqrt{[(x - x_o)^2 + (y - y_o)^2]}$$
(3.5)

where Φ is a constant ensuring that the central location (x_0, y_0) is a positive value and, using a landscape analogy, is the peak of the attractor. Batty, Jiang and Thurstain-Goodwin (1998) build models of pedestrian movement, based on this Euclidean attractor, out of five components, which contribute to a pedestrians heading angle Θ , calculated in radians, where $0 < \Theta < 2\pi$. The field of view is therefore limited to an arc.

The direction for a k pedestrians is updated over time according to:

$$r_k(x, y, t) = \sum_{i=1}^{N=5} \tau_i f_i$$
 (3.6)

where f_1 evaluates a gradient of the attraction surface, f_2 calculates default forward movement, f_3 calculates the perturbations needed to navigate physical barriers, f_4 is a threshold related perturbation function related to congestion and f_5 allows the attraction surface to be updated with respect to the number of pedestrians in a given area. A switch τ_i for each function can determine that $f_i = 0$. A radian direction is returned from the equation and P_i move accordingly.

 f_1 calculates the differences in the attraction surface from the current location v_P to locations in the surrounding area. f_2 perturbs the current heading by 0.314 Radians (18 degrees),

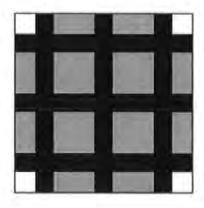






Figure 3.8: A shopping mall model in STREETS. Left: mall configuration. Centre: attraction surface. Right: pedestrian positions after 100 steps. Pedestrian behaviour is determined by a surface of attraction, which is defined globally and without regard for obstacles in the space. Adapted from Batty et al. (1998).

adding a small quantity of noise to the default gradient-based movement. f_3 searches local coordinates for barriers $\in B(x,y)$. If the coordinates returned from $f_1[x_{\pm 1}(t+1), y_{\pm 1}(t+1)] \in B$, pedestrians cannot occupy them, and the f_1 switch is zeroed. A progress variable for each pedestrian determines if the obstacle can be circumnavigated. These variables are incremented each time the pedestrians encounter a new obstacle. In the event that no progress is made the direction is reset to a random direction. The last two functions (f_4, f_5) relate to the interactions between pedestrians. f_4 provides information regarding congestion in a specified area of space. A threshold is set in the model where Θ is again perturbed randomly if this threshold is reached.

In an application to an idealised shopping mall, Batty et al. (1998) use equation 3.5 to define an attraction surface. The pedestrians are then initialised to a surface, which is also populated with obstacles. Pedestrians are attracted and converge on the middle of the shopping mall, avoiding obstacles on the way (see Figure 3.8).

Methods in STREETS clear blockages by brute force, rather than allowing the emergence of naturally occurring blockage avoidance. This approach is diametrically opposed to other pedestrian models, where blockages beget blockage or become frozen by heating (Helbing et al., 2002). We believe this latter approach to be a much more natural representation of real behaviour where, when blockages occur, they are reinforced; an observation made in real

pedestrian crowds (Canter, 1980). We believe that the reason such techniques are employed is because the attractor field is a fixed and globally defined technique, which does not naturally represent obstacles in the space. Any navigation is merely coded by accumulations of random jumps. In more complex configurations than this, where empty Euclidean attraction surfaces map poorly to labyrinthine structures, behaviour might depend largely on random walks. This default behaviour is inefficient and, more importantly, unrealistic.

In an attempt to overcome the problems, Batty et al. (2002) introduce a stigmergy-based approach where complex attraction surfaces are developed via incremental feedback. Each pedestrian is located in a space representing the street layout at Notting Hill, London. The aim for each pedestrian is to find a destination area. At the beginning of the simulation this is done by pure random walking. However, if a pedestrian is initialised close to a destination, then it is more likely to find it. Once a destination is found, the pedestrian moves back to its origin, dropping pheromone on the return journey so that feedback takes hold in the population. The attraction surface of a more complex configuration is built in this way. However, although this approach may better represent a labyrinth, the effectiveness of the surface will be inversely related to the number of agents used. For example, if we choose a single agent for a complex scenario the success of the agent will depend on its initial random position in the space. The more agents we add, the more likely each is to find the destination through pheromone feedback.

Apart from the general criticism already made, are three specific reasons why STREETS is of little use to us:

- 1. We need a method that allows agents to move over spaces of arbitrary complexity. In this context, the methods used by Batty et al. (2002) are brittle—they require a priori knowledge of how many pedestrians would be needed in order to generate surfaces devoid of local minima; highly complex spaces will need many pedestrians. In an evolutionary scenario, this knowledge will not be available because spaces will be genetically coded.
- 2. We cannot tolerate the default randomness in this model. The rules of behaviour seem

very much designed around a random-jump algorithm, to the extent that other dynamics appear arbitrary from a modelling point of view.

3. In relation to point 2, the model does not contain any adaptive mechanism at the level of individual pedestrians. Pedestrians follow stigmergy-based increments, accumulated from the entire population. There is no attempt to model local *processes* in the context of, individual pedestrians, situated in local conditions.

For these reasons, we abandon this approach and go on to review EXODUS. We will see that this latter system contains methods, which are far less arbitrary, and potentially useful for developing realistic behaviours.

3.3.2 EXODUS

The attraction surface uses the path length or depth (see above), from an origin v_o to a destination v_d (e.g., see Figure 3.9) and obstacles are excluded form the attractor field. Therefore, spatial navigation does not depend on random jumping behaviour for obstacle avoidance. This is an important advantage over the methods used within STREETS. From an evolutionary perspective it is important that randomness in a model is minimised and certainly not a dominant feature of the dynamics.

A more general disadvantage is that EXODUS works on very detailed evacuation scenarios⁵. For example, complex representations, like chemical concentrations due to fire and smoke, are advocated (Gwynne et al., 1998) and pedestrian movement depends very much on a number of hand-coded elements. For these reasons, EXODUS does not seem to capture the *general* behaviours required of our fitness functions, which will be used to evaluate populations of arbitrarily complex spaces. In short, the EXODUS system takes a hands-on, engineering approach, which is important for specific applications, but on the basis of their lack of generality, we exclude the use of its behavioral rules.

⁵Of course, in some application domains, this is a strength

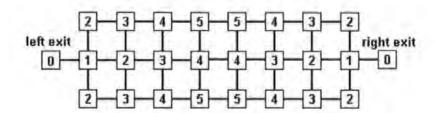


Figure 3.9: Depth information in EXODUS. Depths are created in relation to the nearest exit. In this example the space is constructed from a von Neumann neighbourhood. Adapted from Owen et al. (1996). This representation can contain objects more naturally than representations used in STREETS.

However, the depth or shortest-path method will form the basis of some of our own methods, developed later on the thesis (see Chapter 5).

3.3.3 Space Syntax

Penn and Turner (2002) define sensors as an isovist subgraph I(G) accessible according to the vertex location v_P of a pedestrian P. This is an alternative method of attractor field generation, based on Space Syntax techniques. Because vertices in the isovist can be paired as edges, clustering coefficients can be derived from I(G) and a field can be created to encourage pedestrian motion.

The isovist links v_o to each j^{th} viewable vertex so that, for example, empty space produces fully connected isovists ($\gamma = 1$)—each node is viewable from every other. The clustering coefficients Penn and Turner (2002) compute for isovists are high ($\gamma_{I(G)} > 0.7$)⁶. Once these values have been computed, the wandering dynamics stem from using the local vertex coefficients as attractors. Obstacles in space are not part of the isovist and do not therefore attract pedestrians. It is in this way that avoidance rules are inherent to the clustering information.

Movement rules in this model are said to be very simple and are opposed, according to Penn and Turner (2002), to complex theories involving higher-level, cognitive representations⁷.

⁶In comparison, social networks produce typical values in the range 0.4 - 0.5 (Watts, 1999).

⁷We agree with this general premiss, although not necessarily with Penn and Turner (2002) specific approach. The question of how people move through a configuration can be thought of as separate to why they make certain perception-based choices. For a more cognitive perspective on navigational behaviour see, for example, Raubal (2001).

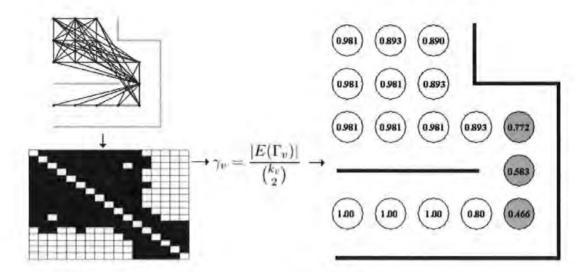


Figure 3.10: Fragmentation of a spatial graph. We demonstrate how, in an example space, the construction of isovist fields, via use of clustering coefficients, can result in local minima. We present a given space with a graph, which maps to a connection matrix with 17 entries, one for each vertices. We then compute the clustering coefficients in order to derive the scalar field values. The greyed-out vertices represent the locally optimal region.

However, although we agree, that to model pedestrian motion we do not need to represent thought processes, the simple clustering coefficient equation, for large N, requires considerable computation. This is what we mean in Chapter 2 when we argued that the complexity has been moved away from the agent-environment interface.

Also, considering the probable lumpiness of space in labyrinthine scenarios, we expect certain problems with using these kinds of methods. For example, where the size of space changes significantly at certain vertices, local optima can result. Consider the scenario where a small square area exits to a winding turn (see Figure 3.10). We record a connection matrix corresponding to the seventeen vertices and use the clustering coefficient equation to calculate the field values. We can see how three of these values represent a locally optimal area, which pedestrians will tend to avoid. Essentially, i.e., as far as movement behaviour is concerned, the use of these clustering coefficients produce local optimum and fragment single component graphs.

These behaviours are therefore inappropriate in terms of any long-range, complex trajectories. By exploiting a clustering technique, lattices will become fragmented and multicomponent. We will need to establish complex trajectories—in real situations pedestrians demonstrate complex navigation—and we therefore abandon the cluster-based methods presented here.

3.3.4 Summary

STREETS, EXODUS, and this latter approach all represent different ways of constraining movement and producing graph walks. The only potentially useful method we may consider is the static field approach used in the EXODUS system. This method may be used to capture the general process of egress and the long-range behaviour in pedestrian crowds i.e., movement from a given origin vertex to a destination vertex. These attractor fields may need to be used in conjunction with local rule sets, which capture pedestrian-pedestrian interaction at a more local level and we now turn our attention to methods that operate at this level.

3.4 Local Dynamics

Instead of using high-level information to direct movement, CA approaches attempt to represent movement with explicit local rules. As a result, rule sets are quite complex because they encompass the agent-environment interface in a dynamic way. As we will see, these approaches have demonstrated considerable success in terms of being able to model movement, and in particular movement interaction between pedestrians. Vector cancellations are local and dynamic and represent interactions solely between pedestrians rather than between pedestrians and obstacles. In this way they do not model complex navigation, but are a method of modelling important *processes* in local pedestrian-pedestrian dynamics. Before we present these models it is important that we introduce the car traffic CA, which inspired their development.

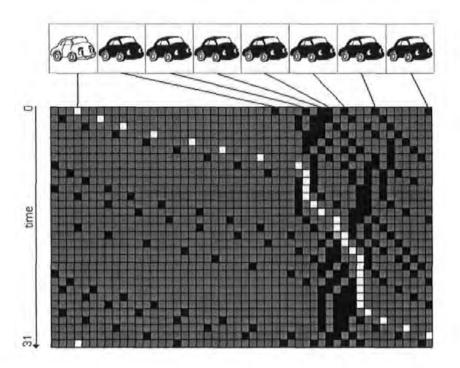


Figure 3.11: Nagel and Rasmussen's vehicle CA. We trace the trajectories of cars in the Nagel and Rasmussen model. The white car allows easy visualisation of the effect of a jam in a single loop trip on a circular lattice with 50 vertices. The car completes a loop in 31 time steps. Boundaries are periodic.

3.4.1 Cellular Automata: Vehicle Traffic

We mentioned in chapter two that CA can model movement through space with very simple, locally computed rules. Vehicle traffic has successfully been modelled with this approach (Nagel and Rasmussen, 1994). The model simulates forward vehicle movement on a 1-lattice graph with periodic boundaries. In this way, an infinitely long road is modelled with a finite memory.

Each vehicle is initialised on a vertex. There is a gap between vehicles; a length of empty vertices, which model vehicle-vehicle distances. Each vehicle is assigned a maximum velocity v_{max} and a current velocity v. The former defines a neighbourhood of searchable cells in a single direction and v is updated in parallel, for each vehicle, according to the following rules:

$$v < v_{max} \text{ with } gap \ge v + 1 : v \to v + 1 \tag{3.7}$$

$$gap \le v - 1: v \to gap \tag{3.8}$$

$$if(r < 0.5)v \to max[v - 1, 0]$$
 (3.9)

The rules model acceleration, slowing due to other cars and random driver effects, respectively. As the simulation unfolds over time, these very simple rules produce quite complex and critical dynamics. When a vehicle reaches a space close to the car in front, the vehicle will slow down or stop. This affects the vehicle behind in a similar way, but at a later point in time and a backward moving jam can occur where density is high enough.

We present the general set-up of the Nagel and Rasmussen (1994) model (see Figure 3.11). To aid visualisation, a single car is painted white. We observe how, although the car initially accelerates forward into open space, in a high-density situation the angle of trajectory through time is altered. This represents a period of time when car motion is inhibited. When the vehicle moves free from the jam, the trajectory changes again and accelerates to its original starting position, completing a single loop of the space.

In a similar way to other MAS, these patterns are "emergent properties from the interacting objects in the system. They are nowhere explicitly represented at the level of interacting objects. They are generated through the dynamics" (p1 Nagel and Rasmussen, 1994).

3.4.2 Cellular Automata: Pedestrian Traffic

Whereas the car traffic CA (Nagel and Rasmussen, 1994) operates on the 1-lattice, the pedestrian CA (Blue and Adler, 2000a; Blue and Adler, 2000b) operates on the 2-lattice and uses a von Neumann neighbourhood as the basis for rule set construction. The rules describe behaviour for 1 and 2-directional pedestrians, but both are very much influenced by the 1-directional rules in the car traffic CA, minus the acceleration component. Indeed, the Blue

and Adler (2000a) model can be seen as an extension of the Nagel and Rasmussen (1994) model. The latter captures interesting properties of vehicle flow in real traffic and the the aim is that pedestrian dynamics be modelled with a similar degree of success.

One frequently observed pattern in real pedestrian systems is the formation of lanes. People make different decisions based on which direction another person is moving. For example, in the simplest case, two people moving in the same direction are likely to have a similar velocity where few or no interactions take place. If two people are moving in opposite directions and in the same lane, then there will be decisions to make regarding collision avoidance. The rule sets of Blue and Adler (2000a) are an attempt to model these kinds of decisions, which optimise forward movement.

Because the space is extended to two dimensions and there are contra-flows, the rule sets are far more complex than the Nagel and Rasmussen (1994) model. We present the Blue and Adler (2000a) rule sets (see Table 3.1).

As a self-organising model, it is difficult to fully appreciate by simply considering printed rule sets. We implement the model and present a graphical snapshot when behaviour has settled for both 1-directional and 2-directional populations (see Figure 3.12 Top). We can see lane formations in the 2-directional case and for 1-directional movements we observe *modelocking* (Blue and Adler, 2000a). In order to present the dynamics in this model, we need to sample a 1-D lane from 2-D space and watch how it changes through time. We observe how lanes develop towards either west or east moving pedestrians (see Figure 3.12 bottom).

3.5 Summary

At the beginning of this chapter, we provided some important definitions, and then provided a more in-depth review of discrete dynamical methods, which we classified according to:

• Attractor Fields

- Fields need to be constructed so that no local optima exist. STREETS and Space

Table 3.1: Pedestrian Cellular Automata Rules (Blue and Adler, 2000a)

1. Lane Change (Parallel Update 1)

- (a) Eliminate Conflicts: two laterally adjacent walkers may not sidestep into each other. A lane is available to one of them only by 50/50 assignment.
- (b) Identify Gaps and Lane Choice: same lane or adjacent (left or right) lane is chosen that best advances forward movement according to $MAX(gap_{centre}, gap_{left}, gap_{right})$ up to v_{max} from the gap computation (row 3 below).
 - i. For dynamic Multiple lanes (DML)
 - Step out of lane of a walker from opposite direction by assigning gap = 0 if within eight cells.
 - Step behind a same direction walker when avoiding an opposite direction walker by choosing any available lane with $gap_{same} = 1$ when gap = 1.
 - ii. Ties of equal gapmax ahead resolved
 - 2-way tie between the adjacent lanes \Rightarrow 50/50 random assignment.
 - 2-way tie between the current lane and single adjacent lane ⇒ stay in lane.
 - 3-way tie \Rightarrow stay in lane.
- (c) Move: Each pedestrian move 1 lateral $(\pm (i, j))$ sidestep Step forward.

2. Step Forward (Parallel Update 2)

- (a) Update velocity: Let v(Pn) = gap where gap is from gap computation (procedure 3 below)
- (b) Bi-directional exchanges: IF gap = 0 or gap = 1 AND $gap = gap_{opp}$ (forward cell occupied by an opposing pedestrian) THEN, with probability $p_{exchange}$, exchange the pedestrians.
- (c) Move: each pedestrian P is moved v_P cells forward to the appropriate cell on appropriate lane.

3. Sub Procedure - Gap Computation

- (a) Same direction: Look ahead max of 8 cells (8 = $2 * v_{max}$) IF occupied cell found with same direction THEN set gap_{same} to number of cells between entities ELSE $gap_{same} = 8$.
- (b) Opposite direction: IF occupied cell found with opposite direction THEN set gap_{opp} to INT (0.5 n cells between entities) ELSE $gap_{opp} = 4$.
- (c) Assign gap = MIN $(gap_{same}, gap_{opp}, v_{max})$.

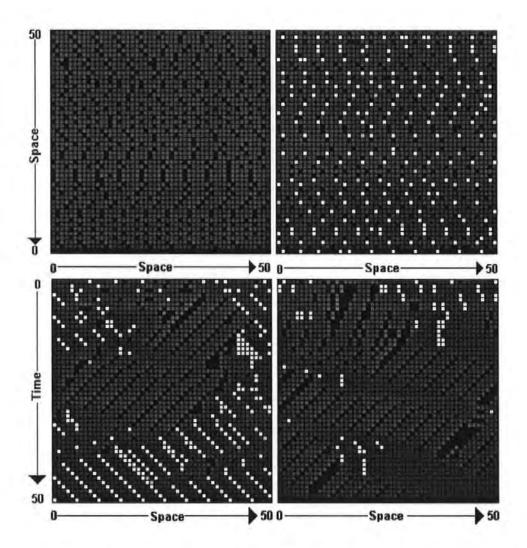


Figure 3.12: Blue and Adler's pedestrian CA. Top Left: East-moving 1-directional pedestrians. Top Right: 2-directional East and West-moving pedestrians. In both snapshots, we present results from the simulation at t=100, as is done in the original publication. Bottom: this time, instead of observing the entire space, we watch two separate lanes over time in two space-time diagrams for the 2-directional scenario. The observed lanes develop so that either east or west pedestrians settle on them over time. The gives us an idea of the developmental dynamics, which are typical in such models.

Syntax fail in this respect. However, methods based on the idea of a shortest-path depth value.

• Cellular Automata

 This method captures interactive processes, which occur between pedestrians on the move. It appears that the CA method is a perfect candidate for the modelling of short-range interactions.

However, it is unclear how we could take the CA rule sets and develop them into more complex navigational behaviours, based on attractor fields. We have mentioned previously that egress and pedestrian dynamics—two different levels of behaviour—need to be integrated. This is a challenge; to incorporate local dynamics into the context of higher-level egress behaviours.

Because the CA method only implements local decisions its potential use, therefore, remains an open question. Moreover, CA's are developmental; it is difficult to get a full appreciation of such models without implementing and experimenting with them. In the next chapter, we provide an in-depth investigation of CA models, in order to discover if above challenge can be met.

Chapter 4

Cellular Automata

The primary aim of this chapter is to explore the usefulness of pedestrian CA models by replicating and investigating them in further detail. We are particularly interested in their robustness and portability. Therefore, our investigations revolve around the following questions: Is behaviour robust across varying levels of density? Do behaviours at varying densities show sensible statistical characteristics? If so, can we transport the model easily to spaces of arbitrary complexity? If not, can we sufficiently improve the CA in order to achieve these ends?

In the first two sections we take a close look at the statistical behaviour of the vehicle CA (see Section 4.1), as a precursor to the pedestrian CA model (see Section 4.2), which we interrogate quantitatively and qualitatively. Conclusions from the investigation lead to attempts at model improvement (see Section 4.3). Furthermore, we attempt to integrate an improved model with attractor field methods of pedestrian movement (see Section 4.4). We present answers to the above questions in the form of empirical results, discussion of findings. We provide an extensive summary and discussion at the end of the chapter (see Section 4.5), which marks a pivotal point in the thesis.

4.1 1-D CA: Vehicles

The reason we implement, test and replicate this model will become clearer later, when we look at the 2-D pedestrian model (Blue and Adler, 2000a). Both models use a similar kind of search to help optimise forward movement, although the pedestrian model extends this search to 2-D scenarios. It is therefore useful to look in more detail at the Nagel and Rasmussen (1994) model.

4.1.1 Statistical Behaviour

We have already seen how jams can form in the Nagel and Rasmussen (1994) model and how this affects vehicle velocity (see Section 3.4.1). Nagel and Rasmussen (1994) study jams over a range of density ρ . Increasing ρ introduces more cars into the system and is associated with different regimes of behaviour. Nagel and Rasmussen (1994) measure the travel time of a car as it passes through a sublattice section of the model road. This produces an array of travel times for each car, as they moves along the 1-lattice, entering and exiting the sampled section. The simulation runs for 10^5 time steps. Travel time is simply the average time spent inside the sampled space and its variation σ is defined as:

$$\sigma(t_l) = \frac{\sqrt{\langle (t_l - \langle t_l \rangle)^2 \rangle}}{t_l} \tag{4.1}$$

where $\langle ... \rangle$ denotes the average travel time for all cars in the simulation. This is the *coefficient of variation* of the array of times recorded for each vehicle.

We interpret the basic intuition of Equation 4.1 as the *shape* of car movement, highlighted again by a white car, in a space-time plot (see Figure 4.1). At low density ($\rho = 0.08$) the white line runs smoothly across the plot where perturbations to movement are small and infrequent. At high density ($\rho = 0.8$) this line changes angle, representing slower movement. Perturbations to this movement are again relatively small, but frequent. In the middle regime ($\rho = 0.16$) the fluctuations are high and fairly frequent. This represents a critical regime

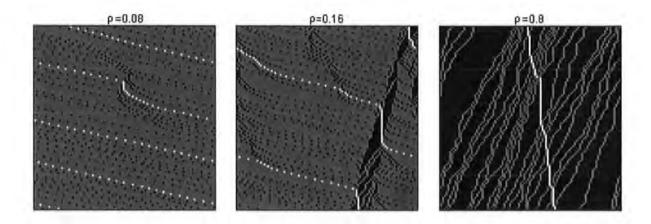


Figure 4.1: Nagel and Rasmussen's model at varying density. The shape of movement through time is different in each regime of density where there are different patterns of jam formation. Notice how the white car trajectory varies most where $\rho = 0.16$.

where travel times are unpredictable.

We plot statistics for travel time and travel time variation against density (see Figure 4.2). The exponential shape of the former reflects feedback between cars—if jams do occur then more jams are likely to occur, creating more jam behaviour ...and so on. As Sole and Goodwin (2000) point out, individual cars are not isolated entities and therefore behaviour is non-linear. We observe a critical density ρ_c , where travel time variation sharply increases. Variation is low at both ends of the density axis, but for opposite reasons. Firstly, when ρ is low, predictability is high, because there are few disturbances to the open road in the form jams—car travel times are predictably fast. Secondly, after the system has gone through the critical regime of high unpredictability, traffic begins to build and travel times become more predictable again, but because jams dominate the system, rather than free road—car travel times are predictably slow.

Variance depends on the stochastic driver effects rule, mentioned in the previous chapter. Although vehicles are not explicitly more likely to be perturbed—r (see Equation 3.9) remains fixed so that individual drivers have a fixed probability of perturbation—with increased ρ there are more cars in the system and any random effects are allowed to ripple through the medium of local car-car interactions. This kind of rippling effect across space is the source of criticality in the model and we can see that is has its greatest effect at $\rho_c \approx 0.09$.

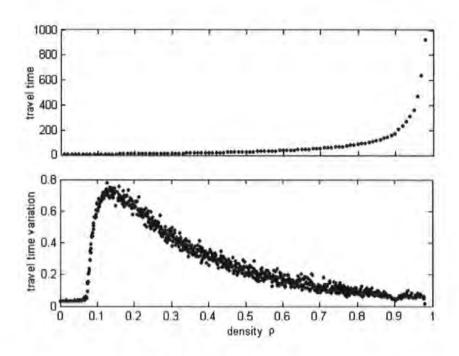


Figure 4.2: Statistical properties of the Nagel and Rasmussen (1996) CA. Each car in the model is tested for travel time according to Nagel and Rasmussen (1994)—also see text. As ρ increases average travel times increase exponentially. Variations of these times sharply increase at a critical point ($\rho_c \approx 0.09$). See Nagel and Rasmussen (1994) for a theoretical analysis.

4.1.2 Summary

This car CA is a simple interactive model. We have replicated it in order to demonstrate some of the higher-level statistical properties and how these result from increasing ρ alone, allowing interactive patters to take hold. We will see later in the thesis that interactive ripples across space are also important for the 2-D pedestrian model. We will now begin our investigation of this model (Blue and Adler, 2000a; Blue and Adler, 2000b), having looked at behaviours in the model that inspired it.

4.2 2-D CA: Pedestrians

In this section we investigate the pedestrian CA model (Blue and Adler, 2000a; Blue and Adler, 2000b) and replicate results of the original work (see Section 4.2.1). However, it is difficult to get a full appreciation of the behavioural range of the model if we rely solely on Blue and Adler's published results and the rationale behind them. Because Cellular Automata unfold over time and typically contain a number of rules, which can be combined in different ways, many different patterns are possible. Small changes in parameter values can have a marked effect on developmental patterns (Wolfram, 1984). We therefore present new experimental results related to different combinations of rules and values of parameters (see Section 4.2.2 and 4.2.3). The aim is to understand more fully how this model behaves in term of parameter variations and various rule combinations.

We combine qualitative observations with statistical ones, with the aim of understanding why certain patterns are found in statistical data. This is therefore a more complete investigation than the original work by Blue and Adler (2000a). We provide an experimental set-up so that we can apply statistical techniques, analogous to those used by Nagel and Rasmussen (1994), to the Blue and Adler (2000a) model (see Section 4.2.3). We show how travel time variation offers good insight into the behaviour of Blue and Adler's pedestrian CA. We also present results from a 4-directional model (Blue and Adler, 2000b), and explain them (see Section 4.2.4), before summarising and concluding this part of our investigation (see Section 4.2.5).

4.2.1 Stable Behaviour

We run each simulation for 1000 seconds ($\Delta t = 1$), but results from the first 100 time steps are excluded—rules need to settle before measurements are taken (Blue and Adler, 2000a). Whereas Blue and Adler use a distribution of 5 : 90 : 5 for walk speeds of 2, 3, and 4, respectively, we use a fixed distribution of 4 for each pedestrian ¹. The 2-directional split is 50/50, so half of the population moves in the opposite direction to the other half. We take results, averaged over 5 trials. In these experiments DML was applied so that forward movement is enhanced (see Table 3.1).

¹Our population can all be interpreted as *fast walkers* according to Blue and Adler. However, we do not consider walk speed to be greatly significant. Rather, we are more interested in the *patterns* that emerge from the model.

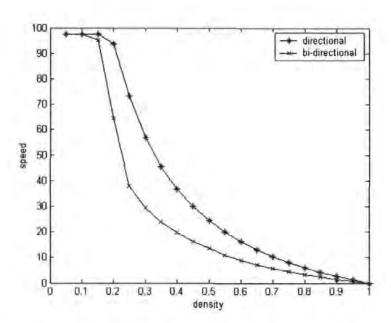


Figure 4.3: Directional and 2-directional speed-density statistics. 2-directional behaviour is less optimal where $\rho > 0.1$. However, the shape of the curves begin to differ where $\rho = 0.2$. $Speed = 4 \forall P_i$.

We observe population average speed for 1 and 2-directional scenarios (see Figure 4.3). Results confirm observations made by Blue and Adler (2000a) where 1-directional populations optimise forward movement more effectively than 2-directional ones. We observe a large decrease in total speed in the 2-directional population compared to the 1-directional population ($\rho = 0.2 - 0.3$). This suggests that the rules function more effectively at different densities—lane formations appear particularly robust at lower densities ($\rho = < 0.3$). At higher density ($\rho = > 0.3$) performance is poorer. However, because speed decays relatively slowly after this density, the lane-formation rules appear to succeed to some extent across the entire density range.

We might conclude that lane formations are robust because forward movement is maintained over this broad range. Only in a special case ($\rho=1$) do we observe zero movement. With these results it is tempting to assume that the lane formation rules will still be at work in situations of high density, particularly when we consider the local intuition of the rule sets. Further, we may be forgiven for thinking that smooth decay indicates a relatively slow deterioration of lane formation patterns.

In order to test these assumptions and uncover the true robustness of lane-formation rulesets, we now investigate the effects of what we believe to be important parameters in the model, particularly with respect to these higher-level patterns of lane-formation. If there are any shortcomings in the model we need to reveal and understand them and the investigations of the following sections are designed with this in mind.

4.2.2 Testing Robustness

Blue and Adler (2000a) only present graphical snapshot diagrams of the kind seen in chapter 2 where mode-locking in 1-directional scenarios, or lane-formation in 2-directional scenarios occurs ($\rho \approx 0.2$). We run the model at higher densities ($\rho = 0.4$) where statistical results (see Figure 4.3) indicate the population to have reached a relatively poor performance level. In order to allow the model to fully escape from the randomness of initialisation, we take snapshots at t = 200 and record results for three different values of $P_{exchange}$.

The reason we do this is because we believe that $P_{exchange}$ is an important source of variation in the model and also an important factor in the formation (or lack of formation) of lanes. Essentially, this is because the $P_{exchange}$ parameter provides the condition on which pedestrians with opposing directions make an exchange and remain in the same lane. This works against the rest of the rules, which allow development away from random initialisation and towards bi-directional lane splits. We present results (see Figure 4.4) and make the following three observations:

1. We observe that the smooth decay of the speed-density statistics (above) do not result from the gradual disappearance of 2-directional lane formations. Most of the population are in a state of conflict, where clusters have formed and dominate the entire lattice (see Figure 4.4, Left) ².

²An interesting dynamical property of these clusters is that they tend to persist by moving around the lattice, sucking in and spitting out different pedestrians. In this way the clusters are persistent entities in themselves, which emerge from the model, but as unintended side effects.

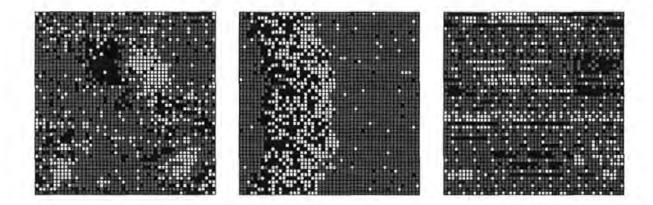


Figure 4.4: More snapshots of the Blue and Adler model. Three examples of patterns for density $\rho = 0.4$. Left: $P_{exchange} = 0.5$. Centre: $P_{exchange} = 0.1$. Right: $P_{exchange} = 0$. Lane formations reappear when we avoid exchanges!

- 2. We also observe the effect of changes in P_{exchange}³. When P_{exchange} = 0.1, behaviour is even less gaseous and pedestrians collect into a thick vertical column. A single cluster dominates the dynamics and pedestrians spend most of their time in mixed lanes (see Figure 4.4, Centre).
- 3. However, if $P_{exchange} = 0$, then lane formations reappear (see Figure 4.4, Right)!

Although counter-intuitive, we can explain these results by looking at how the rules interact. The reappearance of lanes is due to lateral exchanges being reinforced in the $P_{exchange} = 0$ case. As we mentioned previously, forward exchanges allow opposing pedestrians to exchange positions and maintain their current lane. Mixed lanes are not tolerated in the $P_{exchange} = 0$ case; opposing pedestrians cannot remain on the same lane (via an exchange) and the effect of lateral moves are allowed to accumulate so that 2-directional behaviour stabilises again.

An argument might therefore be made in favor of fixing $P_{exchange} = 0$. However, if we increase the density ($\rho = 0.6$) in this case and run simulations for three different random initialisations, we observe three frozen states (see Figure 4.5). In each case the population develops into fixed-point behaviour. It now seems clear why the random exchange rule is part of this model; it prevents fixed-point deadlocks.

Results in this section indicate that, at certain densities, behaviour will suddenly default

 $^{^3}$ Recall that in the pedestrian CA rule sets this is fixed— $P_{exchange} = 0.5$

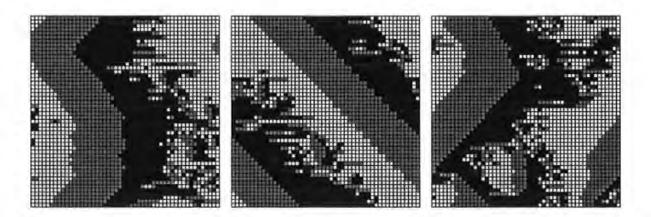


Figure 4.5: Frozen patterns in the pedestrian CA. When east moving pedestrians and west moving pedestrians form into homogenous walls the system reaches fixed point or oscillatory regimes.

largely to random exchange rules⁴. Although the $P_{exchange}$ parameter works against some of the intended higher-level patterns, in order to allow the population to move, it is essential, particularly in higher-density situations. The $P_{exchange}$ parameter is therefore a mixed blessing in the context of ρ . We will return to this issue below when we attempt to build improvements into this model by presenting a method to squeeze out randomness from the rule sets by controlling the probability of exchanges at a local level (see Section 4.3.1).

4.2.3 Statistical Behaviour

We now provide a statistical picture of behaviour across the entire density range. In the vehicle model (Nagel and Rasmussen, 1994) we have observed exponential increases in speed and critical effects of density on variations. We have already seen that density affects the speed of pedestrians in the Blue and Adler model (see Section 4.2.1), but we want to use statistics, which indicate variations in travel times, rather than look purely at average speed behaviour. This will give us a broader picture of how the Blue and Adler (2000a) rule sets perform, particularly relating to the random effects of rules in the model, which as we have just seen, appear to be very important in the model dynamics.

⁴We see later in the thesis (see Chapter 5) that the suddenness of the failure is related to interactions that ripple across space. This kind of rippling is typical on the 2-lattice and is known a percolation.

Because the vehicle model is 1-D and the pedestrian model is 2-D, the experimental setup used by Nagel and Rasmussen (1994) is not directly applicable. We therefore use the following experimental set-up for the Blue and Adler model in order to apply similar statistical techniques.

Experimental set-up: We allocate to each pedestrian P_i^z a value σ_i , which defines a desired number of steps to be made in a given direction. Three separate populations of pedestrians P^l , P^m and P^n are defined where v_{max} is set to 1, 2, and 4, respectively and $\sigma = 100 \ \forall P^z$. With $\Delta t = 1$, we record $t \ \forall P^z$, when the halting condition ($\sigma_i = 100$) is met and apply this scenario for a range of densities $\rho = (0, 0.8)$. Three separate simulations are used for each population. We make four observations:

- As we would expect, travel times in each population exponentially increase (see Figure 4.6, Top), for similar reasons to results obtained from the Nagel and Rasmussen (1994) model.
- 2. However, in contrast to the Nagel and Rasmussen (1994) model, the travel time variations continue to rise (see Figure 4.6, Bottom). This occurs because, with increases to density ρ, random exchanges increase and the variations in travel times reflect increased randomness all the way through the density range.
- 3. Populations P^l , P^m and P^n with different values for v_{max} have different travel-time variation patterns as a function of density. The parameter v_{max} determines the potential amount of space available for search. Therefore, at low densities, for P^n ($v_{max} = 4$), neighbourhood search is more likely to be affected by oncoming pedestrians compared with P^m ($v_{max} = 2$). We also expect P^m to be more likely affected than $P^l(v_{max} = 1)$ for the same reason.
- 4. Convergence of variations at $\rho = 0.58$. Above $\rho = 0.5$, the majority of space is occupied and randomness influences movement regardless of the v_{max} value because each popula-

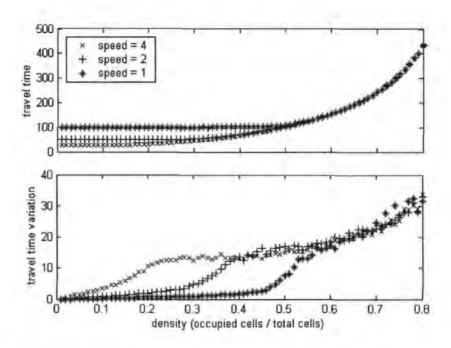


Figure 4.6: Travel time properties in the pedestrian CA. Top: At lower densities ($\rho < 0.5$) it takes roughly $t = \sigma/v_{max}$ to reach the halting condition, then behaviour increases exponentially. Bottom: we measure the variation in travel times taken to Reach the destination. Variations join where $\rho \approx (0.56, 0.59)$.

tions' neighbourhood is equally subject to perturbations from oncoming pedestrians.⁵

These results confirm what we suspected from the previous experiments; it appears as if the self-organising properties of the model progressively loose grip on the dynamics as density is increased. We will attempt to clean-up these dynamics (see Section 4.3), but first we look at some of the clustering properties in the 4-directional model (Blue and Adler, 2000b).

4.2.4 4-Directional Model

So far we have looked at 1-directional and 2-directional movement. Blue and Adler (2000b) extend their model to 4-directional scenarios. This introduces added complexity to interactions between pedestrians, but this complexity is achieved with a relatively simple extension to the 2-directional algorithm and the underlying rules remain the same (Blue and Adler, 2000b). We capture the basic problems with 4-directional behaviour in the following observations.

⁵We will see later that these results can be interpreted in the light of *Percolation Theory*. In the next chapter we interpret these results in the context of 2-lattice Boolean percolation statistics, which appear to define important limits on Cellular Automata behaviours.

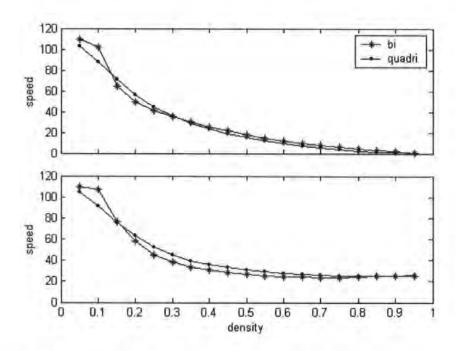


Figure 4.7: 2-directional and 4-directional behaviour. Top: By using the speed calculation, behaviour degrades towards zero. Notice how the speed is higher for the 4-directional case than the 2-directional case at densities between $\rho=0.18$ and $\rho=0.3$. Bottom: We find similar behaviour when measuring the actual changes in positions of pedestrians in these densities, but above $\rho=0.3$ the more complex rules sets appear to perform better until densities are very high. Both 2 and 4-directional cases do not reach a speed of zero at high densities because exchanges cause movement.

We compare 2-directional and 4-directional cases. We set population speed to a uniform distribution ($v_{max} = 4$) for both populations and again run for 1000 seconds ($\Delta t = 1$). We record the average speed of both populations per 60 seconds.

We present results from the same simulation, but using two different measures of speed (see Figure 4.7). We can see that if we measure speed to be the gap between pedestrians (Blue and Adler, 2000a; Blue and Adler, 2000b), then we observe one kind of behaviour (see Figure 4.7, Top). We observe here that 2-directional populations mostly out-perform 4-directional ones. However, if we measure speed as the actual change in position, once all exchanges have been executed, then we observe how 4-directional populations out-perform 2-directional ones above a certain density ($\rho \approx 0.2$) (see Figure 4.7, Bottom). Another strange behaviour is the comparatively poor performance of the 2-directional population ($\rho \approx 0.2 - 0.3$). It seems that for 2-directional cases the population is less robust and conflicts appear more easily. We

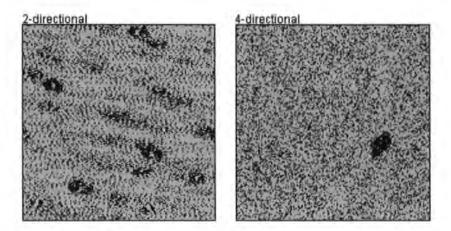


Figure 4.8: Conflicts in 2-directional and 4-directional populations. The snapshots are taken at t = 100 for a 2-lattice (200 × 200). We can see that even after 100 time steps in 2-directional populations clusters form more easily. Density $\rho = 0.2$.

believe this to be the result of 4-directional rules having more potential for exchanges and clusters of pedestrians appear to be less likely to form.

In order to demonstrate this latter point, we create a large lattice (200×200) and take a snapshot of the population at t = 100. We observe clustering patterns in populations of 2-directional and 4-directional pedestrians (see Figure 4.8). It appears that clusters do form more readily in the 2-directional model.

4.2.5 Summary

In this investigation we replicated similar results to Blue and Adler (2000a), but then identified a number of problematic and contradictory behaviours:

- Randomness is necessary in order that rule sets do not hit fixed point behaviours.
 However, the model defaults too easily to random behaviour. We discovered this by manipulating P_{exchange} and observing non-intuitive effects such as lane formation reappearance (P_{exchange} = 0). In this case however, increases in density produces deadlock scenarios.
- Certain rules allow better forward movement in a population, but only up to certain densities. Above these densities behaviour is badly affected by randomness and lane

formation patterns disappear. In contrast to what we might expect with this model that the effect of self-organising rules gradually deteriorate—we observe that breakdown occurs fairly suddenly.

- We further tested the robustness of the 2-directional model with a simple experiment, which provided a statistical picture of the effect density on populations with a different search capacity, defined by the parameter v_{max}. However, at certain densities, there is only one picture; runaway random behaviour.
- We have demonstrated how anomalies are inherent in 4-directional populations, which
 perform equally well or sometimes better than 2-directional populations, depending on
 parameter values. We argued that this is also due to inherent randomness of the rule
 sets.

Despite the problems, we believe that the local approach to modelling pedestrian dynamics has some attractive features. In particular we are reminded of our requirements of a good egress model and that it should "rely upon constituent factors" (Gwynne et al., 1998). We certainly believe that the Cellular Automata approach captures the appropriate level of detail. For this reason, we now attempt to eradicate some of the randomness from the rules sets, which appear to cause pervasive damage to dynamic pattern formations.

4.3 Extension of Pedestrian CA

The aim of this section is to try to salvage any useful components in the model by improving and extending the techniques. We redefine $P_{exchange}$ using density information from local neighbourhoods (see Section 4.3.1). Rather than globally fix the exchange probability, we define it according to a pedestrians local conditions. We then make a case for extending rule sets beyond the constraints of a von Neumann neighbourhood (see Section 4.3.2). We show that Moore neighbourhoods more closely resemble continuous neighbourhoods and we extend

the 4-directional rule sets in order to cope with Moore (k = 8) neighbourhoods (see Section 4.3.3).

4.3.1 New Rules for $P_{exchange}$

We have previously demonstrated a number of weaknesses in the behaviour of the pedestrian CA. The main source of this weakness, we believe, is the global definition of certain parameters. For example, Blue and Adler (2000a) define a global density parameter in their model, which is used to initialise pedestrians in a 2-lattice. In order to define a local definition of density, we will need to determine how many pedestrians are present in a given subset of a lattice L. Because diagonal exchanges are allowed in this model (Blue and Adler, 2000b), a Moore neighbourhood is the most appropriate unit to use. We then define the probability of exchange as:

$$P_{exchange} = 1 - \frac{\sum_{P \in \Gamma_{v_o}}^{8} P}{8} \alpha \tag{4.2}$$

where

$$P = \begin{cases} 1 & \text{present} \\ 0 & \text{absent} \end{cases}$$
 (4.3)

so that P defines a truth condition for the presence of another pedestrian in a Moore neighbourhood Γ_{v_o} , 8(k) scales the second term between 0 and 1 and 0 < α < 1. The alpha parameter is used to tune the probabilistic constraint on exchanges. The value of α determines the range of this function and where it is set to a high value the constraint on movement is correspondingly high.

We implement this in order that local conditions can affect behaviour with feedback. In designing fitness functions, it is crucial that this kind of local, density-related movement is included in a model. Configurations may have local bottlenecks, where density restricts movement. With this redefinition of density we aim to:

1. achieve more acceptable results regarding the relationship between the movement of the population and density ρ , as an alternative to the runaway random behaviour, observed

in the original model.

control the effects of the necessary randomness. If this is successful we expect that
measures of variation will show different patterns, more akin to the variation peaks
found in the Nagel and Rasmussen (1994) model.

We calculate $P_{exchange}$ with the new definition and measure speed and travel time variation, as outlined above. This was done for both a homogenous distribution ($v_{max} = 4$) and a heterogeneous distribution (Blue and Adler, 2000a). We fixed $\alpha = 0.9$ for all simulations so that Equation 4.2 defines a steep, negative gradient; local blockage feedback has a much larger effect in the face of an increasingly dense locality and where there, are dense pockets of pedestrians, exchanges are less likely to occur.

We present results for, 1, 2 and 4-directional populations for both distributions (see Figure 4.9). These results support arguments outlined earlier. We have previously seen how the randomness in exchange rules favours populations with more complex contra-flow. By using this local definition of exchange probability, we can see how 2-directional populations out perform 4-directional populations. It seems that the lane formation rule sets have more effect on how populations organise themselves. By removing some of the randomness from the rule sets, we have given the actual rules for forward movement more chance to express themselves. We also see clear transitions in the variation of travel times, indicating that in critical ranges behaviour becomes highly unpredictable. Patterns compare with the earlier vehicle model (Nagel and Rasmussen, 1994), although transitions are not as sharp.

Results therefore indicate that this local, density-determined movement is an appropriate method for constraining the effects of the randomness in the model identified in the previous investigation. Further, we believe that, regardless of these advantageous effects, local definitions of density are a far more sensible assumption in such models.

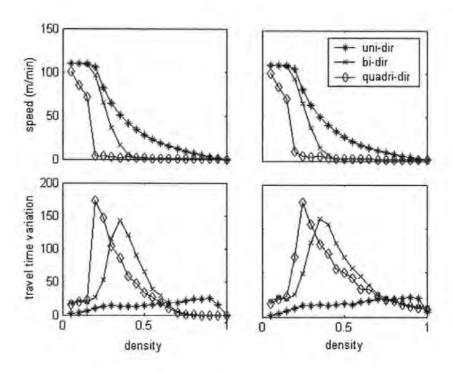


Figure 4.9: Results of local, density-determined exchanges. Left column: results for the Blue and Adler (2000a) distribution. Right column: results for a homogenous distribution $v_{max} = 4$.

4.3.2 Neighbourhoods and Graph Walks

At this point in the thesis we take a brief excursion from the main investigation and consider the properties of graph walks in different kinds of neighbourhoods. This is important because it will effect choices we make relating to rule-set extensions in our attempt to integrate the Cellular Automata approach with attractor fields, which represent the navigation-based behaviours of real pedestrian crowds.

We have introduced von Neumann and Moore spaces as important examples of neighbourhood representations. Ideally, we want to use the most appropriate, i.e., the best approximation to a continuous neighbourhood. We will measure the distance associated with each kind of neighbourhood (i.e., continuous, von Neumann and Moore) in a given optimal-walk scenario. This scenario is a simple walk from an origin vertex v_o to a destination vertex $v_n j_v$ where nj defines the the right-most column of a $n \times n$ 2-lattice and o = 0, 0 is the bottom left vertex. For illustrative purposes, we present an example of the set of walks made where n = 7 (see Figure 4.10).

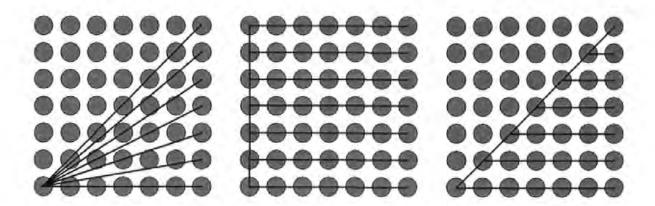


Figure 4.10: Optimal walks for three different spaces. The Euclidean metric cuts across the graph (left), whereas von Neumann (centre) and Moore (right) demonstrate different spatial metrics.

We extend this to a lattice where n=100 and plot the length of each of the 100 corresponding walks (see Figure 4.11). As we can see, walks in Moore neighbourhoods are a better approximation than von Neumann neighbourhoods when compared with continuous ones. We will therefore use Moore neighbourhoods and, for these reasons, we now attempt to extend the Blue and Adler (2000a) rule sets to incorporate these neighbourhoods.

4.3.3 Rule Extension for Moore Neighbourhoods

So far, we have seen 1, 2 and 4-directional Cellular Automata movement. Here, we extend rule sets to encompass 8-directional movement; remember that the significance of the number 8 in this sense is due to the k=8 property of connectivity in this kind of neghbourhood. This extension will introduce further complexity to interactions between pedestrians. We use the local exchange approach defined above (see Section 4.3.1) and the same experimental set-up (i.e., the same measurements for speed and travel time).

We compare results from Figure 4.9 (4-directional) against the new 8-directional population (see Figure 4.12). We observe that the randomness still appears to be favouring movement for more complex rule sets. However, overall, the speed-density and variation-density relationships are quite similar. Again results demonstrate that randomness can be controlled by a local

⁶We refer the reader to the appendices (appendix B) for the extended rule sets.

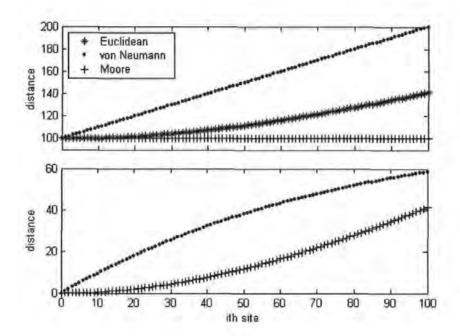


Figure 4.11: Relationships between discrete and continuous spaces. Top: We can see that by plotting the distance metrics from von Neumann and Moore walks and comparing results with the Euclidean metric different relationship are apparent. Bottom: We plot the distance of both discrete metrics against the continuous one. The Moore metric is a closer approximation than the von Neumann one.

approach to density-determined exchange.

4.3.4 Summary

This section had two main parts:

- We introduced a new method for controlling randomness in the pedestrian CA model, based on a local definition of density. We demonstrated that randomness can be controlled by presenting statistical results of the model.
- We showed that Moore neighbourhoods are a better approximation of continuous Euclidean spaces and therefore we extended Blue and Adler's 4-directional rule sets to cope with Moore neighbourhoods (k = 8).

We will return (see Section 4.5) to discuss the efficacy of the Cellular Automata approach after we gauge how well we might be able to integrate this kind of method with another, higher-level technique, i.e., attractor fields.

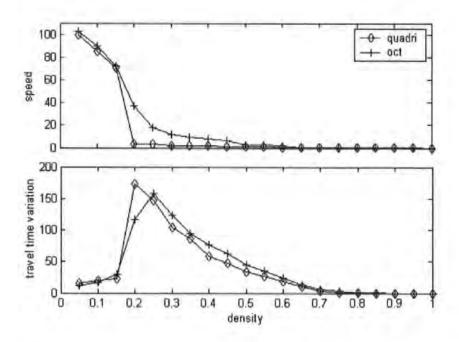


Figure 4.12: Relationship between 4 and 8-directional behaviour. The more directions there are to exchange the more randomness is produced from the rule sets. However, the local definition of exchange controls the randomness in 8-directional populations also as well as 4-directional populations.

4.4 Coupling Cellular Automata with Attractor Fields

So far in this chapter we have concentrated on the investigation and improvement of a given pedestrian CA model (Blue and Adler, 2000a; Blue and Adler, 2000b). This model operates on a simple 2-lattice with periodic boundary conditions and rules are designed from a local perspective in order to model pedestrian-pedestrian interactions. In this section, we are concerned with the portability of the model to spaces of arbitrary complexity.

We show how a complex space is represented in a typical pedestrian and evacuation model, with the use of shortest-path depths to define directional information in complex labyrinthine spaces (section 4.4.1). We then introduce two simple variants of the pedestrian CA, which allow us to simulate movement in these more complex scenarios. We present and interpret results (section 4.4.2) before summarising the section (see Section 4.4.3).

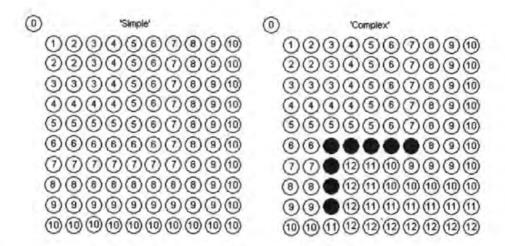


Figure 4.13: Representing complex configurations. Obstacles can be represented by setting the depth to a very high value, here represented as blacked-out vertices, which represent an L-shaped obstacle in space. Any algorithm, which then attempts to minimise depth in a graph walk will naturally avoid these vertices.

4.4.1 Representing Obstacles

We need to explicitly represent obstacles. This can be done using the simple notion of depth from an origin vertex v_o to a destination vertex v_d as outlined above (section 3.3.2). By using this approach, very large values can be used to exclude certain vertices. Depth values often decrease towards the exit node v_d , represented by zero, and any attempt to minimise distance in a walk will naturally exclude vertices with non-representatively large values. We illustrate this general approach (see Figure 4.13).

This kind of representation for movement is common in discrete approaches to pedestrian and evacuation simulation models. In most models this representation is static throughout a run and the dynamics of escape are captured in more local rule sets. We now demonstrate this kind of approach by introducing two variants of the pedestrian CA (Blue and Adler, 2000a; Blue and Adler, 2000b), which incorporate the information contained in the field.

4.4.2 CA-Based Model of Navigation

Experimental set-up: We define a population of pedestrians P_i^z , which move according to a minimisation rule. This rule depends on 2-lattice vertices being represented by scalar

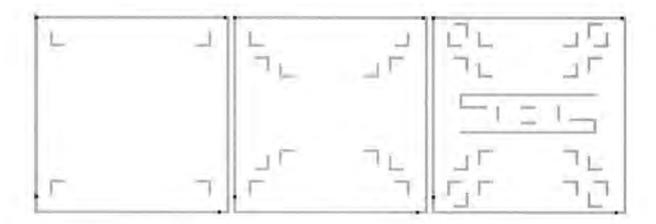


Figure 4.14: Complex, non-periodic configurations. We show three example configurations, increasing in complexity from left to right. Exit vertices are black and contain a scalar value of 0. Obstacles, which contain very large values, are represented as greyed-out pixels.

values (shortest-path depths). The rule: if an array of vertices $A_j \in \Gamma_i < v_o$ the local move A_j is deemed valid, otherwise no move can be made. Once a direction of movement has been determined, we apply the 8-directional, extended rule sets, to update the location of the pedestrian.

We define two population subsets P^l and P^k . The pedestrians P^l update at each time step whereas P^k update directions when, and only when, the *current* direction no longer provides an optimising move in A_j . We can see that the former will tend to perturb the influence of the Blue and Adler (2000b) rule sets, whereas the latter will tend to preserve them. In this way we can test how well the rules perform in more complex configurations.

We run experiments on three different configurations of increasing complexity (see Figure 4.14). After initialising P_i^z randomly to an origin vertex v_{α_i} , a random exit is chosen as the destination v_{d_i} . The direction for each pedestrian at each time step is updated according to the above constraints. For 10 trials we measure the average travel time for a range of densities and plot results for each configuration and for each pedestrian type, k and l (see Figure 4.15).

Results produce a clear pattern— P^k outperform P^l . The P^k pedestrians preserved the influence of the Blue and Adler (2000b) rule sets and therefore this seems to suggest the dynamics in this model appear to contain useful self-organising properties. However, as we have shown in previous sections, it is at lower densities that lane formations are maintained,

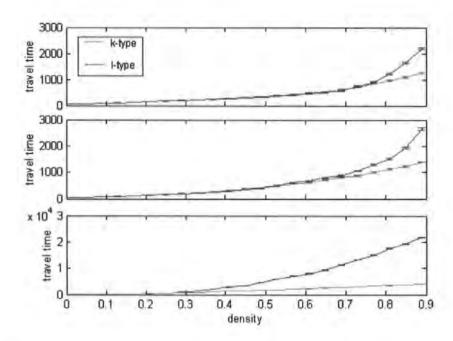


Figure 4.15: Travel time comparison for various configurations. Top: results for the simple configuration. Middle: results for the medium configuration. Bottom: results for the most complex configuration.

whereas it is at higher densities that the benefits seem most prominent. At lower densities the populations perform very similarly.

4.4.3 Summary

In this section we:

- presented a simple extension to a pedestrian CA (Blue and Adler, 2000b) in order to incorporate the concept of navigation.
- established that although the rules in this model seem to be achieving some degree of optimisation, this is again only at certain densities within the model.

In view of the results obtained in the early parts of this chapter, where emergent patterns broke down at relatively early density values, it would be wrong to attribute the beneficial results obtained in the final experiments to the robustness of the rule sets. It might have been significant if benefits had emerged at much lower density values, but as we observed, the two different populations performed very similarly in these conditions.

4.5 Summary and Conclusion

We undertook an in-depth investigation of a particular Cellular Automata, and discovered a number of results, which will prove significant in the work that unfolds in future chapters.

We discovered:

- · density-dependent, lane-formation collapses.
- runaway random behaviour. This affects populations with different search capacities equally, above certain densities, and differently below these densities.
- other anomalies in the model, which relate to random exchange rules.

The findings of this investigation answer two of our opening questions:

- Is behaviour robust across varying levels of density? → No.
- Do behaviours at varying densities show sensible statistical characteristics? → No.

We then attempted to improve behaviour by introducing a new exchange mechanism, based on dynamically changing, local densities. Although randomness will never be completely eradicated in models that require perturbation in the face of deadlock situations, limiting its effects are, perhaps, worthwhile. The exchange probability was constrained by a simple parameter-based function. Statistical results suggest that this is a good method of control.

We made a case for extending the pedestrian CA to Moore neighbourhoods. Based on this extension, the final section considered navigation. We elaborated on techniques introduced in chapter 3, which represent obstacles in space, i.e., attractor fields. We then demonstrated a method with the aim of investigating whether or not the pedestrian CA rules carried over to the more complex spatial scenarios. Results answer two further opening questions:

- Can we transport the model easily to spaces of arbitrary complexity? → No.
- If not, can we sufficiently improve the CA in order to achieve these ends? → No.

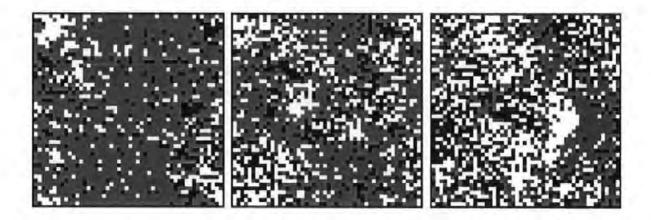


Figure 4.16: Graphical snapshots of the pedestrian model. Left to right $\rho = 0.3$, $\rho = 0.5$, $\rho = 0.7$. Even at quite a low density ($\rho = 0.3$) clusters appear in the pattern formations. We can see that as we increase the density these clusters grow. The dynamics of this model show these kinds of clusters that persists by moving around the space, swallowing and spitting out different pedestrian units on their way.

Overall, we found the CA approach to pedestrian-pedestrian interaction disappointing. CA models do not appear to be a useful approach to pedestrian-pedestrian interaction, at least in the way they are used by Blue and Adler (2000a), and in the way we have extended their work. In this respect, we failed. However, investigations have given us a better appreciation of the general mechanisms and, moreover, an appreciation of why they failed.

4.5.1 Persistent Failure

Before we end this chapter, it is important to present some brief results, which demonstrate the persistence of rule breakdown in the CA model. In section 4.3.1, we attempted to constrain the inherent randomness in the pedestrian model, as a method of reinforcing lane formation rules. We provided statistical results, which demonstrated how the randomness is controlled by this technique. However, closer observation indicates that even with these changes patterns remain brittle. We present some graphical snapshots, which indicate the nature of failure (see Figure 4.16).

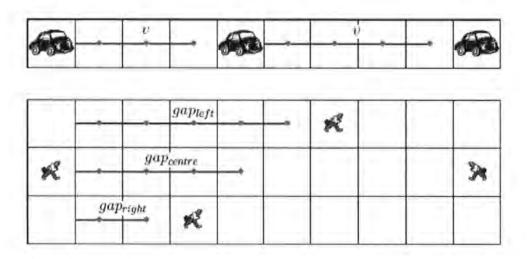


Figure 4.17: Components of search in CA models. The car CA (Nagel and Rasmussen, 1994) depends on search along a 1-D component (1-lattice). We can see how the pedestrian CA (Blue and Adler, 2000a) adopts a similar method of search by decomposing a 2-D space (2-lattice) into 1-D components.

4.5.2 Reasons for Failure

We believe that some of the problems of Blue and Adler's model stem from blindly inheriting components from the car CA. Forward neighbourhood search in the car CA (Nagel and Rasmussen, 1994) is perfectly applicable to a one dimensional *road* scenario. However, because Blue and Adler (2000a) adopt this kind of approach in a 2-D model, 2-D search becomes a search along a collection of 1-D components (see Figure 4.17).

We believe this choice severely limits the model. In the next chapter, we move attention away from the logic of CA rules and focus on processes of search on 2-D graphs.

Chapter 5

Optimal Paths

In order to understand the sudden collapse of lane formation behaviour, we look at Boolean percolation on the 2-lattice graph, and the formation of neighbourhood-connected clusters (see Section 5.1). Statistical results suggest that Cellular Automata models are limited by the fundamental properties of neighbourhood interactions, which catastrophically affect local dynamics. We therefore argue that an alternative approach to purely local dynamics is required.

We then pause to list a number of essential levels of representation (see Section 5.2), before introducing the Pulse Coupled Neural Network (PCNN), which provides a more inclusive search than local neighbourhoods (see Section 5.3). Equation-based mechanisms are presented in the context of image processing, the original application domain for the PCNN, and the relevance of the PCNN to essential levels of representation are outlined (see Section 5.4). We then discuss potential pitfalls in the PCNN, before improving it (see Section 5.5). Equations are pruned in order to improve search robustness and behaviour is tested on simple and complex 2-lattice configurations. Further definitions from graph theory are given, before a chapter summary, where we recap on our main arguments (see Section 5.6).

5.1 Site Percolation on 2-lattice Graphs

We turn our attention to the nature of the 2-lattice graph, which is the single most common construction used in pedestrian and evacuation dynamics modelling. However we believe that, although widely used, the properties of this kind of representation have previously been overlooked. We look at Boolean interactions on a 2-lattice (see Section 5.1.1) and then interpret the meaning of their statistical properties in the light of local CA rule sets (see Section 5.1.2).

This section represents a shift away from the rationale of purely local approaches to pedestrian interaction and those based purely on attractor fields. It is therefore the beginning of a shift in perspective, which will inform the techniques developed throughout the remainder of this chapter, and the next (see Chapter 6).

5.1.1 Percolation Threshold ρ_c

Generally, *percolation* refers to the process of diffusion in a given medium. For example, if we place a drop of liquid on tissue paper, then we will see the liquid spread. Perhaps the liquid will cover the entire tissue or maybe only part. The pattern of spreading will depend on a number of things, one of which is the percolation properties of the paper.

If we use a 2-lattice (analogous to tissue paper), then each vertex is occupied with a given probability. We are interested in the clustering properties of the lattice in terms of occupied neighbouring sites¹. Therefore, consider a 2-lattice with sides N=200 and von Neumann neighbourhoods. We initialise each vertex with independent probability ρ . If a random fraction $\epsilon_{ij} < \rho$, then the ij^{th} site is occupied, otherwise it is empty. The following procedure allows us to gain a picture of the vertex clusters and how interactions percolate across the lattice:

- 1. A set V defines the occupied sites in the bottom row of the lattice at time t=0.
- 2. At each time-step ($\Delta t = 1$), we record the occupied neighbours in $\Gamma_{V(t)}$.
- 3. Once a vertex has been recorded we then determine that it cannot be recorded again.

¹This kind of percolation is known as site percolation (Stauffer and Aharony, 1992).

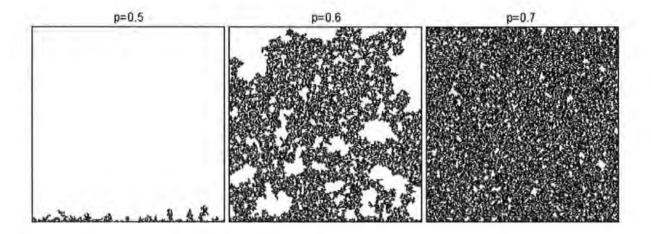


Figure 5.1: Percolation on a Boolean network. Left: At density $\rho < 0.5$ the walk on the graph typically fails after a short amount of time. Centre: At density $\rho = 0.6$ the walk is complex and percolates throughout the lattice. Right: At density $\rho = 0.7$ the walk percolates over the lattice, but does not show very complex patterns.

 We iterate through steps 2 and 3 until there are no further occupied sites accessible from Γ_{V(t)}, and so, at t = t + 1, V = 0.

We present three typical patterns of 2-lattice site percolation for three separate probability values (see Figure 5.1). Notice how relatively small changes in ρ are accompanied by sudden, large changes in the clustering, particularly between $\rho = 0.5$ and $\rho = 0.6$. The interval between these values contains a percolation threshold, which we can identify more easily with more extensive simulations.

We carry out similar experiments over the entire density range, sampling each density value 50 times, and incrementing density values by 0.001. We measure two statistics:

- The 'walk time', i.e., the time at which point there are no longer any occupied sites accessible (i.e., the V = 0 case).
- The 'fraction of walkable vertices', i.e., the total fraction of sites, which at some point in the simulation have been accessed and loaded into V.

Initially, for small values of ρ , percolation shows little progress—few occupied vertices connect via $\Gamma_{V(i)}$. The values for walk time and fraction of walkable vertices are low (see Figure

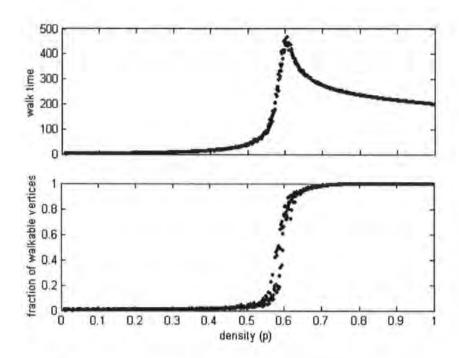


Figure 5.2: Critical percolation transitions. Top: Sharp increases are produced within a given interval ($\rho \approx 0.57 - 0.59$), which corresponds to the regime where clusters are complex (see Figure 5.1).

5.2). However, in a critical range, we observe a sudden increases in the values for both statistics. ²

5.1.2 ρ_c : Implications

Remember, for the pedestrian CA (Blue and Adler, 2000a), we observed fairly sudden changes in behaviour at certain density ranges ($\rho \approx 0.3$). The significance of percolation thresholds to pedestrian models are as follows. To initialise a graph with pedestrians is a Boolean operation because we either place a pedestrian on a vertex or we don't. In this way, all models that use this kind of spatial representation implement a type of Boolean network.

We believe that the results in the previous chapter, where behaviours were observed to suddenly collapse, rather than progressively deteriorate, are the result of unwanted interaction, between neighbourhoods, when they suddenly become connected. In the Blue and Adler (2000a) model, neighbourhood search is defined by a number of variables (gap_{left}, gap_{right},

²With similar kinds of experiments for lattices with very large N it has been shown that the critical point $\rho_c = 0.59275$ (Stauffer and Aharony, 1992; Sole and Goodwin, 2000).

 gap_{centre} , v_{max}), rather than a von Neumann neighbourhood, but according to the statistical properties of the 2-lattice, we should expect sudden, giant-component connections, regardless of neighbourhood type. We believe that the 1-D components in the Blue and Adler (2000a) model are perturbed by the updates of other pedestrians in the lattice. Therefore, when pedestrians join hands and cluster, a single pedestrian move might have effects far beyond the dynamics of its own locality in a single update of the entire population.

In this way, pedestrian updates become limited by a complex, interactive web of overlapping neighbourhoods. For this reason, any self-organising properties of local rules have less effect on the dynamics in the face of increasing density. Such localities therefore make it very difficult to implement sensible behaviours in higher-density regimes.

In order to further demonstrate the significance of the above argument, we illustrate regimes in CA behaviour, which we think correspond to different regimes of percolation (see Figure 5.3, Top). Notice how clusters begin to form in the CA at the point where giant clusters emerge in the boolean network. We also present results for the travel time variation of pedestrians and see that variations for different populations converge within the critical range where a transition in the cluster size is observed. These results lend significant weight to the idea that local rules in CA models get invaded by giant component networks, which then dominate the dynamics of the model.

5.2 Important Representations

This part of the thesis defines a point of departure from some of the methods that are currently used within the field of pedestrian and evacuation dynamics. In the light of observation made thus far, we move our attention away from the work of others, and begin the development of an alternative approach.

We will begin by providing a summary of what we believe are important representations that need to be incorporated into our methods. These should be thought of as related and interdependent objectives, which will determine the nature of the methods we develop.

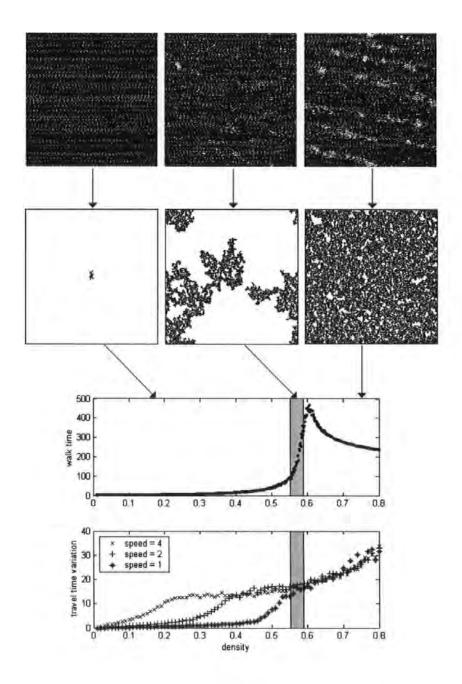


Figure 5.3: Effect of critical point of interaction. Above: to visualise clusters of pedestrians we black out one population and highlight the other as whitespace. Middle: we present percolation away from a single vertex located in the centre of a 2-lattice. We think that the onset of clustering occurs when neighbourhoods begin to connect. Bottom: the critical range $(\rho=0.57-0.59)$ represents an important phase in which the variations of pedestrian travel times converge for different populations. This suggests that the joining of neighbourhoods represents an important limit on the efficacy of search in the Blue and Adler (2000a) CA model.

The following four levels are important:

- 1. Inclusive search: So far in this thesis we have covered a number of techniques, which rely on the concept of search—e.g., the pedestrian CA (Blue and Adler, 2000a) and the Space Syntax approach (Penn and Turner, 2002) and we have identified serious problems with both. The basis of our first criterion is that a search algorithm needs to cover an appropriate local region in order to obtain information from the environment, which allows effective dynamics to be specified. A more inclusive search is therefore required.
- 2. Local dynamics: The reason we need an inclusive search is to find the appropriate vectors of the lattice to cancel. Only when a more inclusive search has been made is this possible. For example, if we only search a subset of vertices in a neighbourhood, then we might be missing essential information. The problem here is again to capture enough information, so that appropriate local dynamics can be specified. We need to produce an algorithm, which can effectively and efficiently compute local moves and model emergent patterns found in real crowds.
- 3. Local-global linking: Using the established attractor field approach guarantees that there is a path, leading from any given origin vertex, to an exit vertex. However, we cannot guarantee that a pedestrian will follow a path, which is simultaneously sensitive to local conditions and longer-range destinations. Therefore, there must be some inherent linkage between the current local situation of the pedestrian, i.e., its origin v_o, its neighbourhood Γ_{v(o)}, and the exit or destination vertex v_d, which will often be a much longer-range target.
- 4. Pedestrian and obstacle representations: Finally, within the context of these criteria, it is important to incorporate representations for other pedestrians and obstacles that define a configuration.

We now introduce the Pulse Coupled Neural Network, which we believe to be an appropriate search algorithm in the light of the above requirements.

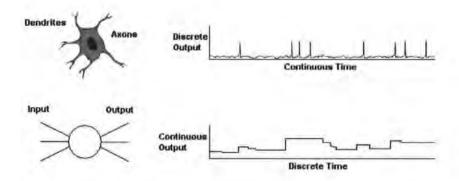


Figure 5.4: Real and artificial neurons. Top: a real neuron, which provides near discrete output in the form of pulses in continuous time. Bottom: a traditional artificial neuron, which produces a continuous output in discrete time.

5.3 PCNN

We introduce Artificial Neural Networks (ANN) and briefly present two alternative kinds of artificial neuron (see Section 5.3.1). We then present and explain equations of a pulsing neuron model from the literature (see Section 5.3.2). We look at the dynamics of a single neuron (see Section 5.3.3) and coupled neurons in 1-lattices (see Section 5.3.4). We then look at the dynamics of coupled neurons in 2-lattices, i.e., network activity and autowave formation (see Section 5.3.5), before a brief summary (see Section 5.3.6).

5.3.1 Artificial Neurons

Image processing is a field of research where biologically-inspired algorithms are widespread because natural visual systems seem to cope easily with the processing of complex, visual patterns. Corresponding to the neural processing of stimuli in a natural system, a widespread artificial approach is to connect artificially represented stimuli to Artificial Neural Networks (ANN).

However, although biologically inspired, the units within such ANN's tend to differ greatly in comparison to biological neurons. We present a few important differences (see Figure 5.4). The most striking difference is the output field. We see that real neurons seem to spike with activity, compared with artificial neurons, which behave continuously. More recently, attempts

are being made to model spiking behaviour, and many examples of these pulsed neurons now exist (e.g., Maass and Bishop, 1998).

Another example is the Pulse-Coupled Neuron, which was first developed on the basis of observations into the mammalian visual system, before being adapted to problems of machine vision (Lindblad and Kinser, 1998). Instead of relying on the adaptation of weighed links to training patterns, as is common with traditional ANN, the pulsed-coupled neuron is an attempt to model spiking behaviour, and exploit patterns of spike synchronisation in neural populations.

We introduce the PCNN in its original image processing context for two reasons. Firstly, the mechanics of the algorithm are better understood in the context of an application. Second, this application requires a similar structure of equations and forms a good basis to explain how the network can be used to search in 2-lattice spaces.

5.3.2 Pulsed Neuron Model

The computational engine relies on local couplings, various kinds of feedback, and stimulation from external pixels in an image plane. This image plane maps to a neural plane, which is a matrix of artificial units. The following equations describe a single, matrix-embedded neuron (Schamschula, Johnson and Inguva, 2000):

$$F_{ij}(t) = e^{-\alpha_F \Delta t} F_{ij}(t-1) + S_{ij} + \sum_{kl} W_{ijkl} Y_{kl(t-1)}$$
(5.1)

$$L_{ij}(t) = e^{-\delta_L \Delta t} L_{ij}(t-1) + \sum_{kl} M_{ijkl} Y_{kl(t-1)}$$
 (5.2)

$$U_{ij}(t) = F_{ij}(t) \left(1 + \beta L_{ij}(t)\right) \tag{5.3}$$

$$Y_{ij}(t) = \begin{cases} 1 & \text{if } U_{ij}(t) > \Theta_{ij}(t) \\ 0 & \text{otherwise} \end{cases}$$
 (5.4)

$$\Theta_{ij} = e^{-\gamma_{\Theta}\Delta t}\Theta_{ij}(t-1) + Y_{ij}(t)$$
(5.5)

where F_{ij} , L_{ij} , U_{ij} , Y_{ij} , Θ_{ij} define the Feeding Field, Linking Field, Internal Activity, Output Field and the Dynamic Threshold, respectively. We now describe the equations in turn and explain how they relate to each other:

- Feeding Field: The first term defines a leaky integrator, which is an accumulative, but leaking value of the previous output. This value is obtained from two external sources;

 1) the stimulus S(0,1) and 2) the weighted couplings (W_{ijkl}) to the output fields (Y_{kl}) of other, local neurons.
- Linking Field: This equation has the same structure as the feeding field, but does not receive any pixel stimulus. Other differences are the constant values, which control the speed at which the leaky integrator leaks. The feeding and linking components have independent values for this, denoted by $\alpha_F \Delta t$ for the feeding field and $\sigma_L \Delta t$ for the linking field.
- Internal Activity: Both the feeding and linking fields link information from surrounding neurons, but a single neuron stimulus is fed directly through the feeding field. Other neuron stimuli are therefore hidden to a neuron. However, both the stimulus and input from other neurons are integrated over time. This results in a build-up of internal activity, which is then compared with a dynamic threshold to determine the value of the output field.
- Output Field: This is a simple if-else, which produces an output of 1, if the internal activity is greater than the dynamic threshold Θ , and 0 otherwise. This is the spike, a simple Boolean approximation to spike behaviour found in real neurons.
- Dynamic Threshold: This value depends on its own previous output. We see that if the neuron has an input strong enough it will pulse. For example, if we initialise the threshold to 0, then, if the stimulus is positive a pulse is given at the initial time step. The feedback from Y_{ij} ensures that a refractory period can follow a pulse.

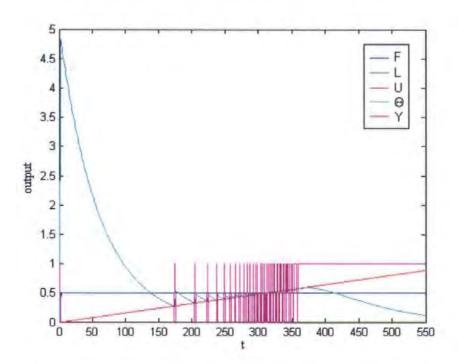


Figure 5.5: Dynamics of a pulse-coupled neuron. Output values for feeding field (F), linking field (L), internal activity (U), dynamic threshold (T) and the output field (Y). The dynamic threshold bounces on the internal activity. This produces an increasing amount of pulses, separated by decreasing periods of refraction, until the point of saturation occurs, where the output is fixed at 1 and the dynamic threshold decays exponentially.

5.3.3 Pulsed Neuron Dynamics

We demonstrate these principles at work by implementing the above equations and presenting the dynamics of a single neuron (see Figure 5.5). As we can see, the dynamic threshold and the output field interact to produce a number of pulses, punctuated by decreasing periods of refraction. The neuron therefore becomes saturated over time; the pulse field continuously fires. We can see that, once saturation occurs, the dynamic threshold decays exponentially. These model neurons tend to become saturated because of the time-dependant decay mechanism of leaky integrators. However, a range of behaviours can still be found in such models by tuning constants appropriately.

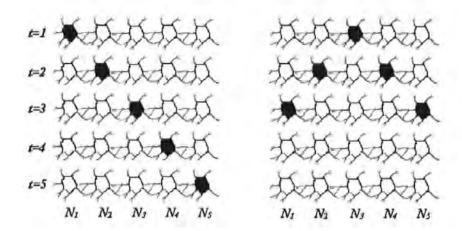


Figure 5.6: Autowaves in non-periodic, 1-lattice media. Two autowaves are presented. Left: a 1-directional autowave. Right: a 2-directional autowave, which departs from a given origin and travels in opposite directions.

5.3.4 Pulse-Coupled Dynamics: 1-lattice

An interesting property of networked artificial neurons, if parameters are set to appropriate values, is the occurrence of a series of spikes, which travel away from the source neuron. This series is known as an *autowave*. This directional transmission of spikes is made possible by the refractory period, which as we have seen, prevents the neuron from spiking immediately after it has done so.

In order to observe autowave formation, we implement a non-periodic 1-lattice network of neurons N with immediate local connections (k = 2, and k = 1 at the boundary). We present results of two simple simulations in a space-time diagram (see Figure 5.6). In the first example, we provide input to N_1 . At t = 2, the boundary demands that only N_2 receives input from N_1 , which enters a refractory period, while N_2 processes the input received form N_1 . At all time steps each neuron that pulses enters a refractory period and therefore the spikes produce a clear wave, which travels through each neuron left to right in 5 = 1 + 1 + 1 + 1 + 1 time-steps. In the second example, we provide input to N_3 . The boundary does not interfere with this neuron and the wave travels in opposite directions away from N_3 . The wave travels through each neuron in 3 = 1 + (4/2) time-steps.

5.3.5 Pulse-Coupled Dynamics: 2-lattice

5.3.5.1 Neighbourhood radius

We organise a 2-lattice network of neurons with Moore neighbourhood Γ connections so k=8. The radius r of a neighbourhood would increase the computational load on the network—e.g., with r=2, the number of connections goes from 8 to 24=(8+16), capturing the immediate Moore neighbourhood $\Gamma(S)$ of a subgraph $S(G)=\Gamma$. However, for our application we only use neighbourhoods where r=1. ³

5.3.5.2 Use of Weights

Most neural networks are associated with the idea of weighted inputs. With a traditional ANN, it is common that the network has a random set of initial weights, which are then trained. For example, with the Multi-Layered-Perceptron, test data is presented and training data is attached to the network to help calculate a network error. This error is then propagated back through the network and weights are adjusted to some degree in order to minimise error. This general process of training is repeated until the error of the network is sufficiently low; i.e., until the distributed encoding in the weights contains a sufficient representation of the training data.

The use of weights in the PCNN is totally different. Here the idea of training is irrelevant. We only consider a predefined set of weights, which then remain fixed throughout a run. The weights used for the current project are simple 1/r connections. The further out a neuron is from the centre neuron, the less weight it will have on the behaviour. However, because our radius r = 1, we see that the weights $W_{ijkl} = r = 1$. As long as there are couplings, the relationships between the various equations produce self-organising, interactive behaviours.

³For more information on details of r for different applications see (Lindblad and Kinser, 1998).

5.3.5.3 Network Activity

From our demonstration of autowaves in 1-lattices, we know that autowave behaviour, as a result of coupled interactions, will vary depending on the source of the spikes (see Figure 5.6). We now demonstrate this with a more complex pattern of input pixels from an image plane. We choose two input images to demonstrate network activity and autowave formation. The input images contain an acronym for Centre for Neural and Adaptive Systems (CNAS). However, the first acronym has a photograph of a section of brain acting as a backdrop (Image 1), while the second has a white backdrop (Image 2). The image plane I corresponds to the neural plane N so that the feeding stimulus $S_{ij} = I_{ij}$.

In order to gain a picture of network activity, we can simply count the number of pulsing neurons over time. In this way, the PCNN converts images into time series (Johnson, 1994):

$$T[t] = \sum_{ij}^{N} Y_{ij} \tag{5.6}$$

The value for T is related to the value of pixels in the image. However, the PCNN is not a passive processor, because the linking field binds local information through Moore neighbourhoods. Local neurons therefore tend to fire synchronously, particularly where input pixels are of a similar value. The 1-D time series thus represents particular properties of the 2-D image and acts as an image signature⁴. We present example network activity for a PCNN according to the above PCNN equations and record T, which we label 'network activity' (see Figure 5.7).

For Image 1, there is a large initial activity, which quickly leads to a refractory period for the entire network of approximately 220 time steps before any more neurons begin to enter a pulsing regime. For Image 2, initial activity lasts longer (approximately 20 time steps), before all neurons have reached the refractory period. Compared with Image 1, which has a network that enters the refractory period earlier, Image 2 has a network with a refractory period of

⁴One useful property of these networks is that they are robust to image rotations—a rotated image produces similar or identical signatures.

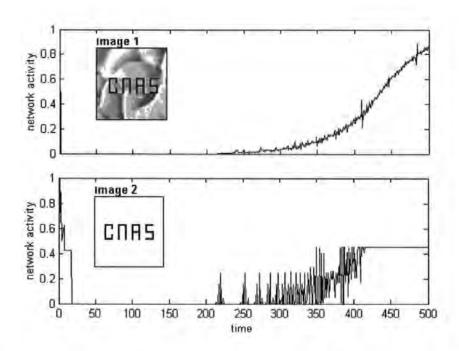


Figure 5.7: Time series signatures. Top: a more intense input image produces a longer period of network refraction. Bottom: the period of refraction is less because the initial behaviour of the network last longer.

only 200 time steps, before any more neurons begin to pulse. These differences represent the different inputs provided to the network. 5

We take snapshots of network activity at different time steps for both $Image\ 1$ and $Image\ 2$ (see Figure 5.8). The PCNN does not produce a pure autowave in the case where there are significant inputs from lots of different areas of the input image ($Image\ 1$). There are many pulsing neurons at t=1 and no clearly identifiable wave is produced. However, the wave phenomena is still active; we can see at t=2 that seven non-pulsing neurons at t=1 are active, whereas all pulsing neurons at t=1 have entered a refractory period. We see the same principles at work for $Image\ 2$; at t=1 all N_{ij} with $S_{ij}>0$ pulse, but the network provides a clean copy of the CNAS acronym. At t=2 these neurons refract and the surrounding neurons pulse.

Here we see the beginning of a clean, well-defined autowave. The wave travels away from

⁵To avoid any confusion, we should mention that the input S_{ij} is received from the image plane I_{ij} where 1 represents black and 0 represents white. These are inverted values compared with image processing applications, but are more consistent with how we have referred to pedestrians being present or absent on a given vertex.

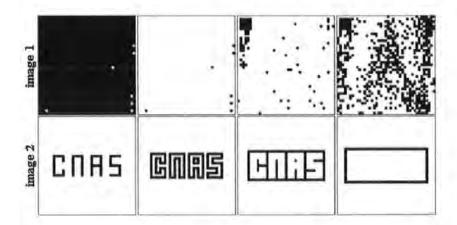


Figure 5.8: Autowave behaviour. In Image 2 we see a clear, expanding autowave spreading from the positive input pixels in the shape CNAS.

the original source until parts of the wave collide and then enter refraction; waves do not pass through each other. The wave then continues to travel outwards passing over other neurons, which have not yet pulsed. These are clean, expanding autowaves, which travel across the entire neural network.

5.3.6 Summary

In this section we have:

- presented and explained a pulsing neuron model.
- demonstrated autowave behaviour in a 1-lattice and a 2-lattice.

The details we have presented will be important when we explain changes we make to the equations. We now take a pause to consider the general relevance of the PCNN, and suggest that this kind of algorithm will form a sound basis for the development of egress models, which capture both the local and global properties of real crowds.

Above (see Section 5.2), we identified four important levels of representation in the modelling of egress and the specification of pedestrian dynamics. We now return to the four levels with brief suggestions about how the PCNN might be used. The aim is not to provide definitive answers, but suggest ideas, which we later implement fully.

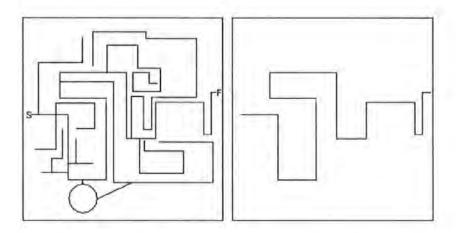


Figure 5.9: PCNN as a shortest path algorithm. Left: the image plane of a complex maze of paths from a start pixel S to a finish pixel F. Right: the trace-back path discovered in the PCNN presents an optimal path between S and F. Adapted from Caulfield and Kinser (1999).

5.4 PCNN: Relevance to Egress Modelling

1. Inclusive search: Unlike search in other models (Blue and Adler, 2000a; Penn and Turner, 2002), the PCNN provides an inclusive search. Rather than relying on a limited subset of space, which we have shown to be problematic, the autowaves explore in all possible directions away from a vertex origin. Caulfield and Kinser (1999) show that this explorative feature of the network can solve complex maze tasks, not by recourse to specific pre-coded rules, but by executing a policy of explore everything (Caulfield and Kinser, 1999). In this approach, the autowave travels over a maze image, from an origin neuron, connected to a 'start' pixel. The wave travels away from the neuron as the simulation progresses and at some point reaches a predefined destination, defined by a 'finish' pixel. When this 'finish' pixel pulses, the simulation halts (Caulfield and Kinser, 1999). By executing a minimal search along the threshold values in the network, a trace-back path is discovered. We present an example of the kind of result (see Figure 5.9). This demonstrates that the inclusive search property of the autowave is potentially capable of coping with complex spaces.

This is an important behaviour, which we will need exploit in order to model egress scenarios in arbitrarily complex spaces.

- 2. Local dynamics: As autowave search moves away from a vertex origin, we will be able to specify occupied pedestrian vertices as the source of autowaves. In this way, local dynamics may be captured during the initial stages of search as the wave progresses through different regions.
- 3. Local-global linking: We will also be able to specify at which vertex a destination is set. In this way, we will be able to combine the longer-range process of egress with local pedestrian dynamics. The distinction between local and global is merged with the image of an expanding autowave.

We will therefore need to ask ourselves at what point in autowave search we apply *local* rules. We showed earlier that dynamics in localities are brittle on 2-lattice spaces. One aim will be to develop a more robust method.

4. Pedestrian and obstacle representations: We have seen above that the PCNN behaves through the synchronization of spikes. This boolean approximation should be very useful for our purposes. We will see in the next section how this behaviour allows a simple approach to the specification of dynamics, which include configurational and pedestrian obstacles.

The following section is intended to demonstrate the use of autowave search, and provide details of techniques, which implement ideas we have suggested here. We will concentrate on relatively simple scenarios, but will build upon them later in the thesis (see Chapter 6).

5.5 Robust Autowaves

Firstly, we consider the possible pitfalls of the PCNN equations for our own domain of application (see Section 5.5.1), and then prune equations to improve search robustness (see Section 5.5.2). Then, as a precursor to how they might be used to model pedestrian navigation, we demonstrate explorative behaviour of the new equations in complex spaces and relate behaviour to further definitions in Graph Theory (see Section 5.5.3).

5.5.1 Pitfalls in PCNN Equations

In the above sections, we have introduced the PCNN in an uncritical light, but before taking the PCNN as a search engine, we need to consider more closely the behaviour of the dynamic threshold. We have mentioned that the leaky integrator of the dynamic threshold allows neural refraction and, therefore, networked neurons can produce autowaves from a given origin v_o to a given destination v_d . Also, the fact that an integrator leaks, provides us with a function, which may be used to trace an optimal path in space (Caulfield and Kinser, 1999).

However, we can only guarantee the trace of a path if we can tune parameters to produce a clean, expanding and full autowave. Autowaves from images do seem to be clean (see Figure 5.8 above), but as Lindblad and Kinser (1998) identify, wave features may fragment. In this kind of neuron model, parameters affect the strength of communications between neurons. Depending on how parameters are set, three general scenarios are possible, which relate to three interpretable pedestrian behaviours:

- 1. In the case where a clean and full autowave expands from v_o to v_d , the function will be a simple, unperturbed decay shape.
 - (a) In this case, a pedestrian will be able to walk along the valid values and reach the exit. We see that the trace-back through the neurons has reconstructed the decay shape of a single leaky integrator (see Figure 5.10, Top).
- 2. If the time decay constants are too strong, then the wave may not reach v_d , because it runs out of energy. The temporal information provided is the same in all directions.
 - (b) The pedestrian location defines a local minimum, out of which all directions appear valid (see Figure 5.10, Middle). The only way out of the minima is, therefore, to search randomly.
- 3. If we set parameters, which allow a neuron to pulse more than once, before the wave front hits v_d , local optima in the network can result.

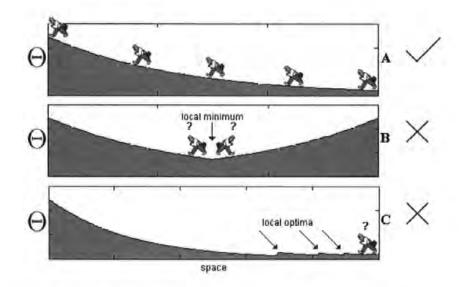


Figure 5.10: Walks determined by threshold functions. Top: the pedestrian will be able to climb towards the exit cell without a problem. Middle: the pedestrian will not be able to follow a particular direction if the wave does not reach the exit, reducing behaviour to a random search. Bottom: if any neurons in the wave are allowed to pulse, more than one local optima could be created, and the pedestrian will not be able to move to the exit.

(c) The pedestrian will not be able to reach the exit because, according to the traceback, it will 'think' that it has already done so (see Figure 5.10, Bottom).

5.5.2 Enhanced Autowave Equations

In order to guarantee desirable autowaves, we prune the equations:

$$F_{ij}(t) = S_{ij} (5.7)$$

$$L_{ij}(t) = e^{-\delta_L \Delta t} L_{ij}(t-1) + \sum_{kl} M_{ijkl} Y_{kl(t-1)}$$
(5.8)

$$U_{ij}(t) = F_{ij} + L_{ij} (5.9)$$

$$Y_{ij}(t) = \begin{cases} 1 & \text{if } U_{ij}(t) > \Theta_{ij}(t) \\ 0 & \text{otherwise} \end{cases}$$
 (5.10)

$$\Theta_{ij} = e^{-\gamma_{\Theta}\Delta t}\Theta_{ij}(t-1) + V_{\Theta}Y_{ij}(t)$$
(5.11)

There are three main changes in the new PCNN equations. Firstly, in the image processing

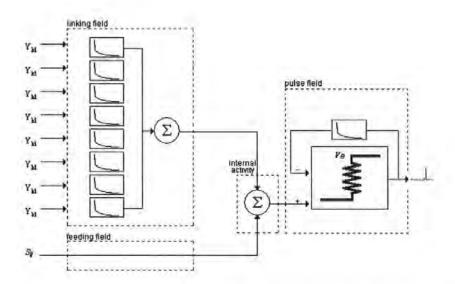


Figure 5.11: PCNN scheme for pure autowaves. The image shows the structural relationships between the equations. Four general areas are shown—the linking field with 8 leaky integrators of input from connecting neurons, the feeding field, which is a single input value, the internal activity and the output field, which contains the spike function and dynamic threshold.

application, F_{ij} sums links to neighbouring neurons with S. Here we only use S, which will represent a pedestrian location v_o . This allows us to reduce feedback to neighbouring neurons. Secondly, instead of U_{ij} being a non-linear combination, it is now the sum of the feeding and linking fields. By turning off the stimulus as soon as a neuron pulses, we ensure that all information used in the threshold comes from the linking field only. This further reduces feedback, and the likelihood of local optima. Finally, we introduce a new term in the dynamic threshold, where V_{Θ} is a large amplification factor. This constant provides a mechanism to ensure that the threshold value is very high as suggested by Caulfield and Kinser (1999). This has the effect of extending the period of refraction in a neuron and hence increasing the robustness of the autowave. We present a schematic relationship between each of the five equations (see Figure 5.11). We also present the full PCNN algorithm (see Table 5.1), which is used to produce the rest of the results presented in this chapter.

With this algorithm, we will be able to explore large spaces without the problems inherent to the PCNN designed to proc. By prolonging the refractory period, we have enhanced the usefulness of the PCNN as an explorer of simple, 2-D graphs.

Before demonstrating how the PCNN will be able to search through complex space, we

Table 5.1: PCNN Algorithm for Clean, Expanding Autowaves

```
/*Initialisation*/
     For every neuron \neq v_o
           SpikeTime = F = S = L = U = Y = \Theta = 0;
     End For
     For neuron vo
           SpikeTime = F = L = U = Y = \Theta = 0;
     End For
/*Main Loop*/
     While Halt == false
           For every neuron (ij)
                Execute the Feeding Field equation;
           End For
           For every neuron (ij)
                Execute the Linking Field equation;
           End For
           For every neuron (ij)
                Execute the Internal Activity equation;
           End For
           For every neuron (ij)
                Execute the Spike Threshold equation;
           End For
           For every neuron (ij)
                Execute the Output Field equation;
                \mathbf{If}(S == 1)
                    S=0;
                End If
                If(Y == 1)
                    SpikeTime = t;
                End If
           End For
           1f(v_d(Y) == 1)
                Halt == true;
           End If
      End While
/*Parameters*/
```

 $\delta = 1$, $\gamma = 0.1$, V = 10000, $\Delta t = 1$;

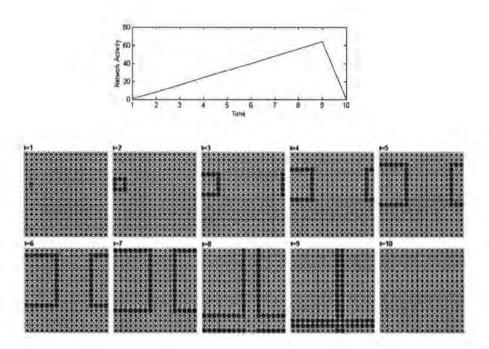


Figure 5.12: Example autowaves on 2-lattice PCNN. In a 17x17 2-lattice neural network an input is initialised to 1 at v_o (x = 2, y = 11) until the whole of the space is covered. At t = 1 the number of pulses T = 1, expanding to T = 8 at t = 2. Then the wave expands according to T = T(t - 1) + 8, the series being 1, 8, 16, 24, 32, 40, 48, 56, 64, until all 289 neurons have entered the refractory period at t = 10.

present a simple, expanding wave scenario to demonstrate the exploratory behaviour of a PCNN with periodic boundary conditions (see Figure 5.12). Notice a simple, linear signature for the network, which contrasts with the image signatures presented above (see Figure 5.7). The clean signature reflects the clean, unbroken nature of the wave. However, were the space to have any complexity, we would expect network activity to look more noisy. We will now look at some more complex examples. ⁶

5.5.3 Complex Search

At this stage we are now confident that, as long as we set the amplification factor in the dynamic threshold to a high value, we get prolonged autowaves, which explore the entire space. An important aspect of these equations is that they be utilised to explore *complex* spaces.⁷

⁶All signatures are produced from equation 5.6.

⁷In the next chapter we present a method to explore arbitrarily complex spaces.

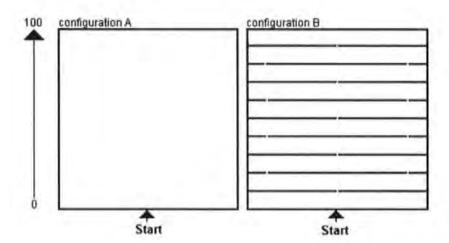


Figure 5.13: Simple and complex configurations. We hand code two 2-lattice configurations both with a border and therefore non-periodic. Configuration A is an empty space, wheareas configuration B contains 9 horizontal obstacles, which connect the East and West boundary. Each obstacle has n slits where n oscillates in a 1-2-1 cycle. We place a start pixel at location x=50, y=2.

We achieve this by fixing the output Y_{ij} of a set of neurons, which correspond to obstacles O in a given configuration. The absence or presence of obstacles in a configuration plane are represented by a Boolean switch $O_{ij} = \{0, 1\}$. We then fix the possible states in the output field of each neuron $Y_{ij} = \{0, \Omega\}$, where $\Omega = 1 - O_{ij}$. This fixes the neuron output field in 'obstacle' neurons to 0 and leaves 'non-obstacle' neurons free to pulse and refract. By doing this, the wave is not able to travel through obstacles, but explores around them instead.

For example, consider the two non-periodic configurations A and B (see Figure 5.13). The first shows an empty space, whereas the second contains horizontal objects in the space with n slits in each which oscillate—(1-2-1-2-1-2-1-2-1)—from the bottom to the top of the plane. We set v_o at the 'start' position and run the network for 380 time steps. This demonstrates the explorative behaviour of our PCNN in different configurations. We plot time series information, which captures the dynamics of the wave fronts as they travel through the corresponding configuration (see Figure 5.14).

We observe that the wave for configuration A is much simpler than the one for configuration B. This is because, in configuration A, there are no obstacles other than the non-periodic border. The wave initially expands in three directions (N, E, W), reaching a peak size, until

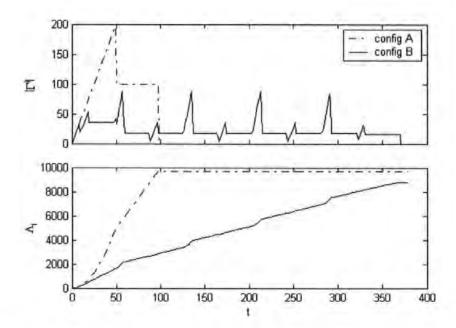


Figure 5.14: Autowave configuration signatures. Top: we plot the signatures over 380 time steps for the autowave, which correspond to the size of the i^{th} neighbourhood $|\Gamma^i|$, where $i = t = \{1, 2, ...n\}$. This is because the autowave explores all paths away from v_o . The shape of the autowaves over time is explained in the text. Bottom: we plot the distribution sequence Λ_i , which corresponds to the sum of the wave front size. See full definitions of Γ^i and Λ_i in text.

two parts (E, W) of the front reach the border, where the output fields are zero (Y = 0). The wave is thus cancelled at these positions on the wave front and the time series suddenly shrinks at t = 50. Then there is a period where the wave remains a constant size, being contained by the boundaries, until all of the neurons enter the refractory period where $T = |\Gamma^i| = 0$ (see Definitions below) at t = 100. With configuration B, the wave oscillates through four cycles as it expands into the space between the obstacles and contracts at the points where slits occur in the obstacles.

In the context of pulsing activity in the neuron population, we see that behaviour naturally corresponds to further definitions within graph theory⁸:

Definition 10 The neighbourhood Γ_S of of a connected subgraph S(G) includes vertices adjacent to, but not including $v_i \in S$. This is the neighbourhood of a neighbourhood.

Definition 11 The i^{th} neighbourhood of v. In the special case where $S = \Gamma(v)$, $\Gamma(S) =$

⁸Again, definitions are based on previous work on graphs (Watts, 1999; Wilson, 1985)

 $\Gamma(\Gamma(v)) = \Gamma^2(v)$ or more generally $\Gamma(\Gamma^{i-1}(v)) = \Gamma^i(v)$, the i^{th} neighbourhood of v, so that $\Gamma^0(v) = v$.

Definition 12 The sequence $\Lambda_j(v) = \sum_{i=0}^j |\Gamma^i(v)|$ for $0 \leq j \leq j_{max}$ is the distribution sequence for v, where $\Lambda_{j_{max}}(v) = |G|$.

Notice that an accumulative value of $T = \Lambda_i$, for a clean, expanding autowave. In this way, the PCNN described in this section automatically implements a 2-lattice search away from any given origin.

5.6 Summary

In this chapter we:

- demonstrated how 2-lattice properties determine limits to local search.
- explained the sudden, catastrophic failure of CA dynamics.
- provided a list of appropriate levels of representation for egress and pedestrian dynamics.
- introduced the mechanisms and behaviour of the PCNN, which is compatible with these levels, and capable of searching beyond limiting localities.
- introduced a more robust PCNN and demonstrated its search capacity within complex configurations.
- presented PCNN network statistics, in the form of search signatures, and pointed out how these relate to further Graph Theory definitions.

We have concentrated on *inclusive search*, *local-global linking* and *obstacle representations*. In the next chapter, we will begin to concentrate more on *pedestrian representations* and *local dynamics*. We will develop techniques derived from autowave search.

Chapter 6

Egress Modelling

One of the challenges in building a model is to produce patterns of pedestrian-pedestrian interaction across all density ranges. Also, a challenge is to produce typical pattern formations between pedestrian crowds and the environment they occupy. These challenges have not yet been sufficiently met by models within the pedestrian and evacuation modelling community. In this chapter we present a set of techniques, which offer an alternative approach to model specification, in order that such models may be developed.

While implementing a more efficient version of autowave search (see Section 6.1), we present the first of these techniques. We look at the limitations of autowave search in multi-agent scenarios and offer a method designed to overcome these limitations, which is based on the exploitation of information in search signatures, and the definition of dynamic boundaries (Section 6.2). We present network-based simulation models to demonstrate the potential power of our tools (Section 6.3), before briefly summarising the chapter (see Section 6.4).

6.1 Efficient Autowaves

In chapter five, we introduced a search algorithm, based on a Pulse Coupled Neural Network, and used it for complex spatial exploration. In this section, we present a much more efficient CA-based autowave, which can compute arbitrarily complex paths, i.e., paths in any single-component graph of any complexity. We introduce 2-D CA (see Section 6.1.1) and present a CA method for the efficient generation of autowaves (see Section 6.1.2). We provide details of a traceback algorithm, which can identify the optimal paths found by the expanding autowave (see Section 6.1.2). We demonstrate search behaviour in large, complex spaces (see Section 6.1.3). The methods in this section should be thought of as tools, out of which pedestrian and evacuation models can be constructed, rather than models per se.

6.1.1 2-D Cellular Automata

For image processing applications of the PCNN, synchronous updates of the entire neural plane are essential, because each pixel is as important as any other¹. In order to significantly reduce processing we now look towards the use of CA rules, which can be very cheap computationally and, therefore, a choice method of implementation. Before we present details of our approach, it will be useful to consider other 2-D CA, in order to introduce the kind of computations we are using.

The assignment of a state, at time t, to cell a_{ij} , is deterministic. For example, with a von Neumann neighbourhood (k = 4), this takes the form:

$$a_{ij}(t) = f[a_{ij}(t-1), a_{ij-1}(t-1), a_{i+1j}(t-1), a_{ij+1}(t-1), a_{i-1j}(t-1)]$$

$$(6.1)$$

where f defines a set of processing rules, which determine the logic of the behavioural transitions and $t\{0, 1, 2, 3, ..., n\}$.

All simple CA of this kind have a similar general form, although the number of inputs may vary. For example, a Moore neighbourhood (k = 8) will have twice as many, although state transitions may be influenced by only a single cell. As an example of the latter, consider the rules in Table 6.1, which describe changes of environmental state, according to the behaviour

¹In chapter 5, the neurons in a PCNN, which at any given time step are members of the wave front, are a small fraction of the network. For example, in the previous chapter (see Figure 5.14), the largest wave front contained 200 neurons, but the number of neurons in the network was 40,000.

Table 6.1: Langtons' 'Ant' Cellular Automata

- 1. If $(a_{ij}(t) = = true)$ Rotate ant 90° anti-clockwise.
- 2. If $(a_{ij}(t) ==$ false) Rotate ant 90° clockwise.
- 3. Move forward one step. Update t = t + 1.
- 4. If $(a_{ij}(t-1) = \text{true})$ Then $a_{ij} = \text{false}$. Else If $(a_{ij}(t-1) = \text{-false})$ Then $a_{ij}(t-1) = \text{-true}$

of a 2-D CA, known as Langton's Ant (Cohen and Stewart, 1994). We provide these rules for two reasons; 1) because they exemplify simple Boolean mapping and 2) because they can be used to introduce interactions between agents via an environment, which will form an important part of work later in the thesis². These kinds of Boolean state-transitions are typical of other CA (Langton, 1990; Suzudo, 2002; Wolfram, 1984).

We implement Langton's Ant and run two separate simulations on a (80×80) 2-lattice with periodic boundaries. The first simulation is a single-ant scenario, whereas the second is a double-ant scenario. In the single-ant case, we observe the formation of a *highway* pattern, which begins at around 10,000 time-steps, but we allow the simulation to continue for 11,000 time-steps, in order that the pattern is easily observable (see Figure 6.1, Left). In the second case, we notice how the highway pattern forms much earlier on in the simulation (see Figure 6.1, Right). This is a trivial example of *stigmergy*—agents (here the ants) affect each other through an environmental medium, rather than by direct contact.

Having introduced simple CA state transitions, we now concentrate on a CA for the production of autowaves.

6.1.2 CA-Based Autowaves

Unlike the refractory period of a pulse-coupled neuron, which was determined by more complex exponential-decay mechanisms, we can determine that once a spike occurs at a given vertex this state can never occur again. This is desirable in order that local optima are avoided

²In Chapter 7 we will use such Cellular Automata in a genotype → fitness mapping

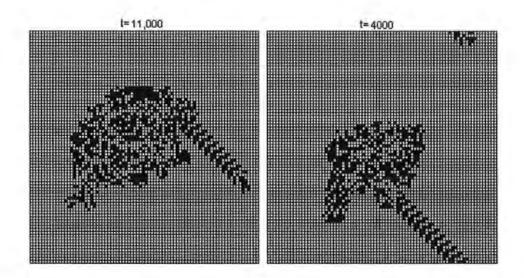


Figure 6.1: Langton's ant. This automaton demonstrates very simple, 2-D state transitions. We provide two examples on an 80×80 2-lattice. Left: a single-ant system. The ant is initialised at cell $a_{40,40}$. Right: a two-ant system. An additional ant is initialised to $a_{40,30}$. The highway forms much earlier in the two-ant system.

(see Section 5.5.1). In order to model autowave properties of the PCNN, we need k = 8 inputs from a Moore neighbourhood Γ . We provide a combinatorial logic diagram (see Figure 6.2), which is also represented abstractly as an s-component. This logic archives similar external behaviour to a pulsed neuron, but with beneficial differences.

Consider the eight-bit input* to be 01000011. If we fix a value of 1 on the output from the XOR, then output* will become 1. We can see from the feedback that at the next time step, because the XOR receives input from a FANOUT operation, the XOR gate can never produce a 'true' value again. The identities in the FANOUT operation are always identical, but a 2-way XOR 'true' output depends on the inputs to it being different, which is impossible. The AND gate can never produce a true value other than in the special case where the output from the XOR is 1. By doing this, we guarantee that a vertex is allowed to produce only a single pulse, which will depend also on processing a single, positive value from the k = 8 inputs of the neighbourhood Γ . These relationships describe the coupling mechanisms, which model similar feedbacks to the pulse-coupled neurons of the previous chapter, but with much more efficient processing elements.

Having described the simple pulse-coupled combinatorial logic of s-components, we arrange

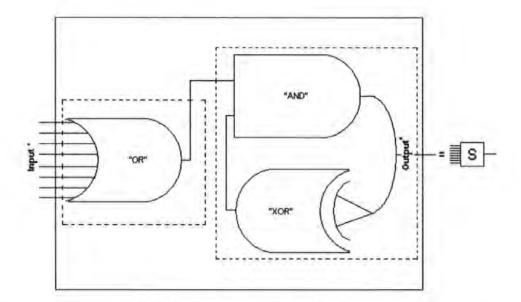


Figure 6.2: Pulse-coupled logic for single vertex. The eight-way OR processes inputs (input*) from a Moore neighbourhood, which map to a single output. This output and output from an XOR are fed into an AND. This has two associated fan-out operations. The first splits to produce output* and another wire, which fans out into an XOR gate, the feedback controller. the output* will feed into another OR—as an element of input* to a neighbouring vertex. We represent these processes more abstractly as s-components.

them into a larger network (see Figure 6.3). Consider a situation where all wires for all vertices are zero. There is no change in this network over time because the first condition of the AND gate will be false for all components. However, if we initialise the output of the XOR to 1 for all neurons and set the output of AND in a single vertex v_o to 1, then, at t + 1, $\Gamma_{v(o)}$ will also have an output* value of 1, whereas the feedback means that output*= 0 in v_o . This is the first step of autowave expansion. Where we iterate this procedure over time $t\{1, 2, 3 ... n\}$, the t^{th} neighbourhood $\Gamma^t(v)$ will experience excitation as the wave expands outwards from the origin.

This autowave expansion can be achieved in a synchronous update, but to improve efficiency it is enough to execute only the neighbours of pulsing vertices at t + 1. Therefore we begin by defining a dynamic set of vertices V. At the first time step, $V = v_o$. For each time step we then execute a minimal search around the neighbourhoods $\Gamma_{V(i)}$ and load any vertex with output*= 1 into V. It now becomes clear why we need the feedback within the s-component. This allows |V| to remain small so that, once we delete the 'pulsed' vertex contents, they

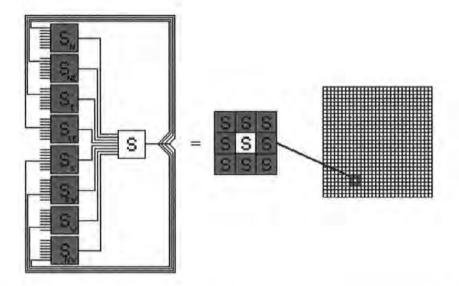


Figure 6.3: Network logic. We see that the network is the same arrangement as for the PCNN. A Moore neighbourhood contains a collection of s-component vertices.

are not reloaded and search remains local to the wave front expansion. This is equivalent to loading the i^{th} neighbourhood at time t, rather than executing computations from the accumulative distribution sequence $\Lambda_j(v)$.³ We now briefly demonstrate search behaviour.

6.1.3 Example Search in Arbitrarily Complex Configurations

Where a vertex corresponds to an obstacle, we fix output*= 0, in the same way as previously (see Section 5.5.3). We use four example configurations with varying degrees of complexity (see Figure 6.4, Top) and demonstrate corresponding search behaviour by plotting $|T(t)| = |\Gamma(t)|$ (see Figure 6.4, Bottom).

Observing the shape of the time series for the different configurations, we see various patterns. The 'Pythagoras Theorem' configuration has a lot of structure, whereas the 'Face or Vase' configuration has slightly less. The 'Cornwall' and 'Circular Complex' configurations have more complex signatures. These signatures represent search performed by the autowave. This demonstration of a range in search capacity is very important in the context of an evolutionary scenario—evolutionary approaches crucially depend on variation and a fitness function

³We defined the neighbourhood of neighbourhoods and the distribution sequence at the end of the previous chapter.

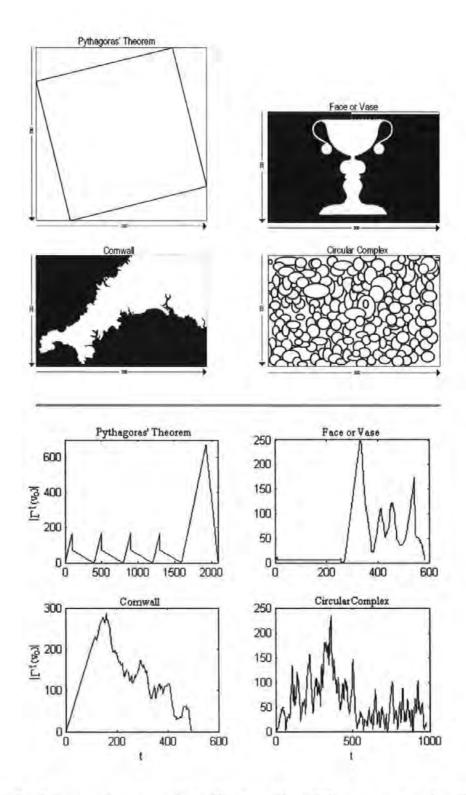


Figure 6.4: Configuration signatures. Top: We choose four different-structured configurations. Each configuration is designed so that the whitespace forms a single component subgraph (Top). Bottom: Four corresponding time series of varying complexity are generated through autowave search. In the top right-hand corner of each configuration we place v_o , at t=0. The wave exploration algorithm searches a configuration while $|\Gamma^t(v_o)| > 0$.

has to cope with the variation available from within a given population (see Chapter 7).

From the presentation of the algorithm in this section, it should be clear that any single-component graph is explored within the constraints of a computational environment. We now present the nature of the trace-back search, which occurs when a wave travels from v_o and 'hits' a destination vertex v_d .

6.1.4 Traceback Algorithm

The aforementioned autowave advances away from v_o at each time-step ($\Delta t = 1$). As the wave advances, we record on each pulsed vertex a value of the pulse time t. When the halting condition is met $(v_{d(output^*)} = 1)$, pulsed vertices represent a time-dependant field. For trace-back search, we start at the destination vertex v_d and search Γv_d . We search for vertices with $t_{v(i)} = t_{v(d)} - \Delta t$, i.e., vertices which pulsed in the previous time step. We then load these vertices to V. We repeat this procedure for $\Gamma_{V(i)}$, until v_o is reached. Vertices $v_{ij} \in V$ define a connected subgraph S(G) from v_o to v_d . It follows from this description that because $\Delta t = 1$, then v_d contains a value D, which is the diameter of S(G).

Definition 13 The Graph diameter is the step distance between two maximally separated vertices. This is equivalent to the time between v_o and v_d , where $\Delta t = 1$.

We illustrate the concept of graph diameter with an example configuration in the shape of a smiley face (see Figure 6.5). The figure shows a graph G with vertices subject to fixed output*= 0, represented in dark grey. We set v_o to the coordinate of the right eye of the face and v_d to a coordinate above the face. near the northern boundary of G. We show the subgraph S(G), which has a diameter of 37.

6.1.5 Summary

In this section we:

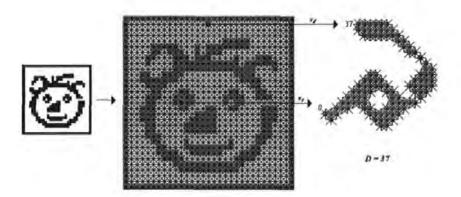


Figure 6.5: Traceback subgraph. We create a smiley-face configuration and set the origin of the pedestrian to the right eye of the face. A subgraph S(G) is obtained from autowave expansion and trace-back search, which links v_o to v_d . In this example, the diameter D = 37.

- introduced a 2-D CA and looked at Boolean, feedback couplings, which combine to produce pulsing behaviour in single-vertex 's-components'.
- removed a large part of the complexity associated with the PCNN and the dependence on synchronous updates across the entire network.
- specified network organisation of s-components, and their ability to search arbitrarily complex configurations, a crucial pre-requisite for developing useful fitness functions for an evolutionary approach.
- outlined details of trace back search, which returns a subgraph, the diameter of which
 is the distance between a vertex origin v_o and a vertex destination v_d.

This section presents the basic methodological components in a network-level approach to the implementation of pedestrian and evacuation dynamics. We believe that more realistic patterns may be achievable with this shift in perspective. Existing models attempt to capture either emergent pedestrian pattern formations or evacuation processes, by using local or global approaches, like CA and attractor fields, respectively.

We believe a synthesis in these levels is required. The following sections begin to offer a way forward in terms of synthesising emergent pedestrian pattern formations and transferring such kinds of patterns to more complex lattice domains.

6.2 Multi-Agent Scenarios

We begin by introducing single-agent movement to illustrate path choices in the face of multiple optima (see Section 6.2.1). We provide a method of representation for multi-agent scenarios (see Section 6.2.2). We identify the fundamental limits that the 2-lattice places on such representations (see Section 6.2.3). In order to address these limits, we identify the nature of statistical information in time-series, search signatures (see Section 6.2.4). This leads to a method based on a dynamic boundary, which captures different regimes of locality, in order to protect against catastrophes inherent to the 2-lattice (see Section 6.2.5). Finally, we summarise our approach (6.2.6).

The aim of this section is to present further alternative methodologies, which we believe will prove useful in terms of model specification.

6.2.1 Movement and Multiple Optima

In order to model movement, we can simply change the position of a pedestrian origin v_o in incremental steps. For example, in the 'smiley face' configuration (see Figure 6.5), if a pedestrian has a step speed of 1, then because D = 37, it will take 37 steps to reach v_d .

However, due to the use of discrete neighbourhoods, a pedestrian will often be presented with multiple optima. For example, lets take the trace-back graph S(G) from the smiley face configuration. This graph can be seen to shrink each time a step is taken by the pedestrian towards v_d (see Figure 6.6). Notice, at t=2, the subgraph S(G) surrounds the eye—the pedestrian has not yet navigated this obstacle. This means that future movement can take a route either below or above the eye. At t=9, we see that the shape of S(G) determines that the former route is now impossible. Therefore, at t=16, movement has been completed around the brow of the eye.

The choice between multiple optima is computed through random assignment, i.e., if there is more than one vertex choice that can minimise depth, then a single choice, sampled from a uniform distribution, is taken. In this way, multiple optima are chosen on an incrementally

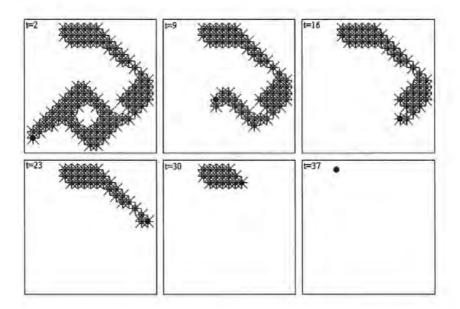


Figure 6.6: Sampled graph trajectory. Six sampled traceback subgraphs are shown, which illustrate how a pedestrian can be directed towards a destination. We can see how |S(G)| shrinks over time until $v_o = v_d$.

random basis.

6.2.2 Multi-Agent Representation

Previously, we have represented obstacles with $output^* = 0$. We can also use this method to allow the expanding autowave to search around other pedestrians. Consider a single north-heading pedestrian P^n , having to avoid two south-heading pedestrians, P^s_a and P^s_b . Incremental search and movement, for P^n , will account for the positions of P^s , which are thus 'detected' by the autowave of P^n . This is done by temporarily fixing $v_o(P^s_a)$ and $v_o(P^s_b)$ to zero, from the perspective of P^n , so that when exploration is executed from $v_o(P^n)$, the trace back subgraph accounts for P^s_a and P^s_b in the local to global link $v_o(P^n) \rightarrow v_d(P^n)$. We provide a graphical depiction of an example scenario (see Figure 6.7).

Notice how the autowave approach provides an adaptive mechanism so that pedestrians do not blindly follow an a priori route throughout the simulation. Dynamic optimal routes are computed in this way, where information regarding the presence of other pedestrians is coded to the lattice, from the frame of reference of each pedestrian. In all models mentioned

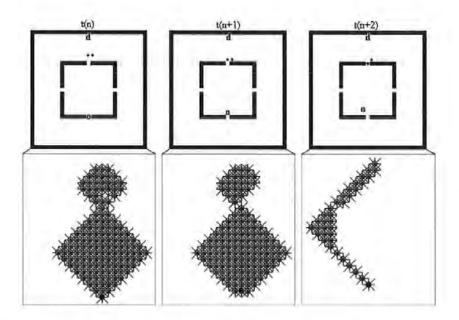


Figure 6.7: Adaptive graph trajectory. Two south travelling pedestrians are detected in the trace-back graph of o, the origin. In the first two time steps of this scenario, the local movement of o is not affected. However, at t=3, one of the south pedestrians blocks the route of o and a different trajectory is then computed. This is an example of how we implement dynamical decision making.

previously, which use a field approach, the paths in the simulation are largely determined before pedestrian motion occurs and any choices are made according to nearby localities.

In more complex scenarios, where the population is larger, the same method can be applied. Each pedestrian will then have a knowledge of N-1 other pedestrians, where N is the population size. As we increase the density of the population, we can imagine that search for P_i will then become more complex as the expanding wave searches through, and avoids, the rest of the population P_j , arriving (hopefully) at the destination vertex v_d .

The word hopefully in the previous sentance is a caveat. As we know, search is limited by properties of the 2-lattice. We now turn to these fundamental properties again, but in the light of our own methods.

6.2.3 Inclusive Search: Limits

We ask the following three questions:

How successful is the autowave algorithm in the face of various density conditions?

- Does the wave link v_o to v_d ?
- ullet How many iterations are needed before the wave reaches or fails to reach v_d from v_o ?

We use an L^2 2-lattice (L=100) with non-periodic boundaries. We place a single destination or 'exit' vertex v_d in the centre of the northern boundary. A population of pedestrians P of size N are placed at vertex locations according to $\rho(0,1)$. For each density increment $\rho_{inc}=0.001$, a wave is executed for each pedestrian P_i and other pedestrians P_j are coded to the network, as described previously. Two statistics are taken:

- the average fraction of success, i.e., we define a success as a case where $v_d(P_i)$ pulses as a result of autowave expansion.
- the average autowave exploration time or walk time, i.e., the number of time-steps required before the autowave dies out by either pulsing at v_d or otherwise exhausting searchable vertices.

We present results for both statistics (see Figure 6.8). We can see how the autowwave is limited in similar density regimes to those identified previously for site percolation (see Section 5.1). This is no suprise. As we mentioned earlier, initialisation of a pedestrian model implements a kind of Boolean network. However, it is important to point out that because we are using an inclusive search, limitations in a model derived from this method should occur when percolation emerges, rather than below this point. Remember, the pedestrian CA (Blue and Adler, 2000a) was shown to collapse at densities $\rho < 0.3$, which is approximately half the value of the critical point ρ_c .

It is import that we produce a picture for the relationship between our two statistics. This is because the first statistic (fraction of success) defines the intention behind the use of the autowave, and the second (walk time) measures the cost. As a trade-off, we need to exploit good results where they don't cost too much, computationally. The following definition of redundancy describes the relationship between these two statistics:

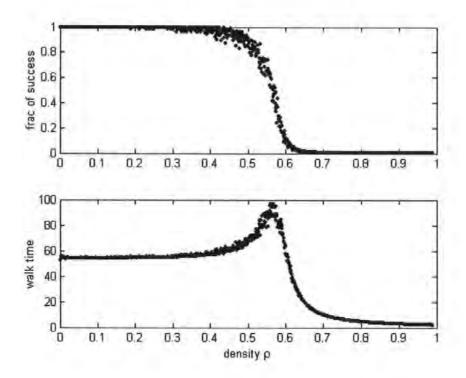


Figure 6.8: Wave exploration in occupied space. The fraction of hits is very high at low densities, but quickly drops at a density of around $\rho \approx 0.55 - 0.56$. At values around $\rho \approx 0.55/0.56$ the walk time of the wave reaches a maximum before quickly falling off.

$$R = \frac{\sum_{i=1}^{N} (1 - P_{i(v_{d(hit)})}) P_{i}(t)}{N}$$
(6.2)

where $P_{i(v_{d(hit)})}$ defines the success of autowave search, which is an either-or result and therefore:

$$P_{i(v_{d(hit)})} = \begin{cases} 1 & \text{hit } v_d \\ 0 & \text{otherwise} \end{cases}$$
(6.3)

and $P_i(t)$ is the walk time for each pedestrian autowave. We plot R across a range of density (see Figure 6.9).

The statistic R demonstrates the complexity of the relationship between $v_o \to v_d$. There is little complexity at low densities; it is a relatively trivial task for the autowave to travel from v_o and successfully to arrive at v_d . There is also little relationship between the vertices at high densities because very few autowaves arrive at v_d ; the $v_o \to v_d$ link does not exist. In between these situations, there is a transition into complex behaviour. These regimes

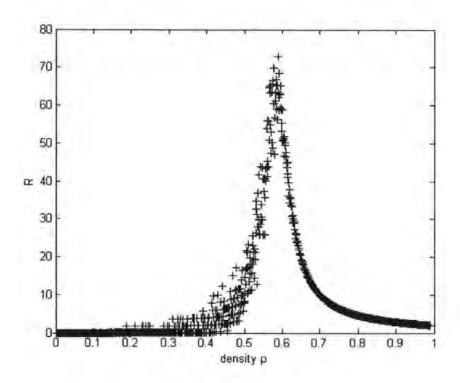


Figure 6.9: Redundancy in wave exploration. Wave redundancy R is a measure of the success and walk time of an autowave. Redundancy peaks at a value $\rho \approx 5.9$, which is in close agreement with results of critical points in 2-lattices (Stauffer and Aharony, 1992).

demonstrate the possible limitations of using the autowave approach. There are therefore practical consequences of this critical range—if there are few 'hits', then few pedestrians can produce a trace-back path, which as we have seen is the backbone of our adaptive search method.

6.2.4 Use of Time Series Signatures

Previously, we demonstrated the importance of S(G). We defined the diameter D(S), which is equivalent to the amount of time spent searching from v_o to v_d (see Section 6.1.4). Notice that the graph diameter $D = P_i(t)$ (equation 6.1). Therefore, for a given pedestrian P_i , where the regime is complex between v_{o_i} and v_{d_i} , we observe a high value for $D(S)_i$.

We present examples of S(G) for two different values of density $\rho = 0.59$ and $\rho = 0.4$ (see Figure 6.10). For $\rho = 0.59$, S(G) is thin, which means that the graph is *brittle*; if another pedestrian P_j were to occupy $v_{o_j} \in S(G)$, then the link $v_o \to v_d$ would probably break. For $\rho = 0.4$, S(G) is fatter, which means that the graph is more *robust*; if another pedestrian P_j were to occupy $v_{o_j} \in S(G)$, this would *not* break the link $v_o \to v_d$.

This poses a problem: how do we tackle the brittleness of autowave search in increasingly high density regimes? The solution to this problem needs to be simple. Any complex algorithm would be outside of computational limitations. We need to try and exploit information in the autowave signature, which as we have demonstrated previously, represents the complexity of search. More specifically, we ask is there information we can use in the time signature of autowave search to control the nature of the search pedestrians execute? What we are looking for is an indicator that the autowave front will die before reaching v_d .

An obvious question to ask about the autowave search is if the wave front shrinks to a value of 1, then what is the probability of quick failure thereafter? Firstly, we will identify whether or not the autowave from v_o to v_d contains a value of 1. If so, we refer to this as a 'one-type' signature. Secondly, we need to know how much time elapses before a one-type signature fails. These two conditions represent a lower limit, and the simplest case, where an heuristic may be introduced into the search method in order to improve robustness in and above critical regimes.

We identify one-type signatures and present the fraction of failed searches that contain a value of 1 in the search signature (see Figure 6.11). We can see that the fraction of one-type signatures quickly increases within the critical ranges identified above. However, an important property is that, above this range, the fall-off of one-type signatures is relatively smooth. The majority of pedestrians still produce one-type signatures in a fairly large density range $\rho \approx (0.6, 0.88)$. This means that, although search is failing in terms of the $v_o \rightarrow v_d$ link, it is likely that a large amount of potentially useful search is being undertaken above p_c .

It will also be useful to know particular properties of this search. We would not expect, given a wave front thats shrinks to 1, much more progress of this front. In order to get a statistical description, we present interval histograms, which represent the critical range of density $\rho = (0.51 - 0.6)$ (see Figure 6.12). Each histogram shows the conditional probability,

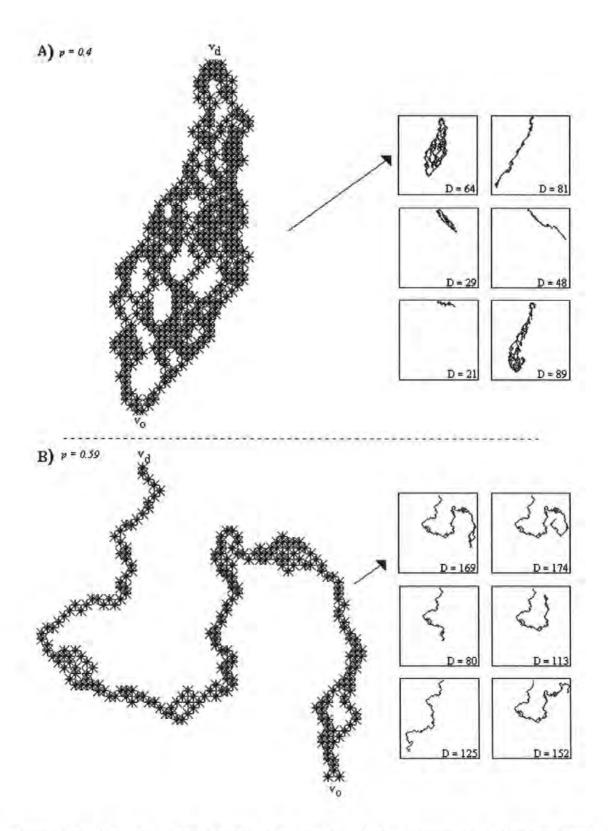


Figure 6.10: Simple and complex trace-back graphs. A) At a density $\rho=0.4$ robust graphs with small diameter are typical. B) At a density $\rho=0.59$ brittle graphs with large diameter are typical.

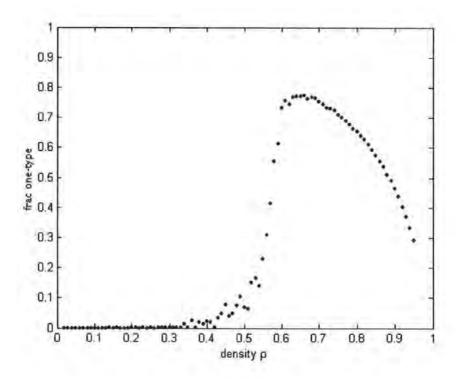


Figure 6.11: Fraction of one-type signatures. In the critical regime we notice a sharp increase in one-type signatures. However, these kinds of signatures do not fall off quickly, remaining in the majority of pedestrian searches until just below $\rho = 0.9$.

given a value of 1 in the signature at a certain time, that the autowave fails at a time interval Δt thereafter. Results suggest that the presence of a one-type autowave is a very good indicator that wave failure is about to occur.

From the results presented in this section, we now go on to introduce a simple heuristic, which allows us to represent a certain level of adaptation to local conditions.

6.2.5 Dynamic Search Boundaries

We believe that the benefit of using an inclusive search, based on a higher-level network approach, is that we can control the type of information processing done in the local to global link, while also respecting fundamental 2-lattice properties. We outline an approach, which exploits the autowave time series information just presented. The aim is to define a general heuristic, which captures catastrophic features of local information, from the perspective of a given pedestrian P_i .

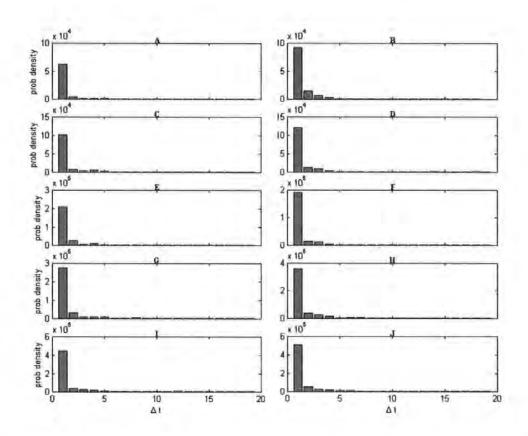


Figure 6.12: Conditional probability of failure in one-type signatures. Given that there is a value of 1 in the signature, at a given point in time t_1 , the probability that failure occurs at Δt from t_1 is given. For ease of presentation we represent $t_1 = 0$. Results indicate that, when a wave front shrinks to a size of 1, a short period of time elapses before the wave fails. These results are taken in the range $\rho = (0.51 - 0.6)$, with $\rho_{inc} = 0.01$ so that each increment corresponds to A-J.

We use time series information to determine likely points of search failure, which, as we have seen, relate to times in autowave search when the wave front shrinks to a value of 1. We then identify the point of failure v_{f_i} . We then define the Manhattan radius r^m from v_{o_i} to v_{f_i} and construct a boundary $B_i = \{v \in G : v = r^m\}$.

We present two examples of this method (see Figure 6.13). We show the times series search of both examples. In example A, failure occurs at the second step (t = 16) after the wave front shrinks to 1 (t = 14). In example B, failure occurs at the fourth step (t = 6) after the wave front shrinks to 1 (t = 2). Underneath each time series, we present corresponding geometrical information. The subgraph $S_r(G) = \{v \in G : v < r^m\}$ corresponds to a robust region of search, i.e., where the front has not yet shrunk to the value of 1. The subgraph $S_b(G) = \{v \in G : v > r^m\}$ corresponds to a brittle region. We identify v_{o_i} (+), walkable vertices (grey), which the wave front can move through, and post type-one search vertices (\times) . The first occurrence of an x-vertex is used to define the radius r^m .

For each pedestrian P_i , a boundary B_i can be used to define which inputs are set to the network. If we do not account for other pedestrians P_j located $v_{o_j} \in S_b(G)_i$, then the autowave will be able to propagate to v_d . The trace back graph $S(G)_i$ would not contain information on all pedestrians between $v_o \to v_d$. However, $S(G)_i$ would still indicate a good move to make in the face of local information, defined inside the boundary, i.e., $v \in S_r$.

In this way, locality is defined dynamically and with respect to the nature of information between the origin of any given pedestrian and the destination of that pedestrian. This method represents a trade-off between gaining useful information $S_r(G)$ and gaining too much information $S_b(G)$. We believe this approach is valid because even in the case where there is high probability of broken $v_o \to v_d$ linkage, there remains a large amount of potentially useful search.

We demonstrate how, at densities above the critical range, search can percolate outside v_o to fairly large areas (see Figure 6.14). The spaces show distributions of post type-one search vertices, but represented differently ($\times = \bullet$). In these simulations, we set v_o to the centre

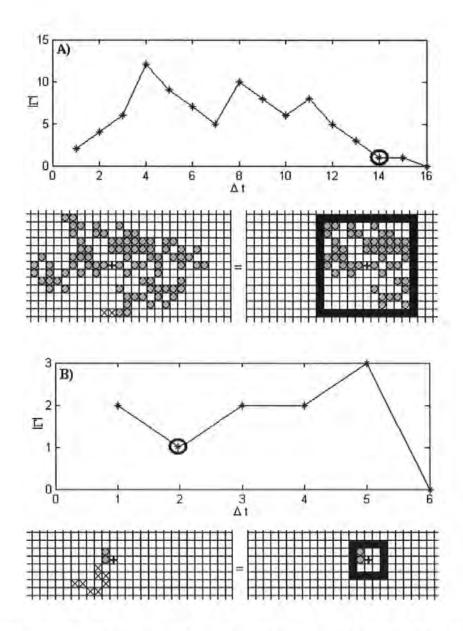


Figure 6.13: Search boundaries using time-series information. We use the Manhattan radius from the point at which the wave shrinks to a value of one to determine the neighbourhood of pedestrian search at the next time step.

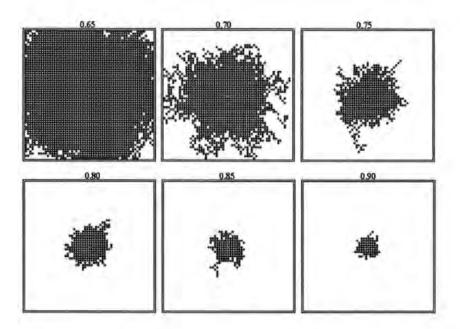


Figure 6.14: Distribution of post type-one search. Here \bullet represents the \times vertices, defined previously. We superimpose on single lattice 10^3 different trials.

of the 2-lattice. At high densities, we see that a significant amount of search is still possible from v_o .

These results are further supported by statistics of average radius r^m (see Figure 6.15). Although previous statistics have indicated that many one type signatures fail early, the boundary method remains sensitive to these kinds of situations. This is an important issue when modelling pedestrian and evacuation dynamics from a Multi-Agent Systems perspective where locality is very important.

We now summarise the results, which led to this locally defined heuristic.

6.2.6 Summary

In this section we:

- demonstrated how the movement of a single pedestrian is akin to the process of shrinking a subgraph, which is derived from autowave expansion and traceback search.
- provided a representation for pedestrian-pedestrian interactions and demonstrated how
 this method captures dynamic 'decision making'; movement through a dynamic sub-

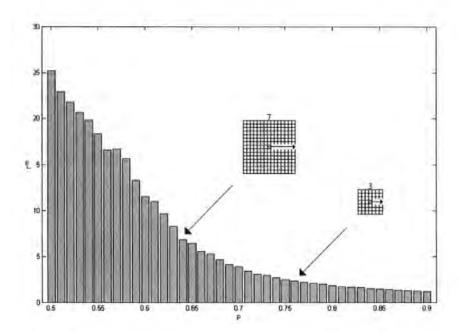


Figure 6.15: Radius against density. We plot the Manhattan radius r^m against density ρ . The smooth decay of r^m indicates that, although for some functions of the graph we have critical behaviour, other kinds of information can be used to specify pedestrian dynamics.

graph.

- presented results, which indicate the generally catastrophic limits on this kind of search by defining autowave redundancy—a product of the failure and computational cost of search.
- showed that, although search is catastrophic according to the v_o → v_d link, there is still potentially useful search, which can be used in order to guide pedestrians through densely populated space.
- presented a dynamic boundary technique, based on simple time series conditions, to
 override catastrophic behaviour. This technique allows the maintenance of v_o → v_d, but
 also captures as much local information as possible.

These techniques are offered as tools for model specification. The methods aim to integrate two important aspects of egress, which have previously been represented with separate levels of abstraction:

- 1. local dynamics of pedestrian-pedestrian interactions (e.g., cellular automata).
- 2. longer-range goal of egress (e.g., attractor fields).

We now test the usefulness of our methods by presenting two computational models.

6.3 Simulation Models: Pedestrian Pattern Formation

We present an alternative model of lane formation behaviour (see Section 6.3.1). We also demonstrate how our techniques are *portable* by simulating a more complex egress scenario, but with only minor model adjustments (see Section 6.3.2).

One of the major motivations for the techniques developed in this thesis rest upon the belief that there are fundamental geometrical patterns formed in crowds and that crowd-flow statistics are determined by these patterns. It is therefore insufficient to present statistical results of, e.g., crowd-flow v density without looking also at the geometrical patterns of pedestrian-pedestrian interaction in a model.

6.3.1 Lane-Formation

One of the most frequently observed phenomenon in real crowds of pedestrians is the development of lane formations (Helbing et al., 2000; Blue and Adler, 2000a; Still, 2000). This is generally regarded as an important emergent property of groups of pedestrians where optimisation of forward movement is thought to be motivating factor.

In order to model this behaviour, it is important to provide each pedestrian with *knowledge* of the positions of surrounding pedestrians. We have previously outlined an approach for representing the positions of other agents (e.g., see Section 6.2.2). In other models, directional movement is achieved by, for example, introducing a relaxation term to a vector (Helbing et al., 2000) or by labelling each pedestrian and creating rules around these labels (Blue and Adler, 2000a). As ours is a discrete model, we also label our pedestrians and then use this label to determine which pedestrians are accounted for in the network. The assumptions behind

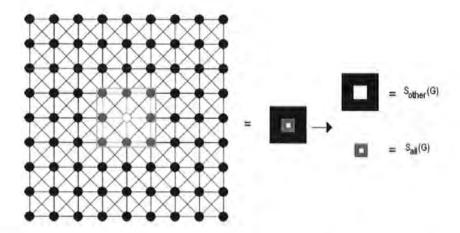


Figure 6.16: Relaxation method. We define the immediate neighbourhood Γ_i^1 where each pedestrian P_i^z has knowledge of all other pedestrians $P_j^z \in \Gamma_i$. Outside of this neighbourhood we account for $P^{opp\{n,s\}} \in \Gamma_i^j$ where Γ_i^j for $0 \le j \le j_{max}$ is the j^{th} neighbourhood of P_i .

our approach reflects the general idea that pedestrians moving in the same direction interact less with each other than with those moving in the opposite direction—i.e., pedestrians are observed to follow paths of least resistance in a crowd of people (Still, 2000).

6.3.1.1 Implementation

We define a population of pedestrians P^z , sub-divided into P^n and P^s . For P^z_i we define the immediate neighbourhood Γ^1_i and apply inputs so that other pedestrians are accounted for in autowave expansion (see Section 6.2.2). In this way, each pedestrian has knowledge of all other pedestrians $P^z \in \Gamma^1_i$. Outside of this neighbourhood, we only apply inputs for $P^{opp\{n,s\}}_j$, so that each pedestrian has a knowledge of other pedestrians, but only those with the opposite sign $n \leftrightarrow s$ (see Figure 6.16). Therefore, the subgraphs $S_{all}(G) = \{v \in G : v = r^m = 1\} = \Gamma^1_i$ and $S_{other}(G) = \{v \in G : v > r^m = 1\}$, define two lattice areas associated with the two different search regimes. Notice that, e.g., for P^n_i , $S_{other}(G)_i$ relaxes the constraints on search associated with other pedestrians of the same kind so that P^n_j are not accounted for outside of the immediate neighbourhood, but P^s_j are.

We define $v_{d(i)}$ as $v_{o(i)} + M_i$, where M_i is a positive value defined relative to the movement direction of a given pedestrian. At each time-step, each pedestrian makes a single step towards $v_{d(i)}$. This ensures that each pedestrian moves forward according to a dynamically changing destination.

Therefore we have a directionally split population of pedestrians. Each pedestrian is trying to maximise its own forward progress, according to simple network-based rules.

6.3.1.2 Statistic

The general behaviour is similar to that defined in the pedestrian Cellular Automata (Blue and Adler, 2000a). In order to present quantitative results for lane formation behaviour, we use the following to calculate the amount of lane formation l in a given lane i:⁴

$$l_i = \frac{max(|P^n|, |P^s|)_i}{max(|P^n|, |P^s|)_i + min(|P^n|, |P^s|)_i}$$
(6.4)

and for the entire lattice:

$$L = \frac{\sum_{i=1}^{N} l_i}{N} \tag{6.5}$$

6.3.1.3 Results

We present results of an N=50, non-periodic 2-lattice (see Figure 6.17). We record L for varying degrees of density ρ , over 500 time steps, both for the Blue and Adler (2000a) CA and our own model. We observe at low densities a relatively high L value for both models. However, the Blue and Adler (2000a) CA clearly produces perfect lane formations, which makes sense when we consider the deterministic nature of search and the tendency to maintain the current lane. In comparison with earlier results from Chapter 4, we notice that at certain density ranges the Blue and Adler (2000a) CA shows a steep drop for L, indicating previously identified lane formation collapse (see Chapter 4).

In contrast to this, we see how our own model produces larger L values as density rises—lane formations gradually increase in the face of increasing density ⁵. Our biased input method pro-

⁴This lane formation measure needs to be defined relative to the parameters given to the model. For example, where we have $P^{z\{n,s\}}$ lanes are defined according to columns 1 or where $P^{z\{e,w\}}$ with rows \leftrightarrow .

⁵It should be mentioned also that the low L values at low densities in our model does not reflect an inability

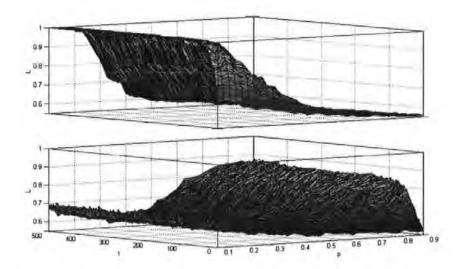


Figure 6.17: Comparison of lane formation behaviour. Top: Blue and Adler's (2000a) model. Bottom: Our model. Typically lane formation disappears in the former at a density around 0.3-0.4. In our model lane formations gradually increase with density.

duces the effect of lane formation, as pedestrians in increasingly crowded regimes, increasingly avoid oncoming individuals. Similarly, real pedestrians are observed to produce 2-directional splits as a *consequence* of increasing interactions (Still, 2000; Helbing et al., 2000). In this way, our model is robust to the density parameter and captures important properties of real crowds. It is therefore a better model than the 2-D pedestrian Cellular Automata.

By simply varying inputs to the network we have managed to model an important feature of real pedestrian crowds. However, the above model does not account for egress behaviour within a more complex configuration and we now turn to this problem.

6.3.2 Egress

It is quite a different problem to capture lane formation, or population-based route-choice preference, within a more complex configurational scenario. We discovered in Chapter 4 how difficult this was, with existing techniques, by failing to synthesise CA rules with field approaches. We now also simulate this aspect of crowd behaviour using the methods presented in this chapter.

to move forward, but rather reflects the non-deterministic features of the autowave search method and the multiple optima typical of 2-lattice graphs. Again, we return to these issues when we discuss future directions to research (see Chapter 8).

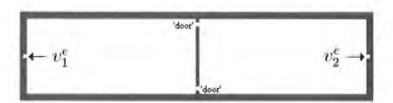


Figure 6.18: Two-door, two-exit configuration. We set the destination of each pedestrian population to different exit vertices. We initialise both populations randomly to the white space. Grey areas represent obstacles or walls.

6.3.2.1 Implementation

The implementation is very similar to that of the previous model, where we use the same relaxation method, but port it to a more complex 2-lattice scenario. We define two sets of pedestrians, P^e and P^w , whose destination vertices correspond to two fixed destination vertices, v_1^e and v_2^e , respectively (see Figure 6.18). We initialise the pedestrians randomly within the white space of the configuration. Each pedestrian P^z then moves towards its designated exit vertex, avoiding other pedestrians, in the same way as described in the more simple model above.

6.3.2.2 Results

We demonstrate the behaviour produced for different time steps (see Figure 6.19). Pedestrians with the same destination vertex are attracted to the same doors. The attraction-to-same-door behaviour is an effect produced by the autowave method—'door preference' reflects the ability of autowave search to exploit and accumulate early fluctuations from random pedestrian initialisation. This accumulation is encouraged by the relaxation method used.

6.3.3 Summary

This simulation section engages with other discrete models of pedestrian and evacuation dynamics. In the literature we find no other methods, which can model both navigation/evacuation processes and higher-level pedestrian pattern formations. We believe this direction is an important challenge for the pedestrian and evacuation dynamics community. In terms of produc-

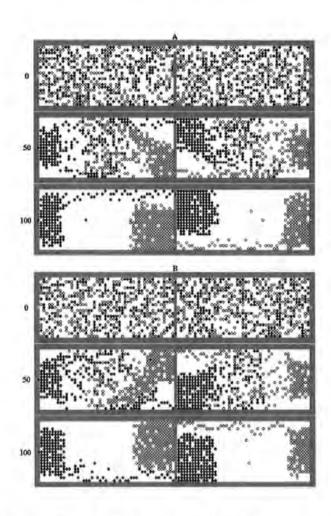


Figure 6.19: 2-directional flows in a configuration. Two different scenarios are possible depending on initial fluctuations in crowd behaviour. This preferential route formation is also widely observed in populations of real crowds (Helbing et al., 2000).

ing evacuation behaviour, while also allowing for the emergence of important developmental patterns, the simulations presented here represent preliminary work and indicate positive results obtainable through the use of simple network-level methods.

We believe this is an important shift in emphasis away from models, which either model pedestrian-pedestrian interactions (and ignore higher-level behaviour) or rely on more global, fixed field representations for escape (and ignore more local interactions).

6.4 Summary

In this chapter we:

- developed a number of techniques to specify pedestrian and evacuation models based on:
 - efficient autowave search.
 - trace-back search.
 - representations for obstacle avoidance.
 - path-choice methods in the face of multiple optima.
 - properties of search robustness.
 - a dynamic boundary to protect local dynamics.
- demonstrated the potential power of these techniques by specifying two simulation models, which define preliminary attempts to integrate two previously separate levels of representation.

This point in the thesis defines an end to work on the development of egress models. For the rest of the thesis we take our model at face value. We will discuss some of its general limitations in our conclusions (see Chapter 8), but firstly we see how these techniques, or improvements to them, might be used in the context of an evolutionary approach to architectural design.

Chapter 7

Evolutionary Architecture

The aim of this chapter is to demonstrate the utility of techniques developed in chapters 5 and 6 in the context of Evolutionary Design. Before this is possible we need to address the specific problem of how to represent valid, single-component graphs inside a genetic code. Previously in the thesis, configurations have been hand-coded. In the context of evolutionary design, however, an *automatic* development of valid designs is needed. As a starting point, we therefore look at two nature-inspired models of self-organising architecture (see Section 7.1). We then present the kind of architectural spaces we seek to develop (see Section 7.2). Based on ideas from the nature-inspired models, we present a self-organising algorithm for the development of 2-D architectures (see Section 7.3).

We then concentrate on artificial evolution. We outline the Simple Genetic Algorithm and discuss evolutionary dynamics (see Section 7.4). We present two evolutionary approaches. In the first we exploit our self-organising algorithm by genetically encoding certain parameters (see Section 7.5). This approach is concerned with the evolution of generative architecture. The second approach is a simpler application of constrained evolution (see Section 7.6). Again, we summarise at the end of the chapter (see Section 7.7).

7.1 Nature-Inspired Growth

In this section, we review two nature-inspired models of generative architecture. We look at nest building (see Section 7.1.1) and crystal formation (see Section 7.1.2), which have social insects and atoms in motion as respective agents of growth. Both models enable us to introduce the kind of methods that we think are appropriate for the automated generation of architectural forms. We identify specific features of these models, which bear on choices made in the design of our own developmental algorithm, presented later (see Section 7.2).

7.1.1 Social Insects

In nature, wasps are able to construct a range of architectural forms on the basis of very simple rules, which are triggered by environmental stimuli. For example, the first act of nest construction begins with a deposit of material on an initial site. The nest then accumulates and expands from this site until nest building is complete. In this way, the nest grows from a small seed into a complex architecture. Potentially, there is a combinatorial explosion of possible forms during growth. As Bonabeau et al. (1999) state: "The number of possible sites where a new cell can be added increases significantly as the construction proceeds". In this scenario, it is easy to imagine a disorganisation of the building process as the number of possible interactions increases, but it appears that the building structure, while enabling growth, at the same time constrains and channels it—"...the building decisions seem to be made locally on the basis of perceived configurations in a way that possibly constrains the building dynamics" (p 212). In short, the potential of diffusion to run away aimlessly is checked by the self-organising properties of the rules.

In the work by Bonabeau et al. (1999) virtual wasps build nest architectures. The original stimulus, akin to an initial deposit, consists of a single brick vertex v_b , embedded within a 3-lattice graph G. Each wasp-like agent has a sensory capacity defined by a 3-D Moore neighbourhood, consisting of 26 vertices. This sensory environment defines a stimulating configuration on which micro-rules operate. The latter trigger local brick-depositing behaviours

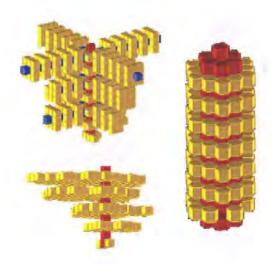


Figure 7.1: Architectural form in a model of nest assembly (Bonabeau and Theraulaz, 1995). and material accumulates in the form of more bricks as the virtual wasp operates over time. At the end of a run, a subgraph of locally connected bricks B(G) defines a *nest*. The variety of nests generated range from random-like structures to highly organised structures reminiscent of real wasp nests. We preset examples of the latter (see Figure 7.1).

This kind of model contains three related properties, which are important for the concerns of this thesis:

- Discrete Stigmergy: reciprocal agent-environment interaction is a good mechanism of
 architectural self-organisation. Specific requirements of self-organisation will need to be
 met and local rule-based interaction is one way to achieve them. For example, we show
 later, with our own generative algorithm, how certain global properties can be handled
 locally.
- 2. Simplistic Rules: the simplistic rule-based action of agents provides algorithmically efficient behaviour. For example, a type-1 stimulus produces action A, a type-2 stimulus produces action B.... and so on. This approach allows complex architecture to grow without the need to define architectural components explicitly. This has implications as to the efficiency with which we can compress a genetic description (genotype) without loosing detailed structure in resulting architectures (phenotypes).

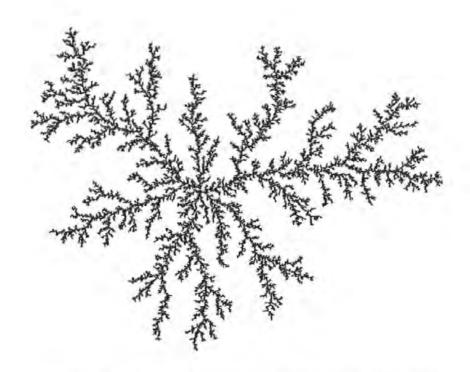


Figure 7.2: Branching architectures of Witten and Sander's DLA.

3. Determinism: the rules are completely defined and stimulus-response in nature. Many kinds of developmental algorithm have elements of randomness. For example, in models of Diffused Limited Aggregation (DLA), the motions of elementary particles are assumed to be random (Witten and Sadler, 1983). Any random effects, not specified parametrically or controlled genetically, are most likely in an evolutionary context to negate the effects of selection. For this reason, simplistic rules should be deterministic.

7.1.2 Crystal Aggregation

The Witten and Sadler (1983) DLA model has catalysed research activity into growth models with applications to many areas of physics (Halsey, 2000). Similarly, the model begins with a single, fixed cell at the centre of a 2-lattice. Particles are released randomly at a distance far away and diffuse as random walks. If the diffusing particle meets another particle within a small radius and set time, then diffusion stops and the particle sticks. Over time, a growing cluster accumulates and intricate branching architectures form as it expands (see Figure 7.2).

We are interested in these kinds of growth models as methods with which to develop our

own generative architectures. Of more specific interest is a parameter-controlled version of DLA where various kinds of morphologies can be tuned into the dynamics of growth (D'Souza and Margolus, 1999). In this model, details of particle interactions are based on less abstract natural processes—exchange of heat and energy conservation—whereas particles in the former DLA are added to a space artificially.

The state of a single vertex in a 2-lattice is defined by a set of 7 bits $N_i(\vec{x}, t)$:

- one crystal $N_c(\vec{x}, t)$ particle.
- two gas $N_g^{\gamma}(\vec{x}, t)$ particles.
- two heat $N_h^{\gamma}(\vec{x}, t)$ particles.
- one random variable $\xi_g(\vec{x}, t)$.
- one random variable $\xi_h(\vec{x}, t)$.

where $\gamma = \{1, 2\}$. There are two steps in the dynamics:

- 1. Diffusion, which consists of two subprocedures:
 - (a) Mixing. This proceeds according to the following rules:

$$N_i^1 = (1 - \xi_i)N_i^1 + \xi_i N_i^2$$

$$N_i^2 = (1 - \xi_i)N_i^2 + \xi_i N_i^1 \tag{7.1}$$

where i = g or i = h—mixing only occurs between the same kind of particle.

(b) Transport. A parameter k defines a distance on the lattice and each of the particles takes on the corresponding value of the k^{th} vertex away from the current site at alternative time steps $\vec{z} + k$, where $\vec{z} = \vec{x}$ at $l\{2, 4, 6, 8...n\}$ and $\vec{z} = \vec{y}$ at $l\{1, 3, 7, 9...n\}$. This completes the diffusion dynamics. The diffusion constant k can take on different values k_h and k_g so that the subsystems can be controlled separately.

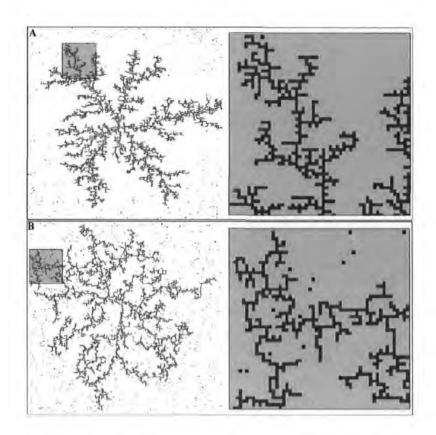


Figure 7.3: Various branching architectures of the D'Souza and Margolus (1999) DLA. Parameters allow different branching structures.

2. Interaction. Particles can either aggregate or evaporate under the following conditions:
A) There is a single crystal present in the von Neumann neighbourhood. B) There is room for a crystal (aggregation) or a gas (evaporation) particle. C) Heat bits can absorb (aggregation) or supply (evaporation).

Parameters in the D'Souza and Margolus (1999) model allow varying kinds of architectures to develop and we present and highlight two examples (see Figure 7.3). Example A is very similar in form to the one presented in Figure 7.2, whereas example B is again branching, but forms a much looser set of branching arms—the crystal almost encloses large areas of the space. In the original model we can see how equivalent areas are less likely to form and are much smaller. Both architectures formed are *organic* and resemble many kinds of natural branching systems such as neural networks, river basins or polymer structures (D'Souza and Margolus, 1999).

For the concerns of this thesis, there are 2 desirable properties of this model:

- Single component graphs: The model forms a single component crystal by aggregating at local sites. For the concerns of this thesis, as previously mentioned, it is essential that we produce single component spaces. To qualify this, what is essential, for us, is not particularly that the crystal, but specifically that the space in which the crystal grows, be single-component. Unlike other kinds of architecture (neural), where recurrent connections might be important (Wheeler, 1996), cyclic connections in the 2-lattice crystal would fracture regions of space. A rule of aggregation (rule A) specifies that a crystal can be aggregated if, and only if, a single crystal is already within the neighbourhood. This rule prevents the formation of cyclic growth in crystals. This property is important for autowave search, which requires connection from v_o and v_d .
- Parameter-Based: The model is parameter-based and allows some variety in architectural form to develop. In an evolutionary context it is essential that a variety of architectures can develop from a single model because evolution requires variation in order to make selection meaningful. We will return to this issue in the next section.

However, there are also two undesirable properties:

- Organic structure: The architectures that develop are very organic in nature and very
 much at odds with the qualitative nature of architectural representation used for built
 human forms (Hillier, 1996).
- Updating: The implementation of this model depends on a synchronous update of the entire lattice. In a population-based approach we need to reduce any unnecessary computation time.

It would be beneficial if we could create an architectural generator, which created more realistic forms, but also did so without the need for this kind of global update.

7.1.3 Summary

In order to evolve architectural configurations we believe it is necessary to design a *generative* algorithm. As a precursor to doing so we have:

- Identified some potentially important methods by reviewing two approaches to architectural form generation.
- Listed some specific algorithmic mechanisms.
- Identified desirable and undesirable properties.

We now briefly consider architectural spaces, before we offer our own generative algorithm, which builds on some of the ideas presented in this section.

7.2 Architectural Spaces

In chapter 3, we introduced the Theory of Space Syntax (Hillier, 1996). We present two example architectural representations from this work (see Figure 7.4). Consider breaking these configurations into a number of components. For example, we can decompose 'A' into two main components; 1) a 1-dimensional *corridor* with seven vertices and 2) a nine-vertex square *room*. These kinds of structures are very easy to hand-design, but, as we have seen, this ease may not be so obvious from the point of view of automatic architectural generation. The examples presented previously have shown overly organic structures, which are a very different form to the examples presented here.

However, we belive it is a generative approach, which we need to apply. Architectural decompositions have been used in some methods of form generation using Genetic Algorithms, where components, represented as grammatical rules are combined into various forms (Rosenman, 1996). These forms are made with a human-in-the-loop process, where Architects evaluate forms at each step in an evolutionary run. However, spaces that result from this process do not maintain any internal structure, but only define a boundary form where all of the

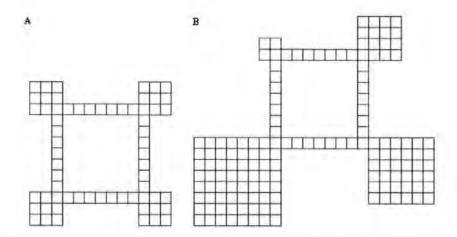


Figure 7.4: Representations of architectural space. In our review of a crystal aggregation model, we showed how architectures that develop have *organic* spatial properties. These are different to the more structured ones presented here. Taken from Hillier (1996).

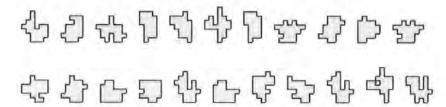


Figure 7.5: Evolved architectural forms. These forms do not show any complex, internal structures. Taken from Rosenman (1996).

interesting structures get lost within the combinations. We present examples of these kinds of spaces (see Figure 7.5).

In the approach that follows, we do not use a *component-based* representation with combination grammars, but a different developmental approach, inspired from the kinds of methods employed in the previous section. In this way we aim to *generate* complex architectural structures rather than simple bounded forms. Therefore, we don't impose architectural components on the process a *priori*.

7.3 Developmental Algorithm

We then present our approach in two stages. Firstly, we outline the methods used and the typical behaviour of the algorithm (Section 7.3.2). Secondly, to satisfy details important for the preservation of single component spaces we outline the specific arrangements made for an evolutionary context (Section 7.3.3).

7.3.1 Diffusion and Construction

Here, we have two aims; 1) to produce architectures, which have internal structure similar to those presented above (see Figure 7.4) and 2) to also avoid the organic structures derived from the generative approaches of others (D'Souza and Margolus, 1999).

A cellular automata of the following general form is used:

$$a_{ij}(t) = f[a_{ij}(t-1), a_{ij-1}(t-1), a_{i+1j}(t-1), a_{ij+1}(t-1), a_{i-1j}(t-1)]$$
 (7.2)

where f defines a set of processing rules, which determine the logic of the behavioural transitions and $t\{0, 1, 2, 3, ..., n\}$. Notice, this is the same form of CA introduced in the previous chapter, where we gave, as an example, Langton's Ant (see Section 6.1.1). Another reason we introduced this CA earlier, apart form being a good general example of 2-D CA's, is that this CA will be use here to derive some useful behaviours.

We start with an empty 2-lattice graph G. The algorithm revolves around two concepts:

1. Ants: We define a set A of ants, which operate on G. The state of a single ant a can be defined by 2 bits. The first bit N₁^a represents the highest-level state where a is on (N₁^a = 1) or off (N₁^a = 0). The second bit N₂^a represents a lower-level, slave state where a is in explore (N₂^a = 1) or construct (N₂^a = 0) mode. The various states are triggered by environmental conditions, which are accessible to a through 2 neighbourhoods Γ_a¹ and Γ_a².

The neighbourhood Γ_a^1 is static von Neumann neighbourhood with a Manhattan radius $r_1^m=1$. The second neighbourhood Γ_a^2 is also a Moore Neighbourhood, but the radius $r_2^m=\alpha$, where α is a user-defined parameter. The von neighbourhood Γ_a^1 is specified by Γ_n^1 , Γ_s^1 , Γ_s^1 , Γ_w^1 so that *opposites* in the neighbourhood are defined by $\Gamma_{n \leftrightarrow s}^1$ and $\Gamma_{e \leftrightarrow w}^1$.

Table 7.1: Modified Version of Langton's Ant

- 1. If $(v_o(t) = \text{true})$ Rotate ant 90^o anti-clockwise.
- 2. If $(v_o(t) = \text{false})$ Rotate ant 90^o clockwise.
- 3. Move forward two steps. Increment $t = t + \Delta t$.
- 4. If $(v_o(t-1) = \text{true})$ Then $v_o(t-1) = \text{false}$. Else If $(v_o(t-1) = \text{false})$ Then $v_o(t-1) = \text{true}$.
- 5. If $(\Gamma_a^1(N^s=1))$ Then enter Construct Mode.

Table 7.2: Construct Mode

- 1. IF($\Gamma_a^2(N^s = 1)$) AND IF($\Gamma_a^1(N^s = 0)$) and = off
- 2. IF($\Gamma_a^1(N^s=1)$) drop seed on v_o and move forward one step

In the event that a seed is found in elements n, e, s or w of Γ^1 , then the *ant* direction d = s, w, n, e, respectively.

2. Seeds: We define a set of seed vertices $S \in G$. Each seed is controlled by a single bit N^s , which allows a seed to be present $(N^s = 1)$ of absent $(N^s = 0)$.

While an ant is on $(N_1^a=1)$ and in an explore $(N_2^a=1)$ mode, it follows rules derived from Langton's ant. This phase of behaviour can be thought of as a general diffusion phase. In this phase a conditional rule, which can trigger a change of state from $N_2^a=1$ to $N_2^a=0$, allows the ant to enter a construct phase. (see Table 7.1). The algorithm, therefore, consists of a set of ants $A(\vec{z},t)$, where the rules in the exploration and build mode determine the interchange of \vec{z} (\vec{x} or \vec{y}). The set of seeds S grows according to the construct mode (see Table 7.2). Each ant diffuses according to these rules and enters a build mode depending on the stimulation condition in Γ^1 or halts depending on the stimulation condition in Γ^2 .

We define G to be a 2-lattice (N = 100). We place A_g randomly on v_{ij} , where $j = i = \{2k + 1 : k \in \mathbb{Z}\}$ and 0 < k < N. We place S_h randomly on v_{lm} , where $l = m = \{2k : k \in \mathbb{Z}\}$. Therefore ants can only be initialised to odd locations in the lattice, whereas seeds can only

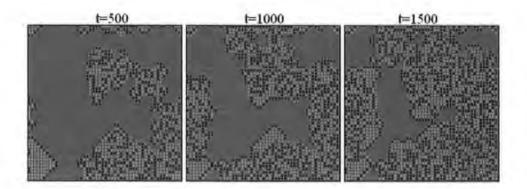


Figure 7.6: Configuration development ($\rho_{ant} = 0.001$, $\rho_{seed} = 0$). From left to right t = 500, t = 1000, t = 1500, respectively. The binary colouring of the Langton's ant is seen in grey and black. White space is progressively covered as the ants diffuse throughout the space. Notice how the diffusion covers only even coordinates on the lattice. This is a result of the initialisation and the modification of rule three to move the ant 2 steps forward. Boundaries are periodic.

be located at even sites.1

For demonstration purposes, we control the number of ants and seeds with global density parameters ρ_{ant} and ρ_{seed} , respectively. In our first demonstration, we set $\rho_{ant} = 0.001$, i.e., given $N^2 = 1000$, $|A| \approx 10$. We set $\rho_{seed} = 0$ and thus determine that |S| = 0. Consider the behaviour. While the environment G contains no seed values, i.e., $N^s = 0 \forall v_{ij} \in G$, then a_i will never enter the construct mode. In this situation there is constant diffusion. We present results from such a scenario (see Figure 7.6).

In a second demonstration, where we set a single seed in the space ($\rho_{seed} = 0.0001$), a given ant $a_i \in A$ senses the seed at some point in time and enters construct mode. This in turn triggers a flurry of constructing behaviour as other ants are similarly triggered by the seeds dropped by increasing amounts of *constructing* ants.

We present results from such a scenario (see Figure 7.7). We can see how, at t = 500, the configuration contains six or seven separate rooms. A large, quite open expanse is separated from the rest of the configuration by a long *corridor*, which leads to other *room* components. At t = 1000, the configuration contains more rooms as constructing activity continues. The configuration is static between t = 1000 and t = 1500 because the proliferating seeds mean

¹This condition ensures that the original lattice is not broken into multiple components. A similar approach is used in the D'Souza and Margolus (1999) DLA model.

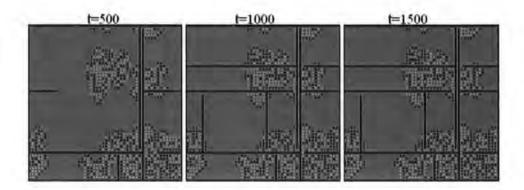


Figure 7.7: Configuration development ($\rho_{ant} = 0.001, \rho_{seed} = 0.0001$)). From left to right t = 500, t = 1000, t = 1500, respectively. A single seed triggers an ant to build and thus drop more seeds, which triggers more building activity in other ants. By the time t = 1000 all ants are in the off state and building activity has stopped. Boundaries are periodic.

that all ants eventually come to be turned off. At this point, the configuration is therefore fully developed because the set of ants have reached fixed point off states.

We demonstrate this kind of internal feedback to the population of ants by recording the fraction of on ants over time (see Figure 7.8, Left). We also demonstrate the tunable nature of the model by increasing the ant density $\rho_{ant} = 0.01$, a ten-fold increase from the previous example. This increases in the number of ant-seed interactions and a corresponding increase in the spatial complexity of the resulting configuration (see Figure 7.8, Right).

7.3.2 Diffusion and Construction: Constraints

In this section, we briefly outline constraints on the developmental algorithm, required by an evolutionary approach (Section 7.2.3.1). We also comment on the desirable variational properties of our method, and the source of these variations (Section 7.2.3.2).

7.3.2.1 The Seeded Boundary

The examples given above (see Figures 7.6-7.8) all show development on a *periodic* 2-lattice. Of course, in real building scenarios, space is bounded, i.e., it is non-periodic. In the case where we firstly execute the developmental model and then create a boundary at the edges of the lattice, afterwards, the lattice will be likely to fragment. We present an example of this

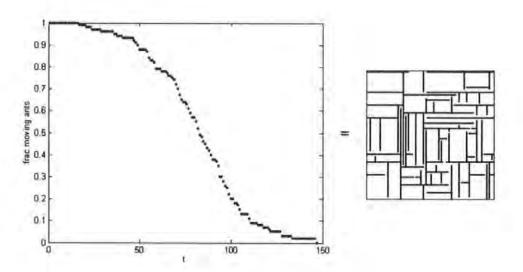


Figure 7.8: Configuration development ($\rho_{ant} = 0.01, \rho_{seed} = 0.0001$). With more ants the model produces a more complex configuration and the feedback determines that all ants are turned off relatively quickly (t < 150), reaching a fixed point. Boundaries are periodic.

kind of fragmentation (see Figure 7.9).

We can see that the space fragments into 8 subgraphs of varying size and complexity. As a fragmented lattice, this space is invalid and inappropriate in the context of pedestrian and evacuation models. We need to develop spaces, which are *connected* in the sense that any given vertex is accessible from any other in the space. One way of ensuring this is to ring the lattice with seeds *before* development.

7.3.2.2 Variations of Configuration

We present some examples of the variety of architectures available under specific parameter settings (see Figure 7.10). We can see that even where the parameters are fixed for each configuration there is potential for a wide range of developmental patterns. This is desirable in an evolutionary context, because evolution itself depends on the availability of enough variation in populations of phenotypes, so that selection has something to select between (Darwin, 1998).

The variety available does not depend only on parameters ρ_{ant} and ρ_{seed} , but also on locations of the ants and seeds in the lattice. The source of variation in our model will be related to

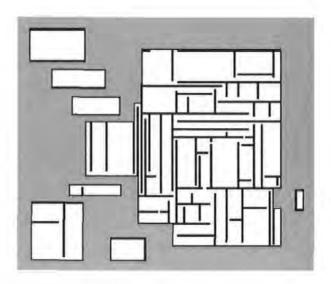


Figure 7.9: Fragmented space. The 2-lattice fragments into a number of components. The undesirable effect of providing a space with a boundary after development has taken place.

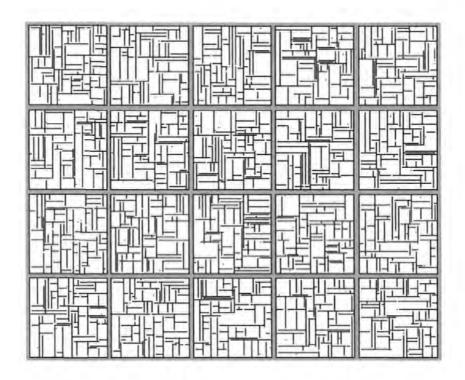


Figure 7.10: Variety of single component configurations. We show twenty configurations each of which is developed according to the technique outlined in the text on N^2 lattices (N = 100), $\rho_{ant} = 0.01$, $\rho_{seed} = 0.001$.

 ho_{ant} , ho_{seed} , which determine $|C_A|$ and $|C_S|$, which are specific sets of location coordinates $C_A = \{(x_1, y_1), (x_2, y_2), \dots (x_n, y_n)\}$ and $C_S = \{(x_1, y_1), (x_2, y_2), \dots (x_n, y_n)\}$ that correspond to $A = \{A_1, A_2, \dots A_n\}$ and $S = \{S_1, S_2, \dots S_n\}$, respectively.

Our algorithm has typical developmental properties including possible sensitivity to initial conditions. On the one hand, if a single ant is initialised at a location, which is sensitive to early patterns of development, then the emerging phenotype may be radically different, where the location of the ant is perturbed, even slightly. On the other hand, it is also possible that a single ant can be placed at very different initial locations, if not critical in this sense, and therefore phenotypes may vary, but perhaps less dramatically, or not at all.

7.3.3 Summary

In this section we

- presented a developmental algorithm, which consists of a process of diffusion. A set of
 virtual ants diffuse in space according to a 2-D cellular automata. Diffusion behaviour
 leads to architectural construction when construction behaviours are triggered by environmental seed information. A natural halting condition is provided through feedback
 to the ants, where all ants switched to an off state.
- presented an approach to the development of valid, single component 2-lattices with varying degrees of complexity. In order to guarantee the development of single-component, bound space, it is important to ring the space with seeds before development proceeds.
 The complexity of a configuration depends on seed and ant density, but also on ant and seed locations, which may vary in terms of their developmental sensitivities.

Before we embed this model within an evolutionary framework, we summarize the algorithm presented in this section in Table 7.3.

Table 7.3: High-Level Description of Developmental Algorithm

```
/*Symbols*/
\Gamma^1(ant sensor 1), \Gamma^2(ant sensor 2), \alpha(radius of \Gamma^2),
d(\text{direction}), \, \rho_{ant}(\text{ant density}), \, \rho_{seed}(\text{seed density}), \, v(\text{vertex}), \, N(\text{size of 2-lattice}), \, G(\text{lattice}), \, R(\text{ringed boundary of lattice}), /*Initialisation*/
Fruitable 1 (G) = N * 4 - 4

v_{i0}(seed) = v_{Nj}(seed) = v_{iN}(seed) = v_{0j}(seed) = 1 \in R_i

|S| = \rho_{seed} * (|G| - |R(G)|)

|B| = \rho_{ant} * (|G| - |R(G)|)

For (|S|)
                                                                                                    //Ring G so that boundary seeds bits are 'on'.
//Determine size of seed array.
                                                                                                    //Determine size of ant array.
            Allocate(s_i \in S, RandEven(x), RandEven(y))
End For
For(|B|)
            Allocate(b_i \in B, RandOdd(x), RandOdd(y))
End For
 /*Main Loop*/
While(|B_{i(on)}| > 0)
                                                                                                      //May replace with For (see Section 7.4.1.3)
            For(B_{i(on)})
                      If ('explore')
                               Run Modified Version of 'ant'
                                                                                                     //Algorithm in Table 7.1
                      End If
                      If ('build')
                               Run 'ant' in build mode
                                                                                                    //Algorithm in Table 7.2
                      End If
            End For
End While
```

7.4 Artificial Evolution

In this section we step back from our own work in order to introduce general concepts related to the behaviour of the Genetic Algorithm (GA) and evolution. We briefly introduce details of a Simple GA as a way of biased sampling of large search spaces (see Section 7.4.1), and the evolutionary dynamics assumed important in applications of such a process (see Section 7.4.2).

7.4.1 Simple Genetic Algorithm

The GA is a sampling procedure. It is genetic in the sense that it processes genotype strings of alleles. It is thus said to operate on a genotype space. For fixed-length genotypes, this space is bound by the genotype length l, and the size b, of the allele alphabet. In our case, we use a typical b=2 alphabet, which therefore produces a 2^l genotype space. Exhaustive enumeration of the genotype space has practical limitations and, for non-trivial values of l, sampling is thus required. Before sampling, The GA employs a number of genetic operators, which are designed for efficiency; after a relatively small sample of the search space good or fit regions are discovered². Of course, this process is a trade-off—without exhaustive enumeration we cannot guarantee the discovery of perfect solutions, but the hope is that good enough ones are found in a short enough amount of time.

1) Alleles in each individual are seeded with randomly selected units from the binary alphabet.
2) Individuals are then evaluated against some function (fitness). 3) A population is then created anew in a sampling-according-to-fitness procedure (selection). Each sampling consists of two child genotypes, constructed from the probabilistic crossover of two parent genotypes

The simple GA is analogous to processes found in Natural Selection is structured as follows:

(inheritance). 4) These offspring are perturbed (mutation), again probabilistically, and then loaded into the new population³. 5) These procedures are iterated until some halting condition

²Detail of the genetic operators are well published (Bently, 1999; Goldberg, 1989; Holland, 1975; Michalewicz, 1999; Mitchell, 1999).

³At this point, where invalid solutions might exist, they are often thrown away or killed (Bently, 1999) and

Table 7.4: Simple Genetic Algorithm

/*Main Loop*/

- 1. Randomly generate an N-sized Population of I bit chromosomes
- 2. Calculate fitness f(x) of each individual chromosome x
- 3. While(|NewPop| < N)

Select two parent genotypes With probability p_c crossover parents to produce two children With probability p_m mutate children and place in NewPop

- Population = NewPop
- 5. If(halting condition == true)End
- 6. Else(goto step 2)

$$\frac{/*Parameters*/}{p_c = 0.7, \ p_m = 0.001, \ |l| = [5 - 1000].}$$

is met. For example, a GA may run for a number of *generations* or until an acceptably fit solution has been reached. The Simple GA is summarized, along with fairly typical parameter values, in Table 7.4.

A number of mappings are usually used to determine f(x). There is a Genotype \mapsto Phenotype mapping and a Phenotype \mapsto Fitness mapping. A simple example of the former is the 'ones' fitness function— $f(x) = \sum_{i=1}^{l} a_i$, where a_i is an allele. Another simple example is the binary to integer decoding, where the genotypes are converted to integer values. The Phenotype \mapsto Fitness mapping is where the Phenotype is assigned a population-bound fitness value. For example, lets assume the binary to integer example, where l=4, population size N=16, and each individual is genetically different to every other. In these artificial conditions, the total fitness of the population is given by the simple formula for an arithmetic progression $S_{16} = \frac{16}{2}(2 \cdot 0 + [16-1] - 1) = 112$. Example Genotype \mapsto Phenotype \mapsto Fitness mappings thus include:

- $(1111) \mapsto (15) \mapsto (0.133)$
- $(0110) \mapsto (6) \mapsto (0.05)$

new solutions generated (often randomly). This is an arbitrary process reflects an insufficient level of design at the genotype level. One of the motivations for our approach is to avoid this kind of arbitrariness.

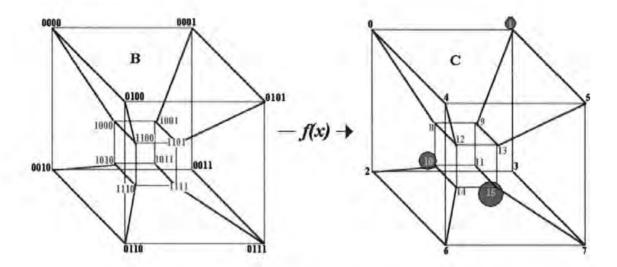


Figure 7.11: Fitness landscape. Each point on a four-dimensional hypercube corresponds to a genotype, which is mapped onto a phenotype value. Three different phenotypes are highlighted by a circle, the size of which corresponds to fitness.

•
$$(1100) \mapsto (12) \mapsto (0.107)$$

These mapping can be visualised in the form of a four-dimensional hypercube (see Figure 7.11).

7.4.2 Evolutionary Dynamics

After the Genotype \mapsto Phenotype \mapsto Fitness mapping is specified, the entire genotype space can then be pictured by analogy to a fitness landscape (Wright, 1932). In our example, we only present 4-bit genotypes, but the landscape analogy is often reffered to in many dimensions. With this picture in mind, the population is more or less converged on a region of the landscape where the relative peaks and troughs represent genotypes of varying fitness values. The crossover and mutation operators explore genotype space and uphill moves are reinforced through the process of selection. The evolutionary dynamics therefore tend to drive the population into areas of progressively higher fitness peaks. In the above sense, the aim of using the GA is to maximise fitness⁴.

In the example mapping provided, the genotypes all map to different phenotypes. Any

⁴In other applications however, f(x) may be designed to minimise energy—in combinatorial optimisation problems this is often the case.

genetic change in a genotype will therefore correspond to a change in fitness. Such an example is at odds with the neutral theory of evolution, which claims that many genetic changes are maintained because selection does not have the power to distinguish between various genotypes with similar, but not identical fitness values. On this view, properties in the phenotype can therefore result from neutral genetic drift rather than selection (Kimura, 1983). The more traditional selectionist paradigm in evolutionary theory has underpinned much work in artificial evolution. Unlike the example provided, artificial Genotype \mapsto Phenotype \mapsto Fitness mappings will almost certainly contain neutrality, but only as an unintended side effect, rather than being intended as a potential adaptive property of the genotype. Only relatively recently has the role of neutrality been taken seriously in the context of artificial adaptation. Genotype \mapsto Fitness mappings, which introduce beneficial neutrality into the search space, have important general implications for artificial evolution as a whole. Investigations into such mappings have exploited the self-organising properties of CA (For example, Shipman, 2003).

It is therefore important to mention that the CA mapping used in this thesis has *not* been investigated along these lines of enquiry, but used only in the context of advantages already outlined above⁵. Having briefly outlined the GA and general conceptions of evolutionary dynamics, we now outline the use of our generative algorithm as a mapping in an evolutionary context.

7.5 Evolution: Generative Architecture

This is the first section of this thesis where we attempt to bring together all the methods presented previously in an approach to evolutionary design. We present a genetic encoding and how this (with the use of our developmental algorithm) maps to phenotypes (Section 7.5.1). We then present the details of the genetic algorithm used (Section 7.5.2). We present details of the phenotype-fitness mapping (Section 7.5.3) and show results from an evolutionary

⁵This is not to say that work in the area of Neutral Networks is not important to our concerns. Open-ended evolution is a desirable property for all Evolutionary Algorithm applications. However, research in this area is in early stages of investigation and beyond the scope of this thesis.

7.5.1 Genotype \mapsto Phenotype

In this section we detail the particular Genotype \mapsto Phenotype \mapsto Fitness mapping in four steps. Firstly, we place ant, seed and exit coordinates under genetic control (see Section 7.5.1.1). Secondly, we outline the overall arrangement and length of the genotype, which corresponds to parameter settings and bit-length allocation for certain blocks (see Section 7.5.1.2). Thirdly, the developmental model outlined in Table 7.3 contains a small, but significant problem, which we highlight and remedy (see Section 7.5.1.3), before proving a brief summary (see Section 7.5.1.4).

7.5.1.1 Genetic Encoding

The set of ants A and set of seeds S each correspond to Cartesian coordinate sets C_A and C_S , which allocate a corresponding vertex v_{ij} bit $v_{ij(ant)} = 1$ or $v_{ij(seed)} = 1$ on the 2-lattice G. For the genetic encoding we specify C_A and C_S as binary strings where neighbouring sections in the string correspond to x-y coordinate components. These coordinates map onto a 2-lattice, placing ants and seeds at even and odd locations, respectively. The first step is to define N the constant of the N^2 lattice—this must be even. Then $n = \frac{N}{2}$ is the upper limit value of the binary representation for each component of the coordinate sets. Each binary segment for the ants and seeds is thus mapped back to the lattice according to $n \times 2$ and $n \times 2 - 1$, respectively (see Figure 7.12).

We constrain the number of exits to four and the location of each to a specific side of a bounding ring subspace R(G). Exit locations are interpreted from a binary encoding as a single value along a predefined ring boundary section. The binary encoding therefore maps to an appropriate boundary along which the GA is free to explore (see Figure 7.13).

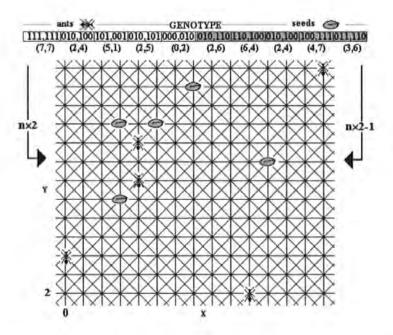


Figure 7.12: Genetic encoding of ants and seeds. The genotype contains a sting of bits, which encode vertex locations on a 2-lattice graph. The values in the genotype are mapped to the space differently according to whether they represent ant or seed coordinates.

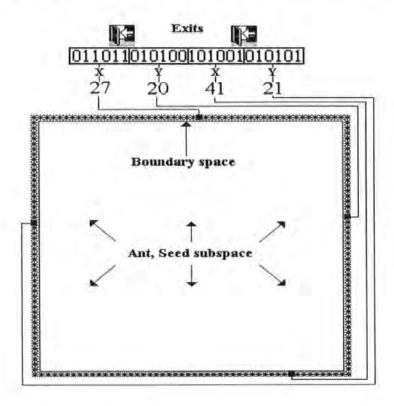


Figure 7.13: Genetic encoding of exits. Four exits are constrained to a single side of a boundary space so that the GA is free to explore either the x or the y component, depending on which side of the boundary an exit resides.

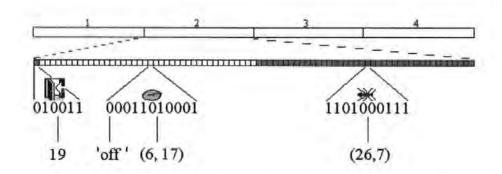


Figure 7.14: Genotype arrangement. Inside a single genotype a single exit and a number of seed and ant coordinates are specified.

7.5.1.2 Genotype Length and Arrangement

Here we provide detail on dimensions and parameter values, how this defines genotype length l and how we arrange the genotype. The size of G is $N^2(N=52)$ and contains two subgraphs U(G) and R(G), which define the main space (for ant and seed initialisation) and the ringed boundary, respectively. Therefore |U(G)| = 2500 and |R(G)| = 204. We set $\rho_{ant} = \rho_{seed} = 0.016$ and therefore |A| = |S| = 40. Six bits represent each of the four exits. Eleven bits represent each of the forty seed coordinates including an extra on/off bit to represent the absence/presence of a seed. Ten bits represent each of the forty ant coordinates. Therefore, the length $l = (6 \times 4) + (11 \times 40) + (10 \times 40) = 864$. We section the genotype into four blocks of 216 bits, which represent a single exit, forty seeds and forty ants (see Figure 7.14). Each exit in the block 1, 2, 3 and 4 is interpreted as n, e, s and w, respectively.

7.5.1.3 New Halting Condition

In Table 7.3 the main loop is controlled by a 'while'—'while ants are 'on' the loop continues'. However, one possible problem with this approach is related to the dynamics of the ant CA. We have seen previously (see Chapter 6) how the rules in Langton's Ant tend towards a highway pattern. This is an example of an attractor. In this case the attractor continues to diffuse through space. However, attractors in the rules can cause oscillations or fixed points at local regions in the lattice and thwart diffusion—in the case where ants are coded to occupy the same coordinates we can observe this.

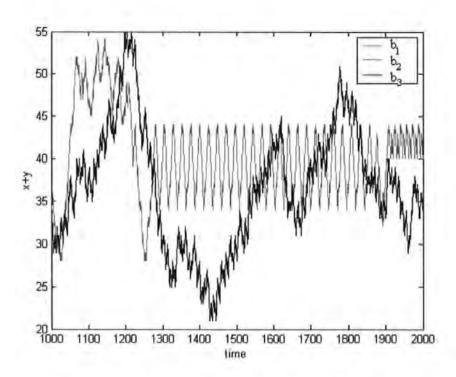


Figure 7.15: Oscillatory patterns in ant CA. The 2-D ant CA can enter into oscillatory patterns, which hold ants inside local regions of the space. The ants as described in the text are labelled b_1, b_2 and b_3 , respectively.

Consider three ants, which occupy coordinates (29, 24), (29, 24) and (24, 28) in U(G). Two ants occupy the same coordinates and tend to enter oscillatory patterns. Where the third ant perturbs these patterns oscillatory behaviour is released, but only temporarily. Without the perturbing influence of other ants, the two oscillators will be confined to a local region in U(G) and will therefore not necessarily be able to diffuse to areas where the turn-ant-off condition is met. Thus the main loop in Table 7.3 can be infinite. In order to avoid this, we replace the 'while' with a 'for' and define ε as the number of iterations through the main loop. We observe a large majority of ants meet the turn-ant-off condition within 400 iterations (For example, see Figure 7.8) and so we set $\varepsilon = 400$. Any ants that enter local attractors are therefore ignored and infinite loops are avoided.

Before introducing details of our genetic algorithm, we present typical phenotype individuals (see Figure 7.16). These phenotypes have not been subject to evolution and so represent typical phenotypes preset an initial evolutionary generation. We now go on to outline our evolutionary method.

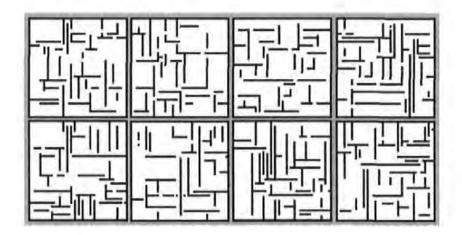


Figure 7.16: Example phenotypes. We define a configuration with an N^2 2-lattice graph G of order |G|=2704. Two sub graphs U(G) and R(G) where |U|=2500 and |R|=204 defines an empty space and a bounding seed ring. We set $\rho_{ant}=\rho_{seed}=0.016$. A chromosome encodes coordinate locations for ants A and seeds S where |A|=|S|=40, so that the chromosome length l=864 (see text). We set $\alpha=2$.

7.5.2 Genetic Algorithm

There are a few details worth highlighting. We implement:

- Tournament Selection: two random individuals are chosen and the fittest replaces a random individual in the population, until an entirely new population has is created.
- Elitism: a single fittest individual replaces a random individual in the population at each generation.
 - In the face of what we think might be quite destructive mutational forces, we include this elitist individual, which also counters some of the possible side-effects of tournament selection, where a uniform distribution is used to choose individuals for a tournament
- the mutation rate $p_m = \frac{0.7}{l}$ so that approximately half of the population experience a single mutation in each generation (Muhlenbein and Shlierkamp-Voosen, 1993).

These general heuristics are not presented as definite 'best-methods' to use—parameter choice is often an art-form and, as this is a preliminary approach, judgements may well be less refined than in other, well-tested GA applications. In this respect, it is not our intention to provide a detailed analysis of GA performance. A more important issue is to provide a useful calculation of fitness in order to allow selective pressure to be produced from our model of pedestrian evacuation.

7.5.3 Phenotype \mapsto Fitness

Each phenotype maps to a fitness value. This is done using the egress model designed in the previous chapter. For the current evolutionary approach we set the density of pedestrians $\rho_p = 0.5$. We are interested in optimising the escape ability of a space and fitness should represent the amount of time elapsed between the beginning of the simulation and the end, when all pedestrians have escaped. We record escape time μ_i and an initial depth φ_i from v_{o_i} to $v_{d_i} \forall P_i$, where v_{d_i} is a randomly assigned exit. Fitness is then calculated according to:

$$f(x) = \frac{\sum_{i=1}^{N} \frac{\varphi_i}{\mu_i}}{N} \tag{7.3}$$

where the minimum possible time, φ_i , for a given pedestrian to escape is divided by the actual time taken μ_i . For example, in the case where each pedestrian moves from v_o to v_d in minimal time, then f(x) = 1 and were each pedestrian to take twice the minimal time f(x) = 0.5. In this way, we maximise fitness by minimising escape time and retain the analogy of the fitness landscape where peaks correspond to fit solutions. We embed this fitness function inside our GA, which is summarized in Table 7.5.

7.5.4 Evolutionary Simulation 1

The GA is free to explore exit positioning and the generation of structures internal to the boundary ring. We run the GA for 6 trials over 500 generations. We observe evolution and how this demonstrates that the fitness function provides evolutionary pressure to the population of genotypes (see Figure 7.17). This is a fairly typical evolutionary plot, but we

Table 7.5: Our Genetic Algorithm

/*Main Loop*/

- 1. Randomly generate an N-sized Population of l bit genotypes
- 2. Develop N phenotypes according to the procedure outlined in Table 7.3
- 3. Calculate fitness f(x) of each genotype x according to Equation 7.2
- 4. While (|NewPop| < N)

Select two parent genotypes according to tournament selection (tournament size = 2) with replacement.

With probability p_c crossover parents to produce two children

With probability p_m mutate children and place in NewPop

End While

- 5. Overwrite an individual in NewPop with best from population
- $6. \ \ Population = NewPop$
- 7. If (generations = 0)

HALT

End If

8. Else

```
generations = generations - 1
(goto step 2)
```

End Else

```
\frac{/{}^{*}Parameters{}^{*}/}{p_{c}=0.7,\;p_{m}=\frac{0.7}{l},\;|l|=864,\;N=100,\;generations=500.
```

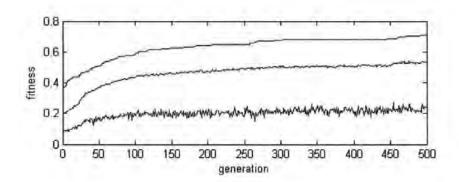


Figure 7.17: GA evolution. Results indicate evolution in the GA averaged over 6 trials. We plot the best (top), average (middle) and worst (bottom) fitness in each generation.

notice that the selection pressure created is not particularly strong. We believe this is related to our choice of pedestrian density in the fitness function and perhaps also so the influence of queuing behaviour at exits. However, we will return to discuss this in the final chapter, in the context of general limitations and future directions to research.

We present each fittest phenotype from the start and at every 100th generation (see Figure 7.18). We can see how the configurations open up in order to improve pedestrian flow behaviour. In particular, one feature very typical in randomly generated structures (see Figure 7.16) is missing—the production of corridor type spaces. This is an obvious way of optimising the fitness of a given configuration—reducing the number of points of conflict within a space so that pedestrians can escape more quickly. However, there may be some trade-off between the openness of space, the structure of the environment, and pedestrian-flows.

We might speculate that the GA, while maximising fitness, is minimising the depth to given exits in the space, because corridor spaces are conducive to increases in depth. We test this possible assumption by plotting the average depth to each exit for four example GA runs. We use the fittest phenotypes at each generation to derive these results (see Figure 7.19).

Results indicate that although the depth is reduced in some examples, in others the depth can change often quite erratically between generations. This would indicate that certain structures are enabling better crowd-flow, and that these structures appear to be relatively independent of depth statistics. The indication is that the reduction of depth is not necessarily

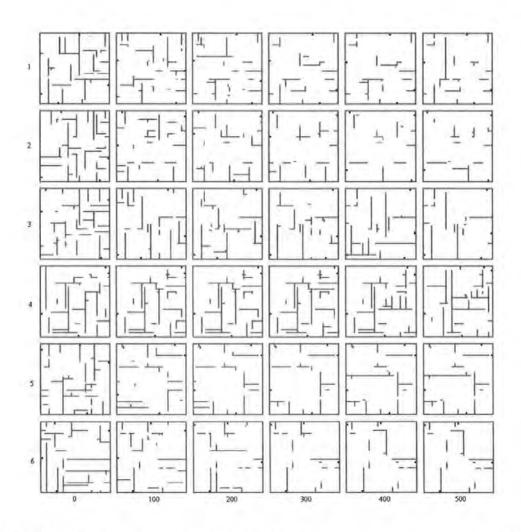


Figure 7.18: GA Evolution (configuration). We plot the fittest genotype at each 100^{th} generation for 6 trials. We can see how the GA opens up the configuration space in order to minimise the amount of conflicting flow.

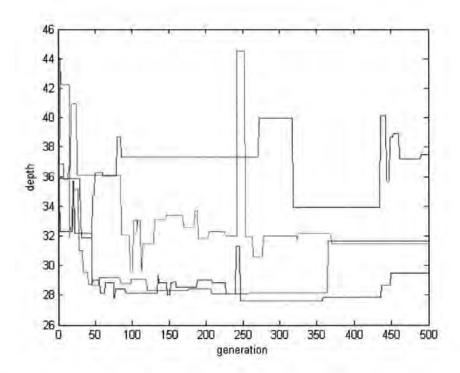


Figure 7.19: GA Evolution (depths). We plot the average depth values for the best phenotypes in each generation for four out of the six trials. Results indicate that this is not a particularly strong force in selection.

the best way to optimise egress behaviours. This is in agreement with other work where the strategic positioning of certain structures can improve egress performance (Still, 2000).

7.5.5 Summary

In this section we:

- introduced a genotype → phenotype mapping, which is derived from the developmental algorithm introduced earlier in the chapter.
- · briefly outlined features in our GA, which differ from the simple GA.
- presented the phenotype → fitness mapping in the GA, which is derived from the Multi-Agent System model presented in Chapter's 5 and 6.
- demonstrated the evolutionary approach be evolving generative configurations. Although evolutionary pressure was slight, we demonstrated one way in which the fitness

functions might be used. Results indicated that depth-from-exit was not continuously optimised and therefore that evolved structures may well be aiding crowd-flows.

This ends our preliminary demonstration of unconstrained evolutionary architecture. We now move onto a more constrained, applied scenario.

7.6 Evolution: Constrained Architecture

This section is intended to demonstrate another potential use of the fitness function, for the evolution of constrained architectures. We thus do not use the developmental algorithm in this section. Instead we choose a configuration layout from a real building (Portland Square Building, University of Plymouth) and attempt to evolve exit sign information by using our pedestrian model. We present details of the Genotype \mapsto Phenotype \mapsto Fitness mapping (Section 7.5.1) and then results from evolutionary runs (Section 7.5.2).

7.6.1 Genotype \mapsto Phenotype \mapsto Fitness

Portland Square Building is a typical office environment. We intend to evolve exit-sign locations and assume these to be placed on corridor sections of the floor-plan. We present the original floor plan and the subset of space taken to define walls on corridor lengths (see Figure 7.20). The subset is mapped to a 1-D array of length L = 209, because there are 209 vertices in the subset.

The genotype length $l = 30 \times 8 = 240$ so that 30 blocks are decoded to values in the range 0 - 208. Each block in the genotype corresponds to an exit, which is thus free to be placed anywhere on the corridor subspace. An extra bit codes for either 'Exit 1' or 'Exit 2'. A random genotype therefore maps to a phenotype, which we represent by a line drawn between a given exit sign set and their respective, coded exit locations. We use a black line for 'Exit 1' and a white line for 'Exit 2' (see Figure 7.21).

We use our pedestrian model in the following way. Each pedestrian P_i is initially fixed inside

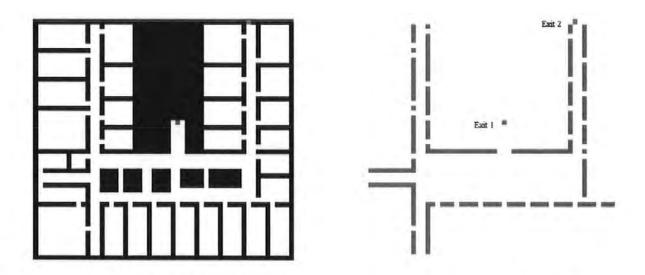


Figure 7.20: Portland Square. This is a birds-eye view of a floor from Portland Square Building, University of Plymouth. The corridor subspace (see text) covers 209 vertices of a 2-lattice graph. We label two exit locations 'Exit 1' and 'Exit 2'.

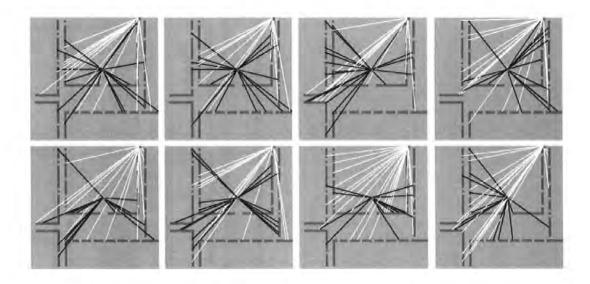


Figure 7.21: Random phenotypes. Each exit sign location is mapped to a corresponding exit bit specified in the genotype. The resulting phenotype is represented as a random network of links to either exit location.

one of the 27 rooms (2 pedestrians per room). The size of the pedestrian population is thus 54. Each pedestrian moves randomly until an exit sign is detected in a Moore neighbourhood Γ_i . On detecting an exit sign, v_{d_i} is then set to the vertex specified by the bit of the genotype, which maps to either 'Exit 1' or 'Exit 2'. The pedestrian can then begin moving towards the exit as indicated by our model. Fitness is calculated in the manner given in Equation 7.2.

7.6.2 Evolutionary Simulation 2

We run 8 trials of the GA, which is the same as the one given in Table 7.5, other than the details outlined in this section. The parameters in the ga are: $p_c = 0.7$, $p_m = \frac{0.7}{l}$, l = 240, N = 100, generations = 500..

We present evolutionary results, which again show the effect of selection over time (see Figure 7.22). We present two example trials in order to show the effect of the random walking in the model, which introduced noise into the fitness function. However, even with this noise, evolution takes hold and fit phenotypes develop. We also present these phenotypes (see Figure 7.23). Notice how the population of exit-signs is split between the two exits in each trial. The pattern of split is such that pedestrians tend to move towards the closest exit, but also pedestrians moving to different exits avoid each other and conflict in the population is minimised.

7.7 Summary

In this chapter we:

- reviewed two models of aggregative growth systems in nature.
- developed an algorithm for the automatic generation of valid, single component labyrinthine configurations.
- presented a method of how to use the algorithm in an evolutionary context as a component in Genotype → Phenotype → Fitness mappings.

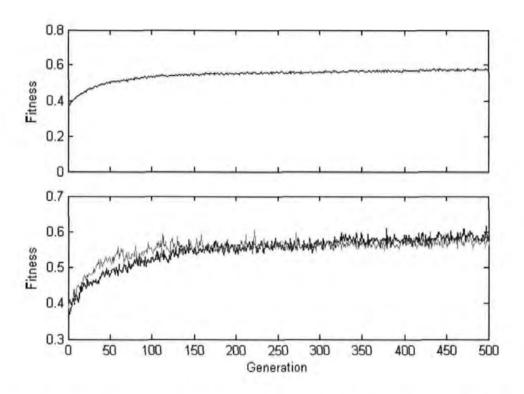


Figure 7.22: Exit Sign Evolution. Top: Average fitness over 8 trials. Bottom: Two example trials, which demonstrate the noise in this evolution.

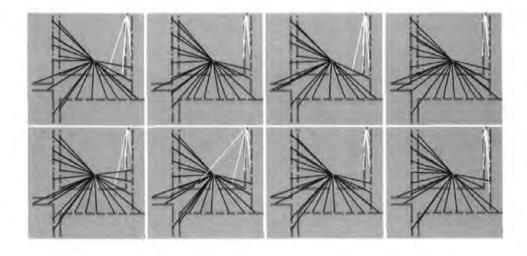


Figure 7.23: Exit Sign Phenotypes. Notice how each phenotype has split the population neatly into two $P^{Exit \ 1}$ and $P^{Exit \ 2}$. This split reduces conflict in the pedestrian model.

• illustrated use of fitness functions for generative and constrained architecture.

This chapter therefore completes our demonstration of how the kinds of MAS fitness functions developed earlier in the thesis might be used in the future. However, technical issues to do with the fitness function itself are relatively independent of evolution. There are problems to be addressed and future directions to be considered before this kind of vision for evolutionary architecture can be taken forward. We will turn our attention to these issues while summarising and concluding the thesis.

Chapter 8

Conclusion

Our long-term objective is to design fitness functions, which introduce useful selective forces, for an evolutionary approach to architectural design. There were two main strands of research, 1) fitness function definition and 2) an evolutionary approach. The first strand defines the core work in this thesis, where we have concentrated on the development of techniques for the design of egress models. We review the important contributions of the thesis (see Section 8.1), before outlining future directions of research (see Section 8.2). Finally, we provide some closing comments to the thesis (see Section 8.3).

8.1 Thesis Summary

8.1.1 Background: Crowd Dynamics

In order to begin thinking about the problem of fitness function definition, we investigated existing techniques from the pedestrian and evacuation modelling community (Chapter's 2 and 3). All of the techniques had limitations, but those based the Multi-Agent System concept used an appropriate level of abstraction—simulation models where patterns emerge through unfolding dynamics—which allow important interactions to be represented.

Given the requirements of labyrinthine spatial complexity, models based on discrete space

were subject to further review. These models implicitly use a 2-lattice graph. After defining important features of this graph, discrete models were reviewed in more depth. Two general methods, which capture two separate levels appeared promising:

- Cellular Automata (CA)
 - local dynamics
- Attractor Fields
 - global dynamics

8.1.2 Limits to Locality: Percolation

Chapter 4 is the point where the core work in this thesis began. We replicated a number of CA models. The first (Nagel and Rasmussen, 1994) allowed us to gain an understanding of the interactive properties of lattice spaces. In the 1-lattice, interactions can be said to *ripple* over time. At critical parameter values, random driver effects can have affect other drivers who are much further away, but at a later point in time.

These chains of interdependence are best appreciated by looking at the statistical dynamics of a model. However, what you see depends on the statistics you choose to observe. Publications on another model, the pedestrian CA (Blue and Adler, 2000a), include average speed-density relationships, which appear unproblematic at face value. We endeavored to gain a fuller understanding of these models by looking at alternative statistical properties and performing an in-depth investigation of the dynamics. Statistical results were complemented by more qualitative observations. Both revealed breakdowns in self-organisation in the face of increasing density. Also, rather than steady deterioration, breakdown was sudden.

One particular experiment revealed underlying patterns of randomness in different populations of pedestrians, which later (Chapter 5) was revealing in the light of 2-lattice percolation statistics. But perhaps this behaviour could have been due to the internal logic of local rule sets? We attempted to investigate this question and control some of the inherent randomness. We also attempted to investigate local dynamics with a view to using them in an attractorfield context, with the aim of integrating the two separate levels. We failed however, and dynamics remained brittle at unacceptably low density values.

The persistent and catastrophic nature of model failure provided a clue as to what was happening. It is known in other areas of graph theory that random edge connections between vertices in a graph undergo phase transitions—sudden, global changes in connectivity with the formation of giant-component clustering (Barabsi, 2002; Watts, 1999; Kauffman, 1995). Could it be that clustering properties were also responsible for the phase transitions observed in the CA?

We argued that 1-D search components of the pedestrian CA, inherited from the 1-lattice car CA, put unacceptable limitations on 2-lattice dynamics. Neighbourhoods were invaded by the sudden emergence of percolation in the 2-lattice, which is analogous to the rippling effects that take root in the car CA. This idea was supported by statistical results, which showed that underlying patterns of randomness converge within critical density ranges, ripe for percolation. These percolation effects are unintended and, rather than capturing interesting properties of real systems, like the car CA, dynamics in the pedestrian CA produced nonsense.

8.1.3 Solution: Inclusive Search, Simple Dynamics

These investigations provided us with an empirical foundation on which to develop a new approach to pedestrian and evacuation modelling. We aimed to provide a solution to the brittle properties of local search and the non-responsive, fixed nature of attractor fields. This solution was based on a more inclusive concept of search.

We proposed the use of a Pulse Coupled Neural Network as a candidate search algorithm, because it automatically encapsulates inclusive search, where all paths away from a given vertex origin can be searched until a specified destination is reached. This allowed us to move model dynamics away from a purely local regions, and to increase model robustness to lattice percolation. We investigated the PCNN and showed how best to arrange neuron equations to produce clean, expanding autowaves, so that the neural plane could contain shortest-path, traceback information. The PCNN is computationally expensive, but the complex behaviours obtainable with simple network logic are a worthwhile trade-off.

In order to improve the trade-off, in Chapter 6, we presented a more efficient algorithm for autowave generation. Pedestrians, it is generally assumed, make more or less local decisions, but attempts to model these mechanisms in discrete space have failed. It is this previously demonstrated, practical difficulty, which motivated inclusive search, rather than any abstract philosophical stance towards human cognition vis à vis navigation. We believe that more inclusive search is the best way to overcome the complex problems inherent to discrete updates on the 2-lattice.

This approach also enables the specification of simple dynamic rules for model specification. A dynamic is a rule, which processes the current state of a system and produces an updated state. In our network-level tools, one rule goes as follows: explore every path away from a given vertex to another vertex. Then: return the shortest path between the two vertices. With these dynamics, we were primarily interested in producing higher-level patterns of emergence, compatible with real observations of crowd populations. We demonstrated how other pedestrians constrain network search and, accordingly, the way to model pedestrian interaction was through a relaxation of these constraints, relative to movement directions.

At the end of Chapter 6, with this simple method, we replicated some higher-level patterns found in real crowds. This was one of our main aims—to develop simple models of complex crowd behaviour. We captured some of the local processes in pedestrian-pedestrian interaction, while also implementing longer-range egress behaviour. This compromise between local and global dynamics is a particular strength of the methods, which are vindicated by the production of more realistic results compared with other approaches.

8.1.4 Evolutionary Approach

The results justify the use of these methods in an evolutionary scenario. The aim of Chapter 7 was to provide a preliminary, motivating framework for evolutionary architecture. We defined a Genotype \mapsto Phenotype \mapsto Fitness mapping. One of the main difficulties in coding a genotype for a valid architectural configuration is to ensure that the resulting phenotype space is not fragmented, i.e., that the lattice remains single-component.

We looked at models of nature-inspired growth to see if we could use any of the processes assumed in natural architectural construction and, therefore, avoid the difficulty of having to specify complex phenotypes in binary strings. The main motivation for this approach was to design a developmental procedure, which acts as genetic-code container. Any of the complexity in phenotype construction could then be handled by the container. The systems that inspired our own—social insects and crystal growth—had some desirable algorithmic and phenotypic properties. We attempted to incorporate these features into our own system.

Our algorithm consisted of two different growth agents—ants and seeds. We showed how this approach could generate architectural spaces without fragmenting them. The method also produced a large variety of forms, which thus met a second evolutionary requirement, i.e., phenotypic variation. In order to map genotypes to variable phenotypes we then placed two sources of phenotypic variation under genetic control.

Evolutionary simulations demonstrated fitness function pressure. In general it appeared that the GA was evolving to spaces of least depth, but further results suggested that the presence of obstacles within the space might be beneficial for an egress scenario; the statistic of average depth was seen to be largely unimportant in the evolution of fit phenotypes. This suggested that evolution may well be exploiting a generally known principle; that obstacles within a configuration may increase, rather than decrease, pedestrian flow (Still, 2000).

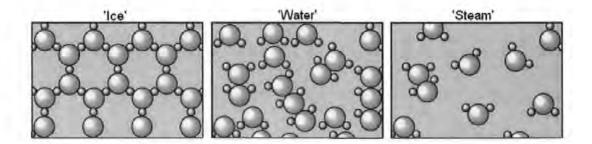


Figure 8.1: Spatial arrangements of hydrogen and oxygen. There are three different behavioural regimes, which we associate with three global properties; 'ice', 'water' and 'steam'.

8.2 Future Directions

8.2.1 Concepts of Space

The immediate concern for the current research is to tackle problems, which stem from 2-lattice representations. Although our method goes some way to relaxing constraints on pedestrian dynamics, we believe that different lattice arrangements should be studied and comparisons made in terms of:

- Search robustness
- Number of multiple optima

Our general thoughts on search robustness are best captured by analogy. Consider typical bond properties of Hydrogen and Oxygen units. 'Ice' is a relatively dense, but tight arrangement, whereas 'water' is more dense and less tightly arranged. 'Steam' is less dense and characterised by relatively weak bonds. We present these three different regimes (see Figure 8.1).

We can think of interacting pedestrians as analogous to interacting atoms. At low density, there are relatively few interactions and self-organisation is allowed to rule the dynamics; e.g., lane formations are allowed to appear. We might think of this regime as steam-like, but rather than randomness ruling motion, as is the case with atoms, specified model dynamics are allowed to work and arrange the global properties of the pedestrian population. As

we increase the density, we reduce the effects of self-organisation. At a critical point we observe a mixed regime; neither self-organisation or percolation wins. This is water-like; pedestrian neighbourhoods begin to merge, but not catastrophically, and some of the dynamics are allowed to unfold between fragmented clusters. Where we increase density above a critical point, we expect that almost all neighbourhoods connect, where once fragmented clusters now form a giant-component. This is ice-like; unintended effects in one local region can percolate far and wide, just like a strong enough force can break hydrogen and oxygen bonds locally, and weaken the global structure of ice by creating long, percolating cracks.

It is possible that pedestrian models need to move away from square lattice concepts of space and towards, perhaps, different kinds of lattices. Certainly, a comparison study of model performance across lattice shapes would be worthwhile. Different vertex connections could be tested for robustness in the face of increasing density. However, it has been shown that most different lattices have a lower percolation threshold than the square lattice (Stauffer and Aharony, 1992). It would, therefore, be unwise to design rule sets on these lattices because we would expect them to be even less robust.

However, there is one exception, the honeycomb lattice (see Figure 8.2). This lattice has a percolation threshold $\rho_c \approx 0.6962$, which is approximately 0.1 greater than the threshold for the von Neumann lattice. We believe this lattice would be more robust for the design of pedestrian dynamics. However, we would not claim that the design of appropriate rule sets for such lattices would be simpler, or even equally as simple, than for more regular lattices.¹

In short, if we use discrete spaces, then we need to look towards the design of underlying representations where percolation thresholds are shifted to much higher densities.

¹This is another reason to use an expanding wave approach; it is more computationally expensive than CA rules, but is a general search method, and should be portable to any kind of neighbourhood. At least with this method we could begin to investigate different kinds of lattices without worrying too much about the design of optimal rule sets.



Figure 8.2: The honeycomb lattice. We can see that each vertex has a neighbourhood size $|\Gamma| = 6$. We might expect that, if a neighbourhood contains more vertices, then percolation might be easier as more vertices are accessible from v_o . However, resultsd show that site percolation in the honeycomb lattice has a percolation threshold $\rho_c \approx 0.6962$ (Stauffer and Aharony, 1992).

8.2.2 Developmental Codings

The methods given in our evolutionary approach are developmental; there is a complex genotype → fitness mapping. Complex mappings and Artificial Embryogenies (AE) are currently of great interest in artificial evolutionary systems. Recent reviews of artificial embryogeny (Stanley and Miikkulainen, 2003) have stated that "...the goal of AE is to be able to evolve phenotypes as complex as biological systems" (p.124). By complex, Stanley and Miikkulainen (2003) seem to mean phenotypically complex. Their review focuses on the benefits of development with respect to 'complexification', where the aim is to produce complex phenotypes without the need to code complexity directly in the genotype. We used a developmental method for the same general reason.

However, as mentioned previously (see Section 7.4.2), there are other more fundamental reasons why we should look at the kinds of developmental algorithms we use. It is thought that natural development improves mutational robustness and allows the maintenance of population variety. Rather than becoming stuck on local optima, as is typical with artificial evolution, biological populations appear to be able to flow away from landscape peaks:

"If no RNA molecule with satisfactory properties is found, a change to high error rate is adequate. Then the population spreads along a neutral network to

other regions in the sequence space, which can be explored in detail after tuning the error rate low again."

(p 54, Schuster, 1997)

Neutral networks thus connect otherwise fragmented local peaks, allowing populations to traverse landscapes and converge on successful solutions without sticking to them. In this way, fault-tolerant development characterises a more open-ended evolutionary search. Recent work in Artificial Evolution is attempting to harness the properties of these natural landscapes in order to provide more open-ended mechanisms for artificial adaptation.

Clearly, this kind of research has important implications to the many approaches, which develop genotypes into phenotypes. It is important to realise that, although developmental codes provide practical advantages for phenotype creation (Stanley and Miikkulainen, 2003), these may not necessarily produce fitness landscapes that are easy to traverse. We would certainly not expect that codes not designed to improve search robustness, in this way, do so. One aim is therefore to encode developmental mechanisms, which maintain the practical advantages, but also allow open-ended evolution. Designing such codes, however, is not necessarily a trivial task.

8.3 Closing Comments

The aim of this research is to develop useful fitness functions for an Evolutionary approach to Architectural Design. Thus far, Evolutionary Architecture has been limited by a narrow vision, where fitness is decided in the context of human decisions about abstract form. We need to move beyond this approach and incorporate realistic models of important crowdflow phenomena, which impinge on the design of human-built forms. In this thesis we have therefore concentrated on developing simple models of complex crowd dynamics.

The usefulness of an evolutionary approach depends very much on the efficacy of these models, an area where there are challenges still to be met. One of the most important challenges is perhaps not only in the design of cellular rule sets, but in the development of more robust cellular spaces. Although we have demonstrated the usefulness of an inclusive search algorithm for producing simple models of complex crowd-flow, we believe that significant progress on discrete space can only be made in this direction.

Appendix A

Glossary

Autowave the occurrence of internally generated waves, which often depend on local couplings between the excitable units.

Egress the process of movement in a population of pedestrians whose goal is to follow paths from their current location to exit points of a configuration.

Cellular Automata lattice based rules determined by local-neighbourhood inputs.

- Clustering Coefficient a value for the clustering $\gamma_v = \frac{|E(\Gamma_v)|}{\binom{k_v}{2}}$ of a given vertex v, where $|E(\Gamma_v)|$ is the degree of v and $\binom{k_v}{2}$ is the maximum possible number of simple connections in Γ_v . For a graph G, the clustering coefficient $\gamma(G) = \gamma_v \forall v \in V(G)$.
- Connected Graph the property of a graph G(E, V) where any vertex $v_i \in V$ is connected by a walk along edge elements $e_j \in E$, such that any other vertex $v_k \in V$ is accessible to v_i .
- Continuous Space a space where coordinates are represented as real, floating point numbers.

Degree the number of connections k_v protruding from vertex v.

¹By simple we mean bi-directional and, therefore, the maximum is derived from a binomial distribution.

- **Destination Vertex** a vertex v_d , which is the last element of a walk away from a vector origin v_o .
- Developmental Algorithm an algorithm, which unfolds over time and often grows in complexity. In the context of a genotype → phenotype mapping, a compressed genotype often expands into a more complex phenotype.
- Discrete Space a space where vertex coordinates are represented as integer numbers. The 2-lattice is an example of a discrete space.
- Dynamic a method which takes as input a given state of a system and then outputs a different state, based on internal rules.
- Field a collection of scalar quantities, which define the global properties of a given space.

 These properties are used in evacuation models to define gradients so that pedestrians can navigate to exit areas.
- Fitness Function a general term used for the calculation of fitness for a phenotype. Fitness is often population-defined in order that individuals can be evaluated against each other. Fitness functions provide artificial pressures akin to Natural selection.
- Genetic Algorithm a nature-inspired algorithm based on Natural Selection. Artificial Darwinian mechanisms, based on *inheritance* and *mutation*, operate on strings of alphabet-constrained alleles, which are interpreted in a Genotype → Phenotype → Fitness mapping and artificially *selected* with the use of a fitness function.
- Genotype → Phenotype → Fitness Mapping the rules that define a mapping from a given Genotype to Phenotype and from a Phenotype to Fitness. The different stages in this process can be more or less complex depending on the application domain and nature of problem description.
- **Graph** defined as G(E, V), where E(G) is the edge-set of the graph and V(G) is the vertex-set. Graphs can be single-component or multi-component.

- Inclusive Search a search where all possible optimal-paths are searched in a link between a vertex origin v_o and a vertex destination v_d .
- Lattice a discrete space where vertices are connected by well-defined edge relationships in simple neighbourhoods. For example, spaces like a 2-lattice can be constructed from von Neumann neighbourhoods or Moore neighbourhoods.
- Local-Global Link $v_o \to v_d$ the paths (or path) defined between the origin vertex v_o and the destination vertex v_d . The link is derived from a traceback algorithm, applied after inclusive search.
- Mult-Agent System any general system composed of many interacting parts. Such systems can also been referred to as 'many-particle'.
- Multi-Component Graph a graph where a single vertex or cluster is separated from other clusters in the graph, i.e., the conditions for a connected graph are not met.
- Neighbourhood the surrounding of a given vertex v denoted by Γ_v , which may define a neighbourhood with arbitrary degree k. However, in this thesis immediate neighbourhoods with degree k=4 (von Neumann) or k=8 (Moore neighbourhood) are particularly important.
- Origin Vertex the vertex v_o currently occupied by a given pedestrian.
- Pedestrian and Evacuation Dynamics the ensemble of rules, which determine the state changes in a given model, designed to represent real pedestrian interactions and evacuation processes.
- Pedestrian Density $\rho(0,1)$ the ratio of occupied lattice vertices to the total amount of vertices in the lattice.
- **Percolation** the property of diffusion in a spatial system, where diffusive interactions are traverse its diameter.

Percolation Threshold the point where percolation is first observed in a system.

Pulse Coupled Neural Network a neural network based on a simple spiking neuron model.

This kind of network has been widely used in image processing applications.

Single-Component Graph see 'Connected graph' (above).

Stigmergy the general mechanism of agent-agent interaction, where the spatial environment is used as a medium of communication.

Vector Canceling the effect of a given procedure where a simple graph, which contains no directional information, is transformed into one that does, so that field information then constrains the bi-directionality of edges.

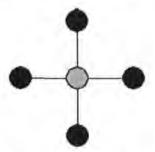
Appendix B

CA Rule Set Extensions

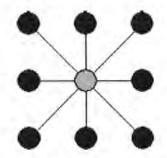
In chapter 3 we listed the rule sets for a pedestrian cellular automata (Blue and Adler, 2000a).

However, for the 4-directional case these rules are extended to account for increased neighbourhood exchanges (Blue and Adler, 2000b).

We apply the same logic in our own extensions to 8-directional scenarios, where the rules are simply rotated again to account for pedestrians in all Moore neighbourhood scenarios. The algorithm in Table B.1 look quite complex, but the logic is simple. Basically, any given pedestrian P_i in a neighbourhood moving in the opposing direction to P_a , or here given as P_n , is exchanged according to a probability $P_{exchange}$. We remind ourselves of the kinds of neighbourhoods being used (see Figure 9.1).



Blue and adler use this neighbourhood



We extend their rule sets to this one

Figure B.1: Neighbourhoods. These are the neighbourhoods, which correspond to the rule extensions.

Table B.1: Pedestrian Cellular Automata Rules (Blue and Adler, 2000a)

1. Step Forward (Parallel Update 2)

See Table 3.1

Step Forward (Parallel Update 2)

- (a) Update velocity: Let $v(p_n) = gap$ where gap is from the gap computation subprocedure (below).
- (b) Resolve cross-directional conflicts: for a desired cell common to two cross-directional pedestrians A and B, cell $L(i+gap_i,j)$, IF rand() < 0.5 pedestrian A gets cell $L(i+gap_A,j)$ and pedestrian B gets cell at $L(i+gap_B,j)$ ELSE pedestrian B gets cell $L(i+gap_B,j)$ and pedestrian A gets the cell at $L(i+gap_A,j)$. IF a conflict at the new cell $L(i+gap_i,j)$, reiterate this procedure.
- (c) Bi-directional exchanges: IF gap = 0 or 1 AND $gap = gap_opp$ (forward cell occupied by an opposing pedestrian) THEN with probability $p_{exchg} v(p_n) = gap + 1$ where new cell is L(i+gap, j) ELSE $v(p_n) = 0$ and gap = 0.
- (d) Bi-diagonal exchanges: IF gap = 0 AND L(i+1,j1) occupied by an opposing direction pedestrian (either diagonally forward left or right cell occupied by an opposing pedestrian) THEN with probability $p_{exchg}v(p_n) = 1$ ELSE $v(p_n) = 0$; IF both diagonally forward cells occupied by an opposing pedestrian in diagonally left and right cells; with probability p_{exchg} and rand() < 0.5 $v(p_n) = 1$ where the new cell is $L(i+1,j1)ELSEv(p_n) = 0$.
- (e) Cross-diagonal exchanges: IF gap = 0 AND L(i+1,j1) is occupied by a cross-directional pedestrian moving forward (either diagonally forward left or right cell occupied by a cross-directional pedestrian that is moving forward relative to the pedestrian under evaluation) THEN with probability $p_{exchg}v(p_n) = 1$ ELSE $v(p_n) = 0$; IF both diagonally forward cells occupied by a crossing pedestrian in diagonally left and right cells; with probability p_{exchg} and rand() < 0.5 $v(p_n) = 1$ where the new cell is L(i+1,j1) ELSE $v(p_n) = 0$.
- (f) Cross-forward-adjacent exchanges: IF gap = 0 AND L(i+1,j) occupied by a cross-directional pedestrian (forward cell occupied by a crossing pedestrian) THEN with probability $p_{exchg}v(p_n) = 1$ ELSE $v(p_n) = 0$. The forward move is to L(i+1,j) and the adjacent-moving pedestrian exchanges laterally to L(i,j1) from its perspective.
- (g) Move: each pedestrian p_n is moved $v(p_n)$ cells forward to the appropriate cell L(i+v(pn), j[0,1]) on the lattice.

3. Sub Procedure - Gap Computation

See Table 3.1

The kind of extensions according to these neighbourhoods are presented here in Table B.1.

Appendix C

Publications

The following publications are included in this order:

Holden, R., and Parmee, I. (2000) Utilising multi-agent based crowd flow models for the evolutionary design of spatial layouts in marine vessels. Proceedings of the 1st European Conference on Marine Technology, Nantes, France.

Holden, R., and Cangelosi, A. (2004) Limits to locality in cellular automata models of pedestrian behaviour. Proceedings of Interflam 2004, 10th International Fire Science and Engineering Conference, Edinburgh.

Holden, R., and Cangelosi, A. (2004) A general, computationally intelligent model for egress simulation. Proceedings of Interflam 2004, 10th International Fire Science and Engineering Conference, Edinburgh.

Evolutionary Spatial Layout Utilising Agent-Based Models of Crowd Behaviours

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Abstract

This paper outlines a proposed approach towards gaining useful measures of fitness for architectural spatial layout by utilising a multi - agent based model of crowd behaviour. Issues relating to computer aided design and the biologically inspired techniques that may underpin them in the future are considered. Then the general concept of an agent is outlined both in terms of intrinsic properties and as a unit within an ecological complex. Finally, the future development of the model is related to long term aims of integration within an evolutionary framework.

Introduction

Perhaps one of the most important issues in architectural design is spatial layout. The concept of space is of fundamental importance and this is reflected in architectural theory (Hillier & Hanson 1984). The implication is that spatial configurations can profoundly influence the nature of people's movements through space and thus certain configurations may be more conducive to the provision of particular movement. A tool capable of providing reliable and useful information on the relationship between spatial layout and people movement may be of great benefit during the process of design.

Inadequate CAD tools

The computer is now an integral part of Architecture courses and just by virtue of its speed the computer is of some use in design. However, it has been pointed out that the use of the phrase Computer Aided Design is more of a misuse because the current software tools do not contribute in any significant way to the actual process of design (Jackson 1998).

The Architects brain still carries the weight of having to sift through the dizzying amount of information available in order to produce designs that satisfy many conflicting constraints. Architects will certainly ignore massive spaces of possible designs and many possibly fruitful avenues in completing their design.

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Biologically Inspired Architecture

Multi-Disciplinary Evolutionary Architecture

Here Evolutionary Architecture has a specific meaning. Some works adopt same terms, but uses them loosely (e.g., Tsui 1999)—Evolutionary Architecture does not necessarily mean that Architecture should look like any particular form that nature herself produced. However, natural algorithms can be abstracted and used as powerful tools in a design framework (Frazer 1995).

The genetic algorithm as a population-based approach might seem like an obvious technique in deploying various solutions and relying on the power of the gas parallelism to produce good combined solutions. However, it is important to remember that creative designs do not decompose into simple representations on which the genetic algorithm will work best.

A key importance to the development of a good tool is that it allows designer interaction as part of the production of architectural solutions. This has always been considered a crucial aspect when considering the potential of evolutionary and adaptive techniques as creative design tools (Parmee 1998).

Also, the power of adaptive search will only be fully harnessed within a multi-disciplinary framework...

Artificial Life Techniques in the Development of Better Tools

The most recent work follows that done by Frazer (1995) and the general theme of an Evolutionary Architecture remains rooted in the following types of techniques: Evolutionary Algorithms (Frazer 1995), Cellular Automata (Broughton, Tan & Coates 1997), and L - Systems (O'Reilly & Testa 2000).

These low-level approaches are considered as a means with which to produce useful architectural forms in order to guide the architect towards informed and creative decisions. In accordance with the experience gained from artificial life in general certain decisions would not be possible or very likely in the absence of these explorative, emergent techniques. In this respect these techniques show the potential to provide powerful computer aided design tools in the future.

Artificial Life techniques have been criticised for remaining useful only in relatively limited domains—theoretical biology. Real world applications of artificial life are still scarce, but it is important to remember that the above examples of research are still in their infancy and are best considered as preliminary investigations into the notion of architectural design by natural principles. This work has brought to the surface similarities between concepts in architectural theory and those in of Artificial Life.

It is thus far from unrealistic to apply techniques of the latter to architectural design...

AGENTS

The term 'Agent' is an inclusive one, used in various contexts. Simple definitions only distort the relevance of the term relating to any particular one of these and

contexts conceptual quicksand often surrounds philosophical debates. Therefore we choose two example approaches relevant to the current project.

Agent-based wayfinding

In this approach (Raubal & Worboys 1999) the concept of affordance is used. Affordance incorporates the notion of subjectivity, but at a reactive level, whereby an object in an environment will allow or 'afford' certain reactions relating to an agent's particular state. It follows that agents will react differently depending on the state they are in. The concept of an affordance seems a good one because it illuminates those fragments of knowledge that exist outside, but not in spite, of agents.

However, in this particular research (Raubal & Worboys 1999), affordances are forced into traditional psychological methodology. People are sampled and tested in virtual reality buildings, the end being a provision of information on how people generally interact with building layouts. Generalisations are made and high-level cognitive models are built in an attempt to isolate an underlying process of wayfinding behaviour.

Reactive agents and ecological constraints

In this alternative approach agents posses simple reactive (Braitenberg 1984) architectures and, acting in unison, these can produce relatively complex *emergent* behaviours. There are many examples of such agents in the field of artificial life.

Although an agent acting independently within an environment can produce relatively complex behaviour, these reactive agents become more interesting as a group. Group behaviour of this sort has been used to great effect in modelling flocking in birds and schooling in fish (Langton n.d.), where collective behaviour emerges from simplistic rules. The purpose is not to imply that the reactive nature of the agents architecture realistically reflects the architecture of an agent being modelled, but that the general dynamics of a group of such agents are similar to those of the group being modelled. In this way social interaction within large numbers of such agents are crucial in producing particular global level patterns.

The environment of an agent will therefore include the existence of other agents and this will profoundly effect the power of an agent to follow its own desires. The agent is embedded in an ecological complex, which is above the level of crude agent architectures, but, at the same time, the agent participates in the production of this complex.

Current Project

The proposed approach will be more related to lower level, reactive perspectives from the field of artificial life. The aim is to develop models whereby local interactions in groups of produce collective behaviour representative of real crowd

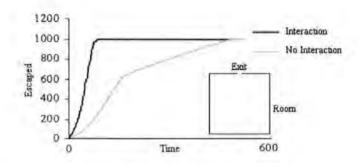


Figure 1: Comparison of flow for interactive and non-interactive agents.

within built forms. In such a system it is assumed that the high level cognitive ability of real people in crowds are not as relevant as the basic interactions between agents and agents with their environment.

Simple example (evacuation)

For illustrative purposes a very simple example of this approach is given (see Figure 1). There are one thousand agents located randomly within an empty room and the aim of each individual is to escape through the exit. In an uon interactive scenario the environment of each agent is simplistic and the performance of the crowd is unconstrained by agent-agent interaction.

In an interactive scenario agents can perceive other agents and the only rule is to *avoid collision* with another agent during the process of completing the task (escape form the room). We see how the interaction of agents on this basis had a large effect on the evacuation speed of the crowd compared to the former scenario.

Future development

The development of the project can be separated in three main areas of interest.

Human Interaction

Relating to:

- obstacles, enclosure configurations, enclosure environment e.g., exits, stairwells, sharp corners, potential bottlenecks etc.
- enclosure specific procedures i.e., alarm system effectiveness, appropriate signing.
- · make-up of the population i.e., age distribution, groupings etc.
- · presence of smoke, toxic gas, fire, remote explosion, noise,

The Modelling of Interaction

- human-environment interaction
- human-human interaction
- human-hazard interaction

The intention is to initially establish simple behavioural agent-based models that take such interactions into account to some extent. Further development of basic models will rely upon calibration to available data relating to fire evacuation and to passenger evacuation data where available. Development and calibration will continue throughout the duration of the project as further information becomes available and knowledge develops.

It is intended that a degree of machine-learning will be incorporated within the behavioural agent models with a longer term objective of establishing autonomous activity to some extent.

Adaptive Design

The development of an evolutionary/adaptive computing design search, exploration and optimisation framework that utilises the developed behavioural models as objective functions in order to investigate:

- spatial layout i.e., the design of exits, passageways, stairwells, muster points etc.
- positioning of signs and lighting.
- information broadcasting and alarm systems

The emphasis will be upon interactive evolutionary design systems that allow a high degree of user-led exploration of possible scenarios.

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Limits To Locality In Cellular Automata Models Of Pedestrian Behaviour

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Abstract

Firstly, we investigate a pedestrian Cellular Automata (CA) model. We show that the model is sensitive to density and seek to control some of the density-dependent randomness in the rule sets, which work against intended pattern formation. The new constraints introduced improve statistical features of the model, but further investigation reveals persistently brittle behaviour. Secondly, we illustrate features of CA spatial representations—interaction properties of the 2-lattice graph. Many models of pedestrian behaviour make implicit use the 2-lattice and we show how this graph determines non-intuitive limits to intended self-organisation. We suggest important implications for local rule-based discrete dynamical models.

1 Introduction

Models of pedestrian movement in discrete spaces have their own domains of application and it is practical to implement specific techniques as needed. However, it is also important that general features of pedestrian behaviour are modelled. In this paper we address CA methods of implementation and their usefulness for modelling the property of lane formation found in crowd populations. CA are increasingly used in this application domain, perhaps due to the success of such kinds of methods in the field of 1-dimensional, 1-directional car traffic modelling (Nagel & Rasmussen 1994). However, once a CA rule set moves into a 2-dimensional domain the number of rules needed increases and the number of permutations expands significantly.

We introduce the connection between a pedestrian CA (Blue & Adler 2000a) and a car CA (Nagel & Rasmussen 1994). Throughout we refer to these models as pedestrian CA and car CA, respectively. We present statistical behaviour of both models and then modify a randomly assigned rule in the pedestrian CA—this rule reinforces mixed lanes and we attempt to conserve the effect of other lane forming rules by constraining its necessarily random nature. Our local constraint controls some of the statistical results, but closer investigation reveals persistent breakdown of self-organised lane formations.

Finally, we present results of Boolean network percolation statistics. We argue that discrete pedestrian models are a specific instance of such networks. We argue that local rule specification is blind to particular effects of density increments—percolation between pedestrian neighbourhoods. We suggest how,

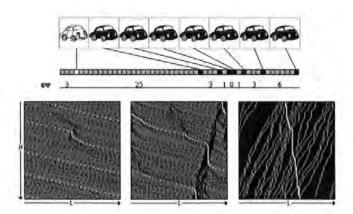


Figure 1: Top: Car Cellular Automata on a 1-dimensional 'road'. Bottom: Space-time plot of car traffic model N(time) = L(space) = 50. Boundary periodic.

at certain densities, pedestrian updates form large connected subgraphs, which shatter intentions built from discrete local dynamics.

2 Cellular automata and movement variation

A CA requires a discrete spatial representation. The coarseness of space can be freely chosen, but once this choice has been made we cannot infinitely fragment the concept of distance. The limits on a single move are chosen a priori and do not, like calculus-based methods, rely on the dynamic aspects of the objects represented, or on infinitely changeable vector quantities. Movements are updated depending on rules that manipulate fixed spatial units. For example, in the car CA, the following four rules apply. 1: Each vehicle of speed $v < v_{max}$ with gap >= v + 1 accelerates to v + 1. 2: Each vehicle (speed v) with gap <= v - 1reduces speed to gap. 3: Each vehicle reduces speed by 1 with probability 0.5. 4: Each vehicle advances v sites. The first three rules represent acceleration of free vehicles, breaking and stochastic driver effects, respectively. The fourth rule is a parallel update. Here, gap represents a discrete value of unoccupied vertices between cars (see Figure 1, Top). The development of the model is controlled by a discrete time variable t where $\Delta t\{1,2,3,4...N\}$. We replicate the car CA and run the model for L=N=50. We observe behaviour under different conditions of density ρ (see Figure 1, Bottom).

One statistic (travel time variation), which Nagel & Rasmussen (1994) use, can be interpreted intuitively as the shape of car movement in the space-time plot (we produce this by highlighting a white car)¹. At low density ($\rho = 0.08$)

¹For details of the car model statistic we refer readers to the literature.

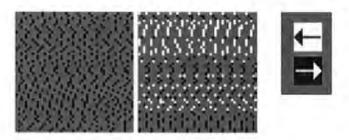


Figure 2: Pedestrian CA. Left: 1-directional behaviour. Right: 2-directional behaviour. $\rho=0.2,\ L=50.$

the white line runs smoothly across the plot where perturbations to movement are small and infrequent. At high density ($\rho=0.8$) this line changes angle, representing slower movement. Perturbations to this movement are again relatively small, but frequent. In the middle regime ($\rho=0.16$) the fluctuations are high and fairly frequent. This represents what a critical point in the behaviour of the model, where unpredictability in travel time shows a sharp peak (see Figure 3, Top).

This car CA forms the basis of the pedestrian CA. Rule sets in the latter are designed for a bi-directional populations, in order that lane formation occurs, so that movement speed is enhanced. Readers are referred to the original publication for details of the rule sets (Blue & Adler 2000a). Like many discrete models of pedestrian behaviour, space is represented by a 2-lattice (L^2). Once this is achieved, rule updates settle to those similar in nature to ones derived from the car CA, but extended to a discrete 2-D plane with movement being 2-directional (the car CA is 1-directional).

We implement the pedestrian CA and present graphical results (see Figure 2). Notice how rules self-organise to produce lanes in 2-directional scenarios. The original statistical analysis by (Blue & Adler 2000a) concentrates on forward moving optimisation over a range of densities. Statistical results show population speed-density relationships as smooth, invested s-shapes, indicating the progressive deteriorations in population performance over density (Blue & Adler 2000a).

However, with this kind of analysis analysis, we believe observations are limited to rather uncritical and 1-dimensional picture of a populations statistical dynamics. We propose a travel time variation statistic. This statistic indicates the nature of randomness built into a CA and provides more insight into the sensitivity of self-organisation—lane formation...

In terms of random effects, whereas the car CA implements stochastic driver effects, the pedestrian CA implements randomised pedestrian exchanges. When two opposing pedestrians meet they swap positions on a random basis ($P_{exch} = 0.5$). For the measure of variation of a pedestrian population P^z we define a

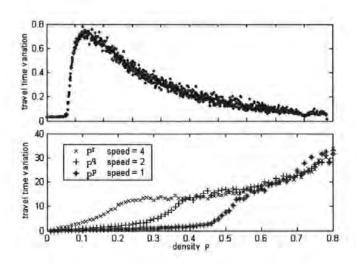


Figure 3: Variance of travel times for CA movement models. Top: car CA Bottom: pedestrian CA, S=100

number of steps S to be advanced in the direction of motion of P^z and record t when this is achieved. We label three different pedestrian populations corresponding to increasing speed P^p , P^q , P^r and run three separate simulations for each to demonstrate the effect of different local neighbourhoods on the variation (see Figure 3, Bottom). In the pedestrian simulations the coefficient of variation of $t \forall P_t$ is plotted.

After reaching a peak the car CA statistic returns to 0 ($\rho=1$). In the pedestrian CA each P^z measure increases with density. We observe in P^p less perturbation at low density because advancement towards S is in small incremental steps—due a smaller range of neighbourhood interactions (we return to the issue of neighbourhoods below). Density effects therefore strike at higher density. Comparing populations, results demonstrate the importance of neighbourhood effects. In the pedestrian CA the walk speed essentially determines the view (search done along collections of 1-D components) or knowledge available from the perspective of a pedestrian automata. We can also see how the pedestrian CA travel time variance continues to rise as density increases in all populations above a given density $\rho\approx 0.58$.

These are effects of the default randomness in the pedestrian CA. When two opposing pedestrians meet, in order to continue advancing forward a random number determines an exchange of position. When we increase the density we increase the number of these interactions, increasing the amount of random influence as a whole. Further, these random exchanges reinforce mixed lanes, which counter the intended dynamics. By allowing two opposing pedestrians to swap positions we allow them to remain on the same lane. In short, when

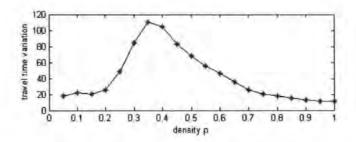


Figure 4: Travel time variation results for a 2-directional population of pedestrians with a walk speed of 2

we increase the density, rules intended for self-organised lane formation are perturbed more often than in lower density scenarios.

3 Constraining default randomness

In this section we aim to control pedestrian CA randomness in order to enhance the development of lane formations. The parameter ρ is global. In a L^2 lattice it can be used to determine a probability that a given vertex be occupied at initialisation and, e.g., where L=50 and $\rho=0.5$ we would expect approximately 1250 pedestrians to occupy the space. We define the local measure of density with the concept of a neighbourhood, which has a parameter k (number of connections to the centre vertex of the neighbourhood) (Watts 1999). A Moore neighbourhood is used, which consists of 9 vertices (k=8). In our definition of exchange we use the following equation to define exchange probability.

$$P_{exchange} = 1 - \frac{\sum_{P \in \Gamma_{\nu_o}}^{8} P}{8} \alpha \qquad (1)$$

where

$$P = \begin{cases} 1 & \text{present} \\ 0 & \text{absent} \end{cases}$$
(2)

so that P defines a truth condition for the presence of another pedestrian in a Moore neighbourhood Γ_{v_α} , 8(k) scales the second term between 0 and 1 and $\alpha=(0,1)$. The alpha parameter is used to tune the probabilistic constraint on exchanges. The value of α determines the range of this function and where it is set to a high value the constraint on movement is correspondingly high. In order to demonstrate the effect of local, density-determined exchange we set $\alpha=0.9$ and we repeat the experiment for pedestrians with a walk speed of 2 (see Figure 4).

Results indicate a control of randomness in the CA. The general shape of the plot resembles that of the car model—single peak. This does not imply



Figure 5: Left to right: Graphical snapshots of the pedestrian model. Density $\rho=0.3,~0.5$ and 0.7, respectively.

that the models behave in a similar way, but only that the randomness inherent in the models follow similar patterns across density. Although we attempted to constrain the inherent randomness in the pedestrian model, as a method of reinforcing lane formation rules, closer observation indicates that perhaps the cause of failure is more fundamental—we still observe pattern breakdown (see Figure 5).

4 Updating on the 2-Lattice

In this section we look to more fundamental reasons why this breakdown may be unavoidable, particularly with local rules as the only means of self-organisation. It is well know that information in 2-lattice networks display certain properties of percolation at given densities, with implications for many kinds of complex systems (Sole & Goodwin 2000, Kauffman 1997, Barabsi 2002). We present similar results here and interpret the pedestrian CA as a specific instance of a boolean network.

Consider the underlying space of the 2-lattice, constructed from a von Neumann neighbourhood Γ . Each pedestrian P with a probability ρ occupies a site (1=on) or they do not (0=off). Each vertex v_{j0} , defining the bottom row of a lattice, at t=0 is loaded to V. At t+1 we search $\Gamma \forall V_{ij} \in V$ and store any immediate on neighbour into V. We then turn the previous members of V off, mark them as a 'walkable vertex' and remove them from V. We iterate this procedure until |V|=0, which defines the halting condition, where there are no more on vertices to be loaded. This procedure defines a type of network walk on the occupied pedestrian locations, which can be used to indicate how the complexity of information on the lattice interacts spatially. We plot t, the 'walk time' when the halting condition is met, and the total fraction of 'walkable vertices' (see Figure 6).

The implications are that at certain densities rule updates will have an effect outside of local neighbourhoods within a single update of the population. If we consider again the results for the pedestrian model in figure 4, we see that at around the critical density, travel time variation for P^p , P_q , P_τ merge—random

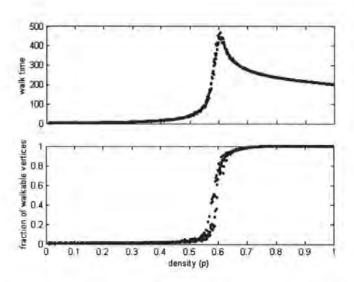


Figure 6: The nature of percolation effects on a 2-lattice graph. At a critical density (0.57) this Boolean network connects into large clusters indicated by a large increase in the walk time and fraction of walkable vertices.

behaviour in all populations reaches a similar level. It appears that rule set based search for optimal forward moves has reached some kind of limit at this point. Boolean network statistics imply that any intended optimal move of a single pedestrian can be perturbed by that of another pedestrian within a single update of the population. We believe that at particular density phases the population will be underpinned by a giant component graph. If this is true the visual representation of pedestrian units as separately updateable entities mislead us (see Figure 2).

When pedestrians' form connected spatial sets we believe that rule updates will show catastrophic failure. We have only shown how this works for an abstract Boolean 2-lattice within a single-step. However, it is seems that $\rho=0.57$ will represent a ceiling of self-organisation for Cellular Automata models like this one, where non-intuitive and spatially extended properties of the 2-lattice maximally thwart self-organising properties of the model.

5 Summary and conclusion

CA methods of implementation are increasing in the pedestrian modelling community and might be a promising way in which models of pedestrian pattern formation can be achieved. However, in a chosen CA, our investigation reveals that some of the rules create runaway random effects at increasing densities. One plausible approach to remedy this is to attempt to control this randomness at its source, locally from within the rule sets. On closer inspection, however, pedestrian CA statistics are shown to hide what appear to be catastrophic failures in self-organisation, indicating that other less apparent dynamic mechanism are running the show, rather than the discrete dynamic rules by design.

We have suggested that the problem could lie in the localisation of rule design in these models. As an instance of a Boolean 2-lattice, the presence of certain quantities of other pedestrians can seriously undermine the knowledge gained from a local neighbourhood. Although vertices are discrete entities, neighbourhoods are not. Changes in one neighbourhood can have effects that percolate outside a given local region. These kinds of behaviours have been known in the field of computational physics and graph theory for some time. The notion of percolation is important for CA models of pedestrian pattern formation because many CA models rely on a graph representation of space.

In short, to maintain pedestrian patterns across the entire density range, it seems that discrete pedestrian models might be forced to use higher-level search algorithms in the face of *limits to locality*.

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A General, Computationally Intelligent Model of Egress Behaviour

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Abstract

We present statistical features of a 2-lattice graph and argue that a search algorithm for pedestrian movement will need to cope with network-level information. As a candidate search algorithm, we introduce a Pulse Coupled Neural Network (PCNN). We improve the search mechanisms in the PCNN equations and show how it can then be used as a general search algorithm for pedestrian models. We present results, which demonstrate the search capacity of these networks in pedestrian occupied spaces. Two discrete-dynamic models are then outlined—pedestrian lane formation and pedestrian egress. Lane formation results are presented and compared to a Cellular Automaton (CA) model. Results show how the network approach is more robust in high-density situations. Egress and exit choices show the potential generality of the approach. We discuss the potential of our technique as a general method for egress model specification for use within the pedestrian and evacuation modelling community.

1 Introduction

The field of pedestrian modelling contains numerous approaches to rule specification. Some models use global information in the form of potential fields to help specify rules of movement (Owen, Galea & Lawrence 1996), whereas others use local rule-sets in order to produce emergent patterns (Blue & Adler 2000a). We have argued elsewhere that the latter is an interesting, but fundamentally limited technique (Holden & Cangelosi 2004). In this paper we are concerned with investigating a new approach to emergent patterns. However, instead of relying on purely local rules we argue that a higher-level network search is necessary in order to provide sufficient information for pedestrians to navigate effectively in complex environments.

We present graphical results of information percolation on a type of 2-lattice graph. The reason we look towards a network approach is related to the effects identified on the 2-lattice. If effects can percolate outside of a given neighbourhood, then it would be a useful approach to search for optimal moves at the level of this information, rather than according to fixed rule sets and limited neighbourhood sizes, as is common in Cellular Automata approaches (Blue & Adler 2000a). We believe that a network-based approach to modelling pedestrian behaviour may be more robust to density parameters. The Pulse Coupled Neural Network (PCNN) is a candidate network approach, identified previously

as a possible search engine for complex maze solutions (Caulfield & Kinser 1999). We investigate the usefulness of the PCNN in the domain of pedestrian modelling. We present equations of artificial neuron units and demonstrate how these units behave in 1 and 2-lattice arrays. A natural property of these networks is the production of autowaves. We present equations that can enhance autowave properties before going on to demonstrate how these networks can produce optimal path information in an occupied configuration. However, as results of 2-lattice percolation indicate, search becomes suddenly constrained at critical density values and we identify situations where redundancy in the search algorithm peaks. We then alleviate these effects by encoding directional information to the network, which produces lane formation effects in a given experimental set-up. We compare results with a Cellular Automaton (Blue & Adler 2000a). Finally, we demonstrate the generality of our approach by testing it in an 'exit choice' configuration, initialised with directional populations. We discuss results and intended future directions.

2 Properties of a 2-Lattice (k = 8)

Properties of Boolean information on a 2-lattice graph are well known (critical behaviour, percolation, fractal properties) and results relating to connectivity have implications for many complex systems (Sole & Goodwin 2000). We have suggested that the reason for catastrophic breakdown of local rule set approaches, such as CA, is because of emerging, giant component sub-networks of pedestrians, which thwart the intended effects of local updates (Holden & Cangelosi 2004). We present behaviour for a 2-lattice with Moore (k = 8) neighbourhoods (see Figure 1). We observe the critical range to be around 4.1, where the walk length and the fraction of walkable sites show large increases 1 .

3 Pulse Coupled Neural Network (PCNN)

The PCNN is a variation of a particular model of the visual cortex. It was originally intended for use in image pre-processing tasks, but has also been applied to a number of other application domains (Caulfield & Kinser 1999, Lindblad & Kinser 1998). The model for a single neuron contains five basic compartments, described by the following equations:

¹ More detailed explanations exist in the litterature (Sole & Goodwin 2000, Holden & Cangelosi 2004)

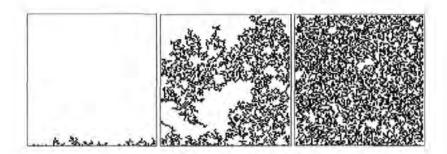


Figure 1: Percolation behaviour on a 2-lattice connected with Moore neighbourhoods (k = 8). From left to right, density $\rho = 0.31$, $\rho = 0.41$, $\rho = 0.51$. respectively.

$$F_{ij}(t) = e^{-i\epsilon_F \Delta t} F_{ij}(t-1) + S_{ij} + \sum_{kl} W_{ijkl} Y_{kl(t-1)}$$

$$L_{ij}(t) = e^{-\delta_L \Delta t} L_{ij}(t-1) + \sum_{kl} M_{ijkl} Y_{kl(t-1)}$$
(2)

$$L_{ij}(t) = e^{-\delta_L \Delta t} L_{ij}(t-1) + \sum_{kl} M_{ijkl} Y_{kl(t-1)}$$
 (2)

$$U_{ij}(t) = F_{ij}(t)(1 + \beta L_{ij}(t))$$
 (3)

$$U_{ij}(t) = F_{ij}(t) (1 + \beta L_{ij}(t))$$

$$Y_{ij}(t) = \begin{cases} 1 & \text{if } U_{ij}(t) > \Theta_{ij}(t) \\ 0 & \text{otherwise} \end{cases}$$

$$\Theta_{ij} = e^{-\gamma_{\Theta} \Delta t} \Theta_{ij}(t - 1) + Y_{ij}(t)$$
(5)

$$\Theta_{ij} = e^{-\gamma_{\Theta}\Delta t}\Theta_{ij}(t-1) + Y_{ij}(t) \qquad (5)$$

where F_{ij} , L_{ij} , U_{ij} , Y_{ij} , Θ_{ij} define the Feeding Field, Linking Field, Internal Activity, Output Field and the Dynamic Threshold, respectively (Schamschula, Johnson & Inguva 2000).

3.1 Decoding the Equations

The Feeding Field: The first term defines a leaky integrator, which contains an accumulative, leaking value of the previous output. This value is obtained from two external sources-the stimulus S and the weighted links WijklYkl to the output field of other local neurons in a 2-D array. The stimulus is a value external to the network and in image-processing applications is often set to the value of an incoming pixel. The Linking Field: This equation has the same structure as the feeding field, but does not receive any pixel stimulus. The feeding and linking components have independent values for time decay constants and are set experimentally depending on a given application. We can see the idea behind these equations—both fields link information from surrounding neurons This results in a build-up of activity within a given neuron. Internal Activity: This is the build up, which integrates the current feeding and linking field valnes. This internal activity is then compared with a dynamic threshold value

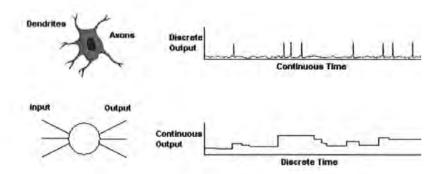


Figure 2: Basic comparison of a biological neuron with a more traditional artificial neural network. The artificial neuron outputs a continuous variable, whereas in the biological neuron the variable is output in pulses.

Output Field: This is a simple if-else rule, which produces an output of 1 if the internal activity is greater than the dynamic threshold Θ and 0 otherwise. This is the pulse or spike information. Dynamic Threshold: The leaky integrator is again controlled by experimental constants. The value of the field depends on the previous value of the output field. We see that if the neuron has an input strong enough it will pulse. However, this does depend on the initial value of Θ . If this is initialised to 0 then any neuron with an input pixel value will pulse at the initial time step if $S_{ij} > 0$.

3.2 Neuron Behaviour

Although the details of these equations are far removed from biological reality, more traditional artificial neural networks use even more abstract neuronal representations. We present a summary of the main differences between the latter and real brain cells on which the PCNN is based (see Figure 2). The most striking difference is the output field. We see that real neurons seem to burst or spike with activity compared with artificial neurons, which show a continuous output. However, more recently, within the artificial neural network community, attempts are being made to model spiking behaviour and many examples of these pulsed neurons now exist (Maass & Bishop 1998). Equations 1-5 define mechanisms, which combine to produce the pulsing behaviour of the neuron. In turn these single pulses can contribute to higher-level patterns, which encode important information. The effect of the PCNN is to convert a static image into a dynamic system so that various informational properties in the dynamics can be exploited and extracted more easily compared to static pattern of pixels in an image.

For our purposes the interesting dynamics are the result of a series of pulses, which can travel away from a source neuron. This series is known as an au-

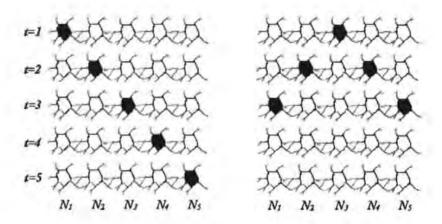


Figure 3: Two examples of autowaves on a neural 1-lattice (k=2) with nonperiodic boundaries.

towave. This directional pulse transmission is made possible by the refractory period in the neuron, which prevents the neuron from spiking immediately after it has done so, implemented above in terms of feedback in the dynamic threshold. We present two example autowaves in a non-periodic network (see Figure 3). The network is a 1-D array of neurons connected to immediate neighbours. We observe behaviour over time in a space-time diagram. A wave of spikes travels away from the source neuron in each example. In the first example we provide input to N_1 . At t=2, the boundary demands that only N_2 receives input from N_1 , which enters a refractory period whilst N_2 processes the input received from N_1 . Each neuron, once pulsed, enters a refractory period and therefore the pulses produce a clear, directional wave, which travels through each neuron left to right in 5(1+1+1+1+1) time-steps. In the second example we provide input to N_3 . The boundary does not interfere with this neuron and two separate waves travel in opposite directions away from N_3 . The wave travels through each neuron in 3(1+(4/2)) time-steps.

3.3 Searching 2-D space with autowaves

One advantage of using these neurons is that, once coupled in some arrangement, this higher-level behaviour naturally occurs. Autowave search with the PCNN has been suggested as a method used to solve complex maze problems (Caulfield & Kinser 1999). The autowave travels over a maze image from an origin neuron connected to a 'start' pixel. The wave travels away from the neuron as the simulation progresses and at some point reaches a predefined destination connected to a 'finish' pixel. When this 'finish' pixel pulses the simulation halts and executing a simple minimal search along the threshold values in the network

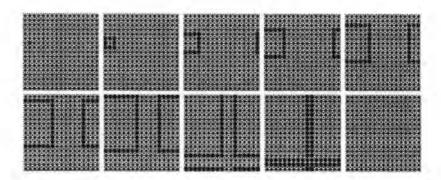


Figure 4: Example autowaves on a 2-lattice PCNN. In a 17x17 2-lattice neural network an input is initialised to 1 at coordinate 2-11, which makes the network pulse at t=1. The autowave then expands away from this single point until all 289 vertices have entered their refractory period. Boundaries are periodic.

produces a trace-back to the origin node and an optimal route solution is found (Caulfield & Kinser 1999). Clearly, the success of this approach will depend on providing a representation of the maze complexity to the network in some way. When we return to the issue of pedestrian navigation we will show how this can easily be done by manipulation of the output field in particular neurons.

In order to demonstrate expanding autowave search, we connect a PCNN as a L^2 2-lattice (L=17) with periodic boundaries and Moore neighbourhoods (k=8). We set a single neuron stimulus S=1 and set all other $S_{ij}=0$. We run the network for ten updates (see Figure 4). The wave propagates outwards from the externally stimulated neuron and it searches the neighbourhood of each successive neighbourhood over time until the entire network has entered a refractory period.

Now consider an origin vertex v_o and a destination vertex v_d in a similar 2-lattice graph. The aim of a pedestrian P in an egress scenario is to traverse the space between v_o and v_d . This can be done by recording the dynamic threshold decay values and tracing back, from v_d to v_o , monotonically decreasing values, which reveal an optimal route. However, in this context there are some pitfalls to the above equations and before we go on to consider the PCNN as a useful search algorithm for pedestrian egress simulation we need to address them.

3.4 Potential problems with original equations

Before accepting the PCNN as a search engine linking v_o to vd, we need to consider more critically the behaviour of the dynamic threshold. The fact that an integrator leaks provides us with information, which may be used to trace a path in space over time. However, we can only do this if we can tune parameters to produce a clean, expanding and full autowave. In the field of image processing,

autowaves from clear images do seem to produce clean autowaves, but depending on the parameters and communication between neurons, wave features may fragment (Lindblad & Kinser 1998). The threshold function, therefore, might potentially produce missing pieces of information. We need to be careful how we set parameters. Three scenarios are possible. 1) In the case where a clean and full autowave expands from v_o to v_d , the function will be a simple, unperturbed decay shape and we will be able to uncover a link between v_o and v_d . 2) If the time decay constants are too strong, then the wave may not reach v_d , because it essentially runs out of energy. In this case, only temporal information is provided, which is the same in all directions and so there is no direction or specific trace back possible. 3) If we set parameters that allow a neuron to pulse more than once before the wave front hits vd, local optima in the network can result. Scenario 1 is desirable, whereas scenarios 2 and 3 are undesirable. In order to ensure the first scenario we enhance the autowave features by pruning some of the pulse-recruitment mechanisms in the original equations:

$$F_{ij}(t) = S_{ij} \tag{6}$$

$$F_{ij}(t) = S_{ij}$$

$$L_{ij}(t) = e^{-\delta_L \Delta t} L_{ij}(t-1) + \sum_{kl} M_{ijkl} Y_{kl(t-1)}$$
(7)

$$U_{ij}(t) = F_{ij} + L_{ij} \tag{8}$$

$$U_{ij}(t) = F_{ij} + L_{ij}$$

$$Y_{ij}(t) = \begin{cases} 1 & \text{if } U_{ij}(t) > \Theta_{ij}(t) \\ 0 & \text{otherwise} \end{cases}$$

$$(8)$$

$$\Theta_{ij} = e^{-\gamma_{\Theta}\Delta t}\Theta_{ij}(t-1) + V_{\Theta}Y_{ij}(t)$$
 (10)

where, again, F_{ij} , L_{ij} , U_{ij} , Y_{ij} , Θ_{ij} define the Feeding Field, Linking Field, Internal Activity, Output Field and the Dynamic Threshold, respectively.

There are three main changes in the new PCNN equations. Firstly, in the image processing application, F_{ij} sums links to neighbouring neurons with S. Here we only use S. We do this because S can be used to identify the pedestrian location v_o on a 2-D plane and itself produce the start of an autowave. The effect of equation 1 is to encourage local pulsing through feedback, which as we have explained is undesirable. The corresponding equation, equation 6, relies only on external stimulus. In addition, by turning off the stimulus as soon as a neuron pulses, we ensure that all information used in the threshold comes from the linking field only, reducing feedback and the tendency of neurons to recruit each other to fire. This reduces the problem of multiple pulses and the creation of local optima because there is less information being input to the neuron. Secondly, instead of U_{ij} being a non-linear combination as in equation 6.3, it is now the sum of the feeding and linking fields. Finally, we introduce a new term in the dynamic threshold, where V^{Θ} is a large amplification factor. This constant provides a means to ensure that the threshold value is very high, with the effect of extending the period of refraction in a neuron. Setting this constant to a large value allows the wave to remain directional because multiple pulses should not occur at a single neuron site. We present a schematic for the

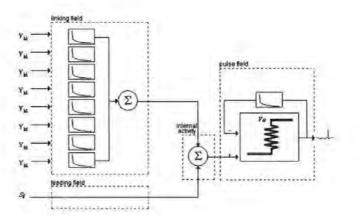


Figure 5: Schematic for the enhanced autowave PCNN in Moore neighbourhood context.

relationships between each new equation for a Moore (k = 8) neighbourhood (see Figure 5).

The same equational structure is maintained as the original equations and the same couplings between neurons. However, the mechanism changes in the equations allow the network to be tuned much more easily to produce clean, expanding and full autowaves, which are conducive to robust exploration. This is achieved by setting the amplification factor to a large value for each neuron. The wave can be stimulated by a given input in order to produce 2-D waves like the example presented above (see Figure 4). However, with a large amplification factor, the wave should maintain enough energy to travel large distances through any given space.

4 Local-Global Linking in Occupied Space

Now consider an occupied space of pedestrians P. It is a simple implementation detail to fix Y=0 if on a corresponding plane a pedestrian occupied that space. If we fix S=1 for a single pedestrian location, a wave will search through space, avoiding all instances where other pedestrians occupy a given location. The trace back search will then provide an optimal route back to the searching pedestrian, through crowds of other pedestrians. We use an N^2 (N=100) 2-lattice space with non-periodic boundaries and place a single 'exit' vertex vd in the centre of the northern boundary. Pedestrians P are placed at vertex locations according to $\rho(0,1)$. A wave is executed for all P_i , as described below (see Table 1). We take two statistics. Firstly, the average wave exploration time or 'walk time' $P_i(T)$ for each pedestrian wave is recorded. This is the cost of search. Secondly, the average fraction of success defined by 'hits' of v_d at each iteration for each

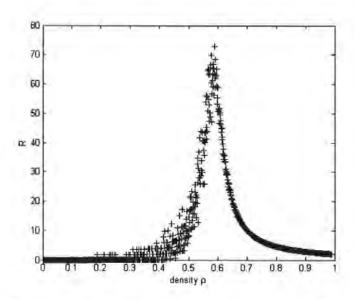


Figure 6: Autowave redundancy. Redundancy is critically effected within the range $\rho=0.5-0.59$.

pedestrian $P_{i(vd(hit))}$ is recorded. This is the *success* of a search. We calculate redundancy R, which indicates the amount of success of a search combined with the cost:

$$R = \frac{\sum_{i=1}^{N} (1 - P_{i(v_d(hit))}) P_i(t)}{N}$$
(11)

where $v_d(hit)$ is a Boolean value and $P_i(t)$ is the walk time for each pedestrian autowave from a given origin location v_i .

We plot R across a range of density (see Figure 6). Notice there is a critical range ($\rho(0.50-0.59)$). This makes sense with respect to previous results in (see Figure 1), where $\rho^*\approx 0.41$ —here we perform a walk on occupied spaces, whereas, using the PCNN, we are exploring unoccupied space. So, the critical points in both sets of results are related because, together, they define the entire space (0.41+0.59=1). Results indicate that the PCNN, like other pedestrian models, are affected by the informational properties of the 2-lattice.

This redundancy measure indicates that, at a value of $\rho \approx 0.59$, there is a large amount of failure and a large cost. However, in the outlined scenario, all other pedestrians are represented by fixed outputs from the network and the wave is maximally constrained. We now turn to 2-directional motion and the formation of lanes with such networks where we relax these constraints and observe interesting patterns of organisation.

Table 1: PCNN Algorithm for Clean, Expanding Autowaves

```
/"Initialisation"/
      For every neuron \neq v_o
            SpikeTime = F = S = L = U = Y = \Theta = 0;
      End For
      For neuron vo
            SpikeTime = F = L = U = Y = \Theta = 0;
      End For
/*Main Loop*/
      While Halt == false
            For every neuron (ij)
                 Execute the Feeding Field equation;
            End For
            For every neuron (ij)
                 Execute the Linking Field equation;
            End For
            For every neuron (ij)
                 Execute the Internal Activity equation:
            End For
            For every neuron (ij)
                 Execute the Spike Threshold equation:
            End For
            For every neuron (ij)
                 Execute the Output Field equation;
                 If(S == 1)
                     S=0;
                 End If
                 If (Y == 1)
                      SpikeTime = t_{\uparrow}
                 End If
            End For
            \mathbf{1f}(v_d(Y) == 1)
                 Halt == true;
            End If
      End While
\frac{/*Parameters*/}{\delta=1, \ \gamma=0.1, \ V=10000, \ \Delta t=1}
```

5 Pedestrian Interaction Model of Lane Formation

Lane formation behaviour is very important in optimising forward movement in crowds (Blue & Adler 2000a, Helbing, Farkas & Vicsek 2000). In order to test the lane formation ability of our model, it is important to provide each pedestrian with knowledge of the movement of surrounding pedestrians. Above we outlined an approach for representing the positions of other agents by fixing Y=0. However, we did not consider here factors relating to the directions of movement. In other models, directional movement is achieved by, for example, introducing a relaxation term to a vector in continuous space (Helbing et al. 2000) or by labelling each pedestrian and creating rules around these labels in discrete space (Blue & Adler 2000a). In our model, a discrete model, we also label our pedestrians and then use this label to determine how pedestrians are accounted for in terms of their relative direction of motion. The assumption behind our approach reflects an idea that pedestrians moving in the same direction physically interact less than with those moving in the opposite direction.

We define a population of pedestrians P^z , sub-divided into two populations P^n and P^s . We define the immediate neighbourhood $\Gamma(vo)_i \forall P^z$ and fix $Y=0 \forall P^z$ in this neighbourhood so that each pedestrian has knowledge of the occupancy of all other pedestrians in its immediate surroundings. Outside of this neighbourhood we only apply inputs $P^{opp[n,s]}$, so that each pedestrian only has a knowledge of pedestrians with the opposite sign. We define v_d as $v_o + M$, where M is a positive value defined relative to the movement direction. At each time-step each pedestrian makes a move towards v_d , so that it continually moves forward in the direction of motion. For a 2-lattice (N^2) , in order to present quantitative results for lane formation behaviour, we use the following to calculate the amount of pedestrian homogeneity l in a given lane l:

$$l_{i} = \frac{max(P^{n}, P^{s})_{i}}{max(P^{n}, P^{s})_{i} + min(P^{n}, P^{s})_{i}}$$
(12)

and for the entire lattice:

$$L = \frac{\sum_{i=1}^{N} l_i}{N} \tag{13}$$

We record L over a range of ρ for the first 500 time steps and compare results with a CA model (Blue & Adler 2000a) for a N=50, periodic 2-lattice (see Figure 7). We observe that at low densities both models produce high values. However, the CA model clearly produces perfect lane formations, which makes sense when we consider the deterministic nature of the search. The relatively high measures for L in our own model are only a result of lower densities having an increased probability that a single pedestrian may be found on a single lane. As we increase the density this effect diminishes. At certain density ranges the CA model produces low values where breakdown in lane formation behaviour occurs (see also (Holden & Cangelosi 2004)). In contrast to this, we see how our own model produces larger values as density increases

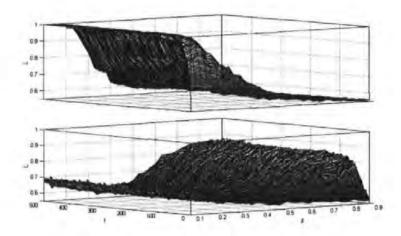


Figure 7: Lane formation performance in pedestrian movement models. Top: CA approach 2. Bottom: Our PCNN approach.

further. The biased input method outlined above produces the effect of lane formation as pedestrians avoid oncoming individuals and tend to become aligned to pedestrians moving in the same direction.

By simply controlling directional inputs to the network we have managed to model a pattern found in real pedestrian crowds. These kinds of interactions and self-organisations are very important because they have a large effect on the overall speed and progress of a crowd of pedestrians. It should be mentioned also that the low values at low densities in our model do not reflect an inability to move forward, but rather reflect some non-deterministic decisions in the face of multiple optima, which are also typical of 2-lattice graphs.

6 Pattern formation within a configuration

In the above simulation we demonstrate lane formation on a simple 2-lattice with periodic boundaries. However, it is quite a different problem to capture lane formation or route choice preference within a configurational scenario. We now simulate this aspect of crowd behaviour. We define two sets of pedestrians P_e and P_w , which have different destination vertices. The rules we apply as network input are similar to the ones above for the lane formation scenario, but here the destination is fixed to a constant location in the configuration. Pedestrians are randomly initialised to white space in a configuration scenario (see Figure 8).

Graphical results demonstrate the behaviour produced for different time steps (see Figure 9). Pedestrians with the same destination vertex are attracted to the same doors. This attraction-to-same-door behaviour is an effect produced

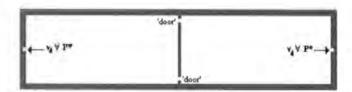


Figure 8: Configuration used for exit choice scenario. Each population has a fixed exit vertex.

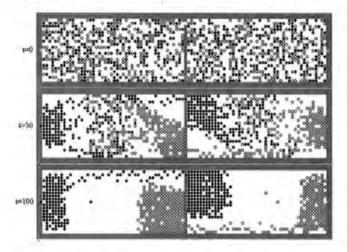


Figure 9: The development of exit-choice behaviour in a 2-directional population $\rho=0.4$.

by the method outlined above, where pedestrians with the same directional label interact less than those with the opposite label. The particular pattern can alternate depending on initial random fluctuations, but the attraction to a given door for a given sub-population is robust.

7 Summary and Conclusions

We began this paper by introducing some of the fundamental limitations of the 2-lattice. We then presented a PCNN and showed how to enhance its autowave behaviour. By fixing inputs to the network, it is possible to allow the wave to navigate around obstacles. We demonstrated applications of this approach with two example scenarios. Firstly, we compared a PCNN model with a CA and found that lane formation behaviour does not suffer from the catastrophic

effects of 2-lattice percolation. This is because, rather than rules being specified locally, an autowave search is allowed to move inclusively through free space, where free space is also defined in relation to directional labels of two opposing populations of pedestrians. The result is a much more robust method for producing these patterns of pedestrian interaction. Secondly, we presented an exit choice scenario where, again, two opposing populations moved through space. The PCNN approach not only allows network specification of other pedestrian locations, but the same method also allows a representation of spatial configuration. An effect of applying this method was the modelling of exit choice, where randomly initialised pedestrians settle to a singe door *en route* to a destination vertex.

This work represents a preliminary investigation into the use of a dynamic, network-level implementation of pedestrian movement. We have used a PCNN because this has already been applied to a related problem. However, there is no reason why the PCNN should be the sole mechanism of implementation for autowave search. Any algorithm designed to produce clean, expanding search waves can be implemented. Future directions include minimising the computation required in this kind of search. As a side effect of pruning original PCNN equations this has been achieved only partially. The long-term aims of this work are to produce a network method of pedestrian movement, which reproduces realistic escape behaviour. Realism should include adaptive decision-making (Gwynne, Galea & Owen 1998). Indeed, to some extent, adaptive decisionmaking is modelled in the second of our scenarios, where exit choices are not coded directly, but result from reinforcement of early clusters in a randomly populated space. Although many models of pedestrian movement exist and are successfully used in specific applications, from a modelling perspective it is an interesting problem to design general methods that can be used in many scenarios without needing to tweak the model. An important future undercurrent in our research is to investigate which network techniques are useful in the design of discrete models of pedestrian behaviour.

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