## EQUALISATION TECHNIQUES FOR MULTI-LEVEL DIGITAL MAGNETIC RECORDING



P. Shah

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## EQUALISATION TECHNIQUES FOR MULTI-LEVEL DIGITAL MAGNETIC RECORDING

A thesis submitted to the University of Plymouth in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

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## Equalisation Techniques for Multi-level Digital Magnetic Recording

Purav Shah

## Abstract

A large amount of research has been put into areas of signal processing, medium design, head and servo-mechanism design and coding for conventional longitudinal as well as perpendicular magnetic recording. This work presents some further investigation in the signal processing and coding aspects of longitudinal and perpendicular digital magnetic recording.

The work presented in this thesis is based upon numerical analysis using various simulation methods. The environment used for implementation of simulation models is C/C + + programming. Important results based upon bit error rate calculations have been documented in this thesis.

This work presents the new designed Asymmetric Decoder (AD) which is modified to take into account the jitter noise and shows that it has better performance than classical BCJR decoders with the use of Error Correction Codes (ECC). In this work, a new method of designing Generalised Partial Response (GPR) target and its equaliser has been discussed and implemented which is based on maximising the ratio of the minimum squared euclidean distance of the PR target to the noise penalty introduced by the Partial Response (PR) filter. The results show that the new designed GPR targets have consistently better performance in comparison to various GPR targets previously published.

Two methods of equalisation including the industry's standard PR, and a novel Soft-Feedback-Equalisation (SFE) have been discussed which are complimentary to each other. The work on SFE, which is a novelty of this work, was derived from the problem of Inter Symbol Interference (ISI) and noise colouration in PR equalisation. This work also shows that multi-level SFE with MAP/BCJR feedback based magnetic recording with ECC has similar performance when compared to high density binary PR based magnetic recording. It has been shown that 4-level PR based magnetic recording with ECC at half the density of binary PR based magnetic recording has similar performance and higher packing density by a factor of 2.

A novel technique of combining SFE and PR equalisation to achieve best ISI cancellation in a iterative fashion has been discussed. A consistent gain of 0.5 dB and more is achieved when this technique is investigated with application of Maximum Transition Run (MTR) codes. As the length of the PR target in PR equalisation increases, the gain achieved using this novel technique consistently increases and reaches up to 1.2 dB in case of EEPR4 target for a bit error rate of  $10^{-5}$ .

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The author has published a paper in the following peer-reviewed international journal:

1. IEEE Transactions on Magnetics(volume 43, No.6 June 2007, pages 2280–2282);

and has presented papers in the following international conferences:

- 1. International Symposium on Physics of Magnetic Materials (ISPMM), Singapore, 14-16 September 2005;
- 2. IMST-EPCOS Joint European Workshop, France 29-31 May 2006;
- 3. 10<sup>th</sup> Joint MMM and Intermag (JMMM), Baltimore, USA 07-11 January 2007;
- 4. The 8<sup>th</sup> Perpendicular Magnetic Recording Conference (PMRC), Tokyo, Japan, 15-17 October 2007;

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This thesis is dedicated to my dearest mother, sister and my supervisor and friend, Zaki, who taught me to never give up!

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## Part I

# **Introduction and Background**

# Introduction

The present age is considered to be the age of ever demanding *information*, *data storage and faster communication*. Thus, the demand in smaller size, high performance, low cost electronic devices is ever increasing. Similar factors like reduction in form factor, better power efficiency, high performance and lower market price influence directly to the magnetic storage devices. There are a variety of information storage systems including magnetic tape drives, magnetic floppy disk drives (nearly obsolete due to limited storage capacity), magnetic hard disk drives, semiconductor memory, holographic optical storage, magneto-optical disk drives and the latest magnetic random access memory (M-RAM). It is seen that different types of magnetic storage devices make a major contribution to the storage technology.

The year 2006 marks the 50<sup>th</sup> anniversary of the introduction of the hard disk drive (HDD) and it is just over 100 years since Valdemar Poulson invented magnetic recording, the base technology for these recording devices (Daniel et al.; 1999) which have become an integral part of our lives. The 2007 Nobel Prize in Physics was awarded jointly to Albert Fert (France) and Peter Grunberg (Germany) for their pioneering discovery of giant magneto-resistance. Their discovery in 1988, has led to an exponential rise in the data storage market, especially hard disk drives. Along with the advances in the conventional magnetic recording and system modelling in order to achieve "smaller", "faster", "high capacity" and "cheaper" magnetic recording systems. Conventional magnetic recording technique is based upon longitudinal recording. This is limited at around 300 Giga-bit per square inch.

Currently, the advancement of perpendicular recording is limited to 1 Tera-bit per square inch. To achieve the highest areal densities, it will be necessary to use a magnetic recording configuration capable of writing and storing data on very small magnetic grains together with a signal processing system capable of recovering data reliably when each bit is recorded on such high density configurations (Wood; 2000).

## 1.1 History of Magnetic Storage Devices

There are a variety of information storage systems with varying degrees of development and usability. Among them are magnetic tape drives, magnetic hard disk drives, magnetic floppy drives, magneto-optic (MO) disk drives, phase-change optical disk drives, semiconductor flash memory, holographic storage, solid-state hard disk drives and magnetic random access memory (MRAM). The information storage hierarchy is as shown in Figure 1.1.



Figure 1.1: A Typical Information Storage Hierarchy

It is seen from the hierarchy that the storage technologies at all levels of the storage hierarchy can be differentiated based upon various characteristics, depending upon their use and development. These characteristics are described below:

- Volatility Whether constant power is required to retain the stored information. Further classified into - volatile, non-volatile and dynamic memories.
- Mutability Capable of constant changes or static. Further classified into Read/Write Storage, Read only storage, slow read fast write storage
- Accessibility Whether the device can be accessed from several locations or static locations as well as mode of access which is either parallel or serial. Further classified into Random Access and Sequential Access.
- Capacity Total amount of stored information that a device can hold. The formatted capacity can be defined as (Wang and Taratorin; 1999):

$$C = \frac{UserBytes}{Track} \cdot \frac{Tracks}{DataBand} \cdot \frac{DataBands}{Head - diskAssembly} \cdot \frac{Assemblies}{Box}, \quad (1.1)$$

where User Bytes is equal to Total Data Bytes-Overhead Bytes(Formatting), Data Bands

= Zones with the same data rate<sup>\*</sup>, Tracks is equal to the number of tracks on each disk, Head-disk Assembly is the air tight assembly which includes a disk pack and read/write heads, Box is the enclosure including the head-disk assembly and electronics for the hard disk drive. Head-disk assembly and box determines the form factor of the hard disk drive.

• Density - The compactness of the stored information. Areal data density  $D_a$  can be defined as the inverse of bit area (bit length times track pitch). It is given in the units of  $bits/mm^2$ ,  $Mb/in.^2$  and  $Gb/in.^2$ .

$$D_a(bit/in.^2) = D_l(bit/in.) \cdot D_t(track/in.), \qquad (1.2)$$

where  $D_l$  is the linear data density defined as the inverse of the smallest bit length and  $D_t$  is the track density defined as the inverse of the track pitch<sup>†</sup>.

• Performance - Can be further classified into - Latency and Throughput.

Depending upon the level of the information storage hierarchy, the type of storage systems are chosen. The primary storage is the semiconductor memory, which is random access. The secondary storage is considered to be hard disk drives. Recently, one of the products of Apple known as Mac book-Air uses a solid-state hard disk drive with 64 Gigabytes has been implemented in order to reduce the size and weight of the laptop. The next in the hierarchy is the offline modes of storage or removable media devices such as floppy drives, magneto-optical drives such as Compact Disc-Read Only Memory (CD-ROM), Compact Disc-Read and Write (CD-RW), Digital Versatile Disc (DVD), Blu Ray Discs, High Definition-DVD (HD-DVD), solid state devices such as flash memory. Further to all of the above mentioned devices, there exists a robotic storage which is used for backups and are known as tape libraries.

Below are the specifications of magnetic hard disk drives and magnetic tape drives, since they are in highest use as secondary storage devices.

#### 1.1.1 Magnetic Hard Disk Storage

Some of the major historical events that took place in the history of Hard Disk Storage are listed below:

• The very first hard disk drive introduced by IBM, San Jose, California in 1956 was called the Random Access Method of Accounting and Control (RAMAC) or IBM350 (Wang and Taratorin; 1999). Since this first introduction to date, magnetic recording technology has made remarkable progress.

<sup>\*</sup>Data rate  $D_r$  is the number of bits per unit time that the write/read head can deliver.

<sup>&</sup>lt;sup>†</sup>Track pitch is the distance between adjacent tracks from their centres

- In 1986, IBM introduced the IBM 9332. It was the first hard disk drive utilising "1-7" run-length-limited (RLL) code efficiently.
- In 1988, Giant Magneto-Resistance (GMR) effect was introduced by M. Biabich et al., which revolutionised the magnetic recording industry (Belleson and Grochowski; 2008).
- In 1991, a major shift occurred in the hard disk drive industry, since the implementation of PRML in rigid disk drives was found (Cocker et al.; 1991).
- Between 1994 and 2002, there was a tremendous increase in the linear densities of magnetic storage devices due to the use of GMR read/write heads.
- Toshiba was the first company to deploy perpendicular magnetic recording in 2005, but it suffered reliability issues in the consumer market (Toshiba Press Release; 2006).
- In early 2005, Hitachi Global Storage Technologies demonstrated an areal density of 230 gigabits per square inch (Gb/in2) on perpendicular recording technology, the highest areal density achieved to date based on vertical recording (Milestones of Magnetic Recording; 2006).
- In early 2006, Seagate started to ship their first laptop-sized, 2.5inch hard drive using perpendicular magnetic recording (Perpendicular Recording; 2007).
- In early 2007, Hitachi announced its first 1 Terabyte Hard Drive using perpendicular magnetic recording, which they delivered in April 2007.
- Fujitsu, in early 2007, also announced the breakthrough of patterned media technology, a process which could make higher densities feasible (Fujitsu Press Release; 2007).

## 1.1.2 History of Magnetic Tape Drive

Advancements in the history of tape drives are listed as below (Wang and Taratorin; 1999):

- In 1951, the first ever computer tape drive called "UNISERVO" was developed and sold with the "UNIVAC".
- In 1974, IBM deployed their first Tape Cartridge. They also announced the Mass Storage Systems (MSS), which is an example of tape library.
- In 1984, the read write assembly was changed to MR heads by IBM. Also, Digital Linear Corporation introduced Digital Linear Tape (DLT) storage system.

- In 1986, first hardware data compression (IDRC Algorithm) was used for data backup.
- In 1995, HP used PRML techniques for tape drives.
- In 1997, Linear Tape Open (LTO), a new standard of open format specifications for network and enterprise storage was announced as a joint effort by HP, IBM and Seagate.
- In 2002, the sales of LTO versus Super-DLT was nearly 2 to 1.
- In 2006, the LTO consortium announced that over 1.5 million LTO drives and 50 million cartridges had been shipped (Linear Tape-Open; 2008).

| Features                               | Hard Disk Drive (HDD)   | Tape Drive   |  |
|--|---|--|--|
| Typical Capacity                       | 500 GB<br>Extended PRML<br>ISI Equalisation   | 300 GB<br>(Uncompressed, with Digital<br>Lempel-Ziv (DLZ) 600GB)<br>PRML (Class 4)<br>ISI Equalisation   |  |
| Data Recovery                          |   |  |  |
| Non-Recoverable<br>error rate          | 1 in 10 <sup>14</sup>   | 1 in 10 <sup>17</sup>  |  |
| Bytes per sector                       | 512=4096 bits   | variable up to 16K   |  |
| Bits per inch                          | 763000<br>120000  | 233000<br>1490   |  |
| Tracks per inch                        |   |  |  |
| Areal Density =<br>Bits/in x Tracks/in | 91560 Mbits/in <sup>2</sup>   | 347 Mbits/in <sup>2</sup>  |  |
| Typical Error<br>Correction Codes      | Interleaved RS(255,239,17)<br>using $GF(2^8)$ symbols<br>or Shortened RS codes (96-bit)<br>using $GF(2^{12})$ symbols | RS 64-bit<br>Cyclic Redundancy Check<br>on each 4kB of data<br>256-bit AES-GCM<br>Advanced Encryption Standard (AES)-<br>Galois Counter Mode (GCM)<br>drive level encryption |  |

A table (Ahmed; 2005) of hard disk drive and tape drive specifications is listed as below:

Table 1.1: Typical Specifications of Consumer Devices

## 1.2 Introduction to Digital Recording

Digital magnetic recording is the process of recording information/data on a magnetic medium and reading it at a later stage. It can be seen as storing "now" and retrieving "later". The "storing now time" is roughly 100 Mb/s (data transfer rate) and the "retrieving time" can be anything up to 3 years. It can be treated as a Linear Time-Invariant (LTI) system in which the readback signal is distorted due to various channel effects. It deals

with two main processes: Write and Read. A digital magnetic recording channel can be modelled as a normal communications channel transmitting data in time and retrieving it later. This involves handling of discrete or quantised units of information for storage and retrieval of digital data. In certain other applications of magnetic recording like audio and video recording applications, the aim is to record and reproduce a replica at a later time. With powerful error correcting capabilities for digital data, all recording is based on digital information. In magnetic recording, data is modulated into a write current, which is passed to the medium through a write head. The write head creates regions of magnetic medium with changing magnetisation by changing the direction of the externally applied magnetic field.

These regions where the magnetisation changes occur are called Magnetic Transitions<sup>‡</sup> on the magnetic medium. These magnetic transitions are written by the write head and can be detected during a readback process by a read head. The read signal is a representation of the transitions written on the magnetic medium.

Magnetic Recording is based on the interaction between a magnetic medium and a magnetic head(transducer), in relative motion with respect to one another (Hoagland and Monson; 1991).

A typical digital magnetic recording channel can be shown as below in Figure 1.2.



Figure 1.2: A Typical Digital Magnetic Recording Channel

Three primary modes of recording can be defined, based on the direction of surface magnetisation relative to the direction of track motion.

These modes are:

- Longitudinal or Horizontal Recording
- Perpendicular or Vertical Recording
- Transverse Recording

<sup>&</sup>lt;sup>1</sup>A real magnetisation transition in magnetic media can be modelled by an arc-tangent function. A magnetisation transition centered at x = 0 can be given as  $M(x) = \frac{2M_F}{\pi} \tan^{-1} \left(\frac{x}{\pi}\right)$ 

The recording directions and magnetisation directions for longitudinal and perpendicular magnetic recording are as shown in Figure 1.3.



Figure 1.3: Definition of Directions of Magnetic Recording

There are various non-linear mechanisms which exist in the recording channel that distort the signal when the data is written as well as when it is read (Melas et al.; 1987). As two transitions come close to each other, the transition walls begin to overlap each other. This happens if the transitions are not perpendicular to the track and have some non-zero width. When parts of the transitions overlap each other, the effective length of the boundary of the transitions is reduced and thus, reducing the amplitude of the transition response. This phenomenon is known as Partial Erasure (Barndt et al.; 1993).

Another write process anomaly, is modelled as Non Linear Transition Shift (NLTS) (Wang and Taratorin; 1999) (Taratorin et al.; 1997) (Bertram; 1994). When a transition is recorded on the medium, it produces a demagnetisation field that opposes its own magnetisation. When the next transition is written, the demagnetisation field from the first causes the second transition to be written earlier (Wang and Taratorin; 1999). This results in the dibit response with a shifted second transition and causes over-writing of previous data. However, these types of write and read anomalies are not considered in this work. Bit Error Rate (BER) performance of the channel is the main criterion of observation in this thesis. The bit error rate is defined as:

$$BER = \frac{Number of Errors}{Total Number of Bits}$$
(1.3)

Figure 1.4 explains various regions of a typical BER curve using a Turbo code performance (Ahmed; 2003a). An error floor appears in the BER versus SNR characteristics of coded systems due to very good performance almost up to the Shannon limit as described in (1.11). Performance of coded systems are bound by the Shannon limit and the typical



Figure 1.4: Typical Bit Error Rate Performance Curve

code properties which are beyond the scope of this thesis. The Waterfall region is due to Shannon's limit for the channel, and the Error floor is due to the the Probability of Error of the code. A typical Turbo code Waterfall and Error floor region are as shown in Figure 1.4.

A magnetic recording channel can be considered to a coded channel, since it is a rate 1 convolutional code channel.

#### 1.2.1 Longitudinal Magnetic Recording

In longitudinal recording, the principle direction of magnetisation is in the plane of the surface and parallel to the direction of surface motion.

The readback voltage from an isolated transition can be approximated by a Lorentzian function as shown in equation(1.4) (Bertram; 1994).

$$h(t) = \frac{1}{1 + \left(\frac{2t}{PW_{20}/T}\right)^2} \tag{1.4}$$

 $PW_{50}/T$  is a measure of how close the transitions are in relation to the minimum length between transitions. It is the pulse width at half maximum, normalised to the bit period T. A typical lorentzian pulse is as shown in Figure 2.2. This measure of  $PW_{50}$  is used in this thesis, when recording density is discussed. As the  $PW_{50}$  increases, the transitions comes closer to each other. As the  $PW_{50}$  decreases, the transitions are consistently apart from each other.

Figure 1.5 shows an LTI equivalence of a longitudinal magnetic recording channel. The

transition response h(t) in (1.4) is the approximation of the readback voltage from an isolated transition on the magnetic medium.  $\{x_k\}$  is a binary sequence having T spaced discrete elements taking values  $\{\pm 1\}$ . Non-Return-to-Zero (NRZ)<sup>§</sup> symbols are written on the magnetic medium by the write head which creates a two-level transition sequence  $\{a_k\}$ .



Figure 1.5: Linear Time Invariant model of Longitudinal Magnetic Recording Channel

The transition sequence can be expressed as:

$$a_k = x_k \cdot (1 - D), \tag{1.5}$$

where D is a T second delay operator.

Since h(t) from (1.4) corresponds to the transition sequence in the media, the overall impulse response of the longitudinal magnetic recording channel s(t), also referred to as a dibit response, can be expressed as:

$$s(t) = h(t) - h(t - T).$$
 (1.6)

The recording channel differs from most communication systems in a number of ways. Reliability requirements are usually much higher for magnetic recording systems. Also, signal power is limited and cannot be increased with respect to noise. As a result, magnetic recording systems work at a much higher signal to noise ratio (SNR) than a normal communications system (Monson; 1987). For example, a typical tape-drive for magnetic recording operates at an SNR of 19 - 22 dB, while a typical communication system downlink with 8 GHz frequency and range of 21.195 nautical miles would require an SNR of 10 - 13 dB to operate.

Increasing the density of the recorded data<sup>¶</sup> stored on the magnetic medium causes the magnetic transitions to interfere with one another. This occurs due to the mechanical limitation and magnetic interactions of the recording system. Magnetic information consists of multiple magnetic transitions. The interaction of closely spaced multiple transitions

<sup>&</sup>lt;sup>§</sup>Digital data {0,1} are recorded such that the symbol 1 represents a positive amplitude of write current, and the symbol 0 represents a negative amplitude

<sup>&</sup>lt;sup>¶</sup>The recorded bit density  $D_c$  can be given by the relation  $D_c = \frac{PW_{50}}{T}$ 

causes bit shift or often known as Inter-Symbol Interference (ISI) which results in pulse crowding, and reduction in signal amplitude causing timing jitter and also incorrect detection (Wang and Taratorin; 1999). Bit shifts which occur due to the linear superposition of multiple pulses, are the peaks of alternating polarity occurring at locations slightly shifted away from the intended locations.

The Signal to Noise Ratio (SNR) is defined by equation(1.7)

$$SNR = 10\log_{10}(\frac{1}{2\sigma^2}),$$
 (1.7)

where  $\sigma$  is the standard deviation of the Gaussian noise distribution. This is a measure of noise only. This definition of SNR is used throughout in this work.

#### 1.2.2 Perpendicular Magnetic Recording

Professor Iwasaki and his colleagues proposed a perpendicular recording system using a new recording medium and magnetic head in 1977, and showed that it was capable of realising high storage densities (Iwasaki and Hokkyo; 1991). The research was characterised by the simultaneous development of new magnetic head designs and recording media. In perpendicular magnetic recording, the principal orientation of the magnetisation is normal to the plane of the surface.

The main difference between perpendicular and longitudinal magnetic recording is that, in perpendicular recording, the adjacent magnetised regions are in anti-parallel states; therefore, an attractive force exists between each pair of the residual magnetisation regions, making them stable (Iwasaki and Nakamura; 1977). Thus, in digital recording, a sharp magnetisation transition can be achieved at high densities, without being affected by demagnetisation. The perpendicular recording process is explained mathematically and analytically in (Middleton and Wright; 1984).

Given current estimates, that would suggest an areal density using PMR as great as one terabit per square inch – making possible in two to three years a 3.5 inch disk drive capable of storing an entire terabyte of data (Wood et al.; 2007). As the areal density is increased, however, the signal processing aspects of magnetic recording becomes more difficult.

A hyperbolic tangent readback signal from an isolated transition s(t) is assumed for perpendicular magnetic recording (Okamoto et al.; 2001). It is given as:

$$s(t) = A \cdot \tanh\left(\ln(3)\frac{t}{PW_{50}}\right). \tag{1.8}$$

where A is the saturation level/amplitude from zero to peak(normalised to unity) and  $PW_{50}$  is the time taken for s(t) to go from -A/2 to +A/2. It is assumed that t and  $PW_{50}$  are normalised to the symbol period, T.

A simulation model of perpendicular magnetic recording channel is as shown in Fig-

ure 1.6. The user data, denoted as  $a_k$ , is a sequence of input symbols taking values of 0, 1. To simulate the write current, the sequence  $a_k$  is mapped to -1, +1. The scaling factor of 0.5 is to ensure the transition takes values of -1, 0, +1. The dibit response p(t) is defined as:

$$p(t) = s(t) - s(t-1).$$
(1.9)

The readback signal r(t) is the convolution of  $a_k$  and p(t) plus some Additive White Gaussian Noise (AWGN):

$$r(t) = \underbrace{\sum_{k} a_k p(t - kT) + n(t)}_{\mathbf{b}(t)}, \tag{1.10}$$

where n(t) is the AWGN with mean of 0 and variance of  $\sigma^2$ . A Maximum Likelihood Sequence Detector (MLSD) is used to provide the decoder solution which is used for performance evaluation. The points of BER measurement are as shown on the simulation model. The errors are calculated between the actual input data and the decoded data.



Figure 1.6: Simulation Model of Perpendicular Magnetic Recording Channel

## 1.3 The problem of Digital Magnetic Recording

The areal density in magnetic recording is increasing faster than Moore's Law<sup>||</sup>, and it has thus been outpacing the semiconductor industry (Moon et al.; 2001). In order to increase the capacity of a magnetic recorder to its maximum, there are a lot of areas which need improvement. Among many of them, better performance media, better head design, magneto-resistive (MR) read heads, scaling of mechanical parts, better signal processing and error correction coding (Moon et al.; 2001) are of utmost importance.

New channel models, equalisation techniques, modulation coding, error correction coding have given a significant boost to the development of magnetic storage technology.

Trellis decoding was introduced by Forney (1967) as a means to explain the Viterbi algorithm. The major milestone achieved in the storage technology was the utilisation of

<sup>&</sup>lt;sup>#</sup>Moore's law states that the number of transistors that can be inexpensively placed on an integrated circuit (IC) is increasing exponentially, doubling approximately every two years

Viterbi algorithm by Kobayashi (1971a)\*\*.

This achievement has led to an increase in linear recording density<sup>††</sup> by allowing bits to be packed closer together (Ahmed; 2003a).

Signal processing and error correction coding have improved a lot since the realisation of Bahl-Cocke-Jelenik-Raviv (BCJR) algorithm in 1974 (Bahl et al.; 1974). The introduction of PRML, led to an improvement of 30 - 40% in the areal density of magnetic recorders in comparison to the standard peak detection systems (Cocker et al.; 1991).

Initial investigation done by Mackintosh and Jorgensen (1981) suggested that the capacity of a digital magnetic recorder could be increased beyond the normally obtainable, by using multi-level encoding, rather than the normal saturated two-level system. Implementation of ternary recording technique has also been shown to be feasible for increasing the information density of the magnetic recording channel (Krueger et al.; 1995). Ahmed (2003b) showed that there is a possibility to reach 10 levels in multi-level magnetic recording although with accountable performance loss. In order to achieve the bandwidth capacity, it is necessary to take into account the performance loss and the number of levels that can be achieved with desirable performance loss. This will be further discussed in Chapter 3.

The PRML technique is a trade-off between performance improvement and noise colouration and enhancement. The front-end PR equaliser modifies the impulse response of the underlying channel and thus causing correlation and enhancement of noise. It is well known that binary codes tend to deviate very quickly from their theoretical capacity. The channel capacity C could be calculated such that if the maximum information rate R, at which the information can be transmitted is less than C, data can be sent error free through a noisy channel. This was proved by Shannon (1948). Channel Capacity is defined in equation(1.11).

$$C(bits/second) = B\log_2(1 + SNR)$$
(1.11)

where, B = Channel Bandwidth (Hz) and SNR= Signal to Noise Ratio = Signal Power (watts) / Noise Power (watts). The above equation holds true only in the case of AWGN.

As a result, in order to achieve the channel capacity at a fixed SNR, multi-level encoding of data is very important for magnetic recording (Ahmed; 2003b). The performance improvement achieved using multi-level encoding in digital magnetic recording was shown in (Shah et al.; 2007).

Noises in digital magnetic recording arise mainly from three sources: Recording medium, readback head and readback preamplifier (electronics). Along with the above noises, there

<sup>&</sup>quot;Omura (1970) and Kobayashi and Tang (1970) both presented the first use of Viterbi algorithm for partial response channels at about the same time.

<sup>&</sup>lt;sup>++</sup>The recorded bit density/channel density is related to the user density  $D_{\mu}$  by  $D_{c} = \frac{D_{\mu}}{r}$ , where r is the code rate

exists unwanted interference and non-linear distortion. Usually, the magnetic recording system is designed to be medium noise limited (Wang and Taratorin; 1999), which comes from the magnetic medium itself. Medium noise is classified into three types of noise:

- 1. Transition Noise It occurs due to the magnetisation fluctuations which are concentrated at the centre of the recorded transitions. In order to get rid of transition noise, the decoding process can be modified in order to include the transition jitter noise in the calculations of the decoding algorithm.
- 2. Particulate/Granularity Noise It occurs due to the random dispersion of magnetic particles during the process of making the magnetic medium. This can be overcome by using the new idea of patterned media (Kikitsu et al.; 2007). Although patterned media does contain particulate noise, it can be controlled depending upon the various patterns of media.
- 3. Modulation Noise It is caused due to the magnetisation fluctuations proportional to the recorded magnetisation between magnetic transitions, i.e. the non transitional areas. This can be overcome by using various equalisation schemes (Wang and Taratorin; 1999).

There exists jitter noise in perpendicular magnetic recording which is caused by transitions, and usually consists of 80-90% of the readback signal (Ahmed et al.; 2005). It is also called Transition Jitter Noise.

The types of noise discussed in this research are as below:

- AWGN or Electronics Noise: is caused by the random motion of electrons in a conductor. It affects the resistive circuit elements. Since the random process (motion) is due to various individual voltages produced by individual electrons, the central limit theorem (Leon-Garcia; 1994) is taken into account and the overall noise is modelled as a zero-mean uncorrelated Gaussian process. This model is chosen since the Gaussian distribution is easier for analysis. This noise arises from the resistive component of the read head and from the pre-amplifier electronics of the magnetic recording system.
- Inter-Symbol Interference (ISI): is caused by overlapping consecutive transitions leading to peak shifting and waveform dropping in the readback waveform. It is know that the magnetic recording channel is a bandwidth limited channel. Thus, when the channel bandwidth is close to the signal bandwidth, the spreading of the pulse will exceed one symbol duration and causes the signal pulses to overlap. This overlapping is called ISI. In the case of magnetic recording channels, the readback signal which is assumed to be Lorentzian, introduces ISI into the channel.

• Transition Jitter Noise: is caused by timing instability or clock recovery instability in readback process. The stochastic nature of the zigzag transitions written into thin film media gives rise to an additional noise mechanism correlated to the transitions (Bertram and Che; 1993) (Tsang and Tang; 1993) (Zhu and Wang; 1995) (Zhu and Ye; 1995) This additional noise mechanism is often referred to as Transition Jitter.

This research is based upon the following aspects of magnetic recording.

- 1. Improvement in equalisation and detection techniques.
- 2. Application of multi-level encoding on longitudinal digital magnetic recording.
- 3. Application of error correction codes and recording codes to magnetic recording.

## 1.4 Methodology

A numerical approach has been undertaken for analysis of the work described in this thesis. The main channel of interest is the magnetic tape drive readback channel. Various numerical models have been simulated using C/C + + programming language under GNU C + + compiler. Longitudinal magnetic recording channel has been simulated using the Lorentzian transition response approximation which is described in (1.4). Perpendicular magnetic recording channel has been simulated using the tanh transition response approximation which is described in (1.8).

Various blocks were developed under C/C + + language and were put together to create a full longitudinal/perpendicular magnetic recording channel. For certain numerical measurements, software packages called *Mathematica* and *MATLAB* were used.

A typical simulation model is as shown in Figure 1.7. Each block shown in the model performs a specific operation and depending upon the simulated channel, it has a specific transition response and equaliser. Other than the ECC block providing Turbo codes which was given to the author by Dr. Marcel Ambroze, all the blocks in C/C + + language by the author himself and an overall simulation model of a typical magnetic recording channel was achieved.

The blocks shown in Figure 1.7 are explained as below:

- 1. Data Generator Block: Random Data 0,1 is generated of length 4096 bits. This random data is generated using the method described in (Press et al.; 1992).
- ECC/Recording Coding Block: Turbo codes or MTR codes are generated. This block also taken into account whether the channel is multi-level or binary and based on that, the data is encoded.
- 3. Data Processing Block: This block converts data into NRZ/NRZI data for transition.



Figure 1.7: General Simulation Model Block

- Transition Response Block: This block provides the transition response approximation based upon whether the simulated channel model is longitudinal or perpendicular.
- After the transition response, AWGN noise is also added for the channel to be considered AWGN. AWGN noise is simulated with random seed using the method described in (Press et al.; 1992).
- Equaliser Block: Depending upon the simulated channel, the equaliser would be PR or SFE. PR equaliser is designed by the author using the methods described in (Watkinson; 1988). SFE equaliser design is the author's original work.
- Decoding Block: This block creates a trellis which is used for application of MLSD/MAP decoding algorithms.
- ECC/Recording Decoding Block: This block provides decoding of Turbo/MTR codes and de-mapping for multi-level data.

The number of frames in error simulated in each case was set to 100, which means for each BER point, there would have been at least 100 frames in error. Each frame consists of 4096 bits. Frame error rate (FER) and BER calculations are done based on the number of frames in error and bits in error in each frame. Unless otherwise stated, the parameters above have been used for the calculations in this thesis.

#### 1.5 Thesis Outline

This thesis covers various topics of interest in longitudinal/perpendicular magnetic recording. It includes equalisation, multi-level encoding for longitudinal magnetic recording, optimisation techniques for GPR targets in perpendicular magnetic recording, application of error correcting and recording codes for longitudinal magnetic recording and improvement in detection techniques for longitudinal and perpendicular magnetic recording. The aims and objectives of this thesis are as below:

#### 1.5.1 Overall Aim

The aim of this thesis is to study the equalisation techniques implemented in magnetic recording in order to achieve near perfect readback from the magnetised media and use various types of coding to improve the performance of magnetic recording.

#### 1.5.2 Objectives

The main objectives of this work are discussed as below:

- 1. Investigation of PR equalisation and development of a new soft feedback equalisation technique for longitudinal magnetic recording.
- 2. Application of multi-level encoding to PR and SFE based longitudinal magnetic recording.
- 3. Application of error correcting and recording codes to PR and SFE based recording channels.
- 4. Investigation of various detection methods and development of asymmetric MAP decoding for magnetic recording channels that experience data dependent noise.
- 5. Study of optimisation techniques for PR targets and development of a new optimisation technique for generalised PR targets for perpendicular magnetic recording channel. This includes the introduction of a new metric that incorporates the PR equaliser euclidean distance and the noise enhancement of PR equalisation.

This thesis is divided into three main parts. The first part (Chapter 2) provides a background to equalisation techniques and detection algorithms applied to magnetic recording channels. The second part (Chapter 3) describes the use of multi-level encoding for magnetic recording. The last part (Chapters 4, 5 and 6) is the major part of this thesis providing background to error correction and modulation coding, and discusses the novel feedback equalisation scheme to obtain optimum performance.

A detailed breakdown of this thesis is as follows.

PR equalisation and the new soft-feedback equalisation (SFE) methods have been discussed for binary magnetic recording in Chapter 2. Trellis based, hard-decision and softdecision detection algorithms have been discussed and their actual calculations have been shown. These algorithms are *maximum-likelihood sequence detection* (MLSD) and *maximuma-posteriori* (MAP). A novel Asymmetric MAP decoding technique for perpendicular magnetic recording is discussed. A new method to design generalised partial response (GPR) targets for perpendicular magnetic recording is also been introduced in this chapter. The limitations of SFE based and PRML based magnetic recording channels are also discussed. Chapter 3 revisits the application of multi-level information over magnetic recording. It essentially shows that, the use of multi-level information is necessary in order to achieve the bandwidth efficiency for magnetic recording channels. Multi-level magnetic recording is applied to channels based with PRML and SFE and their results are compared.

One of the major drawbacks of using SFE based magnetic recording over PRML based magnetic recording is the effect of residual ISI on the performance. Chapter 4 presents the use of MTR codes in order to reduce the effect of residual ISI on the performance. Various MTR codes have been discussed and applied to both PRML and SFE based magnetic recording schemes. A novel class of MTR codes for multi-level magnetic recording is also developed and described in this chapter.

Another major problem of using multi-level information is the increase in requirement of SNR to provide the desired performance. Chapter 5 introduces the use of error correcting codes in order to overcome this problem. Turbo codes have been used in conjunction with PRML and SFE based multi-level magnetic recording schemes and their performance has been presented in this chapter.

Chapter 6 discusses a novel feedback scheme which overcomes the drawbacks of MTRcoded SFE based magnetic recording. This scheme combines PRML and SFE and uses MTR codes as recording codes in order to improve the performance in comparison to standard PRML schemes. PRML is used as the front-end, while SFE-decoding block is used to obtain decoded output. Comparison of various PR targets against the new scheme are shown. This chapter provides results which show that there is a consistent gain of 0.6 dB over standard PRML and SFE based longitudinal magnetic recording.

In Chapter 7, the author concludes work presented in this thesis and proposes some directions for future research.

## **1.6 Contributions to Knowledge**

This thesis covered various topics of interest related to longitudinal and perpendicular magnetic recording. The original work of the author which contributes to the knowledge can be listed as below:

- 1. Investigation and development of a novel soft feedback equalisation for longitudinal magnetic recording
- 2. Study of multi-level encoding for longitudinal magnetic recording using error correction coding
- 3. Development of asymmetric MAP decoding for perpendicular magnetic recording with data dependent noise

4. Investigation of a new optimisation technique for generalised PR targets for perpendicular magnetic recording

A chapter-wise expansion of the above mentioned main contributions is listed below: Chapter 2

- Asymmetric MAP decoding used for perpendicular magnetic recording with transition jitter provides an important conclusion that jitter noise is not strictly Gaussian; but, it is data dependent. The decoder does not provide any gain in terms of channel SNR, which means that it does not reduce the amount of SNR required to achieve the same performance as a typical BCJR decoder, but it does provide a gain of 1 order of magnitude of BER when the transition jitter is considered at 60% in comparison to traditional MAP decoder.
- A different insight into designing of GPR targets is given which provide consistently better GPR targets when compared to the traditional method of designing GPR targets.
- The complexity of the MLSD and MAP decoders are presented and it is shown that, their complexity is also dependent upon the length of the PR target used.
- A new scheme for equalisation known as SFE is presented which does not colour the added noise in magnetic recording channels. This new scheme estimates the amount of ISI introduced in the channel and tries to remove it iteratively.

## **Chapter 3**

- The noise colouration effects due to the PR equaliser have been documented and it is shown that if AWGN is added after PR equalisation, there is an improvement of about 0.7 dB.
- Various algorithms have been used for the feedback loop in SFE based magnetic recording and it is seen that MAP algorithm in feedback provides the best performance.
- It is seen that 4-level SFE based magnetic recording has about 0.7 dB worse performance when compared to 4-level PRML based magnetic recording for a  $PW_{50}$  of 1.2. The advantage of the SFE based magnetic recording channel can be further exploited by using various recording and error correcting codes.

#### **Chapter 4**

• The error event analysis shows that in PRML based magnetic recording channels, the errors are pattern based and this does not exist in SFE-MLSD/MAP based magnetic recording channels.

- MTR codes have been applied to SFE and PRML based magnetic recording channels.
- A new method of designing MTR codes for multi-level magnetic recording is proposed. This method is based on two factors: the number of consecutive transitions and also the step-size of each transition.
- Binary SFE schemes with MTR codes do not perform well in comparison to PRML schemes. This is due to the (1 D) dicode nature of the SFE based channel.

#### **Chapter 5**

- The performance of multi-level schemes for magnetic recording increases the amount of SNR required to reach a BER of  $10^{-5}$  and beyond. In order to reduce this requirement, ECC is applied to these schemes for magnetic recording and their performance is reviewed.
- There is a gain of about 10 dB for binary and about 6 dB for multi-level PR/SFE based magnetic recording.
- The error floor caused by turbo codes is based upon various factors. These factors were concluded to be the type of interleaver used, the effect of inner concatenation of the component codes with the PR target polynomials, amount of ISI introduced at specific densities and the length of PR targets.
- Various results depicting these factors have been presented and discussed.

#### **Chapter 6**

- At higher densities, it was observed that SFE-based binary magnetic recording channels failed to give optimum performance. It was shown that in a thresholding detector, the thresholding point was not fixed and had a high variance in lower SNR regions. As a result, a new technique was developed to replace the thresholding device in a SFE based channel and replace it with a PRML device to provide the best initial ISI estimation.
- This new scheme has a better detection capability than standard PRML detector as it uses the PRML detector for the first initial estimate and then uses the SFE decoding block iteratively for 2 iterations to obtain the optimum performance.
- In terms of complexity of hardware of the new decoder, has an additional feedback block with non-linear MAP algorithm in addition to the standard PRML detector. The SFE-MAP feedback block can be designed as an additional module to the existing electronics of PRML detection.

- The output of the new decoder being *soft*, gives the flexibility to use error correction coding.
- Results show that for a EPR4 magnetic recording channel, the new combined scheme with the use of MTR codes has about 0.6 dB SNR gain at a  $PW_{50}$  of 2.4 in comparison to the EPR4-ML magnetic recording channel with MTR codes. The gain increases to 1.6 dB in comparison to the standard EPR4-ML magnetic recording channel for the same density of  $pw_{50} = 2.4$ .
- Better GPR targets of longer length need to be designed along with the design of MTR decoder for this scheme in order to achieve the maximum gain.
## Part II

# Equalisation Schemes for Digital Magnetic Recording

## Equalisation Techniques and Detection Schemes

## 2.1 Introduction

The magnetic channel can be assumed to be an Linear Time Invariant (LTI) system with AWGN. As a result, the optimum detector for the magnetic channel corresponds to a sampled matched filter, s(-t), with a noise whitening filter and a maximum likelihood sequence detector (G. D. Forney; 1972). The matched filter has the same magnitude response as the channel, and thus, the frequency components where the signal can exist are permitted and where noise exists are attenuated. In the time domain, convolution with the matched filter is equivalent to cross-correlation with the channel response (Sklar; 2001).

The matched filter can be considered to be a template that is matched to the known shape of the signal being processed. It significantly modifies the spectral structure of the signal by gathering the signal energy matched to its template, and, at the end of each symbol time, presents the result as a peak amplitude (Sklar; 2001). The peak amplitude is proportional to the signal energy.

The optimal detector is as shown in Figure 2.1. However, the complexity of this brute force method of searching through all  $2^l$  possible transmitted sequences for a length l input to the channel is very high. Thus, in practise, the number of states required in the MLSD for the magnetic recording channel with long ISI spans and matched filter approach is nearly impossible due to complexity reasons. From the Figure 2.1, if the overall response of the filter is considered including the matched filter and the whitening filter, the combination is called whitened matched filter (WMF).



Figure 2.1: Optimal Detector for Magnetic Recording Channel

When addressing the ISI problems in magnetic recording systems, the combination of matched filter and noise whitening filter is replaced by a continuous time low-pass filter and a partial response equaliser, which shapes the channel to have a response with shorter ISI span (Kobayashi and Tang; 1970). The partial response equalisation scheme and detection methods for magnetic recording are described later in this chapter.

This chapter provides a background to partial response equalisation and MLSD/MAP decoding algorithms. Based on the problem of noise colouration in PR equalisation, this chapter introduces a novel soft feedback equalisation technique which was investigated and developed by the author. In the field of perpendicular recording, the author investigated the problem of jitter noise and developed a new asymmetric MAP decoding technique which is dicussed in this chapter. Also, the author looked at generalised partial response targets and designed a new optimisation technique and investigated its performance over existing optimisation techniques.

Parts of this chapter appear in the Conference Proceedings under the papers titled:

- M. Z. Ahmed, A. Fahrner and P. Shah "Asymmetric MAP Decoding for Perpendicular Magnetic Recording with Data Dependent Noise", International Symposium of Physics of Magnetic Materials, Singapore, September 2005.
- P. Shah, M. Z. Ahmed and Y. Kurihara "New Method for Generalised PR Target Design for Perpendicular Magnetic Recording", Perpendicular Magnetic Recording Conference, Tokyo, Japan, October 2007.

### 2.2 Partial Response Maximum Likelihood

Lender (1966) first introduced duo-binary Partial Response (PR) signalling as a data transmission method. Kretzmer (1966) categorised the characteristics of several PR schemes and compared them on the basis of SNR degradation and speed of tolerance. Kobayashi (1971a), Kobayashi (1971b) and Kobayashi and Tang (1970), in addition to comparing PR scheme with other coding schemes, has provided a good background to *Partial Response Maximum Likelihood* (PRML) system by first introducing *Maximum Likelihood Sequence Detection* (MLSD).

The correlative coding technique, also known as PR Signalling, originated in the 1960's and was initially applied to data transmission over private and switched telephone circuits (Lender; 1966) PRML is a dominant technique in the magnetic recording industry.

The idea of *partial response* is to allow ISI to be present; however; the amount of ISI is well defined and can be taken into account in the detection process. Magnetic recording channels can be transformed into PR channels which satisfy three fundamental properties:

- The superposition of voltage pulses from adjacent transitions is linear;
- the shape of the readback signal from an isolated transition is exactly known and determined;

 the channel operates in a bandwidth limited region, meaning that the spectral components beyond the bandwidth are cut-off.

The above three assumptions make the PR channel very simple and thus considering the magnetic channel, which is a non-linear channel, the assumptions do not completely fulfil. But so far, the optimum solution of magnetic recording channels is to consider them as PR channels. The full-response equaliser is a type of equaliser that would remove ISI completely. The response that the PR equaliser tries to shape the channel is often referred to as a PR target, which is defined by a polynomial using T second delay operators D. Using PR equalisation, the number of states required for state-based detectors to perform optimised detection is manageable. This is possible because the PR equaliser controls the amount of ISI introduced by the lorentzian filter, by using specific length PR targets.

The PRML approach for magnetic recording is the industry's most popular scheme for many years. In the case of a PR4 channel, the PR4 Equaliser does not remove the ISI introduced by the Lorentzian pulse, but instead uses it which leads to zero-voltage at all sampling levels except at the location of transition and the next sample instance from the transition. The lorentzian pulse is defined in (1.4).

A Partial Response filter/equaliser is normally used as a structure of the *Maximum Likelihood Sequential Detector* (MLSD) for a partial response system and it is not as complicated as the matched filter system. The derivation of matched filter output for a Lorentzian filter is discussed in Appendix A.

The partial response equaliser is a sub-optimal approach. The matched filter has complexity which increases exponentially with respect to  $PW_{50}/T$ . Thus, implementing a matched filter for a PR system with higher densities would increase the complexity of the system astronomically. The matched filter output for a Lorentzian pulse is shown in Figure 2.2.

The matched filter response of a Lorentzian pulse can be given as:

$$C(x) = \int_{-\infty}^{\infty} \frac{1}{1 + \frac{2t}{PW_{50}^2}} \cdot \frac{1}{1 + \frac{2(t-x)}{PW_{50}^2}} dt.$$
 (2.1)

Solving the above equation and applying this equation to (Wolfram Mathworld Website; 2005), the matched filter response for Lorentzian pulse is obtained. It is:

$$C(x) = \frac{\pi}{4} \cdot \frac{PW_{50}^3}{PW_{50}^2 + x^2}.$$
(2.2)

#### 2.2.1 PR Equalisation

When the receiving filter is configured to compensate for the distortion caused by *both* the transmitter (or writing head) and the channel, it is often referred to as an *equalising* 



Figure 2.2: Typical Lorentzian Pulse and its Matched Filter Output at  $PW_{50} = 0.7$ 

*filter or a receiving/equalising filter.* The aim of equalisation is to modify the frequency response of a magnetic recording channel so that it matches with the frequency response of the desired *PRML* scheme.

The frequency spectrum of a linear channel is usually defined as a Fourier Transform of its impulse response, i.e. the dipulse response. The Lorentzian pulse in frequency domain is a Fourier Transform of the pulse described in (1.4). It is described as:

$$H(\omega) = \int_{-\infty}^{\infty} \frac{1}{1 + \left(\frac{2t}{PW_{50}}\right)^2} e^{-j\omega t} dt.$$
 (2.3)

Thus, after calculating the isolated lorentzian frequency response, the dibit response  $D(\omega)$ can be given as:

$$D(\omega) = (1 - e^{(-j\omega T)}) \frac{PW_{50}}{2} \pi e^{-\omega} \frac{PW_{50}}{2}, \qquad (2.4)$$

where  $\omega$  is the frequency term and T is the sampling period.

The readback signal consists of high frequency components which corresponds to the the fastest changing slope of the signal. PR Equalisation is performed so that the ISI introduced by the Lorentzian pulse can be controlled. As the  $PW_{50}$  increases, the magnetic channel response changes and more equalisation is required to force the readback waveform to a partial response target.

Determining the number of taps for a FIR filter is somewhat complicated. The number of taps determines the total length of the impulse response of the equaliser and hence provides an estimate of the detail in the frequency response of the filter. The total length of the delay lines in the FIR filter, should not be less than the duration of the pulse to be equalised. Typically 6 - 10 programmable taps are suitable for the FIR filters (Wang and Taratorin; 1999). The filter used in the system modelling is a symmetrical filter and the number of taps decided is 7.

For a PR4 target, the overall channel response is:

$$(1-D)(1+D) = 1 - D^2.$$
 (2.5)

PR4 equalisation can be done in two ways:

- The first approach equalises the Lorentzian pulses to sinc pulses, then performs (1 + D) operation on the resulting samples for a PR4 channel.
- The second technique transforms the Lorentzian pulse directly into a PR4 waveform.

Both of these methods are based on solving a system of simultaneous equations to obtain the filter tap coefficients. The (1 + D) operator in PR4 determines how the transition sample is spread over the neighbouring bit periods.

The PR4 pulse can be given as (Wang and Taratorin; 1999):

$$s(t) = \frac{\sin\frac{\pi t}{T}}{\frac{\pi t}{T}} + \frac{\sin\frac{\pi (t-T)}{T}}{\frac{\pi (t-T)}{T}}.$$
(2.6)

The frequency response of (2.6) can be expressed as:

$$D(\omega) = 2\sin(\omega T) \tag{2.7}$$

where,  $\omega$  is the frequency term and T is the sampling period.

Figure 2.3 shows a basic simulated PRML channel.

For a 7-tap PR equaliser with tap coefficients v,

$$v = \begin{bmatrix} v_0 & v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{bmatrix}.$$
 (2.8)

Using equation(1.4) to evaluate h(t) for a given  $PW_{50}$ ,

#### Chapter 2. Equalisation Techniques and Detection Schemes



Figure 2.3: Simulation Model of a Standard PRML Channel

$$\begin{bmatrix} h(0) & h(1) & h(2) & h(3) & h(4) & h(5) & h(6) \\ h(-1) & h(0) & h(1) & h(2) & h(3) & h(4) & h(5) \\ h(-2) & h(-1) & h(0) & h(1) & h(2) & h(3) & h(4) \\ h(-3) & h(-2) & h(-1) & h(0) & h(1) & h(2) & h(3) \\ h(-4) & h(-3) & h(-2) & h(-1) & h(0) & h(1) & h(2) \\ h(-5) & h(-4) & h(-3) & h(-2) & h(-1) & h(0) & h(1) \\ h(-6) & h(-5) & h(-4) & h(-3) & h(-2) & h(-1) & h(0) \end{bmatrix} \begin{bmatrix} v_0 \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} = i'$$
(2.9)

#### i is the transpose of i matrix.

i is defined as:

$$i = \begin{cases} \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} & Lorentzian \ to \ Sinc \ Equaliser \\ & & & \\ \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} & Lorentzian \ to \ PR4 \ Equaliser \end{cases}$$
(2.10)

The equaliser coefficients are symmetrical about the  $3^{rd}$  tap for the Lorentzian to Sinc equaliser. This is because the equaliser is symmetrical and with linear phase with a group delay of (N-1)/2 samples. Thus, for a 7-tap filter, it is  $3^{rd}$  tap. As shown in Figure 2.3,  $\{x_k\}$ , where  $x_k \in \{0, 1\}$  is the user data. The (1 - D) block is inherent with the magnetic channel, since it provides with a high-pass amplitude response. The (1 - D) block provides a dc-free response that matches the low-frequency response of the dc-free longitudinal magnetic recording channel. The response is moulded to band-pass in order to achieve better bandwidth efficiency and spectral matching with the magnetic channel transfer function (Thapar and Patel; 1987). The (1 - D) block is used to convert the step response of the magnetic

recording channel into pulse (dibit) response. The absolute values of (1 - D) operation provides NRZI data bits. The output of the (1 - D) block is given as:

$$a_k = x_k \cdot (1 - D). \tag{2.11}$$

The N coefficients of the Lorentzian filter are  $h(-\frac{N-1}{2}), \ldots, h(\frac{N-1}{2})$ , where N is a positive odd integer. They are given by the Lorentzian function described in equation(1.4). The value of the coefficients of the Lorentzian filter depend upon the value of  $PW_{50}$ . After passing  $a_k$  through the Lorentzian filter, the readback pulse r(t) is obtained. The channel noise n(t) is assumed to be AWGN whose variance,  $\sigma^2$  is determined by the SNR equation given in equation(1.7). This noise is then added to r(t) giving b(t), where

$$b(t) = r(t) + n(t).$$
(2.12)

The readback pulse b(t) obtained after adding the AWGN is equalised using the best PR or Generalised PR (GPR) target. PR equalisation is performed so that the Inter Symbol Interference (ISI) introduced by the Lorentzian Pulse can be controlled. The PR equalised data  $q_k$  is decoded using a trellis based decoders producing the decoded data  $\{\bar{x}_k\}$ .  $q_k$  is given as:

$$q_k = (r(t) + n(t)) * p(t), \tag{2.13}$$

where, p(t) is the impulse response of the PR equaliser, r(t) is channel readback data with ISI, n(t) is AWGN and \* denotes convolution. The decoders used for PR magnetic channels are MLSD and MAP.

#### 2.2.2 Partial Response Targets

PR equalisation is achieved by a linear filter which shapes the natural response of the channel into a target response. This target response, achieved by the linear filter, whose coefficients are typically integers, is pre-determined. The class of partial response targets based on the polynomial form  $(1 - D)(1 + D)^n$ , where n is to increase with the recording density, and D is a T-second delay operator has been widely accepted in magnetic recording (Kobayashi; 1971a)(Thapar and Patel; 1987). In this class of PR targets, (1 - D) provides a spectral null at DC, which is similar to the magnetic recording channel and,  $(1 + D)^n$  factor matches the high-frequency attenuation of the channel.

The *PR*4 target is used in magnetic recording systems in order to achieve recording densities up to  $PW_{50} = 2.5$ . In order to achieve densities greater than 2.5, the Extended *PR*4 (EPR4) target response is used. The final PR target equation for EPR4 is  $(1 - D)(1 + D)^2$ . The  $(1 - D^2)(2 + 2D + D^2)$  target documented in (Tanner; 1981b) is also a good match for high density magnetic recording.

Although the  $(1 - D)(1 + D)^n$  class of PR targets provide good equalisation, depending upon the operating linear density and the head/disk components used, the natural channel response may be significantly different from the prescribed form (Moon and Zeng; 1995). Thus, a Generalised PR (GPR) target with non-integer valued coefficients, can result in a considerably smaller equalisation loss and the overall gain can be substantial at high linear densities. Given a finite length, several methods to design generalised PR targets have been proposed in (Lee et al.; 1993) and (Qureshi and Newhall; 1973). Minimum Mean Square Error (MMSE)\* equalisation technique in conjunction with a monic generalised PR target has been observed to perform extremely well in practise (Moon and Zeng; 1995).

Along with generalised PR targets, this work focuses on *PR4* and *EPR4* targets for magnetic recording, because of their ubiquity in the recording industry.

The best way of decoding PR samples for magnetic recording is by using the Viterbi Algorithm described by Forney (1973). A PR channel is basically a rate-1 convolutional code and it operates on real numbers instead of binary  $\{0, 1\}$ . Thus, the trellis diagram has a trellis section which repeats up to the data length. The Viterbi detector is the optimum detector for a PR channel.

Figure 2.4 shows the  $PW_{50}$  vs BER performance for binary PRML channel at an SNR of 17 dB. Amongst the PR target performances shown include the GPR targets of the form  $(1-D)(0.4 + D + 0.4D^2)$  and (1-D)(0.9 + 0.5D).

Figure 2.4 shows the variation of BER with respect to changing  $PW_{50}$ . It is observed that different PR targets have individual behaviour at different  $PW_{50}$ . It is seen that at lower  $PW_{50}$ , non-integer GPR targets have better performance in comparison to the standard PR4, EPR4 and EEPR4 targets. It is seen that the GPR target (1-D)(0.9+0.5D)performs better than PR4 target in lower  $PW_{50}$  region. The point after  $PW_{50} = 1.45$  is not simulated as the BER is in the region of  $10^{-7}$ . The variation in behaviour of the targets also exists in the length of the PR targets. As the PR target length increases, their performance improves in the higher  $PW_{50}$  regions. Figure 2.5 shows the variation of BER with increase in SNR at  $PW_{50} = 2.2$ . It is seen that the length 4 PR target EPR4 and the GPR target perform better in comparison to shorter PR target of length 3 and also the EEPR4 target which has length 5. Thus, the previous Figure 2.4 holds true for varying behaviour of PR targets at different densities.

Performance is compromised in the linearly equalised channel because, the extent of ISI in the unconditional channel is very large at higher densities that the linear equaliser always leads to severe noise enhancement. A good spectrum matching between the original channel and the target channel is impossible to achieve at high densities when the target channel is constrained to be of particularly smaller length (Moon and Carley; 1990). The Figure 2.6 shows the difference between the equalised signals of noisy and noiseless longitudinal magnetic recording channels. It is seen from the equalised signals that there

<sup>\*</sup>MMSE criterion is minimisation of the distance between the equalised and the target signals



Figure 2.4: BER vs PW50 Performance for PRML at SNR=17 dB

is noise enhancement in the PR4 equalised signal and at 18 dB SNR, the amplitude variation of the signal is significant. Due to this amplitude variation, there is a noise penalty of about 0.6 dB, which affects the overall performance of the PR4 channel.

Two different type of trellis-based detectors are discussed below:

- 1. Maximum Likelihood Sequence Detection (MLSD)/ Viterbi Detector
- 2. Maximum-A-Posteriori (MAP) Detector.

## 2.3 MLSD Decoder

The novelty of the ML approach is the trellis diagram which simplifies the Viterbi algorithm over the ML approach. The Viterbi algorithm is a recursive structure that does not grow with the length of the message sequence, and has the complexity proportional to  $m^L$ , where m is the size of the input alphabet and L is the length of the impulse response h(t)with T as the unit time-period. The complexity of the Viterbi algorithm can be classified into:



Variation of BER vs SNR for PRML Channels at PW<sub>50</sub>=2.2

Figure 2.5: BER vs SNR Performance for PRML at  $PW_{50} = 2.2$ 

- 1. Memory: The algorithm requires M memory locations, one for each state, where each location must be capable of storing a length L(m) and a truncated survivor listing S(m) of the symbols.
- 2. Computation: In each unit of time, the algorithm must make  $M^2$  additions at most, one for each transition, and M comparisons among the  $M^2$  results.

Thus, the amount of memory is proportional to the number of states, and the amount of computation to the number of transitions. The equivalence of ML in log domain with respect to the classical Viterbi decoder is discussed in Appendix B.

MLSD detection is a "hard" decision detection scheme. It outputs 1's and 0's and conveys no information on the reliability of each decision. It is a Soft Input and Hard Output Detector (Ahmed; 2003b). The ML/Viterbi decoder considers the entire sequence and finds a sequence among all possible sequences that has the highest probability of being correct. The highest probability codeword is the output of MLSD detector and is always a codeword. In the PRML magnetic recording channel as shown in Figure 2.3, using the noisy readback



Figure 2.6: Difference of Equalised Signals from Noisy and Noiseless Channels at 18 dB SNR and  $PW_{50} = 2.2$ 

data *q*, the MLSD detector detects the most likely sequence *x*. Thus from Bayes Rule:

$$P(x|q) = \frac{P(q|x) \cdot P(x)}{P(q)}.$$
(2.14)

P(q) is a normalising constant and is same for all sequences s and thus can be neglected. Thus equation(2.14) becomes:

$$P(q) = \sum_{x} P(q|x) \cdot P(x).$$
 (2.15)

P(x) is the priori probability of the codeword and it is assumed that it has equal probability. Thus, it can be neglected as well. Assuming noise in the magnetic channel to be AWGN and that the channel is memoryless, i.e., each readback sample is affected by noise independently, equation(2.14) can be written as follows for a sequence of length N.

$$P(x|q) = \prod_{k=1}^{N} P(q_k|x_k).$$
(2.16)

The MLSD detector searches for the sequence with highest probability of being correct, i.e.:

$$\max P(x|q) = \prod_{k=1}^{N} P(q_k|x_k).$$
(2.17)

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The probability  $P(q_k|x_k)$  can be obtained from the probability density function (pdf) for Gaussian distribution. It can be calculated as below:

$$P(q_k|x_k) = \frac{k}{\sqrt{2\pi\sigma^2}} e^{-(q_k - x_k)^2/2\sigma^2},$$
(2.18)

where, k is a constant such that the sum of all the possible probabilities for a given sample is 0 and it is of no consequence when the probabilities of all possible sequences are normalised.

Simplifying the above equation(2.17) by applying natural log domain and using the pdf and Gaussian distribution, equation(2.19) is obtained as below:

$$\min \sum_{k=1}^{N} (q_k - x_k)^2.$$
(2.19)

Thus, piecewise minimisation of probability in log domain is Viterbi algorithm, which is equivalent to MLSD. This is the equivalence of the Viterbi algorithm and MLSD decoder. The Minimum Squared Distance (MSD) is calculated by comparing each of the possible sequences to that of the received sequence of samples. Thus, the ML solution means to search for a sequence or codeword that has minimum distance, i.e., maximum likelihood. The decisions made by the MLSD detector consist of some delay. The ML decoder minimises the frame error probability.

#### 2.3.1 Implementation and Example of MLSD Decoding

A binary PR-4 trellis is as shown in Figure 2.7. The process of ML decoding is as below: Assume that the equalised output of the PR4 equaliser  $q_k$  is -0.0084, 0.9911, 1.056, -0.9688. Assume that there are four possible states: 00, 01, 10, and 11. The decoding process is as below:

- The state metric is initialised at the beginning of trellis with 0 for the first state which is called *root* and any positive large number for the rest of the states. The state metrics are as show in the Figure 2.7. The root of the trellis, state node  $a_0$  is initialised to 0, while  $b_0$ ,  $c_0$  and  $d_0$  are initialised to 10000.
- The state metric for each state node is calculated according to equation(2.19). If there are two or more branches converging at a particular state node, the path that has minimum state metric is considered and is stored for further decoding process. This path is called the *survivor* path. At t = 3, at state node  $a_3$ , there are two paths coming from  $a_2$  and  $b_2$ .

#### 2.3. MLSD Decoder



Figure 2.7: MLSD Decoding using Trellis Structure

#### Calculation of State Metric at node a3 and survivor path

$$a_3 = \min\{0.983 + (1.056 - 0)^2, 2.00 + (1.056 - (-1))^2\}$$

It is as shown below with respect to Figure 2.7.

 Traceback begins from the end of the trellis. The state node where it begins from must have the minimum state metric. The process continues until the root of the trellis with state 00 is reached. The dark line in the Figure 2.7 is the traceback path. At t = 4, the final minimum state metric is calculated to start the traceback path. The decoded bits are the input label of the trellis path to that particular trellis segment.

#### Calculation of start point of traceback

 $\min\{a_4, b_4, c_4, d_4\}$ 

#### $\min\{1.1161, 0.00425, 2.0537, 3.879\}$

• The decoded bits  $\hat{x}(t)$  are 0 = 1 = 1 = 0.

| 2   |  |
|-----|--|
| a   |  |
| pt  |  |
| er  |  |
| 01  |  |
| -   |  |
| p   |  |
| nc  |  |
| ili |  |
| SO  |  |
| ti  |  |
| 07  |  |
| 2   |  |
| ec  |  |
| 4   |  |
| n   |  |
| nb  |  |
| es  |  |
| 8   |  |
| n   |  |
| 11  |  |
| De  |  |
| te  |  |
| cta |  |
| 0   |  |
| 2   |  |
| Sc  |  |
| he  |  |
| m   |  |
| es  |  |
|     |  |

| k | $q_k$   |         | State 00 | )        | State 01 |        | 1        | State 10 |        |          | State 11 |         |          |
|---|---------|---------|----------|----------|----------|--------|----------|----------|--------|----------|----------|---------|----------|
| - |         | Path 1  | Path 2   | Survivor | Path 1   | Path 2 | Survivor | Path 1   | Path 2 | Survivor | Path 1   | Path 2  | Survivor |
| 0 | -       | -       | -        | 0        | -        | -      | -        | . —      | -      | -        | -        | -       | 0        |
| 1 | -0.0084 | 0.00007 | -        | 0.00007  | -        | -      | -        | -        | 1.0168 | 1.0168   |          | · · · · | 0        |
| 2 | 0.9911  | 0.9823  | -        | 0.9823   | 2        | +      | 2        | -        | 0.1    | 0.1      | -        | 1.016   | 1.016    |
| 3 | 1.056   | 2.097   | 6.227    | 2.097    | 1.1152   | 5.224  | 1.1152   | 0.9854   | 3.1151 | 0.9854   | 0.0032   | 1.020   | 0.0032   |
| 4 | -0.9688 | 3.0355  | 1,1161   | 1.1161   | 1.924    | 0.0042 | 0.0042   | 5.973    | 2.055  | 2.055    | 4.861    | 3.579   | 3.579    |

Table 2.1: State metrics and survivor path for all sections of trellis for MLSD

### 2.4 Maximum-A-Posteriori Decoder

Bahl et al. (1974) proposed the Maximum-A-Posteriori (MAP)<sup>†</sup> decoding algorithm for trellisbased codes (Bahl et al.; 1974). MAP decoding algorithm is a soft-decision decoding algorithm for general linear codes. MAP decoders tend to have slightly better Bit Error Rate (BER) performance compared to MLSD decoders. MLSD decoders have very slightly better Frame Error Rate (FER) performance. MAP decoders use the A priori probability and the code constraints (trellis) to obtain the A-Posteriori Probability (APP). The BCJR/MAP decoder results in the most likely bit sequence and information on the reliability of each bit. Thus, it is a bit-wise decoder. In order to minimise the bit error probability, one must use a decoder which works bit-wise and does not take into account a sequence of certain length as in the case of the MLSD detector which was described earlier.

The aim of MAP decoder is to compute the APP of the decoded bit  $\bar{x}_k$  being either 0 or 1 for binary levels or 0, 0.33, 0.66 or 1 for 4-levels magnetic recording. Consider the similar trellis as used in mlsd decoding from Figure 2.7 and consider  $q_k$  as the noisy readback samples and  $\bar{x}_k$  as the decoded samples.

$$APP(x_k = 0) = \sum_{S_{k-1}^{n'}, S_k^n} P_{x_k = 0}(S_{k-1}^n, q)$$

$$APP(x_k = 1) = \sum_{S_{k-1}^{n'}, S_k^n} P_{x_k = 1}(S_{k-1}^n, q)$$

The above equations mean that the APP of  $x_k$  being 0 or 1 is equal to the sum of the joint probabilities taken over all trellis transition from state  $S_{k-1}^{n'}$  to state  $S_k^n$  for information bit or trellis input label of 0 or 1. Consider 0 or 1 being *i*, where  $i \in \{0, 1\}$ .  $S_k^n$  means the  $n^{th}$  state at time k.

Considering only AWGN noise with the memoryless magnetic channel, the joint probability  $P(S_{k-1}^{n'}, S_{K}^{n}, q)$  can be written as the product of three probabilities (Bahl et al.; 1974):

$$P(S_{k-1}^{n'}, S_{k}^{n}, q) = P(S_{k-1}^{n'}, q_{j < k}) \cdot P(S_{k}^{n}, q_{k} | S_{k-1}^{n'}) \cdot P(q_{j > k} | S_{k}^{n})$$

$$= \underbrace{P(S_{k-1}^{n'}, q_{j < k})}_{\alpha_{k-1}(S_{k-1}^{n'})} \cdot \underbrace{P(S_{k}^{n}, q_{k} | S_{k-1}^{n'})}_{\gamma_{k}(S_{k-1}^{n'}, S_{k}^{n})} \cdot \underbrace{P(q_{j > k} | S_{k}^{n})}_{\beta_{k}(S_{k}^{n})}$$
(2.20)

where,  $q_{j < k}$  denotes the readback samples  $q_j$  from the root of the trellis to time k - 1 and  $y_{j>k}$  denotes the readback samples from time k + 1 to the toor<sup>1</sup> of the trellis. Hence, APP of

MAP decoding algorithm is commonly known as BCJR algorithm after it's authors

<sup>&</sup>lt;sup>1</sup>Toor of the trellis means the end of the trellis, where the trellis is terminated

information bit  $x_k$  is given as:

$$APP(x_k = i) = \sum_{\substack{S_{k-1}^{n'}, S_k^n}} \alpha_{k-1} (S_{k-1}^{n'} \cdot \beta_k (S_k^n) \cdot \gamma_k^{x_k = i} (S_{k-1}^{n'}, S_k^n)$$
(2.21)

The term  $\alpha_k(S_k^n)$  is known as alpha recursion defined as:

$$\alpha_k(S_k^n) = \sum_{\substack{S_{k-1}^{n'}\\k=1}} \alpha_{k-1}(S_{k-1}^{n'}) \cdot \gamma_k^{x_k=i}(S_{k-1}^{n'}, S_k^n)$$
(2.22)

The term  $\beta_k(S_k^a)$  is known as beta recursion defined as:

$$\beta_k(S_k^n) = \sum_{S_{k+1}^n} \beta_{k+1}(S_{k+1}^n) \cdot \gamma_{k+1}^{x_k=i}(S_{k+1}^n, S_k^{n'})$$
(2.23)

The term  $\gamma_k^{x_k=i}(S_{k-1}^{n'}, S_k^n)$  is called the *path probability* associated with transition from previous state to next state which has information bit of *i*. The path probabilities for the PR channels are calculated using equation(2.18).

#### 2.4.1 Implementation and Example of MAP algorithm

Below is a step-by-step implementation of the MAP algorithm for decoding the PR equaliser output samples. In this case, the PR equaliser output is considered to be the computed readback sample.

- Compute the path/channel/apriori probabilities P(xk = i) for the readback samples qk using equation(2.18). In this case, i ∈ {0,1} for binary, while i ∈ {0,0.33,0.66,1} for 4-levels. The path probabilities are normalised so that their sum is equal to 1.
- Initialise the root of the trellis (α(S<sup>n</sup><sub>α</sub>)) with 1 for first state, n = 0 and 0 for n ≠ 0. Get alpha values for all the state nodes using equation(2.22) and normalise them at each trellis section. Alpha recursion goes from root to the toor of the trellis.
- Once all alpha values are obtained, initialise the toor of the trellis  $\beta(S_N^n)$ , with equal divided value of 1, depending upon the number of states, where N is the length of the readback samples. The equal probabilities initialisation is for PR channel, which is the unterminated trellis, because it is not known where the data sequence ends. In case of a terminated trellis, the initialisation of  $\beta(S_N^n)$  is similar to that of  $\alpha(S_0^n)$ . Compute the beta recursion using equation(2.23) and normalise them at each trellis section. Beta recursion goes from toor to the root of the trellis.

• Once alpha and beta values are computed, compute  $APP(x_k = i)$  at each trellis section using equation(2.21) and normalise them. Once the APP are normalised, perform the thresholding operation to obtain hard-decision output.

Consider the same example as discussed in the MLSD decoding. Assume that the output of the PR4-equaliser  $q_k$  is -0.0084, 0.9911, 1.056, -0.9688. Assume that there are four possible states: 00, 01, 10, and 11. Since, it is a PR4 channel, the possible outputs are 0, 1 and -1.

#### Calculation of alpha recursion, beta recursion and path probabilities:

**Calculation of Path Probability:** Using equation(2.18) for  $q_k = -0.0084$  and assuming the standard deviation  $\sigma$  of the Gaussian noise as 1 :

$$\gamma_1(0) = \frac{1}{\sqrt{2\pi}} e^{-(-0.0084-0)^2/2} = 0.398925$$
$$\gamma_1(1) = \frac{1}{\sqrt{2\pi}} e^{-(-0.0084-1)^2/2} = 0.398925$$

$$\gamma_1(-1) = \frac{1}{\sqrt{2\pi}} e^{-(-0.0084+1)^2/2} = 0.398925$$

After the initial  $\gamma$  is calculated for each possible output, the  $\gamma$  values are normalised.

$$\gamma_{lotal} = \gamma_1(0) + \gamma_1(1) + \gamma_1(-1) = 1.306235$$

Thus,

$$\gamma_1(0) = \frac{\gamma_1(0)}{\gamma_{total}} = \frac{0.398925}{1.306235} = 0.30540$$

$$\gamma_1(1) = \frac{\gamma_1(1)}{\gamma_{total}} = \frac{0.66331}{1.306235} = 0.50780$$

$$\gamma_1(-1) = \frac{\gamma_1(-1)}{\gamma_{total}} = \frac{0.2440}{1.306235} = 0.18679$$

Using the path probability- $\gamma$  table as shown in table(2.2), alpha and beta recursion are calculated using equation(2.22) and equation(2.23).

| k              | 1       | 2       | 3       | 4       |
|----------------|---------|---------|---------|---------|
| $q_k$          | -0.0084 | 0.9911  | 1.056   | -0.9688 |
| $\gamma_k(0)$  | 0.30540 | 0.34972 | 0.33844 | 0.35356 |
| $\gamma_k(1)$  | 0.50780 | 0.57148 | 0.59015 | 0.08138 |
| $\gamma_k(-1)$ | 0.1879  | 0.07879 | 0.07140 | 0.56505 |

Table 2.2: Channel/Path Probabilities being 0, 1 or -1

#### **Calculation of Alpha Recursion:**

$$\alpha_2(00) = \alpha_1(00)\gamma_2(0) + \alpha_1(01)\gamma_2(-1) = 0.13131$$

 $\alpha_2(01) = \alpha_1(10)\gamma_2(0) + \alpha_1(11)\gamma_2(-1) = 0.23107$ 

 $\alpha_2(10) = \alpha_1(00)\gamma_2(1) + \alpha_1(01)\gamma_2(0) = 0.21459$ 

 $\alpha_2(11) = \alpha_1(10)\gamma_2(1) + \alpha_1(11)\gamma_2(0) = 0.35688$ 

 $u_2 total$  for the particular trellis section is:

 $\alpha_2 \text{total} = \alpha_2(00) + \alpha_2(01) + \alpha_2(10) + \alpha_2(11) = 0.93385$ 

Thus, normalised alpha recursion values are:

 $\alpha_2(00) = \frac{\alpha_2(00)}{\alpha_2 \text{total}} = 0.1406$ 

$$\alpha_2(01) = \frac{\alpha_2(01)}{\alpha_2 \text{total}} = 0.2474$$

$$\alpha_2(10) = \frac{\alpha_2(10)}{\alpha_2 \text{total}} = 0.2294$$

$$\alpha_2(11) = \frac{\alpha_2(11)}{\alpha_2 \text{total}} = 0.3821$$

The normalised alpha recursion table is as shown in the table(2.3).

| k              | - 0    | 1      | 2      | 3      | 4      |
|----------------|--------|--------|--------|--------|--------|
| $\alpha_k(00)$ | 1.0000 | 0.3755 | 0.1406 | 0.1083 | 0.1851 |
| $\alpha_k(01)$ | 0.0000 | 0.0000 | 0.2474 | 0.1745 | 0.4744 |
| $\alpha_k(11)$ | 0.0000 | 0.6245 | 0.2294 | 0.2768 | 0.0965 |
| $\alpha_k(11)$ | 0.0000 | 0.0000 | 0.3821 | 0.4401 | 0.2438 |

Table 2.3: Normalised Alpha Recursion

#### **Calculation of Beta Recursion:**

$$\beta_2(00) = \beta_3(00)\gamma_3(0) + \beta_3(10)\gamma_3(1) = 0.10881$$

 $\beta_2(01) = \beta_3(00)\gamma_3(-1) + \beta_3(10)\gamma_3(0) = 0.22983$ 

 $\beta_2(10) = \beta_3(01)\gamma_3(0) + \beta_3(11)\gamma_3(1) = 0.10865$ 

$$\beta_2(11) = \beta_3(01)\gamma_3(-1) + \beta_3(11)\gamma_3(0) = 0.22947$$

 $\beta$ total for that particular trellis section is:

$$\beta_2 \text{total} = \beta_2(00) + \beta_2(01) + \beta_2(10) + \beta_2(11) = 0.66876$$

Thus, normalised alpha recursion values are:

$$\beta_2(00) = \frac{\beta_2(00)}{\beta_2 \text{total}} = 0.1607$$

$$\beta_2(01) = \frac{\beta_2(01)}{\beta_2 \text{total}} = 0.3395$$

$$\beta_2(10) = \frac{\beta_2(10)}{\beta_2 \text{total}} = 0.1605$$

$$\beta_2(11) = \frac{\beta_2(11)}{\beta_2 \text{total}} = 0.3390$$

The normalised beta recursion table is as shown in the table(2.4).

| k             | 0      | 1      | 2      | 3      | 4      |
|---------------|--------|--------|--------|--------|--------|
| $\beta_k(00)$ | 0.4168 | 0.2230 | 0.1607 | 0.2502 | 0.2500 |
| $\beta_k(01)$ | 0.2188 | 0.0987 | 0.3395 | 0.2498 | 0.2500 |
| $\beta_k(11)$ | 0.1838 | 0.4708 | 0.1605 | 0.2502 | 0.2500 |
| $\beta_k(11)$ | 0.1803 | 0.2077 | 0.3390 | 0.2498 | 0.2500 |

Table 2.4: Normalised Beta Recursion

Once the alpha and beta recursions and the path probabilities are calculated, the APP being a 0 or a 1 is calculated. This is done using the above equation(2.21). The APP and the decoded bits received after thresholding the APP's are as shown in table(2.5.

Thus, it is seen that the decoded bits are  $\bar{x}(t)$  are 0 = 1 = 1 = 0, which are similar to that received by MLSD decoding.

| k                  | 1      | 2      | 3      | 4      |
|--------------------|--------|--------|--------|--------|
| $APP_k(0)$         | 0.5758 | 0.2246 | 0.0361 | 0.6603 |
| $APP_k(1)$         | 0.4252 | 0.7754 | 0.9639 | 0.3397 |
| $x_k$ Decoded bits | 0      | 1      | 1      | 0      |

Table 2.5: Normalised APP and Hard Decision Output

## 2.5 Asymmetric MAP Decoding for Perpendicular Magnetic Recording

Previous sections of Chapter 2 discussed the implementation of MLSD and MAP decoders. Based on this discussion, this section introduces a novel idea of a new modified MAP decoding technique which was derived from the jitter problem and MAP decoding for perpendicular magnetic recording. This novel Asymmetric MAP decoding technique has been published in the Conference Proceedings of ISPMM 2005. This section explains the derivation of the novel asymmetric decoding technique and its application to perpendicular magnetic recording.

It is known that the MAP decoder takes the AWGN (electronics noise) into account for its metric computations. This was shown above in the example of MAP decoding. In order to improve the data capacity in magnetic recording, it is necessary to increase the linear density by storing magnetic transitions more closely in each track. However, this results in substantial increase in the transition noise which degrades the performance and thus, reducing the reliability of the decoder to reproduce reliable information. Previous work in longitudinal magnetic recording (Zeng and Moon; 1992) showed the improvement in the performance of the decoder when Gaussian jitter was included in the metric calculations along with electronics noise. Based on this work, a new modified MAP decoder was designed in order to conquer the effects of transition noise occurring in perpendicular magnetic recording channels. In the metric calculations, an additional error term was computed based on the jitter noise term j(t).

Transition noise in magnetic recording cannot be considered to be additive, since it is data dependent. Random jitter in the transition position is believed to be one of the major contributions to transition noise (Zeng and Moon; 1992). The transition jitter noise consists of 80 - 90% of the readback data amplitude. The fact that the jitter noise is greater in regions of transitions compared to where there are no transitions can result in improved reliability of the metric computations. Transition jitter manifests itself as a pulse position jitter and pulse width/amplitude variation. Further accurate description of transition-noise dominant magnetic recording channel can be found in (Moon; 1991).

Asymmetric Decoder: A block diagram of the simulated perpendicular channel is as shown in Figure 2.8. From the simulation model discusses in Chapter 1, Figure 1.7, another block added in this simulation model is the jitter generator block. The data processor block in this case is 0.5 multiplier block. The transition response block is a perpendicular recording channel approximation from (1.8).

The received signal r(t) can be described as:

$$r(t) = h(t) * (s(t) + j(t) + n(t))$$
(2.24)

where, h(t) is the impulse response of the PR equaliser, s(t) is the channel readback waveform including ISI, j(t) is the transition jitter noise and n(t) is AWGN or electronics noise and \* denotes convolution. The transition jitter is generated using an expansion of Taylor series (James; 1999), where the input to the series is the data and it's derivatives which is also an input to generate the readback signal. This proves the data dependency of the transition jitter noise in the simulation model.

Using noise prediction within the decoder, the resulting decoded signal in the Log-MAP domain is:

$$r(t) = [h(t) * s(t)] + j(t) + n(t)$$
(2.25)

Considering the typical expressions of the Viterbi decoder and Log-MAP decoder, there exists a add-compare and select branch metric. The Viterbi decoder has the expression:

$$\min S_a + (r_i - s_j)^2, S_b + (r_i - s_k)^2,$$
(2.26)

where,  $S_a$  and  $S_b$  are the state metrics,  $r_i$  is the received sample and  $s_j$ ,  $s_k$  are ideal possibilities of  $r_i$ .

The modification required in the above (2.26) is to include the precomputed jitter noise





Figure 2.8: Block Diagram of Perpendicular Magnetic Recording Channel with Jitter Noise

statistics. Let us say they are X and Y. X and Y are precomputed based on the statistics of the data dependent noise. The above equation then modifies to:

$$\min S_a + (r_i - s_j)^2, S_b + [(r_i - s_k)^2 \cdot \mathbb{X} + \mathbb{Y}].$$
(2.27)

X depends on the statistics of the noise and Y depends on the noise statistics and the probability of the ideal symbols  $s_i$  and  $s_j$ . If noise prediction is removed, the noise is no longer data dependent and (2.27) is equal to 2.26. A crucial aspect of the asymmetric decoder is the dependence of X and Y on the decoder output, so additional storage of all possibilities are required before any decision is made.

The modified MAP decoder is known as Asymmetric Decoder (AD). The decoder uses the input branch labels to determine if a particular path would include transition noise. As a result, the knowledge of two consecutive states is required before the state metrics are updated. The asymmetric MAP decoder includes 1 addition and 1 multiplication per branch metric computation in addition to the classic Log-MAP trellis based decoder. Thus, the increase in computational complexity will be exponential as more terms around the transition region are considered, and thus, it is considered to be a limitation for higher density.

The asymmetric MAP decoder provides gains in performance over classical MAP decoders that assume the noise to be not data dependent. The jitter noise has been found not to be strictly Gaussian, due to the non-linear effect of ISI.

## 2.6 New Partial Response Target Design for Perpendicular Magnetic Recording

The current method for designing optimised GPR targets is based on the MMSE criterion between the equaliser output and the desired output, subject to a monic constraint<sup>§</sup> (Moon and Zeng; 1995). For a longitudinal magnetic recording channel, the read channel uses a PRML detector with a target polynomial of the form:

$$S(D) = (1 - D)(1 + D)^n$$
 where  $n = 0, 1, 2, \cdots$  (2.28)

For perpendicular magnetic recording, the main signal energy is at low frequencies (Madden et al.; 2004). Thus, if the PR equaliser used in longitudinal magnetic recording is to be used in a perpendicular magnetic recording channel, then the filtering of the low frequency components in order to obtain dc-free target polynomial would actually eliminate the important part of the perpendicular magnetic recording signal energy leading to a loss in the system. As a result, the GPR targets for perpendicular magnetic recording channel do not contain the (1 - D) component of longitudinal magnetic recording channel. The target polynomial for perpendicular magnetic recording channel is of the form:

$$S(D) = (1+D)^n$$
 where  $n = 0, 1, 2, \cdots$  (2.29)

The class of target polynomial described in (2.29) are flexible to match a variety of signal shapes. A length 4 or 5 integer-valued target polynomials provide sufficient improvement in performance for the perpendicular magnetic recording channel.

A new method is described which is based on designing the GPR targets that maximises the ratio of minimum squared euclidean distance of the PR target to the noise penalty introduced by the PR filter.

For selection of the best GPR target, there are two specific requirements that need to be fulfilled.

- The matching of the spectrum of the GPR target to the actual readback waveform
- The presence of low-frequency disturbances

#### 2.6.1 Design Ratio for GPR Targets

Figure 2.9(a) shows the block diagram of a perpendicular magnetic recording channel. This model is equivalent to the model shown in Figure 1.6 in Chapter 1.

<sup>&</sup>lt;sup>6</sup>Monic constraint for a GPR target can be defined as the first target sample to be 1 and rest to be zero, leading to zero forcing solution



Figure 2.9: Simulation Model including GPR Target Search

The readback signal r(t) is a convolution of  $a_k$  and p(t) plus some AWGN. It is given as:

$$r(t) = r(t) = \underbrace{\sum_{k} a_k p(t - kT) + n(t)}_{b(t)},$$
(2.30)

where n(t) is the AWGN with mean of 0 and variance of  $\sigma^2$ .

The present technique in perpendicular recording (Kovintavewat et al.; 2002a) for achieving optimised GPR target is optimal only if the receiver has noise prediction. If there is no noise prediction, then the equaliser leads to additional noise. This is as shown in Figure 2.9(b). Considering a PR equaliser without a monic constraint, where  $h(0) \neq 1$ , the new technique described is based on the calculation of ratio of the minimum squared euclidean distance of the PR scheme under this non-monic constraint to the squared noise penalty introduced by the filter. Consider the N tap coefficients  $h(-\frac{N-1}{2}), \dots, \dots, h(\frac{N-1}{2})$ , where N is an odd integer. The equalised signal y is:

$$y = r(t) * h(t)$$
(2.31)  
=  $b(t) * h(t) + n(t) * h(t)$   
=  $b(t) * h(t) + n(t) + n_f(t)$ 

Here,  $n_f(t)$  is the noise penalty from the filter. The analysis of this system for obtaining optimised GPR targets is shown in Figure 2.9. The simulation model is shown in three sections, where the process of calculating the filter noise penalty is shown in Figure 2.9(c). The first part (a) of the simulation model is a typical perpendicular magnetic recording channel. The second part (b) of the simulation model shows the direction of flow from the FIR filter of the PR equaliser and also addition of AWGN. The final part (c) of the simulation model shows how the filter noise penalty is calculated by using the flow of data through the FIR filter of the PR equaliser.

The rule of optimisation is to find the GPR target that maximises the ratio of minimum squared euclidean distance on the trellis over the noise penalty. The noise penalty is the squared sum of all the filter coefficients, except for h(0). Thus, the effective design ratio is:

$$Design Ratio = \frac{Minimum Squared Euclidean Distance}{Filter Noise Penalty}$$
(2.32)

where, the filter noise penalty is computed as,

Filter Noise Penalty = 
$$\sum_{\forall j, j \neq 0} h(j)^2$$
 (2.33)

The optimised search looks for the PR target that maximises this Design Ratio.

#### 2.6.2 Results

The definition of integer based GPR targets for perpendicular magnetic recording is different than the conventional longitudinal magnetic recording channel. In case of longitudinal magnetic recording channel, the GPR targets are non-integer PR targets, while in perpendicular magnetic recording channel, although the same PR class exists, there is specific class of PR targets and thus, the GPR targets can be integer (un-normalised) or non-integer based polynomials. The table 2.6 provides a list of un-normalised integer based optimum GPR targets at various recording densities.

Figure 2.10 shows the comparison of the performance of the new designed GPR target polynomial for perpendicular magnetic recording channel with the GPR targets previously published in (Madden et al.; 2004) (Kovintavewat et al.; 2002a) (Tan and Cruz; 2005). It is observed that the new integer GPR target has an improvement of about 0.2 dB for a BER of  $10^{-5}$ . For comparison purposes, the definition of density in (Kovintavewat et al.;

| $PW_{50}$ | Target        | Design Ratio gain (dB) |
|-----------|---------------|------------------------|
| 1.1       | [1,4,8,3]     | 26.605                 |
| 1.2       | [1,5,9,4]     | 22.859                 |
| 1.3       | [1,5,9,4]     | 20.157                 |
| 1.4       | [4, 10, 7, 2] | 16.237                 |
| 1.5       | [1,5,8,4]     | 13.749                 |
| 1.6       | [1,4,6,3]     | 12.374                 |
| 1.7       | [1,4,6,3]     | 10.552                 |
| 1.8       | [2,7,9,4]     | 8.721                  |
| 1.9       | [2,7,9,4]     | 6.695                  |
| 2.0       | [2,6,7,3]     | 4.767                  |

Chapter 2. Equalisation Techniques and Detection Schemes

Table 2.6: GPR Target Results for Perpendicular Magnetic Recording

2002a) provides a density of  $PW_{50} = 2.5$ , which is equivalent to  $PW_{50} = 1.4$  according to the density described in (1.8). It is observed that the GPR target for a density of  $PW_{50} = 1.4$  found using the design ratio [4, 10, 7, 2] has a gain of about 0.5 dB over a length-5 GPR target discussed in (Kovintavewat et al.; 2002a). The new target, [4, 10, 7, 2] target is about 0.1 dB worse in comparison to the target [1, 1.72, 1.15, 0.33] discussed in (Tan and Cruz; 2005) because it is an integer target, while the one in (Tan and Cruz; 2005) is a non-integer target of same length.

The GPR targets obtained using this new method for GPR search which are shown in Table 2.6 are the same in performance with traditional method of GPR search for most  $PW_{50}$ . This new method provides consistently equal or better targets for PRML schemes that does not include noise prediction. The same design method can be applied to longitudinal magnetic recording but it needs further investigation.

#### 2.7 Soft-Feedback Equalisation

In the PRML approach for magnetic recording, at the receiver end, there is PR equalisation and a maximum likelihood detector. The filter in the PR equaliser shortens the impulse response of the underlying channel, eventually transforming the channel response into a generalised PR target. There is noise colouring and noise enhancement penalty in this process which grows as  $PW_{50}$  changes. This is because the dependence of noise colouration is non-linear (Ahmed et al.; 2005). The equaliser which is optimal for lower density readback signals, becomes sub-optimal for higher density readback signals, which contain more non-linear distortion. At higher densities, the equaliser needs to force the readback signal more to equalise depending upon the PR target. As a result, the correlation properties of the noise are affected resulting in the colouration of noise. This also affects the performance of the ML detector assuming there is no colouration of noise.

Soft Feedback Equalisation (SFE) is a technique which is based on linear filters with



Figure 2.10: BER vs SNR Performance Comparison of GPR Targets for Perpendicular Magnetic Recording

different algorithms for removing ISI in the feedback of the decoding block. Hence, the complexity of the SFE system grows linearly with the length of these filters (Chesnutt et al.; 2005). The SFE scheme is based upon the decision feedback equalisation principles. A decision feedback equaliser (DFE) is a non-linear equaliser that uses previous detector decisions to eliminate the ISI on the pulses that are currently being demodulated. The ISI being removed was caused by the tails of previous pulses; in effect, the distortion on a current pulse that was caused by previous pulses is subtracted. The SFE is a completely different scheme in which inter symbol interference (ISI) is removed using a feedback error convergence loop. The ISI introduced in the magnetic channel is not used in any way, but instead is cancelled out and as a result, there is no noise colouring or enhancement. Thus, the channel remains in its natural form without any alteration in the correlation properties of noise. As a result, different ISI and noise removing algorithms can be applied in the feedback loop of the SFE system.

In the SFE based magnetic recording scheme, the PR-Equaliser is replaced by a linear filter which estimates the ISI, and the MAP/MLSD detector is replaced by a iterative MLSD/MAP/Threshold/LMS detector. The novelty of the SFE based scheme is that it uses the MLSD/MAP/Threshold/LMS detector iteratively in order to obtain the best performance. The number of iterations in the decoder block are decided based upon the bit error rate performance. The non-linearity of the SFE block stems from the non-linear characteristic of the detector that provides an input to the feedback filter. A basic difference in equalisation models of PRML and SFE based magnetic recording channels is as shown in Figure 2.11.



Figure 2.11: Basic Comparison of PRML and SFE based Magnetic Recording Models

The channel is considered to be a dicode channel in SFE based magnetic recording. The convergence criteria is summarised as follows: Initially the estimates of the noisy data are passed through a linear filter and error is calculated from the original readback data and the estimates. This error is then fed into a loop which uses a different error minimising algorithm in order to locate the transitions. The error calculated eventually tries to estimate and remove the ISI from the channel as it goes through the iteration of the feedback loop. The convergence of error depends upon the amount of ISI and AWGN introduced in the channel. The convergence equations are obtained using MMSE criterion (Proakis; 1995). As the recording density increases, the amount of ISI introduced by the Lorentzian filter increases. Depending upon both the factors, the number of iterations required to achieve the performance of the system is set. A block diagram of a soft-feedback based magnetic recording channel model is as shown below in Figure 2.12.

From Figure 1.7, the equaliser and detection blocks have been combined together and merged into one SFE decoding block. The modifications required to simulate a SFE magnetic recording channel as shown in Figure 2.12 from the standard simulation model are:

- 1. Addition of a zero-hold switch
- 2. Thresholding device for initial iteration
- 3. Merging of the equaliser and decoding block into one SFE decoding block
- 4. Addition of a feedback path and error calculating block

The performance of SFE scheme with various signal processing algorithms was observed. Below is the list of schemes which were investigated:



Figure 2.12: Soft-Feedback Magnetic Recording Channel Model

1. SFE with Thresholding Feedback

2. SFE with LMS feedback

3. SFE with MLSD feedback

4. SFE with MAP feedback

From the above mentioned schemes, the SFE-MAP scheme has the best bit error rate performance. Calculation of the LMS algorithm for SFE with LMS feedback based magnetic recording has been discussed in Appendix C. SFE with MLSD feedback and SFE with thresholding feedback are very similar to SFE with MAP feedback, except for changing the type of decoder. As a result, their discussion is ignored and only SFE with MAP feedback based magnetic recording has been analysed and discussed in depth. Further results of other SFE schemes have been discussed in Chapter 3. The only problem with the SFE scheme is the residual ISI. In the simulation models of SFE with different feedback algorithms, ISI is treated as ISI noise i(t).

Figure 2.13 shows the performance of SFE with MAP feedback in terms of BER versus SNR at different densities. It is seen that the SFE with MAP feedback performs well with lower densities. As the densities increase, the amount of ISI introduced in the channel increases tremendously. This is shown in Figure 2.14. The SFE scheme with MAP feedback fails at higher densities because the thresholding device fails to operate efficiently because of higher amount of ISI. As a result, the overall SFE decoding process fails. In order to achieve better performance at higher densities, the initial estimate made by the thresholding device needs to be as accurate as possible.



Figure 2.13: BER vs SNR Performance for SFE with MAP Feedback



Figure 2.14: BER vs Density Performance for SFE with MAP Feedback

#### 2.8 Summary

In this chapter, various equalisation and detection techniques currently used in longitudinal/perpendicular magnetic recording have been discussed. PR equalisation techniques and MLSD/MAP detection algorithms have been studied and discussed. This chapter presents the introduction to the novel soft-feedback equalisation technique for longitudinal magnetic recording. It also discusses the new Asymmetric MAP decoder for perpendicular magnetic recording. Various results have been presented for the new optimisation technique for GPR targets for perpendicular magnetic recording and it is shown that this new developed technique provides better GPR targets in comparison to the previously published GPR target optimisation techniques.

Below is a summary of key results and contributions discussed in this chapter.

- It is seen from Figure 2.4 that generalised PR targets which are non-integer have better performance in comparison to the standard class of PR targets which observe the  $(1-D)(1+D)^n$  generator polynomial. Thus, for lower and higher density magnetic recording, short length or long length GPR targets can be designed which outperforms the standard PR4, EPR4 and EEPR4 target performance.
- For perpendicular magnetic recording with jitter and AWGN noise, it it seen that the asymmetric MAP decoder has better performance in comparison to the standard MAP decoder. The design of asymmetric MAP decoder only has 1 extra addition and 1 extra multiplication compared to standard addition and multiplications required for a standard MAP decoder.
- A new method for GPR target search for perpendicular magnetic recording is discussed and it is observed that it provides consistently better integer targets in comparison to the previously published GPR targets for perpendicular magnetic recording. Results show an improvement of 0.5 dB over GPR targets found using traditional method for GPR target search.
- A new scheme for equalisation in magnetic recording called the *soft feedback equalisa*tion scheme is discussed which does not colour the added noise in magnetic recording channels and thus, keeps the channel in it's original form.
- It is observed that the SFE scheme with MAP feedback has best performance in comparison to other SFE schemes, although it does not outperform the binary PRML scheme. This is because of residual ISI, which shall be addressed in the later chapters of this thesis.

## Part III

# **Multi-level Magnetic Recording**

# Multi-level Magnetic Recording for Different Equalisation Schemes

## 3.1 Introduction

The traditional longitudinal saturation magnetic recording method where, the medium is magnetically saturated in of the two opposite directions at any given position has already been exploited to its maximum limit where super-paramagnetic\* effect comes into place. It has been suggested that the capacity of a digital magnetic recorder can be increased beyond that normally obtainable, by using linear multi-level encoding, rather than the normal saturated two-level system (Mackintosh and Jorgensen; 1981). Chapter 2 introduced PR and SFE based magnetic recording. This chapter discusses the novel work of multi-level encoding for longitudinal magnetic recording with PR and SFE based equalisation techniques and presents various results.

The whole idea behind multilevel recording is to enable storing of more information bits per transition on the magnetic medium. It has already been shown that ternary magnetic recording channel appears to be feasible for increasing the information density of the magnetic recording channel (Krueger et al.; 1995). It is known that at higher code rates for AWGN channels, binary codes tend to deviate very quickly from their theoretical capacity (Proakis; 1995). This is shown in Figure 3.1. Multilevel techniques operate closer to the channel capacity than binary for a bandwidth limited channel experiencing AWGN at increased SNR (G.D. Forney and G.Ungerboeck; 1998).

Assume that a pulse is of duration T and its bandwidth is W. Thus, the baseband to the first null bandwidth W is 1/T. Let R be the bit-rate/data-rate, and the number of bits per symbol be k. Multilevel encoding of data gives its M set of symbols, a k-bit meaning, where  $M = 2^k$ . Thus, bandwidth efficiency can be measured as a ratio of the bit/data rate to bandwidth (Proakis; 1995).

$$R/W = \log_2 M. \tag{3.1}$$

It is seen from Figure 3.1 that as M is increased, the bit rate -to-bandwidth ratio R/W is increased. However, this increase comes from increase in SNR per bit. The advantage of magnetic recording system is that it works at a higher SNR, as in case of tape drives,

<sup>\*</sup>Super-paramagnetism sets a limit on the storage density of a medium, due to the minimum size of particles that can be used. It is often referred to as "data-density barrier".

Chapter 3. Multi-level Magnetic Recording for Different Equalisation Schemes



Figure 3.1: Bandwidth Efficiency Curve for Binary and Multilevel Signals (Proakis; 1995)

it works in the region of 19 - 22 dB. Thus, multi-level encoding, which shifts the SNR requirement depending upon the number of levels and error-correction codes can be applied to achieve more bandwidth efficiency, i.e. more *information bits* can be stored in the transitions on the magnetic media.

Using multi-level encoding, it is possible to achieve several equally spaced magnetisation levels which could be used to store more information bits on the medium. In case of 4 levels, the levels could be equally spaced to -M/3, -M, +M/3, and +M. As the number of levels of recording increase, the complexity of the ML decoder also increases. As a result, only 4 levels of recording has been discussed in this thesis.

Multi-level encoding of data in magnetic recording is similar to pulse amplitude modulation (PAM). Assuming that the data rate of input data is R bits/sec, the amplitudes of the input data can be defined as:

$$X_m = (2m - 1 - M)d$$
, where  $m = 1, 2, 3, \cdots, \cdots, M$  (3.2)

where,  $\{X_m, 1 \le m \le M\}$  denotes the set of M possible amplitudes corresponding to  $M = 2^k$  possible k-bit blocks of symbols. 2d is the distance between the adjacent signal amplitudes. The symbol rate for a PAM signal can be given as R/k, where k is the number of bits taken together to map them to another signal amplitude. In case of 4-level magnetic recording, 2 data bits are taken and mapped into a single recording bit. Table (3.1) shows the equally spaced levels achieved by mapping 2 and 3 bits together to achieve 4 and 8 levels of recording respectively.

| 3 mapped bits | 8 equally spaced levels | 2 mapped bits | 4 equally spaced levels               |
|---------------|-------------------------|---------------|---------------------------------------|
| 0.0.0         | 0                       | 0.0           | n                                     |
| $0 \ 0 \ 1$   | 1/7                     | 0 1           | 1/3                                   |
| 0 1 0         | 2/7                     | 1 0           | 2/3                                   |
| 0 1 1         | 3/7                     | 11            | 3/3                                   |
| 1 0 0         | 4/7                     |               |                                       |
| $1 \ 0 \ 1$   | 5/7                     | ~             | × .                                   |
| 1 1 0         | 6/7                     |               | ÷                                     |
| 1 1 1         | 1                       | R             | · · · · · · · · · · · · · · · · · · · |

Table 3.1: Mapping of 2 and 3 bits for respective 4 and 8 equally spaced levels

The above mapping of bits is done using finite fields called *Galois Field* and denoted as GF(q). These finite fields contain a finite number of elements. The finite field can only be constructed if q is a prime or a power of prime number.

Multilevel Recording shows that the required SNR for low error rate is relatively higher for multilevel recording (Sklar; 2001). An increase in the number of output levels in contrast to binary levels inherent in higher order systems could provide added flexibility in designing better codes for magnetic recording (Ungerboeck; 1982). The rate at which transitions would occur in multilevel magnetic recording would be much higher because the amplitude of the readback signals would be much smaller in comparison to the binary recording scheme. If the transition rate could be kept the same as binary using signal processing schemes, an improvement in packing density with a factor of 2 for four levels and a factor of 3 for eight levels recording could be achieved (Mackintosh and Jorgensen; 1981). As seen from Figure 3.1, 4-PAM modulation method, which is 4-level magnetic recording, is closer to the channel capacity limit C/W in comparison to 2-PAM, which is binary magnetic recording.

The next sections discuss the application of multi-level encoding to PR and SFE based longitudinal magnetic recording channels.
# 3.2 PRML Approach with Multi-level Encoding

Based on the previous work on PRML discussed in Chapter 1, this section introduces the novel multi-level magnetic recording technique using PR equalisation. The simulation model of a multi-level PRML magnetic recording is similar to the standard PRML magnetic recording model with an addition of  $GF(2^m)$  encoder and decoder blocks. The standard PRML magnetic recording model is as shown in Figure 2.3. Figure 3.2 shows a multi-level approach of a basic simulated PRML magnetic recording model. This simulation model is based on the general simulation model discussed in Chapter 1, in Figure 1.7. In this figure, the difference from the general simulation model is that the transition response is shown as a convolution of the Lorentzian pulse and the equaliser block is expanded into a Lorentzian to Sinc equaliser and a PR filter.

As shown in Figure 3.2,  $\{x_k\} \in \{0, 1\}$  is the user data. The mapping output  $\{d_k\}$  depends upon the number of mapping levels used based on  $GF(2^m)$ . Here, m is the number of bits mapped together. For 4-levels,  $\{d_k\} \in \{0, 0.33, 0.66, 1\}$  and for 2-levels,  $\{d_k\} \in \{0, 1\}$ .

The transition sequence can be expressed as below:

$$a_k = d_k \cdot (1 - D).$$
 (3.3)

When  $\{a_k\}$  is passed through the Lorentzian filter whose impulse response is described in (1.4), the multi-level readback pulse r(t) is obtained. The readback pulse b(t) obtained after adding the AWGN given in (2.12) is equalised using the best PR or GPR target. The equalised data given by  $\{q_k\}$  is decoded using a ML decoder. Finally, the decoded data  $\{\tilde{d}_k\}$  is soft de-mapped depending upon the  $GF(2^m)$  mapping. The points of error rate calculations are as shown in the simulation model block diagram.



Figure 3.2: Simulated Multilevel PRML Magnetic Recording System

A 4-level dicode trellis is as shown in Figure 3.3 with all the input/output labels and state transitions.



3.2. PRML Approach with Multi-level Encoding

Figure 3.3: 4-level PR4 Channel Trellis

The trellis structure for the PR-MAP multi-level system is based on the number of levels used and also on the length of the GPR targets. For a 4-level PR-MAP system with GPR target of length 3, each segment of the trellis has 64 paths and the maximum number of states is 16, while for a binary PR-MAP system with GPR target of length 3, each segment of the trellis has 16 paths and the maximum number of states is 8. The targets shown below for longitudinal magnetic recording channel have been designed using the MMSE solution for searching optimal GPR targets. The MMSE criterion is discussed in Appendix B. Table (3.2) shows the GPR targets for binary and 4 level longitudinal magnetic recording. These targets have been used for various simulations which are discussed later. Chapter 3. Multi-level Magnetic Recording for Different Equalisation Schemes

| Binary $PW_{50} = 1.5$           | 4-level $PW_{50} = 0.75$ |
|----------------------------------|--------------------------|
| (1 - D)(1 + 0.5D)                | (1-D)(1+0.2D)            |
| (1-D)(0.9+0.5D)                  | (1-D)(1+0.3D)            |
| (1-D)(0.8+0.4D)                  | (1-D)(0.9+0.1D)          |
| $(1-D)(0.1+0.35D+0.1D^2+0.3D^3)$ | A sugar sugar b          |

Table 3.2: GPR Target Results for Longitudinal Magnetic Recording using old scheme

### 3.2.1 Results

The results for BER calculations have been obtained using the simulation model shown in Figure 3.2. The simulation parameters used for getting the bit error rate versus SNR performance are as follows shown in table (3.3)

| Levels             | Two     | Four             | Eight                                |
|--------------------|---------|------------------|--------------------------------------|
| SNR Range (dB)     | 10 - 18 | 10 - 25          | 10 - 35                              |
| $PW_{50}$          | 1.5     | 0.75             | 0.375                                |
| Mapping            | [0, 1]  | [0, 1/3, 2/3, 1] | [0, 1/7, 2/7, 3/7, 4/7, 5/7, 6/7, 1] |
| Data/Sector Length | 4096    | 4096             | 4096                                 |

Table 3.3: Simulation Parameters for PRML Based Magnetic Recording

Based on the simulation model, an exhaustive search for the suitable GPR targets of maximum length 3 was carried out in order to obtain the desired BER vs SNR performance for the multilevel magnetic channel. The mapping scheme used for four and eight levels is as shown in the table (3.1). Results shown in Figure 3.4 show the dependence of BER on SNR and GPR-target length for a binary, 4-level and 8-level magnetic channel. The results show a comparison of multilevel performance in terms of BER versus SNR.

It is seen that the GPR target channel performs better than the PR4 channel in the lower SNR region, but as the SNR increases, and reaches to 19 dB, performance of the PR4 channel comes closer to the GPR target performance. GPR targets are used for multi-level recording channels, since it is seen in case of binary, the performance of GPR targets is better at lower density of  $PW_{50} = 1.5$ . For a binary level magnetic channel, the desired base error rate of  $10^{-6}$  was achieved at 18 dB SNR. For the best GPR target for 4-levels and 8-levels recording, the desired base error rate of  $10^{-6}$  was achieved base error rate of  $10^{-6}$  was achieved at 24 dB and 38 dB respectively. The point for 8-levels is not shown in the figure. Hence, there is a loss of 6 dB for the 4-level and 18 dB for the 8-level recording channel considering the best GPR target.

Performance of BER versus  $PW_{50}$  for four levels at SNR = 27 dB is as shown in Figure 3.5. The performance of this system in the desired range of  $PW_{50} = 1$  to 1.5. The error base rate achieved in this range varies from  $10^{-3}$  to  $10^{-5}$ . As the  $PW_{50}$  goes from 0.45 to 0.1 (not shown), the performance starts to degrade. This can be seen for the EPR4 channel, that as the  $PW_{50}$  goes below 1.3, the BER versus  $PW_{50}$  curve starts to bend upwards.



Figure 3.4: BER vs SNR Performance for 2, 4, and 8 level PRML



Figure 3.5: BER vs PW<sub>50</sub> Performance for 4-level PRML at SNR=27 dB



Figure 3.6: BER vs SNR Performance for various 4-level PRML Channels

Figure 3.6 shows the BER vs SNR performance of various PR targets for 4-level magnetic recording. It is seen that for a lower density of  $PW_{50} = 1.2$ , the required length of the PR target reduces for 4-level magnetic recording. It is observed that the PR4 target performs better in comparison to the EPR4, EEPR4 and length-4 GPR targets. The amount of SNR required is higher in comparison to the 2-level (binary) magnetic recording is higher. This was observed at a  $PW_{50}$  of 1.2. As the  $PW_{50}$  changes, the performance of 4-level magnetic recording channel also changes. This is as shown in Figure 3.5.

Figure 3.7 shows the effect of noise colouration introduced by the PR equaliser on the performance of the PRML channel. If AWGN is added after PR equalisation is performed, then there is no noise colouration and the noise is still considered to be AWGN. Thus, the noise is not coloured with the effect of PR equaliser. But, AWGN comes from the read head and pre-amplifier electronics, thus needs to be treated before it is equalised causing noise colouration. This leads to loss in performance. From the BER vs SNR performance, it is observed that for a coloured noise channel (magnetic recording channel), the performance is about 0.6 dB worse than the un-coloured noise channel for a BER of  $10^{-5}$ .

The loss in performance due to noise colouration can also be calculated as the filter



Figure 3.7: Noise Colouration Effect on BER Performance for 4-level PRML Scheme

noise penalty discussed in 2. It is computed as:

Filter Noise Penalty = 
$$\sum_{\forall j, j \neq 0} h(j)^2$$
, (3.4)

where,  $h_j \ j \neq 0$ , are the coefficients of the filter in PR equaliser. It was observed that the calculation of filter noise penalty and the difference in performance were in the same region of about 0.6 dB.

Thus, in order to remove the noise colouration effects for multi-level magnetic recording, SFE based multi-level recording is chosen which is discussed in the next section.

### 3.3 SFE Approach with Multi-level Encoding

Chapter 2 introduced the novel SFE scheme with MAP feedback for longitudinal magnetic recording. This section discusses the application of multi-level encoding for various SFE based magnetic recording channels. This section presents results based on various SFE based magnetic recording channels and compares them. Multi-level SFE based magnetic recording is similar to the PR based multi-level magnetic recording, with a variation in their equalisation and detection techniques. The previous chapter discussed SFE based magnetic recording and showed that soft MAP output feedback outperforms the previous SFE schemes with:

- 1. Thresholding feedback
- 2. LMS feedback and
- 3. MLSD feedback

The SFE scheme with MAP feedback provides a better estimate of ISI in comparison to the other schemes, and thus, provides better performance in terms of BER.

This scheme is similar to the SFE scheme with MLSD feedback, but the only difference is in the way the readback data is treated. In the SFE scheme with MAP feedback, the readback data is decoded bit-wise. In this scheme, the front end PR equaliser of the receiver in PRML is replaced by a linear filter which estimates the amount of ISI introduced by the Lorentzian filter in the magnetic channel. This linear filter is a modified Lorentzian filter. The design of the SFE block is independent of the number of levels used for recording and the overall process of SFE is non-linear.

A dicode multi-level recording channel is simulated using SFE with MAP decoder in the feedback loop. The block diagram is as shown in Figure 3.8. Figure 3.8 is similar to the block diagram shown in Chapter 2, Figure 2.12. The only variation is the addition of  $GF(2^m)$  mapper and de-mapper. Also, the block diagram shown in Figure 3.8 is similar to the general simulation model discussed in Chapter 1. The parameters of this simulation model are discussed below.

As shown in the model, the channel is a multilevel dicode SFE-MAP channel. The random data generator generates  $\{x_k\}$ , where  $x_k \in \{0, 1\}$ . The  $GF(2^m)$  encoder uses m = 2 for 4-level and m = 3 for 8-level encoding. Thus,  $\{d_k\} \in \{0, 0.33, 0.66, 1\}$  for 4-levels, which are equally spaced. Here, m is the number of bits mapped together. The output of the dicode is  $\{a_k\}$ .  $\{a_k\}$  is no longer binary, instead it has multiple levels, depending upon the levels used in the  $GF(2^m)$  encoder. The values of the Lorentzian samples produced by the Lorentzian filter depends upon the value of  $PW_{50}$ .  $\{a_k\}$  is passed through this filter, resulting in the output r(t) of the lorentzian filter being a convolution of the lorentzian samples h(t) and  $\{a_k\}$ .



Figure 3.8: Simulation Model of SFE-MAP Multi-level Magnetic Recording Scheme

Filtering of  $\{a_k\}$  with the first Lorentzian filter introduces ISI in the channel. This is the point where the ISI noise i(t) comes into account. Thus, the noisy channel output has two additional components: ISI noise i(t) and AWGN n(t), where i(t) is a function of  $\{a_k\}$ .

$$b(t) = a_k(t) + i(t) + n(t).$$
(3.5)

The noisy channel output b(t) is then thresholded, using a thresholding device and the output of the thresholding device is used as an initial estimate of the dicode output given as  $\{\hat{a}_k\}$ . This thresholding process is controlled by an open switch which closes only during the 0th iteration. The thresholding is performed depending upon the number of levels used. The thresholding process introduces some quantisation noise, since it quantises the data based upon the number of levels of recording. This noise is very minimal and thus, ignored in the calculations. The thresholded data  $\{\hat{a}_k\}$  consists of correlated noise n(t) and i(t). Thus,

$$\hat{a}_k = \Gamma(a_k(t) + i(t) + n(t)),$$
(3.6)

where

$$\Gamma(f(t)) = \arg\max_{a} \{ \Pr(f(t) = a_k) : a_k \in \operatorname{GF}(2^m) \},$$
(3.7)

where,  $Pr(f(t) = a_k)$  is the a-priori probability of f(t) being  $a_k$ , which can be simply obtained from the probability density function of a Gaussian distributed random variable (Proakis; 1995). Once the estimates are achieved, the estimates go through another lorentzian function filter, where the middle lorentzian sample, i.e. the peak value is set to 0. It is set to 0 in order to estimate the ISI introduced by the first lorentzian filter. This modified Lorentzian function is denoted as

$$w(t) = h(t) - \delta(t), \qquad (3.8)$$

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where  $\delta(t)$  is a unit amplitude impulse at t = 0 and zero elsewhere. The N tap coefficients of the second filter are  $w(-\frac{N-1}{2}), \ldots, w(\frac{N-1}{2})$  with w(0) = 0, where N is a positive odd integer. The output from the second filter  $\hat{r}(t)$  is cancelled from the output b(t), resulting in error output e(t).

This error sequence is passed through the MAP detector and passed through (1 - D)and the estimates are updated as  $\{\hat{a}_k\}$ .

As the MAP output  $\{d_k\}$  enters into the convergence loop, it tries to minimise the effect of ISI completely since it is dependent upon the error event e(t). After the convergence has reached, which is usually after 5 iterations, BER and FER are calculated at the points shown in the block diagram.

The equations explaining the above process are as follows: For the first lorentzian filter, consider  $\{a_k\}$  as its input sequence, N tap coefficients  $h(-\frac{N-1}{2}), \ldots, h(\frac{N-1}{2})$  and output r(t).

$$\dot{r}(t) = \sum_{j=-\frac{N-1}{2}}^{\frac{N-1}{2}} a_k(t-j) \cdot h(j).$$
(3.9)

Expanding the above equation,

$$r(t) = a_k(t) \cdot h(0) + \underbrace{\sum_{\substack{j=-(N-1)/2\\N \neq 0}}^{(N-1)/2} a_k(t-j) \cdot h(j)}_{\text{ISI term = i(t)}}.$$
(3.10)

The second term of (3.10) after the + sign is the ISI term i(t).

For the second lorentzian filter with its highest coefficient value equal to 0, consider the estimates  $\{\hat{a}_k\}$  as its input sequence, N tap coefficients  $w(-\frac{N-1}{2}), \ldots, w(\frac{N-1}{2})$  with w(0) = 0 and output  $\hat{r}(t)$ .

$$\hat{r}(t) = \sum_{j=-\frac{N-1}{2}, N \neq 0}^{\frac{N-1}{2}} \hat{a}_k(t-j) \cdot v(j),$$
(3.11)

where,  $\hat{r}(t)$  is the estimated ISI term.

Error e(t) is

$$e(t) = b(t) - \hat{r}(t).$$
 (3.12)

Thus, from (3.8) and (3.11), the error e(t) is

$$e(t) = a_k(t) \cdot h(0) + n(t)$$

$$= \sum_{\substack{j=-(N-1)/2\\N \neq 0}}^{(N-1)/2} v(j) \cdot [a_k(t-j) - \hat{a}_k(t-j)]$$
(3.13)

The soft-feedback equaliser tries to converge using the minimum mean squared error criterion (Proakis; 1995), given as :

$$\min(|a_k(t) - \hat{a}_k(t)|) \tag{3.14}$$

Numerical analysis of the SFE-MAP based magnetic recording is discussed next along with other SFE based schemes. The author simulated various SFE based magnetic recording channels and observed their performance. The next section provides a detailed analysis of how the SFE-MAP scheme outperforms the other SFE based schemes for longitudinal magnetic recording. The next section also presents a comparison of PR and SFE based multi-level magnetic recording.

# 3.4 Results for SFE Based Magnetic Recording

In order to obtain the performance of a typical SFE based magnetic recording channel, a lot of questions needed answering. Among those questions were:

- 1. How many iterations would be required to obtain the MMSE solution?
- 2. Which SFE based feedback scheme would work better at a certain PW<sub>50</sub>?
- 3. Which feedback scheme would remove the maximum amount of ISI, leaving minute residual ISI?
- 4. Which feedback scheme would provide the best performance in terms of BER?

In order to answer the above questions, the simulations were carried out on a stepby-step basis. The simulations started with the SFE with thresholding feedback and were carried out through the optimum SFE based scheme with MAP feedback. The results below consist of all four types of feedback based SFE schemes.

### 3.4.1 Results for SFE with Thresholding Feedback

By modifying the simulation model shown in Figure 3.8, the simulation model for SFE with thresholding feedback is obtained. The modification is to place a threshold detector with specific levels based upon the number of levels used for magnetic recording in place of the MAP detector. The SFE scheme with thresholding feedback was simulated using the following parameters:

| Levels             | Two     | Four             |
|--------------------|---------|------------------|
| SNR Range (dB)     | 10 - 22 | 10 - 30          |
| $PW_{50}$          | 1.5     | 0.75             |
| Mapping            | [0, 1]  | [0, 1/3, 2/3, 1] |
| Data/Sector Length | 4096    | 4096             |
| Iterations         | 5       | 5                |

Table 3.4: Simulation Parameters for SFE with Threshold Feedback Magnetic Recording

Results obtained using the modified simulation model are shown in Figures 3.9, 3.10, 3.11.

It is seen from the results that the performance of the system using SFE with thresholding is not very good in comparison to the PRML system discussed in the previous section. Even at a high SNR of 22 dB, in case of two levels, the base error rate is still  $10^{-3}$ , where as in case of PRML, this base error rate is achieved at 14 dB. The BER versus SNR curve does not follow the water-fall approximation for two levels. It means that there is a very



Figure 3.9: BER vs SNR Performance for SFE-Thresholding Scheme with 5 iterations

minor change in BER with changing SNR. This is caused by the thresholding device which does not pick up all the ISI and a result, the effect of residual ISI is observed.

It is seen from Figure 3.10 that as the number of iterations in the feedback loop increases, the performance also improves but after 5 iterations, the performance almost remains the same. This is because of the quantisation noise that is introduced by the thresholding device which is used iteratively in the system. As the number of iterations increase, the quantisation from the previous iteration keeps on adding to the next iteration and as a result, once the convergence is reached, there is no improvement in the performance. The noise is carried forward to the next iterations and this causes the problem of residual ISI. This is usually after 5 iterations.

The thresholding device is used once in order to get the initial estimate of  $\hat{a}(t)$ , where as the second time it is used as a detector in the feedback loop. The quantisation noise introduced by the initial thresholding of b(t) stays within the feedback loop and as a result, the performance is not as what is expected. The quantisation noise is introduced because it is not possible to fix the number of quantisation levels accurately.

Hence, for a binary level channel, the thresholding rule is as follows: if r(t) < -0.5then estimated  $\hat{a}(t) = -1.0$  else if  $r(t) \ge -0.5$  and  $r(t) \le 0.5$ , then estimated  $\hat{a}(t) = 0.0$ else estimated  $\hat{a}(t) = 1.0$ . As from the rule, it can be seen that, if there is a slight error, the estimated value could be wrong. As the number of levels increase, the problem of



Figure 3.10: BER vs SNR Performance

Figure 3.11: BER vs PW<sub>50</sub> Performance

quantisation noise increases because the levels of thresholding also increase.

Performance of BER versus  $PW_{50}$  for four levels at SNR=25 dB is shown in Figure 3.11. The performance of this system in the desired range of  $PW_{50}=1$  to 1.5 for four level recording is not very good. The error base rate achieved in this range varies from  $10^{-1}$  to  $10^{-4}$ .

The next section discusses the results obtained using LMS feedback for SFE based magnetic recording.

### 3.4.2 Results of SFE with LMS Feedback

The simulation model shown in Figure 3.8 was modified to obtain a simulation model for SFE with LMS feedback. The modification is to use the LMS algorithm in place of the MAP detector. The SFE-LMS scheme was simulated using the following parameters:

| Levels             | Two    | Four             |
|--------------------|--------|------------------|
| SNR Range (dB)     | 10-18  | 10 - 25          |
| $PW_{50}$          | 1.5    | 0.75             |
| Mapping            | [0, 1] | [0, 1/3, 2/3, 1] |
| Data/Sector Length | 4096   | 4096             |
| Iterations         | 5      | 5                |

Table 3.5: Simulation Parameters for SFE with LMS Feedback Magnetic Recording



The results obtained from the modified simulation model are presented in Figures 3.12, 3.13, 3.14. Based on these results, the performance of SFE with LMS feedback is discussed.

Figure 3.12: BER vs SNR Performance for SFE-LMS Scheme with 5 iterations

It is seen from the results that the performance of the system using SFE with LMS is not very good in comparison to the PRML system. But in comparison to the thresholding scheme, the performance is much better with less iterations and complexity. It is seen from Figure 3.13 that as the number of iterations in the feedback loop increases, the performance also improves.

Despite the thresholding device introduces quantisation noise in the estimates  $\hat{a}(t)$ , the LMS being an adaptive algorithm, tries to remove the quantisation effect, by forcing equalisation and thus reducing errors.

### 3.4.3 Results of SFE with MLSD Feedback

The simulation model shown in Figure 3.8 was modified to obtain a simulation model for SFE with MLSD feedback. The only difference between SFE-MLSD and SFE-MAP based magnetic recording is the way the decoders work , which was discussed in Chapter 2. The SFE-MLSD scheme was simulated using the following parameters:

Figure 3.15 shows the performance of SFE-MLSD based magnetic recording channel with respect to BER versus SNR performance. The performance represents both binary and 4-level magnetic recording with SFE-MLSD.



Figure 3.13: BER vs SNR Performance

Figure 3.14: BER vs PW<sub>50</sub> Performance

| Levels             | Two     | Four             |
|--------------------|---------|------------------|
| SNR Range (dB)     | 10 - 22 | 10 - 25          |
| $PW_{50}$          | 1.5     | 0.75             |
| Mapping            | [0, 1]  | [0, 1/3, 2/3, 1] |
| Data/Sector Length | 4096    | 4096             |
| Iterations         | 5       | 5                |

Table 3.6: Simulation Parameters for SFE with MLSD Feedback Magnetic Recording

Several different simulation techniques were applied and the best were chosen. Amongst those techniques were:

- The thresholding device was completely removed from the system and performance was obtained. The performance was slightly degraded and as a result the thresholding device was kept in the simulations.
- A four-level dicode trellis was decoded using a 2-level scheme, but the performance again degraded, so it was kept as it is.

It is seen from the results shown in Figure 3.17 that the performance of the system using SFE with MLSD is not very good in comparison to the PRML system. But in comparison to the previous SFE schemes, the performance of SFE MLSD is better.



Figure 3.15: BER vs SNR Performance for SFE-MLSD Scheme with 5 iterations

From Figures 3.15 and 3.12, it can be observed that the performance of SFE-LMS almost matches to that of SFE-MLSD based magnetic recording. As seen from the results in Figure 3.15, it is seen that the dicode performance for binary levels is much worse than a PR channel with best GPR targets and is as expected. The probability of error is minimised by MLSD in case of an AWGN. The channel which is simulated is an dicode AWGN channel as the noise remains unaltered.

A comparison of BER versus SNR for different number of iterations is as shown in Figure 3.16. It is seen that after 5 iterations, there is not much improvement. This is because after 5 iterations, there is very nominal change in the estimated values and effectively the errors caused are the floating point errors.

The comparison of BER versus  $PW_{50}$  shown in Figure 3.17 shows that a SNR of 25dB is not enough to work with high densities for 4-levels. As the density increases, gradually the SNR requirement for a decent base error rate increases. It is seen that again in the region of interest, i.e.  $PW_{50}$  in the range of 1-1.5, the base error rate is quite poor. It is in the range of  $10^{-1}$  to  $10^{-4}$ . All the systems have been compared in this Figure 3.17 and it is seen that the PRML and the SFE-ML have the best performance. The curves show that the SFE-ML system has a much better waterfall behaviour and that up to  $PW_{50}$ =0.1, it behaves naturally and there is no specific limitation as seen in the PRML curve. The PRML curve, after a  $PW_{50}$ =0.45 up to  $PW_{50}$ =0.1, starts to degrade. Hence, it can be concluded that as



Figure 3.16: BER vs SNR Performance

Figure 3.17: BER vs PW<sub>50</sub> Performance

the densities are reduced, the performance of SFE-ML over PRML is slightly better.

### 3.4.4 Results of SFE with MAP Feedback

The simulation parameters used for the SFE with MAP feedback based magnetic recording are same the SFE with MLSD feedback, as given in table (3.6). The performance of SFE-MAP and SFE-MLSD schemes are similar which can be seen from Figures 2.13 and 3.15. Thus, the performance explanation is similar to that of SFE-MLSD. The only difference between the results is that the performance of SFE-MAP is about 0.2 dB better than SFE-MLSD. This is because the SFE-MAP is a bit-wise decoding solution, which provides better information of the previous bit information.

Since, the performance of the SFE with thresholding feedback is not very impressive in comparison to the SFE LMS and SFE MLSD, it is not worth looking at depth.

The error floor seen in Figure 3.18 is caused by the amount of residual ISI left and not by AWGN. It is seen from Figure 3.21, that the performance of any SFE schemes is not as good as the GPR targeted PRML scheme. But, when error event analysis is performed, it is found that there is repetition of certain error patterns in PRML scheme, while in SFE-MAP scheme, the error events are completely random. Also, there is no colouration of noise in the SFE scheme, although there is a drawback of residual ISI, which quite significantly reduces the performance of the SFE scheme in comparison to the PRML scheme.



Figure 3.18: BER vs SNR Performance for SFE-MAP Scheme with 5 iterations



Figure 3.19: BER vs SNR Performance



Figure 3.20: BER vs PW<sub>50</sub> Performance



Figure 3.21: BER vs SNR Performance for PR-MAP and SFE-MAP Schemes

In order to overcome the effects of large amount of ISI as well as repetitive/consecutive transitions that cause most error events, it is necessary to use coding. Sequence detection can be enhanced for magnetic recording channels by reducing the set of possible recording sequences in order to increase the minimum distance (Karabed et al.; 1999). A comparison of error performance of SFE-LMS, SFE-MLSD, SFE-MAP and PRML with best GPR target of length 3 for 4-levels and at  $PW_{50} = 0.75$  is shown in Figure 3.21.

The theory of Viterbi detector proves that longer error events have smaller probability to occur in the trellis. The probability of error event typically falls off almost exponentially with the length of the error event, meaning that the shortest error events are the main contribution to the total error rate of the system (Wang and Taratorin; 1999).

### 3.5 Summary

This chapter introduces multi-level magnetic recording using PR and soft feedback equalisation techniques. Various results have been presented for different configuration of the simulation model for multi-level magnetic recording. The novel results provide a background to the results which will be discussed in Chapter 5. A novel idea of using soft feedback equalisation for multi-level magnetic recording and its benefits and drawbacks have also been presented. Below is a summary of key results and contributions discussed in this chapter.

- By application of multi-level encoding, it is found that although, 4-level magnetic recording channel operates closer to the capacity, it requires about 6 dB more SNR compared to binary magnetic recording channel in order to achieve similar BER performance of 10<sup>-5</sup>.
- It has been shown that binary channel does not operate closer to channel capacity, and due to super-paramagnetism effect, there is no other alternative than multi-level magnetic recording in order to improve the data density.
- Although short length PR targets operate better than longer PR targets for 4-level magnetic recording at lower densities, as the density increases, longer targets operate better.
- In order to reduce the amount of SNR required to achieve better performance for multi-level magnetic recording, it is necessary to use error correcting codes like Turbo codes, which is discussed in the later chapters.
- It is shown that the PR equaliser modifies the correlation properties of added noise and that degrades the performance of the PRML based 4-level magnetic recording. Figure 3.7 shows that there is a loss of about 0.7 dB due to the colouration of noise.
- In order to remove the colouration of noise, SFE with MAP feedback is applied to multi-level magnetic recording.
- It is observed that the SFE scheme does not have better performance than PRML for 4-level magnetic recording. In fact, it is 0.7 dB worse than PRML based magnetic recording. But, the problem of worse performance is pin-pointed at the initial estimate done by the thresholding device. As the number of levels of recording and the density increases, the quantisation noise introduced by the thresholding device is more and as a result, the initial estimation made by the device is incorrect.
- In order to overcome the problem of residual ISI and in-efficiency of the thresholding device, there are various schemes developed with the use of Maximum Transition Run (MTR) codes which are discussed in the next chapters.

# Part IV

# **Application of Coding**

# Application of Maximum Transition Run Codes

# 4.1 Introduction

Digital communication systems employ codes to transform the information bits into a form which is suitable for transmission. Digital magnetic recording is no exception in this case. It is a common practise in the data storage systems to add redundancy to control errors and the implementation of various complex modulation codes. Additional sectors are included on the disk that can be used in place of those that are determined to be unusable when the drive is initially formatted.

For a disk drive, it is common to produce one error for every 10 million reads; i.e. a BER of  $10^{-7}$ . ECC is used to correct a large number of such errors and thus reduces the BER to  $10^{-12}$  or less. If ECC cannot detect an uncorrected error, the drive controller re-reads the data and improves the error rate to  $10^{-14}$ . In this case, for a 1 gigabyte hard drive, it needs to be read more than 12,000 times to produce a single error and thus, making the reliability of the drive very high. Reed-Solomon codes are the typical error control codes devised in digital magnetic recording systems (Lin and Costello; 1982) (E. J. Weldon; 1992).

ECC is used to correct errors, while the modulation code tries to prevent the errors by eliminating some repetitive transition patterns. This is done by adding more information to be used in readback. Run length limited (RLL) codes which are a type of modulation code have been used by the magnetic industry for many years. The use of RLL codes was pioneered in late 60's by Franaszek (1969). These codes are known as recording codes. Recording codes have been used with great success in magnetic disk storage to increase linear density and improve performance (Siegel; 1985b).

From the results obtained by using multi-level encoding for PRML and SFE schemes for magnetic recording, it is seen that the uncoded performance of these schemes have a very high SNR requirement to achieve a BER of  $10^{-5}$ . For a binary magnetic recording system with PRML, the required SNR to achieve a BER of  $10^{-5}$  at a high density of  $PW_{50} = 2.4$  for a *PR*4 target is 19 dB. For a 4-level magnetic recording system with PRML, at a density of  $PW_{50} = 1.2$  for the same target, the required SNR is 25 dB in order to achieve a BER of  $10^{-5}$ . Thus, there is an additional 6 dB SNR requirement for the 4-level magnetic recording system.

For a 4-level magnetic recording system with SFE, at a density of  $PW_{50} = 0.75$ , the

required SNR is 23 dB in order to achieve a BER of  $10^{-5}$ . For a binary magnetic recording system with SFE, at a density of  $PW_{50} = 1.5$ , the required SNR is 21 dB in order to achieve a BER of  $10^{-5}$ .

It is known that in order to achieve better bandwidth efficiency, there exists a trade-off between SNR requirement and BER performance.

In order to obtain a good BER performance  $(10^{-5})$  in the operating region of 19-22 dB, it is necessary to use error correcting codes and maximum transition run (MTR) codes. Error correcting codes would target the errors that occur in the decoding process and reduce the amount of error propagation in the PRML and SFE based multi-level magnetic recording systems. While the MTR codes would target certain error events which occur often and would eliminate them.

Error event plots for a 4-level PRML system with the best GPR target at SNR=20dB are shown in Figure 4.1 and Figure 4.2. Similar error event plots for a 4-level SFE-MLSD system with 5-iterations are shown in Figure 4.3 and Figure 4.4.

- In the PRML system, the error events are plotted as the difference between the ML output and the desired output, which in our case is the multi-level data.
- In the SFE-MLSD system as shown in figure 3.8, the error events are plotted as the difference between the estimated data and the actual differentiated data, i.e. a(t) â(t).
- Looking from the error correction point of view, the SFE-ML system is more flexible for application of error correction codes, since the error bursts are more random in nature than the PRML system. This would provide better input to Turbo and Low Density Parity Check (LDPC) codes as they are random error correcting codes.
- The PRML system would require recording codes (also known as constrained codes) in order to reduce the repetitive transition patterns that cause specific error patterns. The application of constrained/recording codes to both the SFE and PRML systems would prove beneficial.
- Application of MTR codes modifies the data in a way that there are not many consecutive transitions in the data pattern. This results in spreading of the data. As a result, the data patterns that cause specific error patterns can be broken and spread out in a way that they do not let the error to occur.

In this chapter, a background to MTR codes is provided and various published MTR codes have been used to investigate the performance of PR/SFE based magnetic recording. A new class of multi-level MTR codes has been introduced which can be used for PR/SFE based multi-level magnetic recording. The next section introduces the use of MTR codes for magnetic recording.



Figure 4.1: Error Event for 4-level PRML for First Frame



Figure 4.2: Error Event Plot for 4-level PRML for Second Frame



Figure 4.3: Error Event Plot for 4-level SFE-MLSD with 5 iterations for First Frame



Figure 4.4: Error Event Plot for 4-level SFE-MLSD with 5 iterations for Second Frame

# 4.2 MTR Codes for Magnetic Recording

Modulation codes are used to add or remove certain characteristics from the recorded waveform in order to improve data recovery. Maximum transition run codes are a type of modulation code that provide significant minimum distance gains when used with sequence detectors operating at high linear densities (Brickner and Moon; 1997). The basic idea of using MTR codes is to eliminate certain bit patterns which occur repetitively at the input so that most certain error patterns can be eliminated from being produced. Specifically, the MTR code eliminates input patterns that contain 3 or more consecutive transitions in the corresponding input waveform. Thus, the performance of near optimal sequence detectors can be substantially improved at high linear densities (Moon and Brickner; 1996).

It is known that in the detection of the readback data in the magnetic recording channel, the errors are produced by bursts of transitions at the channel input (Brickner; 1998). The above only holds true when there is no noise correlation involved in the recording channel. For the magnetic recording channel, it is known that the most likely error events are of the form:  $e_k = \pm \{2, -2, 2 \cdots - 2, 2\}$  (Brickner; 1998). Error events  $e_k$  in magnetic recording occurs due to consecutive transitions or bursts of transitions at the input of the channel which causes the decoding to fail and produce specific error events. The shorter error events are a subset of the longer ones. The Figure 4.5 below shows the pairs of NRZ write current waveforms which generate  $\pm \{2, -2, 2 \cdots - 2, 2\}$  error sequence (Immink; 1989).

In NRZI format of magnetic recording, the above pairs become:  $\cdots 1110 \cdots$ ,  $\cdots 0111 \cdots$ ,  $\cdots 1111 \cdots$  and  $\cdots 0110 \cdots$ .

A code constraint that could provide a restriction to the number of consecutive transitions would yield a minimum distance gain. The probability of the error event occurring at j, where the true signal  $a_k$  is misconstrued for some other signal  $\bar{a}_k = a_k + e_k$  arising from



Figure 4.5: Pairs of NRZ Write Current Generating  $\pm \{2, -2, 2\}$  Error Sequence

other valid path allowed by the detector (Barbosa; 1989) is :

$$P(eventj) = Q(d_j^2/2\sigma_j^2), \tag{4.1}$$

where,  $d_j^2 = \sum e_j^2$  is the Euclidean distance between the real and the incorrectly detected sequence in the detection space.  $\sigma_j^2$  is the variance of the noise projected into the error event subspace.

The pair of constraints, similar to the run length limited (RLL) code constraints has been developed for MTR codes. Run-length is the number of bits for which the signal remains unchanged.

The RLL code has the following constraints : m/n(d, k) (Proakis; 1995). where,

- d is the minimum number of consecutive 0's allowed between two 1's and  $d \ge 0$ .
- k is the maximum number of consecutive 0's allowed between two 1's and  $k \ge 0$ .
- *m* is the number of user bits required to map and *n* is the encoded bits, where  $n \ge m$ .

In RLL coding for disk drives, a change in magnetisation is considered to be a 1, while no change is indicated by a 0. Thus, RLL coding for a disk is considered to be the higher and lower limits on how many 0 there must be between consecutive 1's. In saturation recording, this means that the write current must not reverse the polarity for at least dperiods following a transition and can remain constant for at most k periods following a transition. The d constraint in RLL codes provides a minimum physical distance between transitions and thus, reduces the amount of ISI introduced. The k constraint ensures that a sufficient amount of information is present for timing recovery. Various types of RLL codes include the rate 1/2 modified frequency modulation (MFM) codes used in the early products of digital magnetic storage and they have (1,3) RLL constraints (Siegel and Wolf; 1991). Another example of codes are the (0, G/I) codes, which is a (d, k)=(0, G) code with a new constraint I applied to interleaved sequences. Amongst such codes was the rate 8/9RLL (0, 4/4) code developed by Eggenberger and Patel (1987) which was used commercially. A new set of modulation codes which were designed to work around the channel impairments were the DC free codes. To prevent the distortion in the written waveform, the power spectrum of the code must have a null at zero frequency. Such codes are DC free codes and have been discussed by Patel (1975) and Soljanin (1997).

A traditional (1, k) RLL code eliminates all the eight transition patterns shown in Figure 4.5, but with very high rate penalties and thus, coding gain is very minimal (Moon and Brickner; 1996).

The MTR code has the following constraints : m/n(j;k) where,

- j is the maximum number of transitions permitted at the channel input.
- k is the maximum number of consecutive 0's in NRZI format of recording, and it is the maximum number of consecutive bit periods in which there is no transition.
- *m* is the number of user bits required to map and *n* is the encoded bits, where  $n \ge m$ . This provides the code rate R = m/n for the MTR code.

For the MTR code constraints, k is also considered from the RLL code constraints in order to prevent long periods without a transition, which can hamper the timing recovery schemes. It provides adequate information for the timing recovery algorithm. For MTR j = 1, only isolated transitions can occur, since a run-length consisting of multiple consecutive transitions would violate other constraint. This code corresponds to the d = 1 RLL codes. For d = 0 RLL code,  $j = \infty$ , which means that the MTR code would allow any number of consecutive transitions. Thus, an MTR code with a finite  $j \ge 1$  is a subset of the RLL d = 0 codes.

The capacity of a code is an upper bound on the amount of information (bits) that can be conveyed with each code bit given a particular set of constraints (Shannon; 1948). It is also an upper bound on the code rate R. For a code where there is no code constraint, the capacity of that code is 1, since each code bit represents one information bit. When a code constraint is applied, as in the case of MTR codes, when j is applied, the capacity of the code lies between 0 and 1, i.e.  $0 \le C \le 1$ .

The capacity C of an MTR code with j constraint can be found as a base-2 logarithm of the largest eigenvalue in the adjacent matrix A for the finite state transition diagram (FSTD) describing the code (Brickner; 1998). Thus,

$$C = \log_2 \left[ \lambda_{max}(A) \right], \tag{4.2}$$

where,  $\lambda_{max}(A)$  is the largest real eigenvalue of A. The capacity C calculation is similar to the calculation for RLL codes described in (Siegel; 1985a).

A FSTD is a state diagram with a finite number of states, the transition between which corresponds to the code bits. It is a graphical representation of the code constraints, and is useful in the code capacity calculations.



Figure 4.6: Finite State Transition Diagram for MTR Codes

The Figure 4.6 (a) shows a general FSTD for the MTR j constraint. The labels on the transitions between the two states correspond to NRZI bits. Any sequence can be formed by moving along the arrows is allowed by the code constraint that the diagram represents. Figure 4.6 (b) is an FSTD example for d = j = 1 and  $k = \infty$  MTR code. The adjacent matrix A for capacity calculation contains elements  $a_{i,j}$ , where the element is 1 if an arrow exists from state i to j, and 0 otherwise.

**Example of Code Capacity Calculation:** Consider a j = 2 MTR code with NRZI sequences. The FSTD can be shown in Figure 4.7. Thus, the adjacent matrix for the MTR



Figure 4.7: Finite State Transition Diagram Example for MTR j = 2 Code

code is:

$$A_{j=2} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} .$$
(4.3)

The capacity C is:

$$C = \log_2(1.8393) = 0.8791. \tag{4.4}$$

Calculation of the capacity of a code is very useful in order to evaluate the amount of data that can be stored in the coded bit sequence. It also provides information for designing actual code by providing the maximum achievable code rate.

It is shown in (Moon and Brickner; 1996) that k should be chosen to be  $k \ge 6$  in order to reduce the loss in capacity resulting from the constraint. Also, limiting the number of consecutive transitions to j = 2 is a sufficient condition to yield this gain.

#### 4.2.1 Valid Codeword Selection for NRZI representation

A block code describes a code for which data words of a fixed length m are mapped to codewords with a fixed length n without ambiguity. Each data word exactly maps to one codeword only, and each codeword from the encoder mapping can be decoded to the same data word.

For a block length of n, there are  $2^n$  possible codewords. The set of valid codewords is obtained by removing codewords that violate the (j; k) constraints within the codeword or at the boundary when combined with another codeword, i.e.; a serial concatenation of two or more codewords making a sequence.

The codewords that do not hold the MTR j constraint true are removed. These include all the codewords that contain three or more consecutive transitions (1's). Next, the codewords that begin/end with a 11 are removed because they violate the MTR j = 2 constraint if they are concatenated with a codeword that placed a single one next to the trailing edge of 11. This would provide a set of codewords that observe the sufficient condition for an MTR  $(2; \infty)$  code.

The next step is to apply the RLL k constraint. Any codeword that contains more than k consecutive 0's is eliminated. If  $k \ge n$ , then no codeword can be eliminated. Any codewords that violate the k constraint at the end of one codeword are eliminated. This is achieved by removing the codewords that begin with  $k_1$  0's and/or with  $k_2$  0's, where

$$k_1 + k_2 = k + 1 \tag{4.5}$$

and

$$|k_1 - k_2| \in 0, 1.$$
 (4.6)

In any case, for a finite k, the all 0's codeword is always removed from the codeword list. The remaining codewords can be combined in any order and used in order to obtain the resulting sequences that will satisfy the MTR (2; k) constraints. Commercial products use codes with k values ranging from 4 to 7. Table (4.1) (Moon and Brickner; 1996) below shows the important code parameters for representative block codes. The efficiency was calculated by dividing the code rate m/n by the capacity C for a given value of k and the MTR constraint j.

| m  | n  | k | Efficiency | No. of Codewords Available | No of Codewords Needed |
|----|----|---|------------|----------------------------|------------------------|
| 4  | 5  | 8 | 0.91       | 16                         | 16                     |
| 8  | 10 | 6 | 0.92       | 282                        | 256                    |
| 9  | 11 | 6 | 0.94       | 514                        | 512                    |
| 10 | 12 | 8 | 0.95       | 1066                       | 1024                   |
| 14 | 17 | 6 | 0.95       | 18996                      | 16384                  |
| 16 | 18 | 7 | 0.96       | 69534                      | 65536                  |
| 24 | 28 | 8 | 0.98       | 17650478                   | 16777216               |

Table 4.1: Parameters for MTR Block Codes with j = 2

The above table shows that increasing n allows for better code rates since the boundary conditions affect a very small number of the codewords. It is suggested that the existence of state dependent codes with smaller block sizes is realisable (Moon and Brickner; 1996). Thus, the 10/12 block code can be realised as a 5/6 state dependent code. Various types of MTR codes with different code constraints and encoding/decoding methods such as time varying MTR codes, MTR codes for high density  $E^2$ PRML magnetic recording channel, boolean algebraic encoding/decoding methods and FDTS/DF decoding have been described by Brickner and Moon (1997), Karabed and Siegel (1995) and Moision and Siegel (1998). This work only focuses on the standard rate 4/5 MTR (2:8) and rate 5/6 MTR (2:6) codes.

Table (4.2) shows the realisation for rate 4/5 MTR (2;8) code and table(4.3) shows the realisation for rate 5/6 MTR (2;6) code.

| DATA | CODEWORD | DATA | CODEWORD |
|------|----------|------|----------|
| 0000 | 10000    | 1000 | 01000    |
| 0001 | 00001    | 1001 | 01001    |
| 0010 | 00010    | 1010 | 01010    |
| 0011 | 10001    | 1011 | 10010    |
| 0100 | 00100    | 1100 | 01100    |
| 0101 | 00101    | 1101 | 00101    |
| 0110 | 00110    | 1110 | 10100    |
| 0111 | 10110    | 1111 | 10101    |

Table 4.2: Rate 4/5 MTR Code

| DATA  | CODEWORD | DATA  | CODEWORD |
|-------|----------|-------|----------|
| 00000 | 000000   | 10000 | 010010   |
| 00001 | 000001   | 10001 | 100100   |
| 00010 | 000010   | 10010 | 010100   |
| 00011 | 100000   | 10011 | 010101   |
| 00100 | 000100   | 10100 | 010110   |
| 00101 | 000101   | 10101 | 100101   |
| 00110 | 000110   | 10110 | 011000   |
| 00111 | 100001   | 10111 | 011001   |
| 01000 | 001000   | 11000 | 011010   |
| 01001 | 001001   | 11001 | 100110   |
| 01010 | 001010   | 11010 | 101101   |
| 01011 | 100010   | 11011 | 101100   |
| 01100 | 001100   | 11100 | 101000   |
| 01101 | 001101   | 11101 | 101001   |
| 01110 | 010000   | 11110 | 101010   |
| 01111 | 010001   | 11111 | 110100   |

Chapter 4. Application of Maximum Transition Run Codes

Table 4.3: Rate 5/6 MTR Code

### 4.2.2 MTR Code Design for Multi-level Magnetic Recording

This section introduces a new class of MTR codes for multi-level magnetic recording and presents some preliminary results. It is know that the MTR constraint means restricting the number of consecutive transitions in a sequence of transitions. In multi-level recording, MTR constraints are dependent upon two factors:

- 1. The maximum number of consecutive transitions.
- 2. The step-size of the transitions. The step-size can be defined as the spacing between the levels. Assuming that M being the saturation magnetisation, the 4-levels of recording 00, 01, 10 and 11 are represented as -M, -M/3, +M/3 and +M respectively.

The step-size changes for transitions in 4-level recording are defined as:

- Step-size 1: Step change from transition of 00 to 01 or 01 to 10 or 10 to 11 and viceversa.
- Step-size 2: Step change from transition of 00 to 10 or 01 to 11 and vice versa.
- Step-size 3: Step change from transition of 00 to 11 and vice versa.

A block diagram of an MTR coded PRML/SFE system is as shown in Figure 4.8.

The constraint of maximum number of consecutive transitions was set to 3. Figure 4.9 shows the effect of including MTR constraints in uncoded PR-MAP and uncoded SFE-MAP



Figure 4.8: Block Diagram of MTR coded PR/SFE Multilevel Magnetic Recording System

schemes. The effect of the MTR step-size is shown and smaller step-size in SFE-MAP scheme outperforms PR-MAP scheme.

Preliminary investigation of using MTR codes showed a noticeable gain of 2 dB using SFE-MAP scheme when compared with the PR-MAP scheme. This gain was achieved when the simulations were performed without ECC and a step-size of 1 was used for the MTR constraint. It is seen that the problem of residual ISI in the SFE scheme is overcome by using smaller step-size MTR constraint. Thus, SFE-MAP performs better with step-size 1 MTR constraint than PR-MAP.

A step-size of 2 MTR constraint is also as shown in Figure 4.11. As the step-size increases, the amount of ISI increases and as a result, the MTR constraint of step-size 2 does not have equivalent performance as achieved with the MTR constraint of step-size 1. Thus, in order to improve the performance of the SFE-MLSD/MAP based magnetic recording scheme, it is necessary to use MTR recording codes. The investigation was performed without using an MTR decoder which is shown in the block diagram. Only a MAP decoder was used for initial investigation. The points of bit error calculations were shown in the block diagram.

### 4.2.3 Results of MTR-coded PRML/SFE based Magnetic Recording

Figure 4.11 shows the BER versus  $PW_{50}$  performance of a rate 4/5 MTR (2:8) code over PR-MAP and SFE-MAP channels at SNR of 16 dB. It is observed that as  $PW_{50}$  increases and reaches towards 2.5, the performance of PR-MAP with *EEPR4* target is better in comparison to other PR targets including the optimum GPR target.

The BER versus SNR performance of various PR-MAP channels at  $PW_{50} = 2.4$  is shown in Figure 4.12. From Figure 4.11, it is seen that at  $PW_{50} = 2.4$ , the optimum performance of PR-MAP channel is with *EPR4* and [2/5, 11/20, -11/20, -2/5] GPR targets. This is verified in Figure 4.12 although, after 20 dB, the PR-MAP performance with GPR target observes a flooring effect. It is seen that the *EEPR4* target PR-MAP performance also has a flooring effect after 20 dB.



Figure 4.9: BER vs SNR Performance, MTR constraint with step-size=1



Figure 4.10: BER vs SNR Performance, MTR constraint of step-size=2

It is seen that the SFE dicode channel does not perform well in the region beyond  $PW_{50} = 1.3$ . This limitation is because of the dicode channel performance beyond that density and it is not dependent upon the MTR code design. Figure 4.13 shows the BER versus SNR performance of standard *PR4* and *EPR4* PR-MAP channels and also of the dicode SFE-MAP channel. It is observed that even at  $PW_{50} = 1.5$ , the SFE-MAP dicode channel has poor performance in comparison to the standard PR-MAP channels. Although, in comparison to an SFE-MAP channel without MTR code, it is observed that the SFE-MAP with rate 4/5 MTR (2; 8) code has improvement in performance at  $PW_{50} = 1.5$ .



Figure 4.11: BER vs PW<sub>50</sub> Performance of Binary PR-MAP and SFE-MAP Channels with rate 4/5 MTR(2;8) Code at SNR=16 dB

## 4.3 Summary

This chapter introduces the use of MTR codes for PR/SFE based multi-level magnetic recording. The results presented in this chapter are based upon the previously published MTR codes as well as the new MTR codes for multi-level magnetic recording. Based upon the results presented in this chapter, below are some key results and discussions which summarise this chapter.

 The error event plots show that in order to improve the performance of PRML channels which have consecutive transitional errors, it is necessary to break up the transitions in a manner which also restricts the non-transition regions. As a result, it



Figure 4.12: BER vs SNR Performance of Binary PR-MAP Channels with rate 4/5 MTR(2;8) Code at  $PW_{50} = 2.4$ 



Figure 4.13: BER vs SNR Performance of Binary PR/SFE - MAP Channels with rate 4/5 MTR(2;8) Code at  $PW_{50} = 1.5$ 

is necessary to use MTR codes which spreads the data in a way that specific error patterns leading to decoding errors can be eliminated.

- The difference between the use of MTR codes as recording codes and error correcting codes is clearly explained and the benefits of each of them have been discussed. The pairs of data causing the major error sequences have been shown and the related MTR code design has been used over PRML/SFE channels.
- A procedure for valid codeword selection for MTR codes is illustrated and preliminary investigation on the design of multi-level MTR codes is discussed.
- Results showing variation in BER versus SNR and BER versus  $PW_{50}$  are shown for both PRML and SFE channels and it is seen that PRML channels with the use of rate 4/5 MTR (2:8) code has better performance when compared to SFE channel at higher  $PW_{50}$ .
- The SFE channel does not have significant improvement in comparison to PRML channels, although, the MTR coded SFE channel has improvement of about 1 order of magnitude when compared to uncoded SFE channel performance at  $PW_{50} = 1.5$ . The performance of the SFE channel is restricted by the dicode nature of the magnetic channel and also on the use of thresholding device for initial estimation of ISI.
- Chapter 6 discusses these drawbacks of the SFE channel and provides the optimal solution to it.
## Application of Error Correction Codes

## 5.1 Introduction

Errors are inevitable in digital magnetic recording. They could be caused by noise, inter symbol interference, transition jitter, modulation and media defects. Error correction coding provides a means to either correct or at least detect such errors to the disk drive controller. The error correction code adds redundancy to the data in a systematic way which allows the error patterns to be corrected by the decoder.

The data in the magnetic recording channel is encoded at different stages before it is sent to the read/write head. Initially, it is encoded with an error correcting code (ECC). After that, it is encoded with a modulation/recording code. The modulation/recording codes were discussed in the Chapter 4. A typical magnetic recording channel with different stages of encoding is shown in Figure 5.1.

The first step of encoding user bits is applying error correction encoding (Mallinson; 1993). The ECC introduces additional redundancy bits into the user bits in a systematic manner. This produces a combined block of redundant and user bits which is known as a codeword. In the ECC encoding process, a block of k user bits are taken at a time and are mapped to a unique n bit sequence, producing an n-length codeword. The amount of redundancy is measured as a ratio of n/k and its reciprocal k/n is called the *code-rate*. The ECC encoding is followed by channel encoding which means modulation/recording codes. Modulation/Recording codes have been previously discussed in Chapter 4.

There are two types of error that could occur in magnetic recording: single-bit errors and burst of errors (Wang and Taratorin; 1999). Single-bit errors occur due to a specific short-duration noise event, resulting in a shifted or missing pulse. Burst errors occur when a group of bits is incorrectly detected or due to media defects. The ML decoder used in the PRML channel can also cause burst errors, since the detector's output is based on a group of bits. Any error event in the ML detector will result in a wrong sequence of bits, thus, producing burst of errors.

The use of ECC in magnetic recording boosts the reliability of recovering/reading the data. Similar to channel coding, ECC requires additional redundancy bits introduced in the stream of user bits, thus requiring more storage space on the magnetic medium. Once it is ascertained that ECC can correct certain number of errors, then the recording densities



Figure 5.1: Stages of Encoding in Magnetic Recording

could be increased until reaching a required BER. This can be explained with the following example.

#### **Example on Calculation of Coding Gain using ECC:**

- Consider a single-bit error correcting code which requires 8 bits of information for each 16-block code. Block codes have been discussed later in this chapter. Thus, it requires 50% more space on the medium to write the data. An error will only occur if 2 or more bits out of 16 are in error.
- If the probability of error for a single-bit error is  $P_c$ , then if there are 2-bits in error out of 16, the probability of error equals to  $P_c^2$ . There are  $\binom{16}{2} = 120$  possible combinations of 2-bit error events in a 16-bit block code.
- Thus, the probability of error using ECC is equivalent to:

$$P_{ecc} = 120 \cdot P_e^2 (1 - P_e)^{14}$$

 If the probability of error using ECC is fixed at 10<sup>-9</sup>, then the probability of single-bit error is P<sub>e</sub> = 3 × 10<sup>-6</sup>.

In order to achieve a BER of  $10^{-9}$  in an uncoded channel, an SNR of about 22 dB is required. To achieve  $3 \times 10^{-6}$  BER requires 19 dB SNR, thus, providing a gain of 3 dB. The reduction in SNR dB required to achieve a specified error performance due to ECC is called the Coding Gain. This assumes that the errors are random and not bursty in nature.

The coding gain in SNR under same target error performance allows additional user density (Watkinson; 1990).

Parts of this chapter appear in the Conference and Journal Proceedings under the papers titled:

- P. Shah, M. Z. Ahmed, M.A. Ambroze, C. J. Tjhai and P. J. Davey "Novel Multilevel Magnetic Recording using Modern Error Correction", IMST-EPCOS Conference, Grenoble, France, May 2006.
- P. Shah, M. Z. Ahmed, M.A. Ambroze, C. J. Tjhai and P. J. Davey "Novel Soft-Feedback Equalisation Technique for Multi-level Magnetic Recording", 10<sup>th</sup> Joint Intermag/MMM Conference, Baltimore, USA, January 2007.
- The extension of the paper titled: "Novel Soft-Feedback Equalisation Technique for Multi-level Magnetic Recording" by authors P. Shah, M. Z. Ahmed, M. A. Ambroze, C. J. Tjhai and P. J. Davey has been published in IEEE Transactions on Magnetics, volume 43, no. 6, June 2007, pages 2280-2282.

Different types of error correcting codes are introduced in this chapter. In this chapter, the investigation on application of Turbo codes to magnetic recording is depicted using various simulation models designed by the author. Turbo codes with different interleavers have been used and their performance over PR/SFE based multi-level magnetic recording is presented. This chapter also justifies the novel use of multi-level magnetic recording over binary magnetic recording, which is discussed in the later sections.

#### 5.2 Introduction to Error Correction Coding

Shannon (1948) showed that the capacity of a channel C of bandwidth B, perturbed by AWGN is given by:

$$C = B \log_2(1 + \frac{S}{N}) \quad \text{bits/second}, \tag{5.1}$$

where, S is the average received signal power and N is the average noise power. The above equation assumes that the block length of codes is infinite and the code is defined in an arbitrary field. This fundamental equation represents the *Shannon Limit*, which is confined between the *sphere packing bound* and the *random coding bound*. These two bounds are a lower and an upper bound on the block error rate, given the channel characteristics and the block length n. Shannon's limit represents the channel capacity C in terms of bit rate and is dependent upon the statistical model of the channel.

It is shown in (Shannon; 1948) that the limiting factors of reliable communications are the signal power S and channel bandwidth B. The above limiting factors compensate for one another, meaning that, there exists a trade-off between the two in order to achieve reliable communication. Increasing the signal power S can lead to reduced bandwidth requirements B, while reducing the signal power requires an increase in the bandwidth requirement. One can achieve the same capacity by either of the above mentioned methods. There is a limit on the amount of increasing signal power S. Increasing the channel bandwidth B also leads to further problems. This is because the average noise power is proportional to the bandwidth (Sklar; 2001).

$$N = \frac{N_o}{2} \cdot (2B) = N_o B.$$
 (5.2)

 $N_o/2$  is the two-sided noise power-spectral density of the AWGN channel. Thus, applying the above to (5.1),

$$C = B \log_2(1 + \frac{S}{N_0 B}) \text{bits/seconds.}$$
(5.3)

This work of Shannon (1948) showed that reliable means of communication can be achieved using error correction coding, which is defined as channel coding in communication systems. In this work, channel coding means modulation/recording codes.

Considering S is the average received power and we are transmitting information over a B Hz bandwidth channel with channel capacity C,

$$S = E_b C, \tag{5.4}$$

where,  $E_b$  is the energy per bit. Thus, the (5.3) becomes,

$$\frac{C}{B} = \log_2 \left( 1 + \frac{E_b}{N_o} \frac{C}{B} \right).$$
(5.5)

Thus,

$$\frac{E_b}{N_o} = \frac{2^{C/B} - 1}{C/B}.$$
(5.6)

When C/B = 1,  $E_b/N_o = 1.0$  dB. When  $C/B \rightarrow \infty$ ,

$$\frac{E_b}{N_o} = \frac{2^{C/B}}{C/B}.$$
(5.7)

Thus,  $E_b/N_o$  increases exponentially as  $C/B \to \infty$ . On the contrary, when C/B = 0,

$$\frac{E_b}{N_o} = \lim_{C/B=0} \frac{2^{C/B} - 1}{C/B} = \ln 2 = -1.6 \text{ dB}.$$
(5.8)

This limit of  $E_b/N_o$  being -1.6 dB is represented by Shannon (1948) as the *ultimate* Shannon's limit for error free communication for an AWGN channel. Thus, for any communi-

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cation system to have reliable communication, the  $E_b/N_o$  operating point must be larger than -1.6 dB.

There are various trade-offs attached to the use of error correction coding. They are:

- Bandwidth versus Error Probability In order to improve the performance of a system by reducing the probability of bit-error, error correction coding is used. The use of ECC adds redundant bits, which results in increased bandwidth.
- 2. Bandwidth versus Signal Power ECC provides reduction in the required  $E_b/N_o$  or power. An uncoded system working on  $S/N_o$  of 19 dB could be incorporated with proper ECC to reduce the required amount of  $S/N_o$  to 14 dB keeping the probability of bit-error the same. This would come at an additional cost of increased bandwidth in terms of SNR, because the distance between the sequences is much larger than the uncoded channel and thus, requiring more power SNR to operate.
- Coding Gain It is defined as the measure of the amount of power saving that can be obtained by coding compared to without coding for a fixed probability of bit-error (Sklar; 2001). It is expressed in dB as:

$$G_{-}(dB) = \left(\frac{S}{N_o}\right)_{uncoded} - \left(\frac{S}{N_o}\right)_{coded} \qquad (dB)$$
(5.9)

4. Bandwidth versus Achievable Data Rates - Considering that the received power  $P_r$  is in the information bearing signal, it is known that for a data rate R:

$$\frac{E_b}{N_o} = \frac{P_r}{N_o} \left(\frac{1}{R}\right) \tag{5.10}$$

ECC allows to obtain the same bit-error probability at lower  $E_b/N_o$  and higher data rate R. But due to this, there is an increase in the bandwidth.

5. Code performance at lower  $E_b/N_o$  - If there are more errors than a designed code can correct in a block, it will perform poorly. Thus, reducing  $E_b/N_o$  for such code systems would mean that the system will degrade continuously since the decoder is overwhelmed with many errors.

When considering magnetic recording channels, which are bandwidth limited channels, a good design of error correcting codes is necessary in order to obtain good data rates as well as reliable retrieval of data. The magnetic recording systems work on a much higher signal to noise ratio in comparison to the communication systems. Both bandwidth and SNR for the magnetic recording system are fixed.

#### 5.3 Evolution of Error Correcting Codes

A block code consists of a set of fixed-length vectors called codewords. The length of these vectors is denoted by n and is the length of the codewords. The elements of a codeword are selected from an alphabet of q symbols, where q = 2 for binary codes. There are  $2^n$  possible codewords in a binary block code of length n. From these possible codewords,  $M = 2^k$  codewords where (k < n) are selected to form a code, where k is the length of information bits mapped into a codeword of length n. The code is denoted as (n, k) code and the ratio k/n is called the code rate  $R_c$ .

Each codeword of length n has its own *weight* which is defined as the number of nonzero elements that the codeword contains. The set of all weights in a code constitutes the *weight distribution*<sup>\*</sup> of the code. Codes are constructed from fields with a finite number of elements. These finite fields are called *Galois field* and denoted as GF(q). The finite field can only be constructed if q is a prime or a power of prime number. If  $q = p^m$ , where p is a prime and m is any positive integer, it is possible to extend the GF(q) field to  $GF(p^m)$ . This is called extended Galois field of GF(p). The binary codes are made of GF(2) finite field and are of concern in this thesis.

The codes are called *linear block codes* from the fact that a codeword can be constructed from linear combinations of other codewords and the component-wise sum of all codewords is an all zeros vector.

In order to understand the characteristics of various codes, it is necessary to understand the definitions below:

- If C<sub>i</sub> and C<sub>k</sub> are two codewords in an (n, k) block code set, the measure of difference between them is the number of differing elements in their respective positions. This is called the *Hamming distance* between two codewords and is denoted by d<sub>ik</sub><sup>†</sup>.
- Let  $w_j$  denote the weight of the  $j^{th}$  codeword. The distance  $d_{ik}$  between any pair of codewords  $C_i$  and  $C_k$ , where  $C_k$  is a all-zero codeword, is equal to the weight of the code formed by taking the difference between  $C_i$  and  $C_k$ . It is given as:

$$d_{ik} = w_i - w_k = w_i$$

 $w_i$  is the Hamming distance between the codewords  $w_i$  and  $w_k$ . Since the code is linear, the difference (equivalent to taking the modulo-2 sum for a binary code) between  $C_i$  and  $C_k$  is also a codeword having a weight included in the set  $w_i$  (Proakis; 1995). Hence, the weight distribution of a linear code completely characterises the distance properties of the code.

<sup>†</sup>Hamming distance is a measure of the separation between pairs of codewords which relates to the crosscorrelation coefficients between corresponding pairs of waveform generated from the codewords (Proakis; 1995)

<sup>\*</sup>The weight distribution of a linear code gives information on the distance properties of the code.

The smallest value of the set {d<sub>ik</sub>}(Hamming distance) for M codewords is called the minimum distance of the code and is denoted by d<sub>min</sub>. A code with Hamming distance of d<sub>min</sub> is able to correct all 0 ≤ t ≤ [(d<sub>min</sub> - 1)/2] hard decision symbol errors, where t is the number of errors the code can correct and [x] is the largest integer contained in x. Minimum distance<sup>1</sup> of the code can be given as:

$$d_{min} = \min_{j, j \neq 1} \{w_j\}$$

• For a given vector  $\mathbf{v} \in GF(q^n)$ , the vector  $\mathbf{s} \in GF(q^{n-k})$  defined by

$$\mathbf{s} = \mathbf{v} \mathbb{H}^T$$

is the syndrome of a code with parity-check matrix  $\mathbb{H}$ . If the vector  $\mathbf{v} \in \mathcal{C}$ , then  $\mathbf{s} = 0$ , otherwise at least one coordinate of s has non zero value.

- An (n, k) code with d<sub>min</sub> has a generator matrix G, which contains k linearly independent codewords of C. A codeword C<sub>i</sub> ∈ C can be obtained by any linear row combination of G, an k × n matrix.
- The parity-check matrix H of a code C is an (n − k) × n matrix. It contains (n − k) linearly independent vectors over GF(q), such that GH<sup>T</sup> = 0, where H<sup>T</sup> denotes the transpose of matrix H. Each row in the H matrix defines the parity-check equation of the code.
- A code that satisfies the property

$$\sum_{t=0}^{\lfloor (d-1)/2 \rfloor} \binom{n}{l} (q-1)^t = q^{n-k}$$

is called a perfect or closely-packed code. The (23, 12) Golay code with  $d_{min} = 7$  over GF(2) field and (11, 6) code with  $d_{min} = 5$  over GF(3) field and Hamming codes are perfect codes.

There are many types of codes, but the author has restricted the description to certain popular codes which are described below.

#### 5.3.1 Specific Linear Block Codes

#### 1. Hamming Codes:

The upper bound of the minimum distance is  $d_{min} \leq (n - k + 1)$  for an (n, k) block code

Hamming codes (Hamming; 1950) are a class of binary codes with a property:

$$(n,k) = (2^m - 1, 2^m - 1 - m)$$
  $m = 1, 2, 3, \cdots$  (5.11)

For example, m = 3 provides a (7,4) Hamming code which was interpreted in (Shannon; 1948). Hamming codes with  $m \ge 2$  are able to correct any single errors.

#### 2. Golay Codes:

Golay codes (Golay; 1949) are 3-error correcting codes rather than Hamming's single error correcting codes. In Golay codes, there is an addition of 11 redundancy bits for every 12 bits of information. They are denoted as (23, 12) codes with  $d_m in = 7$ . Golay codes are perfect 3-error correcting codes. All the 3 error patters are uniquely represented by the  $2^{11}$  syndromes. The Golay code is either generated by:

$$g_1(X) = 1 + X^2 + X^4 + X^5 + X^6 + X^{10} + X^{11}$$
(5.12)

or by

$$g_2(X) = 1 + X + X^5 + X^6 + X^7 + X^9 + X^{11}$$
(5.13)

Both  $g_1(X)$  and  $g_2(X)$  are factors of  $X^{23} + 1$ .

$$X^{23} + 1 = (1+X) \cdot g_1(X) \cdot g_2(X) \tag{5.14}$$

#### 3. Cyclic Codes:

Cyclic codes are a subset of linear block codes that follow a cyclic shift property. All possible cyclic shifts for a C code are possible codewords. Various number of efficient methods for encoding and hard-decision decoding have been devised for cyclic codes in order to implement long block codes with large number of codewords.

#### 4. BCH Codes:

BCH codes have been named after their authors, *Bose*, *Ray-Chaudhuri and Hocquenghem* (Bose and Ray-Chaudhuri; 1960). Peterson (1960) discussed the cyclic nature of BCH codes. This class of codes provides a large range of block lengths and code rates. The generator polynomial of BCH codes are specified in terms of its roots from the Galois field  $GF(2^m)$ .

Binary BCH codes can be constructed based on the property:

$$n = (2^m - 1), where, m \ge 3.$$
 (5.15)

$$(n-k) \le (m \times t), \tag{5.16}$$

$$d_{designed} = (2 \times t + 1). \tag{5.17}$$

 $(2 \times t + 1)$  is the designed distance of BCH codes and it not necessarily the true minimum distance is equal to the designed distance.

For p being a prime number, and q being a power of p, there are codes with symbols from the Galois field GF(q). These are q-ary codes. For given positive integers s and t, there exists a q-ary BCH code of length  $n = q^s - 1$ , which is capable of correcting any combination of t or fewer errors and requires no more than  $2 \times s \times t$  parity-check bits.

#### 5. Reed-Solomon Codes:

Reed-Solomon (RS) codes are a sub class of non binary BCH codes. They have s = 1 in *q*-ary BCH codes with  $2 \times t$  parity check bits. They are also named after their authors Irving Reed and Gustave Solomon (Reed and Solomon; 1960). RS codes tend to have the highest error correcting capability amongst linear codes with the same finite field. The efficient decoding algorithm developed by (Massey; 1969) for BCH codes can be applied to RS codes.RS codes are optimum for applications that require burst error correction since they are symbol-wise decoding codes. This gave an important milestone for magnetic recording channel, which requires burst error correction. The coding gain achieved by application of RS codes over magnetic recording channels was described in (E. J. Weldon; 1992).

In the early days of coding, hard decision decoding did not give enough information on the reliability of the information symbol. This resulted in a loss of about 2 dB due to thresholding of the received signals to the levels which are equal to the field size of the code used before being decoded. The complexity of the matched filter decoder was often prohibitive to realise a large coding gain. In order to come closer to Shannon's limit and reduce the gap between the ideal and the achievable limit, it was realised that hard decision decoding was not the optimal solution. Soft decision decoding is a type of decoding in which the symbol reliability information is accounted for. Soft decision decoding was introduced by (Wozencraft; 1957) for convolutional codes using sequential decoding. The application of Viterbi algorithm using trellis diagrams by (Forney; 1973) was a major milestone in soft-decision decoding. This decoding technique overcame the sub-optimality of the previous sequential decoding. The Viterbi algorithm returns a codeword which has the highest probability of being correct among all possible transmitted codewords. The algorithms introduced by Bahl et al. (1974) and Hartmann and Rudolph (1976) are symbol-by-symbol optimum decoding algorithms. These soft-decision symbol-by-symbol decoders return a symbol that has the highest probability of being correct when compared to other symbol possibilities for a given coordinate in the received sequence. The sequential decoding based on the Viterbi algorithm and symbol-by-symbol decoding based on BCJR algorithm have been previously discussed in Chapter 2. There was a great effort dedicated towards the search for good codes with efficient decoding structures. This resulted in the finding of convolutional codes.

#### 5.3.2 Convolutional Codes

Convolutional codes,<sup>§</sup> when compared to block codes, do not separate data into blocks, but encode it into infinite streams of encoded data (theoretically). In convolutional encoding, a part of the information symbols  $\hat{k}$  is used to generated a block of length  $\hat{n}$  symbols. In the encoding procedure, the encoder is input with a part of information k symbols, and generates a part of n symbols. The generation of  $\hat{n}$  symbols is not only dependent upon the current  $\hat{k}$  symbols, but also on some previous symbols. The number of previous symbols that are involved in generating the current  $\hat{k}$  symbols are based upon the memory or constraint length K of a convolutional code. These codes are different from block codes in the sense that the encoding shift register in convolutional encoding has memory.

The encoding procedure is a function of the previous K - 1 inputs.  $\hat{n}$  and  $\hat{k}$  are small integers, while K is variable according to the complexity and capability of the code. The constraint length K represents the number of  $\hat{k}$ -bit shift over which a single information bit can influence the encoder output. At each unit of time,  $\hat{k}$  bits are shifted into the first  $\hat{k}$  stages of the shift register; all bits in the register are shifted  $\hat{k}$  stages to the right, and the output of the  $\hat{n}$  adders are sequentially sampled to provide the code symbols. These code symbols are then used by the modulator to specify the waveforms to be transmitted over the channel.

In the case of a magnetic recording channel, the modulator uses the error correction code symbols for them to be converted into NRZI representation and/or into recording (MTR/RS) code symbols.

The involvement of the short memory in the construction of convolutional code allows the implementation of soft-decision decoding algorithms efficiently. The soft-decision decoding of convolutional codes is based upon trellis based decoding algorithms.

**Example of Convolutional Encoding:** Consider a binary convolutional encoder with constraint length K = 3,  $\dot{k} = 1$  and  $\dot{n} = 3$ . Assume that the initial state of the shift registers is all zeros. The encoder and a table showing the register contents and the input/output sequence is shown in Figure 5.2. Each stage of convolutional encoder represents the data

<sup>&</sup>lt;sup>5</sup>Convolutional codes were often referred to as *Recurrent Codes*in the early days



Figure 5.2: Convolutional Encoding Stages for a rate 1/2 code with K = 3

present in the shift register at that time instant. The shift register can also be considered to be a K - 1-tap delay structure, with the delay denoted as D.

The convolutional encoder belongs to a class of devices known as *finite state machines*<sup>¶</sup>. The state consists of a small amount of information which can be used together with the current input at a particular time instant to provide the output of the machine. Convolutional codes can be described in three ways:

- 1. Tree diagram
- 2. State diagram
- 3. Trellis diagram

The focus of this thesis is on the description of convolutional codes based on trellis diagram. Apart from the above mentioned ways of representing convolutional codes, they can also be represented in polynomial form, similar to that of cyclic codes. The polynomial form consists of  $\dot{n}$  generator polynomials, each representing the  $\dot{n}$  modulo-2 adders in the encoding structure. Each polynomial is of K - 1 degree or less and describes the connection of the encoding shift register to that of the modulo-2 adder. The coefficient of each term in the (K - 1) degree polynomial is either 1 or 0, depending upon the connection exists or not.

The optimum decoder for a convolutional code is *Maximum Likelihood Sequence Estimator*, which is described in Chapter 2. There are various other decoding algorithms developed for convolutional codes which are described in (Sklar; 2001). Although in this thesis, only MLSD and MAP decoders are discussed. Optimal decoding of a convolutional code involves a search through the trellis for the most probable sequence. The minimum distance of a convolutional code is referred to as the *minimum free distance* denoted by  $d_{I}$ . It is the set of arbitrarily long paths that diverge and re-merge on a trellis diagram for a

<sup>&</sup>lt;sup>1</sup>Finite state machines can be described as machines that have a memory of the previous signals and have a finite number of states that it can encounter

all-zero path. It can also be defined as the weight of the minimum weight sequence which deviates from the all-zero path and returns to it a number of frames later. The number of frames in the non-zero segment of the path used to calculate minimum free distance is called the free length,  $n_{free}$  of the code.

A systematic convolutional code is one in which the input  $\hat{k}$  symbols appear as part of the output branch word  $\hat{n}$  symbols associated with that  $\hat{k}$  symbols. In the systematic convolutional encoding structure, there exists a direct connection with the  $\hat{k}$  input to the shift register and the one of the code symbols which generate the branch word.

A recursive systematic convolutional code(RSC) code is one in which the encoder has a feed-forward and a feed-back polynomial describing the connection of the shift register to the modulo-2 adders in the encoding structure. These codes are characterised by their property that the code bits at a given instant do not depend only on the information bits at the present instant and the previous instants, but on all the previous bits, as the encoder exhibits a structure with feedback. From a 1/n non-recursive non-systematic convolutional code, it is possible to obtain a recursive systematic convolutional code with the same rate and the same codewords, and thus, with the same minimum free distance  $d_1$ . Obviously, for a given input word, the output code words will be different for both the cases. The encoding structure of a rate 1/2 RSC (4, 5/7) code is as shown in Figure 5.3. Table (5.1) shows the input, start and end state of the encoder and output of the convolutional encoder. In the (4, 5/7) RSC code, the constraint length is K = 3. Thus, the number of states is  $2^{K-1} = 2^2 = 4$ . The start state is defined by points c and d. The end state is defined by points b and c. It is assumed that the initial state of the encoder is all zeros. The output codeword symbols are defined by a and  $b \oplus d$ . Recursive systematic convolutional codes form the component codes for Turbo codes, which shall be discussed later in this chapter.



Figure 5.3: Encoding structure of (4,5/7) RSC code

| Input | Start State | End State | Output | Encoder State |
|-------|-------------|-----------|--------|---------------|
| 0     | 0.0         | 0.0       | 0.0    | 0 0 0         |
| 0     | 0.0         | 0.0       | 0.0    | 0 0 0         |
| 1     | 0 0         | 1 0       | 11     | 1 0 0         |
| 1     | 10          | 1 1       | 11     | 1 1 0         |
| 1     | 11          | 1 1       | 1 0    | 1 1 1         |
| 0     | 1 1         | 0 1       | 0 1    | 0 1 1         |
| 1     | 0 1         | 1 0       | 1 0    | 101           |
| 0     | 1 0         | 0 1       | 0.0    | 0 1 0         |
| 1     | 0 1         | 1 0       | 1 0    | 1 0 1         |
| 1     | 1 0         | 1 1       | 1 1    | 1 1 0         |
| 0     | 1 1         | 0 1       | 0.1    | 0 1 1         |
| 1     | 0 1         | 1 0       | 1 0    | 1 0 1         |
| 1     | 1 0         | 1 1       | 11     | 1 1 0         |
| 1     | 1 1         | 1 1       | 1 0    | 1 1 1         |
| 1     | 1 1         | 11        | 1 0    | 111           |
| 0     | 11          | 0 1       | 0 1    | 0 1 1         |
| 0     | 0 1         | 0 0       | 0 1    | 0 0 1         |

Table 5.1: Rate 1/2 RSC (4,5/7) Code Encoding Process

#### 5.3.3 Concatenated Codes

Concatenated codes were introduced by Forney (1967) in order to obtain longer block lengths with lower decoding complexity. Forney (1967) suggested the combining two codes<sup>||</sup>, a nonbinary outer code and a binary inner code to be cascaded together to form a longer and more powerful binary code. This cascaded coding scheme, which is called *Concatenated codes*, restrict the decoding complexity to that of the component codes. These codes are known as *serially concatenated codes*. The decoding process of concatenated codes is performed in two stages: the inner code is decoded first, then the output is decoded by the outer code.

The information that each decoder receives is incomplete when compared to the overall code, and thus, there is a loss in performance as compared to the decoding of the concatenation as a single overall code.

Turbo codes (Berrou et al.; 1993), first introduced in 1993 by Berrou, Glavieux and Thitimajshima are described in the next section, and are of the main interest in this work. They are discussed in the next section.

MacKay and Neal (1996) revisited the original work of Gallager (1963) which showed that low-density-parity-check (LDPC)<sup>\*\*</sup> codes were also capacity-approaching codes in the AWGN channel. During the time of their invention, the decoding of LDPC codes was prac-

<sup>&</sup>quot;These codes are often known as component or constituent codes

<sup>\*\*</sup>LDPC codes were initially found by Gallager (1963)

tically impossible due to technology constraints at that time. The development of iterative decoding during the time of turbo codes made it possible once again to use LDPC codes today. In Gallager (1963), the decoder described is commonly known as *sum-product* decoder. It consists of a bank of BCJR decoders for (n, n-1) single parity-check codes with  $d_{min} = 2$ . Each of the BCJR decoders produces soft-output information which is processed and used in the next iteration.

Both turbo and LDPC codes are very popular in today's industry. Although, the industry prefers LDPC codes, the complexity of encoding irregular LDPC codes is considerable and LDPC codes that have some regularity are preferred (Tjhai; 2007). The work produced in this thesis is based upon turbo codes.

## 5.4 Turbo Codes

The use of Turbo codes for magnetic recording are of main interest to the author and thus, this section provides a discussion on the design and implementation of Turbo encoding, decoding and interleaving processes. The name *Turbo* is actually tied to the decoding part of the codes, since it is an iterative process with a feedback loop trying to converge to the optimal decoded output which would be produced by a single code. The process is similar to turbo engines, hence, the name turbo codes. Turbo codes have a concatenated encoding structure including an iterative algorithm to decode the codewords. They are parallel concatenated convolutional codes with interleaving and can be thought of as an improvement of the code sequence. This iterative decoding was one of the major milestone in the development of error correction codes.

In (Berrou et al.; 1993), a scheme is described that achieves a bit-error probability of  $10^{-5}$  at  $E_b/N_o = 0.7$  dB. The encoder is a *parallel* concatenation of convolutional codes. The scheme uses a rate 1/2 code over an AWGN channel and *binary phase shift keying* (BPSK) modulation. The block length is set at  $n = 256 \times 256 = 65536$  bits. The information block is encoded directly by the first code and through an *interleaver* by the second code. The interleaver modifies the order of information bits in the block. A block diagram of a turbo coding scheme is as shown in Figure 5.4

The decoding is done in two stages. For each received block, the first code is decoded using its corresponding received values, and produces its version of the information bits and also a new type of information called *extrinsic information*. Extrinsic information in turbo decoding is defined as a reliability measure that is determined by the redundancy in the transmitted sequence. This contribution improves the reliability of transmission over noisy channels using the parity check bits. The second code is decoded using its corresponding received value and the extrinsic information from the first code (interleaved), producing its version of the information bits and extrinsic information. The first code is decoded again by



Figure 5.4: Block Diagram of a Turbo Coding Scheme (Ambroze; 2000)

making use of the extrinsic information from the second code and producing a new version of the information bits and extrinsic information. The process continues iteratively. By iterating this information exchange, the decoded values should converge to the values that would be produced if the overall code were decoded as a single code.

The extrinsic information needs to be *soft* in order to reduce the restrictions in possible values, and to allow a smooth convergence (Ambroze; 2000). If the extrinsic information was *hard* or *coarsely quantised*, there is a possibility that the steps the decoder needs to reach towards convergence would not be possible. Thus, the decoder for each component code needs to be a *Soft Input Soft Output* (SISO) decoder.

The complexity of the turbo decoder is low due to the separate decoding of the two codes. The block length of the overall code is determined by the interleaver. Although, the decoding complexity for each code depends upon its constraint length, which in (Berrou et al.; 1993) was 5, where the memory was 4, which was 1 less than the constraint length. Another advantage of the turbo codes over previous classical linear block codes is that the component codes are simple convolutional codes and the interleaver is a random chosen permutation. The interleaver length is variable in order to achieve the desired BER performance.

#### 5.4.1 Encoding of Turbo Codes

The encoding of turbo codes is done using two convolutional (component) codes in parallel concatenation and an interleaver. Figure 5.5 shows the turbo code encoder with two parallel concatenated convolutional codes (RSC I and RSC II) and an interleaver (INT). The information symbols are designated as x(n), where  $x \in 0, 1$ . The output of the first RSC

encoder is designated as  $y_1(n)$ , where  $y_1 \in 0, 1$ . The output of the second RSC encoder is designated as  $y_2(n)$ , where  $y_2 \in 0, 1$ . RSC codes have been discussed earlier in convolutional codes section. The turbo code encodes binary data in a continuous or block fashion, depending upon the structure of the interleaver. The second RSC encoder receives the interleaved version of the information symbols. The output of the turbo code is a multiplexed version of information bits and parity bits of the two RSC encoders.



Figure 5.5: Turbo Code Encoder with Two RSC Codes and an Interleaver

#### 5.4.2 Interleaver Design

In turbo coding, two or more bit errors in an information sequence can be permuted into different bit patterns whose encoded output bears no resemblance to the encoding of the unpermuted information (Dolinar and Divalsar; 1995). This process of achieving permutations is known as *Interleaving*. In turbo coding, it is necessary to match a low weight codeword of one permutation with a high weight codeword of another, so that the total weight is significantly higher than the low weights that are possible from each of the component codes individually. The interleaver is designed to reduce the probability of associating low code weight codewords of the two codes. Robertson (1994) presented an iterative method in which a given interleaver finds the codeword association with the lowest code weight and breaks it by modifying the interleaver. The process is repeated until the minimum code weight is increased. Interleavers increase the constraint between the code bits so that a code bit depends upon many more bits than the short constraint imposed by the component codes. Amongst several methods of interleaver design, the  $S^{\dagger\dagger}$  interleaver, presented in (Dolinar and Divalsar; 1995) is the most popular. Interleaver designs based on a cost function are presented in (Daneshgaran and Mondin; 1997).

<sup>&</sup>lt;sup>11</sup>A S interleaver is often referred to as S-random or spread interleaver

#### S Interleaver

Dolinar and Divalsar (1995) stated that for a turbo code using q and q-1 permutations, the probability that a weight-n data sequence will be reproduced somewhere within the block by all q-1 permutations is of the form  $1 - [1 - (\beta/N^{n-1})^{q-1}]^N$ , where  $\beta$  is a number that depends on the weight-n data sequence but does not increase with block size N. For large N, this probability is proportional to  $(1/N)^{nq-n-q}$ , which falls of rapidly with increasing N, when n, q > 2. Semi-random permutations are designed by generating random integers  $i, 1 \leq i \leq N$ , without replacement. S-random permutation is defined as: Each randomly selected integer is compared to the S previously selected integers. If the current selection is rejected. This process is continued until all N integers are selected. The searching time increases with S and it is not guaranteed to finish. Although, choosing  $S \leq \sqrt{N/2}$  usually produces a solution in definite time. For S = 1, the S-random permutation becomes a random permutation. Some of the recent interleaver types are discussed in (Hokfelt et al.; 1999) and (Crozier et al.; 1999).

#### **Dithered Relative Prime Interleaver**

Crozier and Guinand (2001) introduced a new method of designing high-performance, lowmemory interleaver banks for turbo codes. The new interleavers are called *Dithered Relative Prime* (DRP) interleavers. *Relative Prime* (RP) interleavers described in (Crozier et al.; 1999) are defined by just one parameter p, the modulo-N index increment, where N is the block length. RP interleavers provide good performance for short block length turbo codes, while their performance deteriorates as the block length increases from short to medium/long. This is because as the block length increases, the number of low-weight codewords generated by the repetitive structure increases. The interleaver design is based on three stages. Initially the input to the interleaver is permuted locally (dithered) using a small dither vector of a certain length. The first dithered output is permuted using an RP interleaver and finally again dithered to provide the DRP interleaver output.

#### 5.4.3 Decoding of Turbo Codes

The decoding of turbo codes is done using SISO decoders and de-interleavers. A MAP decoder is used as a SISO decoder which is discussed in Chapter 2. The decoding process of turbo codes is an iterative process. An iterative turbo decoder is as shown in Figure 5.6. In the figure,  $\mathcal{R}$  is defined as the received stream which is de-multiplexed and sent to the two SISO decoders and the interleaver and de-interleaver to produce apriori information.  $\mathcal{R}_{y1}$  is the de-multiplexed data sent to the first SISO decoder along with the recursive information from the encoding process, which is given as  $\mathcal{R}_x$ . The interleaved  $\mathcal{R}_x$  information known as  $\mathcal{R}_{x1}$  is sent to the second SISO decoder along with the information from the second encoder

given as  $\mathcal{R}_{y2}$ . The received signal is de-multiplexed and grouped into blocks of length N. These blocks are then passed into the iterative decoder and each block is iterated several times before the decoded output is obtained.



Decoding Process

Figure 5.6: Turbo Code Decoder with Two SISO Decoders

The iterative decoding process can be described as follows: The received values are transformed into probabilities as the decoders used are SISO. The received values for the systematic bit are passed directly to the first SISO decoder and its interleaved counterpart to the second SISO decoder. Both decoders act on the received values for the corresponding parity bit. The *a priori* probabilities are initialised to equal probabilities.

The extrinsic information and the decoded values based on received values and *a priori* information are produced by the first decoder. The interleaved extrinsic information is passed on to the second decoder as *a priori* information for the second decoder.

Based upon the received values and its own *a priori* information, the second decoder produces its own extrinsic information and decoded values. This extrinsic information is de-interleaved and passed on to the first decoder as *a priori* information. This process is continued for a number of iterations. The decoded values from the first decoder or the interleaved decoded values from the second decoder are thresholded as the decoder output. This thresholding is necessary in order to obtain hard decision decoded output, since Turbo decoding process provides soft values. In order to achieve nearly optimal performance, it is crucial that the extrinsic information exchange occurs properly from one iteration to the next.

The extrinsic information depends upon all the channel inputs and the *a priori* probabilities, excepting the systematic value and *a priori* probability of the current bit. As shown in Figure 5.6, the extrinsic information from the first decoder is passed on to the second decoder via an interleaver. This extrinsic information is generated by the probabilities of the received values and the *a priori* probabilities, except the current bit *a priori* probabilities. Likewise, the second decoder generates its own set of extrinsic information in the same manner.

#### Convergence

Barbulescu (1998) gave a qualitative proof for the convergence of the iterative decoder to the transmitted data. It relies on the MAP algorithm property to minimise the BER to show that the MAP functions are *contractions* and thus, the output must converge to the transmitted data. Optimal component codes have better distance properties, making the steps of the iterative decoder bigger in comparison to non-optimal codes. Thus, if one of the component codes is optimal and the other is not, it could create disagreement between the two decoders leading to non-convergence. Berrou et al. (1993) suggested the use of *simulated annealing*, which weighs the extrinsic information with an empirical factor dependent upon the statistics of the extrinsic values. Although, this method was not used again in (Divalsar and Pollara; 1995).

In order to save computing time, the number of iterations required for the iterative turbo decoder is based upon the fact that if after certain iterations, there not much significant gain, then the iterations could be terminated (Hagenauer et al.; 1996).

## 5.5 Application of Turbo Codes to Magnetic Recording

The previous sections of this chapter introduced various error correcting codes and Turbo codes were discussed in depth. Based on the previous background on Turbo codes, this section introduces the novel work of application of Turbo codes over various magnetic recording channels. Amongst the investigated recording channels are, PR/SFE based binary magnetic recording channels and multi-level magnetic recording channels. This section provides some key results which have been presented in 2 conferences and 1 article is published in the *IEEE Transactions on Magnetics* journal.

Wolf and Ungerboeck (1986) introduced trellis coding for partial response channels. The authors Wolf and Ungerboeck (1986) discussed various techniques to obtain codes which increased the Euclidean distance between permitted sequences of channel outputs and avoid the occurrence of unlimited runs of identical outputs. The method used convolutional codes in conjunction with precoding. A lot of literature has been produced as a result of the need to improve the performance of magnetic recording channels and to reduce the SNR requirement. Turbo codes and similar other error correcting codes have been found to give significant improvement in performance at lower SNR in magnetic recording (Siala et al.; 1995)(Ryan et al.; 1998) and (McPheters et al.; 1998). Various concatenated schemes in order to obtain optimal code rates as well as performance have been developed and discussed in (Wu et al.; 2000), (Ryan; 2000).

Figure 5.7 shows the block diagram of the simulated magnetic recording channel with ECC. This simulation model is based on the general model presented in Chapter 1. In this simulation model, it is seen that the ECC encoder and  $GF(2^m)$  mapper blocks have been shown separately to introduce the effective code rate. Also, the equaliser block shows both PR/SFE based equalisation, since, both types of equalisation methods have been investigated.



Figure 5.7: Block Diagram of Magnetic Recording Channel with ECC

#### 5.5.1 Results

The simulation model shown in Figure 5.7 is used in various combinations of different types of interleavers, different rate Turbo codes, different types of equalisation and binary or multi-level, in order to investigate the performance of various channels and make a comparison of them. These results have been presented and discussed in this section. The results described in this section are based upon various parameters of ECC of which the main parameters used unless otherwise stated are as below.

For simulation purposes, it is necessary to define the ECC parameters. These specifications are described below.

#### **ECC Specifications:**

- The outer ECC code used is a rate 1/3 turbo code. The design of this code is achieved using tail-biting recursive systematic convolutional codes with feed-forward polynomial  $F_f = [37]_8$  and feed-back polynomial  $F_b = [23]_8$  for an overall rate 1/3 turbo code.
- The turbo decoder is an iterative parallel concatenated MAP decoder with extrinsic information exchange (Berrou et al.; 1993).
- The interleavers used are S-random and DRP interleavers.
- The block length is set to 500 information bits and the maximum number of iterations is set to 50.
- At least 100 error blocks were collected for each BER point.
- In the simulation results, TC stands for Turbo Code, PR stands for Partial Response channel, SFE stands for Soft Feedback Equalisation channel.
- The SNR considered for the simulation results is the channel SNR. For the 4-level scheme the channel SNR is offset by +1.72 dB, which is calculated based upon the overall code rate R.
- The results presented and discussed are based on PR-MAP and SFE-MAP, binary and multi-level channels.

Denoting the code rate of the ECC as  $R_1$  and the code rate for the 4-level PR/SFE magnetic recording channel as  $R_2 = 2$ , the overall code rate R of the 4-level channel is computed as:

$$R = R_1 \times R_2 = 2 \times R_1. \tag{5.18}$$

Thus, if the rate 1/3 turbo code is used for binary PR/SFE magnetic recording channel, then the overall rate for a 4-level PR/SFE magnetic recording channel would be 2/3.

Figure 5.8 shows the performance of binary and 4-level PR-MAP channel with rate 1/3 outer turbo code using S-random interleaver. It is observed that for a BER of  $10^{-4}$ , the performance of coded PR-MAP channel is nearly 12dB better in comparison to the uncoded PR-MAP channel. Also, for 4-level coded PR-MAP channel, the performance improvement is nearly 8dB. The error floor region beyond  $10^{-6}$  BER for the coded performance comes from various perspectives. This could be because of the type of interleaver used (S), the inner concatenation of the component codes used for turbo codes and the non-integer PR-target polynomials, the length of the PR target used or the amount of 1SI introduced due to the higher density of binary coded channel. It is observed that the error floor of the binary



Figure 5.8: BER vs SNR Performance of Multi-level PR-MAP with rate 1/3 Turbo Code



Figure 5.9: Comparison of PR-MAP and SFE-MAP with rate 1/3 Turbo Code



Figure 5.10: BER vs SNR Performance of Binary PR-MAP with rate 1/3 Turbo Code

coded channel and the 4-level coded channel is the same and as a result, it is better to use 4-level coded channel in order to work closer to the channel capacity.

Figure 5.9 shows the performance comparison of PR-MAP and SFE-MAP magnetic recording channels with rate 1/3 turbo code. It is observed that the 4-level SFE-MAP channel with turbo code has similar error floor as that of binary PR-MAP channel at  $PW_{50} = 2.4$ . The performance of 4-level SFE-MAP at  $PW_{50} = 1.2$  with turbo code is slightly better that 4-level PR-MAP with turbo code. Also, the gain in terms of magnitude is of 1.5 order.

Figure 5.10 shows the performance of various PR targets over rate 1/3 turbo code with a DRP interleaver. It is observed that the performance in the error floor region for GPR targets has improved in comparison to the performance shown in Figure 5.8 for the same  $PW_{50}$  of 2.4. This improvement comes from the use of a different interleaver as well as different GPR target. The previous GPR target [1, -0.5, -0.5] also has slight improvement in the error floor region. Furthermore, the new GPR target [0.4, 0.6, -0.6, -0.4] has better performance and it does not reach the error floor region until  $10^{-8}$ . This improvement in performance is achieved by the inner concatenation of component codes with the noninteger GPR target polynomial. More GPR targets need to be search based upon the turbo codes used in order to achieve better performance.

Figure 5.11 shows the performance of rate 1/3 turbo code with DRP interleaver over SFE-MAP channel at various densities. It is observed that as the density increases, the performance degrades and this is because of the amount of ISI left even after the feedback



Figure 5.11: BER vs SNR Performance of Binary PR/SFE-MAP with rate 1/3 Turbo Code

operation for removal of ISI is performed. The amount of residual ISI is much larger at higher densities and as a result, even the turbo decoder cannot cope with it. The ideal density for the SFE-MAP channel with turbo code to outperform PR-MAP channel with the same turbo code is 1.8. There is an improvement of 1 dB for the coded SFE-MAP channel in comparison to the coded PR-MAP channel for a density of 1.8 and at a BER of  $10^{-4}$ . The improvement in the SFE-MAP channel for higher densities could be obtained if MTR codes are used in conjunction with turbo codes.

Figure 5.12 shows the performance of 4-level PR/SFE-MAP channels with rate 1/3 turbo code and DRP interleaver for various densities. It is observed that the coded 4-level SFE-MAP channel has an improvement of 1.8 dB over coded 4-level PR-MAP channel. Comparing figures 5.8 and 5.9, it is observed that coded 4-level SFE-MAP channel has better performance when compared to coded 4-level PR-MAP channel with optimum GPR target at  $PW_{50} = 1.2$ . This results from the fact that the turbo decoder is able to remove the residual ISI efficiently than at higher densities for 4-level SFE-MAP channels. Thus, using multi-level SFE-MAP channel with turbo codes is more beneficial when considering the operation of multi-level magnetic recording channel in terms of closeness to channel capacity.



Figure 5.12: BER vs SNR Performance of 4-level PR/SFE-MAP with rate 1/3 Turbo Code with DRP Interleaver

### 5.6 Summary

This chapter introduced the use of error correcting codes for magnetic recording and presented the novel work of application of Turbo codes over various configurations of magnetic recording channels. An introduction to different error correcting codes was provided along with a detailed discussion on Turbo codes, which were of main interest to the author. Some of the key results and discussions from this chapter can be summarised as below.

- Error correction coding is used at the disk drive controller end to correct/detect errors caused by AWGN, ISI, transition jitter, modulation noise and media defects.
- The main features of parallel concatenated turbo codes design, S-random and DRP interleavers were discussed.
- The effect of both the interleavers was observed on PR/SFE-MAP channels along with the effect of GPR targets on PR-MAP channels.
- An interesting result observed in Figure 5.10 which showed the effect of inner concatenation of non-integer GPR target with the component codes on the improvement in performance was discussed.
- The improvement in performance of SFE at lower densities was observed and it was

seen that 4-level SFE-MAP channel has better performance with turbo codes than 4-level PR-MAP channel with optimum GPR targets.

 In order to improve the performance of binary SFE-MAP channels with turbo codes at higher densities, it is necessary to include MTR codes as recording codes and constraint the dominant error events from occurring.

# Combined PRML and SFE based Magnetic Recording

## 6.1 Introduction

In binary magnetic recording systems, the SFE scheme did not provide any gain in comparison to the PRML scheme. This was due to the fact that the modified filter which was used to estimate the amount of ISI could not detect the ISI properly if there were more than 3 consecutive transitions. As a result, there was a large amount of ISI left even after 10 iterations. The solution to this is to use MTR codes which was discussed in Chapter 4. But even after using MTR codes, although the amount of ISI reduced, there is significant amount of ISI left. This is due to the use of the "hard" thresholding device in order to generate the initial estimates. The initial estimates are incorrectly thresholded because, the thresholding point shifts as the  $PW_{50}$  increases and also as SNR changes.

In case of a magnetic channel considering no ISI and only AWGN, the thresholding point T can be considered to be a Marcum function  $Q(T/\sigma)$  of Gaussian nature. This is given as:

$$\mathcal{Q}(\mathbb{T}/\sigma) = \frac{1}{2}\operatorname{erfc}\left(\mathbb{T}/(2\sigma)\right),\tag{6.1}$$

where, erfc is the complementary error function defined in (Benvenuto and Cherubini; 2002),

If ISI is taken into consideration, the operating threshold point will change depending upon the overlapping of the transition pulses. The assumptions made in the calculation of the optimum thresholding point are that there is no ISI, that the probability density function (pdf) is Gaussian and that the probability of a 0 is twice as likely as +1 and -1. It is also known that *there exists one and only one minima* for threshold detection. The pdf of 1 is divided into two: +1 and -1, both having equal probability. Thus, the number of samples being 0 is 1/2, while the number of samples being +1 is 1/4 and -1 is 1/4 in the total number of sampled data.

Taking into account, the adjacent pdf's of 0,  $\pm 1$  and -1,

$$\frac{d}{d\mathbb{T}}\left[\mathcal{Q}(\mathbb{T}/\sigma) + 2\mathcal{Q}(1 - \mathbb{T}/\sigma)\right] = 0.$$
(6.2)

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#### Chapter 6. Combined PRML and SFE based Magnetic Recording

The differentiation of the erf function is given as:

$$\frac{d}{dz}\operatorname{erf}(z) = \frac{2e^{-z^2}}{\sqrt{\pi}}.$$
(6.3)

Substituting (6.3) and (6.1) into (6.2),

$$\frac{d}{d\mathbb{T}} \left[ \frac{1}{2} \left[ 1 - \operatorname{erf}(\frac{\mathbb{T}}{\sqrt{2}\sigma}) \right] + \left[ 1 - \operatorname{erf}(\frac{1 - \mathbb{T}}{\sqrt{2}\sigma}) \right] \right] = 0.$$
(6.4)

Solving the above equation, the optimum thresholding point is obtained which is dependent upon  $\sigma$ .

$$\mathbb{T} = \frac{1}{2} \left[ 1 + 2\sigma^2 \ln(2) \right] - 1.$$
(6.5)

Figure 6.1 shows the variation of the thresholding point versus SNR. It is seen that at lower SNR, there is a variation in the thresholding point from 0.5, but as the SNR increases, the point comes closer and closer to 0.5. Thus, in the lower SNR region, the shift in the thresholding point causes many errors in the initial estimation of the readback signal, and overall affecting the performance of the feedback mechanism of the SFE scheme. Figure 6.1 provides the optimum thresholding point at a specific SNR. The improvement in performance is not significant due to the fact that the improvement can only be seen in the lower SNR region.

If the thresholding device was replaced by something which could provide a better estimate of ISI, would provide an improvement in the SFE decoding process. It was seen that PR equalisation provided the near optimum estimate of the readback response of the magnetic recording channel. Thus, the thresholding device in the SFE with MAP feedback is replaced with PRML device.

The research documented in this chapter has been patented under "Novel MTR-coded Soft Feedback Equalisation Scheme for Magnetic Recording", whose authors are **P. Shah** and M. Z. Ahmed. The UK Patent Application number is **GB 0720196.5** and keyword is "shah-med decoder".

This chapter investigates the drawbacks of SFE based magnetic recording which was discussed in Chapters 2 and 3. Based on this investigation, this chapter introduces the novel work of combining PRML and SFE-MAP based magnetic recording in order to improve the performance of longitudinal magnetic recording. The next section discusses the simulation model of the combined scheme which was derived from the simulation model discussed in Chapter 1.



Figure 6.1: Optimum Thresholding Point for SFE Scheme

## 6.2 Simulation Model of Combined PRML and SFE based Magnetic Recording

This new combined PRML and SFE based scheme incorporates PRML - *the industry standard technique* with the concept of SFE. The new scheme combines PRML along with a feedback algorithm for removing ISI from the magnetic recording readback channel data. The feedback algorithm used is the BCJR algorithm and it is soft output algorithm. The novelty of this scheme is:

- Use of PRML for best initial estimate.
- Use of MAP decoding during SFE iterations.

The design of the SFE block is independent of the GPR target length. This technique is in contrast with the popular PRML technique, since it does not use the introduced ISI, but instead it tries to remove the ISI using a feedback loop. As a result, there are no penalties of noise colouration in the decoded data. The SFE scheme is very similar to the Interference Cancellation (IC) Schemes described by Raphaeli and Saguy (2002) and Dejonghe and Vanderdorpe (2002). The main difference between the IC schemes and the present scheme is that the SFE combines the modified filter output and the apriori information from the PRML block to form the most reliable estimate of ISI.

The scheme described in (Chesnutt; 2005) combines turbo iterations with a SFE-based equaliser. During the 1<sup>st</sup> iteration, the scheme described in (Chesnutt; 2005) lies somewhere between a linear equaliser and a decision feedback equaliser (DFE), depending upon the SNR and the severity of the ISI. On the contrary the proposed scheme is optimal, since it provides the best possible estimate of ISI during the first iteration itself. As a result, it does not require many iterations to achieve the MMSE solution.

To achieve similar coding performance with the proposed scheme, error correcting codes like Turbo and LDPC codes, could be used as outer codes and not as turbo-based equalisation as described in (Chesnutt et al.; 2005). The ECC block is completely independent of the SFE iterations and thus, the decoding complexity is much less.

#### 6.2.1 Simulation Model of the Combined PRML-SFE Scheme

Figure 6.2 shows the simulation model of the combined PRML-SFE based magnetic recording system. This simulation model is in conjunction with the general simulation model which was discussed in Chapter 1. Comparing the simulation models shown in Figures 6.2 and 2.12, it is seen that the thresholding device is removed from the initial estimation loop and replaced by a PRML device.

A linear modified Lorentzian filter is used to estimate the amount of ISI introduced by the magnetic channel and the Maximum A Posteriori (MAP) decoder in the iterative loop determines the best estimate of the data bit sequence. The feedback in the SFE block results in no alteration within the correlation properties of noise. The problem of SFE is residual ISI which is obtained by linear super-positioning of certain error patterns. This can be overcome by the use of Maximum Transition Run (MTR) codes.



Figure 6.2: Block Diagram of combined PR-SFE-MAP Scheme with MTR Codes

The convergence criteria for the feedback loop is as follows: Initially the estimates of the noisy data are passed through a modified Lorentzian filter and differentiated. The error is calculated from the original readback data and the estimates from the differentiated PRML output. This error is then fed into a loop which uses an error minimising MAP algorithm in order to locate the transitions. The error calculated tries to estimate and remove the ISI from the channel as it goes through the iterations of the feedback loop. The convergence of the error depends upon the amount of ISI, the pattern of transitions and Additive White Gaussian Noise (AWGN) in the channel. Depending upon these factors, the number of iterations required to achieve the performance of the system is set. The convergence equation is obtained by Minimum Mean Squared Error (MMSE) criterion (Proakis; 1995). It is found that after 2 iterations, the amount of ISI within the data is minimal and thus, there is no requirement for additional iterations.

MTR codes improve the minimum distance properties of sequence detectors operating at high linear densities (Moon and Brickner; 1996). MTR codes eliminate the data patterns producing three or more consecutive transitions, while imposing the usual *k*-constraint necessary for timing recovery.

A rate 4/5 MTR code is used for the simulation. The MTR codewords used are as described in Table 4.2. MTR codes have been discussed in detail in Chapter 4.

The data is precoded with  $\left[\frac{1}{1\oplus D}\right]$ . Due to the nonlinear characteristics of the magnetic read channel, the write waveform consists of two symbols +1, -1. Since the magnetic channel is differentiating, only the transition positions are relevant for the received signal from the disk/tape. The precoder shifts these transitions inside the bit window. This puts a limitation on the available precoding equalization. Thus precoding as a stand-alone equalizer, seems unable to handle all the equalization needs of a high-density magnetic channel (Feyh; 1998). It is useful in conjunction with a constrained, finite dimensional equalizer (PR equaliser).

#### 6.3 Results

Performance of the combined SFE-PR-ML based magnetic recording channel is evaluated using the simulation model shown in Figure 6.2. Rate 4/5 MTR (2;8) and rate 5/6 MTR (2;6) codes have been used in the simulation to evaluate BER performance at various densities. This scheme can be considered to be an ultimate solution to high density recording problem, which means that it provides a better SNR gain in comparison to the existing magnetic recording techniques.

The existing scheme is considered to be the PRML scheme with rate 4/5 MTR (2;8) code as outer recording code. The new scheme is SFE-MAP with PRML estimation and rate 4/5 MTR (2;8) code as outer code. The trellis structure for the PRML system is based on the length of the GPR target. As in the case of SFE-MAP block, it is not based on





Figure 6.4: FER vs SNR Performance

the length of the GPR target. As seen from the Figure 6.3, the MTR coded SFE-MAP scheme performs slightly better than PRML scheme in the operating region(19 - 22 dB) with rate 4/5 MTR code. The difference in performance for a *PR*4 target is about 0.2 dB gain in the region where commercial products operate. Figure 6.4 shows the performance of the combined SFE-PR4-ML scheme with rate 4/5 MTR (2;8) code with respect to frame error rate versus SNR. It is observed that the FER improves consistently and as the SNR increases, the improvement is significant. For shorter length PR targets, as in the case of PR4, it is observed that MTR does not have significant improvement on the MTR coded PR4-ML channel. This is because the error events occurring in the PR4-ML channel are different than the restricted events of MTR.

The new scheme provides soft output from the MAP decoder and thus, it increases the possibility to use error correction codes. Figure 6.5 shows the EPR4 magnetic recording channel simulation results. It is seen that the new scheme outperforms the MTR-EPR4 ML scheme and the standard EPR4 ML scheme. The gain of the combined scheme in terms of dB at  $PW_{50} = 2.4$  is about 0.6 dB against MTR-EPR4 ML scheme and about 1.6 dB against a standard EPR4 ML scheme.

Comparing figures 6.3 and 6.5, it is observed that at higher densities and with longer PR targets, the new combined SFE-PRML-MAP scheme with MTR codes has a higher gain in comparison to lower/higher densities with shorter PR targets.

Furthermore, it is seen that the improvement in performance of the combined scheme



Figure 6.5: BER vs SNR Performance

Figure 6.6: BER vs SNR Performance

is a function dependent upon the SNR requirement as well as the length of PR targets.

A performance comparison with application of rate 5/6 MTR (2; 6) code to the combined SFE-PRML scheme and PRML scheme are shown in figures 6.6, 6.7 and 6.8.

The performance comparison in terms of BER versus  $PW_{50}$  shown in figures 6.7, 6.9 and 6.8 provides essential information that at what  $PW_{50}$ , a specific PR target would provide better estimation of ISI for the SFE block to work in an optimum manner. Comparison of figures 6.7 and 6.8 clearly shows the SNR dependency on the performance of the combined SFE-PRML scheme with MTR codes.

### 6.4 Summary

This chapter discussed the drawbacks of SFE based magnetic recording and presented a novel technique to overcome them. The combined PR-SFE-MAP scheme provided a consistent gain over standard PR-MAP and SFE-MAP schemes for longitudinal magnetic recording. This chapter also emphasised the use of MTR codes in order to achieve SNR gain. Below is a summary of some of the key results and important discussions presented in this chapter.

• The problem of thresholding is discussed characterising the difference in the thresholding point at various SNR. It is found that the optimum thresholding point is a



Figure 6.7: BER vs PW<sub>50</sub> Performance

Figure 6.8: BER vs PW<sub>50</sub> Performance

function of SNR. At lower SNR, the thresholding point is varying and at higher SNR, it becomes consistent.

- The use of PRML as an initial estimation device for the SFE scheme is shown and it is observed that it performs better than the SFE scheme discussed in Chapter 3.
- The combined binary SFE-PR-ML channel is discussed and it provides better performance than PRML or SFE based channels for magnetic recording.
- The use of MTR codes is emphasised and it is observed that for short length PR targets, MTR codes tend to be insignificant and there is not much improvement in performance of PRML channels.
- The problem of binary SFE based magnetic recording at higher densities is overcome using PRML as initial estimation instead of thresholding.
- Results shown in figures 6.3, 6.5, 6.6 dictate that the use of SFE-PR-ML scheme with various rates MTR codes is beneficial when compared to standard PRML scheme with MTR codes. For a BER of 10<sup>-5</sup>, the SFE-EPR4-ML scheme with rate 4/5 MTR (2;8) code has roughly about 0.5 dB SNR gain over EPR4-ML with the same MTR code.
- The gain in SNR comes from the length of the PR target chosen, as well as the amount of SNR required to reach the BER of 10<sup>-5</sup>.



Figure 6.9: BER vs  $PW_{50}$  Performance of combined PR-SFE-MAP scheme with rate 4/5 MTR (2;8) Code

- The output of the combined scheme being *soft*, it is expected that the performance of this scheme will be improved significantly with error correction.
- Application of Turbo and LDPC codes and implementation of multi-level magnetic recording using this scheme will be the future work of this scheme.

## Part V

# Conclusions
## Conclusions and Future Research Work

The present age of electronics which include various portable and large storage devices are a very good example of the ever improving magnetic recording technology. The best comparison that could be made is of the storage capacity of 3.5 in floppy drive and today's 3.5 in hard disk drive.

A major milestone in the fast development of magnetic recording was set by the work of Kobayashi and Tang (1970). Since then, the increase in areal densities achievable by magnetic recording has been much faster than Moore's Law. The development in the magnetic recording industry over a long period of time is supported by various channel model designs, equalisation techniques, development in error correction coding, head and media designs and detection techniques.

When considering the signal processing aspects of digital magnetic recording, there are various areas of interest which include the modulation and equalisation techniques, decoding processes, transition responses, consideration and treatment of noise and ISI in the channel, and various other factors. In this thesis, an in depth work on various signal processing and coding aspects has been carried out for both - longitudinal and perpendicular digital magnetic recording. This thesis discusses a major development, analysis and derivation of novel channel-based soft-feedback equalisation technique (Chapter 2 and Chapter 6) which addresses the noise colouration problem for magnetic recording channel.

Furthermore, it provides a background investigation into the use of multi-level magnetic recording, application of turbo codes as outer error correcting codes for magnetic recording and use of MTR codes as recording codes. Various developments in search for generalised partial response targets and decoding techniques for perpendicular and longitudinal magnetic recording have also been discussed.

Transition noise (jitter noise) occurs due to the magnetisation fluctuations concentrated at the centre of the recorded transitions. As the magnetic transitions are closely packed in each track in order to increase the linear density of a magnetic recorder, this noise increases tremendously causing problems at the decoding end of the channel. Transition jitter manifests itself as a pulse position jitter and pulse width/amplitude variation, and it contributes 80 - 90% of the readback data amplitude.

The transition jitter problem for perpendicular magnetic recording is studied and it is found that it is data-dependent and not additive as in the case of Gaussian noise. A new model of the MAP decoder called the *Asymmetric MAP decoder* is designed which includes jitter noise calculations in its branch metrics. It is observed that in conjunction with Turbo codes, for a maximum jitter of 60%, the asymmetric MAP decoder provides a substantial gain of the order of 1 magnitude over classical MAP decoders. This novel work was discussed in Chapter 2.

It is known that a generalised PR target with non-integer valued coefficients, can result in a considerably smaller equalisation loss and the overall gain can be substantial at high linear densities. In perpendicular magnetic recording, the main signal energy from the readback signal is at low frequencies. Thus, the GPR target for PRML based perpendicular magnetic recording do not contain the (1 - D) component of the PRML based longitudinal magnetic recording channel. In order to reduce the effect of equalisation loss, a new design of GPR targets is studied which is based upon maximising the ratio of minimum squared euclidean distance of PR targets to the noise penalty introduced by the PR filter (equaliser). Results indicating the improved performance in terms of BER have been presented and at a high density of  $PW_{50} = 1.4$ , the new [4, 10, 7, 2] optimised target is about 0.5 dB better than previously published length-4 integer GPR targets. Key results for the new optimised scheme have been presented in Figure 2.10.

In longitudinal PRML based magnetic recording, it is well known that the filter in the PR equaliser shortens the impulse response of the magnetic readback channel, and transforms the channel response into a generalised PR target, thus introducing noise colouration and enhancement. This noise colouration is dependent upon the  $PW_{50}$ , i.e. the density of the magnetic channel. This dependence is proportional to  $PW_{50}$ , i.e. as the density of the magnetic channel increases, the noise colouration increases. The noise colouration introduced by the PR filter grows as the  $PW_{50}$  changes due to the non-linearity of the magnetic recording channel. Assuming there is no colouration of noise, this also affects the performance of Viterbi-based decoding algorithms used in magnetic recording channels.

In order to overcome these non-linearities and dependencies, a new approach is discussed which uses linear filters with different non-linear algorithms to estimate and remove ISI in an iterative manner. This new approach is called *Soft Feedback Equalisation (SFE)* technique for longitudinal magnetic recording. This new technique for readback process in magnetic recording is based upon decision feedback principles. In this process, a linear modified Lorentzian filter is used to estimate the amount of ISI introduced in the magnetic channel which is iteratively removed in the feedback process. The novelty of the SFE based scheme is that it uses both the MLSD and MAP decoder iteratively in order to obtain the best performance. The benefits of using SFE based magnetic recording is that there is no noise colouration as in the case of the PRML based magnetic recording and the channel remains in its original form, providing a good input to the outer error correction block. Although, due to the non-linearity of the magnetic channel, the SFE based magnetic recording faces the problem of residual ISI. Various results have been discussed and it is observed that for densities higher than 1.8, PRML based magnetic recording outperforms SFE based magnetic recording. This drawback comes from the thresholding device used to obtain the initial estimate of ISI in the feedback process and the dicode nature of the SFE channel. These results have been discussed in Chapters 2 and 3.

For a long time since Mackintosh and Jorgensen (1981), multi-level magnetic recording was forgotten assuming that it was not possible due to the increase in the SNR requirements. After careful considerations to the previous work, the work described in this thesis answers the question to why multi-level magnetic recording is beneficial and discusses the improvement in performance obtained. Various PRML and SFE based multi-level techniques have been studied and it is observed that in order to improve the performance of multi-level magnetic recording, it is necessary to use powerful error correction codes. Turbo codes have been used as error correcting codes for both - binary and multi-level PRML/SFE based magnetic recording. Chapter 5 provides a background to error correcting codes and discusses Turbo codes in detail. Application of Turbo codes over various longitudinal magnetic recording channel configurations is investigated and various observations have been made.

Two interesting observations were made during the study of application of turbo codes with various PRML/SFE based techniques. They are:

- The error floor region beyond 10<sup>-6</sup> BER for the turbo coded performance could be because of the type of interleaver used S-random, the inner concatenation of the component codes used for turbo codes and the non-integer PR-target polynomials, the length of the PR target used or the amount of ISI introduced due to the higher density of binary coded channels.
- For PRML based turbo-coded magnetic recording, it is observed that the GPR targets found using the new GPR target design method, provides a type of inner concatenation with the component codes which improves the performance of multi-level and binary PRML based magnetic recording channel significantly.

The above mentioned observations can be found in Figures 5.10, 5.11 and 5.12. The types of interleavers used for the study of turbo codes for magnetic recording are S-random and DRP interleavers. It is seen that DRP interleaver has better performance than S interleaver over the PRML/SFE based magnetic recording channel. Another observation made is that the error floor of the binary coded magnetic recording channel and the 4-level coded magnetic recording channel is in the same region and as a result, it is better to use 4-level coded channel in order to work closer to the channel capacity for magnetic recording. For a typical magnetic recording channel with the operating region SNR of 19 - 22 dB, it is observed from Figure 3.1 that 4-level magnetic recording is closer by 2.17 dB in comparison to binary magnetic recording to the channel capacity.

The use of Maximum Transition Run (MTR) codes for PRML/SFE based magnetic recording channels is studied in Chapter 4. MTR codes are used in order to eliminate the bit patterns which occur repetitively at the input so that most certain error patterns can be eliminated from being produced. They eliminate input patterns that contain 3 or more consecutive transitions in the corresponding input waveform. Results show that rate 4/5 MTR (2;8) code performs better than rate 5/6 MTR (2;6) code for binary PRML based magnetic recording. The performance comparison between MTR coded PRML and SFE magnetic recording indicates that beyond  $PW_{50}$  of 1.3, the SFE-MAP magnetic recording channel does not operate well. This limitation is due to the dicode nature of the SFE based magnetic recording channel. In PRML based magnetic recording, the  $(1 + D)^n$  operation in equalisation, where  $n = 1, 2, 3, \cdots$  provides information about the transition sample being spread over neighbouring bit periods. In SFE based magnetic recording, this information is not provided and hence the (1 - D) dicode nature of the SFE based magnetic recording channels is a drawback which needs investigation.

The thresholding device used in SFE based magnetic recording does not have a fixed point at different SNR's. The thresholding point is fluctuating based upon the SNR of the magnetic channel. This discussion is made in Chapter 6. The novel idea of combining PRML and SFE based magnetic recording together to achieve significant gain over standard PRML based magnetic recording is studied in this thesis.

In order to overcome the loss incurred by the thresholding device used for initial estimation of ISI in SFE based magnetic recording, a new type of method is designed which involves the use of the PRML detector as an initial estimation device and use of SFE-MAP algorithm for final detection. The PRML detector replaces the thresholding device as an initial estimator of ISI and thus provides a gain of about 0.6 dB over standard PRML/SFE based magnetic recording. This gain comes with the use of MTR codes as well as the length of the GPR target chosen for the PRML detector to make its initial estimate. Figures 6.3 and 6.6 present the above mentioned SNR gain achieved using the novel combined PR-SFE-MAP scheme for longitudinal magnetic recording. It can be said that this novel technique of longitudinal magnetic recording is the ultimate solution to achieve high density recording.

Several problems have arisen that have not been answered during the course of this work. Also, new ideas which suggest several areas for further investigation have not been discussed in depth. These topics have been proposed as potentially important topics for future work and are listed as below:

 In perpendicular magnetic recording, the transition jitter noise can be accounted to be 80 – 90% of the readback data amplitude. The transition jitter noise has been studied and has been found that it is not strictly Gaussian, due to the non-linear effect of ISI. Future work can investigate on the use of more accurate probability density functions of transition jitter in order to obtain accurate performance. This results in further investigation into transition jitter's data dependency and modification to the current implementation of asymmetric MAP decoder.

- Media noise in perpendicular magnetic recording comes from the imperfections of the media and its effect is significant in the magnetic transition regions. Typically, media noise can be accounted to be approximately four times the electronics noise in the transition regions. In order to obtain more accurate GPR targets, it is necessary to take into account media noise and based upon the calculations of AWGN and media noise, the overall filter penalty can be calculated and optimum GPR targets can be achieved. This direction of research in media noise for search of optimum GPR targets is the extension of the work presented in this thesis.
- A new method of equalisation described as Soft Feedback Equalisation is studied using longitudinal magnetic recording channel model. The same SFE scheme can be applied to perpendicular magnetic recording channel model and it is expected that it would perform better than longitudinal magnetic recording since the main readback signal energy for perpendicular magnetic recording lies in the low frequency components. This new strategy of using SFE based perpendicular magnetic recording is worth investigation in order to obtain higher recording densities.
- A major area of investigation for the work presented would be to understand the error patterns produced by various PRML/SFE based multi-level longitudinal magnetic recording channels and to design new MTR codes for them. This would in return provide with a new class of MTR codes specifically for multi-level magnetic recording, which could be presented to the tape-drive industry for verification purposes.
- Various schemes for SFE/PRML based longitudinal magnetic recording have been discussed in this work. It is worth continuing further investigation into the combined PRML and SFE based MTR coded magnetic recording which would facilitate the magnetic tape-drive industry to obtain higher recording densities and thus, progress in overall tape-data storage capacity. Application of multi-level magnetic recording, error correcting codes for this scheme would be an interesting area of future investigation.
- The ease of implementation for the combined PRML-SFE based MTR coded magnetic recording comes from the fact that the existing system already has a PRML detector, and thus, an additional SFE block is required as the final decoding block. In addition to the SFE block, another requirement is to use a buffer, in order to treat the PRML detected data to estimate the amount of ISI introduced in the channel. Thus, implementation of the combined PRML-SFE based MTR coded magnetic recording channel into a programmable chip and investigating the performance with real tapedrive data would be essential in order to fully explore this advantageous scheme. The

reduced noise colouration of the combined PRML-SFE based scheme could result in better performance from the overall concatenated coding scheme.

## Part VI

# Appendices

## Matched Filtering and Frequency Response

### A.1 Matched Filter Output for Lorentzian Response

The lorentzian response can be defined as:

$$p(x) = \frac{1}{1 + (\frac{2x}{PW_{50}})^2}$$

The matched filter output for the lorentzian response can be given as:

$$C(X) = \int_{-\infty}^{\infty} \frac{1}{1 + (\frac{2t}{PW_{50}})^2} \cdot \frac{1}{1 + (\frac{2(t-x)}{PW_{50}})^2} dt$$

Let  $\alpha$  be  $\frac{2t}{PW_{50}}$ . Thus,  $d\alpha = \frac{2}{PW_{50}}dt$ . Let a be  $\frac{2x}{PW_{50}}$ .

$$C(X) = \frac{PW_{50}}{2} \int_{-\infty}^{\infty} \frac{1}{1+\alpha^2} \cdot (\frac{1}{1+(\alpha-a)^2}) d\alpha$$
$$= \frac{PW_{50}}{2} \int_{-\infty}^{\infty} \frac{1}{\alpha^4 - 2a\alpha^3 + \alpha^2(a^2+2) - 2a\alpha + (1+a^2)} d\alpha$$

Applying the above equation to (Wolfram Mathworld Website; 2005),

$$\begin{split} C(X) &= \frac{PW_{50}}{2} \cdot \left[ \frac{-1}{4a + a^3} \left[ a \cdot \arctan(a - \alpha) - a \cdot \arctan(\alpha) \right] \right]_{-\infty}^{\infty} \\ &- \frac{PW_{50}}{2} \cdot \left[ \frac{-1}{4a + a^3} \left[ \log(1 + \alpha^2) - \log(1 + a^2 - 2a\alpha + \alpha^2) \right] \right]_{-\infty}^{\infty} \end{split}$$

Applying the limits of  $\infty$  and  $-\infty$ , the log terms cancel each other.

$$C(X) = \frac{PW_{50}}{2} \cdot \left[\frac{-1}{4a+a^3} \cdot \left[-a\frac{\pi}{2} - a\frac{\pi}{2} - a\frac{\pi}{2} - a\frac{\pi}{2}\right]\right]$$

Thus,

$$C(X) = \pi P W_{50} \cdot a \cdot \frac{1}{4a + a^3}$$

Substituting back a to be  $\frac{2x}{PW_{50}}$ ,

$$C(X) = \frac{\pi}{4} \cdot \left[ \frac{PW_{50}^3}{PW_{50}^2 + x^2} \right]$$

### A.2 Frequency Response for a Lorentzian Dibit Pulse

The frequency response of dibit lorentzian pulse can be obtained using (1.4). The dibit response can be given as:

$$d(t) = \frac{1}{1 + (\frac{2t}{PW_{50}})^2} - \frac{1}{1 + (\frac{2(t-1)}{PW_{50}})^2}$$

The Fourier transform for an isolated lorentzian waveform is given as:

$$H(\omega) = \int_{-\infty}^{\infty} \frac{1}{1 + (\frac{2t}{PW_{50}})^2} \cdot e^{-j\omega t} dt$$

Substituting  $\frac{2t}{PW_{50}}$  as  $a, dt = \frac{PW_{50}}{2}da$ .

$$\therefore \qquad H(\omega) = \frac{PW_{50}}{2} \int_{-\infty}^{\infty} \frac{e^{-j\omega x} \frac{PW_{50}}{2}}{1+a^2} da$$

Substituting  $e^{-j\omega t}$  as  $\cos(\omega t) - j\sin(\omega t)$ , the above equation becomes,

$$H(\omega) = \frac{PW_{50}}{2} \int_{-\infty}^{\infty} \frac{\cos\left[\frac{PW_{50}}{2}\omega a\right]}{1+a^2} da$$
$$-j \quad \frac{PW_{50}}{2} \int_{-\infty}^{\infty} \frac{\sin\left[\frac{PW_{50}}{2}\omega a\right]}{1+a^2} da$$

Considering the anti-symmetrical properties of the lorentzian response (Ahmed; 2003a), the imaginary term is 0.

$$H(\omega) = \frac{PW_{50}}{2} \int_{-\infty}^{\infty} \frac{\cos\left[\frac{PW_{50}}{2}\omega a\right]}{1+a^2} da$$

Thus, from (Ahmed; 2003a), the above equation becomes,

de l

$$H(\omega) = \frac{PW_{50}}{2}\pi \cdot e^{-\omega} \frac{PW_{50}}{2}$$

Thus, the Lorentzian dibit response  $D(\omega)$  can be obtained using the above equation.

$$D(\omega) = (1 - e^{-j\omega T}) \frac{PW_{50}}{2} \pi \cdot e^{-\omega} \frac{PW_{50}}{2}$$

where,  $\omega$  is the frequency term and *T* is the sampling period.

### A.3 Frequency Response for Partial Response Equalisation

The PR4 frequency response described in equation(2.7) can be obtained as shown below.

The standard class of PR systems is defined by Thapar and Patel (1987). The further generalisation of the class of PR systems can be described as below:

$$S(D) = (1 - D)^m (1 + D)^n$$

where m and n are integer values  $\in \{0, 1, 2, 3, \dots\}$ .

The Frequency response to the above equation can be given by substituting |D| as  $e^{-j\omega T}$ , where  $\omega$  is the frequency term and T is the sampling period.

$$\begin{split} S(\omega) &= (1 - e^{-j\omega T})^m (1 + e^{j\omega T})^n \\ &= \left[ (e^{+j\frac{\omega T}{2}} - e^{-j\frac{\omega T}{2}}) e^{-j\frac{\omega T}{2}} \right]^m \cdot \left[ (e^{+j\frac{\omega T}{2}} + e^{-j\frac{\omega T}{2}}) e^{+j\frac{\omega T}{2}} \right]^n \end{split}$$

Using trigonometric identities,

$$\begin{split} S(\omega) &= \left[ +j2\sin\left(\frac{\omega T}{2}\right) \right]^{m} \cdot e^{-jm\frac{\omega T}{2}} \left[ 2\cos\left(\frac{\omega T}{2}\right) \right]^{m} \cdot e^{+jn\frac{\omega T}{2}} \\ S(\omega) &= e^{+j(n-m)\frac{\omega T}{2}} \cdot \left[ +j2\sin\left(\frac{\omega T}{2}\right) \right]^{m} \cdot \left[ 2\cos\left(\frac{\omega T}{2}\right) \right]^{n} \\ &|S(\omega)| = \left[ 2\sin\left(\frac{\omega T}{2}\right) \right]^{m} \cdot \left[ 2\cos\left(\frac{\omega T}{2}\right) \right]^{n} \end{split}$$
(A.1)

$$\arg(S(\omega)) = (n-m)\frac{\omega T}{2}$$
(A.2)

The above equations (A.1) and (A.2) can be used to evaluate the frequency response of any PR system, given m and n.

For a PR4 system, the frequency response can be calculated based on m = 1 and n = 1. The magnitude of the frequency response is given as :

$$S(\omega) = \left[2\sin(\frac{\omega T}{2})\right] \left[2\cos(\frac{\omega T}{2})\right]$$
$$= 4\sin(\frac{\omega T}{2})\cos(\frac{\omega T}{2})$$
$$= 2\sin(\omega T)$$

### Maximum Likelihood and Mean Squared Error

### **B.1 Equivalence of MLSD and Viterbi Decoding**

The ML decoder is defined as the decoder which minimises the error probability for the case where all transmitted sequences are equally likely (Sklar; 2001). If the information sequence is defined as  $\{c_j\}$  and the output from the equaliser is defined as  $b_j$ , which is the MLSD input, then according to the MLSD rule, the decoder chooses the trellis path which defines the sequence such that:

$$\max\{\prod_{i=0}^{n-1} P(b_i = c_j)\}$$

where,  $c_j \in \{0, 1\}$  and n is the number of input bits. Applying probability density function equation, the above equation is equivalent to:

$$\max\left[\prod_{i=0}^{n-1} \frac{k}{\sqrt{2\pi\sigma}} \cdot e^{\frac{(b_i - c_j)^2}{2\sigma^2}}\right]$$

where,  $\sigma$  is the standard deviation of the Gaussian noise.

The value of  $\frac{k}{\sqrt{2\pi\sigma}}$  is insignificant since it a constant and it does not affect the maximum value in the equation. Applying log to the above equation, we obtain:

$$\max\left[\log\left[\prod_{i=0}^{n-1}e^{-\frac{(b_i-c_j)^2}{2\sigma^2}}\right]\right]$$

Assuming  $2\sigma^2$  as constant and since it will not affect the maximised value, the above equation simplifies to:

$$\max\left[\sum_{i=0}^{n-1} -(b_i - c_j)^2\right]$$

 $(b_i - c_j)^2$  will always be positive and  $-(b_i - c_j)^2$  will always be negative.

Thus,  $\forall j$ , the maximisation of a set of negative numbers will be the minimisation of positive numbers. Thus, the above equation becomes:

$$\min\left[\sum_{i=0}^{n} (b_i - c_j)^2\right]$$

Thus, piece-wise minimisation of probability in log domain is Viterbi algorithm, which is equivalent to maximum sequential probability, where maximum sequential probability solution leads to MLSD solution.

The output from the MLSD decoder is always a codeword/sequence with highest probability. MLSD contains a list of all possible codewords leading to the sum of the probability of the possible codewords being not equal to 1.

### **B.2 Mean Squared Error Criterion**

Consider the block diagram of a channel with a zero forcing equaliser.



Figure B.1: Block Diagram of Channel with Equivalent Zero-Forcing Equaliser (Proakis; 1995)

Assume an information sequence as  $\{a_k\}$ . Let the output of the filter be denoted as  $\{\tilde{a}_k\}$ .

The tap weight coefficients  $\{c_j\}$  of the filter are adjusted such that they minimise the mean square value of the error:

$$e_k = a_k - \hat{a}_k$$

where,  $a_k$  is the information symbol transmitted in the *k*-th signalling interval and  $\hat{a}_k$  is the estimate of that symbol at the output of the equaliser.

This estimate of the information symbol is defined as:

$$\hat{a}_k = \sum_{j=-K}^{K} c_j \cdot v_{k-j}$$

 $\{c_j\}$  are the (2K+1) complex-valued tap weight coefficients of the filter. The estimate  $\dot{a}_k$  is quantised to the nearest information symbol to form the decision  $\vec{a}_k$ . If  $\vec{a}_k$  is not identical to the information symbol, then an error occurs.

The performance index of the mean squared error (MSE) criterion is denoted by J and defined as:

$$J = E|e_k|^2 = E|a_k - \hat{a}_k|^2$$

For an infinite-length equaliser, the estimate  $\hat{a}_k$  can be expressed as:

$$\bar{a}_k = \sum_{j=-\infty}^\infty c_j \cdot v_{k-j}$$

Applying the orthogonality principle in mean square estimation, a set of linear equations can be obtained. The selection of  $\{c_j\}$  is done in such a way that the error  $e_k$  is orthogonal to the signal sequence  $\{v_{k-l}^*\}$  for  $-\infty < l < \infty$ .

Thus,

$$E(e_k v_{k-l}^*) = 0$$

Substituting the above equation into the performance index equation,

$$\sum_{-\infty}^{\infty} c_j \cdot E(v_{k-j} v_{k-l}^*) = E(a_k v_{k-l}^*)$$
(B.1)

where,  $-\infty < l < \infty$ .

The output from the noise whitening filter  $v_k$  can be expressed as:

$$v_l = \sum_{n=0}^{L} f_n a_{k-n} + w_k$$

where,  $\{w_k\}$  is a AWGN sequence and  $\{f_k\}$  is the set of tap coefficients of an equivalent discrete-time transversal filter having the transfer function F(z).

Thus, the performance index becomes:

$$E(v_{k-j}v_{k-l}^*) = \sum_{n=0}^{L} f_n^* f_{n+l-j} + N_0 \delta_{lj}$$

and

$$E(a_k v_{k-l}^*) = \begin{cases} f_l^* & (-L \le l \le 0) \\ 0 & (otherwise) \end{cases}$$

Substituting the above equation into (B.1) and applying z transform on both sides of the equation,

$$C(z)\left[F(z)F^{*}(z^{-1}) + N_{0}\right] = F^{*}(z^{-1})$$

Thus, the transfer function of the equaliser based on the MSE criterion becomes:

$$C(z) = \frac{F^*(z^{-1})}{F(z)F^*(z^{-1}) + N_0}$$

When the noise whitening filter is incorporated into the equaliser and the equivalent equaliser transfer function is obtained. The equivalent equaliser transfer function is given as:

$$C(z) = \frac{1}{X(z) + N_0}$$

where,  $F(z)F^*(z^{-1})$  is equivalent to X(z).

When  $N_0$  is very small in comparison to the signal, the coefficients that minimise the peak distortion are approximately equivalent to the coefficients that minimise the MSE performance index J (Proakis; 1995).

The measure of residual ISI and additive noise is obtained by evaluating the minimum value of J, given as  $J_{min}$ . Applying orthogonality principle and previously defined  $J = E|e_k|^2$ ,  $J_{min}$  is defined as:

$$J_{min} = E(e_k a_k^*)$$
  
=  $E|a_k|^2 - \sum_{-\infty}^{\infty} c_j E(v_{k-j} a_k^*)$   
=  $1 - \sum_{-\infty}^{\infty} c_j f_{-j}$ 

Applying contour integral and a transforms, the above equation simplifies to:

$$Jmin = \frac{N_0}{1+N_0}$$

The above equation only holds true when there is no ISI.

For a finite-length equaliser, the minimum value of J(K) is given as  $J_{min}(K)$ , where the filter has (2K + 1) tap weight coefficients.

$$J_{min}(K) = 1 - \sum_{j=-K}^{0} c_j f_{-j}$$

## LMS Algorithm

### C.1 Derivation of LMS Update Equation

The derivation of the LMS update equation is based upon the simulation model below.



Figure C.1: Simulation Model of SFE-LMS Multilevel Magnetic Recording System

Consider the filter input sequence  $\bar{r}(t)$  with N tap coefficients  $w(-\frac{N-1}{2}), \ldots, w(\frac{N-1}{2})$ , where N is a positive odd integer and output  $\{\hat{a}_k\}$ . Thus,

$$\hat{a}_k(t) = \sum_{k=0}^{N-1} \hat{r}(t-k) \cdot w(k)$$
(C.1)

From Figure C.1, the output from the second lorentzian filter is  $\hat{r}(t)$ , the aim is to minimise the  $E\left[e^2(t)\right]$  varying the filter coefficients only, where  $E\left[\cdot\right]$  is the instantaneous expectation value. In the above Figure C.1, the filter coefficients remain the same, but the estimated input sequence  $\{\bar{a}_k\}$  is changed/equalised. The assumption of LMS algorithm is that there is only one global minima of the expected squared error from the filter (Proakis; 1995). In the SFE-LMS scheme, this minima exists in the expected estimated sequence  $\{\hat{a}(k)\}$ .

$$\hat{a}_{k}(t+1) = \tilde{a}_{k}(t) - \frac{E\left[e^{2}(t)\right] - \min\{E\left[e^{2}(t)\right]\}}{\frac{d}{d\hat{a}_{k}(t)}\{E\left[e^{2}(t)\right] - \min\{E\left[e^{2}(t)\right]\}}$$
(C.2)

#### Appendix C. LMS Algorithm

Assuming min  $E[e^2(t)]$  is a constant  $c_r$ 

$$\ddot{a}_{k}(t+1) = \ddot{a}_{k}(t) - \frac{E\left[e^{2}(t)\right] - c}{\frac{d}{d\hat{a}_{k}(t)}E\left[e^{2}(t)\right]}$$
(C.3)

#### **Assumption 1:**

The square of the current error sample is approximately equal to the average of all the squared error samples. Thus,

$$\frac{\mathbf{d}}{\mathbf{d}\hat{a}_k(t)} E\left[e^2(t)\right] \approx \frac{\mathbf{d}}{\mathbf{d}\hat{a}_k(t)} e^2(t) = 2\frac{\mathbf{d}}{\mathbf{d}\hat{a}_k(t)} e(t)$$

#### **Assumption 2:**

Assuming the expected value of  $e^2(t)$  is same as the instantaneous value of  $e^2(t)$ , the equation(C.3) becomes:

$$\hat{a}_{k}(t+1) = \hat{a}_{k}(t) - \frac{e^{2}(t) - c}{\frac{d}{d\hat{a}_{k}(t)}e^{2}(t)}$$

$$= \tilde{a}_{k}(t) - \frac{e(t) - \frac{c}{e(t)}}{2\frac{d}{d\hat{a}_{k}(t)}e(t)}$$

$$= \tilde{a}_{k}(t) - \frac{e(t) - \frac{c}{e(t)}}{2\frac{d}{d\hat{a}_{k}(t)}e(t)}$$
(C.4)

b(t) is independent of  $a_k$ .

**Assumption 3:** Replacing the derivative with partial derivative, and bringing the negative sign of  $\hat{r}(t)$  in denominator to numerator, the equation(C.5) becomes:

$$\begin{aligned} \hat{a}_k(t+1) &= \hat{a}_k(t) + \frac{e(t) - \frac{c}{e(t)}}{2\frac{\partial}{\partial \hat{a}_k(t)}\hat{r}(t)} \\ &= \bar{a}_k(t) \\ &+ \frac{e(t) - \frac{c}{e(t)}}{2\frac{\partial}{\partial \hat{a}_k(t)} \left[ \hat{a}_k(0)w(-(N-1)/2) + \dots + \hat{a}_k(N-1)w((N-1)/2) \right]} \end{aligned}$$

Simplifying the above equation, following equation is obtained:

$$\hat{a}_k(t+1) = \hat{a}_k(t) + \mu e(t) \left[ w(-(N-1)/2) \dots w((N-1)/2) \right] - \kappa$$
(C.5)

where,

$$\kappa = \frac{c}{e(t) \left[ w(-(N-1)/2) \dots w((N-1)/2) \right]^T}$$
(C.6)

and

$$\mu = \frac{1}{2\left[w(-(N-1)/2)^2 + \ldots + w((N-1)/2)^2\right]}$$
(C.7)

**Assumption 4:** Assuming that there will be no error in the new estimated value of  $\hat{a}_k$ , the assumptions c=0 makes the constant  $\kappa$  equal to 0, and this simplifies the expression. Thus, the final LMS update equation becomes:

$$\hat{a}_k(n+1) = \hat{a}_k(t) + \mu e(t) \left[ w(-(N-1)/2) \dots w((N-1)/2) \right]$$
(C.8)

Since the above update equation equates to a constant which is between 0 and 1, in the simulations, just a constant was multiplied with error and the results were obtained for different values of this constant and the best value was chosen. As such from the above equation (C.7), it is seen that  $\mu$  depends upon the  $PW_{50}$ , since in its equation, the lorentzian filter coefficients are used. As a result, in order to remove this dependency, a different technique which was independent of  $PW_{50}$  and just used a constant between 0 and 1 was designed and implemented. Of all the constants, the best one found was 0.7. In order to justify the use of 0.7 as the constant, a simulation was performed with different values of k for a SFE based dicode channel with LMS feedback at  $PW_{50}$  of 1.5. Figure C.2 shows the BER vs SNR performance of the channel with different values of k.

The comparison between these two constants is as shown below in table(C.1). In the table, the constant used in the simulated system is considered to be 0.7.



Figure C.2: Error Rate vs SNR Performance of SFE with LMS Feedback for Different values of k

| Constant  | Actual $\mu$                          | Actual Constant Used in system |
|-----------|---------------------------------------|--------------------------------|
| $PW_{50}$ | · · · · · · · · · · · · · · · · · · · |                                |
| 0.5       | 0.492565                              | 0.7                            |
| 1.0       | 0.421500                              | 0.7                            |
| 1.2       | 0.372605                              | 0.7                            |
| 1.5       | 0.295506                              | 0.7                            |
| 2.0       | 0.192119                              | 0.7                            |

Table C.1: Comparison of actual  $\mu$  and constant used in simulated system at different  $PW_{50}$ 

### Part VII

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### Part VIII

## **Publications**

## Conferences

#### ASYMMETRIC MAP DECODING FOR PERPENDICULAR MAGNETIC

#### RECORDING WITH DATA DEPENDENT NOISE

M.Z. Ahmed<sup>1</sup>, A. Fahrner<sup>2</sup> and P. Shah<sup>1</sup> <sup>1</sup>CR1ST, University of Plymouth, PL4 8AA Devon, UK

<sup>2</sup>TAIT, University of Ulm, D-89081 Ulm, Germany

Introduction : This paper presents the Asymmetric Decoder (AD) ; The AD application of a new MAP algorithm to perpendicular recording in the presence of jitter Respecting the limit on references, the noise. following four references are provided as the fundamental basis for this work. For Maximum A Posteriori (MAP) decoding on trellis's[1] and previous work on jitter noise for longitudinal magnetic recording[2]. For the perpendicular channel model, we assume a hyperbolic tangent readback signal from an isolated transition u(t), given by

$$u(t) = \tanh\left(\ln(3)\frac{t}{D_{50}}\right) \tag{1}$$

, where  $D_{50}$  is the normalised user density [3] and the target Partial Response given in [4].

We will present a modified trellis based algorithm that accounts for the differences between electronics and jitter noise in the metric computations. The fact that the jitter noise is greater in transitions compared to where there are no transitions can result in improved reliability of the metric computation. It has been found that this provides gains in performance over classical MAP algorithms that assume the noise is not data dependent.



Fig. 1 : Simulation Block Diagram

The error correction code (ECC) is a (4096,3072) LDPC code. The received signal r(t) can be described by the following equation

$$r(t) = h(t) * (s(t) + j(t) + n(t))$$
(2)

where h(t) is the impulse response of the PR equaliser, s(t) is the channel readback waveform (with ISI), j(t) is the transition jitter noise and n(t) is AWGN, or the electronics noise and \*denotes convolution. We use noise prediction within the decoder, resulting in the Log-MAP decoded signal being described as

$$r(t) = h(t) * s(t) + j(t) + n(t)$$
(3)

includes 1 addition and 1 mul per branch metric computation in addition to the classic Log-MAP trellis decoder. Additionally, the decoder uses the input branch labels to determine if a particular path would include transition noise, so two consecutive state needs to be considered before the state metric is updated. This increases the computational complexity will be exponential as more terms around the transition are considered, and is seen as a limitation for higher  $D_{50}$ . The results compare fixed electronics SNR, and varying the maximum transition jitter within a sector of 4096 bits are shown for GPR[0.74, 0.83, 0.33, 0.08, 0.01] at  $D_{50} = 1.0$ .





BER vs. maximum jitter, measured after ECC. The transition jitter is specified by  $t_{max}/T$  where  $t_{max}$  represents the maximum deviation in a bit period T. The variance of n(t) for Fig 2a and Fig 2b are different, and were chosen for convenience. Conclusion and Future Work : Results show a consistent improvement over BCJR with the use of ECC, however improvements are also dependant on the percentage of transitions, and show a maximum at 60%. Of interest is the fact that although the improvement in channel BER is not very much, the gain in decoded BER (Fig 2b) is 1 order of magnitude for large  $t_{max}/T$ . The jitter noise has been found not to be strictly Gaussian, due to the non-linear effect of ISI, and the future work will investigate on the use of more accurate probability density functions to decode it.

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# NOVEL MULTI-LEVEL MAGNETIC RECORDING USING MODERN ERROR CORRECTION

P. Shah<sup>1</sup>, M.Z. Ahmed<sup>1</sup>, M. Ambroze<sup>2</sup>, C. Tjhai<sup>2</sup>, P.J. Davey<sup>1</sup>

<sup>1</sup> Centre for Research in Information Storage Technology, University of Plymouth, Plymouth PL4 8AA, UK, <sup>2</sup> Fixed and Mobile Comms. Research, University of Plymouth, Plymouth PL4 8AA, UK.

Email: {puray.shah.m.ahmed.m.ambroze.ctihai.p.davey}@plymouth.ac.uk

# Introduction

It has been shown that multilevel techniques operate closer to the channel capacity than binary for a bandwidth limited channel experiencing Additive White Gaussian Noise (AWGN) at increased Signal to Noise Ratio (SNR) [1]. Previous work[2] suggested that multi-level techniques, offered little, if any improvement of the magnetic recording capacity compared to the binary(two-level) system, and is eventually limited by amplitude irregularities in the magnetic channel. This paper looks at a new approach of applying powerful Error Correction Codes (ECC) on the multi-level magnetic recording channel and investigating the improvement in the performance.

The main idea behind multilevel recording is to enable storing of more information bits per transition on the magnetic medium. Several magnetisation levels could be used with the multi-level channel. It is known that at higher code rates for AWGN channels, binary codes tend to deviate very quickly from their theoretical performance[1]. In order to achieve very low error-rates at a particular SNR, it is necessary to use state of the art ECC like Turbo Codes. This paper examines the use of multilevel data in conjunction with Turbo codes, for a high density magnetic recording channel, to achieve increased channel capacity for a particular SNR in the operating region of the magnetic recording devices.

# Simulation Model

Figure(1) shows a multi-level approach of a basic Partial Response - Maximum-A-Posteriori(PR-MAP) system with ECC. A complete software simulation of the PR channel with AWGN noise is performed using turbo codes as the ECC. The recording channel being assumed to be longitudinal, where the readback voltage from an isolated transition is approximated by a Lorentzian function with  $PW_{50}/T$  as the normalised recording density. The readback pusles obtained after adding the AWGN are equalised using the best Generalised Partial Response (GPR) target. This controls the Inter Symbol Interference(ISI) introduced by the Lorentzian function. Finally the PR equalised data is decoded using a trellis based MAP(BCJR) decoder [3].

**Turbo Code Specifications:** The outer ECC codes used for the simulation is 1/3 rate turbo code. The design of turbo codes is achieved using tail-biting recursive systematic convolutional codes with feed-forward polynomial  $F_f = [37]_8$  and feed-back polynomial  $F_b = [23]_8$  for an overall rate 1/3 turbo code. The turbo decoder is iterative parallel concatenated MAP decoder with extrinsic information exchange. The interleaver used is a S-random interleaver[4]. The block length is set to 500 information bits and the maximum number of iterations is set to 50. At least 100 error blocks were collected for each BER point.

Denoting the code rate of the ECC as  $R_1$  and the code-rate for the 4-level system as  $R_2 = 2$ , the overall code rate of the 4-level system is

$$R = R_1 \times R_2 = 2R_1 \tag{1}$$

The channel SNR definition used for the system in the simulations is

$$SNR_{channel} = 10\log_{10}\left(\frac{1}{2\sigma^2}\right) \tag{2}$$

where  $\sigma$  is the standard deviation of the Gaussian Noise distribution.

# **Results and Discussions**

The initial exhaustive search for the best GPR target gave the polynomial of the type  $(1 - 0.8D - 0.2D^2)$  for 4-level recording for a  $PW_{50}=1.2$  and  $(1 - 0.5D - 0.5D^2)$  for 2-level(binary) recording for a  $PW_{50}=2.4$ . Results



Figure 1: Simulated Multilevel Magnetic Recording System



Figure 2: Error Rate vs SNR Performance Comparison of Multilevel Magnetic Recording Systems

shown in figure(2) show the variation of error rate on SNR for different configurations of the multi-level recording system. For an uncoded 4-level system, the desired base error rate of  $10^{-5}$  is achieved at 25 dB SNR. To achieve the BER, the 4-level system with ECC requires almost 10 dB less channel SNR compared to the uncoded 4-level system. Also, it is seen that the coded binary system needs 6 dB less SNR than the coded 4-level system. The error floor of binary coded and 4-level coded systems is similar. At lower density, the colouration of the AWGN introduced by the PR equaliser would be less. Also, the use of multi-level signalling enables the use of lower rate ECC with larger minimum distances. In the normal operating region (19-22dB channel SNR) of the magnetic recording systems, it is seen from figure(2) that if both the binary and 4-level system performance curves are extended for a higher SNR, the error flow rate merges. This error floor merge is caused by the ECC properties and not by the modulation technique used. Hence, since both binary and multi-level coded systems have similar performance, at higher SNR, multi-level system works much closer to the capacity than the binary system.

# **Conclusions and Future Work**

Simulation results were presented for the binary and multilevel, coded and uncoded PRML systems. Application of multilevel signalling has been shown to be beneficial, when the operating region of magnetic devices and low error rates were discussed. The error floor region which is the operating region of magnetic recording devices was shown and it was presented that the binary coded system had similar error floor to that of 4-level coded system. Lower rate codes with better ECC properties used with multilevel signalling and lower recording densities could have better performance compared to higher rate binary coded systems with high recording densities. Also the use of GF(4) codes could give better performance than the GF(2) codes used in the simulations. This could be a possible enhancement of the system described in this paper, resulting in improvement for multi-level magnetic recording systems incorporated with ECC.

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# Novel Soft-Feedback Equalisation Method for Multilevel Magnetic Recording

P. Shah\*, M. Ahmed\*, M. Ambroze<sup>†</sup>, C. Tjhai<sup>†</sup>, P. J. Davey\*

\*Centre for Research in Information Storage Technology, University of Plymouth, UK.

Fixed and Mobile Communications Research, University of Plymouth, UK.

Email: {puray.shah,mahmed,m.ambroze,ctjhai,p.davey}@plymouth.ac.uk

Abstract-This paper investigates the use of multi-level modulation for magnetic recording using a novel Soft-Feedback Equalisation (SFE) approach. Different aspects of investigation are: 1) Multilevel Recording, 2) SFE and 3) Application of Turbo Codes. The SFE scheme is a model in which the partial response (PR) equaliser and Maximum A Posteriori (MAP) decoder are replaced by a linear filter with an iterative MAP decoder. Error correction codes (ECC) are applied to the multilevel recording system in order to achieve very low error-rates. Implementation of the SFE scheme for multi-level recording shows a reduction in complexity in comparison to various PRML schemes. The simulation results show a clear performance gain of multi-level-coded against binary-coded recording systems. At higher Signal-to-Noise Ratio (SNR), the coded multi-level SFE scheme overcomes the error floor effect produced in the coded multi-level PRML scheme, which is caused by minimum distance error events. Overall, this paper proposes the use of coded multilevel recording with SFE scheme at lower rates rather than coded binary recording at higher densities in order to achieve similar performance.

Keywords: Soft Feedback Equalisation, Multilevel, Magnetic Recording, Longitudinal Recording, PRML, MAP, Error Correction Codes, noise colouration.

#### I. INTRODUCTION

It has been shown that multilevel techniques operate closer to the channel capacity than binary for a bandwidth limited channel experiencing Additive White Gaussian Noise (AWGN) at increased Signal to Noise Ratio (SNR) [1]. Previous work [2] suggested that multi-level techniques, offered little, if any improvement of the magnetic recording capacity compared to the binary(two-level) system, and is eventually limited by amplitude irregularities in the magnetic channel.

Multilevel encoding of data gives its M set of symbols, a k-bit meaning, where  $M = 2^k$ . Since the magnetic recording system works at a higher SNR, we can take advantage of it and use multi-level encoding and ECC to achieve more bandwidth efficiency, i.e. more "information bits" can be stored in the transitions on the magnetic media. It is known that at higher code rates for AWGN channels, binary codes tend to deviate very quickly from their theoretical performance [1]. In order to achieve very low error-rates at a particular SNR, it is necessary to use state of the art error correction codes like Turbo Codes. SFE was introduced in [3], in which Turbo Equalisation was discussed with linear complexity.

SFE based multilevel recording is discussed which is based on decision feedback channel [4] principles and is in contrast to the popular PRML technique. It is not the same as Turbo Equalisation which was discussed at length in [5]. In this work, turbo codes are used as outer ECC codes and they are not in conjunction with the SFE process.

The design of the SFE block is independent of the number of levels used for recording and the overall process of SFE is non-linear. This paper examines the use of multilevel data in conjunction with powerful ECC, for PRML and SFE magnetic recording channels, to achieve increased channel capacity for a particular SNR in the operating region of the magnetic recording devices.

### II. SYSTEM SIMULATION MODEL FOR SFE APPROACH OF MULTI-LEVEL RECORDING

Soft Feedback Equalisation is a technique which is based on filters with feedback algorithm for removing ISI from the magnetic recording readback channel data. In the PRML approach, the filter in the PR equaliser shortens the impulse response of the underlying channel. There is noise colouration and noise enhancement penalty in this process which changes as PW50 changes. This is because the dependence of noise colouration is non-linear [6]. The noise in magnetic recording channel is uncorrelated, and the equaliser modifies the correlation properties of the noise resulting in colouring of noise. The noise correlation affects the error rate of the PRML channel. In the proposed SFE scheme, a linear modified Lorentzian filter is used to estimate the amount of ISI introduced by the magnetic channel and the MAP decoder in the iterative loop tries to remove this estimated ISI. The overall decoding process is non-linear. This technique is in contrast with the PRML technique, since it does not use the introduced ISI, but instead it tries to remove the ISI using a feedback loop. As a result, there are no penalties of noise colouration or enhancement.

The feedback approach in the SFE system is an adaptive approach and thus the channel data remains in its original form without any alteration in the correlation properties of noise. The only problem is residual ISI obtained from linear super-positioning of certain error patterns. In this simulation model, ISI is treated as ISI noise i(t).

The convergence criteria for the feedback loop is as follows: Initially the estimates of the noisy data are passed through a non-linear filter and error is calculated from the original readback data and the estimates. This error is then fed into a loop which uses an error minimising MAP algorithm in order



Fig. 1. Simulation Model of SFE Multilevel Magnetic Recording System

to locate the transitions. The error calculated tries to estimate and remove the ISI from the channel and as it goes through the iteration of the feedback loop. The convergence of the error depends upon the amount of ISI, the pattern of transitions and AWGN in the channel. Depending upon both the factors, the number of iterations required to achieve the performance of the system is set. The convergence equation is obtained by minimum mean squared error criterion [7].

Here,  $\{x(k)\}$ , where  $x(k)\epsilon\{0, 1\}$  is the user data. The ECC encoder output is  $\{w(n)\epsilon\{0, 1\}\}$  and the mapping output d(t) depends upon the number of mapping levels used based on  $GF(2^m)$ . Here, m is the number of bits mapped together. The mapping for 4-levels is done by taking 2 bits at a time and mapping them as: 00 becomes 0, 01 becomes 1/3, 10 becomes 2/3, and 11 becomes 1. All the levels are equally spaced. For 4-levels,  $d(t)\epsilon\{0, 0.33, 0.66, 1\}$  and for 2-levels,  $d(t)\epsilon\{0, 1\}$ .

The N coefficients of the Lorentzian Filter are  $h(-\frac{N-1}{2}), \ldots, h(\frac{N-1}{2})$ , where N is a positive odd integer. The value of the coefficients of the Lorentzian filter depends upon the value of  $PW_{50}$ . After passing a(t) through the Lorentzian filter, the readback pulse r(t) is obtained. The channel SNR definition in dB used for the system in the simulations is

$$SNR_{channel} = 10 \log_{10} \left(\frac{1}{2\sigma^2}\right) dB$$

where  $\sigma$  is the standard deviation of the Gaussian Noise distribution. The channel noise n(t) is assumed to be AWGN whose variance,  $\sigma^2$  is determined by the above SNR equation. This noise is then added to r(t) giving b(t).

Filtering of a(t) with the first Lorentzian filter introduces ISI in the channel. This is the point where the ISI noise i(t) comes into account. Thus, the noisy channel output has two additional components: ISI noise i(t) and AWGN n(t), where i(t) is a function of a(t).

The noisy channel output b(t) is then thresholded, using a threshold device and the output of the threshold device is used as an initial estimate of the dicode output given as  $\hat{a}(t)$ . This thresholding process is controlled by an open switch which closes only during the 0th iteration. Thus, it is only used in the beginning of the feedback process. The thresholding is performed depending upon the number of levels used. The thresholded data  $\hat{a}(n)$  consists of n(t) and i(t). Thus,

where

$$\Gamma(f(t)) = \arg\max_{a} \{ \Pr(f(t) = a) : a \in \operatorname{GF}(2^m) \}$$

 $\dot{a}(t) = \Gamma(a(t) + i(t) + n(t)),$ 

and Pr(f(t) = a) is the a-priori probability of f(t) being a, which can be simply obtained from the probability density function of a Gaussian distributed random variable [7]. Once the estimates are achieved, the estimates go through another lorentzian function filter, where the middle lorentzian sample, i.e. the peak value is set to 0. It is set to 0 in order to estimate the ISI introduced by the first lorentzian filter. This modified Lorentzian function is denoted as

$$v(t) = h(t) - \delta(t), \qquad (2)$$

(1)

where  $\delta(t)$  is a unit amplitude impulse at t = 0 and zero elsewhere. The N tap coefficients of the second filter are  $v(-\frac{N-1}{2}), \ldots, v(\frac{N-1}{2})$  with v(0) = 0, where N is a positive odd integer. The output from the second filter  $\hat{r}(t)$  is cancelled from the output b(t), resulting in error output e(t). This error sequence is passed through the MAP detector and passed through (1 - D) and the estimates are updated as  $\tilde{a}(t)$ .

As the MAP output  $\hat{d}(t)$  enters into the convergence loop, it tries to minimise the effect of ISI completely since it is dependent upon the error e(t). After the convergence has reached, which is after 5 iterations, Bit Error Rate (BER) and Frame Error Rate (FER) are calculated at the points shown in the block diagram.

When ECC is introduced in the system, the input to the SFE channel is given from the ECC encoder output w(n). After convergence has been reached, the output of the MAP decoder d(t) is given to the ECC decoder. The points of overall BER and FER calculation with ECC are also shown in the block diagram. The equations explaining the above process are as shown in figure(1).

### III. ECC SPECIFICATIONS

The outer ECC code used for the simulation is a 1/3 rate turbo code. The design of turbo codes is achieved using tail-biting recursive systematic convolutional codes with feed-forward polynomial  $F_f = [37]_8$  and feed-back polynomial  $F_b = [23]_8$  for an overall rate 1/3 turbo code. The turbo decoder is iterative parallel concatenated MAP decoder with extrinsic information exchange [8]. The interleaver used is an S - random interleaver [9]. The block length is set to 500 information bits and the maximum number of iterations is set to 50. At least 100 error blocks were collected for each BER point.

Denoting the code rate of the error correcting code as  $R_1$ and the code-rate for the 4-level system as  $R_2 = 2$ , the overall code rate of the 4-level system is

## $R = R_1 \times R_2 = 2R_1$

### IV. RESULTS AND DISCUSSIONS

The simulation parameters for the different coded and uncoded schemes are as below:

- Binary PR-MAP: PW<sub>50</sub>=2.4, GPR=[1,-0.5,-0.5], ECC used is rate 1/3 Turbo Code as specified in the ECC specifications.
- 4-Level PR-MAP: PW<sub>50</sub>=1.2, GPR=[1,-0.8,-0.2], ECC used is rate 1/3 Turbo Code as specified in the ECC specifications.
- 4-Level SFE-MAP: PW<sub>50</sub>=1.2, maximum number of iterations in feedback loop is set to 5, ECC used is rate 1/3 Turbo Code as specified in the ECC specifications.
- Uncoded 4-Level PR-MAP: PW<sub>50</sub>=1.2, GPR=[1,-0.8,-0.2]

Results shown in figure(2) show the variation of bit error rate on SNR for different configurations of the multi-level recording system. For an uncoded 4-level PR-MAP system, the desired BER of  $10^{-5}$  is achieved at 25 dB SNR. To achieve the same BER, the PR-MAP system with ECC requires almost 10 dB less channel SNR compared to the uncoded system. In comparison, the coded binary system needs 6 dB less SNR than the coded 4-level system. The coded 4-level SFE-MAP system has similar performance as the coded 4-level PR-MAP system. As seen from the figure(2), the coded SFE-MAP



Fig. 2. Performance Comparison of Different Multilevel Recording Channels

scheme performs better than PR-MAP scheme in the operating region(19-22 dB) with rate 1/3 turbo code. The difference in performance is about 1.5 orders of magnitude in the error floor region. Also from figure(2), the error floor of binary coded system is same as the error floor of 4-level coded systems.

In terms of complexity, the SFE-MAP has less complexity than PR-MAP. This is because of the removal of the PR-Equaliser. The trellis structure for the PR-MAP multi-level system is based on the number of levels used and also on the length of the GPR targets. As in the case of SFE-MAP multilevel system, it is only based on the number of levels used. For a 4-level PR-MAP system with GPR target length of 3, each segment of the trellis has 64 paths and the maximum number of states is 16, while for a 4-level SFE-MAP system, each segment of the trellis has only 16 paths and maximum number of states is 4.

At lower density, the colouration of AWGN introduced by the PR equaliser would be less. The loss in terms of SNR dB calculated for a 4-level PRML system is about 0.5 dB at  $PW_{50}$ = 1.2 for the 4-level GPR target. Similarly at  $PW_{50}$  = 2.4, the calculated loss is about 2.10 dB for the binary GPR target. The use of multi-level signalling enables the use of lower rate error correction codes with larger minimum distances. In the normal operating region (19-22dB channel SNR) of the magnetic recording systems , it is seen from figure(2) that if both the binary and 4-level system performance curves are extended for a higher SNR, the error floor merges. It is observed that this error floor merge is caused by the ECC properties and not by the modulation technique used. The SFE scheme performs better for the multi-level recording system with ECC than PR-MAP.

The use of rate 1/3 turbo code show that lower rate codes with better ECC properties used with multilevel signalling and lower recording densities can have equivalent performance compared to binary coded systems with high recording densities.

### V. CONCLUSION

The proposed new SFE scheme performed better than PRML scheme in magnetic recording. Simulation results were presented for the comparison of both the schemes. This paper gives an insight into a new type of magnetic recording which is better in terms of complexity, performance and implementation. Equations were presented to clearly explain the convergence criterion of the SFE scheme. Application of multilevel signalling in magnetic recording has been shown to be beneficial, when the operating region of magnetic devices and low error rates were discussed. The SFE scheme performed better than PR-MAP scheme when ECC was implemented because the noise colouration effect was reduced which was caused by the PR-equaliser in the PR-MAP scheme. The error floor region which is the operating region of magnetic recording devices was shown and it was presented that the binary coded system had similar error floor to that of 4-level coded system.

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# New Method for Generalised PR Target Design for Perpendicular Magnetic Recording

# P. Shah<sup>1</sup>, M.Z. Ahmed<sup>1</sup> and Y. Kurihara<sup>2</sup>

<sup>1</sup> Centre for Research in Information Storage Technology, University of Plymouth, Plymouth PL4 8AA, UK, <sup>2</sup> Department of Electronic Control Engineering, Niihama National College of Technology, Niihama, 792-8580, Japan, Email:{purav.shah,m.ahmed}@plymouth.ac.uk, kurihara@ect.niihama-nct.ac.jp

# 1. INTRODUCTION

In recent years, perpendicular magnetic recording (PMR) has been the main topic of interest in the industry. Given current estimates, that would suggest an areal density using PMR as great as one terabit per square inch – making possible in two to three years a 3.5-inch disk drive capable of storing an entire terabyte of data [1]. As the areal density is increased, however, the signal processing aspects of magnetic recording becomes more difficult.

The present technique for finding the optimised GPR targets is based on the minimum mean squared error (MMSE) between the equaliser output and the desired output, subject to the monic constraint [2]. In this paper, we present a new method of designing GPR targets for PMR. This method is based on maximising the ratio of minimum squared eucledian distance of the PR target to the noise penalty introduced by the PR filter. The description of the channel model and the new method follows in the next section and the results and comparison follows after that.

### **II. SIMULATION MODEL**

Figure(1(a)) shows the block diagram of the PMR system model used in this paper. The user data, denoted as  $a_k$ , is a sequence of input symbols taking values of 0, 1. To simulate the write current, the sequence  $a_k$  is mapped to -1, +1. The scaling factor of 0.5 is to ensure the transition takes values of -1, 0, +1. We approximate the single-transition step repsonse, denoted as s(t), using the hyperbolic tangent function [3][4]:

$$s(t) = A \cdot \tanh\left(\ln(3)\frac{t}{PW_{50}}\right) \tag{1}$$

where A is the saturation level/amplitude from zero to peak(normalised to unity) and  $PW_{50}$  is the time taken for s(t) to go from -A/2 to +A/2. It is assumed that t and  $PW_{50}$  are normalised to the symbol period, T. The dibit response p(l) is defined as:

$$p(t) = s(t) - s(t-1)$$
(2)

The readback signal r(t) is the convolution of  $a_k$  and p(t) plus some Additive White Gaussian Noise (AWGN):

$$r(t) = \underbrace{\sum_{k} a_k p(t - kT)}_{b(0)} + n(t) \tag{3}$$

where n(t) is the AWGN with mean of 0 and variance of  $\sigma^2$ . A Maximum Likelihood Sequence Detector (MLSD) is used to provide the decoder solution which is used for performance evaluation.

## III. GPR TARGET SEARCH METHOD

The present technique [2] for achieving optimised GPR target is optimal only if the receiver has noise prediction. If there is no noise prediction, then the equaliser leads to additional noise. This is as shown in figure(1(b)). Considering a PR equaliser with a monic constraint, where h(0) = 1, the new technique described in this paper is based on the calculation of ratio of the minimum squared eucledian distance of the PR scheme under this monic constraint to the squared noise penalty introduced by the filter. Consider the N tap coefficients  $h(\frac{N-1}{2}), \dots, \dots, h(\frac{N-1}{2})$ , where N is an odd integer. The equalised signal y is:

$$y = r(t) * h(t)$$
(4)  
=  $b(t) * h(t) + n(t) * h(t)$   
=  $b(t) * h(t) + n(t) + n_f(t)$ 



| $PW_{50}$ | Target    |  |
|-----------|-----------|--|
| 1.3       | [1,4,7,3] |  |
| 1.4       | [1.5,8,4] |  |
| 1.5       | [1.5,8.4] |  |

Figure 1: Simulation Block including GPR Target Search

Figure 2: GPR Target Results

Here,  $n_f(t)$  is the noise penalty from the filter. The analysis of the system is shown in Figure 1.

The rule of optimisation is to find the GPR target that maximises the ratio of minimum squared euclidean distance on the trellis over the noise penalty. Thus, the effective design ratio is:

$$Design Ratio = \frac{Minimum Squared Eucledian Distance}{Filter Noise Penalty}$$
(5)

where, the filter noise penalty is computed as,

Filter Noise Penalty = 
$$\sum_{\forall j, j \neq 0} h(j)^2$$
 (6)

The optimsed search looks for the PR target that maximises this Design Ratio. Results from this are in Figure 2.

## **RESULTS AND DISCUSSIONS**

The GPR targets obtained using this new method for GPR search are the same with traditional method of GPR search for most  $PW_{50}$ . This new method provides consistently equal or better targets for PRML schemes that does not include noise prediction. Future work will focus on investigating the effect of media noise.

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# Journals

# Novel Soft-Feedback Equalization Method for Multilevel Magnetic Recording

P. Shah<sup>1</sup>, M. Ahmed<sup>1</sup>, M. Ambroze<sup>2</sup>, C. Tjhai<sup>2</sup>, and P. J. Davey<sup>1</sup>

### <sup>1</sup>Centre for Research in Information Storage Technology, University of Plymouth, Plymouth PL4 8AA, U.K. <sup>2</sup>Fixed and Mobile Communications Research, University of Plymouth, Plymouth PL4 8AA, U.K.

This paper investigates the use of multilevel modulation for magnetic recording using a novel soft-feedback equalization (SFE) approach. Different aspects of investigation are 1) multilevel recording, 2) SFE, and 3) application of turbo codes. The SFE scheme is a model in which the partial response (PR) equalizer and maximum *a posteriori* (MAP) decoder are replaced by a linear filter with an iterative MAP decoder. Error correction codes (ECCs) are applied to the multilevel recording system in order to achieve very low error rates. Implementation of the SFE scheme for multilevel recording shows a reduction in complexity in comparison to various PRML schemes. The simulation results show a clear performance gain of multi-level-coded against binary-coded recording systems. At higher signal-to-noise ratio (SNR), the coded multilevel SFE scheme overcomes the error floor effect produced in the coded multilevel PRML scheme, which is caused by minimum distance error events. Overall, this paper proposes the use of coded multilevel recording with SFE scheme at lower rates rather than coded binary recording at higher densities in order to achieve similar performance.

Index Terms—Error correction codes (ECCs), longitudinal recording, magnetic recording, multilevel, maximum a posteriori (MAP), noise coloration, PRML, soft feedback equalization (SFE).

# I. INTRODUCTION

T has been shown that multilevel techniques operate closer to the channel capacity than binary for a bandwidth limited channel experiencing additive white Gaussian noise (AWGN) at increased signal-to-noise ratio (SNR) [1]. Previous work [2] suggested that multilevel techniques offered little, if any, improvement of the magnetic recording capacity compared with the binary (two-level) system and is eventually limited by amplitude irregularities in the magnetic channel.

Multilevel encoding of data gives its M set of symbols, a k-bit meaning, where  $M = 2^k$ . Since the magnetic recording system works at a higher SNR, we can take advantage of it and use multilevel encoding and ECC to achieve more bandwidth efficiency, i.e., more "information bits" can be stored in the transitions on the magnetic media. It is known that at higher code rates for AWGN channels, binary codes tend to deviate very quickly from their theoretical performance [1]. In order to achieve very low error-rates at a particular SNR, it is necessary to use state of the art error correction codes like turbo codes. SFE was introduced in [3], in which turbo equalization was discussed with linear complexity.

SFE-based multilevel recording is discussed which is based on decision feedback channel [4] principles and is in contrast to the popular PRML technique. It is not the same as turbo equalization which was discussed at length in [5]. In this work, turbo codes are used as outer ECC codes, and they are not in conjunction with the SFE process.

The design of the SFE block is independent of the number of levels used for recording and the overall process of SFE is nonlinear. This paper examines the use of multilevel data in conjunction with powerful ECC, for PRML and SFE magnetic recording channels, to achieve increased channel capacity for a particular SNR in the operating region of the magnetic recording devices.

### II. SYSTEM SIMULATION MODEL FOR SFE APPROACH OF MULTILEVEL RECORDING

Soft feedback equalization is a technique that is based on filters with a feedback algorithm for removing intersymbol interference (ISI) from the magnetic recording readback channel data. In the PRML approach, the filter in the PR equalizer shortens the impulse response of the underlying channel. There is noise coloration and noise enhancement penalty in this process which changes as PW 50 changes. This is because the dependence of noise coloration is nonlinear [6]. The noise in the magnetic recording channel is uncorrelated, and the equalizer modifies the correlation properties of the noise, resulting in coloring of noise. The noise correlation affects the error rate of the PRML channel. In the proposed SFE scheme, a linear modified Lorentzian filter is used to estimate the amount of ISI introduced by the magnetic channel and the MAP decoder in the iterative loop tries to remove this estimated ISI. The overall decoding process is nonlinear. This technique is in contrast with the PRML technique, since it does not use the introduced ISI, but instead it tries to remove the ISI using a feedback loop. As a result, there are no penalties of noise coloration or enhancement.

The feedback approach in the SFE system is an adaptive approach, and thus the channel data remains in its original form without any alteration in the correlation properties of noise. The only problem is residual ISI obtained from linear superpositioning of certain error patterns. In this simulation model, ISI is treated as ISI noise i(t).

The convergence criteria for the feedback loop is as follows: Initially the estimates of the noisy data are passed through a nonlinear filter and error is calculated from the original readback data and the estimates. This error is then fed into a loop which uses an error minimizing MAP algorithm in order to locate the transitions. The error calculated tries to estimate and remove the ISI from the channel and as it goes through the iteration of the feedback loop. The convergence of the error depends upon the amount of ISI, the pattern of transitions and AWGN in the channel. Depending upon both the factors, the number of iterations required to achieve the performance of the

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Fig. 1. Simulation model of SFE multilevel magnetic recording system.

system is set. The convergence equation is obtained by minimum mean-squared error criterion [7].

Here,  $\{x(k)\}$ , where  $x(k) \in \{0, 1\}$  is the user data. The ECC encoder output is  $\{w(n) \in \{0, 1\}\}$ , and the mapping output d(t)depends upon the number of mapping levels used based on GF(2<sup>m</sup>). Here, m is the number of bits mapped together. The mapping for four levels is done by taking 2 bits at a time and mapping them as follows: 00 becomes 0, 0 becomes 1/3, 10 becomes 2/3, and 11 becomes 1. All the levels are equally spaced. For four levels,  $d(t) \in \{0, 0.33, 0.66, 1\}$  and for two levels,  $d(t) \in \{0, 1\}$ .

The N coefficients of the Lorentzian filter are  $h(-N - 1/2), \ldots, h(N - 1/2)$ , where N is a positive odd integer. The value of the coefficients of the Lorentzian filter depends upon the value of PW<sub>50</sub>. After passing a(t) through the Lorentzian filter, the readback pulse r(t) is obtained. The channel SNR definition in decibels used for the system in the simulations is

$$SNR_{channel} = 10 \log_{10} \left( \frac{1}{2\sigma^2} \right) dB$$

where  $\sigma$  is the standard deviation of the Gaussian noise distribution. The channel noise n(t) is assumed to be AWGN whose variance  $\sigma^2$  is determined by the above SNR equation. This noise is then added to r(t), giving b(t).

The filtering of a(t) with the first Lorentzian filter introduces ISI in the channel. This is the point where the ISI noise i(t)comes into account. Thus, the noisy channel output has two additional components: ISI noise i(t) and AWGN n(t), where i(t)is a function of a(t).

The noisy channel output b(t) is then thresholded, using a threshold device, and the output of the threshold device is used as an initial estimate of the dicode output given as  $\bar{a}(t)$ . This thresholding process is controlled by an open switch that closes only during the zeroth iteration. Thus, it is only used in the beginning of the feedback process. The thresholding is performed depending upon the number of levels used. The thresholded data  $\bar{a}(n)$  consists of n(t) and i(t). Thus

$$\dot{a}(t) = \Gamma(a(t) + i(t) + n(t)) \tag{1}$$

where

$$\Gamma(f(t)) = \arg\max\{\Pr(f(t) = a) : a \in \operatorname{GF}(2^m)\}$$

and  $\Pr(f(t) = a)$  is the *a priori* probability of f(t) being *a*, which can be simply obtained from the probability density function of a Gaussian-distributed random variable [7]. Once the estimates are achieved, the estimates go through another Lorentzian function filter, where the middle Lorentzian sample, i.e., the peak value is set to 0. It is set to 0 in order to estimate the ISI introduced by the first Lorentzian filter. This modified Lorentzian function is denoted as

$$v(t) = h(t) - \delta(t) \tag{2}$$

where  $\delta(t)$  is a unit amplitude impulse at t = 0 and zero elsewhere. The N tap coefficients of the second filter are  $v(-N - 1/2), \ldots, v(N - 1/2)$  with v(0) = 0, where N is a positive odd integer. The output from the second filter  $\tilde{r}(t)$  is canceled from the output b(t), resulting in error output c(t). This error sequence is passed through the MAP detector and passed through (1 - D), and the estimates are updated as  $\tilde{a}(t)$ .

As the MAP output d(t) enters into the convergence loop, it tries to minimize the effect of ISI completely since it is dependent upon the error e(t). After the convergence has been reached, which is after five iterations, the bit error rate (BER) and frame error rate (FER) are calculated at the points shown in the block diagram.

When ECC is introduced in the system, the input to the SFE channel is given from the ECC encoder output w(n). After convergence has been reached, the output of the MAP decoder  $\hat{d}(t)$  is given to the ECC decoder. The points of overall BER and FER calculation with ECC are also shown in the block diagram. The equations explaining the above process are as shown in Fig. 1.

### **III. ECC SPECIFICATIONS**

The outer ECC code used for the simulation is a 1/3 rate turbo code. The design of turbo codes is achieved using tail-biting recursive systematic convolutional codes with feedforward polynomial  $F_f = [37]_8$  and feedback polynomial  $F_b = [23]_8$  for an overall rate 1/3 turbo code. The turbo decoder is iterative parallel concatenated MAP decoder with extrinsic information exchange [8]. The interleaver used is an S-random interleaver [9]. The block length is set to 500 information bits, and the maximum number of iterations is set to 50. At least 100 error blocks were collected for each BER point.



Fig. 2. Performance comparison of different multilevel recording channels.

Denoting the code rate of the error correcting code as  $R_1$  and the code rate for the four-level system as  $R_2 = 2$ , the overall code rate of the four-level system is

## $R = R_1 \times R_2 = 2R_1$

#### IV. RESULTS AND DISCUSSIONS

The simulation parameters for the different coded and uncoded schemes are as follows:

- Binary PR-MAP: PW<sub>50</sub> = 2.4, GPR = [1, -0.5, -0.5], ECC used is rate 1/3 turbo code.
- Four-Level PR-MAP:  $PW_{50} = 1.2$ , GPR = [1, -0.8, -0.2], ECC used is rate 1/3 turbo code.
- Four-Level SFE-MAP: PW<sub>50</sub> = 1.2, maximum number of iterations in feedback loop is set to 5, ECC used is rate 1/3 turbo code.
- Uncoded Four-Level PR-MAP: PW<sub>50</sub> = 1.2, GPR = [1, -0.8, -0.2].

The results in Fig. 2 show the variation of BER on SNR for different configurations of the multilevel recording system. For an uncoded four-level PR-MAP system, the desired BER of  $10^{-5}$  is achieved at 25-dB SNR. To achieve the same BER, the PR-MAP system with ECC requires almost 10 dB less channel SNR compared with the uncoded system. In comparison, the coded binary system needs 6 dB less SNR than the coded fourlevel system. The coded four-level SFE-MAP system has similar performance as the coded four-level PR-MAP system. As seen from Fig. 2, the coded SFE-MAP scheme performs better than the PR-MAP scheme in the operating region (19–22 dB) with rate /3 turbo code. The difference in performance is about 1.5 orders of magnitude in the error floor region. Also from Fig. 2, the error floor of the binary coded system is same as the error floor of the four-level coded systems.

The SFE-MAP has less complexity compared to the PR-MAP, because of the removal of the PR-equalizer. The trellis structure for the PR-MAP scheme is based on the number of levels used and the length of the GPR target, while in the case of the SFE-MAP, it is only based on the number of levels used. For a four-level PR-MAP system with a GPR target length of 3, each segment of the trellis has 64 paths and 16 states, while for the SFE-MAP system, it has only 16 paths and four states.

At lower density, the coloration of AWGN introduced by the PR equalizer would be less. The loss in terms of SNR dB calculated for a four-level PRML system is about 0.5 dB at  $PW_{50} = 1.2$  for the four-level GPR target. Similarly, at  $PW_{50} = 2.4$ , the calculated loss is about 2.10 dB for the binary GPR target. The use of multilevel signaling enables the use of lower rate error correction codes with larger minimum distances. In the normal operating region (19–22-dB channel SNR) of the magnetic recording systems, it is seen from Fig. 2 that if both the binary and four-level system performance curves are extended for a higher SNR, the error floor merges. It is observed that this error floor merge is caused by the ECC properties and not by the modulation technique used. The SFE scheme performs better for the multilevel recording system with ECC than PR-MAP.

The use of rate 1/3 turbo code shows that lower rate codes with better ECC properties used with multilevel signalling and lower recording densities can have equivalent performance compared with binary coded systems with high recording densities.

#### V. CONCLUSION

The proposed new SFE scheme performed better than PRML scheme in magnetic recording. Simulation results were presented for the comparison of both the schemes. This paper gives an insight into a new type of magnetic recording, which is better in terms of complexity, performance, and implementation. Equations were presented to clearly explain the convergence criterion of the SFE scheme. Application of multilevel signaling in magnetic recording has been shown to be beneficial, when the operating region of magnetic devices and low error rates were discussed. The SFE scheme performed better than the PR-MAP scheme when ECC was implemented because the noise coloration effect was reduced, which was caused by the PR-equalizer in the PR-MAP scheme. The error floor region, which is the operating region of magnetic recording devices, was shown, and it was presented that the binary coded system had similar error floor to that of four-level coded system.

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# **Patent Applications**

# A Novel MTR-Coded Soft-Feedback Equalisation Method for Magnetic Recording

# Specification

# Background:

Partial Response Maximum Likelihood (PRML) is a dominant technique in the magnetic recording industry. The recording channel is assumed to be longitudinal, where the readback voltage from an isolated transition is approximated by a Lorentzian function [1], with  $\frac{PW_{50}}{T}$  as the normalised recording density. The Lorentzian function is defined as:

$$h(t) = \frac{1}{1 + (\frac{2t}{PW_{50}/T})^2} \tag{1}$$

The Lorentzian function is the isolated step response of the recording channel. The channel SNR definition used for the system described is:

$$SNR_{channel} = 10\log_{10}\left(\frac{1}{2\sigma^2}\right)$$

where  $\sigma$  is the standard deviation of the Gaussian Noise distribution. The channel noise is assumed to be Additive White Gaussian Noise (AWGN) whose variance,  $\sigma^2$  is determined by the above SNR equation. Partial Response equalisation is achieved by using an equaliser with the best Generalised Partial Response (GPR) target, which eventually forms the readback data.

PR Equalisation is performed so that the Inter Symbol Interference (ISI) introduced by the Lorentzian filter can be controlled. The PR equaliser in the PRML scheme shortens the impulse response of the underlying channel and matches it according to the PR target. There is noise colouration and noise enhancement penalty in this process which changes as  $PW_{50}$  changes. This is because the dependence of noise colouration is non-linear [2]. The noise in the magnetic recording channel is uncorrelated, and the equaliser modifies the correlation properties of the noise resulting in noise colouration. The noise colouration affects the error rate of the PRML channel. Thus, in order to overcome the noise colouration penalties introduced by the PRML technique, the new Soft Feedback Equalisation (SFE) technique is proposed which is described next.

# Statement of Invention:

This invention incorporates PRML - "the industry standard technique" with the concept of SFE. The new scheme combines PRML along with feedback algorithm for removing ISI from the magnetic recording readback channel data. The novelty of this invetion is:

- Use of PRML for best initial estimate.
- Use of MAP decoding during SFE iterations.

The design of the SFE block is independent of the GPR target length. This technique is in contrast with the popular PRML technique, since it does not use the introduced ISI, but instead it tries to remove the ISI using a feedback loop. As a result, there are no penalties of noise colouration in the decoded data. The block diagram and the equations are as shown in figure(1).

The SFE scheme is very similar to the Interference Cancellation (IC) Schemes [3]-[4]. The main difference between the IC schemes and the present scheme is that the SFE combines the modified filter output and the apriori information from the PRML block to form the most reliable estimate of ISI.

The scheme described in [5] combines turbo iterations with a SFE-based equaliser. During the  $1^{st}$  iteration, the scheme described in [5] lies somewhere between a Linear Equaliser (LE) and a Decision Feedback Equaliser (DFE), depending upon the SNR and the severity of the ISI. On the contrary the proposed scheme is optimal, since it provides the best possible estimate of ISI during the first iteration itself. As a result, it does not require many iterations to achieve the MMSE solution.

To achieve similar coding performance with the proposed scheme, error correcting codes (ECC) like turbo codes, could be used as outer codes and not as turbo-based equalisation as described in [6]. The ECC block is completely independent of the SFE iterations and thus, the decoding complexity is much less. A figure explaning this is shown in fig(2).



Figure 1: Block Diagram of the Proposed SFE Model for Magnetic Recording Systems

# Simulation Model of the Proposed SFE Scheme:

A linear modified Lorentzian filter is used to estimate the amount of ISI introduced by the magnetic channel and the Maximum A Posteriori (MAP) decoder in the iterative loop determines the best estimate of the data bit sequence. The feedback in the SFE block results in no alteration within the correlation properties of noise. The problem of SFE is residual ISI which is obtained by linear super-positioning of certain error patterns. This can be overcome by the use of Maximum Transition Run (MTR) codes. MTR codes are discussed later.

The convergence criteria for the feedback loop is as follow: Initially the estimates of the noisy data are passed through a modified Lorentzian filter and differentiated. The error is calculated from the original readback data and the estimates from the differentiated PRML output. This error is then fed into a loop which uses an error minimising MAP algorithm in order to locate the transitions. The error calculated tries to estimate and remove the ISI from the channel as it goes through the iterations of the feedback loop. The convergence of the error depends upon the amount of ISI, the pattern of transitions and Additive White Gaussian Noise (AWGN) in the channel. Depending upon these factors, the number of iterations required to achieve the performance of the system is set. The convergence equation is obtained by Minimum Mean Squared Error (MMSE) criterion [7]. It is found that after 2 iterations, the amount of ISI within the data is minimal and thus, there is no requirement for additional iterations.

MTR codes improves the minimum distance properties of sequence detectors operating at high linear densities [8]. MTR codes eliminate the data patterns producing three or more consecutive transitions, while imposing the usual k-constraint necessary for timing recovery.

A rate 4/5 MTR code is used for the simulation. The data are precoded with  $\frac{1}{1\oplus D}$ . The table showing the codewords is below [8]:

| DATA | CODEWORD | DATA | CODEWORD |
|------|----------|------|----------|
| 0000 | 10000    | 1000 | 01000    |
| 0001 | 00001    | 1001 | 01001    |
| 0010 | 00010    | 1010 | 01010    |
| 0011 | 10001    | 1011 | 10010    |
| 0100 | 00100    | 1100 | 01100    |
| 0101 | 00101    | 1101 | 00101    |
| 0110 | 00110    | 1110 | 10100    |
| 0111 | 10110    | 1111 | 10101    |

Table 1: Rate 4/5 MTR Code



Figure 2: Block Diagram of the Proposed SFE Model for Magnetic Recording Systems with ECC

# Claims

The existing scheme is considered to be the PRML scheme with rate 4/5 MTR code as outer code. The new scheme is SFE with MAP decoder (SFE-MAP) and rate 4/5 MTR code as outer code. The trellis structure for the PRML system is based on the length of the GPR target. As in the case of SFE-MAP block, it is not based on the length of the GPR target. As seen from



Figure 3: Bit Error Rate Performance for PRML (PR4) and SFE-MAP schemes with 4/5 rate MTR code

the figure(3), the MTR coded SFE-MAP scheme performs better than PRML scheme in the operating region(19-22 dB) with rate 4/5 MTR code. The difference in performance for a PR4 target is about 0.2 dB gain in the region where commercial products operate. The new scheme provides soft output from the MAP decoder and thus, it increases the possibility to use error control coding. Figure(4) shows the EPR4 magnetic recording channel simulation results. It is seen that the new scheme outperforms the MTR-EPR4 ML scheme and the standard EPR4 ML scheme. The gain of the proposed scheme in terms of dB at  $PW_{50} = 2.4$  is about 0.6 dB against MTR-EPR4 ML scheme and about 1.6 dB against a standard EPR4 ML scheme. Thus, it is seen that at higher densities and with longer PR targets, the proposed scheme has a higher gain in comparison lower densities and shorter PR targets.

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Figure 4: Bit Error Rate Performance for PRML (EPR4) and SFE-MAP schemes with 4/5 rate MTR code

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