

HARMONIC INTONATION AND IMPLICATION (ANALYSES AND COMPOSITIONS): Harmonic perception and intonation in the reception and performance of alternative tuning systems in contemporary composition.

Volume 1 of 2

by

JOHN PAUL SWOGER-RUSTON

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ABSTRACT

John Paul Swoger-Ruston

HARMONIC INTONATION AND IMPLICATION (ANALYSES AND COMPOSITIONS):
Harmonic perception and intonation in the reception and performance of alternative tuning systems in contemporary composition

Most composers and theorists will acknowledge that some compromise is necessary when dealing with the limitations of human performance, perception, and the realities of acoustic theory. Identifying the thresholds for pitch discrimination and execution is an important point of departure for defining workable tuning schemes, and for training musicians to realise compositions in just intonation and other alternative tuning systems.

The submitted paper '*HARMONIC INTONATION AND IMPLICATION (ANALYSES AND COMPOSITIONS): Harmonic perception and intonation in the reception and performance of alternative tuning systems in contemporary composition*' is a phenomenological study of harmonic perception and intonation through the analysis of recordings, scores, theoretical papers, and discussion with practicing musicians. The examined repertoire covers western 'art' music of the late nineteenth to early twenty-first centuries.

I approach my research from the composer's point of view though filtered through the ears and eyes of the performer, who is here considered 'expert listener'. It is considered that intonation is a dynamic experience subject to influences beyond just intonation or equal temperament (the two poles for intonational reference)—the performance is assumed 'correct', rather than the idealised version of the composer.

My goal is to relate the performance to the intentions of the composer and raise questions regarding the choice of notation, resolution of the tuning systems, the complexity of the harmonic concept, etc. and perhaps to suggest how to extend a general theory of harmony that embraces both musical practice and psychoacoustics.

It is with the understanding that harmonic implication affects intonation, but that intonation is subject to several other forces making intonation a complex system (and therefore not fully predictable).

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BOUND IN SCORES (IN VOLUME 2 OF 2) AND RECORDINGS (CD)

I drew a line in the sand, and it goes from here to there...

Workshop performance by members of Black Hair: Anna Myatt, voice; Sharon Lyons, clarinet; Catherine Laws, piano; Damien Harron, vibraphone; Emma Welton, violin; Charlotte Bishop, cello. Dartington College of Arts, Devon, 03 March 2002.

The Beaten Path

Performed by Ning: Tora Ferner Lang, flute; Erik Daehlin, vibraphone; Maja Bugge, cello. Parkteateret, Oslo, Norway, 26 November 2003.

1024 to 1

Realized with Csound.

For Muted Piano

Performed by John Lenehan, piano. London Spitalfields Festival, Wilton's Music Hall, 12 June 2004.

Track and Field

Workshop performance by members of [rout]: Emma Welton, violin; David Arrowsmith, electric guitar; Catherine Laws, keyboard; Philip Howard, keyboard; Richard Pryce, contrabass; James Woodrow, electric bass. Dartington College of Arts, Devon, 09 October 2003.

Eventide

Workshop performance by members of The Barton Workshop: Frank Denyer, piano; Tobias Liebezeit, vibraphone; Marieke Kezer, violin; Alex Geller, cello. Dartington College of Arts, Devon, 02 May 2003.

The Crow, the Road, and the Ramble

Workshop performance by members of Icebreaker: Christian Foreshaw, saxophone; James Woodrow, electric guitar; Andrew Zolinsky, keyboard; Audrey Riley, cello. Dartington College of Arts, Devon, 12 March 2004.

Corrections and Amplifications

Performed by Zephyr Quartet: Wiek Hijmans, electric guitar; Lydia Forbes, violin; Jacob Plooj, violin; Elisabeth Smalt, viola; John Addison, cello. Concertgebouw, Amsterdam, Netherlands, 23 December 2004.

This Mnemonic Machine

Performed by Wiek Hijmans, electric guitar. De Link, Tilburg, Netherlands, 24 October 2006.

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I would like to credit here the influence of several people that have had a great impact on my understanding of the subjects of intonation and tuning, psychoacoustics, and composition. Through informal discussion, formal classes and tutorials, I have absorbed many ideas from Bob Gilmore, James Tenney, Rob Wannamaker, Patrick Ozzard-Low, and many of the musicians who have played my compositions. It would be almost impossible to confidently isolate and give proper credit to the appropriate individual for many of the ideas expressed in this paper. That said, I also cannot say that these people will unconditionally agree with everything presented in this thesis, and I assume full responsibility for any dubious argument or interpretation.

My family has supported my musical career for a long time now (emotionally, financially, and with their babysitting services), and I hope I am able to one day reciprocate. Thank you first of all to my wife Janet Swoger-Ruston who has supported my work unconditionally for the entire time we have been together and deserves an equal share of the credit for my work, and to my parents Derek and Rollande Ruston, my siblings Marc Ruston and Michéle Gowda, her family, and to my in-laws, Gordon, Barbara, Kate, Jim, and Tom Swoger.

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I would like to thank the following persons and ensembles for commissioning and/or premiering my compositions: Ning—*The Beaten Path* (Norway), The Zephyr Kwartet—*Corrections and Amplifications* (Netherlands), Wiek Hijmans—*This Mnemonic Machine* (Netherlands), John Lenehan—*For Muted Piano* (United Kingdom). And similar thanks goes to the following bodies for lending further support to my work: the Society for the Promotion of New Music, who short listed my piece *For Muted Piano* and arranged for its première by John Lenehan at the London Spitalfields Festival, which subsequently

resulted in my being awarded the 2004 George Butterfield Award. Thank you to The George Butterworth Memorial Fund for supporting this award, to Deirdre Gribbin for nominating me, and to John Lenehan for his excellent performance of this piece. Thank you also to the Dartington College of Arts Artist in Residence programme, which gave me the opportunity to workshop my pieces with international calibre musicians. Thank you to Ultrasound, Black Hair, [rout], The Barton Workshop, Catherine Laws, and Icebreaker for their participation in this initiative and for performing my music. And to Christopher Best, Catherine Laws and Frank Denyer for organising and overseeing these workshops—these were great learning experiences for me.

Several bodies and initiatives supported my research and practical work as well. I would like to thank Dartington College of Arts for providing me with a fee bursary, without which it would have been impossible for me to carry out my work at Dartington. Dartington College of Arts also supported my travel expenses for the attendance of premières in Oslo, London, and Amsterdam, and my attendance at the Huddersfield Music Festival and The Royal College of Music's 2001 RMA Conference. The Canada Council for the Arts also helped in supporting my travel to premières in Norway and The Netherlands, for which I am very grateful.

For their support and friendship, stimulating conversation, and for inspiring me through the example shown in their own work, I would also like to thank Michael Bassett, Maja Bugge, Theresa and Catherine Grimaldi, Tim Hodgson, Rosie Jones, Michael Kane, Matt Lord, Tim Oram, Elisabeth Smalt, Andy Smith, and Josh Thorpe. My apologies to anyone I may have missed here.

Further more, Thank you to the support staff and departments at Dartington College of Arts: Sharon Townsend, Corrie Jeffery, Dartington College of Arts Library and the Service and Production Unit.

AUTHOR'S DECLARATION

At no time during the registration for the degree of Doctor of Philosophy has the author been registered for any other University award.

This study was financed with the aid of a studentship, fee bursary, and travel grants from Dartington College of Arts.

PERFORMANCES OF COMPOSITIONS

All of the compositions presented here have been performed and recorded at a public event, with the exception of the computer piece *1024 to 1*:

This Mnemonic Machine, premièred by Wiek Hijmans, De Link, Tilburg, Netherlands, 24 October 2006.

Corrections and Amplifications, premièred by Zephyr Kwartet and Wiek Hijmans:

Theater Kikker, Utrecht, Netherlands, 28 January 2005

Galerie Marzee, Nijmegen, Netherlands, 23 January 2005

Kleine Zaal Concertgebouw, Amsterdam, Netherlands, 23 December 2004

Korzo Theater, Den Haag, Netherlands, 19 December 2004

Theater Romein, Leeuwarden, Netherlands, 12 December 2004

The Crow, the Road and the Ramble, workshop performance by members of Ice Breaker, Dartington College of Arts, 12 March 2004.

Eventide, Workshop performance by The Barton Workshop, Dartington College of Arts, 02 May 2003.

Track and Field, workshop performance by [rout], Dartington College of Arts, 26 April 2003.

The Beaten Path, premièred by Ning Ensemble:

Parkteateret, Oslo, Norway, 26 November 2003

Borealis Festivalen, Bergen, Norway, 27 March 2004.

For Muted Piano, premièred by John Lenehan, Spitalfields Music Festival, Wilton's Music Hall, London, UK, 12 June 2004.

I drew a line in the sand, and it goes from here to there, workshop performance by Black Hair, Dartington College of Arts, 02 May 2002.

PRESENTATIONS AND CONFERENCES ATTENDED

The Hammond Organ and Electronic Instrument Technology in the 20th Century, University of Surrey, Surrey, UK, 03 February 2004.

Three New Pieces: My Recent Compositional Practice, undergraduate seminar, Dartington College of Arts, 04 November 2003.

Practical Issues in Tuning Theory and Microtonal Performance, Royal College of Music, RMA Conference, 14 December 2001.

Huddersfield Music Festival (attendant), Huddersfield, UK, 30 November – 02 December 2001.

AWARDS

For Muted Piano, Society for the Promotion of New Music George Butterworth Award, London, UK, 07 December 2004.

Canada Council for the Arts Travel Grants, 2003 and 2004.

Society for the Promotion of New Music Shortlist, 2002/03.

Signed..........

Date*20 DEC 2006*.....

INTRODUCTION

The use of microtonal intervals has become increasingly more commonplace in contemporary classical music. Over the course of the twentieth century, musicians¹ have extended and continue to extend the concept of *harmony*, and more specifically *pitch*, in a variety of ways.

The earliest adventures away from twelve-tone equal temperament explored microtones based on further equal divisions of the octave. The obvious first expansion of the octave is quartertones (half a semitone), most notably explored in the music of Charles Ives, Alois Hába, and Ivan Wychnegradsky.

It now seems safe to say that quartertones do not represent any real threshold of harmonic or melodic perception; we hear changes in pitch of this magnitude quite distinctly. Since the introduction of quartertones, pitch has been extended in several directions by exploring melodic, harmonic (irrational² and proportional), and also through inharmonic possibilities. Approaches to extending pitch resources include the use of various equal temperaments (equal divisions of the octave in most cases, both greater than and less than twelve tone equal temperament); just intonation (the use of whole number ratios as the expression of proportional pitch relations); satellite tones (just or irrational microtonal intervals based around any equal tempered pitch set or other base system); spectral simulation (the orchestrated simulation of the spectra of complex sounds); melodic embellishment and inflection; glissandi, pitch clusters and sound masses; the modelling of the natural inflections of speech; and perhaps other as yet unidentified approaches (which might also include extended well- and mean-tone temperaments based on higher-limit systems, although I am unaware of any contemporary examples).

In my own music, the tuning system of each piece is one of several variable parameters. Microtonal intervals result from a variety of compositional strategies. They can emerge from the use of the harmonic series as a musical resource, the simulation of subjective tones (sum and difference) and resonance effects, equal divisions of the octave, melodic

¹ I use the term *musician* in the broadest sense possible. Where I wish to be more specific I use the terms *player*, *composer*, *theorist*, *acoustician* instead.

² For the purpose of this paper, an 'irrational' tuning system is any system not based on an underlying acoustic principle or property of tone relations, even though, for example, 12tet equal temperament is based on a rational *mathematical* concept, it is here considered irrational (but not as a matter of judgement).

embellishment, as a means to create beating or increased dissonance, or to usurp or challenge concepts of consonance and 'in-tune-ness'.

It seems obvious that considering the vast number of approaches to musical composition in the past century that a model of harmonic theory can no longer be simply a model for musical style. James Tenney, in "John Cage and the Theory of Harmony", imagines a model of harmonic theory that is objective and not stylistically specific, based as a subset of acoustics and psychoacoustics rather than a recipe for musical style.

...the "continued evolution of the theory of harmony" might depend—among other things—on a broadening of our definition of "harmony".

...and perhaps, of "theory" as well. By "theory" I mean essentially what any good dictionary tells us it means—e.g.:

...the analysis of a set of facts in relation to one another...the general or abstract principles of a body of fact, a science, or an art...a plausible or scientifically acceptable general principle or body of principles offered to explain phenomena...³

...which is to say, something that current textbook versions of the "theory of harmony" are decidedly not—any more than a book of etiquette, for example, can be construed as a "theory of human behavior," or a cookbook a "theory of chemistry" (Tenney 1982: 57).

Such a theory will of course include the traditional western system where as few as only twelve tones can create complex and ambiguous harmonic situations—to a greater extent than addressed in most music theory textbooks. Of particular relevance to this thesis is the fact that complex and ambiguous harmony raises complex and ambiguous intonational issues. And 'twelve-note' music should necessarily receive some special attention as new notation systems are often (but not exclusively) based in this system, where a twelve-note model is adapted or augmented for more precise control of intonation rather than new notation systems created from scratch.

If twelve-tone serial composition is regarded as the last step in the development of harmony and melody based on the twelve notes of the equal tempered scale⁴—although I would argue that Debussy represents a parallel trajectory—then everything that follows is

³ Tenney quotes here Webster's New Collegiate Dictionary, Thomas Allen & Son, Ltd., Toronto, 1979.

⁴ However, Schoenberg states in the article *New Music, Outmoded Music, Style and Idea* that "a superficial judgement might consider composition with twelve tones as an end to the period in which chromaticism evolved, and thus compare it to the climaxing end of the period of contrapuntal composition which Bach set by his unsurpassable mastery...But...I believe that composition with twelve tones and what many erroneously call 'atonal music' is not the end of an old period, but the beginning of a new one" (Schoenberg 1946: 120)

either a redefinition of *first principles*, or an expansion of the tonality concept itself (which Schoenberg also acknowledged as a possible trajectory). Schoenberg's *emancipation of the dissonance* roughly coincided with the emancipation of pitch from twelve-tone equal temperament. Edgard Varèse, Charles Ives, Harry Partch, and Henry Cowell all recognised pitch as a continuum rather than a musical parameter with discreet states, and expressed this to varying degrees in their music.

This thesis aims to address *intonation* as it relates to issues of *harmonic perception* (or a broad theory of harmony) in practical musical contexts—that is, away from, although informed by, reduced scientific research (where testing conditions are necessarily limited to a conditioned listening environment), and away from theoretical models that presuppose specific historical western art music styles or the style of any particular composer.

The most basic question this thesis asks is **what can be heard as *harmonic***, or harmonically in a psychoacoustical sense within contemporary systems of pitch, including both rational and *irrational* concepts for extending the western twelve-note system? This question has important implications for intonation concerning notation (the composer), interpretation (the player), and reception (the listener). Do the composer's intentions match what is implied in the score? To what extent does intonation in the performance reflect the intentions of the composer (and what does this say about harmonic construction or concept)? And, to what extent is the listener able to understand or rationalise what s/he hears harmonically (and *how* does s/he rationalise it)?

Substantial work has been done in the field of pitch perception and intonation within a scientific model. The fields of acoustics and psychoacoustics have developed models of pitch perception and extracted theories of consonance and dissonance, both of which are particularly important to the present discussion. The reductive approach is an important method in isolating component phenomena and cannot be disregarded.

However, the scientific approach leaves the musician wanting. The musician does not normally work within the abstracted parameters of sine tones, isolated dyads, fixed durations, and idealised listening environments. A more comprehensive theory of musical

harmony or harmonic perception must take into account this research, but must also extend it to include the influence of the complex musical environment; phenomena that occur within the reductive model do not necessarily retain the same properties or influence in complex models.

Acoustic/psychoacoustic models considered in this work include (but are not limited to) those of: Georg Ohm, Hermann von Helmholtz, Carl Stumpf, Ernst Terhardt, Ray Meddis and Michael J. Hewitt, Carol Krumhansel, and Akio Kameoka and Mamoru Kuriyagawa.

Musical-theoretical models of melodic and harmonic perception are also lacking in generality, but offer many clues to what might be developed into a more comprehensive theory of harmony or harmonic perception. More often than not, the theorist engages notions which are stylistically specific, or when developed by a composer, quite specific to her or his own work (to be fair, these models are not always intended as prescriptive or objective). Again, the observations made in many of these models provide a basis for a more general phenomenological approach.

Musical theoretical models influencing or considered in my research include (but are not limited to) those of: Jean-Philippe Rameau, Arnold Schoenberg, Harry Partch, Paul Hindemith, Ben Johnston, and James Tenney.

I will engage many of these theories and models throughout this paper, but my agenda is more general, and aims to contextualise much of this work into living musical environments. To pretend that the development of a general theory of harmony (which I certainly do *not* aspire to in this dissertation) will produce clear and concrete answers is hugely naïve. This research, I feel, necessarily requires a phenomenological approach that considers previous scientific and musical research, musical experience, borrowed models from other disciplines, and individual subjective responses. The analysis of complex systems requires both intuitive/subjective and reductive/objective models, and I consider music exactly that—a *complex* system.

Through score analysis, spectral analysis of recorded material, and discussions with composers and players, the complex issues surrounding harmonic perception are explored, contemplated, and to the extent possible, rationalised. However, the attempt to rationalise involves both acoustic and psychoacoustic research as well as my own

subjective experiences and those of other 'expert listeners' (composers, theorists, players, musicologists, sound engineers), and is therefore phenomenological (in the broader sense of that term) in nature.

I try to keep my own agenda as a composer to a minimum in Section I, and to concentrate as much as possible on the purely theoretical and perceptual issues. However, it will be obvious through the chosen examples that my focus lies within the twentieth and twenty-first centuries and takes on a particularly western-centric, 'literate', 'art' music bias. It is expected, though, that many of the issues discussed have implications for most methods and styles of music making. The lack of attention given to jazz, rock, folk, early European music through the Romantic era, and the many musics of the world is a factor of scale and scope rather than interest and relevance.

The attention given to piano and string music will also be apparent. Again, this is a matter of scale and scope. The piano and the violin are representative of two extremes in the classical tradition with regards to intonation; the piano is the symbolic 'voice' of fixed twelve-tone intonation (12tet), and the violin family is the most flexible in terms of *dynamic* intonation. Instruments falling within these two extremes (*guided intonation* or *fixed-but-variable*⁵) face unique intonation problems that can only be brushed upon in a paper of this size, but many of the issues raised can be applied to these other instrument groups through considered and qualified interpolation or extrapolation.

One further bias will be evident. All of the score examples use some form of standard western notation, either used as is, or extended and adapted to serve the purpose of the tuning system. Graphic scores and other unique notation systems have not been considered here, again simply due to the scale of the project.

Borrowing from the Gestalt Law of *Prägnanz* (simplicity)⁶, I make the underlying assumption that players and other listeners interpret musical materials through the simplest rationalisation possible, which may include sub-laws of *Prägnanz* such as the Laws of *Closure* (if a portion of something is missing, we add it), *Similarity* (we group

⁵ Patrick Ozzard-Low identifies three broad categories of intonational characteristics for instruments: *fixed*, *fixed-but-variable* and *variable* intonation. "Fixed-but-variable instruments differ from the variable in the sense that conventional woodwinds and valved brasses are designed to *guide the reliable production of a specific scale or system of tuning*" [emphasis my own] (Ozzard-Low 1998: 4).

⁶ Gestalt theory is generally considered out of date with current theories of perception; but in what it fails to *explain* it remains relevant at least as a *description* of several modes of organisation.

similar things together), *Proximity* (we perceive things that are close together as belonging together), *Symmetry* (we tend to group symmetrical entities regardless of the distance separating them) *Continuity* (once a pattern stops, we continue it), and *Common Fate* (things moving together are grouped together) (*The New Encyclopaedia Britannica*, 1998).

[A] listener will always try, whatever the situation or listening strategy, to structure the acoustic world that confronts his or her ears. The creation of a structured representation is what allows music to be more than a simple succession of percepts (Pressnitzer & McAdams 2000: 49).

I assume the models of the harmonic series and twelve-tone equal temperament as the two important references concerning intonation and the interpretation of harmonic sonorities. The use of the harmonic series has been much discussed, but I take the proportions found in the harmonic series to be analogous (or, at the very least, descriptive or metaphorical) to the cognitive processes of the inner ear and brain, and equal temperament is the model we are conditioned to that most obviously confuses this interpretation, but is often the model to which composers, players, and listeners are most likely to relate intonation.

These models and tendencies are confounded by many factors, particularly context and complexity. Thus, often a conflict or ambiguity arises which must be reconciled by the player/listener in order to make decisions regarding intonation and interpretation. The extent to which any of this might occur consciously is a matter of attention, which in itself is dependent on many factors (rhythmic activity and pulse, complexity, musical precedence and anticipation, conditioning, etc.), but it is also likely that a great deal of intonation occurs below the conscious level.

After a brief consideration of the traditional harmonic structures of western music, Section I continues with the analysis of ambiguous harmonic structures found in twelve tone equal temperament compositions, in particular, symmetrical structures (which are unique to equal temperaments) first with fixed-pitch instruments with respect to harmonic *implication*, and then for variable-pitch instruments with respect to intonation. The sections continues similarly with discussion and analyses of chromatic and spectrally derived 12tet harmony, quartertones (24tet), eighth-tones (48tet), twelfth-tones (72tet), other n-tets, just intonation, and finally pitch clusters and sound masses.

Section II focuses on my own compositional work, and relates each piece to many of the issues raised in Section I. But the discussion is not limited to the harmonic concept, and engages other relevant structural parameters, extra-musical issues, and any other parameter deemed central to the aims of the individual piece. I do not attempt to suggest any grand narrative linking all of the pieces, but simply accept that there is some shared *family resemblance*⁷ as they have all emerge over a limited space of time, when certain issues and interest have been at the forefront, and back, of my mind.

There are many concepts important to the discussion and analysis of both sections. Some are explained as they arise in relation to the topic at hand, but others are referred to without detailed explanation. Most special terms and concepts that are not addressed in the body of the paper are addressed in Appendix 1 – Glossary.

This paper addresses at least **three modes of the musical process**. Complex hearing and intonation issues have important implications regarding **notation** (composition), **interpretation** (performance), and **perception** (listening). It is important for the composer to have realistic expectations regarding intonation, and to be aware of ambiguous and/or complex harmonic constructions. These same issues might also suggest approaches to pitch structure that take advantage of ambiguity and complexity.

The method of notation is a contentious issue in microtonal music. While the notation of quartertones is relatively well established, this is not the case for any other intonation system. For some composers, notation is also a compositional parameter and the chosen system to some extent follows intent and may reveal a high level of *self-analysis*⁸. However, many microtonal composers have developed their own notations from a very particular approach to intonation (that is, from what approach do microtonal inflections result?), and are therefore not transferable. And, some composers are attached to a single method that, while sometimes theoretically well defended, often does not consider function or the needs of the performer and does not vary depending on compositional intent. An understanding for complex issues of intonation may suggest approaches to notation that address both theoretical clarity and efficient decipherability by the musician.

⁷ See discussion of Wittgenstein's concept of *Family Resemblances* in the Introduction to Section II.

⁸ See Section I – Introduction for an explanation of *self-analysis*.

There has been little study carried out on how performers confront microtonal material. For the player, sensitivity and knowledge of harmonic conditions may aid in interpretation and alleviate frustration where what is implied is more demanding than is perceptually possible. But perhaps composers have more to learn from players than the other way around, as composers must also acknowledge the player as expert listener.

The importance to the listener is less obvious, but it is with the listener in mind that music is made (most of the time), and the importance of the listener may be tied up in a feedback loop where the composer or performer can assess to what degree, and in what way, the listener is able to hear the piece, suggesting refinements to the notation system and the theoretical harmonic/melodic basis of the music.

Finally, in the age of auto-pitch correction tools, the creative use of intonation should be informed by an understanding of the addressed issues, and other developments in harmonic perception and theory, and thus this paper may be particularly significant to the recording engineer/producer working in various musical styles. Again, time and space will not allow for a detailed investigation of this area of music production, but the applicability of this study to the field of sound recording should be obvious.

I cannot pretend that it is possible to resolve the many parameters involved within the context of a complex system such is the act of composing, playing, and listening to music, and suitably this thesis will likely raise more questions than it will answer. It is hoped, however, that the discussion will provide a framework for understanding the various influences on intonation, and expose where we lack knowledge, and provoke further research by the interested scientist, psychologist, musician, and, most directly, myself.

SECTION 1 HARMONIC INTONATION AND IMPLICATION

SOME PRELIMINARY NOTES

ORGANISATION OF SECTION 1

Section 1 is organized hierarchically, on one level, beginning with 12tet as the least 'resolved' tuning system investigated but also as a the base system most commonly adapted to tuning systems of higher resolution. Elaborations on each system are indicated through a decimal point numbering system where each further decimal place relates hierarchically to each place holder to the left (e.g. in a subheading labelled 1.3.2, the material relates to heading 1.3, which subsequently relates to heading 1. This material is presented in the Arial font.

Unfortunately, not all of the material fits nicely into such a configuration and therefore some occasional cross-referencing will be necessary. For example, under certain conditions, the augmented triad might just as easily occur as a subheading of 1.3 ('chromatic harmony') or 1.5 ('suggestion of higher harmonics in 12tet') rather than where it lies as subheading 1.4 (symmetrical harmonic structures in 12tet)

Other sections of the paper, including this one, are similarly organized without the numbering system but adhere to the typeface of the font for each hierarchical level. The highest level is the Section, of which there are only two, plus the Introduction, Table of Contents, Appendices, etc (bold 16 point):

SECTION (LEVEL 1)

In each section, the highest heading level is labelled in bold 12-point typeface using capitals and small caps:

HEADING (LEVEL 2) x

The next level is in bold 11-point typeface in small caps without capitals:

SUBHEADING (LEVEL 3) x.x

The next level is in italicised bold 11-point typeface:

SUBHEADING (LEVEL 4) x.x.x

Concurrently, I attempt to develop the discussion through a series of analyses, which generally speaking follow the most relevant heading or subheading (indicated through the

label 'A' for a 'analysis' and followed by the related subheading—A1.2—and italicised and, along with the body of the analysis, in the Times New Roman font for easier recognition.

Although partly influenced and informed by the work of other musicians, I consider these analyses as representative of the significant portion of my 'original work'. However, I feel that I have also contributed to the discussion regarding the theoretical bases of each tuning system and to the analysis of vertical harmonic chords and structures.

FIXED VS. DYNAMIC INTONATION

Intonation is addressed from two perspectives. The first is from the standpoint of *fixed intonation* where the implication of certain harmonies is considered from the theoretically perfect state of the tuning system. Twelve-tone equal temperament (12tet) is the obvious example where the resolution of the system limits harmonic implication. However, 12tet may also act as a referential system for instruments capable of *dynamic intonation*, and is considered from this perspective as well.

I discriminate between two approaches to dynamic intonation, although the categories have considerable overlap. *Harmonic intonation* refers to, in this thesis, intonation that responds to the acoustic and psychoacoustic qualities of the harmonic situation—that is, where the player makes adjustment in order to reflect harmonic proportion more precisely and to maximise *smoothness, fusion, or sensory consonance*. However, other factors contribute to intonational tendencies which have little to do with the acoustic/psychoacoustic properties of the vertical sonority, such as voice leading, the influence of accidentals, and expressive approaches. To all of these tendencies, I apply the term *expressive intonation*⁹.

My terms are more general than those terms proposed by Janina Fyk (1995)¹⁰, and some confusion may arise where our terms are differently applied. As the most general category, Fyk uses the term *expressive*, where I use the term *dynamic*. She identifies "four types of expressive tuning: harmonic, melodic, corrective and colouristic" (Kanno

⁹ James Tenney's term 'melodic intonation' is also appropriate here, however I want to keep open a concept of the expressive intonation of vertical chords and sonorities as well as melodic expression.

¹⁰ Referenced by Mieko Kanno in her *Contemporary Music Review* article "Thoughts on How to Play in Tune: Pitch and Intonation" (2003: 35).

2003: 35). My model differentiates between harmonic and more general expressive tuning, which is due to the emphasis given to harmonic function in this paper.

A hierarchical model, which integrates Fyk's terms, might look something like fig 1 (my terms in bold), but acknowledges that there is a great deal of overlap between categories and subcategories.

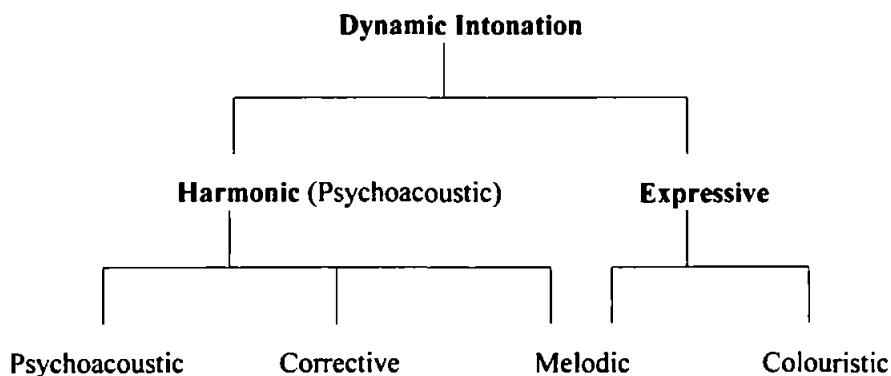


Figure 1 – Dynamic Intonation Tree (my terms in bold)

In my model, dynamic intonation refers to any real time intonation adjustments, large or small, conscious or unconscious. Expressive intonation refers to any intonational adjustment made for emotional or colouristic reasons, such as vibrato or portamento. Harmonic intonation occurs where the musician makes adjustments in order to reflect some sense for the acoustic or psychoacoustic properties of sound, such as the maximization of *fusion* or *smoothness*. Corrective intonation may also refer to adjustments made in reference to some overriding ideal (a particular temperament or system of intonation that directs or overrides intonation based on sensory consonance). Melodic intonation does not fall neatly under Expressive or Psychoacoustic intonation as melodic intonation may result from a number of concerns and phenomena—I might just as easily have positioned the term below the Harmonic heading, as could Corrective fall under both Melodic and Harmonic.

METHODS OF ANALYSIS

HARMONIC SPACE (TUNING LATTICES)

I use a variation of James Tenney's *harmonic space model* and Ben Johnston's *tuning lattices* throughout this paper as a means for describing harmonic relationships—both implied and real. In these models, melodic/harmonic pitch relations are represented graphically in a Euclidean n-dimensional space in which, typically, the horizontal plane

corresponds to 3-limit¹¹ relations, i.e. perfect fifths and fourths ($3/2$ and $4/3$), and the vertical axis to 5-limit interval relations, i.e. major thirds and minor sixths ($5/4$, $8/5$). The inclusion of higher-limit relations requires the addition of a new plane (or dimension) for each additional prime number generator (for example the 7-limit introduces the $7/4$ and its inversion $8/7$ and requires a 3-dimensional space)—in my models, distinct oblique angles represent each additional ‘dimension’. These models generally ignore the 2-limit as *octave equivalence* (explained by *the affinity of tones*) is acknowledged or suggested in many of the world’s musical systems, and considered harmonically redundant and “identical in chroma” (Terhardt 1984: 279). However, the omission of the 2-limit is purely for ease of representation and should not suggest that note spacing, register, and voicing are perceptually insignificant.

In the lattices that follow, dashed lines are used to indicate speculative harmonic relationships. And square bracketed pitch classes indicate implicit relations that are not actually a part of the analysed sonority, but connect two harmonically related pitch classes (Gestalt law of closure). For example, a $9/8$ major second shows the root $1/1$ connected horizontally to a bracketed perfect 5th ($3/2$), which is in turn connected to the $9/8$ major second, thus showing the harmonic basis of the $9/8$ (which is the result of two stacked perfect fifths condensed to within an octave (see figures 9, 10, and 11 for example)).

¹¹ Please see Glossary of Terms in Appendix 1 for a definition of the term ‘limit’.

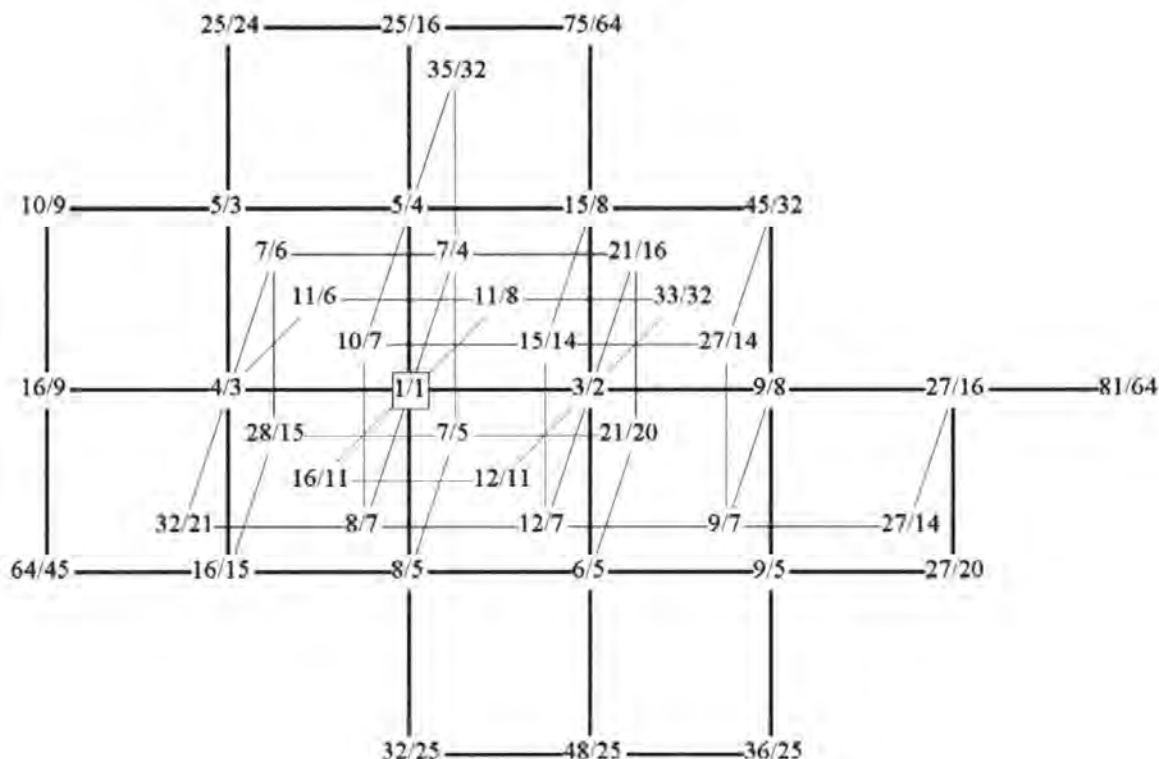


Figure 2 – Example of an 11-limit harmonic space in four ‘dimensions’

GESTALT INFLUENCE

In these lattice models, the simplest relations fall closest within the space, and it is the relative compactness of the space that is thought to be an indicator of complexity (or dissonance within a limited definition of that term). Although each pitch class appears in several positions, each position suggests a unique intonation and implication.

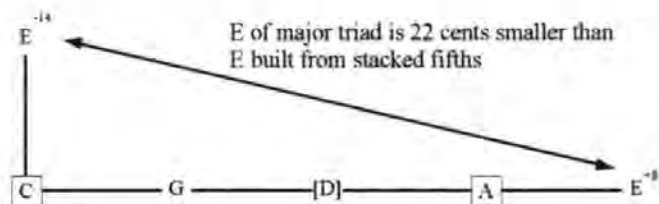


Figure 3 – Intonation of E in major triad compared to intonation of E found in a quintal chord

To what extent vertical sonorities may be suggestive of some acoustical ideal depends on conditioning, acoustics/psychoacoustics, and the implied resolution of the system. That is, as the tuning system becomes more refined, the implied source of certain harmonies become more explicit. For example, an F[♯] in the key of C in 12tet might serve any number of harmonic premises (Messiaen uses it as suggestive of the 11th partial of the harmonic series, whereas a more traditional harmonic vocabulary might consider it a

tonicisation of the dominant), in a quartertone temperament the implication is more explicit because the composer has a choice of $F\sharp$ or $F\flat$, the choice of which clears up the ambiguity of this particular example. This also means that what is heard as 'in-tune' depends upon similar parameters and by what is meant by 'in-tune'.

The Gestalt *Law of Prägnanz* (simplicity) encompasses five main laws of grouping: proximity, similarity, good continuation, closure, and common fate, and "can be briefly formulated like this: psychological organization will always be as 'good' as the prevailing conditions allow. In this definition, the term 'good' is undefined. It embraces such properties as regularity, symmetry, simplicity and others..." (Koffka 1963: 110).

If three points in space are considered, an observer will create a relationship that connects the points with three straight lines (a triangle) regardless of the fact that...

[i]n principle, [those] connections could take any shape among an infinite number of curves, twists, and loops of the most irrational kind. The economical choice of the shortest connection is an elementary application of Gestalt psychology's principle of simplicity: any pattern created, adopted, or selected by the nervous system will be as simple as the given conditions permit (Arnheim 1977: 11)

James Tenney has extended the Gestalt law of simplicity into the harmonic realm in order to explain the perception of tone relations as described in the harmonic space model. In a 1985 interview, answering Brian Belet's question "What are your thoughts regarding harmonic organization and the relationship of just and tempered intervals?" Tenney states:

My hypothesis is that our ears will interpret things in the simplest way possible. Given a set of pitches, we will interpret them in the simplest way possible. This can be translated into harmonic space terms by saying that it will be the most compact arrangement in harmonic space. Well, I think compactness, in that sense, could be measured somehow, and could be made very explicit by speaking of the sum of harmonic distances among these various points. So you could go through a piece and say, 'Alright, we've heard in the beginning of the piece two pitches. You take the simplest ratio representative of that interval—tempered. Now we hear the third pitch. What specific, rational intonation for that approximate pitch will give us the simplest configuration in harmonic space? Let's call it that.' And then analyse the music on that basis. It leads to some very interesting harmonic discoveries.

[Brian Belet: Even if that means re-evaluating the first two pitches?]

I believe so. Of course you have to do that with a lot of sensitivity towards how we hear and to what extent we refer backwards, and revise our interpretation as we go on. The notion is that we are going to interpret it, again, as simply as possible. This process could be developed into an analytical tool. (Belet 1987: 161)

If we consider three pitches, a simple interpretation is often available. For example, a major triad sounding in equal temperament suggests its simplest just ratio form: 4:5:6. However, some situations are much more ambiguous (just as a rough or random shape may not immediately suggest a particular geometric ideal). Whether or not an equal tempered dominant seventh chord suggests 4:5:6:7 is more difficult to answer because the equal tempered seventh is closer to a 16/9 than to a 9/5 or to a 7/4, and the implied function differs depending on the musical context—particularly in its traditional use which requires its resolution to a more ‘stable’ chord. Or a vertically dense cluster of pitches is likewise difficult to model in harmonic space. To what degree the simplicity principle can overcome these distortions and level of complexity is difficult to determine.

In passages that are more complex, our perception of harmony may also be revised backwards, within some qualified timing threshold.

The auditory mode of recall is remarkably powerful...The hypothesis advanced [by Crowder and Morton (1969)]...is the existence of a sensory storage: a sort of ‘echoic’ memory specific to hearing that conserves the stimulus trace for a brief period of time. This hypothesis has since been refined and there are most likely several different retention intervals (Cowan, 1984). One of these intervals would be on the order of several hundreds of milliseconds and another one on the order of several seconds. The first storage would be related to sensation, constituting a sort of ‘perceptual present’, while the second one would serve as a basis for what is called working memory. These fairly short durations raise all kinds of questions concerning the possibility of apprehending structures extended through time and carried by sound. The stimulus trace vanishes within a few milliseconds of the echoic memory, and the working memory cannot hold more than a few items (Pressnitzer and McAdams 2000: 54).

Echoic memory makes possible the application of harmonic space analysis to melodic material as well.

In tonal harmony, the listener is quickly able to apprehend a sense of key. But, as David Butler states in *The Musicians Guide to Perception and Cognition*, no one has successfully proposed a convincing theory of tonal harmony. He suggests, “Key identification must be a simple process, which is ironic considering that discussions of it tend to be so complicated”;

It seems improbable that the listener determines the tonal center of a group of tones by matching that group to all possible scales or pitch sets or fifths cycles and rejecting the poor matches. This sort of brute-force thinking would be very costly in terms of time and attention. A likelier assumption is that any tone one hears will suffice as a tonal center until the listener is prodded by better tonal evidence to opt for a more plausible choice. For example, if one hears only the tone C (the choice of octave may make a difference here), there is no reason to suppose that it represents any tonal center other than C. If the C is followed by the tone B \flat and G, it becomes less convenient for the listener to assume that

the key center is C¹², and the intervallic evidence now reduces the ambiguity to the extent that the choices are (at least for major mode, and not allowing the possibility of chromatic harmony) that the tonic is F, B^b, E^b, or A^b. Alternative choices of tonal center are further constrained as intervallic information is added (Butler 1992: 121).

Somewhere along this passage, Butler seems to slip back into a comparative model, which he dismissed earlier as “costly”, but if we apply his thinking to a harmonic space model, something more efficient may emerge; the ‘tonic’ would be the common fundamental, found in the lowest, leftmost and forward-most corner of the pitch set (Tenney 1983: 78)¹³. Of course, the listener is not comparing a pitch set to harmonic space, but harmonic space attempts to reflect the cognitive process of harmonic organisation.

But some long-term learned (cultural, historic, stylistic, etc.) memory, to which we compare the perceptual present, must also be included in this model.

Memory would more likely be distributed, as a by-product of cognitive processing, in the form of *potential representations*. In other words, in the presence of a stimulus, the brain actively results in the extraction of the relevant features, making generalizations, and forming categories...When a new stimulus is perceived, if it is similar enough to a potential representation already memorized, it will be categorized as a member of the same family (Pressnitzer and McAdams 2000: 55).

Another way that we impose order on [tone groups] arises from our knowledge of tonal harmony—a knowledge that may be quite sophisticated, even though we may have never had any formal training in music (Butler 1992: 114).

HARMONIC DISTANCE

In the above interview, Tenney refers to the measure of *harmonic distance*. Several theorists, including Tenney himself, have taken up this concept and suggested methods for calculating harmonic distance (Tenney first talked about harmonic distance in “John

¹² This particular example is problematic in relation to the development of this paper. The interpretation of this set of pitches (B^b, C, and G), depending on stylistic factors, may actually result in the opposite result, and be heard as further reinforcement of C as the perceived tonal centre (particularly if it occurred in a piece by Debussy or Messiaen).

¹³ “Regarding the ‘tonic phenomenon,’ our model does not, in itself, suggest either an explanation or a measure of it, but we can incorporate into the model the simple observation that there is a kind of directed ‘field of force’ in harmonic space, such that a tone represented by a given point will tend to ‘become tonic’ with respect to tones/points to the ‘right’ of it (...i.e. in the 3/2 or ‘dominant’ direction), and to a lesser extent, ‘above’ it (in the 5/4 direction). Such a tone seems capable of absorbing these other tones into what might be called its ‘tonic field,’ and to be absorbed, in its turn, into the tonic field of another tone to the ‘left’ of it (i.e. in the 2/3 or ‘subdominant’ direction), or ‘below’ it” (Tenney 1983: 78).

Cage and the Theory of Harmony”), which is defined as a measure of relative complexity (‘dissonance’ in a very broad definition of that term).

The basic formula for the harmonic distance between any two pitches is $Hd(a/b) = k \log_2(a/b)$, where a/b is the frequency ratio representing the interval (in its maximally reduced, “relative prime” form¹⁴), and k simply determines the unit of measurement (with base-2 logarithms, if $k = 1$, Hd is in “octaves”) (Tenney 1987: 81).

Other measures have been suggested by theorists such as Ervin Wilson and Clarence Barlow, for example. I will not adopt any particular measurement for harmonic distance (complexity) here, as this level of precision (and debate) is not necessary for the analyses that follow. I will instead adopt a simple and intuitive approach to discussing relative complexity/simplicity, informed more generally from the shared properties of the above methods.

TERHARDT – SUBHARMONIC COINCIDENCE

Further to analyses in terms of Tenney’s harmonic space model, as both an alternative and elaboration, some 12tet harmonic sonorities will be compared with Ernst Terhardt’s *subharmonic coincidence* analysis. Terhardt maintains that roots can be found for all chords according to a method based on a virtual-pitch algorithm. Pitch classes based on the first five odd-numbered subharmonics of each pitch in the chord are generated and compared. The pitch class that occurs most often is the root of the chord. In some cases there is no ambiguity, such as with the major triad. Other chords are more or less ambiguous depending on whether there are two or more pitch classes with an equal number of occurrences (or no common occurrences at all)

...one can deduce from the theory of virtual pitch a root-finding algorithm that operates on the score—just as in conventional music theory. The principle is, that all candidates of roots must be subharmonics of the spectral pitches elicited by the actual sound, and that the prominence of any root is enhanced by “subharmonic coincidence”. When one translates this concept into musical notation, one gets a simple algorithm that shall be explained by the following example.

<u>Chord notes:</u>	c	f	a
same	C	F	A
- 1 fifth	F	B \flat	D
- 1 maj 3rd	A \flat	D \flat	F
+ 1 maj 2nd	D	G	B
- 1 maj 2nd	B \flat	E \flat	G

¹⁴ See Glossary of Terms: *Frequency Ratio*.

The sample chord of which the root(s) are to be determined is written into the first line (2nd to 4th column, lowercase letters), i.e., c-f-a. The root candidates derived from the notes are written into the pertinent columns (uppercase letters). The first candidate corresponds to both the first and second subharmonics, i.e., it is just the same note (second line), because, by definition, octave-equivalent pitches are not distinguished. The second candidate (third line), corresponding to the third subharmonic, is obtained by stepping one fifth down from the chord note, as octaves are not distinguished. The third candidate (fourth line), corresponding to the 5th subharmonic (the fourth is omitted, due to octave-equivalence) is obtained by stepping one major third down. The fourth candidate (fifth line), corresponding to the 7th subharmonic (the 6th is omitted, due to octave-equivalence to the third), is obtained by stepping either down by a minor seventh or stepping up one full tone (due to octave equivalence). The 8th subharmonic is omitted due to octave equivalence, such that the last candidate (6th line) corresponds to the 9th subharmonic, and it is obtained by either stepping up a 9th interval or stepping down one full tone (octave equivalence). As one can see in the table, there is one and only one candidate, namely F, that occurs in all three columns (full subharmonic match). This, by definition, indicates that F is the most prominent root of the chord.

As is apparent from this example, the determination of root candidates cannot fail for any type of chord. What may happen just is that there is no "full match", i.e., that there does not occur any candidate which is found in all columns, such that the root is less pronounced and more ambiguous than in a major triad such as above (Terhardt 2000).

Terhardt acknowledges that an analysis of harmony using this method has little to do with functional harmony although we might expect some correlation because of the partial acoustical basis of functional harmony.

As to the term "harmonic analysis", there is danger of more misunderstandings. In my work on pitch, I am referring to "subharmonic coincidence analysis" (or...detection) in a fairly technical sense which a priori does not bear any relationship to music, especially to "harmonic analysis" in the sense of music theory. When the "sound" of the Tristan chord F-B-D \sharp -G \sharp is analyzed by my theory, i.e., by subharmonic coincidence detection (of the acoustic sound spectrum), one of its pertinent virtual pitches turns out to be the low C \sharp ¹⁵. This finding then is elucidated from the viewpoint of music theory, i.e., saying "Aha, this makes sense because the chord is a subset of..." see above (Terhardt, e-mail to the author, 05 February 2005)¹⁶.

PITCH SPELLING ANALYSIS

Malcolm Gillies (1993) and George Perle (1984) both argue for the significant contribution of pitch spelling to musical analysis; that the composer provides a *self-analysis* of her or his work through pitch spellings that "distinguish[es] the functions of notes within their

¹⁵ Please see the subharmonic coincidence analysis of the Tristan chord (section 1.3) for more detail of this analysis.

¹⁶ I believe that by "low C \sharp ", Terhardt simply means the 'root' of the chord found a major third below the F.

melodic, harmonic, contrapuntal and, ultimately, tonal contexts” (Gillies 1993: 42). George Perle offers the example of Beethoven's Fifth Symphony.¹⁷

...even though they did not give us analytical surveys of their compositions, the composers of an earlier age were constantly making explicit and detailed analytical assertions, in the very act of writing the notes down. When Beethoven, in the slow movement of the Fifth Symphony, spells Forte's pc set 4-27 as $A^b C E^b F^\sharp$ in bar 29 and as $A^b C E^b G^b$ in bar 206, he is engaged in an act of analysis, as well as an act of composition (Perle 1984: 101).

Such an approach to analyses is steeped in the harmonic/melodic tradition of European art music. Where the composition is firmly based in a tonal or extended tonal language, this approach to analysis is particularly revealing. However, with the advent of atonality, it was thought that much of the meaning of pitch spelling was lost, or should be lost.

Faced with the growing freedom of use of all twelve chromatic notes in twentieth-century music, and the widespread abandonment of triadic harmony, many analysts have assumed the dissolution of those notational conventions of earlier times. They have therefore concluded that distinctions of pitch spelling have little or no significance in twentieth-century music. Indeed, the survival into the twentieth century of five different accidentals (\times , \sharp , \natural , b , bb) and three different spellings for most pitches (E^\times , F^\sharp , G^b) has been seen by some as a misleading encumbrance which can easily foster false notions because of the suggestions of traditional function inherent in the signs themselves. In the early years of the century Busoni (1910), Bartók (1920), Schoenberg (1924), and Cowell (1927)—to name but a few—called for or proposed new systems of pitch representation. Their cries of dissatisfaction with traditional notational methods were merely in the vanguard of a concerted movement amongst musicians to stress the *aural* nature of music, apparently to redress what was perceived as an excessive concentration on *pictorial* facets of music in earlier times. Alexander L. Ringer (1980: p. 123) has, for instance, written of ‘the extent to which musical notation, with all its blessings, has narrowed Western man's understanding of a cultural phenomenon that is always aural in essence and rarely if ever graphic (Gillies 1993: 43).

With twelve-tone serial music, in theory, all meaning in pitch spelling is lost. The pitch spelling is derived from a fixed gamut with no suggestion of harmonic/melodic function.

Despite this widespread dissatisfaction with existing methods of visual representations, most composers did continue to employ traditional pitch spellings. Some chose to use the different chromatic inflections randomly in an attempt to cancel out any conventional implications. Others allowed their use of accidentals to be guided by readability alone, preferring, according to the dictates of the moment, a vertical or a horizontal homogeneity in their spellings. A few, notably Scriabin and Bartók, sought to develop new rules, to allow for consistent representation of their music within the limits of traditional pitch spellings (Gillies 1993: 43).

¹⁷ This passage is also quoted in Gillies 1993: 42.

In the discussion that follows, it emerges that for music composed throughout the twentieth century pitch spelling approaches are both variously vague or specific, and determining the level of 'self-analysis' embedded in the score is given consideration prior in each analysis.

Self-analysis can also be extended into other tuning systems to varying degrees. For example, a standardized notation system for quartertones has been more or less established, which has removed some of the self-analytic potential in these scores (because the *motivation* for quarter tones is *not* standardized) but the notation of other extended systems is not so well-established and, depending on the composer, some self-analysis may be available; in just intonation systems, pitch spelling is inextricably linked to harmonic/melodic meaning.

Each analysis below considers the implications of pitch spelling and notation, not only with regards to the composer's intent but also with the player in mind—to gauge the effectiveness of the chosen notation given the composer's intent versus the musician's performance.

SPECTRAL ANALYSIS

For analysing recorded performances, I use the spectral analysis software included with Sound Forge 4.5e in order to determine the intonation of single pitches and to extrapolate the intonation of tones within a harmonic sonority.

Spectral analysis is far more subjective than one might expect, particularly where strong vibrato can create *sidebands* louder than the centre frequency, and lower frequencies measured in Hz are less refined than higher ones (whereas the difference between 200 and 201 Hz is perceptually much greater than the difference between 500 and 501Hz). The expressive nature of the performance also means that some average frequency must be decided upon—in this case, I often give preference, following attack, to the early onset of the tone, prior to the onset of vibrato. But it is also important to allow the pitch to 'settle' in order to consider adjustments and corrections, which presumably reflect some response to the harmonic and melodic situation.

The problems in extracting individual tones from a vertical sonority also requires a good deal of interpretation and analysis in itself—the fundamental tone is often not the strongest component of a single tone, nor is the root of any give chord, and a harmonic of

one chord tone may not coincide with the fundamental of another but may be very close in pitch. So, I cannot say categorically that the following analyses are wholly definitive, but should suffice for a general description of intonational tendency.

Although in some circumstances these parameters are changed in order to focus on a particular detail or anomaly in any recorded performance, the following are my default spectral analysis settings¹⁸:

FFT size:	32,768
FFT overlap:	30 percent
Smoothing window	Blackman-Harris
Logarithmic graphing	
Freq. Min. / Max.:	variable depending on score range

PSYCHOACOUSTIC CONCEPTS OF CONSONANCE AND DISSONANCE

I have integrated my psychoacoustics research into my analyses more implicitly than explicitly, but certain concepts and theories retain an important influence on my approach and thought.

Measures of sensory consonance, using simple tones, predict that intervals falling within a *critical band* of unisons and octaves will be perceived as more dissonant than high number frequency ratios (intervals) falling outside a critical band, and in this sense, sensory consonance is not fully synonymous with musical concepts of consonance and dissonance (which are themselves historically specific), although they are strongly related¹⁹.

¹⁸ The *Fast Fourier Transformation (FFT)* size determines the resolution of the frequency analysis. The number of FFT points divided by the sampling rate of the audio sample (CD quality = 44,100 Hz) determines the number of frequency bands the sample rate is divided into and therefore the frequency width of each frequency band (it also determines the time length of each sample). Large point values return smaller frequency bands for larger time segments, small point values return larger frequency bands for smaller time segments. For example, 128 points / 44100 = analyses in 0.003 second increments 64 frequency bands each 345 Hz wide. In my analysis, 32,768 points / 44100 = .74 seconds and frequency bands of 2.7 Hz.

FFT Overlap determines the overlap of each time segment. Higher values provide a more detailed analysis.

The *Smoothing Window* function affects the sharpness of the peaks in the graph. The Blackman-Harris function limits side-band leaks but loses sharpness in the frequency peaks. (a square function results in sharp peaks with high leakage).

Logarithmic Mapping displays the X-axis frequencies logarithmically rather than linearly (which more closely reflects the psychoacoustic perception of frequency change).

Freq. Min. / Max. determines the frequency range of the analysis. I set this value according to the register of the instrument(s) I am analysing.

¹⁹ See James Tenney's *A History of Consonance and Dissonance* for a detailed investigation into the historical meanings of consonance and dissonance

A graph that plots relative measures of sensory consonance—that is, one that plots dissonance (or consonance) against interval—is known as a ‘dissonance curve’. Hermann von Helmholtz’s graphs show maximal consonance for the unison and the octave, followed (in order of consonance) by the 5th, 4th, major 6th and major 3rd, and so on, confirming traditional expectations of a hierarchy of consonant and dissonant intervals described by music theory. The exact points of maximal consonance correspond to the just fifth, the just fourth, etc.; but the curves may also be taken to describe ‘zones’ rather than ‘points’ of consonance—that is, a small portion either side of the apex of a curve shows an intervallic ‘zone’ or band which is ‘close to maximally consonant’ (or ‘maximal relative to that part of the curve’) (Ozzard-Low 1998: 43).

FIG. 60 A.



FIG. 60 B.



Figure 4 – Helmholtz dissonance curves (1877/1985: 193)

Plomp and Levelt, in 1965, discovered that the dissonance curves of Helmholtz and Harry Partch are dependent on harmonically rich tones, and that dissonance curves using sine tones produce only one region of maximal dissonance (roughness) around “an interval just smaller than a semi tone”, and that intervals outside the critical band are essentially all equally consonant (a just perfect 5th is equally consonant to a 12tet tritone, for example). As the harmonic spectra of the tones are enriched, dissonance curves begin to resemble those of Helmholtz, et al. (Ozzard-Low, 43-44).

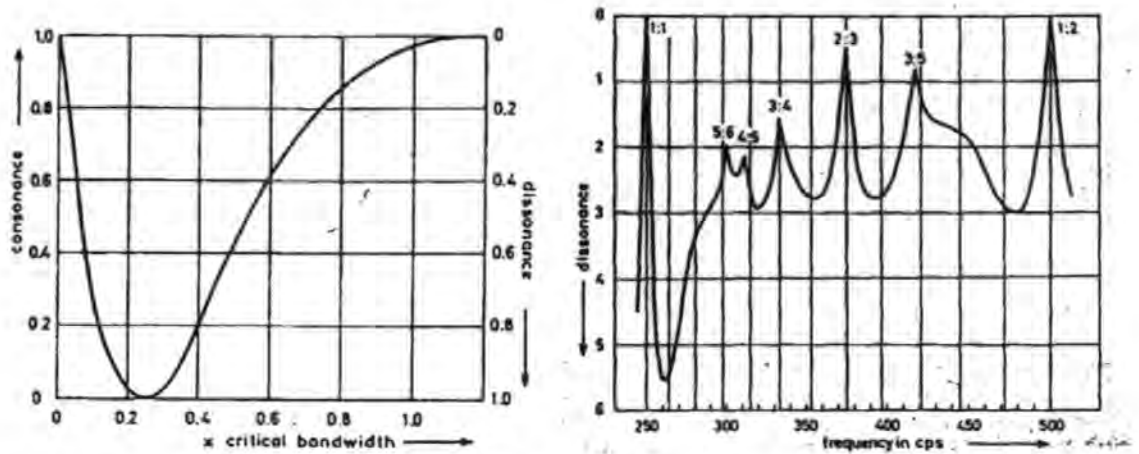


Figure 5 – Plomp and Levelt dissonance curve with a) simple tones, and b) complex tones (1965: 556)

Ernst Terhardt points out that with complex tones, a fine structure emerges where consonant peaks exist at musically significant interval points, although not necessarily in perfect accordance with music theory (for example, “the consonance rating of a major seventh may be significantly higher than that of a fourth”) (Terhardt 1984: 280).

Although psychoacoustic experiments may conflict with music theory, the two do not exist in a vacuum. Musical consonance shares some psychoacoustical basis of perception. This is important to the current study as much new music is concerned with the expansion of musical materials and often the integration of other acoustic and psychoacoustic phenomena, and in this regard, both music theory and psychoacoustic theory should be considered in the development of new musical languages. The player is very often aware of where traditional training may be at odds with the current practice, but also where connections remain intact.

1. TWELVE-TONE EQUAL TEMPERAMENT

Discussion regarding the relationship between equal temperament and harmony is hardly straightforward. Twelve-tone equal temperament (12tet) evolved as a response to developments in music and music theory, but music and theory also responded, and continues to respond, to the development of tuning systems, leading towards and moving from 12tet. In this sense, the relationship between tuning system and music theory/composition is set within a continuous feedback loop informed by their independent *and* related histories.

Certainly by the mid-nineteenth century 12tet had become the *de facto* means for musical expression, but from the end of the century the intentions of the composer gradually, and sometimes drastically, moved away from the founding principles of equal temperament; and developments in tuning systems since the establishment of 12tet can be seen as a further response to these developments.

But 12tet retains a certain hold on musical thinking that cannot be ignored at the present time. This section acknowledges and analyses the influence of 12tet on intonation and harmonic implication in various musical contexts within the twentieth century.

Increased chromaticism and extended and reinvented harmonic approaches meant that 12tet further loosened its ties to its acoustical basis, which is at its simplest the expression of minor and major triadic harmony. The move towards equal temperament, from Pythagorean intonation through mean-tone and well-temperaments, meant that as the system of tuning shifted so to did the demands put on that system—from the modal to the atonal. The system evolves in response to musical style, but style also adjusts in response to the system, which means that 12tet is used in the expression of a variety of compositional first principles (harmonically speaking).

For example, the exploration of symmetrical structures is a direct response to the advent and acceptance of 12tet, as symmetry is a unique product of equal temperament. But symmetry is based on a harmonic premise unrelated to the roots of equal temperament.

Atonal music is also a direct response to the unique properties of equal temperament. The possibility for the ambiguity of key is particular to equal temperament, as a chromatic scale based on just intonation, in theory, will always point to a fundamental based on the common denominator of each interval, thus negating the tonal ambiguity necessary for atonalism. Indeed, it is difficult to imagine the progression towards increased

chromaticism and “the emancipation of the dissonance” without the premise of equal temperament.

Many other ambiguous harmonic situations, effects, and illusions are also uniquely afforded by equal temperament, as will be further explored in the analyses that follow.

It must be acknowledged that, in many ways, equal temperament is a rather elegant solution to the problems that arise in particular harmonic/melodic styles of music making (roughly from the mid-18th to the late 19th century), and that meaningful musical idioms have been made possible through the invention of equal temperament (Impressionism, Serialism), but that in other cases equal temperament is perhaps less than ideal as an intonational standard (modal²⁰, spectral—although exceptions follow). A critique of equal temperament must respond to the first principles of the musical system in question and acknowledge what is both gained and lost in the use of the tuning system, and as a by-product, the notation system.

There are at least two ways to address equal temperament: as a fixed set of pitches where intonation cannot shift (keyboards and fretted instruments—*fixed intonation*); or as a reference, though not necessarily an *ideal*, around which variable-pitch instruments base their intonation (*dynamic intonation*).

This section considers equal temperament as not only a fixed system, but also as a referential system for dynamic intonation, and in this second sense 12tet is inversely suited to other modes of music making: as a system for atonal and ambiguous harmonic systems, the premise, when used with instruments capable of dynamic intonation, is somewhat weakened, where intonation might be adjusted contrary to the principles guiding the music (ambiguity, atonality). On the other hand, dynamic equal temperament is more suited than its fixed intonation counterpart to spectral simulation and expanded tonality or modal systems as intonation may shift according to the implied harmony or to other psychoacoustic influences.

So, a discussion of 12tet cannot begin and end with a critique of its ability or inability to imply, explore, or demonstrate a system of major and minor triads, but must also consider the first principals of other harmonic/melodic musical systems that use 12tet as its

²⁰ The term ‘modal’ is here based on a particular conception which suggests that each mode should have a unique series of interval step-sizes based on, in the case of the Greek modes, a Pythagorean intonation, the character of which is lost in equal temperament.

starting point. This discussion will generalise the first principles of particular compositions and composers for the sake of example, but it is rarely the case that a composition is singularly dedicated to the expression of a clearly stated premise.

1.1 DIATONIC HARMONY IN FIXED 12TET

Twelve-tone equal temperament developed as a means for approximating 5-limit harmony through the tempering of 3-limit intervals. The nature of 12tet, in terms of intonational accuracy, gives preference first to octaves (2-limit, where the tuning is theoretically perfect), then to fifths and compounds (3-limit or Pythagorean, where the tuning error is only 2 cents for each fifth compound from a given root), then to 5-limit (mistunings range from approximately 10 to 16 cents), and then, as a few personalities argue, 7-limit, in the particular case of dominant seventh chords (where the just minor seventh (7/4) is 31 cents smaller than the 12tet minor seventh)²¹.

This preference reflects some psychoacoustical observations, namely that our tolerance for mistuned intervals is lesser for simple ratios and greater for more complex ones. We are less tolerant to the mistuning of unisons and octaves (2/1) than we are major thirds (5/4), and less tolerant to the mistuning of major thirds than minor sevenths (7/4, 16/9, or 9/5). This tolerance is loosely related to the concept of *sensory consonance* where the sense of *smoothness* and the lack of *beating* or *roughness* are indicators of consonance levels. Small number interval ratios have higher levels of sensory consonance, and fuse easily, *but* deviations in the tuning show up more readily in small number interval ratios; that is, changes in intonation, resulting in increased beating or roughness, are most noticeable here.

Where 12tet fails is in the reciprocal effect that, in order to recognise a particular ratio as intended, greater latitude in intonation is available to simple ratios and less for complex ratios. That is, a major third must be tuned more accurately than an octave in order to be recognised as such. Therefore, *harmonic suggestibility* in twelve-tone equal temperament becomes less functional as ratio-limit increases—it is perhaps impossible to suggest the interval 11/8, which is about a quartertone flat of a 12tet augmented fourth, but the equal tempered major third is highly suggestive of the just 5/4 major third even though there is a

²¹ James Tenney is one advocate for this.

difference of 14 cents between the two. And an octave mistuned a quartertone sharp may still be identified as an octave, albeit unacceptably out-of-tune for most ears²².

1.1.1 MAJOR TRIADS

There is little argument regarding the implication of the traditional usage of major triads in equal temperament; theorists and psychoacousticians from Helmholtz through Hindemith to Tenney support this. A 12tet major triad *implies* the just ratio of 4:5:6. The equal tempered major triad lies within some threshold of intonational accuracy, and is not a particular challenge to the limits of harmonic suggestibility. But this is not so clearly the case in other sonorities.

[A]lthough the harmonic space model is ratiometric, a tempered system may be mapped onto it by relating its pitches to the simplest ratios within their tolerance range. Another [feature of the harmonic space model] is that although the model is a theoretically infinite n-dimensional lattice—like [Ben] Johnston, Tenney too delimits the number of dimensions in any given model by the number of different prime factors required to specify the frequency ratios of a given set of pitches—the tolerance principle puts a significant constraint on the unlimited proliferation of points in harmonic space, and of dimensions in the lattice: beyond a certain point, large-number ratios furthest along the axes from the centre point, Tenney contends, become redundant, because the ear interprets them as mistuned versions of small-number ratios (Gilmore 1995: 487).

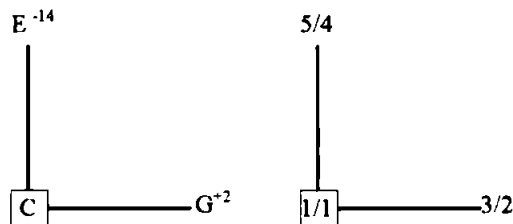


Figure 6 – Major triad in harmonic space

AUDIO TRACK 1: 12TET MAJOR TRIAD COMPARED TO 5-LIMIT JUST MAJOR TRIAD

The major triad is one of the simplest (most compact) sonorities described in harmonic space. In applying these models to equal tempered harmonic relations, something specific is implied about resolution and tolerance. Where lattices used to represent just intonation extend infinitely along all planes, as a description of equal temperament each plane is in effect circular, curving back upon itself. 3-limit planes will have 12 steps, one for each tempered fifth; 5-limit planes will only have three steps, one for each major third

²² This would depend a great deal on backwards listening. If there is a precedence of quartertones, then a quartertone flat or sharp octave may be perceived as such.

of the augmented triad: root – third – augmented fifth – root). If 12tet is considered capable of suggesting 7-limit intervals, then the 7-limit plane would consist of seven steps (but as it is such a special case in 12tet, it might only occur as a satellite of one of the 5-limit chord components; a stacking of minor sevenths in 12tet is not likely to be heard as a chain of 7/4's). But within lattice diagrams, it is simply understood that note names indicate identical pitch classes when speaking specifically about equal temperament.

1.1.2 MINOR TRIADS

While the source of the major triad is generally attributed either to sensitivity to the lower partials of the harmonic series, or to small number divisions of a string length (in either case, naturally ('nature-ly') justified), the source of the minor triad has been more contentious. Harry Partch and Henry Cowell both proposed the “theoretically dubious concept [of undertones as a] ‘natural’ justification for the minor triad” (Anderson 2000, p.9) where the minor triad emerges between the third and fifth subharmonics—the second and fourth undertones— $(1/3, 1/5)^{23}$.

The minor triad may alternatively be the result of the 10th, 12th, and 15th harmonics of a missing fundamental. Both the undertone and this overtone theory generate a minor triad described by the ratio 10:12:15 (in its most compact form).

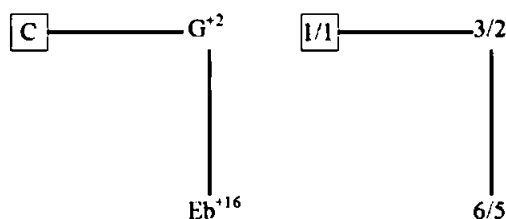


Figure 7 – Minor triad in harmonic space

AUDIO TRACK 2: 12TET MINOR TRIAD COMPARED TO 5-LIMIT MINOR TRIAD

In determining the acoustical root of the minor triad, Paul Hindemith²⁴ in *The Craft of Musical Composition* includes, after a detailed explanation of the use of combination

²³ In *Genesis of a Music*, Harry Partch ascribes the implication of the 300 cent dyad of 12tet to the ratio 32/27 rather than to 6/5 of the minor triad (and the inversion 900 cents to 27/16 rather than 5/3).

²⁴ I turned to Paul Hindemith's *The Craft of Musical Composition, Book 1: Theory* with an eye for discrediting what has proved to be an influential but flawed text. To be sure, much of chapters I and II suffer from circular reasoning, misinformation, conjecture, and contradiction, but upon rereading I discovered that behind the inaccuracies lay some interesting fodder worthy of at least consideration for inclusion in a general theory of harmony; and the appeal to and influence of this text on the French spectral composers became more apparent to me.

tones (I believe he actually means *difference* tones) in justifying a ranking of intervals and chords, a description of two further theories: one based on the coincidence of overtones, which he discounts because the use of this measure makes the major triad subordinate to the minor in terms of acoustic justification²⁵; and one which considers the minor triad a clouding of the acoustically clear major triad.

What, then, is the minor triad, in reality? I hold, following a theory which again is not entirely new, that it is a clouding of the major triad. Since one cannot even say definitely where the minor third leaves off and the major third begins, I do not believe in any polarity of the two chords. They are the high and low, the strong and weak, the light and dark, the bright and dull forms of the same sound. It is true that the overtone series contains both forms of the third (4:5 and 5:6) in pure form, but that does not alter the fact that the boundary between them is vague. Pure thirds furnish us with pure forms of both major and minor triads. But the ear allows within the triads, too, a certain latitude to the thirds, so that on one and the same root a number of major triads and a number of minor triads can be erected, no two alike in the exact size of their thirds. Triads in which the third lies in the indeterminate middle ground can, like the third itself, be interpreted as major or minor, according to context. But why the almost negligible distance between the major and minor thirds should have such extraordinary psychological significance remains a mystery.

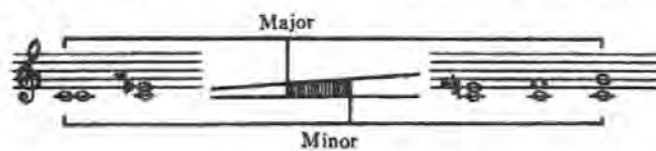


Figure 8 – from Paul Hindemith *The Craft of Musical Composition*

It seems as if this middle ground between the thirds were a dead point in the scale, to which another similar but less significant dead point corresponds—the middle ground between the two species of sixths (Hindemith 1942: 79).

AUDIO TRACK 3: ELEVEN CONSECUTIVE TRIADS WITH DIFFERENTLY SIZED THIRDS (FROM MINOR THIRD -33 CENTS TO MAJOR THIRD +50 CENTS IN 1/12TH TONE INCREMENTS)

While this theory is not widely regarded, and may seem a stretch in the particular case of the minor triad, applying this to other sonorities may be useful and finds a parallel in the work of Charles Ives where he conceives of 'the neutral 3rd' in his 'primary chord'. (There may also be precedence in other musics of the world.) If the minor triad is considered an extreme case of distortion, testing the limits of a poorly resolved or ambiguous fundamental, then other less extreme cases may conceivably be regarded as such.

²⁵ Harmonic coincidence occurs at partials 9, 8, and 7 (root, 3rd, 5th respectively) for a major triad; and at partials 6, 5, and 4 for the minor triad. Of course in the major triad the seventh harmonic of the fifth does not strictly coincide with the ninth harmonic of the root—it is 31 cents flatter, and for the minor triad the sixth harmonic of the root does not exactly coincide with the fifth harmonic of the third, which is 14 cents flat.

However, the usefulness of the harmonic space model becomes severely challenged if Hindemith's theory is considered (and that is no reason in itself to abandon Hindemith's theory).

(To date, I have not been able to find any scientific research that considers a clouding theory of harmony.)

1.1.3 DOMINANT SEVENTHS

The implication of the equal tempered dominant seventh depends on the function of the harmony, the difference of which is most clearly exemplified through the examples of Wagner and his predecessors (functional harmony), who used the dominant seventh as a consonant triad with a dissonant interval requiring resolution²⁶, and Debussy, who used dominant chords as a stable sonority not requiring resolution.

There is no theoretical precedence for this first interpretation of the dominant seventh chord, but it is useful for demonstrating how the addition of prime numbers leads to compact sets in the harmonic space model. A 3-limit Pythagorean interpretation mimics somewhat intonation tendencies influenced by voice leading (a slightly raised leading tone – B ^{+17 cents})²⁷. But arguments against this interpretation point to the rough sonority and complexity of the ratio (729:864:1024:1152), which is reflected by an extended set in harmonic space (fig. 9). The roughness is improved if the third is substituted for a 5-limit option: 32:36:45:54 (fig. 10).

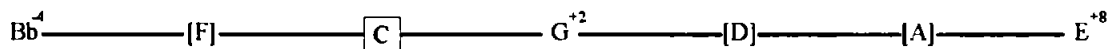


Figure 9 – Dominant seventh chord in 3-limit harmonic space

AUDIO TRACK 4: 12TET DOMINANT SEVENTH CHORD COMPARED TO 3-LIMIT DOMINANT SEVENTH CHORD

²⁶ By using chains of unresolved dominant sevenths, Wagner plays with the prolongation of tension.

²⁷ In performance, the third of the dominant seventh tends to be sharpened as it ascends melodically to the root of the tonic, but the similarity in magnitude in the melodic intonation of the leading tone is likely only coincidentally related to the harmonic intonation of the 3-limit dominant seventh.

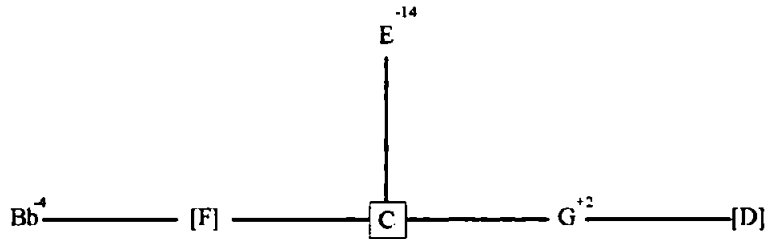


Figure 10 – Dominant seventh chord in 5-limit harmonic space

AUDIO TRACK 5: 3-LIMIT DOMINANT SEVENTH CHORD COMPARED TO 3-LIMIT PLUS 5/4 MAJOR THIRD DOMINANT SEVENTH CHORD

The second interpretation includes the major triad (4:5:6), and remains within a 5-limit ascribing the seventh to the ratio 9/5 (a minor third from the 3/2) creating the chord ratio of 15:18:20:25 in its simplest form (fig. 11). A slightly less complex but disperse set in harmonic space is created, and the vertical sonority becomes, relatively, slightly less complex (rough). This interpretation is consistent with the use of the dominant seventh in a functional harmonic language (17th to 19th century), where a stable triad has an added dissonant tone (the seventh) requiring resolution.

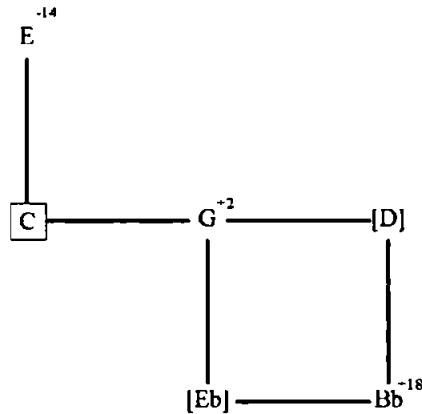


Figure 11 – Dominant seventh chord in 5-limit harmonic space (alternative)

AUDIO TRACK 6: 3-LIMIT DOMINANT SEVENTH CHORD COMPARED TO 5-LIMIT DOMINANT SEVENTH CHORD

The third interpretation introduces the 7-limit and ascribes the flat seventh to the seventh harmonic (7/4) for a chord ratio of 4:5:6:7 (fig. 12). Arguments in support of this implication include the fact that it is based on a low occurrence in the harmonic series, and a smooth sonority (and therefore its compactness in harmonic space). Arguments against include the fact that the 7/4 is 31 cents smaller than the 12tet minor seventh and perhaps requires too great a perceptual stretch in connecting these two intervals, and that the source of the dominant seventh in traditional Western harmonic theory has

nothing to do with the seventh harmonic. This interpretation is more in accordance with a harmonic language that considers dominant structures as stable and unified, not requiring any resolution.

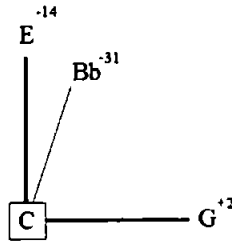


Figure 12 – Dominant seventh chord in 7-limit harmonic space

AUDIO TRACK 7: 5-LIMIT DOMINANT SEVENTH CHORD COMPARED TO 7-LIMIT DOMINANT SEVENTH CHORD

(Notice that with the addition of each new prime the compactness of the harmonic space increases.)

Hindemith explicitly denounces this structure as the basis of the dominant seventh chord:

The seventh overtone in the series based upon C ($-b\flat^1$) does not make the triad into a dominant seventh chord such as we know in practice. It is flatter than the $b\flat$ that we are used to hearing as the seventh of c...

And the quote continues in the typical Hindemith fashion of obscuring solid points within bold generalisation and contradiction:

Like the seventh overtone, the high prime-numbered members of the series and their multiples do not fit into our tonal system. They also are either too flat or too sharp... although we must remember that it is for simplicity's sake that we can let that statement stand. The natural tones of the overtone series cannot of course be 'too sharp' or 'too flat' in themselves. It is just that our tonal system, which strives to bring incomprehensible multiplicity within our grasp, cannot find any simple and clear place for them. In acoustical reckoning, so far as it serves as a basis for considerations of composition, one does not need these prime numbers, or the 'pure' tones which lie above the 16th in the overtone series. No theory of music that is to be taken seriously has ever gone beyond the series 1-16, and we shall see in the course of our investigations that an even smaller portion of this series suffices to represent all the tonal relations used in music²⁸ (Hindemith 1942: 24).

But what can be acknowledged is that while the dominant seventh arises through a conflation of tonal harmonic construction and voice leading based on the first six harmonics of the series, what the ear might *interpret* is a structure which *suggests* the

²⁸ The contradiction here is that Hindemith acknowledges the first 16 harmonics as the source of valid musical material but that the seventh harmonic is somehow not a part of that set.

harmonic ideal of 4:5:6:7 regardless of the chord's theoretical source. Here the source is not of concern—the perceptual issue is.

1.1.4 TRITONES (DIATONIC)

The discussion of the seventh chord has important implications regarding the harmonic function of an isolated tritone dyad. The tritone can be particularly ambiguous as it results from several harmonic conditions. The tritone found in the 5-limit seventh chord is 25/18 (569 cents). However, within the just intonation literature, this interval is rarely pointed to as having any basis for the tritone dyad (it is 31 cents narrower than the equal tempered tritone). In the 7-limit seventh chord, the tritone is 7:5 (582 cents), 18 cents narrower than equal temperament. Coincidentally, the intonation of the 7-limit tritone is quite close to the most common 5-limit interpretation of the tritone: 45/32 (590 cents).

Because the equal tempered tritone equally divides the octave, inversions are identical. Attempts to interpret the tritone in just intonation therefore include inversions of each of the above (which are not identical): 7/5 inverts to 10/7, 45/32 inverts to 64/45, and 25/18 inverts to 36/25.

AUDIO TRACK 8: SEVEN TRITONES IN ASCENDING ORDER OF MAGNITUDE (25/18, 7/5, 45/32, 12TET, 64/45, 10/7, 36/25)

The theoretical basis of the 45/32 is the difference between 5/4 and 16/9, and arises theoretically in traditional harmony through the secondary dominant chord (the II⁷ chord) between the root of I and the major third of II. $9/8 \times 5/4 = 45/32$. Twelve note just intonation scale construction requires a single tritone and therefore must reconcile one of these six options, and most often appears as a 45/32²⁹. But when considered as a consonant and stable dyad, 7/5 seems to me the most convincing implied just ratio.

²⁹ In continuing from Hindemith's 'clouding' of the major and minor triad, the logical extension of this might provide a speculative theory where the tritone could function as a 'neutral perfect'—neither a perfect fourth or fifth—in a similar vein to Ives' concept of the neutral third, which appears in his 'fundamental chord', or the neutral Major 2nd/min 3rd, from his 'secondary chord' (see Ives analysis in section A2 below). If such an interval exists perceptually, it would not be the result of some rational harmonic scheme but a unique result of equal temperament where ambiguity is desired. The basis of this theory would be the equal division of the simplest ratios simplified into 12tet: 2/1 (octave) is first broken into a perfect fifth and fourth ($3/2 + 4/3 = 2/1$ or vice versa), with the tritone being the 'neutral perfect'; the perfect 5th is then broken into a major and minor third ($5/4 + 6/5 = 3/2$ or vice versa) and the quarter-tone flat major third being the 'neutral third'; the 4/3 is also broken down into $6/5 + 10/9$ with a quartertone sharp major second being the neutral minor third/major second (some theorist will need to come up with a fancy name for this); then the 5/4 is broken into $9/8 + 10/9$ and a 193 cent major second, approximated by the 12tet major second remains a 'a major second'; the minor third (6/5) breaks in two and gives a 'neutral second'; etc.

1.1.5 MAJOR SEVENTHS

The simplest interpretation of an equal tempered major seventh chord in harmonic space is 8:10:12:15 (fig. 13).

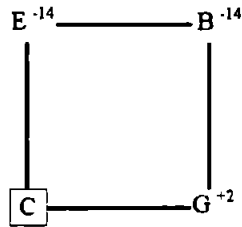


Figure 13 – Major seventh chord in 5-limit harmonic space

It seems safe to say, that as an isolated chord, this is the implied space of the equal tempered version. A sounding of each version reveals no significant difference in harmonic quality, only a difference in smoothness. There is little ambiguity regarding the interpretation of each chord tone, unlike the significance of the minor seventh in the dominant seventh chord.

AUDIO TRACK 9: 12TET MAJOR SEVENTH COMPARED TO 5-LIMIT MAJOR SEVENTH

However, a subharmonic coincidence analysis results in an ambiguous root—either 'C' or 'A', although 'C' is the earliest common occurrence.

major seventh chord:	c	e	g	b
	C	E	G	B
	F	A	C	E
	A^b	C	E^b	G
	D	F[#]	A	C[#]
	B^b	D	F	A

Figure 14 – Major seventh chord subharmonic coincidence analysis

1.1.6 MINOR SEVENTHS

Minor seventh chords are slightly more problematic than the major seventh chord as it contains both the minor triad and the minor seventh interval, which are each problematic in their own right. Both the 5-limit and 7-limit versions shown in figs 14 and 15 are relatively compact and smooth. Context will determine the validity of either configuration, although the 5-limit version is closer in intonation to equal temperament.

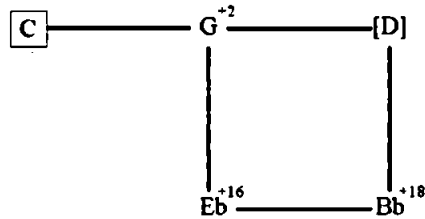


Figure 15 – Minor seventh chord in 5-limit harmonic space (9:10:12:15)

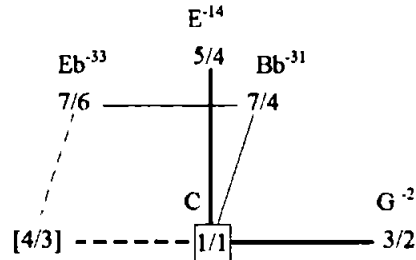


Figure 16 – Minor seventh chord in 7-limit harmonic space (9:12:14:21)

AUDIO TRACK 10: 12TET MINOR SEVENTH CHORD COMPARED TO 5-LIMIT MINOR SEVENTH CHORD COMPARED TO 7-LIMIT MINOR SEVENTH CHORD (REPEATED ONCE)

A subharmonic coincidence analysis (fig. 17) results in an ambiguous root, with root possibilities that include 'C', 'Eb', 'F', and 'Ab'. This confirms some of the ambiguity which results from a harmonic space analysis; that is we can see the source of these roots in the above harmonic space sets: a 'C' root may correspond with a clouiding theory, as might the 'Eb'; and the 'F' and the 'Ab' are consistent with the two missing fundamental possibilities.

minor seventh chord:	c	eb	g	b
	<u>C</u>	<u>E^b</u>	G	<u>B^b</u>
	<u>F</u>	<u>A^b</u>	<u>C</u>	<u>E^b</u>
	<u>A^b</u>	B	<u>E^b</u>	<u>G^b</u>
	D	<u>F</u>	A	<u>C</u>
	<u>B^b</u>	<u>D^b</u>	<u>F</u>	<u>A^b</u>

Figure 17 – Minor seventh chord subharmonic coincidence analysis

I will return to the discussion of fixed 12tet harmonic chords and intervals that imply higher-limit systems after addressing issues of interpretation in dynamic 12tet suggestive of the 5-limit (and perhaps the 7-limit).

1.2 DYNAMIC DIATONIC INTONATION IN TWELVE TONES

Historically, twelve-note notation has served several intonational systems including just intonation, meantone-, well-, and 12tet temperament. In this way, we might consider the function of twelve-note notation as an intonational reference for a dynamic tuning system, where it serves as a reference rather than an ideal. The inflections for the twelve pitch classes serve several different functions and the careful use of accidentals in many cases carry information about the harmonic function of a given pitch. In diatonic music, there are no 'true' enharmonic equivalents. E^b as the third of a C minor triad does not carry the same intonational implication that the D^\sharp of a B major triad does. This is true regardless of whether or not the music is played in fixed intonation or dynamic intonation (on piano or violin, for example); the difference is that the first may *imply* a certain harmonic relationship, and the second that intonation may actually *express* the harmonic relationship or other conditions of the musical moment.

But harmonic and melodic implication often conflict. For example, the B^b of a G7 chord in the key of C harmonically implies 15/8 (in C), a B 12 cents lower than in 12tet, but is also melodically a leading tone and thus subject to an expressive inflection which is likely to sharpen the tone as it resolves to C.

One way to discover what is implied by equal tempered harmony is to look at the dynamic (harmonic and expressive) intonation of compositions written in twelve tones. Here, equal temperament may be regarded as a reference point for a variety of complex harmonic systems. Identifying the harmonic space of an isolated 4-part chord is telling, but the same chord in musical context may reveal other psychoacoustic factors influencing intonation (and thus telling us something further about harmonic implication). In the analyses that follow, the player is also regarded as *expert listener*, and while the naïve listener may be able to say something about their own listening state, by analysing the performance, we may gain a more direct knowledge of the dynamic function of harmonic implication and intonation.

Intonation, even in the 12-note western tuning system, is a complex battle between several psychological issues including at least: the preconditioning of equal tempered intonation, voice leading, harmonic implication, and kinaesthetic memory. Experiments have been unable to establish that common practice intonation follows any specific tuning system such as Pythagorean, equal temperament, or just intonation. What occurs instead

appears to follow a complex set of rules (Rasch 1985: 442).

Putting major and minor triads into musical context immediately complicates dynamic intonational issues. Advocates for just intonation are likely to state that, given certain conditions, just intonation is the intonation towards which musicians are likely to gravitate. This may be true where the harmonic tempo is slow and homophonic, and where the musician understands the concept of just intonation, but there is little research to support this within a musical context. It appears that conditioning within equal temperament can affect intonation even where harmonic implication may seem strongest.

Furthermore, if indeed the musician is inclined toward just intonation, it is not clear that 5-limit just intonation is the intonation of choice. Richard Parncutt explains that...

...the ear is remarkably insensitive to frequency ratios between simultaneous and successive pure tones (Allen, 1967; Plomp, 1967; Plomp & Levelt, 1965). Western musical intervals are perceived *linearly* and *categorically*, and intervals are identified by the center and boundaries of the category—not by ratios such as 5:4 or 81:64. Intervals can vary in size by up to a semitone (e.g. a major 3rd ranges from 350 to 450 cents: Burns, 1999) Typical intonations deviate systematically from frequency ratios (major 3rds larger than 4:5, 8ves larger than 2:1) and the size of the deviation depends on register (Rosner, 1999). The exact size of a performed interval is the result of a compromise between partially conflicting constraints (Terhardt, 1974a) such as roughness, temporal context, musical style, emotion, and melodic emphasis: intonation thus depends not only on sensitivity to the musical surface but also on cultural knowledge (Burns, 1999). Frequency ratios do not *directly* affect or determine intonation; instead, pure intonation minimizes *roughness* between harmonic complex tones (Hagerman & Sunderberg, 1980; cf. Mathews & Pierce, 1980)—but most intonation is closer to equal temperament or Pythagorean than pure (Burns, 1999). These empirically based arguments cast doubt on ratio-based, abstract mathematical theories of the nature and origins of scales (Parncutt 2001).

We should take the above quote with some caution even though the issues are indicative of the complex issues surrounding intonation. There is an unstated bias in the referenced research that includes musical style and tonal resolution (12 intervals of the tempered scale to use as signs for the sounds heard). It seems somewhat obvious, given that the subject only has a limited set of signs and a conditioned experience of equal temperament (which might explain the unlikely conclusion that “a major 3rd ranges from 350 to 450 cents), that the subjects only identify “centers and boundaries” of the categories when only 12 discrete intervals are available (enharmonics are not relevant here)—like so many things in nature, science attempts to categorise and makes discrete phenomena that function as a continuum.

1.2.1 THE EFFECT OF CONDITIONING

Conditioning through 12tet is an important factor in intonation preference. Franz Loosen in “The Effect of Musical Experience on the Conception of Accurate Tuning” established that the “conception of accurate tuning is determined by musical experience rather than by characteristics of the auditory system” (Loosen 1995: 291), and is determined more specifically by the instrument on which the musician is trained.

Loosen found that: violinists tend to prefer Pythagorean intonation over just intonation for the tuning of a major scale; that pianists prefer equal temperament over just intonation; and that untrained musicians show no preference between equal temperament, just intonation, or Pythagorean intonation. While Loosen notes that no significant preference exists between equal temperament and Pythagorean intonation for pianists and violinists, when direct comparisons are made, biases show up as predicted—that pianists prefer equal temperament more often than Pythagorean intonation and vice versa for violinists.

A 1.2 DIATONIC HARMONY IN PHILIP GLASS’S *STRING QUARTET NO. 2* ‘COMPANY’ (1984)

Philip Glass’s diatonic but non-traditional harmonic language offers a good opportunity to scrutinize intonation tendencies of diatonic chords that avoid the compounding intonational problems of voice-leading and functional harmonic concepts of tension and release.

Company is structured in cells and short segments that repeat an indicated number of times. For the purpose of the analysis, each bar is identified first by section (roman numerals), then by system number, and finally by the bar number of that system. For example, the opening bar is I-1-1; the final bar of the piece is IV-5a-2 (which is played six times).

Figure 18 – score reduction of *Company* by Philip Glass (section I)

PREDICTIONS FOR SECTION I

We can imagine Section I of *Company* as if an opening up in harmonic space³⁰. The piece begins with a perfect fourth built from E. In I-1-2 the cello moves from E to F making an F maj⁷ (no 5th), then to G in I-1-3 [G-A-E]; and then descends in quarter-notes: F-E-B^b while the viola moves to G with the B^b. Fig 19 demonstrates this in harmonic space.

³⁰ James Tenney suggests that the harmonic space model might be used as a compositional tool where harmonic and melodic development is conceived actively in harmonic space. Although this is most certainly not how Glass conceives his harmonic development, harmonic space aptly describes the process.

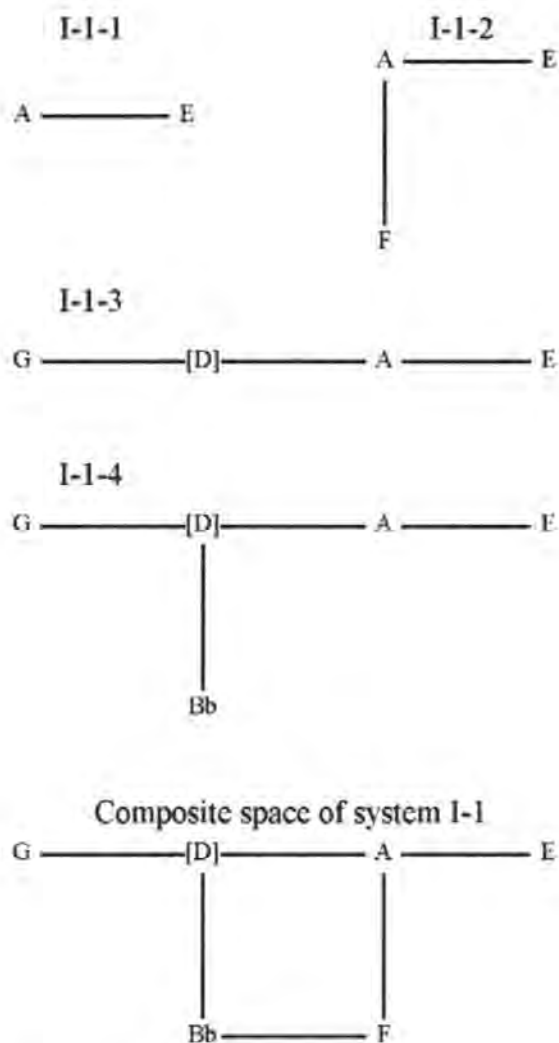


Figure 19 – Harmonic space analysis of system I-1 of *Company*

In I-2, the harmonic structure opens up similarly, but newly voiced, except for in I-2-4 where an F is added to the texture [~~Bb~~-G-F-A-E] (fig 20).

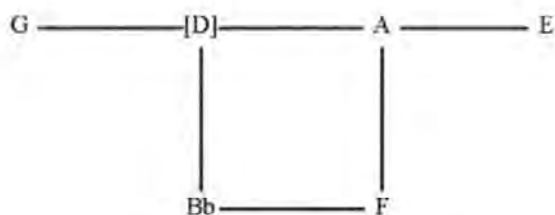


Figure 20 – Composite harmonic space of I-2 through I-4 of *Company*

In I-3, in the viola part, a C is added to each bar (fig 21):

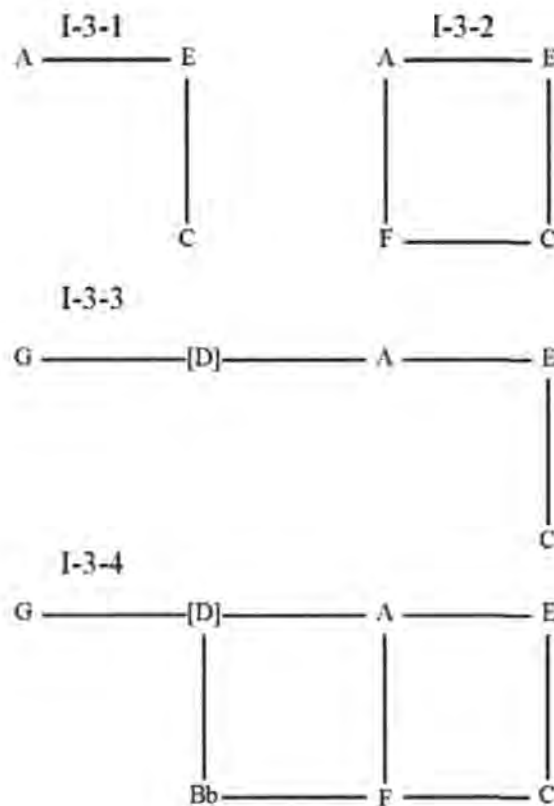


Figure 21 – harmonic space analysis of system I-3 of *Company*

This seems to me the simplest and most compact configuration in harmonic space.

In a performance reflecting this analysis, shifts on the order of the syntonic comma (from Pythagorean intonation) might be expected for the pitches B^b, F, and C, while the other pitches should remain close to Pythagorean intonation.

AUDIO TRACK 11: 5-LIMIT REALISATION OF 'COMPANY'

AUDIO TRACK 12: EXCERPT FROM RECORDING OF 'COMPANY' BY THE KRONOS QUARTET

PERFORMANCE ANALYSIS OF KRONOS QUARTET PERFORMANCE OF 'COMPANY' BY PHILIP GLASS

In the end, I abandoned a rigorous spectral analysis because it proved to be too inaccurate and subjective for this particular piece. My reasons for abandoning the analysis say a lot about harmonic theory and the factors that influence the notion of what 'ideal' intonation might mean, and the close relationship between *diatonic harmony* and the harmonic series.

The first problem I encountered is the fact that measurements in the lowest voices are too rough to be accurate. The margin of error is close to the magnitude of intonation tendencies I hoped to measure (at least as large as the syntonic comma). In other analyses, I can rely on the spectral

analysis of harmonics of a fundamental, which are more resolved.

The second problem was, due to the nature of the harmonic material, that I could not rely on measuring the frequency of harmonics of the cello voice to identify the intonation more precisely because the cello voice harmonics often coincide with either the fundamental or a harmonic of another voice. This is a problem for the analysis of any chords strongly tied to the lower partials of the harmonic series, which is true of most European art music until at least the Romantic era.

The use of vibrato in the violin I voice also caused a great deal of trouble. The side bands created from the vibrato disturbed the isolation of a centre frequency. And, the pitch was often further obscured by the harmonics of lower voices.

As well, in the voices with measured tremolo, resonance effects further clouded the isolation of a stable tone.

Very often, the spectral analysis revealed clusters of frequencies rather than singular frequency peaks (fig 22).

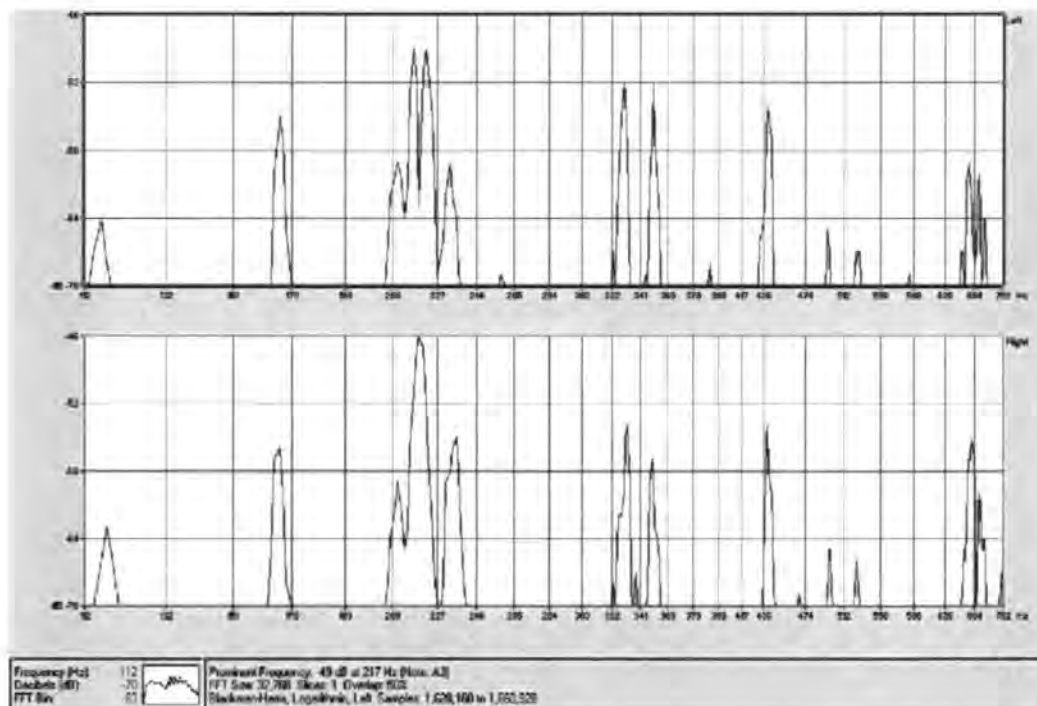


Figure 22 – example of clustering in spectral analysis of Philip Glass’s *String Quartet no. 2 ‘Company’*, performance by Kronos Quartet

But to speak generally and intuitively of the performance, the analysis does seem to reveal a tendency toward Pythagorean intonation or 12tet rather than what my predictions suggest. Slight shifts in intonation where third relations are introduced do not occur as might be predicted by the harmonic space analysis. The use of open strings is likely a strong influence on the intonation and

helps the ensemble to maintain their reference to Pythagorean intonation.

Theories of auditory streaming support Kronos's choice of intonation for this passage. In order for the ear to discriminate between voices, an intonation that is not 'just' will aid the listener in isolating individual voices. This is both important to minimalist textures but also here important as the harmony is full of octave, fifth, and fourth relations, as well as some tertian relations, which would be most susceptible to fusion effects, thus reducing streaming.

1.3 CHROMATIC HARMONY

Due to the huge body of chords that this category covers, it is not feasible to provide a lexicon of chord analyses in harmonic space. Instead, the following presents examples and suggests an approach to tackling this type of harmonic material in the analysis of chromatic works.

A 1.3A CHROMATIC HARMONY ANALYSIS – THE 'TRISTAN' CHORD



Figure 23 – 'Tristan' chord from 'Prelude' to *Tristan and Isolde*

The 'Tristan' chord has drawn the attention of many theorists for its elusive harmonic function. While it is beyond the focus of this section to go into much detail regarding traditional harmonic function except where voice-leading influences intonation, three interpretations of this chord are common: 1) considers the chord as an inversion of the half diminished seventh chord ($\text{min}7^{(b5)}$ in jazz terminology), with A as a passing tone leading through A^\sharp to B; 2) where the G^\sharp is considered a suspension resolving to A, and therefore a French sixth of sorts ($7^{(b5)}$); or 3) a 'vagrant' chord from E^\flat minor (Schoenberg 1911/1983: 257-258), which Schoenberg at once defends and challenges in *Theory of Harmony*. I include here the following quote for its entertaining exuberance, but also to foreshadow further discussion regarding harmonic ambiguity and to support an analysis of the Tristan chord in and out of its musical context—first as a static vertical event rather than from the standpoint of voice-leading.

Of course I do not actually wish to say that this chord has something to do with \flat minor. I wanted only to show that even this assumption is defensible and that little is actually said whenever one shows where the chord comes from. Because it can come from everywhere. What is essential for us is its function, and that is revealed when we know the possibilities the chord affords. Why single out these vagrant chords and insist that they be traced back at all cost to a key, when no one bothers to do so with the diminished seventh chord? True, I did relate the diminished seventh to the key. That relation is not supposed to restrict its circle of influence, however, but should rather show the pupil systematically its range of practical possibilities, so that he can find out through inference (*Kombination*) what his ear has recognized long ago through intuition. Later, the pupil will best take all these vagrant chords for what they are, without tracing them back to a key or a degree: homeless phenomena, unbelievably adaptable and unbelievably lacking in independence; spies, who ferret out weaknesses and use them to cause confusion; turncoats, to whom abandonment of their individuality is an end in itself; agitators in every respect, but above all: most amusing fellows (Schoenberg 1911/1983: 258).

The implied harmonic space of the Tristan chord, if we take the half-diminished chord interpretation, is difficult to resolve in the 5-limit. The top three tones form a minor triad, but a compact 5-limit set that includes the tritone is not available. Introducing the 7-limit provides new possibilities. It seems important to retain the B minor triad set and find a close F—the 8/7 from $D\sharp$ seems like a good candidate. The set is compact, suggesting harmonic integrity and simplicity, and sounds relatively smooth (fig 24).

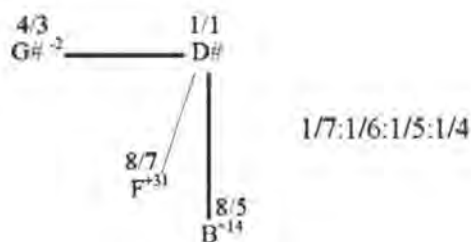


Figure 24 – ‘Tristan’ chord as *Utonality*³¹ in 7-limit harmonic space

AUDIO TRACK 13: ‘TRISTAN’ CHORD (AND DOMINANT) IN 12TET COMPARED TO 7-LIMIT ‘UTONALITY’ JUST INTONATION

This space happens to be the exact inversion of the 7-limit dominant seventh chord (4:5:6:7), and shares a number of pitch categories (as opposed to ‘classes’) where the intonation of F and B are significantly different (fig 25).

³¹ Please see Glossary for a definition of the terms ‘Otonality’ and ‘Utonality’.

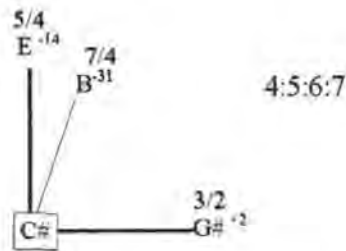


Figure 25 – Inversion of ‘Tristan’ chord (7-limit dominant seventh chord)

This suggests at least one other candidate for the implied harmony of the Tristan chord; that is 5:6:7:9 (or 5:7:9:12 as voiced), essentially a rootless 7-limit ninth chord (fig 26).

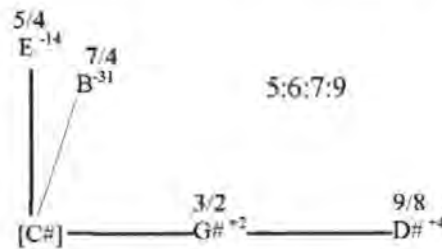


Figure 26 – ‘Tristan’ chord as *Otonality*³² in 7-limit harmonic space

A Subharmonic Coincidence Analysis supports this last harmonic space, which Ernst Terhardt deals with directly on his webpage:

As another example let us consider a chord that by conventional theory has been regarded to be "at the borderline to atonality", i.e., the famous Tristan chord. Here is the corresponding table:

Tristan chord	f	b	d#	g#
	F	B	D#	G#
	A#	E	G#	<u>C#</u>
	<u>C#</u>	G	B	E
	G	<u>C#</u>	F	A#
	D#	A	<u>C#</u>	F#

The algorithm tells us that actually there is a full match for the root C#. Thus the chord, when considered in isolation, is far from being atonal. One can easily verify that the root C# indeed "makes sense", i.e., by playing it in the bass register together with the Tristan chord. So, the subharmonic matching algorithm has found out what one may as well explain in terms of the conventional theory: The Tristan chord f-b-d#-g# can be said to be a major 9th³³ chord with root C#, of which the root note itself is missing (Terhardt 2000).

AUDIO TRACK 14: 12TET ‘TRISTAN’ CHORD COMPARED TO 7-LIMIT ‘OTONALITY’ JUST INTONATION

³² Please see Glossary for a definition of the terms ‘Otonality’ and ‘Utonality’.

³³ I contacted Ernst Terhardt in regards to this nomenclature, to which he replied: “Sorry about the ambiguity concerning the ‘major 9th chord’. ‘Major’ was merely meant to indicate the opposite of ‘minor’. I intended to say that F-B-D#-G# is a subset of C#-F-G#-B-C#-D#-F-G#. The latter can basically be termed a C# major chord” (Ernst Terhardt, email to the author, 05 February 2005).

But how musicians actually tune this chord is another matter. For the implied space above, the 'B' must be tuned 31 cents lower than in equal temperament, and the intonation of this chord in context will likely be more influenced by the strong voice-led nature of the passage.

SPECTRAL ANALYSIS OF THE 'TRISTAN' CHORD IN PERFORMANCE

In the excerpt from the 1959 recording by the Hallé Orchestra conducted by Sir John Barbirolli, there is very little indication that intonation is consciously adjusted from 12tet. Most pitches fall within a few cents of 12tet (although the 'smear' on each pitch is quite significant as can be expected in orchestral music). The only significant deviations from 12tet is on the D[#] of the actual 'Tristan chord', which may reflect a 'sweetening' of the major third (B—D[#]), here 10 cents smaller than a 12tet major third; and in the final E7 chord of the first phrase (the entire chord has shifted upward by about 10 cents from the opening pitch) the D (7th of the chord) is 15 cents flat relative to the rest of the chord, which may indicate a harmonic shift influenced by the seventh harmonic, although this is hugely speculative.

AUDIO TRACK 15: 'TRISTAN' CHORD BY HALLÉ ORCHESTRA CONDUCTED BY SIR JOHN BARBIROLLI

There is nothing to indicate that voice leading has significantly influenced intonation in the Hallé version, but the excerpt from the Norton Anthology³⁴ (audio track 16) shows at least one example of this influence. In the second phrase, the G^b resolves to the F[#] of the 'Tristan Chord', where this minor second interval is only 72 cents wide.

More striking in the Norton example is the intonation of the G7^(b5) chord, where the major third G—B in the bass clef is 32 cents wider than 12tet (close to the 7-limit 9/7 major third) and the F above that a further 9 cents sharp making for a very wide minor seventh interval (1041 cents!). The sonority sounds strikingly spectral.³⁵

G⁻¹⁸ B⁺¹⁴ F⁺²³ C^{#+18}

Figure 27 – Intonation of chord components in Norton 'Tristan' chord

AUDIO TRACK 16: NORTON INTONATION OF 'TRISTAN' CHORD

³⁴ This example came from a set of CDs which accompanies the Norton Scores. Nowhere in any of the publication is the performing orchestra or conductor mentioned. My best guess based on previous Sony releases suggests that the performing ensemble may be the Beyreuth Festival Chorus and Orchestra, conducted by Karl Böhm.

³⁵ It is difficult to define precisely what I mean here. In sounding 'spectral', I suspect that I am responding to a level of fusion in the chord, or to a sensorial experience which resembles that of a chord built from selected upper partials of the harmonic series. But it is just that, a 'resemblance', as it can be seen that I have not convincingly reconciled the intonation of this chord within the harmonic series.

A possible harmonic space set based on the 'Norton' Intonation is presented in Figure 28.

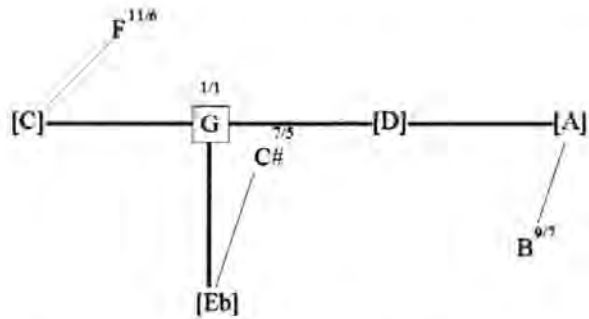


Figure 28 – Speculative harmonic space analysis of the Norton intonation of ‘Tristan’ chord

If an acoustical root of F is considered (lowest, left-most, front-most common root) then the chord is based on partials $G^{(35)}—B^{(45)}—F^{(1155)}—C\#^{(49)}$, which is not particularly convincing. If indeed this is heard as ‘spectral’, then the intonation is likely a rougher approximation of a more compact space and therefore a lower portion of the series.

Another possibility maintains the exactly tuned $9/8$ between the B and $C\#$, and a good $25/16$ between B and G, leaving a more mysterious F to deal with (we could choose from several B—F tritones). The space would therefore be based on partials: $G^{(25)}—B^{(1)}—F^{(?) }—C\#^{(9)}$ (fig 29).

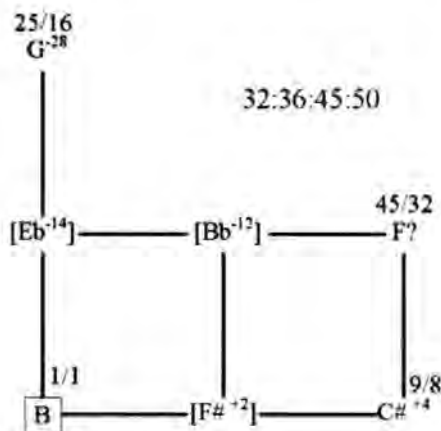


Figure 29 – Alternate harmonic space analysis of Norton ‘Tristan’ chord intonation

In the recorded example, the detuning is not as apparent as in the simulated example, and balance seems to make a huge difference in the perception of the intonation. Compare the synthesized Audio Tracks 17 and 18, which are tuned according to the Norton excerpt. The first is balanced with all lines at equal volume; the second mimics more closely the balance of the recording. The extreme intonation in the second of these two simulations is far less noticeable or ‘strange’ than in the first. Audio Track 19 provides a synthesised 12tet version for comparison.

AUDIO TRACK 17: NORTON INTONATION OF TRISTAN CHORD WITH ALL VOICES AT ROUGHLY EQUAL VOLUME

AUDIO TRACK 18: NORTON INTONATION OF TRISTAN CHORD WITH VOICES BALANCED ROUGHLY TO ORIGINAL RECORDING

AUDIO TRACK 19: A 12TET VERSION OF THE TRISTAN CHORD

This analysis suggests at least two things: 1) that balance may also be an important contributor to intonation, as it affects the perception of consonance³⁶; 2) that deviations greater than 15 cents (5-limit inflections) are not out of the realm of possibility in an extended 12-note diatonic language, therefore opening up the possibility for the implication of 7- and perhaps higher-limit sonorities within an extension of traditional harmonic vocabulary.

A 1.3-B CHROMATIC INTONATION IN WEBERN'S *FIVE MOVEMENTS FOR STRING QUARTET*, OP. 5

The musical score shows four staves: Violin I, Violin II, Viola, and Cello. Each staff is marked 'mit Dämpfer'. The first two measures are silent. In the third measure, all four parts play a chord of six notes (F#4, G4, A4, Bb4, C5, D5) with dynamics 'ppp'. In the fourth measure, the same chord is played with dynamics 'pp > ppp'. The Cello part has a chromatic line starting in the second measure, with dynamics 'pp' and 'ppp' indicated by arrows.

Figure 30 – Opening to Webern's *Five Movements for String Quartet*, Op. 5 - V (1909)

The following analysis compares a recorded performance of the opening bars of the fifth movement of Webern's *Five Movements for String Quartet*, Op. 5 (1909) by The Julliard Quartet to three other quartets: the Emerson, Artis, and Kronos Quartets. I selected this passage first for

³⁶ “A dyad showed better consonance when the higher-frequency component is stronger than the lower-frequency component ($P_1 < P_2$) than the case of its opposite spectrum form ($P_1 > P_2$)” (Kameoka and Kuriyagawa 1969: 1456).

the isolated melodic passage in the cello part, and second for the duration and structure of the vertical chords³⁷. The raw data for these analyses can be found in Appendix 2.

INITIAL PREDICTIONS FOR THE IDEA INTONATION OF OPENING CELLO PASSAGE OF WEBER'S FIVE MOVEMENTS FOR STRING QUARTET, OP. 5 - V

I have attempted to guess at the implied harmony of this passage through the positioning of each note in a 5-limit harmonic space (and considering some special 7-limit possibilities). Taking the passage in three note groups [(F#-B-G) (B-G-G#) (G-G#-C) (G#-C-E) (C-E-C#)] each is analysed in its most compact version.

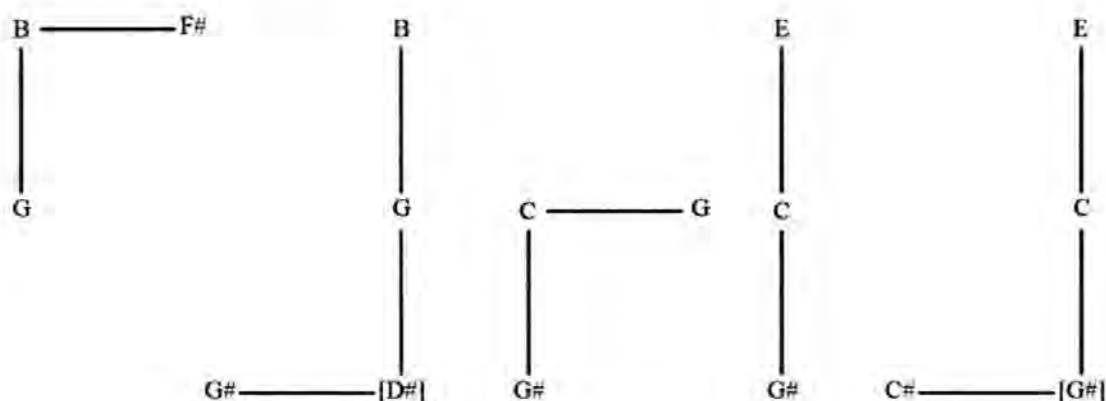


Figure 31 – Three note configurations of cello in *Five*

The first three pitches fall compactly within a 5-limit just intonation harmonic space. But the addition of G# creates a difficult question as to the ideal intonation for the descending major seventh. The simplest harmonic choice is 16/15 from G, but in relation to the starting tone F#, this is relatively quite complex (128/75) and deviates significantly from equal temperament (25 cents sharp). Perhaps a player in this situation will make an intonational choice based on fingering habits influenced by equal temperament and perhaps voice leading rather than acoustic integrity. When the player arrives at C descending to E, a strong harmonic choice is 5/4, but this again takes the intonation further from the initial pitch F#, more than 20 cents from a 12tet fingering. If the player executes every interval in the simplest pure form, he/she will be almost a quartertone out from

³⁷ Thresholds regarding the intonation of melodic and harmonic sonorities are not identical. Where many theorists identify just noticeable differences (JND) for pitch discrimination anywhere from 2 to 10 cents, the testing conditions are hugely influential and depend on the quality of tone (simple or complex), tone duration, whether the materials are presented consecutively (melodic) or as a dyad or chord (where other factors contribute to JND such as beating).

12tet. There is likely to be a struggle between harmonic purity, rehearsed fingering, voice leading, and tonal pull.

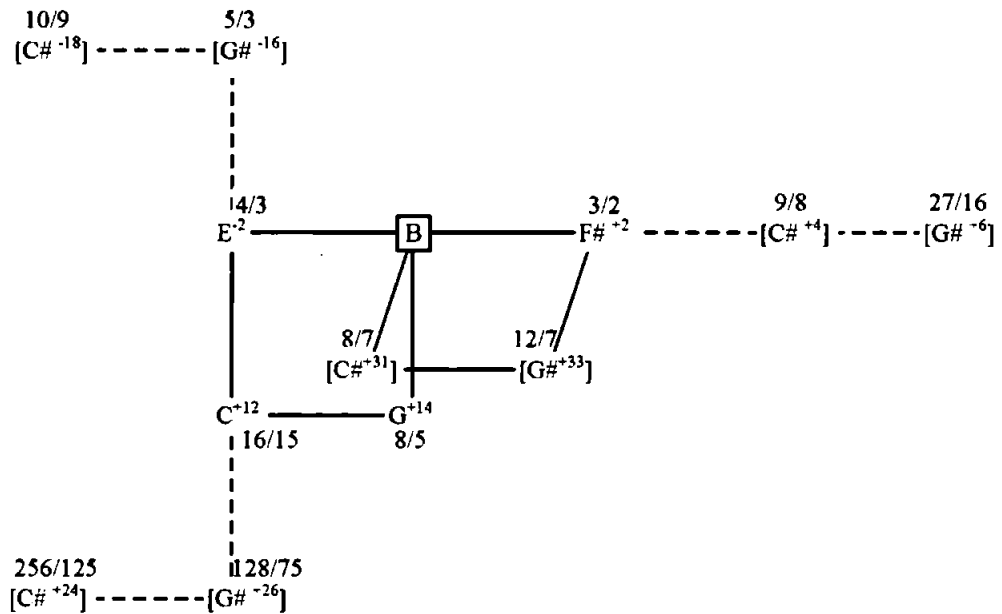


Figure 32 – Projected harmonic space of Webern cello part

Although the unbracketed elements of the diagram suggest a coherent and compact set, the $C\sharp$ and $G\sharp$ prove problematic. The need for the re-evaluation of previous pitch relations, which Tenney speaks of, is apparent in the construction of this harmonic space diagram. If we simply find the closest relation through each consecutive pitch, a sprawling, and therefore non-compact (complex), harmonic space emerges. But re-evaluation makes possible a relatively compact space, which—not coincidentally—matches the foundations of the western tonal scale; that is if the $9/8$ and $5/3$ are indeed implied by $C\sharp$ and $G\sharp$.

It should be acknowledged that there is a good deal of circular reasoning occurring here—chromatic music offers a challenge to the historical tonal basis of pitch relations, and here I have re-imposed that structure onto a chromatic passage of music, which in many ways is intended to subvert the tonal premise. The fact that the pitch relations are not easily reconciled suggests that Webern has succeeded in breaking down a clear or prolonged sense of tonality. The inability to reconcile the space implies something of the ambiguity of chromatic, and later, atonal music, and may perhaps provide one of several stylistic indicators. That is, the relative difficulty in reconciling a particular harmonic approach in the harmonic space model may be tied to the aesthetic of the musical style.

The first analysis is of a performance by the Julliard quartet. The cellist's intonation of the opening melodic passage may be interpreted as a battle between just intonation and equal temperament, and perhaps Pythagorean intonation; that is if the premise of acoustic simplicity is meaningful to this sort of musical material. The passage presents the player with some difficult intonational decisions.

AUDIO TRACK 20 – JULLIARD QUARTET RECORDING OF WEBERN'S FIVE MOVEMENTS FOR STRING QUARTET, OP.5 – V (CELLO PASSAGE)

The cellist's intonation is quite consistent over the duration of any one given pitch (+/- 3 to 6 cents), but the relative size of each melodic interval reveals significant discrepancies from equal temperament (which should *not* be considered inaccuracies).



Figure 33 – Julliard Quartet, Cello intonation from opening of 'V' from Webern's Five Movements for String Quartet, Op. 5 (1909)

A rough pattern emerges here that lends support to a hypothesis³⁸ that the player may tend toward just intonation when that choice falls within approximately 20 cents of equal temperament, and when the simplest possible frequency ratio interpretation of the next interval might take him/her further than 20 cents from 12tet, a compromised tuning takes place that usually falls near 12tet or a more distantly related just interval tuning (Pythagorean).

³⁸ Based, admittedly, on a very small sampling.

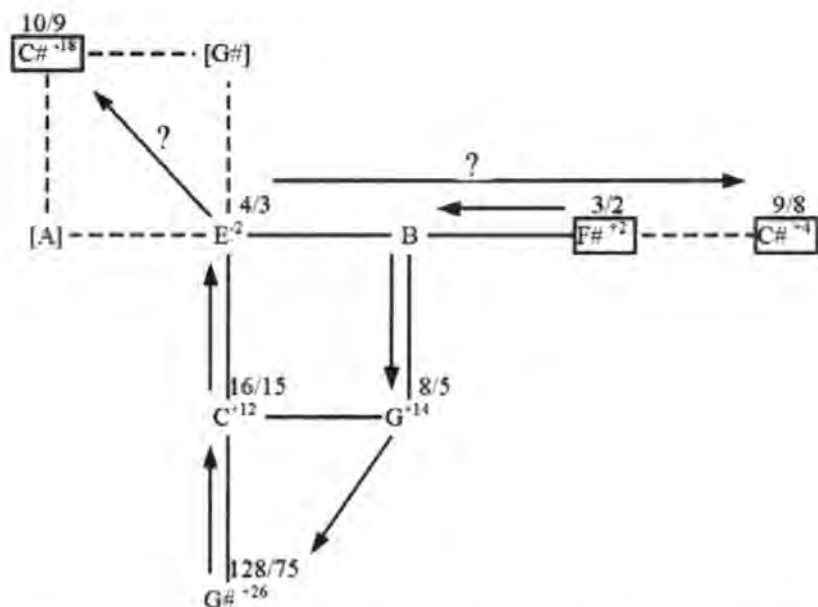


Figure 34 – Predicted intonation of cello in Julliard Quartet performance of *Five* (speculative)

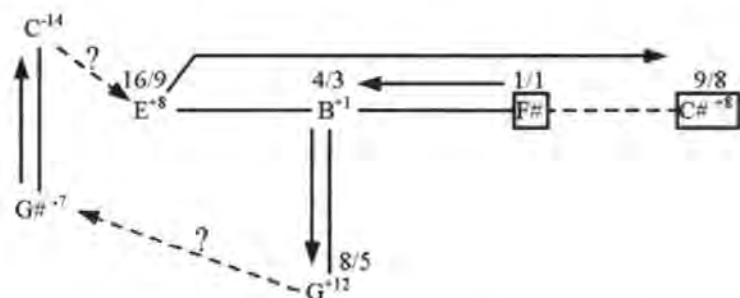


Figure 35 – Actual intonation of cello Julliard Quartet performance of *Op.5 no 5* (the implied ratios can only be considered speculative)

I have compared the Julliard intonation to three other recorded performances.

Quartet	Pitch	F#	B	G	G#	C	E	C#
Julliard		372 Hz	497	397	416	522	333	280
A = 440 Hz		+10 cents	+11	+22	+3	-4	+18	+18
corrected to F#		0	+1	+12	-7	-14	+8	+8
Artis		369 Hz	489	388	413	524.5	323	273
A = 440 Hz		-5 cents	-17	-13	-10	+4	-35	-26
corrected to F#		0	-12	-8	-5	+9	-30	-21
Emerson		373 Hz	495	393	415	518	330	279
A = 440 Hz		+14 cents	+4	+4	-1	-17	+2	+11
corrected to F#		0	-10	-10	-15	-31	-12	-3
Kronos		371 Hz	494	392	419	520	328	279
A = 440 Hz		+5 cents	0	0	+15	-11	-9	+11
corrected to F#		0	-5	-5	+10	-16	-14	+6

Table 1 – comparison of intonational tendencies in four performances of the opening melodic phrase in *Five Movements for String Quartet, Op. 5 – V* by Anton Webern

In the chords that follow in bars 3 and 4, the musicians face a unique challenge. Generally, the intervallic content of a chord may suggest an ideal intonation determined by the simplest interpretation of the relationship between chord components. In this passage, the cello changes pitch in the middle of each chord sounding in the remaining upper voices. The two cello pitches suggest two unique harmonic situations, and require a re-evaluation of the upper chord. The same is true of the inverse: in relation to the C \sharp – E ostinato, the pitch set of the upper chord is unique in each circumstance.

As with the melodic passage, there is no compact configuration available in harmonic space (at least not in a 5- or 7-limit space), suggesting again that there are no strong tonal connotations.

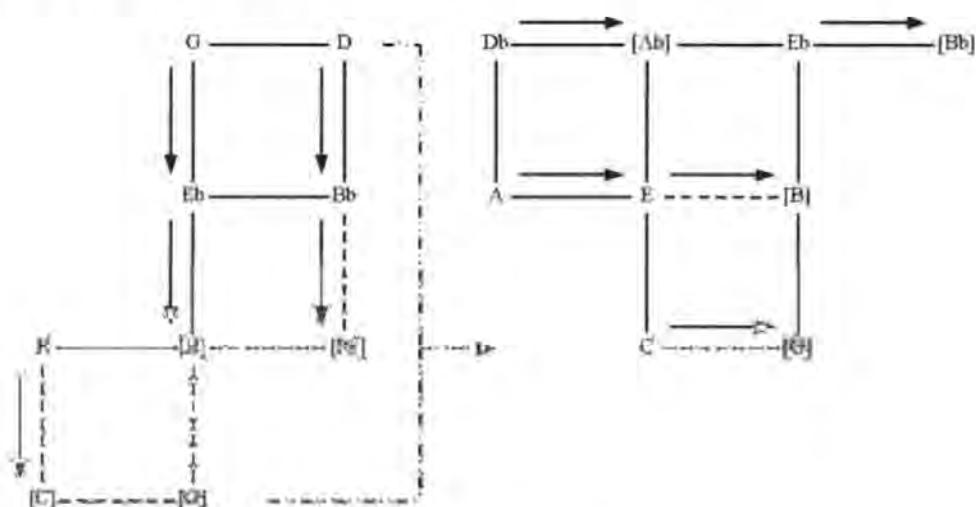


Figure 37 – Speculative harmonic space of Webern chords (excluding cello pitches)

Excluding the cello pitches, a subharmonic coincidence analysis predicts an only slightly ambiguous C root for Chord 1, and A \flat for Chord 2 (Chord 2 is a transposition and revoicing of chord 1), occurring in four out of five possible subharmonic sets; and only ambiguous roots for chords 3 and 4, with the two common subharmonics occurring each three times (and are further ambiguous as the two candidates form a tritone). The addition of the cello ostinato increases further the ambiguity of chord root.

Chord 1	Chord 2	Chord 3	Chord 4
f b \flat e \flat g d	c f \sharp b e \flat b \flat	e \flat a d \flat e c	b \flat e a \flat b g
A E \flat A \flat C G	F B E A \flat E \flat	A \flat D G \flat A F	E \flat A D \flat E C
C G \flat B E \flat B \flat	A \flat D G B G \flat	B F A C A \flat	G \flat C E G E \flat
F \sharp C F A E	D G \sharp C \sharp F C	F B E \flat F \sharp D	C F \sharp B \flat C \sharp A
D A \flat D \flat F C	B \flat E A D \flat A \flat	D \flat G B D B \flat	A \flat D G \flat A F

Table 2 – subharmonic coincidence analysis of Webern chords

AUDIO TRACK 24 – EMERSON QUARTET RECORDING OF WEBERN'S FIVE MOVEMENTS FOR STRING QUARTET, OP.5 – V (CHORDS)

The deviation from equal temperament in the Emerson recording is relatively mild. Although the fourth chord sounds most noticeably 'strange', with the E to G interval particularly wide at 34 cents, the first chord sounds unremarkable with regards to intonation despite the fact that B \flat to D is comparably wide at 33 cents greater than 12tet. Overall, there appears to be very little adjustment in the tuning of the upper chord as the cello moves from C \sharp to E.

AUDIO TRACK 25 – ARTIS QUARTET RECORDING OF WEBERN'S FIVE MOVEMENTS FOR STRING QUARTET, OP.5 – V (CHORDS)

In the Artis Quartet recording, the deviation from 12tet is also somewhat mild in the upper voices, varying between +/- 10 cents (with the exception of the C in Chord 2). However, the cello is significantly flat (-33 cents) for the first two chords, and makes a minor adjustment at the third chord (-20 cents). The second chord, in particular, sounds tentative, with the C \sharp (cello) to the C (viola) being more than a quartertone wider than a 12tet major seventh plus an octave.

AUDIO TRACK 26 – KRONOS QUARTET RECORDING OF WEBERN'S FIVE MOVEMENTS FOR STRING QUARTET, OP.5 – V (CHORDS)

The Kronos Quartet recording exhibits more 12tet qualities than the previous two recordings, particularly between the cello and the lower half of the upper chord, which tends to be slightly compressed.

Overall, there does not seem to be an obvious trend in the intonation of these chords, suggesting that, first of all, there is no *ideal* intonation to strive toward, and secondly, that this music successfully subverts the sense of tonality.

1.4 SYMMETRICAL HARMONIC STRUCTURES IN 12TET

1.4.1 TRITONE (AS A SYMMETRICAL STRUCTURE)

The largest equal division of the octave is the tritone. This region of the octave is perhaps the most ambiguous of all the equal tempered intervals in terms of harmonic implication.

Equal temperament makes possible the use of ambiguous and symmetrical harmonic relationships. In such situations where ambiguity is desired, the choice of accidental is less important; however, spelling may provide some clues regarding intent in harmonically and melodically derived compositions.

The traditional harmonic language is quite clear as to the source of a tritone; inversions are usually spelt distinctly, and voice leading governs the melodic function. The tritone may occur in the V^7 chord between the third and seventh where the spelling will adhere to the key signature and is (arguably) described by the ratio 7/5; it may also occur between the root of the key and the third of a subdominant seventh chord— II^7 (45/32).

But here I am addressing more specifically cases where the tritone is used *because* it equally divides the octave, not where it emerges as a bi-product of tonal function.

The spelling and harmonic/melodic source of the tritone becomes gradually obscured as we move forward in the history of western music. Beyond its traditional functions, it may also emerge through the use of the whole tone scale (irrational ratio), Messiaen's *chord of resonance* (11/8), and in jazz, the use of the Lydian mode and the Lydian $b7^{39}$ mode as the basis of a tonic chord introduces the $\sharp 11$ or the $b5$ chord (Maj.7 $^{\sharp 11}$, Maj.7 b5).

In each of case, the tritone functions ambiguously or contradictorily with respect to harmonic implication. The whole tone tritone is most ambiguous and will depend greatly on context; Messiaen uses the equal tempered tritone as suggestive of the 11/8, which is a great deal out of tune from 12tet and it is therefore questionable as to whether it is heard as such; and in the two Lydian modes it is again unclear exactly how the interval functions harmonically and melodically⁴⁰, even though the Lydian mode may also be

³⁹ Nomenclature in the jazz and classical traditions is not consistent here. C Lydian $b7$ is C—D—E—F \sharp —G—A—B \flat ; also similar to Messiaen's 'chord of resonance' but missing the major seventh, but this is not derived from the same theory.

⁴⁰ The twentieth century sees an increase in the use of the free vertical structure where voice leading is not a consideration in the construction of harmonic progression. Debussy is a prime example in the classical repertoire, and in jazz becomes most common in the modal experiments of the 60's and from the Hard Bop

derived from a mode built from the fourth degree of the major scale and the Lydian $\flat 7$ from the fourth degree of the ascending melodic minor scale.

AUDIO TRACK 27— 12TET WHOLE TONE TETRACHORD AND TRITONE COMPARED TO 11-LIMIT JUST INTONATION TETRACHORD (1/1, 9/8, 5/4, 11/8) AND TRITONE (1/1 – 11/8)—THE HARMONIC RATIONALE FOR MESSIAEN'S CHORD OF RESONANCE

1.4.2 AUGMENTED TRIAD

The augmented triad is perceptually a tricky situation. How do we rationalise the equal tempered basis of such a chord? The first occurrence of anything resembling an augmented triad in the harmonic series is between partials 7, 9, and 11. While this structure fuses easily, the intonation of the components are significantly removed from equal temperament and it is therefore questionable whether 12tet can suggest this proportion in any isolated context. The next occurrences fall between partials 8-10-13 and partials 9-11-14. These all require some stretch of the imagination, especially if the equal tempered version is taken as our reference. The first believable occurrence of an augmented-like triad in the harmonic series is between partials 12-15-17. The deviation from equal temperament is relatively small: 15/12 is a just major third (15/12 reduces to 5/4) and is 14 cents smaller than the equal tempered equivalent; and the 17th partial falls only 5 cents sharp of equal temperament. But applying a harmonic distance measurement makes the 17th partial quite remote⁴¹.

AUDIO TRACK 28— FIVE AUGMENTED TRIADS ([12TET]; [PARTIALS 7,9,11]; [8,10,13]; [9,11,14]; [12,15,17])

However, in his harmonic space model, James Tenney argues that the simplest (most compact set) occurrence, and therefore the likely psychoacoustical ideal, is 25:20:16—two stacked 5/4 major thirds (Tenney 1987: 80). In comparing this structure with other *ideal* structures, it is only on a matter of voicing where questions may arise.

movement and forward (for example, Joe Henderson's *Inner Urge* is a composition constructed entirely of parallel Major $7^{(11)}$ chords).

⁴¹ A question arises here. While the 25th partial is fairly remote, its prime root '5' is not. Whether or not the less remote 17th partial (but large prime) is more compact than the 25th is difficult to determine. Harmonic distance models attempt to consider both prime number generator, and harmonic remoteness. In this situation, we must also consider that the 17th harmonic sounds less remote due to its close intonation to 12tet, and it is hard to speculate whether or not it would be heard as remote or exotic without the conditioning of 12tet.

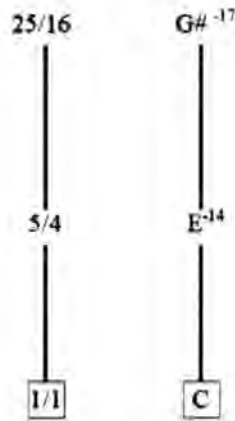


Figure 38 – Augmented triad in harmonic space

Generally, in discussions of just intervals and chords, voicing is not explicitly dealt with. This is due partly to the degree to which the topic becomes complex; the variables increase significantly. But in the case of symmetrical structures, it seems necessary to deal with this at least superficially.

To compare, a major triad is usually described as 4:5:6. In root position, the simplest interval exists between the root and the fifth of the chord ($6/4$, which reduces to $3/2$). The interval is then divided arithmetically into a $5/4$ interval between root and third, and a $6/5$ interval between third and fifth (the allocation is reversed for a minor triad). The tendency is to think of the first three degrees of a major scale in a similar fashion; a major third is arithmetically divided into a $9/8$ and $10/9$ ($10:9:8$). Sounded as a chord, this sonority fuses easily.

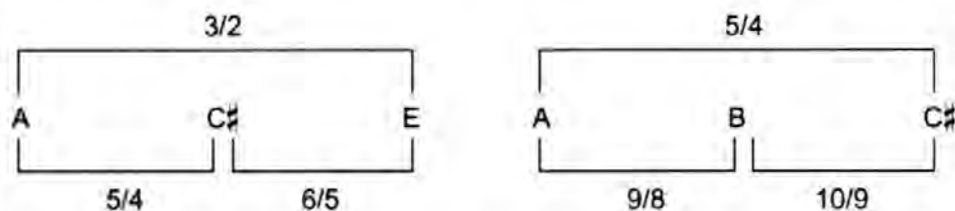


Figure 39 – Arithmetic division of perfect fifth and just major third

Is an exception made then for an augmented triad? Should the first consideration be the outside interval? This makes for a structure built from an $8/5$ minor sixth with a $5/4$ major third extending from the root. The resultant interval between the $8/5$ and $5/4$ is $32/25$. Mapped out in harmonic space, this is ratio-metrically identical to the proposed 25:20:16 but sonically is perhaps more stable due to the relationship between the outer voices. Inversions are difficult to compare directly as register has an affect on roughness. But note that Tenney is not specifically saying anything about voicing in harmonic

space, and that the 16:20:25 is still the simplest representation in harmonic space of these three tones, although a root position for the augmented triad is implied—which may seem like a contradiction.

AUDIO TRACK 29 – FOUR VOICINGS OF THE 25:20:16 AUGMENTED TRIAD – OUTSIDE INTERVAL AS: 12TET; 8/5; 32/25; AND 25/16

If this structure best represents the augmented triad, then each sounding of subsequent inversions would be reinterpreted from a new fundamental. But equal temperament maintains the ambiguous fundamental through all inversions.

A 1.4 WHOLE TONE STRUCTURES IN DEBUSSY’S ‘VOILES’

In “Voiles” from *Preludes, Livre I*, Debussy begins with two groups of major third intervals descending in whole tone steps (fig 40). This passage introduces questions regarding the composer’s intent, psychoacoustic interpretation, and function.



Figure 40 – major third intervals in “Voiles”

AUDIO TRACK 30 – OPENING FOUR BARS OF DEBUSSY’S “VOILES” FROM PRELUDES LIVRE I, PERFORMED BY WALTER GIESEKING

An isolated major third interval is generally regarded as an implied 5/4 just major third; it is difficult to imagine an argument contradicting this. However, the spelling of the intervals in “Voiles” may suggest otherwise. In bars 2 and 4, four intervals are spelled as diminished fourths.

A ‘Debussy as proto-microtonalist’ explanation (see A 1.5) for this may state that Debussy was thinking of a scale built from partials 8-9-10-11-13-14-16 based on a root C. Although the spelling suggests a type of major second, the step-size between partials 11 and 13 is very large and closer to a minor third, and is therefore not a good candidate for the implied harmonic source of the whole tone scale⁴².

⁴² In his *Music Theory Spectrum* article “Brilliant Colors Provocatively Mixed: Overtone Structures in the Music of Debussy”, Gary Don makes a believable argument suggesting the opposite with respect to the pitch structure of Debussy’s *L’isle joyeuse* which includes a partial wholetone scale (Don 2001: 61-62)

AUDIO TRACK 31 – JUST WHOLE TONE SCALE BASED ON PARTIALS 8-9-10-11-13-14-16

This explains most of the spellings with the exception of G^\sharp/Ab . There is a vague though unlikely possibility that Debussy is thinking of $Ab-C$ as a major third (from C's theoretical undertone series) and $G^\sharp-C$ as the interval $13/16$ ⁴³? If we ignore the tendency to think of $13/8$ as a type of 6th rather than an augmented 5th, then perhaps this is a satisfying interpretation if not Debussy's intention. It may also be $25/16$ but is most simply and likely a part of an augmented triad.

AUDIO TRACK 32 – JUST WHOLE TONE SCALE BASED ON 8-9-10-11-25-14-16

Further clues arrive in bars 15 through 21 where the whole tone structures are supported with a Bb in the bass and augmented triads built on Ab and Bb .

Otherwise, Debussy may simply have been applying to the augmented triad a standardised notation for the whole tone scale⁴⁴, in which the whole tone scale is spelt with sharps as the scale ascends, and flats as it descends, and consistently spelling major seconds rather than allowing any diminished thirds:

$Eb - Fb - Gb - Ab - Bb - C - D - E - F^\sharp - G^\sharp - A^\sharp - B^\sharp - C^\times - D^\times$

Figure 41 – A common spelling for the whole-tone scale in Debussy's era

While the recorded examples and preceding arguments are at least plausible, listeners who are unfamiliar with hearing a piano tuned to the harmonic series will likely find the examples extremely exotic. Whether or not it is possible to establish Debussy's intent, the important question arises: What do we actually think we hear in an equal tempered realisation of "Voiles"? Just because the piece can be rationalised into a just tuning does not mean that this is what our ears and brains interpret in a normal listening situation.

I return to Debussy below in the section on *Proto-microtonality*

1.4.4 DIMINISHED TRIAD

The diminished triad has at least two theoretical bases, one that results from equal divisions of the octave, and a second that comes from its function as a dominant substitute.

⁴³ When and where the idea that the 13th partial as a type of $b6$ th rather than $\sharp 5$ appears in music theory would make for an interesting tangent of inquiry, but seems too far of topic to pursue here.

⁴⁴ Both Scriabin and Bartok applied the diatonic method of notating successive scale degrees with successive letter names (in both whole tone and octatonic scales) (Gillies 1993: 43).

The second case has the same problems of interpretation as the dominant seventh chord. But because of its use in the western harmonic language as a chord that may resolve to any number of roots, backward listening is particularly relevant here. Depending on the voicing of the chord, a 'B' diminished triad may alternatively resolve to C, E \flat , G \flat , and depending on the resolution, its implied harmonic space will shift to G, B \flat , or D \flat roots respectively, and the implied intonation of the diminished triad will first be influenced by expectation, and secondly by backwards listening.

If we follow the approach for other triads where the outside interval is arithmetically divided, then an isolated diminished triad will be situated in harmonic space as shown in figure 42. In this case, the diminished triad comprises the upper portion of a 7-limit dominant seventh chord with the acoustical root a major third below the chord root⁴⁵:

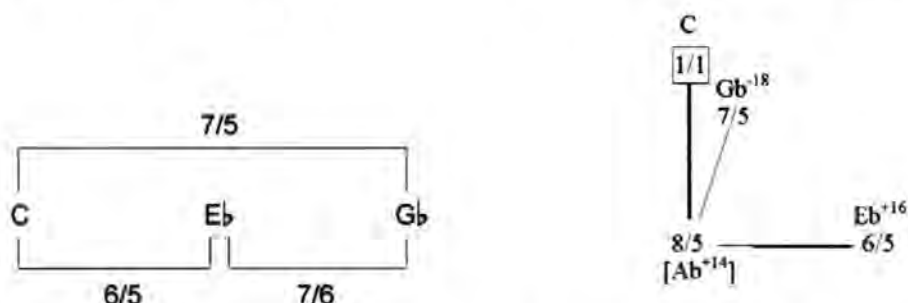


Figure 42 – arithmetic division of diminished fifth and diminished triad in harmonic space

But the situation is more complicated than this because in a traditional harmonic language, the diminished triad can resolve in any number of directions; this configuration implies a resolution to D \flat (the I of V (A \flat)).

AUDIO TRACK 33 – 12TET C DIMINISHED TRIAD RESOLVING TO FOUR DIFFERENT ROOT CHORDS (E \flat , F, A \flat , C) COMPARED TO A JUST DIMINISHED TRIAD (1/1 C – 6/5 E \flat – 7/5 G \flat) RESOLVING TO THE SAME FOUR JUST ROOT CHORDS.

1.5 SUGGESTION OF HIGHER HARMONICS IN 12TET

12tet has also been used for music based on harmonic concepts drawn from beyond the 5-limit. The relative success is a matter for debate.

⁴⁵ James Tenney identifies the intervals of the diminished chord similarly in his *Perspectives of New Music* Article 'About Changes: Sixty Four Studies for Six Harps' (Tenney 1987: 80).

A 1.5 'PROTO MICROTONALITY' - DEBUSSY AND MESSIAEN

It may be arguable that Debussy, Messiaen, and some spectrally inspired composers⁴⁶ have been able to suggest higher-limit harmonies using only 12tet (it is important to note that the spectralists routinely use microtones where Debussy and Messiaen did not). Although the compromise in tuning accuracy is considerable, using any system carefully can at least *suggest* harmony based on higher-limit ratios.

A1.5.1 DEBUSSY AS 'SPECTRAL' COMPOSER

We can only speculate as to harmonic intent in the music of Claude Debussy. He spoke little of his compositional approach and less so of any harmonic concept. Nonetheless, some musicians describe Debussy as a *proto-microtonalist*—his harmonic structures seemingly borrowed from the harmonic series (Johnston 1988: 236). There is speculative knowledge of Debussy's familiarity with the acoustic research of Hermann Von Helmholtz⁴⁷, and we can further speculate that this may have informed his compositional practice. This tidbit of information has been absorbed by the just-intonation community as evidence in support of Claude Debussy as the grandfather of just-intonation composition; had he had the resources, surely he would have used just intonation...

Debussy...seems motivated by an expansion of harmonic resources and a greatly widened horizon. But for Debussy's revolution to have been achieved fully, the tempered scale would have had to go, in favor of extended just intonation, so that the overtone structures would have been unambiguously recognizable as such. The sense of brilliant colors provocatively mixed which generated the comparison to impressionist painting would have been enormously heightened (Johnston 1988: 236-237).

But, to play devil's advocate, Debussy regularly takes advantage of the ambiguity afforded by equal temperament and derives a good deal of harmonic freedom through the exploitation of these ambiguities. Debussy's use of extended dominant structures, which sometimes include both minor and major sevenths (as suggestive of the 7th and 15th harmonics respectively) within a single structure (Don 2001: 61)⁴⁸, suggests the intent to which Johnston refers, but Debussy's music, in my mind, is as much a product of equal temperament as that of the serialists.

⁴⁶ I am thinking in particular here of the French tradition: Messiaen, Grisey, Murail and other composers such as Varèse and Ligeti.

⁴⁷ Ben Johnston states in "Beyond Harry Partch" – "Among the early strong impressions pushing me toward becoming a musician was a lecture that I heard at the age of twelve at Wesleyan Conservatory of Music in Macon, Georgia. It concerned the importance of the acoustical findings of Helmholtz in the development of Debussy's music" (Johnston 1983/4: 225-6)

⁴⁸ A Lydian scale with an added flat seventh is often regarded as the equal tempered version of a scale built from partials 8 through 16 despite the gross variations in intonation with regard to the 11th and 13th partials

**AUDIO TRACK 34 – EXTENDED 12TET DOMINANT 13TH (PLUS MAJOR 7TH) COMPARED TO JUST INTONATION
DOMINANT 13 CHORD (PLUS MAJOR 7TH)**

Equal temperament, in particular, affords ambiguous harmonic situations through several means including the use of symmetrical and parallel harmonic structures. In my mind, the blurring of intervals in 12tet is more indicative of the Impressionist sensibility than the focussed clarity of just intonation. Debussy said little of his composition technique, although speculative knowledge says that he was intrigued by the interaction between the upper partials of his sonorities, and furthermore, his interest in Balinese gamelan music and bell sounds is well noted. 12tet affords this interference, or beating, of upper partials, where just intonation specifically seeks to minimise these effects.

That Debussy may or may not have been a frustrated microtonalist does not preclude an analysis of his work within a just intonation framework.

In the Preface to the G. Henle Verlag edition of Debussy's *Images I*, François Lesure quotes Debussy as saying about replacing 'Reflets dans l'eau' that "I have decided...to compose another piece in its stead, this time with a completely new approach and in accordance with the most recent findings of harmonic chemistry". This new piece was 'Hommage à Rameau' (and 'Reflets dans l'eau' remained in the set).

The image shows a musical score for two staves, likely piano and right hand. The top staff is in treble clef with a key signature of two flats (B-flat and E-flat). It begins with a forte (ff) dynamic. The first measure (bar 58) contains a chord with a triplet of eighth notes on the upper line. The second measure (bar 59) continues with a similar chord structure, featuring a triplet of eighth notes. The bottom staff is in bass clef and contains a complex rhythmic pattern of sixteenth and thirty-second notes, with a sixteenth-note triplet in the final measure. The score is annotated with '3' above the triplet groups and '6' above a sixteenth-note group in the second measure.

Figure 43a – Dominant structures in 'Reflets dans l'eau' (bars 58 and 59)

(the 11th is almost exactly a quartertone flat of a tritone and the 13th actually closer to a minor 6th than to a major 6th), and to a lesser extent, the 7th partial.

Figure 43b – Dominant structures in ‘Hommage à Rameau (bar 52 – 55)

Both pieces directly suggest the harmonic series in the spacing of several of the dominant structures. Bar 58 of ‘Reflets dans l’eau’ is a carefully spaced $E\flat^{13}$ chord approximating partials 1-2-3-4-5-6-8-10-12 in the left hand, and in each of the right hand vertical structures, partials [12-16-20-24], [13-26], [16-20-24-32], [14-28], [13-26]. In the following bar, an altered dominant chord includes an $A\flat$ (11th partial?), $C\flat$ (13th partial?), and D (15th partial), but they occur an octave lower than would otherwise be expected.

AUDIO TRACK 35 – DOMINANT CHORD VOICINGS FROM DEBUSSY’S ‘REFLETS DANS L’EAU’, BARS 58 AND 59 – PERFORMED BY IVAN MORAVEC

In ‘Hommage à Rameau’ in bar 52, a G9 chord is spaced in accordance with the harmonic series, followed by a D9 chord, C maj 9, and D9 in bars 54, 55, and 56 respectively.

AUDIO TRACK 36 – DOMINANT CHORD VOICINGS FROM DEBUSSY’S ‘HOMMAGE À RAMEAU’, BARS 52 THROUGH 55 – PERFORMED BY IVAN MORAVEC

A1.5.2 MESSIAEN’S ‘CHORD OF RESONANCE’

If Olivier Messiaen’s *The Technique of My Musical Language* were written as a general theory of harmony, it may not easily withstand scrutiny. On one level, I believe it aspires to this, but on another (more relevant) level, it explains the composer’s own approach and thus we gain insight into Messiaen’s craft. This text has been hugely influential and in it are the seeds of many assumptions regarding the relationship between harmony and the harmonic series. It also says a

great deal about Messiaen's acceptance of 12tet as adequately resolved to suggest harmony based on the higher components of the harmonic series. It is also a fascinating insight into a well thought-out system of composition (even if it is not a consistently convincing theory of music).

Messiaen appeals first to Debussy in establishing the added sixth chord as a naturally derived chord.

With the advent of Claude Debussy, one spoke of appoggiaturas without resolution, of passing notes with no issues, etc. ... These notes keep a character of intrusion, of supplement: the bee in the flower! They have, nevertheless, a certain citizenship in the chord, either because they have the same sonority as some classified appoggiatura, *or because they issue from the resonance of the fundamental*. They are *added notes*... The most used of these notes is the added sixth. Rameau foresaw it; Chopin, Wagner made use of it (and also some writers of a facile and light temperament, notably Massenet and Chabrier, which proves to what point it is natural!). Debussy and Ravel installed it definitively in the musical language [**bold italics my emphasis**] (Messiaen 1944: 47).

Because of the claimed inherent naturalness of the added sixth, it is considered completely functional with 'the perfect chord' (the major triad) or the dominant seventh. To these structures, Messiaen adds the augmented fourth, which is justified as follows:

In the resonance of a low C, a very fine ear perceives an F-sharp... Therefore, we are authorized to treat this F-sharp as an added note in the perfect chord, already provided with an added sixth...and there will be an attraction between the F-sharp and the C, the former tending to resolve itself upon the latter (Messiaen 1944: 47).

Messiaen's logic requires several stretches of the imagination in order to accept the above as musical truths (but not to accept it as interesting musical material). There is nothing particularly 'natural' to the added sixth as an acoustical or psychoacoustical phenomenon⁴⁹ except its precedence in many theoretical texts including Rameau and Zarlino⁵⁰ (Tenney 1980/82: 96); "In the resonance of a low C" one may hear a *quartertone* sharp F (but not likely an F \sharp); and the attraction between F \sharp and C is most likely imagined or conditioned here—no doubt important to the composer but not an indisputable truth.

Amongst Messiaen's list of special chords, the composer explains the 'chord of resonance', which is a chord built from partials 8 through 15—a dominant seventh with a major ninth, an augmented

⁴⁹ The most compact set of a C^{add6} chord in 5-limit Harmonic Space suggests an 'F' acoustical root; Subharmonic Coincidence Analysis predicts an ambiguous root ('D', 'F', 'C', and 'A' occur each three times); and in the harmonic series of 'C' a pitch close to 'A' does not occur until the 27th harmonic (taking into consideration that the 13th harmonic is closer to a 12tet 'A^b' than an 'A').

⁵⁰ "If the second and seventh, though dissonant, are tolerable in syncopation, how much more tolerable is the sixth, which far from being dissonant, is accepted by all as consonance" (Zarlino, Gioseff, 1588, quoted in Tenney 1980: 96)

fifth, and a major seventh for extensions. Again, the use of tempered intervals to suggest these upper partials implicitly states Messiaen's acceptance of the resolution of 12tet⁵¹.

AUDIO TRACK 37 – MESSIAEN'S 'CHORD OF RESONANCE' IN EQUAL TEMPERAMENT COMPARED TO JUST INTONATION

⁵¹ Scriabin uses a similarly derived six- and later seven-tone mode: "With the composition of *Prométhée*, Scriabin's musical language undergoes a profound mutation, the true meaning of which can be understood only in the context of Scriabin's spiritual life, preserved by the entries in his journal. The harmonic foundation of *Prométhée* is based on a mode of six tones: C, F-sharp, B-flat, E, A, and D. This mode represents a transposition, necessarily approximative, of the upper partials of the natural harmonic series. It is simultaneously a chord and a tone color, a timbre. In his last sonatas, Scriabin adds to this chord-timbre a seventh tone, G, which provides the twelfth harmonic partial" (de Schloezer 1987: 331).

2 QUARTERTONES

2.1 USES OF QUARTERTONES

Quartertone usage arises from a great variety of compositional strategies. Historically, they have occurred: 1) melodically to increase the chroma of the 12tone scale; 2) referentially to suggest microtones based on higher harmonics of the series, and more specifically; 3) harmonically in the suggestion of 11-limit harmony, where the notation is quite precise; 4) to simply be 'out of tune' from equal temperament

Depending on the source of quartertones, the resolution of the system widely varies from composer to composer and from composition to composition. It seems important to distinguish between a quartertone that is the result of an equal division of the semitone, a microtonal embellishment, one that represents a frequency ratio whose intonation falls significantly outside of equal temperament ($7/4$, $11/8$, $13/8$, etc.), or a description of a frequency ratio that is specifically very close to a quarter-tone (usually an 11-limit ratio).

2.2 QUARTERTONE NOTATION

Gardner Read makes a convincing argument for the use of the following quartertone accidentals based on popularity of use, clarity, and simplicity, although he acknowledges that in the case of the flats a more arbitrary decision was necessary (Read 1990: 25).

3/4 tone sharp	♯
semi-tone sharp	♯
1/4 tone sharp	♯
natural	♮
1/4 tone flat	♭
semi-tone flat	♭
3/4 tone flat	♭

Table 3 – Gardner Read's list of the most common quartertone accidentals in a wide sampling of 20th century compositions (Read 1990: 25)

He does not, however, specify whether these are context specific. That is, the source of the composer's microtonal material seems irrelevant. The adoption of a fixed set of quartertone symbols limits the means for *self-analysis* and thus may affect performative aspects in some circumstances, or leave questions of interpretation.

In discussion with Dr. Bob Gilmore and Patrick Ozzard-Low, both feel that quartertone

notation has an established practice in Europe. While their consensus matches Read's conclusions for the use of sharps, their opinion differs with regard to flat notation, where they recognise the most common use of the following accidentals:

1/4 tone flat	♭
semi-tone flat?	♭
3/4 tone flat	♭

Table 4 – some other common quartertone flat symbols suggested by Bob Gilmore and Patrick Ozzard-Low

Although it is not my intent to engage in the notation debate too deeply, this system of flats incorporates a similar logic to that of the sharp symbols, in that a flat plus a quartertone flat equals a 3/4-tone flat.

I include this set only to the degree to which it represents a style of, or a trend in, notation for quartertones. I do not consider it definitive, nor do I wish to advocate or dismiss it. What I wish to do is to question the purpose of attempting to standardise any system of notation. (I will say more about this in my conclusions.)

Read does not avoid the performative issue completely. He notes that the use of this set of accidentals should follow spelling conventions for diatonic adjacencies in tonal music—that is, using the progression of altered sharp symbols when a line ascends chromatically and flats when descending. He also questions the use of the combination of arrowheads with traditional accidentals. "The lesser number of composers inclining to the use of arrows attached to sharps and flats may be the result of a certain unease many feel in fractionally raising an already flatted pitch (♭) and similarly lowering a previously sharpened note (♯). By placing the arrow-signs before, over, or underneath the noteheads affected this notational ambivalence is at least partially alleviated" (Read 1990: 46).

A 2 THE HARMONIC IMPLICATION OF QUARTERTONES IN IVES' CHORALE

The following analysis explores further the idea of harmonic implication within the expanded pitch gamut of fixed quartertones (24tet) through an analysis of the 'Chorale' from Ives' *Three Quarter-Tone Pieces* (1924) for two pianos tuned a quarter tone apart. (Quartertones as a referential notation arise in other sections as the base system of a larger pitch gamut.)

In Ives' *Chorale*, quartertones can be heard in at least two ways: as suggestive of seven- and eleven-limit harmony, or as intentionally harmonically ambiguous. If we accept that 12tet is successful in suggesting five-limit harmony, then a quartertone temperament (24tet) should be

successful in suggesting much higher limit harmonies, perhaps beyond the 19-limit.

But proponents of other equal divisions of the octave do not generally consider 24tet an improvement on 12tet; although 11-limit intervals are quite accurate, the 5-limit is no better resolved and the 7-limit is only marginally improved upon. However, if we also consider that our tolerance for mistunings increases as limit increases, then perhaps 24tet can be seen as an improvement on 12tet within specific contexts.

The allowances for 3- and 5-limit harmony in 12tet suggest a continuum that would allow for larger deviations in 7-limit harmony, i.e. if 12tet is successful in suggesting 5-limit harmony where deviations are on the order of 12 to 16 cents, then 24tet should successfully suggest 7-limit harmony where the deviations are on the order of 20 cents.

In his article, “Some Quarter-Tone Impressions”, Ives describes the chords that became the basis for *Chorale*. The first he calls “primary” or “fundamental” and is constructed by interlocking two perfect fifths a *neutral* third apart. A neutral third is halfway between a major and minor third. The resulting chord also produces a neutral seventh, one that is neither major nor minor. Whether it was Ives’ intention or not, this chord very closely approximates an eleven-limit chord in which the neutral third is represented by the ratio 11/9 and the neutral seventh 11/6⁵². In fact, no component of this chord is more than 2.5 cents out of tune from an eleven-limit system, which may suggest that Ives’ ears were sensitive to these harmonic relations⁵³. A harmonic space analysis of a C primary chord predicts an acoustical root of B \flat (fig 44).

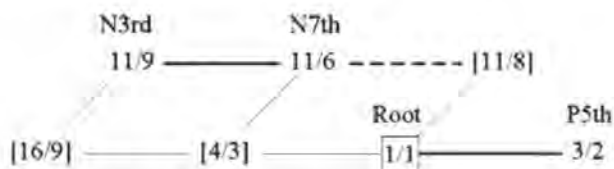


Figure 44 – Ives ‘Primary chord’ in 11-limit harmonic space

AUDIO TRACK 38 – IVES’ ‘PRIMARY CHORD’ IN 24TET

Ives’ “secondary” or “minor” chord is constructed with two interlocked perfect fourths separated by a quartertone sharp major second. There is no simple eleven-limit harmony that this chord might suggest; however, the quartertone sharp major second approximates two seven-limit

⁵² James Tenney identifies the intervals of Ives’ primary chord similarly in his *Perspectives of New Music* Article ‘About Changes: Sixty Four Studies for Six Harps’ (Tenney 1987: 80).

⁵³ Indeed, I have found that over time, my own ears have gradually adjusted to higher limit intervals and I now hear the 11/8 as perceptually fused where I heard it as rough when I first came to just intonation.

possibilities: an $8/7$ major second, or a $7/6$ minor third. The quartertone flat minor seventh that occurs may suggest a $7/4$ minor seventh. Here, a C secondary chord has G as the acoustical root.

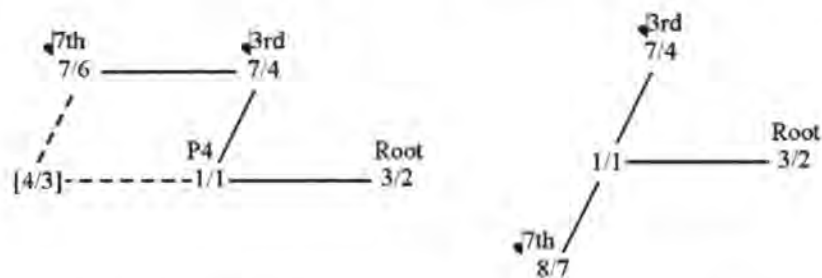


Figure 45 – Ives ‘secondary chord’ in 7-limit harmonic space (two possibilities)

This secondary chord appears to be problematic; the seven-limit intervals are more poorly approximated than are five-limit intervals in 12tet, and it appears that there are two equally viable options for the interpretation of the quartertone sharp major second. However, Ives suggests that this chord is somehow less stable than the fundamental chord. He may have been responding to the inharmonicity of the chord, or to its relative complexity.

But Ives’ secondary chord, to my ears suggests a $7/6$ minor third rather than an $8/7$ major second, which is likely due to the stronger fifth relation between the $7/6$ and $7/4$. Comparing the quartertone chord to a just intonation version reveals no significant difference in harmonic quality, but a noticeable, although more slight than one might expect, improvement in fusion.

AUDIO TRACK 39 – IVES ‘SECONDARY’ CHORD IN 24TET COMPARED TO A 7-LIMIT JUST INTONATION ‘SECONDARY’ CHORD

The first interpretation in fig 45 is the same space I suggested in the discussion of the 12tet minor seventh chord (7-limit prediction), but here we can say that while one ambiguity is resolved (whether or not a 5- or 7-limit chord is implied), a new one arises (which 7-limit chord is implied?).

In harmonic space, both versions of the chord (one with $7/6$, the other with $8/7$) produce a compact space and the argument for the preference of the $7/6$ interpretation is: a) that this space includes two $3/2$ relations rather than the complex relation between the $7/4$ and $8/7$ ($49/32$); and b) $7/6$ allows the $1/1$ to continue functioning as the root (or at least as the fifth of a missing fundamental ($4/3$), whereas $8/7$ would be the implied root in the second possibility.

11/6	11/6-----13/8?	7/6 or 8/7	11/6
11/9	7/6 or 8/7	3/2	11/9
3/2 ----- 7/5	4/3	11/9	3/2
1/1	1/1	1/1	1/1
primary	<u>primary</u> secondary	<u>secondary</u> primary	primary

Figure 46 – Reduction of Ives *Chorale*

AUDIO TRACK 40 – CHORALE REDUCTION IN 24TET COMPARED TO INTONATION IN FIGURE 45 (WITH 7/6 SMALL MINOR THIRDS)

It is also interesting that in *Chorale* Ives almost completely avoids five-limit intervals—major and minor thirds and sixths, minor seconds, and major sevenths—for the obvious reason that he simply wanted to explore the less familiar quartertone sonorities. Ives also states that he considers the fifth a sort of immutable law of harmony or nature but the major third has too much “diatonic expectancy” embedded in its sound (Ives 1961: 113)⁵⁴.

A second interpretation of Ives’ primary and secondary chord constructions may connect to Hindemith’s clouding theory—that thirds and seconds are mutable, and that Ives simply chose to work with the most ambiguous forms of thirds and seconds. Or even more generally, where perfect fourths and fifths cannot be ‘clouded’, intervals that divide fourths and fifths are fully mutable or available for clouding.

⁵⁴ Ives discards the chord C E G# A# because the major third creates a “kind of diatonic expectancy which the upper intervals resist” (Ives 1961: 113).

3 EIGHTH-TONES

3.1 FRENCH SPECTRAL MUSIC

The music of the so-called 'spectral' composers in France from the 1970s forward embraces the use of varying levels of intonational refinement dependent on the individual composition and the instrumentation, and therefore, for the purpose of this thesis, straddles the sections on 12tet, 24tet, and 48tet. In planning a composition, some composers associated with the *French Spectral School* regard the tuning system as a secondary consideration to the harmonic concept (Fineberg 2000b: 84).

I will use the term *Spectral Music* quite specifically here, tied most directly to the music of Horatiu Radulescu, Gérard Grisey, and Tristan Murail rather than in reference to the many composers for whom spectral information is important and of central concern⁵⁵ but is otherwise the result of a different musical heritage and aesthetic.

'Recourse to nature' and the 'natural laws of acoustics' (Anderson 2000: 10) are recurring themes in many theories of harmony from the Ancient Greeks through Rameau, to Partch, Hindemith, and Messiaen in the early to mid-twentieth century (and many others since). Grisey cites Hindemith as an important influence in the application of sum and difference tones "as generators of harmonic fields (Anderson 2000: 10), and Messiaen and Varèse are seen as "forerunner[s] of spectral composition" (Anderson 2000: 11).

Hindemith places great emphasis upon the derivation of his scales from not only the harmonic spectrum but most especially from sum and difference tones...In common with Cowell and Partch, Hindemith repeatedly cites his acoustical researches as 'natural' justification for his theories, implying that they are therefore inherently superior to other theoretical conceits of the time such as the twelve-tone system, for which he presumably felt no such natural justification could be found. This of course is a fallacy: whoever said that art music, by nature constructed and the product of complex socio-cultural phenomena, had to be based in nature? But it is a fallacy which will recur frequently throughout the second half of the twentieth century (Anderson 2000: 10).

That Hindemith could use 'recourse to nature' and simultaneously embrace equal temperament is a contradiction that Partch and his progeny might find impossible to

⁵⁵ "A Provisional History of Spectral Music" in *Contemporary Music Review* by Julian Anderson includes the music of Edgard Varèse, Harry Partch, Olivier Messiaen, Henry Cowell, James Tenney as a sampling of composers inspired by harmonic spectra, but there are important factors which separate the French school from others—most important here, the assumption that quartertones (and in some cases 12tet) are sufficiently resolved tuning systems in which the realise tones based on harmonic and inharmonic spectra, and the use of the orchestra as the ideal synthesising palette.

reconcile. But what is important to take from this is that music, at the very most, is an abstraction of nature, and necessarily makes all sorts of concessions which are the choice of the composer—indeed an integral part of the creative process and a more general component of 'style'.

3.1.1 TOLERANCE AND RESOLUTION

At least two of the influences⁵⁶ on spectral composition—Varèse and Hindemith—both accepted equal temperament as a suitable vehicle for the realisation of spectrally derived harmony (although Varèse adapts 12tet further than Hindemith does).

For spectral composers, microtones are not the result of scales built on frequency ratios, nor even one of tuning. Instead, the microtones in spectral music are simply approximations of a set of frequencies to the nearest available musical pitches. In most cases, quarter-tones are used for instrumental music (with some eighth-tones in very slow tempos and occasional reversions to semitones in very fast tempos or for keyboard instruments). This approximation is often a last step, allowing the musical structure to be generated in its most precise form (frequencies), then approximated to the nearest available pitch depending on the details of the instrumental abilities and context. This also allows many spectral composers to tailor difficulty to individual realizations, adding or removing difficult notes in a way that does not change the underlying structure, but merely refines or coarsens the approximation of the abstract musical structure. Since the ear analyses structures based upon their frequency structure, the ear is able to hear past these approximations and hear the underlying frequency structure whenever the approximation is within tolerable limits (Fineberg 2000b: 84)

What is considered "within tolerable limits" is of central concern in tuning theory; the implied 'tolerable limits' of French Spectral Music in the above quote are not universally accepted.

One measure of tolerance may emerge from the *resolution of the virtual fundamental*. Sets of related tones produce a virtual fundamental, which is "explained essentially as the greatest common denominator of a harmonic spectrum" (Fineberg 2000b, 98), although by what mechanism this phenomenon occurs is a matter for debate. With small number ratios the virtual fundamental is well defined (resolved); a just tuned 6/5 (say, (C[#]₄ – E₄) will produce a virtual fundamental of A₂, and is predicted by the difference tone calculation: 6 – 5 = 1; or in Hz: 660Hz – 550Hz = 110Hz. If A₅ is added to the above ratio (5:6:8), the common denominator remains 1, and thus produces the same virtual

⁵⁶ Hindemith is a relatively minor influence in comparison to composers such as Stockhausen, Xenakis, and Ligeti.

fundamental. Taking all the first order difference tones gives: $8 - 6 = 2$; $8 - 5 = 3$; $6 - 5 = 1$, all of which point to or reinforce the same fundamental.

In equal temperament, the fundamental will not resolve so easily as intervals are not harmonically (in the acoustic sense) related. A 12tet minor third relation ($C\sharp_4 - E_4$) produces a first order difference tone of 104.89Hz

$$659.26 - 554.37 = 104.89\text{Hz} (A_2^{-82.4 \text{ cents}} \text{ or } G\sharp_1^{+17.6 \text{ cents}})$$

Adding the A_5 (880Hz) to the dyad (554.37:659.26:880) causes problems in extracting a common denominator:

$$880 - 659.26 = 220.74\text{Hz} (A_3^{+5.8 \text{ cents}});$$

$$880 - 554.37 = 325.63\text{Hz} (E_4^{-21.1 \text{ cents}});$$

$$659.26 - 554.37 = 104.89\text{Hz} (G\sharp_1^{+17.6 \text{ cents}})$$

Predictably, three unrelated difference tones, thus no fully resolved virtual fundamental.

But the ear has a tolerance that tries to reconcile intervals that are *close* to small number harmonic relations.

Joshua Fineburg, in 'Appendix I' to the issues of *Contemporary Music Review* devoted to Spectral Music, states that "since the ear analyses structures based upon their frequency structure, the ear is able to hear past these [quarter-tone] approximations and hear the underlying frequency structure whenever the approximation is within tolerable limits" (2000b: 84). Unfortunately, or wisely, Fineburg does not speculate upon what that level of approximation might be.

For a distorted, shifted, non-harmonic, or modulation-based spectrum...the ear still tends to find a fundamental...Many psychoacoustic algorithms have been proposed which attempt to model this effect by which the ear creates a sort of 'virtual' fundamental in spectra lacking a real one. These algorithms depend on the tolerance of the ear...With slight variations, they calculate the greatest common denominator to a given tolerance (which is often user specified). The virtual fundamental has often been used by spectral composers as an ad-hoc measure of harmonicity (lack of tension) or inharmonicity (presence of tension), equating higher virtual fundamentals with greater harmonicity or less inharmonicity and lower ones with less harmonicity, or greater inharmonicity. The motivation for this is that harmonic spectra start with a real fundamental and as they are distorted the virtual fundamental moves in various directions; when these distortions become more and more noise-like, the virtual fundamental descends until, for white noise, the virtual fundamental approaches zero Hz (Fineburg 2000b: 84).

The above passage refers to distortions to a harmonic spectrum calculated before being orchestrated within a quartertone temperament, but the act of orchestration also represents a distortion of, in the case of *spectral music*, harmonic and inharmonic sonorities, and thus a further shift of virtual fundamental within the acceptable tolerance of the musical *style*. This level of tolerance is both greater and less than those allowed in other musical styles, and is thus one defining characteristic of this music.

The idea that the refinement of tuning can be a function of style is a novel one, and a controversial one, and is central to many arguments between advocates of various temperaments and tuning systems.

In suggesting spectral intonation, voicing and spacing becomes important. The closer the voicing mimics portions of the harmonic series, the less ambiguous the harmonic implication will be, which may extend the tolerance of a given tuning system.

As shown in Debussy's dominant structures, with fixed intonation the careful use of voicing and spacing may allow for the suggestion of higher-limit harmony, and in dynamic intonation the spacing may actually influence the musician toward an intonation that reflects the harmonic concept.

A3 **PROLOGUE FOR VIOLA, GERARD GRISEY (1978)**

Where quartertones improve the accuracy of 7- and 11-limit intervals but not 5-limit intervals, eighth-tone temperament improves 5-limit intervals. However, the 5/4 major third is 11 cents flat in an eighth-tone temperament as opposed to 14 cents sharp in 12tet, so the resolution is only marginally improved upon, and may actually not be for the better as there is some speculation that the ear is more tolerant to sharp thirds than flat ones. Whether this is simply a matter of conditioning is unclear. (We are used to hearing thirds sharp, both in fixed 12tet where they are 14 cents sharp from pure and in dynamic intonation in a traditional harmonic/melodic vocabulary where the third of the dominant tends to be slightly sharpened as it leads to the root of the tonic.)

In *Prologue for viola (1978)*, Gérard Grisey uses a subset of the quartertone symbols, and uses arrowheads in conjunction or on their own to indicate further eighth-tone inflections, resulting in the following set of accidentals (descriptions to the right are in Grisey's own words):

3/4 tone sharp	♯♯	sharp raised exactly one quarter-tone
semi-tone sharp + 1/8 tone	♯♯	slightly higher (1/8 tone)
semi-tone sharp	♯	
1/4 tone sharp + 1/8 tone	♯♯	slightly lower (1/8 tone)
1/4 tone sharp	♯	sharp lowered exactly one 1/4-tone
1/8 tone sharp	♯	slightly higher (1/8 tone)
natural	♮	
1/8 tone flat	♭	slightly lower (1/8 tone)

Table 5 – Grisey’s system of eighth-tone accidentals used in *Prologue*

The exclusion of any flat symbols is worth mention, as are the instructions that quartertones are to be exact but eighth-tones only approximate.

The avoidance of the flat symbol is most obviously advantageous because the musician has to decode a smaller pallet of accidentals. This piece is particularly suited to this because its construction is not melodic or chromatic in a traditional sense. The piece is based on an E harmonic series, and is therefore concerned with a fixed relationship to a single fundamental (although that relationship becomes increasingly complex). This means that each pitch serves a specific function separate from any notions of voice-leading or traditional harmonic context. However, there may be a psychological disadvantage with the use of the ♯ where the musician receives the conflicting message to flatten a sharpened pitch, as opposed to the alternative ♭ which asks the musician to flatten further an already flattened pitch (for example, compare A♯ to B♭ for clarity)

The notation for the A♯, the 11th harmonic, raises a first question. Grisey specifically states that “The note A♯ should be thought of as a lowered A♯ and not as a raised A(♯). The 11th harmonic is almost exactly a quartertone flat of an equal tempered augmented fourth (tritone)⁵⁷ from the eighth harmonic. It really should not matter whether this pitch is notated as a perfect fourth plus a quartertone or as an augmented fourth less a quartertone. But Grisey may be concerned for the psychological adjustment required to detune what is normally a very smooth interval in the perfect fourth as opposed to the already ambiguous tritone. Or, he may simply have been adhering to the

⁵⁷ I will use the term tritone to describe the augmented fourth, regardless of the extra meaning attached to this term (as an interval that exists within a dominant seventh chord). This is to avoid the awkward problem of referring to partials and intervals within the same discussion. For example, the 3rd partial is a perfect fifth plus an octave from the fundamental and the 5th partial is a major 3rd plus two octaves from the fundamental.

well-established practice of notating the 11th harmonic as a type of tritone—a convention that can be traced back at least to Messiaen and Hindemith.

The second question regarding why Grisey requests exact quartertones but not eighth-tones in the score instructions has implications that are more important.

Early Twentieth Century usage of quartertones was specifically concerned with an increase in the chroma of the equal tempered scale. The addition of twelve extra tones had no specific harmonic motivation, or at least it was not thought of as an extension of Western harmonic practice. While Ives was interested in chords of 'strength' and variations on tertian chords, he was not explicitly motivated by theories of harmony based on acoustic models such as the harmonic series (at least he does not state so directly). He was able to create sonorities that sounded ambiguous in relation to traditional harmonic practice.

Grisey and other spectral composers of the late Twentieth Century are more often interested in the harmonic implication of microtonal intervals. Quartertones can be used to suggest 11-limit harmony. Other divisions of the semitone, tone, and octave can be used to explore sonorities based on other prime numbered partials. The fact is that only quartertones happen to express accurately the 11th partial. Further whole number subdivisions of the semitone (within reason) do not improve upon the 11th partial's special condition. Improvements in approximating higher partials occur sooner than the improvement of lower partials.

Grisey is saying something very specific in the score for Prologue. He is using quartertones as a specific and accurate notation for the 11th partial, and using eighth-tone inflections to approximate other partials to varying degrees of exactitude. Table 6 compares eighth-tones to the ideal tuning of partials 8 through 16, measured in cents.

Partial	1/8 tone temperament	Just-intonation	Deviation from just-intonation in cents
15	1100 or 1075	1088	12 or 13 cents
14	975	969	6
13	825 or 850	841	16 or 19
12	700	702	2
11	550	551	1
10	400 or 375 ⁵⁸	386	14 or 11
9	200	204	4
8	0	0	0

Table 6 – comparison of 1/8th tones to the basic just intonation intervals

Clearly, in *Prologue*, Grisey uses eighth-tone notation to *push* the musician in the direction of the proper intonation of each pitch—in effect, combining a fixed 24tet notation with referential eighth-tone inflections. He further expects that uninflected notes also be tuned to their partial equivalent (specifically partials 5, 15, and to a lesser extent 17 and 19). For the most part, Grisey chooses the closest eighth-tone equivalent, but makes a curious choice with the 13th partial. From the third beamed grouping in the second system on the first page onward, Grisey notates the 13th partial with C^{\sharp} . A perfect eighth-tone puts this at an interval of 825 cents from the nearest octave of the fundamental. A just 13th partial is 841 cents, which is most closely approximated by C^{\sharp} .

As the piece progresses Grisey’s expectations for intonation gradually loosen as the source of the microtones move away from a clear and simple relationship to the lower portions of the harmonic series. The beginning of the piece is clearly based on an E harmonic series but past a certain point “la justesse n’a plus d’important”, as the piece becomes increasingly chaotic.

The important question to all of this is; how does the performer respond to this notation? The following analyses a 1997 recording by Gérard Caussé. I have selected moments in the score for analysis, identified as “Passage A, B, C, and D”, where each is more melodically complex than the previous, but where the harmonic motivation remains relatively clear.

⁵⁸ I have, at some point, come across a premise which states that while 1/8-tones improves the accuracy of the tuning of the 5th partial, the ear somehow has a harder time rationalising major thirds tuned flat than those tuned sharp.

A3.1 PASSAGE 'A'



Figure 47 – Passage 'A' from Grisey *Prologue*

AUDIO TRACK 41 – PASSAGE 'A' FROM GRISEY'S PROLOGUE – PERFORMED BY GÉRARD CAUSSÉ

The opening passage (A) is repeated four times. The notation suggests pitches based on, respectively, harmonics number 9, 8, 12, 18, 14, and 6. With the exception of the D?, each repetition comes closer to a 12tet or Pythagorean intonation, which is appropriate to the implication of the notation.

However, the intonation of D?, which we should expect to sound 31 cents flat of 12tet, is actually raised by 14 to 20 cents (a difference of nearly a quartertone, and further exaggerated by the following B, which consistently sounds 7 cents flat of 12tet). The result is a diatonic passage with one strange pitch, which on some level maintains a stylistic aspect of the piece, although not quite as expected.

A3.2 PASSAGE 'B'



Figure 48 – Passage 'B' from Grisey *Prologue*

AUDIO TRACK 42 – PASSAGE 'B' FROM GRISEY'S PROLOGUE – PERFORMED BY GÉRARD CAUSSÉ

The second passage selected is the sixth beamed group in the first system. It contains the first occurrence of the 11th partial (A[♯]). The uninflected pitches are all performed slightly sharp, but close to Pythagorean intonation relative to itself. However, the A[♯] sounds only slightly flat relative to the surrounding pitches, and at best, merely sounds like a tritone. However, the large leap required to arrive at the tone not only makes intonation challenging, but contributes to the exoticness of the pitch, and is perceived as odd even though it is not intonationally so.

A3.3 PASSAGE 'C'



Figure 49 – Passage 'C' of Grisey *Prologue*

AUDIO TRACK 43 – PASSAGE 'C' FROM GRISEY'S PROLOGUE – PERFORMED BY GÉRARD CAUSÉ

The third excerpt begins after the second beamed group in the second system. A three-note pitch set is transposed down by an eighth-tone four times, shifting in total by one full semitone.

The pitches of each cell form a ratio set based on 6:8:11.

In the first instance, the $A\sharp$ sounds simply as a tritone rather than a quartertone flat tritone. No apparent shift is heard in the first eighth-tone transposition; although $E?$ is slightly higher than the E from the first group, and makes $A?$ look more like a quartertone in the spectral analysis data (but I don't hear it). However, the duration of each of the pitches here is very short.

In the third transposition, a shift is audible but is not related to score. And, much like the first and second transpositions, the difference between the third and fourth transpositions is negligible.

Overall, the quartertones are roughly distinguished in this passage, although not accurately so, but the 25 cent shifts (the second and fourth transpositions) are not reflected by any significant shift in intonation. What remains important is that the *effect* of the passage is transmitted even though it is not even remotely accurate with regards to the indicated intonation. Perhaps we can say that the performance is intonationally stylish though inaccurate.

A3.4 PASSAGE 'D'



Figure 50 – Passage 'D' of Grisey *Prologue* (for viola)

AUDIO TRACK 44 – PASSAGE 'D' FROM GRISEY'S PROLOGUE – PERFORMED BY GÉRARD CAUSÉ

In Passage D, the intonation is variously accurate and inaccurate. The wide intervallic material is likely a contributor to the inaccuracies.

4 TWELFTH TONES (72TET)

Twelfth tones are very precise, to a degree that effectively simulates high-limit just intonation systems. From eighth tones to twelfth tones, there is no improvement upon the 5/4, the 7/4 improves and is only 6 cents out, 13/8 is most poorly represented at 8 cents out, and 11/8 is, as with quartertones, 1 cent out.

		numerator									12 th tone (size-in-cents)
		8	9	10	11	12	13	14	15	16	
n u m e r a t o r	8	0	200	383.3	650	700	<i>833.3</i>	966.6	1016.6	1200	
	9	204	0	183.3	350	500	<i>633.3</i>	766.6	883.3	1000	
	10	386	182	0	166.6	316.6	450	<i>583.3</i>	700	816.6	
	11	649	347	165	0	150	283.3	416.6	533.3	650	
	12	702	498	316	151	0	133.3	266.6	383.3	500	
	13	841	637	454	289	139	0	133.3	250	366.6	
	14	969	765	583	418	267	128	0	116.6	233.3	
	15	1012	884	702	537	386	248	119	0	116.6	
	16	1200	996	814	647	498	359	231	112	0	
		just ratio (size-in-cents)									

Table 7 – comparison of just intervals to twelfth tones (basic 5-limit intervals in bold; italics indicate tuning errors greater than 5 cents; shaded values are inversions or repetitions of other ratios)

Historically, the size of the pitch gamut has been a consideration in the adoption of a temperament, which makes twelfth-tones (72-tet) extremely impractical. Imagining a keyboard or a staff system that can accommodate 72 tones per octave demonstrates the point. But in a dynamic system, twelfth tones can be accommodated through inflections of the basic twelve tones.

Because a twelfth-tone temperament encompasses both 12tet and 24tet, thinking of twelfth-tones as a satellite system is effective with dynamic intonation. That is, any pitch in 12tet may have a satellite tone that is 1/6, 1/3, or 1/4 of a semitone away, or in a 24tet, each tone requires only one satellite tone of 1/6 of a semitone. Using a quartertone notation with inflections provides a learnable and decipherable scheme for notating twelfth tones.

The resolution of the system means that almost all of the just intervals (well beyond the intervals found in the first sixteen partials of the harmonic series indicated in the above table 7) are resolved to within a 5 cent tuning error. Exceptions occur occasionally with some complex intervals (high-limit and large numbers) and with the 13/8.

A twelfth tone notation can be used to create an unambiguous extended just intonation

system because there are few (if none) pitches that can have multiple interpretations (unless the composer is exploring extreme regions of the harmonic series). A further advantage of twelfth tones over a just intonation tuning is that it is fully transposable to any pitch in the system; a single pitch may be used in the approximation of a number of just intervals from 72 distinct roots, whereas in a just intonation tuning, each pitch is exclusive to its function in the harmonic series.

An example of the use of twelfth tones is James Tenney's *Sixty Four Studies for Six Harps*, where the composer uses 72tet to approximate an extended 11-limit tuning system. There is little ambiguity regarding the implication of any individual pitch.

The harps are tuned a sixth of a semitone (16.66...cents) apart, so the ensemble is capable of producing a tempered microtonal set of seventy-two pitches in each octave. This tuning system (which I call the 72-set) provides very good approximations of most of the important just intervals within the 11-limit, with the worst case being the three-cent error for the 5/4 major third (383¢ instead of 386¢). The relations between some of these just intervals and their nearest approximations in the 72-set are shown in Table 1 (where interval sizes are rounded off to the nearest cent).

ratio	size	pc number		error
		in 72-set	size	
8/7	231¢	14	233¢	+2¢
7/6	267¢	16	267¢	±0¢
6/5	316¢	19	317¢	+1¢
11/9	347¢	21	350¢	+3¢
5/4	386¢	23	383¢	-3¢
9/7	435¢	26	433¢	-2¢
4/3	498¢	30	500¢	+2¢
11/8	551¢	33	550¢	-1¢
7/5	583¢	35	583¢	±0

(etc.—larger intervals which are octave-complements of these have the same absolute value for error)

(Tenney 1987: 65)

72-tet is also adaptable to systems where intervals might be compressed or expanded according to a scaling algorithm. The twelfth tone system may also be practical for cluster and simulated noise inspired composition within a limited degree of complexity (clusters of seven tones per semitone are available)

Where twelfth tones are least practical are with fixed intonation instruments. A 72 pitch per octave keyboard is extremely impractical. However, some recent developments with

the tuning of synthesizers may alleviate some of this impracticalness—the development of 'paratactical' tuning by Larry Polansky for example⁵⁹.

⁵⁹ See: Polansky, Larry (1987) 'Paratactical tuning: An agenda for the use of computers in experimental just intonation', *Computer Music Journal*, Vol. 11, No. 1 (Spring) pp. 61-8.

5 OTHER '*N*'-TETS

N-tets refer to a large gamut of equal temperaments based on equal divisions of the octave; where *n* can be any whole number with a general preference for values greater than 12, which of course includes temperaments based on multiples of twelve, but for the purpose of this section, these are considered special cases and are not included.

5.1 RESOLUTION OF VARIOUS *N* -TETS

Proponents of *n*-tets generally embrace this method of tuning in contrast to the premise of Just Intonation. In effect, the goals of *n*-tets are similar to those of 12tet—the good approximation of 3- and 5-limit harmony through a practical number of pitch-classes. However, supporters of various *n*-tets feel that 12tet is too arbitrary and rough a system for continued use, and that other *n*-tets might not only improve on the accuracy of 3- and 5-limit intervals but also make possible the use of higher-limit intervals within an acceptable tolerance range.

...In the face of seemingly boundless freedom of choice, what is needed is a basis for selection that will tell us which systems offer the greatest resources and will thereby be the most likely to reward our exploration. In fact, there is a deeper question than this, and that is the question of how one might appropriately describe the resources of a pitch system. To be sure, the ultimate resources of a pitch system are some function of its intervals, the primitive pairwise relation between pitches. So the question really boils down to one of how to conceive of intervals...

(This is the primary basis for disagreements between proponents of just intonation and proponents of higher-division equal temperaments.)

The commonly accepted answer is that the canonical definition of an interval is to be couched in terms of frequency ratio, moreover a ratio of powers of small integers, a mathematical object of the form $2^p, 3^q, 5^r \dots$, with *p, q, r* ranging over positive and negative integers. The resources of an equal-tempered *n*-fold pitch system of octave divisions are then a function of the "goodness-of-fit" between the equal log-frequency grid of the system and some set of ratios (Mandelbaum 1961; Stoney 1970; von Hoerner 1974; 1976). Certain ratios may be set aside as special in the sense that it is particularly important to approximate them closely, for example, $2^{-1}3^1$ (p5) or $2^{-2}5^1$ (M3) (Balzano 1980: 66)⁶⁰.

It strikes me that this last premise puts the use of *n*-tets in an awkward relationship to contemporary musical practice throughout the 20th century, which saw the "emancipation of the dissonance", and approaches to harmony that consciously sought to usurp the

⁶⁰ Balzano continues with an argument for "another way of assessing the resources of a pitch system, one that is independent of ratio concerns and that considers the individual intervals as transformations forming a mathematical group" (Balzano 1980: 66).

importance of the perfect fifth and the major third and systems derived from those two intervals. Of course, there are contemporary styles that embrace the small prime intervals, but it is not apparent that these composers have been particularly drawn to *n*-tets as the basis of their tuning systems.

Adriaan Fokker, one of the foremost proponents of *n*-tets, provides a comprehensive analysis of several *n*-tets up to 94tet, which takes into consideration both the resolution of the basic intervals within the system (plus higher-limit intervals up to 13/8) and the vastness of the system (or relative practicalness). He concludes that “[f]or the nearest future it seems to be the best policy to switch over from twelve[tet] to thirty-one. Music today seems ready for that” (Fokker 1966: 202).

Supporters of various *n*-tets include Nicola Vicentino (1555) and Christiaan Huygens (1655)—31tet; Woodhouse and Yasser—19tet; Von Janko—41tet; Mercator (17th c.)—53tet)

Second to 31tet, 19tet is probably the next most often championed *n*-tet. The following chart provides a comparison of tuning error (absolute value) for the most important intervals to the 13-limit (according to Fokker).

	48tet	31tet	24tet	19tet	12tet
3/2	2 cents	5	2	7	2
5/4	14	1	14	7	14
7/4	6	1	14	22	14
11/8	1	7	1	19	49
13/8	9	10	9	20	41

Table 8 – comparative chart of tuning errors associated with common *n*-tets (given in absolute values in cents deviation)

AUDIO TRACK 45 – 13-LIMIT ‘HARMONIC’ CHORD IN FIVE DIFFERENT *N*-TETS (48-, 31-, 24-, 19-, 12-)

Overall accuracy may not be the most important concern in comparing these intervals—of course musical intent is most important—but if we are to extend Fokker’s methods of analysis, tolerances for tuning based on interval complexity should also be considered. That is, simple ratios should be given preference over ratios that are more complex because our ears are most sensitive to the mistuning of simpler ratios—but with consideration and sensitivity for the need for resolution in avoiding the ambiguity of complex ratios (these decisions must remain context specific). With the exception of the 3/2 perfect fifth, 31-tet maintains its preferential status according to the parameters in table 8.

But it is easy to imagine how any of these temperaments might be applied strategically in contemporary composition where the composer may chose to explore higher-limit harmony exclusively or selectively.

6 JUST INTONATION

Just Intonation is any system of tuning in which all of the intervals can be represented by ratios of whole numbers, with a strongly-implied preference for the smallest numbers compatible with a given musical purpose...

Just Intonation is not a particular scale, nor is it tied to any particular musical style. It is, rather, a set of principles which can be used to create a virtually infinite variety of intervals, scales, and chords which are applicable to any style of tonal music (or even, if you wish, to atonal styles). Just Intonation is not, however, simply a tool for improving the consonance of existing musics; ultimately, it is a method for understanding and navigating through the boundless reaches of the pitch continuum—a method that transcends the musical practices of any particular culture (Doty, David B. 1994—From The Just Intonation website).

6.1 JUST INTONATION THEORY

Throughout the twentieth century, a growing body of musical composition based on just intonation has emerged primarily from the influential work of Harry Partch, who 'rediscovered' the language of ratios as a return to the principles of the ancient Greeks, and who was, in turn, inspired by that other influential work: Hermann von Helmholtz's *On the Sensations of Tone*.

Just intonation is a language of tone relations, which is described through the language of frequency ratios, and provides an organisation of a particular conception of consonance and dissonance—where ratios based on low prime numbers and containing small numeric values are considered most consonant (1/1, the unison, might here be considered maximally consonant), and ratios based on larger primes and/or composed of large numeric values (in reduced form) are considered more dissonant. Here, consonance and dissonance are not opposites but relative measures.

Using ratios as a measure of pitch relation allows the notation of precise intonation values; a 5/4 major third is 386 cents in magnitude (rounded to the nearest whole number).

Just intonation composers have adopted several conventions. While ratios describe the magnitude of a musical interval, octave condensed ratios are often used to describe *pitch classes*. For example, a scale may be described in relation to a single root: (F)1/1—(G)9/8—(A)5/4—(Bb)4/3—(C)3/2—(D)5/3—(E)15/8.

Just intonation is also an open system. Where equal temperament is a closed system analogous to a circle (Gilmore 1995: 461)—any equal tempered system's "lattice

structure is *periodic* in harmonic space" (Tenney 1987: 72)—just intonation may extend infinitely along any number of branches (each branch associated with a particular prime number) allowing for an infinite number of pitch relations and classes. However, Bob Gilmore points out in "Changing the Metaphor" that ratios significantly removed from a fundamental (we can think in harmonic space for convenience) are more likely to be recognised as a simpler ratio that is intonationally close to the more complex one. Ambiguity grows as distance from the fundamental increases, and therefore it becomes increasingly unlikely that a complex ratio will be recognised as such. Particularly, it is here that the terms consonance and dissonance lose their meaning completely as a complex or dissonant ratio might easily be confused with a more simple/consonant ratio. Or, even where differences can be heard, the association may be so strong with the simpler ratio that the more complex is simply heard as a slightly mistuned version of the first (imagine the octave, 2/1 and an extremely complex ratio such as 1000/999 which is 1.7 cents larger than the octave).

However, this section considers certain composers as more indicative of the Just Intonation scene, which, as stated in the above quote from the Just Intonation Network, prefers small numbered ratios to large (complex) ratios. For this reason, the spectral composers such as Radulescu, Grisey and Murail (who delve into more extreme territories of the harmonic series), and composers like James Tenney (who uses a variety of just intonation and temperaments) and Ligeti (who freely mixes tempered and just tones) are not here considered, strictly speaking, just intonation composers.

6.2 JUST INTONATION SCALES

The ancient Greeks⁶¹ built scales based on perfect fifth relations (3/2). A Pythagorean mixolydian scale may be built from a chain of 3/2 relations (and its inverse 4/3) condensed into a single octave:

$$\begin{array}{cccccccc} 16/9 & - & 4/3 & - & 1/1 & - & 3/2 & - & 9/8 & - & 27/16 & - & 81/64 \\ Bb & & F & & C & & G & & D & & A & & E \end{array}$$

Figure 51 – Pythagorean Mixolydian scale in harmonic space

AUDIO TRACK 46 – PYTHAGOREAN MIXOLYDIAN SCALE

⁶¹ I use the Greek example and the subsequent historical development very loosely here and more for the purpose of allegory; other cultures and earlier societies probably used a similar language of number and proportion in the description of pitch relations, and I will simplify the historical use of different prime numbers as a simple linear progression leading up to the twentieth century even though there is a good deal of evidence of, for example, early theoretical scales built using the prime numbers 7 and 13.

If the prime number 5 is introduced into the system, the $81/64$ major third can be replaced by the simpler $5/4$ major third (386 cents) and the $27/16$ major sixth with $5/3$ (884 cents), and we can introduce the $15/8$ major seventh (1088 cents):

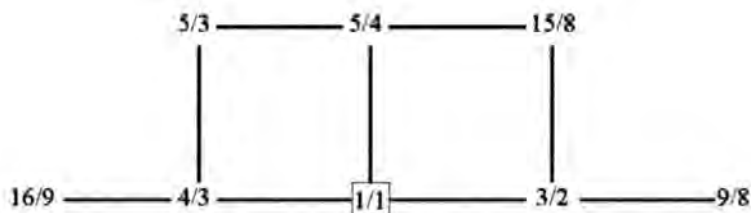


Figure 52 – 5-limit Mixolydian (plus major seventh) scale in harmonic space

AUDIO TRACK 47 – 5-LIMIT MIXOLYDIAN SCALE (PLUS MAJOR SEVENTH)

If we continue along the historical path Partch lays out...

In terms of consonance man's use of musical materials has followed the scale of musical intervals expressed as Concept One; from the earliest times it has progressed from the unison in the direction of the great infinitude of dissonance (Partch 1949/1974: 94).

...and continue adding new primes through the 7-limit, the $16/9$ minor seventh can be replaced by the $7/4$ minor seventh (969 cents):

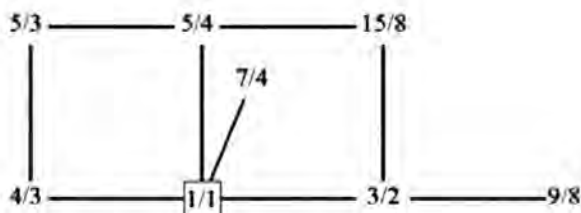


Figure 53 – 7-limit Mixolydian (plus major seventh) scale in harmonic space

AUDIO TRACK 48 – 7-LIMIT MIXOLYDIAN SCALE (PLUS MAJOR SEVENTH)

Partch introduces the 11-limit in the early twentieth century, and Johnston the 13-limit later in the century. Below, I replace the $4/3$ with the $11/8$ and the $5/3$ with $13/8$. Perceptually, there is little connection here; however, there is some precedence with Messiaen and Hindemith—the resultant scale being the just intonation version of their 'harmonic' scales (i.e. Messiaen's *chord of resonance*):

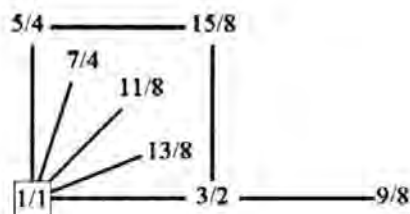


Figure 54 - implied space of 'harmonic scale'; or Lydian $\flat 7$ ⁶² (plus major seventh); or Messiaen's chord of resonance in harmonic space

AUDIO TRACK 49 – HARMONIC SCALE IN JUST INTONATION (LYDIAN $\flat 7$ (PLUS MAJOR SEVENTH))

While many just intonation composers use a fixed set of pitches (which may result in the use of the above materials as the pitch set for an entire movement or composition—for example, Ben Johnston's *Suite for Microtonal Piano* which is based on partials 8 through sixteen of the harmonic series), others may use just intonation openly, allowing for pitch relations extending from any other pitch class (Johnston's *Sonata for Microtonal Piano* in contrast). We could imagine a piece which explores the just augmented triad, which could extend continually along a chain of 5/4 relations: 1/1—5/4—25/16—125/64—625/512—etc. (C—E⁽⁻¹⁴⁾—G \sharp ⁽⁻²⁸⁾—B \sharp ⁽⁻³¹⁾—D^{*} ⁽⁻⁵⁵⁾—etc.) and never return precisely to the fundamental.

Within a relatively compact set of just intervals ('relative' will be influenced by cultural conditioning, exposure, experience, memory, and such things and may be an individual experience), there should be, in theory, no ambiguity regarding harmonic/melodic intent. And an *ideal* intonation is always implicit. To what extent the ideal is achievable is one subject of my analyses.

6.3 JUST INTONATION TODAY

My sense is that a looser relationship to just intonation is emerging in the new music repertoire. Where some early proponents of just intonation are outspoken about the superiority of just intervals, stating that "just intonation is the best intonation" (Harrison 1993: 88), contemporary composers are adopting a language of ratios, or pitch sets

⁶² The Lydian $\flat 7$ scale has occasionally been referred to as the 'natural' scale, but the function of the major sixth is dubious depending on whether the thirteenth partial is considered as a type of major or minor sixth (which it is neither). Although the thirteenth partial is often represented as a major sixth, the frequency is actually closer to a minor sixth (+41 cents).

based on just intonation, and are coming at the subject with less specialized focus—or at least with just intonation as a less pertinent parameter in the scheme of the compositional process—and therefore just intervals are used less preciously or without the political or spiritual agenda of, for example, Harry Partch, Ben Johnston, or Lou Harrison.

6.4 JUST INTONATION NOTATION

While just intonation affords the expression of an unambiguous harmonic language, notation is conversely contentious.

The number of notation systems for just intonation is large and not even remotely standardized, although there are a few trends that may be identified. Ben Johnston has developed an extensive system of new and adopted symbols (see A 6.2), James Tenney uses precise cents-deviation indicators above the standard twelve-note notation system (see A 7.2), Partch developed his own idiosyncratic notations (fig 57), and others use a variety of symbols borrowed from some of the equal temperaments discussed above and below.

Figure 55 – example of Harry Partch's notation of just intervals from *Barstow: Eight Hitchhiker Inscriptions from a Highway Railing in Barstow, California (1941-1968)*

A6 JUST INTONATION IN ANALYSIS

A 6.1 ANALYSIS OF LIGETI'S "HURA LUNGA" FROM SONATA FOR SOLO VIOLA

Hura lunga, the first movement of Ligeti's 'Sonata for Viola Solo', offers a good opportunity for scrutinizing intonation tendencies in a microtonal piece based on the harmonic series while reflecting the intentions of the composer. Dedicated to Tabea Zimmermann, the violist's recording may be considered definitive given the composer's endorsement and supervision of the recording session.

Ligeti is an interesting example as he is not strictly speaking a just intonation composer, but is a composer experienced in the use of microtonal intervals; and here, in contrast to Ben Johnston (A 6.2), we observe how a composer external to the just intonation 'scene' deals with just intonation material.

Ligeti states in the 'Preface to Sonata' that "The movement is played exclusively on the C-string and in it make[s] use of natural intervals (pure major third, pure minor seventh and also the 11th harmonic)".

Although executed exclusively on the C-string, the harmonic reference is a mix of a 'C' and 'F' series. The melodic/harmonic organisation is quite traditional except for the microtonal inflections; the harmonic centres of 'C' and 'F' take on a functional dominant-tonic role, although leading tone tendencies are, in particular, usurped here.

The image shows a musical score excerpt from Ligeti's *Hura lunga* (1994). It consists of four staves of music, numbered 1 through 10. The notation includes various dynamics such as *p* (piano), *mf* (mezzo-forte), *f* (forte), *meno f* (diminuendo), and *più f* (crescendo). There are also microtonal markings, including downward-pointing arrows and symbols like $\downarrow \frac{1}{5}$ and $\downarrow \frac{1}{6}$, which indicate downward microtonal departures from normal intonation. The music features complex rhythmic patterns and melodic lines.

Figure 56 – score excerpt from Ligeti’s *Hura lunga* (1994)

6.1.1 USE OF ACCIDENTALS

In a footnote to the first movement, Ligeti states:

*) \downarrow , \downarrow , \downarrow indicate downward microtonal departures from normal intonation: $\downarrow \frac{1}{5}$ about a quarter tone lower as with the 11th harmonic (which is 49 cents lower); $\downarrow \frac{1}{6}$ about a sixth of a tone lower, as in the 7th harmonic (which [sic] is 31 cents lower); \downarrow the very slight deviation (14 cents lower) which is the difference between the major third of the tempered scale and the natural scale. (The harmonics of the C string serve here as a model for the harmonic series F).

With only three symbols an 11-limit tuning system is represented as it is approximated by what is essentially a referential twelfth-tone temperament (72tet), although Ligeti does not mention explicitly that the ‘ \downarrow ’ is close to 1/12th of a semitone.

The \downarrow appears on the pitches ‘B’ below middle ‘C’ (the 11th harmonic in an ‘F’ series), and on ‘F \sharp ’s’ beginning at bar 29 (the 11th harmonic in a ‘C’ series)⁶³. ‘ \downarrow ’ also occurs in the penultimate bar (37) on a ‘B \flat ’⁶⁴. There is no quartertone inflection on the ‘B \sharp ’ in bar 7, but the fact that a natural is used to cancel the ‘ \downarrow ’ suggests that Ligeti had something else in mind here (perhaps the 15th harmonic of a ‘C’ series, although we might then expect ‘ \downarrow ’).

⁶³ Although the notation for the ‘F \sharp ’ in bar 20 may be an error.

⁶⁴ Which again must be an error as the previous five bars are clearly based on a C series—in which case we should expect the ‘ \downarrow ’ indicating a deviation of 31 cents.

The '↓' appears on pitches representing the 7th harmonic of an 'F' series ('E^b'), and a 'C' series ('B^b'). It appears once more in the penultimate bar on an 'A', possibly indicating the 13th partial of a 'C' series but most likely an error as a quartertone flat would more accurately indicate the pitch of this harmonic. (13th partial sounds 40.5 cents sharp from an equal tempered minor sixth).

The '↓' appears consistently on 'A's' on the first page as the 5th harmonic of an 'F' fundamental, and on 'E's' in the series built from 'C'. As mentioned above, it appears in bar 20 on 'F[#]' where a quartertone inflection might seem more appropriate. It also functions in the penultimate bar as the 15th harmonic of a 'C' series ('B'), in which case the ideal intonation is -12 cents.⁶⁵

Although Ligeti is quite clear about the intonation of inflected pitches, he is not clear as to the function of uninflected pitches, except for the roots of each harmonic series ('F' and 'C'). There is at least one occurrence of each chromatic pitch, with the exception of 'E[#]', appearing without inflection, and thus with no clear rational harmonic intent.

I attempt here to analyse the function of each note in order to determine an ideal intonation for the passage. In particular, I am curious as to the function of many of the uninflected tones, and am curious to compare my predictions with the choices (deliberate or intuitive) made by Tabea Zimmermann.

6.1.2 HARMONIC ANALYSIS AND PREDICTIONS

The harmonic intent is strong in the opening phrases of *Hura lunga*. 'C' tonicises the following melody based on an 'F' harmonic series (partial number indicated in parenthesis):

$$C^{(6)} - F^{(8)} - G^{(9)} - A \downarrow^{(10)} - B \downarrow^{(11)} - C^{(12)}$$

Figure 57 – pitches from an 'F' harmonic series used in Ligeti's *Hura lunga*

But 'D', in the turn leading to 'C', is uninflected and unclear as to harmonic function or intent. The most obvious and likely interpretation bases the pitch on the 9th partial of a 'C' series, which occurs as the 27th partial of an 'F' series. This may suggest that uninflected notes are considered from a Pythagorean intonation. Where 'F' is the generating tone, this 'D' should actually sound approximately 6 cents sharp from 12tet. In a 72tet notation system, leaving this pitch uninflected is the most appropriate choice.

⁶⁵ The inflection '↓' may also be missing from the second 'A[#]' in bar 18.

An alternative is that all uninflected notes are simply intended as 12tet pitches, or that intonation is left to the taste of the player.

Similarly, in bar three, an uninflected 'B^b' immediately follows a dotted quarter 'C'. Again, Pythagorean intonation seems the best explanation for this pitch, ideally sounding 2 cents flat from 12tet. As the melody is actually quite traditional, an interpretation of the 'B^b' as the subdominant of 'F' is not out of the question. However, because the line leads to a 'G' it is possible that the 'B^b' functions as a pure minor third (6/5). In this case, we should expect '↑', reflecting an ideal intonation of +20 cents.⁶⁶

At the end of bar 7, an 'E^b ↓' extends the opening phrase upwards into the series to the 14th (7th) partial, skipping the 13th.

In bar 7, the 'B^b' and the 'A^b' are more difficult to reconcile. They are remote pitches in Pythagorean intonation, and seem unlikely to function as such. If the pitches come from a 5-limit system ('B' - a major third above 'G'; 'A^b' - a major third below 'C') then we should expect the inflections ↓ and ↑ respectively. A further possibility for the 'A^b' is that the composer intends to imply the 19th partial of an 'F' series, which falls just 2.5 cents flat of equal temperament, but because Ligeti has chosen not to explore the 13th harmonic, this seems unlikely.

The 'G^b' following the 'A^b' in bar 11 is more perplexing, but is possibly related to 'D'.

Although it is possible that Ligeti intends 12tet intonation for uninflected pitches, a more convincing explanation, which would be consistent with Ligeti's use of microtones in other pieces (see *Double Concerto* analysis—A 7.1), is that he is actually unconcerned with any specific implication for uninflected notes and that the performer is at liberty to find an intonation that works well within the context of the surrounding harmonic/melodic structure.

With this in mind, it may be necessary to reconsider other less reconcilable pitches and present the possibility that Ligeti had no specific intention for those pitches that do not occur within the first 11 harmonics of an 'F' or 'C' series. What emerges is a mix of traditional voice-leading and just intonation.

⁶⁶ A pure 6/5 minor third is 16 cents wider than its equal tempered correlate, but because we are thinking from an 'F' root, 4 cents must be added to account for the Pythagorean 'G'.

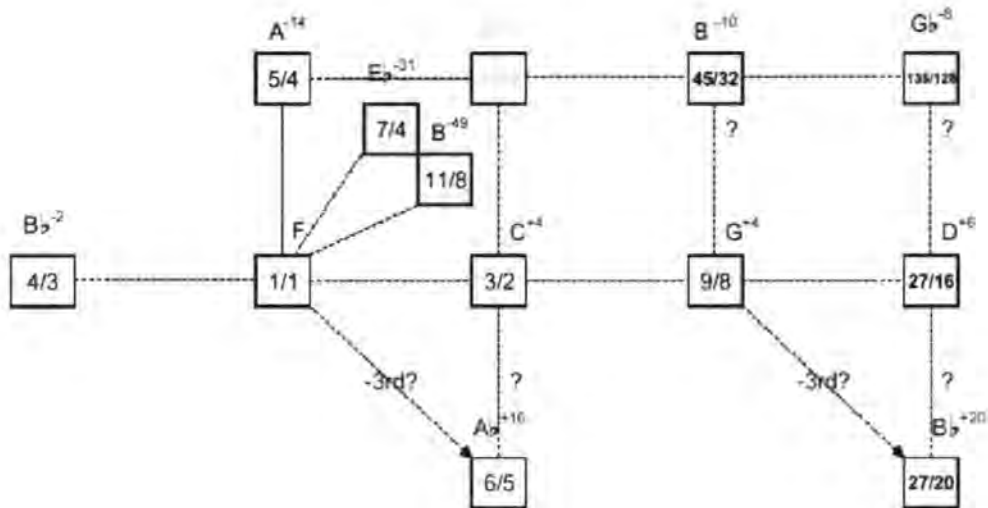


Figure 58 – Speculative harmonic space of bars 1 to 14 of *Hura lunga*

6.1.3 ZIMMERMAN ANALYSIS—EXPRESSIVE VS HARMONIC INTONATION IN JUST INTONATION

The Zimmerman performance of the *Viola Sonate* (here considered ‘definitive’—see above *A6.1*) may be considered a hybrid just/expressive intonation as it deviates noticeably from the tuning indicated in the score; it is influenced by some tendencies of voice leading; and it employs heavy vibrato and expressive pitch bends. For those familiar with just intonation, the deviation from the inflections indicated in the score are clearly audible in the recording, but I feel that the contour of the intonation is somehow maintained in this performance and thus the motivation for microtones convincingly, if not accurately, reflected.

I have generated several examples based on the first eight bars of the Zimmermann recording. Audio Track 50 is the original, Audio Track 51 is a pitch corrected version⁶⁷ where each pitch is adjusted to its equivalent in the harmonic series of ‘F’, and Audio Track 52 presents the original (panned left) and the pitch corrected version (panned right) simultaneously. Audio Track 53 presents the original with an F drone in order to hear the harmonic intervals vertically, and Audio Track 54 is the pitch corrected version with the F drone.

⁶⁷

Pitch correction software is Antares Audio Technologies Auto-Tune 3 DirectX

AUDIO TRACK 50 – EXCERPT OF RECORDING OF LIGETI'S "HURA LUNGA" – PERFORMED BY TABEA ZIMMERMAN

AUDIO TRACK 51 – PITCH CORRECTED RECORDING OF LIGETI'S "HURA LUNGA" – PERFORMED BY TABEA ZIMMERMAN

AUDIO TRACK 52 – LEFT CHANNEL (ORIGINAL) – RIGHT CHANNEL (PITCH CORRECTED) COMPARISON OF LIGETI'S "HURA LUNGA" – PERFORMED BY TABEA ZIMMERMAN

AUDIO TRACK 53 – ORIGINAL RECORDING WITH F DRONE OF LIGETI'S "HURA LUNGA" – PERFORMED BY TABEA ZIMMERMAN

AUDIO TRACK 54 – PITCH CORRECTED RECORDING WITH F DRONE OF LIGETI'S "HURA LUNGA" – PERFORMED BY TABEA ZIMMERMAN

The example in Audio Track 52 is most revealing. Here, most simultaneous pitches simply create a mild chorusing effect due to slight deviations in tuning, and in a melodic passage such as this, I think these differences are rather inconsequential as without vertical relations any lack of harmonic fusion is reduced between adjacent tones.

However, deviations in the 'B \downarrow ' and the 'E \flat \downarrow ' are far more obvious. The mistuning of the 'B \downarrow 's' in the original version should be obvious to the expert listener. This pitch occurs as part of a tetrachord scale run from 'F' to 'C' and, in a traditional vocabulary, as the leading tone of 'C'.

As a leading tone, we might expect a slight sharpening of the pitch as it leads to 'C'. Combined with the ' \downarrow ', the mixed datum present a psychologically contradictory situation which asks the performer to simultaneously raise (as a conditioned voice leading response) and lower (as a conscious response to the inflection) the pitch. The result here is that the pitch is indeed lower than 12tet (remarkably consistent at -14 cents), but actually a further 1/12th tone away from what might be expected in referential 12tet where the leading tone is likely to be on the order of a 1/12th tone sharp. Perceptually, this perhaps means that we hear a deviation of approximately 1/6th tone (33.3 cents) rather than only -14 cents. Although likely only coincidental here, a B^{-14 cents} is extremely close to the 5-limit major seventh 15/8 in the key of 'C'— strict advocates of just intonation often claim that this is the 'correct' intonation for this pitch in a traditional harmonic/melodic language.

In the scalar passages, 'B \downarrow ' often sounds between -21 cents and -28 cents from 12tet. And, significantly, the following 'C' often sounds between +16 cents and +22 cents, which means that although both are significantly out-of-tune from the score, the interval size between these adjacent tones is, in many cases, remarkably close to the indicated 150 cent minor second interval.

The other pitch that is noticeably out-of-tune from the pitch corrected version is the 'E \flat \downarrow '. In the first two occurrences of the original version, this pitch sounds at +5 cents. Adjacent pitches do not maintain the proportion of the intended interval, and it is perhaps only with this pitch that we can say that the intention of the score is not convincingly represented. The ' \downarrow ' indicates a 33.3 cents

microtonal inflection. The inaccurate intonation of this interval by Zimmermann may suggest that the $1/6^{\text{th}}$ of a tone sits between some awkward threshold of conscious intonation. Where a twelfth-tone (16.6 cents) requires little conscious adjustment (it is about the smallest adjustment a player can make) and the quartertone requires a substantial, or at least easily relatable adjustment, the sixth-tone may lie awkwardly between these two states, where the adjustment is slightly larger than ‘as small as possible’ and slightly smaller than the more easily measurable quartertone (on a fingerboard, the player can physically see or feel a quartertone as half the size of a semitone).

The analysis also reveals the important influence of bow pressure on intonation. Each phrase begins with the open C-string, but while the average frequency of the pitch is about 262Hz, the variation in pitch spans 260Hz to 264Hz, a bandwidth of about 25 cents—this despite the removal of vibrato and finger position.

While some just intonation composers might demand a more accurate overall performance, Ligeti’s involvement in the recording of this performance must suggest that Zimmermann’s intonation satisfies the intentions of the piece.

6.2.1 BEN JOHNSTON’S SYSTEM OF ACCIDENTALS

The notation in the music of Ben Johnston contains a high level of self-analysis; it is perhaps maximally high concerning harmonic/melodic intent. Through the development of an elegant, although extensive system of accidentals, Ben Johnston’s scores leave no question as to harmonic/melodic intent for the informed reader.

Johnston uses traditional diatonic notation as the basis of his notational system by using the standard system of accidentals, which have embedded in them intonational cues that are easily adapted into a 5-limit just intonation systems.

In Johnston’s system of accidentals, uninflected pitches refer to a 5-limit diatonic pitch space based on C major:

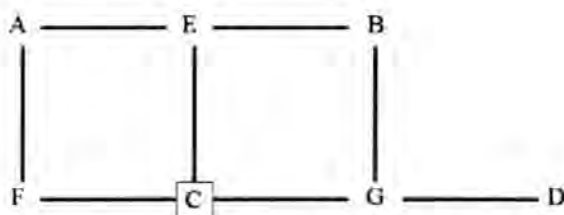


Figure 59 – 5-limit major scale in harmonic space, which is the basis of Ben Johnston’s system of extended just intonation accidentals

A '+' or '-' raises or lowers a pitch by a syntonic comma—approximately 22 cents—which is the difference between a 9/8 major second and a 10/9 major second (or the difference in the size of step between the first and second scale degrees and the second and third scale degrees of a just intonation major scale)

A '#' raises any pitch by approximately 70 cents and a 'b' lowers the pitch by 70 cents—which is the size of the just intonation diatonic half step (25/24)

By combining these accidentals, any extended 5-limit just intonation system can be represented while maintaining the pure tuning required of the system:

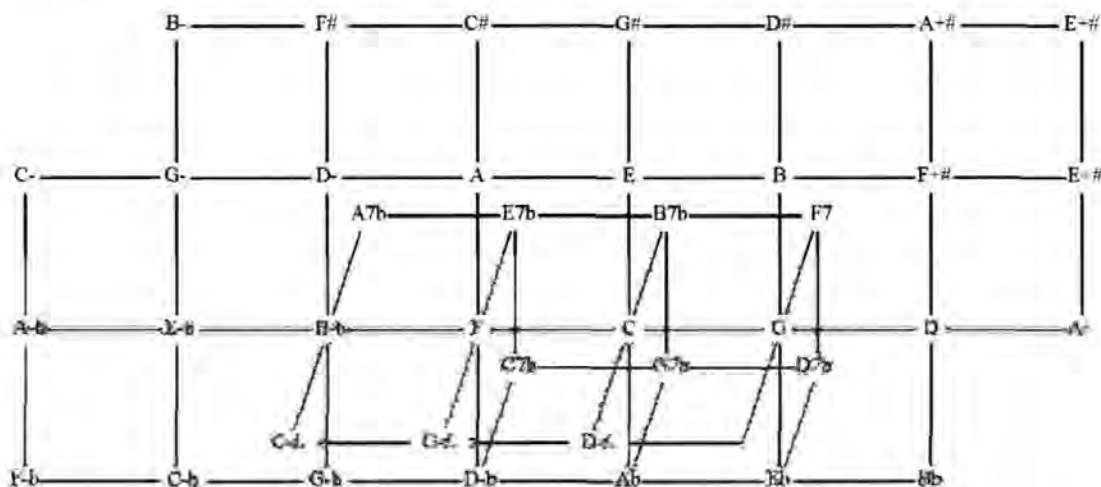


Figure 60 – an extended harmonic space using Ben Johnston's system of accidentals

To look at $A\flat$ for example, the flat indicates that the pitch is to be lowered by 70 cents from the uninflected A (5/3), which is by default 16 cents flat, which makes the $A\flat$ +14 cents from 12tet—a pure 8/5 in C.

To show higher limit systems, Johnston introduces a new symbol for each new prime number. The symbol '7', for the 7-limit, represents a shift of -49 cents, the *septimal chroma*, which is the size of the interval 36/35, or the difference between a 5-limit minor seventh (9/5) and a 7-limit minor seventh (7/4) [$9/5 \div 7/4 = 9/5 \times 4/7 = 36/35$]. An inverted 7 – 'L' indicates a shift of +49 cents.

These accidentals may be combined in various ways. For example, to indicate the 7/4 minor seventh from a root C, the '7' accidental must be combined with a 'b':

'B' (-12cents) 'b' (-70 cents) '7' (-49 cents) = B (-131 cents) or $B\flat^7$

An '↓' and '↑' indicate shifts of 53 cents for the 11-limit and the system continues as

necessary up the sequence of prime numbers simply using the limit number symbol and its inversion as the accidental.

Johnston's system is theoretically elegant, but can be extremely complex to read and execute in some of his scores. Where there is a small and limited set of pitches, the problems are not particularly daunting, but Johnston uses this system of accidentals for extended just intonation systems that must be a huge challenge to the player.

A 6.2 BEN JOHNSTON – *STRING QUARTET NO.4 (1973)*

Ben Johnston's *String Quartet No.4* is a theme and variations on the hymn *Amazing Grace*, based on (among other compositional devices) an expanding just intonation tuning systems. The opening section is in Pythagorean intonation, followed by the 5-limit, then the 7-limit. Each instrument is retuned according to figure 61:



Figure 61 – open string tuning for Ben Johnston's *String Quartet no. 4*

While the following analysis will look at the intonation in a recording by the Kronos Quartet, the Kepler Quartet have taken on the daunting task of recording all of the Johnston string quartets, and I've had access to some of the correspondence the Kepler Quartet has had regarding the challenges involved in interpreting the Johnston scores, which will inform some of my analysis.

One of the problems the Kepler Quartet initially had with the Johnston notation is the confusion that arises from Johnston's system being based on a 5-limit C major scale while the strings of a quartet are based on a Pythagorean intonation in C (C⁻ – G⁻ – D⁻ – A – E). In an email from Brek Renzelman (vln) to Andy Stefik (who was hired to help the quartet with interpreting the scores), Brek notes that while Johnston's 1/1 is C, the string quartet is based on A roughly 440Hz (1/1). This requires that A is tuned to -16 cents.

This creates one further problem, which is that the resonance of the instruments are affected when they are retuned to C

"It changes our instruments' basic tonal qualities and their technical responsiveness too drastically from the way they were built to sound, causing challenges for the left-hand orientation" (email from Andy Stefik to Brek Renzelman. 28 January 2003)

A6.2.1 KRONOSQUARTET RECORDING OF BEN JOHNSTON STRING QUARTET NO. 4

The following analyses the Kronos Quartet’s intonation of the first four bars of page 6 of Ben Johnston’s *String Quartet No. 4*.

The first instance of 7-limit intervals in Johnston’s *String Quartet No. 4* occurs at the top of page 6. The pitch set for the first four bars is laid out in harmonic space in Fig 64.

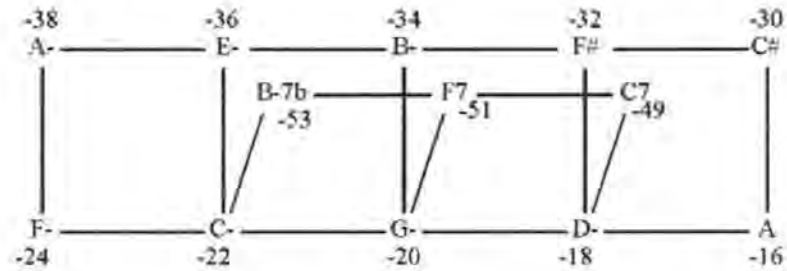


Figure 62 –pitch set from page 6 of Ben Johnston’s *String Quartet No.4* in harmonic space

AUDIO TRACK 55 – EXCERPT FROM PAGE 6 OF BEN JOHNSTON’S STRING QUARTET NO. 4 – PERFORMED BY THE KRONOS QUARTET

It seems safe to say that the ensemble has chosen to ignore the ‘ - ’ inflection, which would otherwise shift the entire passage down by 22 cents in relation to a 12tet C. Adjusting the tuning reference in this manner seems an appropriate and intelligent choice that lessens the problems of instrument resonance (mentioned above) without interfering with the intervallic content of the piece (the intonation remains relative).

However, the average intonation throughout the passage is further sharp, with both the inflected ‘ - ’ and uninflected A’s sounding between 8 and 16 cents sharp of A = 440Hz. The 7-limit pitches are variously intonated, most often played sharp rather than flat.

The complexity of Johnston’s notation, I feel, is a great stumbling block here. For example, in order to decode the required intonation for B - $\frac{7}{b}$, the player first must process that an uninflected B sounds -12 cents from 12tet in Johnston’s system, then adjust by -22 cents for the ‘ - ’, -70 cents for $\frac{b}{b}$, and -49 cents for ‘7’. Furthermore, this is one of the sparser moments in the piece, and the source of the 7-limit intervals becomes increasingly complex as the piece progresses.

Objectively, the intonation on the Kronos recording bares little resemblance to the suggested intonation in the score. However, that is not to say that the performance necessarily sounds poorly

intonated – at least some of the aesthetic harmonic qualities implied by the notation are maintained even if they might not match Johnston’s usual strict expectations⁶⁸.

See Appendix 5 for details of the intonation of this passage.

⁶⁸ I have no information regarding Ben Johnston’s opinions of this recording, only the knowledge that he is dedicated to the pure intonation of just intervals.

7 PITCH CLUSTERS AND SOUND MASSES

In the physical world, frequency, time, and intensity are considered as continuous dimensions. Music, on the other hand, has been built on discrete scales of pitch and duration made necessary, among many other reasons, by the desire to notate events and by instrumental playing constraints. The different representations [of frequency analysis]...associated with sound synthesis, give us access to the physical continua. This allows us, as Varèse noted, to catch a glimpse of an alternative to the 'fil à couper l'octave'. From this point onward, between each degree of the scale there can be a continuous thus infinite world to be discovered and organised.

Another continuum is revealed by these mathematical representations. What difference is there between the spectrum of a note associated with a timbre and the spectrum of a chord considered as an element of harmony? The answer is to be found on the computer screen: at first sight there isn't any! A simple note is a collection of partials, thus a timbre. Sound synthesis allows the organization of the note itself, introducing harmony into timbre, and reciprocally sound analysis can introduce timbre as a generator of harmony (Pressnitzer and McAdams 2000: 39).

Pitch clusters represent a connecting point between harmony and noise, and connect the two extremes as endpoints on a continuum. Treating pitch clusters as harmony is possible to a certain degree, but analysing them from a harmonic standpoint is generally complex (depending on the density of the chord and the magnitude of the component intervals) whether that is in harmonic space, subharmonic coincidence, or spectral analysis.

A7 PITCH CLUSTERS IN ANALYSIS

A 7.1 MICROTONAL PERFORMANCE IN LIGETI'S *DOUBLE CONCERTO FOR FLUTE, OBOE AND ORCHESTRA*, 1972.

An approach that suggests an intermediate step between equal divisions of the octave and just intonation uses loosely defined microtones to various ends, an example of which is Ligeti's Double Concerto for Flute, Oboe and Orchestra, composed in 1972.



Figure 63 – Double Concerto Clusters

In the score instructions, the composer notes that the microtone accidentals are instructions to lower or raise the intonation of the respective pitch by no more than a quartertone, although he does not demand precise quartertones. Ligeti speaks of creating “dirty patches” which result from blurring the tempered chromatic scale with microtonal inflections.

In the first movement of the Double Concerto, Ligeti presents an expanding cluster of pitches that also increase in vertical density. In the score, it is apparent that the composer is working with quasi-symmetrical structures that are further divided by microtonal intervals. The resultant harmonic sonorities have no easily analysable configuration in harmonic space due to the complexity of the structures, however Ligeti still expects that there is a ‘good’ note between the equal tempered ones and that the musician should find a tuning that works. In the recording, the tuning is not nearly as rough as one might expect from such dense harmony. The intonation of both the microtonal pitches and the referential 12tet pitches creates sonorities of relatively high fusion considering the extreme harmonic complexity.

BERLIN PHILHARMONICA INTONATION

AUDIO TRACK 56 – OPENING SECTION OF LIGETI’S DOUBLE CONCERTO – PERFORMED BY THE BERLIN PHILHARMONICA

time:	23.96	25.17	29.07	32.32	35.57	38.82	45.88	48.39	54.15	1:02.04	1:10.22	1:16.72	variation	range
nearest 12tet pitch	in Hz												in Cents	
A#												470	n/a	+17
A										441	436	435	24 cents	-20 to +4
G#										419	419		0	+15
G										394	401	397	30 cents	+9 to +39
F#		371	371	369	370	369		369	369	369			9 cents	-5 to +5
F	351	351	345	347	350	353	350	351		350	351	354	35 cents	-21 to +23
E		330	327	324	324	331	335	332	326	329	331		58 cents	-30 to +28
D#	307	307	309	311	310	312					310	311	28 cents	-23 to +5
D				(291)	296	296	296	295	296	295		295	6 cents	+8 to +14
C#	276	276	280	279	276	276	276	277	277		279	277	25 cents	-7 to +17
C											264	260	26 cents	-11 to +16

Table 9 – selected chord intonation at various time points in Berlin Philharmonic recording of Ligeti’s Double Concerto for Flute and Oboe

In the recording by the Berlin Philharmonic, certain pitches act as anchors for the surrounding pitches. The intonation of the pitches D and F# are the most stable tones throughout and are generally close to a just major third (5/4). C# is also relatively stable, tending toward a slightly flat minor seventh (relative to 12tet). Other pitches vary more so, but in such ways that seem to minimise roughness.

SCHOENBERG ENSEMBLE INTONATION

AUDIO TRACK 57 – OPENING SECTION OF LIGETI’S DOUBLE CONCERTO – PERFORMED BY THE SCHOENBERG ENSEMBLE

time:	03.25	04.46	17.74	20.81	24.24	27.86	30.19	32.42	34.46	40.77	43.65	45.60	51.83	57.59	1:06.69	1:12.91	variation	range
nearest 12tet pitch																		
in Hz																		
A#																		
A																		
G#																	428	
G																	389	
F#					370	370	370	371	371	371	371	371	370	373	369			
F	350	351	349		346	347	351	349	347	351	351	350	350	351				352
E				331		328	328	328	328	328	331	331	330	332	330			
D#	311	312	312	314	314	308	310	310	311	312	314	313						311
D										293	293	293	295	295	299			
C#	275	276	276	277	276	275	276	276	278	279	276	276	279	279	277	277		
C																		

Table 10 – selected chord intonation at various time points in Schoenberg Ensemble recording of Ligeti’s Double Concerto for Flute and Oboe

In the beginning of the Schoenberg Ensemble performance the three-note cluster shifts upward slightly but maintain a proportional intonation (a wide major second and major third). Even though this is not a common intonation, it somehow sounds harmonic.

Overall, there is a more subtle intonation based around 12tet in this performance. The influence of 12tet is very strong but the intonation suggests some psychoacoustical adjustment, even if subconscious.

A 7.2 JAMES TENNEY’S CRITICAL BAND, 1988/2000.

In the ensemble piece *Critical Band* by James Tenney, the composer requests precise microtonal deviations from equal temperament that are indicated in the score with cents values above each note head. The piece begins with a single pitch A = 440Hz and gradually expands toward a sounding of the first eight harmonics of a fundamental A = 110Hz.

While the method of notation may seem the exact opposite of Ligeti’s *Double Concerto*, the two approaches are actually not so disparate. When Tenney indicates precise intonation by means of cents deviation, he is not necessarily expecting that the performer will be capable of such precision but that the notation will push her/him towards the correct intonation. Indeed, Tenney expects and allows for deviations of up to five cents in many of his pieces. This tolerance range allows for satisfactory intonation and interval class identification for most ratios, the main exceptions being octaves, unisons, perfect fifths and fourths (which are likely to be tuned more accurately anyway in dynamic intonation; and in the tempered systems favoured by Tenney, these intervals are highly

refined). Although the precise notation represents an ideal toward which the performer must strive, Tenney also trusts that having been pushed in the right direction, the performers' ears will guide them to an ideal, or close to ideal, intonation; that is, when the harmonic situation is relatively clear.

The early stages of *Critical Band* involve complex harmonic relationships situated within a critical band. Theoretically, within this band, the individual elements of a sonority cannot be isolated, that is, we cannot hear two or more pitches as separate entities if they all fall within a bandwidth less than a quarter of one critical band (the critical band around A = 440Hz is approximately a minor third); and tones that are separated by more than approximately a quarter of one critical band are perceived as *rough* (maximally roughness occurs at approximately one quarter of a critical band) (Plomp 1976). In the second section of the piece, due to the harmonic complexity and proximity of the pitches, the musicians must rely on tuning clues other than harmonic smoothness or fusion. In this case, while measuring the beating that should occur between components is a possibility, the player might also simply make a minor adjustment to her/his embouchure or fingering. As I stated earlier, the complexity of a harmonic relationship is inversely proportional to the refinement of the tuning. In this case, the complexity is extreme, and it is unlikely that anyone would be capable of perceiving the second and third sections harmonically. Therefore, an approach that relies on kinaesthetic adjustment may be more appropriate and successful in these situations.



Figure 64 – Beginning of James Tenney's *Critical Band*

THOUGHTS, OBSERVATIONS, HYPOTHESES, AND CONCLUSIONS

My intentions in pursuing this subject are not necessarily to uncover a clear set of conclusions. Rather, my interest is to expose the complexity of the subject and to demonstrate that results are contextually specific. That is, there is very little that can be said conclusively about intonation and harmonic perception that is not context specific.

THREE QUESTIONS

In the Introduction, I posed three questions:

DO THE COMPOSER'S INTENTIONS MATCH WHAT IS IMPLIED IN THE SCORE?

In general, most of the chosen scores demonstrate a high level of 'intention'. Either the intentions of the composer can be deduced from the score, or the score reflects the harmonic aesthetic of the composer (which may be tied to ambiguity).

However, what should be given special attention is the level of self-analysis the composer wishes to include in the notation—with the understanding that generally, a high level of self-analysis is likely correlated to harder work for the performer (but in the long-run, for the player willing to put in the work, a more profound understanding of the harmonic materials may emerge), and alternatively, the use of standardised models or the minimal adaptation of 12tet notation may speed up the rehearsal process but with a lower level of understanding (and perhaps a less accurate performance, though little evidence of this emerged in the analyses).

TO WHAT EXTENT DOES INTONATION IN THE PERFORMANCE REFLECT THE INTENTIONS OF THE COMPOSER (AND WHAT DOES THIS SAY ABOUT HARMONIC CONSTRUCTION OR CONCEPT)?

The answer to this question is perhaps the most significant hypothesis of the paper. It seems to me that the greater the harmonic agenda and the more refined the notation system, the less the intentions of the composer are satisfied. It is a bit ironic that the pieces based in referential 12tet are performed more 'microtonally' than the explicitly microtonal compositions.

In the explicitly microtonal scores, the degree of precision indicated in the score is not directly proportional to the accuracy of intonation in the performance. In fact, in some circumstances, composers achieve higher intonational precision from a rough inflection system, whereas a more highly refined notation does not ensure a corresponding level of

precision in performance.

There appears to be a conflict between the level of self-analysis embedded in the score and the ease of decipherability for the performer. However, I do not believe that a standardised system of microtonal inflections is the answer.

Gardner Read's *20th-Century Microtonal Notation* details the development of notation for extended tuning systems in an attempt to establish a definitive system of microtonal notation.

Read organises his analysis based according to *interval size*. While this is a logical means for investigation, an alternative would organise an analysis based on the *function* of microtonal inflection. For example, Johnston is interested in the explicit expression of just ratios, Tenney uses variously refined referential notations to push the performer toward just intonation, whereas Ligeti is often simply interested in indicating intervals smaller than a semitone, and Ives blurs the perception of otherwise familiar tonal material.

Although quartertones may be emerging as a new base system, this may not be best way forward.

It is my feeling that notation is a compositional parameter and must be addressed as best fits both the intentions of the composer and the needs of the performer. An approach informed by the physiological, psychological, acoustical properties of harmony and knowledge of well-established systems is necessary for efficient realisations of musical works containing considerable intonation demands.

The composer must weigh the conflicting values of self-analysis versus efficient decipherment for the performer.

Tenney says:

One of the exciting things about being involved with new tuning systems is that it is relatively new, again. And, although there are more and more people working in this area, it's still in that experimental stage. It's not stuck, it hasn't become rigid. And I really am glad of that. I'm also glad that nobody has found a perfect notation, because then I'd feel compelled to use it. I'm glad Ben [Johnston]'s got his, which does a certain thing, and does it beautifully. I've used several, depending on the nature of the piece. It's an exciting situation, because these are notations that are innate to the composers, but they're not just arbitrary personal signatures. Each of us has thought through the problems and worked out a solution that seems to work for us. And each accomplishes certain things, maybe better than another's because you have developed it for

your own work (Belet 1987: 461)

**TO WHAT EXTENT IS THE LISTENER ABLE TO UNDERSTAND OR RATIONALISE WHAT S/HE HEARS HARMONICALLY
(AND HOW DOES S/HE RATIONALISE IT)?**

This broad question cannot really be answered. Or, it can be answered by saying: “objectively, not a lot” and “subjectively, a lot”.

Objectively, the results of the spectral analyses show that it is in fact rare that the performance reflects the harmonic/melodic intentions of the piece to any moderate to high degree of accuracy (relative to the intentions of the composer). There are too many other factors interfering.

Subjectively, the measured intonational inaccuracies do not seem to affect significantly the listening experience. The intonation often sounds ‘good’, and demonstrates that ‘smooth’ intonation need not only arise from a rational and simple harmonic structure.

Despite the objective measures of intonation, the pieces generally *sound* as expected; the referential 12tet recordings don’t actually *sound* microtonal, and the microtonal recordings subjectively seem to reflect the intentions of the composer. This perhaps reveals that a number of other factors influence intonation, and that microtonal inflection responds to an ideal or reference other than 12tet. For example, perhaps voice-leading expectations push the ideal intonation in a direction opposite to the inflection indicated in the score, therefore requiring a smaller inflection than objectively described.

The broader question raised at the beginning of this paper—what can be heard as harmonic?—can only be vaguely answered. And just as consonance and dissonance are relative terms along a continuum, perhaps what can be heard as harmonic is also a relative state between harmonicity and inharmonicity (the furthest extremes being pure tone and noise). The experience for the listener is conditional upon a number of factors including experience, conditioning, and expectancy.

TOWARDS A GENERAL THEORY OF HARMONY

In the Introduction, I made reference to James Tenney’s desire for a general theory of harmony. While this paper on one level aspires to the consideration for an expansion of the textbook notion of ‘harmony’, what it demonstrates is that the issues are incredibly complex to unravel. For the time being, what might be necessary is that we need a growing body of *alternative* theories of harmony, ones that follow certain historical/cultural

trajectories. We might imagine one inspired by Debussy, that follows through Messiaen to the French spectral composers; another inspired by Edgard Varèse that is connected to the inharmonicity of percussion sounds and noise, and connects harmony to timbre; a third and fourth might follow the legacy of Harry Partch—one inspired by speech and corporeality, another inspired by just intonation (including Ben Johnston, Lou Harrison, and LaMonte Young); etc.

A NUMBER OF CONCEPTS WHICH AFFECT INTONATION AND HARMONIC PERCEPTION

One further, somewhat unrelated observation I would like to make is that in the sampling of recordings presented in this paper there seems to be a historical trend in intonation. The degree of expressive intonation seems much greater in the older recordings, and the influence of 12tet greater in the more recent recordings. Perhaps we are losing an important element of musical style and expressive music making by limiting our expectations to either the precise and explicit execution of microtonal intervals, and/or to the concept of 12tet as an 'ideal' state rather than as a referential system for intonation (and this would extend to quartertones as well).

To demonstrate the complexity of harmonic listening, I offer a parting list of some, but certainly not all, of the issues that affect intonation and harmonic perception, and requiring acknowledgement in a speculative general theory of harmony (in no particular order).

Inharmonicity of strings (pianos for example), consonance and dissonance is dependent on timbre, avoidance of fusion to increase stream segregation between melodic parts, just noticeable difference (JND) of frequency is dependent on context, balance, sensory consonance, fusion, critical band, subjective tones (combination and difference), resolution of the missing fundamental (root detection), historical concepts of consonance and dissonance, register, voicing, chord spacing, orchestration (tied to timbre), size of melodic intervals, psychoacoustical stretching and shrinking of intervals, voice-leading tendencies, harmonic 'direction' (root progressions in harmonic space), harmonic relatedness (in harmonic space), the number of distinct pitch classes in a chord or chord complexity (dyads, three-part chords, four-part, etc.), critical band effects on roughness and relative complexity, 'zonality' (see Ozzard-Low 1998), conditioning, register (and its affect on critical band), 'limit'-number of the harmonic system, size of the system, familiarity with the sound world of the actual piece, harmonic/melodic expectation,

refinement of tuning system (e.t., J.I., etc.), precedence (both culturally, stylistically, and internal to the piece), pitch density, timbral richness, resolution of the tuning system, pitch duration...

SECTION II COMPOSITIONS

INTRODUCTION

If I could say one thing about my creative goal, it is that I seek to embrace the strange—to find the beautiful in the strange and the strange in the beautiful.

I also consider the act of composition essentially a problem-solving task. And the problems are unique to each composition, and function from large-scale structural problems to the smallest level of detail.

The impulse for many of the compositions here begins with a question or anomaly of perception, and most often a question of harmonic perception. The pieces explode and explore oddities and quandaries that arise through my psychoacoustical research.

In my mind, one of the main objectives in all of the compositional work presented here is to ask the listener to reconsider their listening habits—to listen for elements that may otherwise be ignored or obscured in traditional styles. All of the pieces begin with a perceptual question—a “what happens if”? And I attempt to conceptualise the process into a form that I consider musical—something I *want* to listen to on an aesthetic level, rather than as mere curiosity; each piece should not simply be justified by the fact that it is the result of a process, but that some other aesthetic concerns allows me to engage with it on an musical level.

A chronological progression of development, observed in retrospect, has also informed my practice. My approach has shifted from strict process oriented pieces; to pieces in which the process is confounded by secondary processes; then to pieces where the process is broken intuitively; and finally to where some of the details of the process are deduced intuitively—that is, instead of using strict calculations for parametric values I allow my intuitive understanding of the process to deduce some of the details (usually following some experimentation within a strict process).

The evolution of this approach, I feel, better mimics my understanding and interpretation of the world around me. Nature, while explained partially by codes and laws, is continually confounded by many perturbations. Aberrations on small levels can have profound affects on the development of many systems. For example, while fractal models may be capable of simulating the contours of mountains, trees, and clouds, there is a level of perfection in these models that seems removed from our day-to-day experience

of systems occurring in nature.

I believe that the reflection of nature in art should not be a measure of artistic integrity. But it may be a useful means for discussing art—the level to which a piece of art aspires to the reflection of nature may be a useful analytical tool. For my music, I am variously interested in reflecting some properties of nature but also confounding them. Nature is simply the model to which we compare, but is not a justifying feature.

I also like oddities. And simple processes ensure that oddities do not arise, except where consideration is given for states that are sensitive to small-number variability (chaotic systems).

I attempt to use the terms 'analogy' and 'metaphor' selectively. In order to consider two concepts as analogous, I believe that some basic perceptual function must be maintained between the source and the musical result. Transferring scientific data arbitrarily to musical pitch (for example) holds no analogous meaning, nor does transferring the physical properties of an object to pitch contour, dynamics, etc. The physical rarely convincingly transfers to the temporal—we simply don't see the way we hear.

A common 'analogy' made in spectral and just intonation music is that timbre and rhythm are analogous because in both, numerical simplicity is equated with perceptual comprehensibility. But the idea that there is a perceptual similarity between 3 over 2 rhythms and a 3/2 perfect fifth seems a stretch to me, except on the level of metaphor. This is not to say that meaningful musical material cannot be derived by mathematical functions which govern at both levels, but that justifying a system based on the physical connection between rhythm, harmony and timbre requires a large perceptual stretch of the imagination—except where attention is drawn to the thresholds between these states and the listener is thus helped to make the perceptual leap. Otherwise, the connection between rhythm and harmony is not maintained perceptually because the mode of reception engages two different psychoacoustic hearing mechanisms

In my music, I often work at two levels, one of analogy where I am exploring some anomaly or curiosity of acoustics or psychoacoustics, and one on the level of metaphor, which more often informs structural attributes and time varying functions.

MY BACKGROUND AND INFLUENCES

ROCK AND JAZZ

My early influences came through the rock world. In particular, I was drawn to the harmonic approaches of Andy Summers (The Police) and Jimi Hendrix: Summers' use of suspended chords, and Hendrix's use of extended and altered dominants. Robert Fripp⁸⁹ and Mark Ribot inspired my interest in playing 'outside' and the use of angular melodic phrases. I also became more aware of the minimalist school through Fripp and Summers' connection to Brian Eno, and through art-rock bands such as Talking Heads, which led me to Philip Glass, Charles Ives, and Edgard Varèse—which was the extent of my knowledge of 20th century art music in my teenage years.

After touring and recording from 1990 to 1992 as a guitarist with King Apparatus, a Canadian ska band, I felt a need for an informed approach to playing outside and extending my knowledge of harmony. I thought that jazz might provide that source. I became interested in learning jazz more as an intellectual pursuit rather than a genuine love for the music, although this subsequently (and temporarily) developed when I began jazz guitar and composition studies at Humber College in Toronto.

A few names and concepts popped up during my jazz studies that provoked me to look into the academic influences in jazz. George Russell's *Lydian Chromatic Concept of Tonal Organisation*; references to the *Lydian flat seven* mode as the 'natural harmonic scale' (derived from the harmonic series); John Coltrane's use of Nicolas Slonimsky's *Thesaurus of Scales and Melodic Patterns*; Ritchie Beirach's polychordal harmony; and Ornette Coleman's *The Shape of Jazz to Come* was influential to my interest in heterophonic and contrapuntal textures that freely mixed perfect and imperfect consonances, and dissonant intervals (variously taught to me as '20th century counterpoint' and 'dissonant counterpoint'—although I did not learn about Carl Ruggles, Charles Seeger, and Ruth Crawford-Seeger until I met James Tenney at York University in 1996).

⁸⁹ Particularly Fripp's playing on David Bowie's *Scary Monsters* LP, and in his foray into pop and punk with his band The League of Gentlemen.

In reading Brian Belet's dissertation *An Examination of the Theories and Compositions of James Tenney, 1982-1985* (I studied with Tenney from 1996-2000), I was surprised to see how much of Tenney's influence remains in my musical thinking. One of my goals in coming to Dartington was to loosen my compositional approach from strict processes—an approach I attributed directly to the influence of Tenney—in order to allow the intuitive back into the compositional stage. I am not critical of an algorithmic approach, but I recognised in my own work a lack of sophistication that stemmed from a lack of finesse in physics, logic, and mathematics; I did not feel that I could compose the music I wanted to hear through this approach. But I mistakenly assumed that the process piece was the extent of James Tenney's influence.

What remains are some similarities in attitude. Some of these similarities are indeed a result of influence, but I think these concurrences say more about why I was drawn to the work of Tenney, and to studying with him, rather than about me adopting a portion of Tenney's set of aesthetic values.

One quote in particular struck me as a connection I had forgotten:

I think we're all phenomenologists. The basic idea in phenomenology is making a more strenuous effort to see things as they are, depending on whatever one is focusing on. I think the best scientists and the best artists are precisely that—phenomenologists. We want to know, what is it ... what is it really (Belet 1990: 6)

I read this passage not long after a conversation in Oslo with artist Andy Smith and cellist Maja Bugge where I said something to the effect that "I think we're all systems analysts and system builders, whether or not we do this consciously, or unconsciously, rationally or intuitively; and this is not particular to the artist or to the scientist".

The most obvious idea that Tenney introduced to me was the concept of the harmonic series as a musical resource. In September of 2004, I had the opportunity to play on Glenn Branca's recording of his *Symphony no. 13 'Hallucination City'* for 100 electric guitars in New York. Branca's infatuation with the harmonic series is similar to my own, and the following Branca quote expresses well what I feel James Tenney instilled in my own thought:

I was interested in what the series had to tell me about structure itself, because it is an organic structure which somehow emerges as a natural system, but at the same time contains all kinds of incredibly interesting structural ideas and patterns... Humans love to find patterns in everything. For me, it was like opening up a treasure chest

and seeing what was inside it...I had never been interested in mathematics before, but the harmonic series taught me what it was, and I realised that this was the deepest philosophy, the seed of all philosophy. This simplest system that we all learn when we're children, contains complexity beyond our comprehension at this point...You see, it's a non-linear system, and because this system is also the vibration of a string, within the vibration of a string is the entire harmonic series...It's fabulous stuff (Duguid 2005)

But the most musically important influence Tenney had on me is the emphasis on large-scale structure, which for him must be elegant, justified, or a logical extension of the musical material. This is something that always concerns me, and is a problem that I have not fully resolved (or expect to ever fully-resolve)—in fact, I am interested in forms that may not seem elegant on the surface or logically perceived, but must still emerge from the musical material and must at least *feel* right when all is done⁷⁰. With strict process pieces, the large-scale form may emerge as a foregone conclusion, but with compounded processes and intuitive interventions, the problem is more difficult for me to resolve. In particular, my interests in applying the concept of *family resemblances* (see below) has not led to an obvious approach to large-scale structure, and is something I will need to pursue further in the future.

⁷⁰ As the reverse of this problem, I am also inspired by the structural concepts of the music of Horatio Radelescu, which are often derived by a number of nested Fibonacci relations, and to me feel boxy and awkward in a uniquely satisfying way.

I. I DREW A LINE IN THE SAND, AND IT GOES FROM HERE TO THERE...

For flute, clarinet, vibraphone, piano, violin, and cello

I drew a line, and it goes from here to there... was composed for the Black Hair ensemble, and was performed during their residency at Dartington College of Arts at a workshop performance of student compositions on 03 March 2002.

STRUCTURE

For *I drew a line, and it goes from here to there...* (from here forward simply *I drew a line...*), I had a concept for an extended just-tuned piano that would also inform the structure of the piece. The crucial problem with retuning keyboards is the limited availability of unique pitches (twelve per octave). I used this problem as an organising principle for the piece (*physicality*).

I began by generating a series of scales from the 22-interval relations found within the sixth to twelfth partials of a 'G' harmonic series, listed here in declining order of magnitude:

Deviation from 12tet	Closest 12tet	Pitch Class Ratio	Cents
+49	F	11/6	1049
+18		9/5	1018
-31		7/4	969
+33	E	12/7	933
-16		5/3	884
+14	E ^b	8/5	814
-18		11/7	782
+2	D	3/2	702
+17	C [#]	10/7	617
-17		7/5	583
-49		11/8	551
-2	C	4/3	498
+35	B	9/7	435
-14		5/4	386
+47	B ^b	11/9	347
+16		6/5	316
-33		7/6	267
+31	A	8/7	231
+4		9/8	204
-18		10/9	182
-35		11/10	165
0		G	1/1

Table 11 – pitch gamut for *I drew a line...*

The broken symmetry and unevenness of the scale gamut that emerges from just intonation constructions is a further element that I embrace here, and is compounded by my approach to tuning the piano (see below), which is connected to the concept of *lines and measurement* outlined above.

The ratio classes within each scale are related either by prime numerator or by denominator. The first scale is an octave condensed 11-limit overtone scale related by the prime denominator 2 (factors of 2).

$$9/8 - 5/4 - 11/8 - 3/2 - 7/4 \text{ (ascending)}$$

The next scale descends and is generated from 9/8 but related by the prime numerator 3 (factors of 3).

$$9/8 - 9/5 - 12/7 - 3/2 - 9/7 - 6/5$$

The next scale is built from the second degree of the previous scale and alternates ascending (related by denominator) – descending (related by numerator).

Next scale is, ascending: $9/5 - 11/10 - 6/5 - 7/5 - 8/5$

Then, descending: $11/10 - 11/6 - 11/7 - 11/8 - 11/9$

Etc.

The piano tuning gives preference to the earliest generated scales. If a piano key for a particular scale is not available (i.e. already retuned), then it is octave transposed (up for ascending scales, down for descending) to the next available pitch class (i.e. one that has not already been retuned).

Eventually, the scales lose their stepwise structure and increasingly disperse across several octaves.

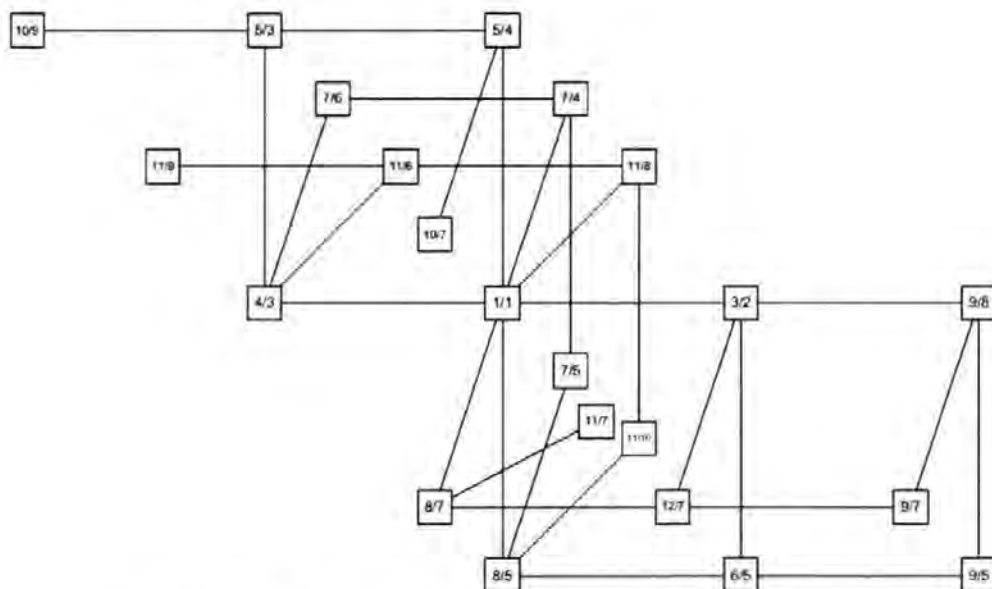


Figure 65 - I drew a line... piano tuning in harmonic space

The gradual fragmentation and displacement of the piano part provides a simple melodic framework for the entire piece, which functions as a thread upon which the other parts 'dangle'.

RELATION OF PARTS TO PIANO

Each section (woodwinds, percussion, piano, strings) takes a distinct approach to harmony.

VIBRAPHONE

The vibraphone is rhythmically closely connected to the piano part. The piano and vibraphone are for the most part vertically aligned, but not entirely homophonic. I apply what I consider a variant or a simplified version of Varèse's *crystallisation* analogy, which connects his approach to musical form to the phenomenon of crystallisation and "the adoption of multiple viewpoints" (Bernard 1987: p.12)⁷¹.

⁷¹ Varèse states: "Conceiving musical form as a *resultant*—the result of a process—I was struck by what seemed to me an analogy between the formation of my compositions and the phenomenon of crystallization. Let me quote the crystallographic description given me by Nathaniel Arbiter, professor of mineralogy at Columbia University: 'The crystal is characterized by both a definite external form and a definite internal structure. The internal structure is based on the unit of crystals which is the smallest grouping of the atoms that has the order and composition of the substance. The extension of the unit into space forms the whole crystal. But in spite of the relatively limited variety of internal structures, the external forms of crystals are limitless.' Then Mr. Arbiter added in his own words: 'Crystal form is a *resultant* (the very word I have always used in reference to musical form) rather than a primary attribute. Crystal form is the consequence of the interaction of attractive and repulsive forces and the ordered packing of the atom.' This, I believe, suggests, better than any explanation I could give,

In this case, I imagine looking at musical material as if embedded in a glass globe, where if looked upon from one perspective, every thing is aligned, but if the perspective is rotated, the pitches shift in their alignment. Here, I only apply this 'filter' rhythmically, but this thinking could just as easily be applied to pitch height as well (see *The Beaten Path* for example). In the end, the vibraphone and piano parts represent mild shifts of perspective so that they remain roughly aligned but their respective attacks are not always simultaneous.

The vibraphone is harmonically related to the piano and plays available (close to) pure harmonic relations around each piano note. The vibraphone explores pitch materials based on the following intervals found between partials 16 through 24: 3/2, 4/3, 9/8, 17/16, 19/16, 19/17, 19/18, 18/17. But occasionally a margin of error of approximately 15 cents is allowed in order to include some 5-limit ratios.

The vibraphone may also be temporarily related to an individual pitch on the piano, which becomes a temporary root or a partial of a root other than G. For example, in the opening three bars, all pitches relate to a G root:

Vibes:	$B^{(5/4)}$	$C^{(4/3)}$	$D^{(3/2)}$			
	$G^{(17/16)}$	$D^{(3/2)}$	$G^{(1/1)}$			
Piano:	$A^{(9/8)}$	$B^{(5/4)}$	$C^{(11/8)}$	$D^{(3/2)}$	$F^{(7/4)}$	$A^{(9/8)}$

But in bar 4, the piano's $D^{(3/2)}$ temporarily functions as 19/16 of B, and the vibraphone tones are $B^{(1/1)}$ and $C\sharp^{(9/8)}$.

STRINGS

The strings often use a 'pinching' technique where two tones are squeezed together around the harmony in the piano and vibraphone parts. This creates an effect where the string tones slide in and out of pure sonorities—in relation to each other and in relation to the other parts.

the way my works are formed. There is an idea, the basis of an internal structure, expanded and split into different shapes or groups of sound constantly changing in shape, direction, and speed, attracted and repulsed by various forces. The form of the work is the consequence of this interaction. Possible musical forms are as limitless as the exterior forms of crystals" (from: Schwartz, Elliot and Barney Childs (eds) (1967) "The Liberation of Sound" *Contemporary Composers on Contemporary Music*, New York: Holt, Rinehart, and Winston quoted. Quoted in (Bernard 1987: 17))

FLUTE⁷² AND CLARINET

The winds are given no precise microtonal inflection markings but are expected to adjust their intonation intuitively in relation to the other parts. For the most part, the flute and clarinet play in unison, but it is expected that the onset of each tone will at first be rough due to slight differences in intonation, but settle as each player makes adjustments.

TRANSFORMATION

At several points in the piece, the flow is interrupted with material that is more rhythmically active. The notes that would normally be generated for the piano are passed around the other parts creating a further fragmented melodic sequence. These sections represent a sort of alternative reality, or an exaggerated distant family resemblance (or perhaps a finer level in a self-similar structure akin to Varèse's crystallisation analogy). The material is loosely related as we hear it, due to the acceleration of rhythmic activity, but the relation is distant as compared to the relationships that occur between the other sections of the piece.

FAMILY RESEMBLANCE

On a sectional level, I feel that the concepts of family resemblance and common thread roughly apply—and that is a contradiction in terms—although I composed this piece before I had identified family resemblance as a meaningful concept in my compositional practice.

⁷²

The submitted recording uses soprano voice in place of flute.

II. THE BEATEN PATH

For flute, percussion, and cello

This piece exaggerates the nuances of unison playing but is exploded to unfamiliar extremes. It is also a metaphor for the way in which paths are built: small and sometimes unnoticeable gestures combine to create a well-defined path.

(from programme notes for Ning Premiere)

The Beaten Path (for flute, vibraphone, and cello) was requested by the Norwegian ensemble Ning and received its premiere performance on 26 November 2003, Parkteateret, Oslo, Norway.

In this piece, I explore an exaggerated heterophonic texture and the sharing of a scaled melody (where the intervals of a melody are proportionately stretched and shrunk according to a mathematical function).

SECTION I

In Section I, the intervals of the scale of the original source melody⁷³: D—F[#]—G—A—B^b—C—D, are at first stretched and then gradually squeezed to a point where all pitches converge on a centre frequency of G=392Hz by the end of the section, through the following function⁷⁴ (Fineberg 2000b: 94):

$$f_p = p^{(x)} \times (f_1)$$

where p = a ratio (or a partial number) and $f_1 = 392\text{Hz}$

(when $x = 1$, no scaling occurs)

For example, the piece begins with the intervals stretched by a factor of approximately 6.2, generating the initiation of the cello glissando beginning on the open C. The formula is expressed as $f = 3/4^{(6.2)} \times 392\text{Hz}$.

The scale (which is treated as a chord in this section) is interpreted as:

$$D^{(4/3)} \text{—} F^{\#(15/8)} \text{—} G^{(1/1)} \text{—} A^{(9/8)} \text{—} B^b(6/5) \text{—} C^{(4/3)} \text{—} D^{(3/2)}$$

(see Appendix 6 for a table of calculated scalings)

⁷³ I originally intended to use the tune *Jeg råde vil alle*, a religious folk melody from Romsdalen, Norway, as the basis for these transformations but instead only used the pitch-set from the tune.

⁷⁴ At the time of conceiving of this process, I was not familiar with the work of the French spectral composers. Bob Gilmore introduced me to this body of work, at which time I found this formula in Fineberg, J. (2000b) 'Appendix I - Guide to the Basic Concepts and Techniques of Spectral Music', *Contemporary Music Review*, vol. 19, part 2: 81-114.

The cello performs a long glissando, beginning on the open C string, and climbs an octave plus a fifth to G below middle C over 19 bars, and then holds the G and gradually moves to sul ponticello over the remaining duration of the section (17 bars).

The cello part represents the interpolation of all values for x , beginning with a value of 3.4 through to a value of 0. Whereas, as x changes, the vibes and flute only play those pitches that fall within their temperament, along an imaginary continuous glissando, and at points where x is rounded to the nearest tenth of an integer: in the case of the vibraphone, 12tet, and in the case of the flute, 24tet plus 5-limit inflections (approx. 12 to 16 cents from 12tet).

SECTION II

In Section II, the melodic contour is preserved and a less extreme scaling than in the first section is applied, with $x > 1$ and < 2.4 . But the melody is passed around the parts freely and some intuitive intrusions are allowed, including harmonics of the melodic components.

SECTION III

Section III explores unison playing within a scaling factor of $x < 1$ and the centre frequency transposed an octave up.

The section is composed with the intention that the intonation between the parts will not be easily coordinated. It intentionally explores micro-deviations in pitch that colour the sound and texture of the melody.⁷⁵

⁷⁵ “Although a great amount remains to be learned about the perceptibility of melodic contour, some interesting work has been reported. Several earlier studies (Emerson, 1906; Werner, 1940; White, 1960) have demonstrated amply that melodies can remain recognizable after drastic alterations have been performed on their component intervals, provided that melodic contour is retained. Werner (1940), for instance, derived a tuning system in which adjacent members of a twelve-tone set were said to be 16.5 cents apart; this amounts to just less than one-sixth of an equal-tempered semitone (see Problem 10 at the end of this chapter). After becoming acclimated to this tuning system, participants in the study could not only recognize intervals in the set quite accurately, but could recognize and evaluate contours of Werner’s “micro-melodies,” and even characterized intervals in this “micro-scale” as qualitatively similar to analogous intervals in the equal-tempered set. For example, the skip from scale degree 1 to degree 13 elicited the judgment “feeling of returning to the same point of departure” (p. 155), the essential perceptual attribute of the octave. White (1960) found that familiar tunes could still be recognized even after their component intervals were stretched or shrunk proportionately; provided that the tunes remained unchanged rhythmically, enough of the contour was evidently preserved that the melodies did not lose their identities” (Butler 1992, 111).

III. 1024 TO 1

For computer

This is a simple computer experiment/piece generated in Csound. 1024 sine tones, based on the harmonic series of 20Hz (the 1024th harmonic is 20480 Hz), converge on a single frequency of 10240Hz midway through the piece, and continue a trajectory that finishes on the undertone series of 20480 Hz (the 1/1024th tone is 20Hz).

I am particularly interested here in the way that a smooth and simple transformation is not perceived linearly. That is, attention shifts abruptly to changes in the texture which seem to arise irregularly, for example, in the second half of the piece where individual tones in the upper register are (suddenly) noticeable—or where different bands of noise emerge from a texture that was previously somewhat harmonic.

This type of experiment has been important to my perception of linear and non-linear changes in determining the structure of my compositions—allowing relatively abrupt changes to emerge from a static or linear formal process. This, I believe, is also quite common in nature and other everyday 'systems' (traffic flow, weather, boiling water, etc.).

IV. FOR MUTED PIANO

For solo piano

For Muted Piano received its premiere performance by John Lenehan at the London Spitalfields Festival, Wilton's Music Hall, on 12 June 2004.

For Muted Piano explores the potential for use of the piano as a spectral instrument. Although there is precedence for use of the piano towards this end, the 12tet tuning of the piano suggests an instrument particularly ill suited to spectral simulation due to the resolution of the system and its relation to the harmonic series. To justify my premise, I adopted the following assumption: If 12tet is capable of the suggestion of three- and five-limit harmony, then it should be equally capable of suggesting pitches based on the 3rd, 5th, 9th, 15th, 17th, and 19th harmonics, as both the 17th and 19th harmonics deviate less than 5 cents from equal temperament. I also allow for the 7-limit, which is justified by the speculative acceptance of the dominant seventh chord in traditional harmony and the use of 12tet to suggest the 7/4 ratio that occurs in this chord.

In each of the two sections of this piece, I apply an imaginary or *virtual* filter to a short harmonic passage. In the first case, the filter gradually opens up, emphasising the upper regions of the virtual harmonic spectrum of each harmonic dyad. In the second case, upper regions of the spectrum are gradually emphasised as the lower regions are attenuated, leaving behind only the high virtual spectra of the original source musical material—the *residual* music.

The harmonic concept for each section is discussed in further detail below after the more general compositional parameters are addressed.

GENERAL ISSUES

This piece was originally intended for upright piano with practice pedal (where a felt damper is lowered between the hammers and strings), which suppresses the strength of upper partials while maintaining sustain, but if approached delicately it can work effectively on an untreated grand piano. Alternatively, a Rhodes or piano with the treble attenuated may be used.

(from *For Muted Piano* performance notes)

I composed this for upright piano with practice pedal because of the distinct way the harmonic spectrum is suppressed while the amplitude envelope shape of the struck string is maintained (other methods of muting the piano dampen the decay).

The opening passage of each section is 'filtered', allowing only low partials to arrive at the

listeners' ears. This of course is only analogy; the goal is not to fully imitate electronic filters, but to allow the concept to inform the harmonic content of the piece.

The muted piano allows for greater control of register and tone. In much of my earlier (pre-Dartington) composition work, I was drawn towards consorts, individual instruments, or muted ensembles. Although it may seem somewhat contrary to my interest in tuning theory and the harmonic series, which often evolves from an interest in complex timbres, the use of muted or harmonically dull sounds allows for greater control of the harmonic spectra while limiting some degree of complexity that would arise through the use of harmonically rich tones.

AESTHETIC ISSUES

This piece is partially justified by the idea that all (musical) material is inherently workable. For the purposes of this composition, I was not particularly concerned with my starting materials, only in a constant or consistent treatment that would lead to an integrated whole. In *For Muted Piano*, I explore variations on an extended "theme"—not several variations on one theme but a progression, or linear chain of variations.

'Part I' develops in a linear fashion and the source material is retained throughout the section, and might therefore be considered an example of 'common thread', which finds its analogy in the gestalt law of *good continuation*.

'Part II' is not strictly an example of family resemblance, but due to the degree that the source material is obscured through the process, the effect resembles what I wish to achieve through the analogy of *family resemblance*.

HARMONIC APPROACH IN 'PART I'

In 'Part I', a single note functions in 12 different harmonic contexts to which are added upper partials based on the acoustical root of each interval (Performance notes, Swoger-Ruston).

The first section of *For Muted Piano* follows a formal procedure for generating harmonic material. Twelve distinct dyads are created with the pitch middle C, based on a conventional just interpretation of the chromatic scale (see bass clef of fig 69).

In each subsequent section, a pitch based on a harmonic of the acoustical root⁷⁸ of each dyad is added to the vertical structure. In the first instance, a pitch based on the 9th harmonic is added to each dyad (9/8 - major second); in the second, the 15th (15/8 - major seventh) or 17th (17/16 - minor second) harmonic is added; and in the third, the 17th or 19th (19/16 - minor third) harmonic is added (fig 68).

acoustical root:

1/1 3/2 9/8 5/3 5/4 7/5 7/4 6/5 8/5 16/15 15/8 4/3

○ - dyad component
 ● - 9/8
 ○ - 15/8
 ▲ - 17/16
 ▲ - 19/16

Figure 66 - harmonic scheme for *For Muted Piano* 'Part I'

The resolution adopted here allows for deviations of up to 16 cents from just intonation with the exception of the 7/4, which is 31 cents smaller than 12tet. I am exploring here the possibility of expressing high-limit harmonies based only on those harmonics that fall close to equal temperament. Therefore, the 11- and 13-limit is skipped in favour of sonorities based on a 3-, 5-, 7-, 17-, 19-limit harmonic space (fig 70).

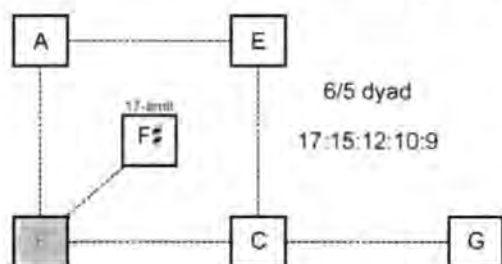


Figure 67 - chord built from 6/5 dyad in harmonic space

⁷⁸ The acoustical root is here based on the simplest or most common occurrence of each interval in harmonic space. Terhardt's subharmonic coincidence analysis generally supports the same results or matches at least one component of an ambiguous result, with the exception of 16/15 and 15/8, which have no subharmonic coincidences.

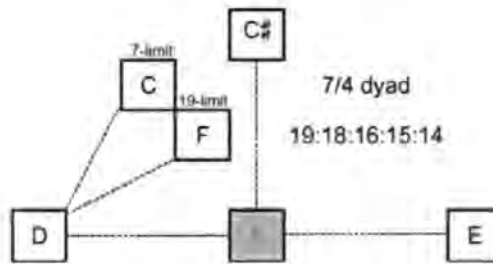


Figure 68 - chord built from 7/4 dyad in harmonic space

The approach also takes into account voicing, and the effect that spacing may have in implying higher-limit harmonic sonorities. To my ears, the minor ninths and major sevenths are not heard as dissonances in any traditional sense of that word, but are heard as spectral components, and relatively smooth (but perhaps not fused). (The clusters that occur in Section II are similarly understood in my mind.)

ANOMALIES IN PART 1

In general, I am not particularly concerned with score 'mistakes'—to the extent that composition does reflect nature, 'mistakes' are a *natural* occurrence. The mistakes confound the process and reflect the inconsistencies found in nature and within natural systems—in this sense I prefer to call them *anomalies*⁷⁷. There are several anomalies in the score for Section I, which I feel no need to rectify: bar 7 – the 6/5 dyad is missing; bar 14 – an F^{\sharp} occurs rather than an F^{\natural} ; bar 18 – a D^{\sharp} occurs rather than a D^{\natural} ; bar 31 – a B^{\flat} is missing.

HARMONIC APPROACH IN 'PART 2'

In Section II, an intuitively composed passage is harmonically altered through the application of a more selective virtual filter (performance notes).

I began with an intuitively composed section for the low register of a piano with the practice pedal engaged. The passage is mildly dissonant and I have accepted it, rather arbitrarily, as source material for the rest of the piece.

I have isolated from the original version of the opening passage intervallic gestalt units—two or three icti that seem to me harmonically and rhythmically connected. From these

⁷⁷ *anomaly*: "2. Irregularity, deviation from the common order, exceptional condition or circumstance" (from *The Compact Edition of the Oxford English Dictionary*, Oxford University Press, New York: 1971). Compare with: *mistake* - "1. A misconception or misapprehension of the meaning of something; hence an error or fault in thought or action" (ibid); or *error* - "4. Something incorrectly done through ignorance or inadvertence; a mistake, e.g. in calculation, judgement, speech, writing, action, etc." (ibid).

intervals, the harmonic series of each is extrapolated, and each variation isolates a different portion of these series. (The first variation sounds pitches that most closely match the 16th, 17th, 18th, and 19th partials of each dyad. The second variation "filters" a narrow bandwidth around C5 to G5, and only those partials which fall within this bandwidth are approximated with a close equal-tempered pitch.) I allow for partials 1 – 20, except for the 11th and 13th partials, which are poorly approximated in 12tet. However, this piece is not *about* just-intonation, so I allow for 7- and 5- limit intervals to be included based on the assumption that there is at least a chance that the intervals may be interpreted as such. This also allows for a greater range of harmonic complexity.

The theme in Section II (which is relatively long) is rather banal. However I do *like* this material (and is therefore not entirely arbitrary) because it seems to me somehow familiar but not entirely consistent with the harmonic language of any specific musical style, and remains somewhat homogenous due perhaps to the unfamiliar register and tone of the muted piano. Harmonic sonorities suggesting dissonant, jazz, triadic, and modal concepts exist equally in this environment.

On top of the variations, a more subtle process takes place over the duration of the piece. Rhythmically, the piece becomes simpler (entropy decreases/reverse time arrow metaphor) and the texture changes from a polyphonic texture to something more homophonic.

V. TRACK AND FIELD

For violin, electric guitar, two synthesisers, electric bass, and contrabass

Track and Field was composed for the ensemble [Rout], and was performed during their residency at Dartington College of Arts at a workshop performance of student compositions on 09 October 2003.

This piece developed out of an experimental piece I composed in Csound for a downward sweeping complex tone and plucked string algorithm⁷⁸. In this piece, I hoped to demonstrate that a sweeping tone could be made to sound continually in tune with an accompanying equal tempered bass and harmony based on the harmonic series of each bass tone.

LINE AND GLISSANDO

What I am particularly interested in here is the perception of what is metaphorically called a line in music. I do not believe the straight lines exist in music on a perceptual level, either metaphorically or analogously. Harmony (or simply, tone relations) affects the perception of linearity, pulling and pushing pitch perception, which is dependent upon context.

In *Track and Field*, a violin plays a single tone which glissandos down an octave over the duration of six minutes (with a couple of interruptions and a change of strings).

In the computer experiment, a single tone sweeps the octave, and is mathematically precise. However, the pitch does not appear to descend evenly. Instead, it seems to alternatively (but not regularly) pause and accelerate dependent on the relation to the harmonic accompaniment in the plucked strings section; I speculate that as the tone gets close to a simple relation in the harmonic field, it seems to slow down because we accept a certain amount of latitude with the tuning. And the degree to which the tone appears to slow down is related to the simplicity of its harmonic function—that is, if it functions as a 5/4 relation to the fundamental then the pause seems greater than when it functions as an 11/8 (for example).

In the performance of *Track and Field*, the opposite occurs. The violin is not completely steady in executing a smooth and even glissando over the duration of the piece. *But*, it

⁷⁸

The 'Karplus-Strong Plucked String Algorithm' packaged with Csound was used.

appears to descend much more evenly than in the computer piece. I speculate that this is because the musician is responding to the pushing and pulling that the ensemble generates and the violin adjusts in a way that is contrary to our perception of the computer piece—because she can speed up the descent when the harmonic “pull” is greatest (thus minimising the effect) and can slow down when furthest outside the gravitational pull of the preceding and forthcoming harmony.

INTERRUPTION AS STRUCTURAL MARKER

In *Track and Field*, the physical properties of the violin meant that I either had to rethink the piece in order to allow a single sweeping tone to last the duration of the piece—I could have transposed the piece so that an entire octave would be available on a single string, or I could have made the magnitude of the sweeping tone smaller. Instead, I decided to allow the moments where the violinist runs out of string to provide two moments of interruption in the process (at sections B and C).

Along with a more intuitive approach to the bass parts (see below), the inclusion of these interruptions feels important in that it takes the piece away from simply being an experiment and provides structure and an element of unpredictability.

HARMONIC BACKGROUND

The remaining instruments harmonise the violin part, providing a harmonic field and context for every moment of the sweeping tone (except where the tone falls perceptually halfway between each successive harmonic moment).

All of the other instruments use a referential 12tet notation, and all but the electric bass use tones that slide into the indicated 12tet pitch. The guitar achieves this using a slide and e-bow^{TM79}, using the frets as references rather than for stopping notes. The keyboards are programmed to use a monophonic portamento which takes about 2 seconds to sweep from one pitch to the next; meaning that the appropriate 12tet pitch sounds only during beats 2 and 3 of each bar.

⁷⁹ An ebow is an electronic device which uses an electromagnetic field to cause the vibration of a metal string when held slightly above—acting as an ‘electric bow’.

Adding the referential 12tet to the harmonic concept means a further margin of error for tuning (and an increase in the complexity of the system). But not 'further' in the sense that the margin of error is greater, only more complex to unravel.

Each bar represents approximately a change of $1/12^{\text{th}}$ of a tone in the violin. On average the tone is harmonised according to the midway point in the bar, meaning that if played precisely, the tone is never more than 8 cents out of tune from its ideal harmonic intonation—likely, this is not what happens in performance.

For example, in bar 1, the guitar plays a G to the violin's $A^{(+/- 8 \text{ cents})}$. The violin makes a perfect 9/8 when it is at $A^{(+4 \text{ cents})}$. In bar 2, the guitar slides to an F and the violin slides from $A^{(-8)}$ to $A^{(-25)}$, arriving at $A^{(-14)}$ near the middle of the bar—a perfect 5/4 major third from the guitar).

THE BASS PART

The two basses alternately provide the root (as opposed to the fundamental) of each chord. Initially I thought to use only one bass, as that was all that was required to complete the experimental concept of the piece. But in the end I decided to use both as it created one further musical problem, thus increasing the complexity of the piece (and perhaps making it more musical in a traditional sense).

The physical differences between the bowed bass and the plucked electric bass means that the first is capable of a slow attack, infinite sustain, and dynamic intonation and the second is capable of a fast attack, shorter decay (although the attack can be slowed through the use of a volume knob or pedal and the decay can be lengthened through various means), and fixed 12tet intonation.

The basses play off each other by exploiting the differences described above. For example, the electric bass may audibly pluck a note and shorten its decay with a volume pedal, while the bowed bass silently attacks and swells on the same pitch, which sweeps into the root of the next chord (bars 18 and 21 for example).

VI. EVENTIDE

For piano, violin, cello, and bowed vibraphone

Literally: "The evening space-in-time".

Eventide was composed for the Barton Workshop ensemble, and was performed during their residency at Dartington College of Arts at a workshop performance of student compositions on 02 May 2003.

Two concepts inform this piece, both of which extend the harmonic language of hymns. I use a chorale-like piano part that elicits the virtual resonance of imaginary objects (simulated with violin, cello, and bowed vibraphone), thus extending the mostly triadic harmonic language of the hymn. And, I have organised the harmonic progression such that the concept of root-relatedness is further extended around the circle of fifths. (Thus far, this is the only piece I have written that has such an overt external musical reference.)

THE CHORALE

KEY

Although in the end I took a much more intuitive route, I initially thought to create a simple process piece that interleaved the harmonic progression of several hymns, with the idea in mind that certain keys would be preferenced, thus weighting the key centres close to C. And that as each hymn wanders into related keys, an extended language of subdominants, deceptive cadences and relative minor/major keys would develop. By interleaving several hymns an extended harmonic language that is still related, though more distantly so, to some central tonal centre could be developed. As each hymn moves toward its final cadence, a sense for the tension and release of the hymnal vocabulary would still be maintained, albeit with a polychordal point of rest centred on the key of C and its immediate neighbours in the circle of fifths (in actual fact, F is the central key—see below).

A Look at the 'Evening' portion of the *Methodist Hymn and Tune Book*⁸⁰ suggests that 'G' is actually the most common key, followed by 'E \flat ' and 'D' and other keys are roughly less

⁸⁰ My copy of the *Methodist Hymn and Tune Book* is missing several pages, including publishing and date information, and is therefore not included in my References.

common the further away they lie from 'G' in the circle of fifths, although this is more true in the direction of flat keys rather than sharp keys. This suggests an organisation that sits in a moderately compact harmonic space. However, the distribution is slightly different in each section of the Hymnal; in the first 500 Hymns, E^b is the most common key, followed by G then F. To look at the whole hymnal as a system of music, the potential exists to compound that system by randomly interspersing material from several hymns.

Key	'Evening' hymns	first 500 hymns
E	1	21
A	5	39
D	6	52
G	14	86
C	0	41
F	4	74
B ^b	3	42
E ^b	9	91
A ^b	4	36
D ^b	0	14

Table 12 – distribution of key centres in *The Methodist Hymn and Tune Book*: 'Evening' section compared to the first 500 hymns

HARMONIC PROGRESSION

Looking at each hymn in isolation reveals a structure that begins closely tied to the home key, which remains established usually to the end of the first phrase; and then a brief movement to related keys and finally a return to the home key. As well, the likeliness of isolated instances of increased dissonance is more likely towards the middle portions of the hymn where suspensions, appoggiaturas, etc., occur more often.

Interleaving hymns exaggerates this trend because now, not only does the occurrence of distantly related chords increase, but dissonances are also exaggerated because they are now taken out of context, unprepared, and likely to be more distantly related in harmonic space.

Combining hymns exaggerates the rules of the existent tonal system. It maintains a central key, exaggerates the consonance/dissonance concept, and maintains the harmonic shape through expanded rules of tension and release.

Hymn 237 (second tune)

Hymn 225 (first tune)

Hymn 237 (first tune)

Hymn 238 (second tune)

Figure 69 – Example of Hymnal samples used in experiments for *Eventide* – octave transposed for reduction

Figure 70 – Example of compounding experiment using four hymns from Figure 69

HYMNAL VOICING

The voicing of chords in chorale texture is often easy to reconcile with the harmonic series. To some degree chord voicing reflects the spacing of the harmonic series, and the

most commonly used chords exist within the lower regions of the harmonic series—specifically major triads, dominant sevenths, and minor triads. Less so are the prepared dissonances, although these often link two harmonic series related by strong root progression (also related to harmonic space and the circle of fifths).

In the end, I suppressed the choice of materials, and used only four hymns for experimentation (which lessens the natural strengthening of a single key centre) and then proceeded to compose intuitively based on my understanding of the experiment. However, a central key is further emphasised using a 'virtual' sympathetic resonator...

SYNTHETIC RESONATORS

Three bowed instruments (violin, cello, and bowed vibraphone) are used to simulate the sympathetic vibrations of a resonant body in close proximity to the piano (which plays the hymnal material).

The resonant body is based on an F-harmonic series, which further strengthens the prominence of this key.

The acoustical root was extracted for each chord in the piano part, which is often simply the root of a major chord. Because the chord is already a part of the lower portions of the harmonic series, the bowed instruments are responsible for emphasizing higher odd-numbered harmonics: specifically 9, 11, 13, 15, 17, and 19.

Any pitch in the piano part falling close to a harmonic of F elicits the resonance of one of the three bowed instruments, which swells and decays as smoothly as possible (in the case of the vibraphone, the tone decays naturally when the bowing stops).

VII. THE CROW, THE ROAD, AND THE RAMBLE

For saxophone, keyboard, electric guitar, and cello

The Crow, the Road, and the Ramble was composed for members of the ensemble Icebreaker, and was performed during their residency at Dartington College of Arts at a workshop performance of student compositions on 12 March 2004.

LINES

The title refers to the multiple paths one may take from point A to point B. "As the crow flies", while not a straight line, takes smaller deviations from a direct path than might a series of roadways (the "road"); and "rambling", where the goal is to walk in as straight a line as possible despite whatever obstacles may impede the hikers path. This is also related to the measuring of coastlines (for example), where the size of the measuring instrument will influence the measure enormously: a coastline becomes infinitely long as the ruler becomes infinitely short—to the point where micro-twists and turns are measured, and at the atomic level, no coast "line" is present at all.

In *The Crow, the Road, and the Ramble*, I imagine that each part is essentially the same but functions at a different scale of resolution. This is not literal or systematic, merely metaphorical.

Perhaps the keyboard part, which is an abstraction of the harmony in Prelude VII, Book 1, "Ce qu'a vu le Vent d'Ouest" from Debussy's *Preludes* (bars 23 through 32), is the only part functioning in 'real world' time. The guitar, which explores an imaginary microscopic sonic world (*microsound*) that lies behind the sounds of the keyboard part, catches some of the minor deviations and fluctuations found within the metaphorical microsound of the piano part, and is therefore more jagged and unsettled, and includes higher harmonic information. The tenor saxophone is viewed up close; we only hear a few select harmonics and hear the variation and unsettled nature of the microscopic sonic image⁸¹. The Cello is often roughly aligned with and supports a pitch in the keyboard part, but has fewer intonation cues than the other parts (and is often intentionally put in an ambiguous intonational situation).

⁸¹ The recording does not live up to this as, despite numerous requests to play as quietly as possible, which I hoped would take away the stability of the tone, the player here makes the part quite lyrical and sits far too upfront in the texture.

FORM AND TIME ARROWS

The form is somewhat ergodic but also cyclical with the keyboard performing four literal repeats of the Debussy reduction. This suspension of the time arrow is contrasted in the guitar part by a slow increase and subsequent decrease in entropy (forward, then reverse time arrow). The other parts remain relatively ergodic (suspension of the time arrow).

INTONATION

KEYBOARD

The keyboard remains in 12tet, but (hopefully) suggests an extended harmonic system as all of the pitches correspond with Messiaen's *chord of resonance* based on a C and E harmonic series (connected by a brief chromatic passage connecting the two series). Bar 1 through beat 2 of bar 6 is in C, the second half of bar 6 holds the chromatic connection, and bars 7 and 8 are in E (although the major 6th—which should represent the 13th harmonic in Messiaen's chord of resonance—is here spelt as a diminished seventh).

GUITAR

The guitar uses a microtonal scordatura based on an intonation I explored in my MA Thesis: *The Fifteen Cent Guitar: Retempering the standard six-string guitar*, where a 15 cent satellite intonation is used to approximate just intonation system to a very high level of resolution. In this system, pitches found on a horizontal plane of harmonic space are tuned in a 15-cent increment from 12tet, although I here replaced -45 cents with a quartertone:

E^(-15 cents) G D F⁽⁻³⁰⁾ B⁽⁻¹⁵⁾ C^{♯(-50)}

Figure 71 – guitar scordatura for *The Crow, the Road, and the Ramble*.

..an 11-limit G harmonic chord with an added sixth:

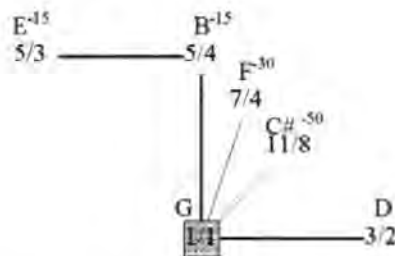


Figure 72 – pitch space of guitar scordatura in *The Crow, the Road, and the Ramble*

The guitar explores a further extension up the series of the two harmonic centres of C and E and is intonationally very precise (thus representing a metaphorical refinement in the measurement of the 'path', which includes finer twists and bends, etc.)

For example, the first six icti are pitch classes related to a C root: $D^{(9/8)} - F^{(21/16)} - C\#^{(33/32)}$ – $F^{(21/16)} - C\#^{(33/32)} - B^{(15/8)}$. These are the same pitch classes found on the open strings but are related to a C root.

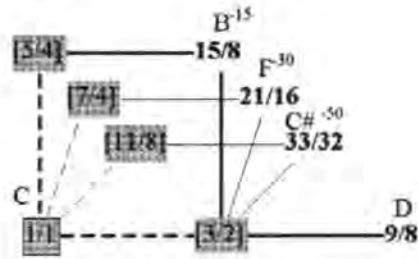


Figure 73 – pitch space of first six icti in guitar part of *The Crow, the Road, and the Ramble*

In the following two compositions, I add a third system to the guitar part, which represents the resultant pitch of the scordatura guitar notation.

SAXOPHONE

The saxophone provides an unstable reinforcement of low partials in the C and E series. The instruction allows the player to choose various methods of producing the indicated pitch, most of which serve to add a mild chaotic element to the harmonic sonority. Because the pitch may be a part of a multiphonic, a harmonic, or produced with alternate fingerings, the resultant intonation is not expected to be stable, and is further emphasised through the instruction to play as quietly as possible (which makes the tone less stable). This, again, represents the mild fluctuations of pitch found in the microsound world of the tone.

CELLO

The cello has the intentional but unfortunate task of reconciling a dynamic 12tet intonation with the ambiguous combined intonation of the other three instruments. As Ligeti requests in his *Double Concerto*, the cellist is left to her own devices to find a suitable intonation for the passage, except that it is expected here that there is no easily reconcilable intonation available. We should here a continuous adjustment as the player struggles to smooth the harmonic sonority.

RHYTHM

As in *I drew a line...*, the rhythmic alignment in *The Crow, the Road, and the Ramble*, goes through a mild Varèsian filter, particularly during the chromatic passage that links the two tonal centres. Otherwise, the players received the further instruction not to be particularly concerned with the vertical alignment where the notation may suggest otherwise. This proved to be a miscalculation as the ensemble continued to be concerned, in particular, with the precise execution of the opening ictus. I consider this a problem with the notation, which will be reconsidered the next time I intend this sort of sound world.

VIII. CORRECTIONS AND AMPLIFICATIONS

For string quartet and electric guitar

Corrections and Amplifications was requested by the Dutch ensemble Zephyr Kwartet to include in their programme *Vampyr!*—a series of concerts in collaboration with guitarist Wiek Hijmans. The piece was premiered on 23 December 2004 at the Concertgebouw, Amsterdam.

This piece is organised by a series family resemblances related through the physical attributes of the tuning, harmony, and finger shapes used in the guitar part. The strings enhance the spectrum of the guitar by sum and difference tones, properties of the harmonic series, or as 'virtual' resonators (from performance notes).

NEWSPAPER ERRATA AND FAMILY RESEMBLANCE

Initially, I was interested in a string of variations that would gradually become compounded through a metaphorical application of various newspaper errata, such as corrections, apologies, retractions, addendum, amplifications, etc., to a single news story where each section represents a further variation on the previous section's (article) errata.

I saw that if the chronologic concept was obscured, that a family resemblance between the sections may emerge that would not be traceable to the ersatz 'article'. In the end, I took a more intuitive approach and dedicated myself more fully to the family resemblance metaphor than to the newspaper errata metaphor, but retained the title as it was still meaningful to the structural and material generating concept—a sort of non-linear theme a variation based on family resemblance.

GUITAR

The guitar part is the source of much of the variation. Four basic chords/chord-shapes (*chord forms* for the remainder of this section) are the source of most of the material in the guitar part, and some of the string parts. But there are no definitive or ersatz chord forms.

Any given chord form is connected to the rest through the intervallic structure of the chord, the harmonic structure of the chord, the physical *shape* as it sits on the guitar fretboard, or through mild distortions of any element of the chord form. For example, an inversion of a chord form could be either an inversion of the intervallic structure, or a

mirror of the actual fingering shape (fig 76). Any variation can be the source for further transformations, but because we cannot point to a single common element, no chord form can be said to be the *source* for the rest of the material.

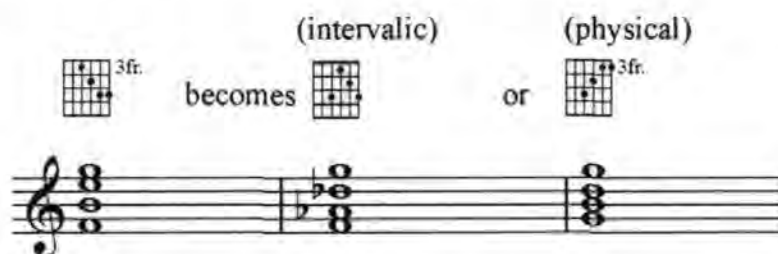


Figure 74 – example of a possible transformation of a guitar chord form in *Corrections and Amplification* (in standard 12tet tuning)—here, first an interval inversion, and secondly a physical inversion of the chord shape as it sits on the guitar fretboard.

The guitar is tuned in a quartertone scordatura:



Figure 75 – guitar scordatura for *Corrections and Amplifications*

STRINGS

The strings often act as a virtual resonator to the guitar, but with a more complex structure than the resonators used in *Eventide*. The harmonic structure of the virtual resonator results from the calculation of sum, difference, or harmonic tones deduced from the guitar part.

Because the guitar is tuned in a quartertone scordatura, the interpretation of any given chord form is subjective and subject to the resolution of the system. Quartertones may variously imply 7-, 11-, or 13-limit intervals, and 12tet tones 3-, 5-, 7-, 17- and 19-limit intervals. The calculations of the virtual resonance are dependent on my subjective analysis of the guitar chord forms, and are therefore context specific. A chord form that occurs in more than one place may not necessarily be interpreted the same way each time.

The strings also occasionally appear without the guitar, where we hear only the virtual resonance of an inaudible guitar source. And, the strings occasionally have material that functions as a transformation of the guitar material—most often with the further

distortion of being translated into a referential 12tet rather than maintaining quartertones.

IX. THIS MNEMONIC MACHINE

For solo electric guitar

This piece was requested by Dutch guitarist Wiek Hijmans and has not yet received its premiere performance.

This piece is the most intuitively composed of all of the included compositions. Much of it is a result of a circular method of composing where I transcribe my improvisations, and then edit the material according to what seems to me the organising principle of the improvisation. I then return to playing with the newly transcribed material, adding another round of improvisation to the process, etc.

SETUP AND TUNING

This Mnemonic Machine uses a delay pedal to create a resonance effect. By setting the delay time of a digital delay unit to 16ms, and using a long feedback setting, the harmonics of a B^{+7.6 cents} fundamental are emphasised through the resonant features of the delay unit settings. This effect occurs because the audio signal is repeated at a rate within the threshold of hearing, rather than at a rate where it would be perceived rhythmically as the delay pedal is traditionally used.

$$16/\text{ms} = 1000/16 \text{ Hz} = 62.5 \text{ Hz} = \text{B}^{+7.6 \text{ cents}}$$

The guitar is in a just intonation scordatura based on a B harmonic series (a capo is used across strings 6 through 3 in order to minimise the degree that the guitar requires retuning (which is a requirement for Wiek who will need to retune quickly between pieces in his live set):

F ^{#+6}	B ⁺⁸	D ^{#+6}	A ⁻²³	B ⁺⁸	E ⁺⁵⁷
4/3	1/1	5/4	7/4	2/1	11/8

Figure 76 – guitar scordatura for *This Mnemonic Machine*

Each open string is emphasised by the delay pedal—in either the same octave it sounds or as a harmonic of the open string (E⁺⁵⁷ is an octave lower than the 11th harmonic emphasised by the delay unit, therefore the second harmonic of the open E is emphasised rather than the fundamental).

Along any string, other close to pure just intervals are also available. For example, on the high B-string, the 16th, 17th, 18th, 19th, 24th, 27th harmonics are all close enough to pure to

be emphasised by the delay pedal resonance.

STRUCTURE

The piece is in five sections. A gradual exploration of increasingly remote harmonics evolves over the duration of the entire piece. At some point, the relation of the pitch material to the harmonic series is so remote that it is not emphasised by the delay pedal resonance. This creates an effect where as dissonance increases resonance decreases.

The opening section is simply a drone. By tapping on the capo, the quite vibration of the open strings are emphasised through the resonance created by the delay pedal. The harmonic series of B gradually opens up over the course of the section, with the lower portions of the series gradually attenuated.

The second section is a sort of crude *Alap*⁸²—here a quasi-improvised section that explores melodic material based on the B harmonic series, with blues references. In my own guitar playing, I conceived of the blues scale as an amalgamation of the minor pentatonic scale and the Lydian $\flat 7$ scale with an ascending natural seventh (similar to Messiaen's *chord of resonance*, although I arrived at this through independent and difference means over a long period of playing).

The third section is more composed and more strictly measured. This section explores more distant portions of the harmonic series, where the harmonics of a pitch may illicit the resonance of upper partials, rather than the fundamental.

The fourth section explores similarly distant harmonic relations, but through the repetition of several short contrapuntal cells that are repeated as the volume pedal is used to emphasise gradually the resonance of the delay pedal.

The final section uses a simple descending sequence, which is based on the physical finger-shapes rather than the melodic shape, to randomly (or coincidentally) elicit the resonance response of the delay pedal. That is, there is no formal structure for the resonance, but if a pitch or one of its harmonics happens to coincide with the B harmonic series, it will be emphasised.

⁸² The *alap* is the improvisatory portion of a classical Indian composition where the mode of the *raga* is explored while avoiding giving a sense of pulse. (Sadie, S., ed. (1980). *Indian. The New Grove Dictionary of Music and Musicians*, vol. 9.

OBSERVATIONS REGARDING MY COMPOSITIONAL PRACTICE

COMPLEXITY

COMPLEXITY AS A CONTINUUM (COMPLEXITY, CHAOS, AND DYNAMICAL SYSTEMS)

The first creation of new-paradigm science is the shift from the parts to the whole. In the old paradigm it was believed that in any complex system, the dynamics of the whole could be understood from the properties of the parts. In the new paradigm the relationship between the parts and the whole is reversed. The properties of the parts can be understood only from the dynamics of the whole. Ultimately there are no parts at all. What we call a part is merely a pattern in an inseparable web of relationships (Capra 1985: 83).

I am interested in a sound world in which order is perceived yet not easily explained. As with my harmonic material—which considers harmonic materials based on relative complexity rather than categories of consonance and dissonance—the textures and sectional developments are concerned with relative levels of complexity, and seek to limit the extremes to those reflected in models found in nature (metaphor).

I enjoy the complexities that arise through the compounding of several simple systems. I feel that this allows for an intuitive understanding of the musical system while maintaining unpredictability on the fine-scale. A good demonstration of this in the visual world was the installation “Say Parsley” (Exeter 2001) by Ciaran Maher and Caroline Bergvall, where a large number of pendulums were hung, evenly spaced, inside a studio. The movement of each pendulum is simple⁸³, i.e. regular, predictable, and easily describable (mathematically). However, the room viewed as a whole appears relatively complex but not overwhelming—there still exists a sense of the placid movement of a single pendulum within the whole.

In working with complex harmonic structures, using harmonically rich tones are sometimes too complex for my interests, requiring attention to too many parameters not directly related to the intentions of the composition. While the harmony may be strictly controlled, the timbral element is controlled only as a bi-product of the harmonic concept. But I am also interested in some of the uncertainty which arises in using acoustic instruments rather than composing with sine tones, and in this sense I consider my work to be concerned with complexity, but as a continuum not as a state—I don’t consider my

⁸³ I can’t recall who said it, but someone once said that “there is no such thing as *simple* harmonic motion”.

music to be *complex* in the sense that this term is normally used in contemporary music discourse.

To summarise, in general I am interested in ideas of complexity and unpredictability but expressed through limited means—for example, extreme complexity applied to many factors or parameters rarely occurs in nature, usually complex processes exist in rather suppressed forms—unlike total Serialism and the New Complexity which exist to express higher levels of chaos.

TRANSFORMATION

TIME AND FORM (LINEAR, CYCLICAL, AND STATIC MODELS OF MUSICAL TIME)

I am also concerned with concepts of time on a macro-scale—as a form shaping parameter. Musical connections to time on micro-scales are implicit in the discussions of harmonic/melodic materials using frequency ratios and micro-sound variations related to intonation and timbre.

Some concepts of time inform my approach to form. But reflections of scientific time in musical time, I believe, can only function on the level of metaphor.

In classical physics and common experience, the properties of objective/scientific time are often thought to include the following notions of time (taken from Jeff Pressing's *Contemporary Music Review* article *Relations between Musical and Scientific Properties of Time* (Pressing 1993: 105-106):

Time provides [a unique] ordering of events

This ordering has a unique direction [which manifests itself in the rise of entropy in an isolated system (the Second Law of Thermal Dynamics)]

Time separates events into three distinct categories: past, present, future

Time is measurable

Time is continuous (but also discrete)

Pressing continues to apply these notions to musical time⁸⁴. Music's intimate connection with time is undeniable. It is almost impossible to discuss music without the function of time, and almost all parameters have time as an important function on both macro and micro scales. In my music, I consider linear processes that move from relative order to chaos as a metaphor for the forward time arrow, and other formal approaches subvert this to varying degrees. For example, a linear process that moves from chaos to order reverses the time arrow; ergodic and cyclical forms suspend the time arrow; and repetition may suggest some degree of backward listening. This all, of course, can only function on the level of metaphor. And music depends on the ordering of events in time, which we experience in one direction only, although with many references to the past, either within the structure of the piece but also along a longer cultural/historical time line. That is, we probably relate all musical sound to what we have heard before, on any number of simultaneous scales.

LINEAR VS CYCLICAL FORMS

To focus on the second property of scientific time, the most direct musical example of time's unique directional arrow is metaphorically suggested in strict linear transformations. However, examples suggest two directions based on the entropy of the system. If a musical system moves from order to chaos, it may suggest a forward time arrow. However, this can be usurped (however weak the analogy) through a linear transformation that works from chaos to order. Regardless, time is directional in both cases (though not necessarily uniformly so).

But in musical structures it is almost impossible to present a form that changes linearly at all levels. Any piece that deals with pitched tones will be at least cyclical at the waveform level (perhaps this is too pedantic for what is a tenuous metaphor to begin with). To be truly linear, all parameters would need to transform with time (with rise in entropy): rhythm

⁸⁴ "The five properties of scientific time...have ready parallels in music...[1]the musical events have a unique time ordering...[2]the unique *direction* of time is "accepted" in nearly all music, but may be vitiated to some degree by various techniques [retrograde]...[3]past, present and future remain useful concepts, within whose ambit the recurrence and development of sonic material operates. Backwards and forwards hearing is how Kramer (1973) has described it...[4]all realized musical events are subject to clock measurability and much of their effect is gained through this kind of temporal perception...[5]the continuity or arbitrary divisibility of time applies without doubt to sound perception. But quantization of time enters in a much more universal way in music than in science...pulse, meter, rhythm, phrase and subdivision...also a temporal course-graining in perception" (Pressing 1993: 108).

and horizontal density, timbre, harmony, melody, vertical density, etc. What occurs most often is a combination of linear, non-linear, and cyclical (trans)formations.

If linear transformation suggests the Second Law of Thermal Dynamics, then cyclical forms, ergodic forms, and repetition may suggest a suspension of the time arrow. However, extreme and strict repetition is rare in musical composition, and the relative use of cyclical forms may suggest a more complex relationship between stasis and motion⁸⁵.

But our subjective experience of repetition is not so easy to explain. The perception of cyclical forms is dependent on the forward progression of time, and each repetition is heard uniquely in respect to backward listening and memory. So to say that cyclical and ergodic forms suggest a suspension of the time arrow is a tenuous metaphor to say the least, but remains useful as a compositional strategy.

PROCESSES AND CONFOUNDING PROCESSES

A good deal of my work is generated at first from an algorithmic process, which is explained in the discussion for each of the submitted pieces. But during my time at Dartington I began to introduce ways to confound the processes that underlie my approach to composition. The most obvious example is a loosening up of the moment-to-moment details of the piece, which are a result of the process, by allowing intuitive intrusions. That is, allowing myself to make an aesthetic decision to adjust a particular musical moment according to what I want to hear as opposed to what the process dictates. *I drew a line...* is a good example of this.

A second method is to allow a secondary process to confound the primary process, thus creating a level of complexity greater than the sum of those two concepts (*Eventide*, for example).

A third method, *physicality*, is detailed below.

⁸⁵ “The recurrence of the same or related sonic events in music is often considered to create a kind of cyclic time that stands in contrast to linear time. This idea is widespread in non-Western music (e.g. the West African time-line, the Indian *tala*) and in the music of our own culture (e.g. passacaglia, ostinato, strophic form, theme and variations, rondo form). Since it is repetition that allows cyclicity to be perceived, it can be useful heuristically to classify the nature of repetition used in music, as an index to the degree of cyclicity of time. Thus we may view a certain passage as being periodic, quasi-periodic, or aperiodic. It may feature exact repetition of all parts, exact repetition of some parts, varied repetition, exact repetition of abstract properties of all/some parts, or contrast. Exact repetition has often been considered to suspend musical time. Perhaps this would be better viewed as reducing the scale of time to the scale of repeated pattern” (Pressing 1993: 111).

PHYSICALITY

I am becoming increasingly interested in aspects of form that are governed by the physical nature of the instruments for which I am composing. More generally, what I often find musically intriguing are the limitations of a particular musical system, instrument, or other piece of technology⁸⁶. In several of my pieces, the physical nature of the instrument dictates some aspect or confounding of the form. In *Track and Field*, the problem of performing a descending octave glissando from 'A' on the E-string of the violin meant that it was split into 3 parts, thus giving the piece some sectional structure. In *Corrections and Amplifications*, the physical shape of the guitarist's fingers on the fretboard became the springboard for harmonic and melodic transformations.

FAMILY RESEMBLANCE

Over my time at Dartington, I was at first interested in the idea that the source material (usually a harmonic or melodic structure) for a piece may not necessarily occur at the beginning or ending of a piece, but might occur at some other structurally significant point (a conversation with composer Frank Denyer brought this to my attention). Or, later, simply be buried elsewhere where the source material may not be recognised as significant or as 'the source'. Eventually, this expanded further into the idea that the source material may not present itself at all, only its offspring or variations or at some point before or after the onset of a process (moments in-between but not inclusive of the logical starting and end points of a process).

I am also interested in stretching this approach to allow for non-linear transformations—so the "theme" may appear anywhere in the piece surrounded by any subsequent variation though not directly connected. Possibly, two or more chains could extend in different directions (along manipulations of different parameters, or different manipulations of the same parameter(s))

⁸⁶ An analogy I like to use is the technology behind the Hammond organ. While the Hammond organ simulates to some extent the production of sound in acoustic instruments, what makes it unique are the ways in which acoustic theory is *not* accurately reflected in the mechanism. Specifically, the additive approach to timbre control via the drawbars creates a distorted harmonic spectrum that is tuned to equal temperament, limited to 9 partials and missing the seventh harmonic, and has no time varying spectral information (although the drawbars may be manipulated during the performance). While these 'limitations' reduce the instrument's ability to simulate acoustic instruments—it was first introduced as a replacement for church pipe organs and then later claims suggested its ability to mimic the sounds of the orchestra—these same factors contribute to the unique sound of the Hammond organ.

This has gradually led me to the application of the concept of *family resemblance* as described by Ludwig Wittgenstein in his *Philosophical Investigations* through his discussion of language games. This strikes me as a meaningful way to think of musical material and resembles the nature of many systems more directly than a straightforward theme and variations or sonata form (for example). Objects, words, numbers, do not fall neatly into fixed and well-delineated categories.

To date, I have not systematically applied this concept—though I conceive of doing so in the future; I have let it inform my composition somewhat intuitively although not rigorously.

Wittgenstein, in Aphorism 65, states:

Instead of producing something common to all that we call language, I am saying that these phenomena have no one thing in common which makes us use the same word for all,—but that they are *related* to one another in many different ways. And it is because of this relationship, or these relationships, that we call them all "language" (Wittgenstein 1953/2001: 27e).

And in speaking about games (Aphorism 66), and what commonality may be found in all "games":

—don't say: "There *must* be something common, or they would not be called 'games'"—but *look and see* whether there is anything common to all.—for if you look at them you will not see something that is common to *all*, but similarities, relationships, and a whole series of them at that...

Wittgenstein continues to examine 'games' in more detail, and "the result of this examination is: we see a complicated network of similarities overlapping and criss-crossing: sometimes overall similarities, sometimes similarities of detail" (Wittgenstein 1953/2001: 27e).

To these similarities, in aphorism 67, Wittgenstein assigns the term 'family resemblances'; "for the various resemblances between members of a family: build, features, colour of eyes, gait, temperament, etc., etc. overlap and criss-cross in the same way" (Wittgenstein 1953/2001: 28e).

In my own music, family resemblances occur in a few pieces; less consciously in the earlier works, but as a specific structural concept in the most recent—in particular *Corrections and Amplifications* and *This Mnemonic Machine*.

COMMON THREAD

In contrast to *family resemblances*, several of my pieces function by a *common thread* which runs throughout the piece (*Track and Field; I drew a line...; The Crow the Road, and the Ramble*). In these pieces, one element remains constant (or develops linearly) throughout while other elements develop around the central thread (although if the constant were removed a family resemblance may still exist between the remaining material).

STRAIGHT LINES AND MEASUREMENTS

A theme that comes up in several of my compositions is the problem of measurement, where the result from measuring a contour depends on the length of the measuring device. For example, the measurement of a coastline depends greatly on the size of the measuring device. A short ruler will measure more of the undulating surfaces than a long ruler will and therefore result in a much larger measurement.

In my music, the concept of the 'straight line' is put to similar means of measurement, but the level of detail varies resulting in lines that are more, or less, coarse. One 'line' may be jagged in contrast to another that is finer.

MICROTONALITY

I consider the fact that most of my music contains microtonal intervals a by-product of other compositional concerns. I do not feel that my music is *about* microtonality, or just intonation, or expanded tuning systems; the microtonal material simply emerges or is necessary from the other formal issues I wish to explore.

Microtonality arises from the extension of my concept of harmony, which may be informed by any number of things: the exaggeration of traditional harmonic concepts (*Eventide*); the exploration of subjective tones (*Corrections and Amplifications*); resonance (*Eventide, This Mnemonic Machine*), pitch set scaling (*I drew a line...*), glissandi (*Track and Field*), indeterminate intonation (*The Crow, the Road, and the Ramble*), and other psychoacoustical phenomena.

I am also in the process of developing an approach to intonation that embraces the variance in the intonational refinement of different instrument families, which rather than limiting the harmonic/melodic resources actually expands them. A rough system, such as

12tet, leaves much open to interpretation, and by allowing multiple interpretations of the same harmonic material, or related harmonic material, this suggest a relationship to parts for more intonationally refined instruments can respond in a variety of ways⁸⁷.

FUTURE GOALS

Since writing the majority of this thesis, my compositional objectives have changed slightly (as compared to my opening statement to this section that I seek to “embrace the strange”). My most current efforts are concerned with removing *effort* – that is, I am seeking honesty and the removal of my ego from composing. What may seem contradictory to this is that I feel that the best way for me to achieve this is to compose for myself. To continually seek to express what *I* want to hear—by stripping away the layers of influence that confound my honest expression. This means, for the most part, suppressing my ego, or my desire for external acceptance, and being steadfastly attached to my own will.

This is not at odds with my opening statement, but more a refinement or simpler reflection of that idea. The strange and the beautiful are the same thing and occur in everything. I don't think that we can say that Mozart is any more strange or beautiful than Cage (for example).

This is also a spiritual (for lack of a better word) quest that informs my personal life. I am not very good at it, but I try. Or, rather, I try not to try.

⁸⁷

The clearest example of this occurs in *Corrections and Amplifications*

APPENDIX 1 - GLOSSARY

AUDITORY STREAMING

Auditory streaming is a term used to describe the ear/brain's ability to hear sonic events independently. In the case of a fugue, we are able to hear the separate melodic lines individually rather than as a progression of vertical sonorities.

Several factors influence auditory streaming. Tonal fusion is thought to reduce the effect of auditory streaming, and may in some cases be undesirable in certain styles of composition. David Huron speculates that Bach avoids intervals that tend toward tonal fusion in favour of intervals that only possess tonal consonance (Huron 1991: 153). This, he sees, as important to the streaming of musical information in polyphonic textures. (This statement supports the common rules of voice-leading from the Baroque and Classical eras.)

"A variety of both "horizontal" and "vertical" factors have been identified as contributing to auditory streaming (references Bregman, 1990). Horizontal factors that enhance the perception of independent streams include the maintenance of close pitch proximity within voices (Dowling, 1967) and the avoidance of part crossing (Huron, 1991). Vertical factor[s] that encourage stream segregation include asynchronous tone onsets (Bregman, 1990) and the avoidance of tonal fusion (McAdams, 1982, 1984)" (Huron 1991: 136).

CENTS

The most common means of describing interval magnitude uses the measurement of the *cent*, and was proposed by A. J. Ellis in the appendices to his translation of Hermann Von Helmholtz's On The Sensations of Tone. This logarithmic measurement divides the octave into 1200 equal parts, and thus an equal tempered semitone into 100 parts, or cents.

This allows us to compare easily the size of various intervals that result from different tuning systems. For example, a just 5/4 major third has a magnitude of approximately 386 cents. An equal tempered major third is 400 cents. So, we can easily calculate that the just major third is 14 cents narrower than its equal tempered correlate.

To convert a ratio to cents:
$$\frac{\log(f_1/f_2) \times 1200}{\log(2)}$$

CHAOS

In this paper, the term "chaos" is used both analogously and metaphorically to its scientific sense:

Unpredictable and seemingly random behaviour occurring in a system that should be governed by deterministic laws. In such systems, the equations that describe the way the system changes with time are nonlinear and involve several variables. Consequently, they are very sensitive to the initial conditions, and a very small initial difference may make an enormous change to the future state of the system (Oxford Concise Science Dictionary, 1996).

COMMA

Commas are the pitch differences that occur between intervals derived from different limit tuning systems. The thirteenth pitch in a chromatic Pythagorean intonation (531441/524288—twelve 3/2 steps transposed into a single octave) exceeds the octave (2/1) by 23.46 cents and is called the "Pythagorean comma". The difference

between a Pythagorean third (81/64) and a 5-limit major third (5/4) is called the Syntonic comma (81/80)—the Pythagorean third is 21.5 cents larger than the 5/4 major third.

COMPLEXITY

In this paper, the term *complexity* is used roughly and metaphorically to its scientific sense and as it relates to Chaos Theory. Complexity is a description of the relative “levels of self-organisation of a system...It is not necessary for a system to have a large number of degrees of freedom in order for complexity to occur” (Oxford Concise Science Dictionary, 1996).

CRITICAL BAND

A critical band is a frequency span between which the ear is unable to separate two (or more) simultaneous frequencies. In musical context, the critical band contributes to the effect of beating (which is tied to concepts of consonance and dissonance) between pitches or between the partials of simultaneous tones. The magnitude of a critical band is dependent on register and is widest in the lower registers (in mid-register, the critical band is approximately a minor third in magnitude). Maximal roughness/dissonance occurs at about 1/4 the critical band.

The frequency decomposition realized by the basilar membrane is mechanical: the displacements of the membrane are not limited to specific points but are spread out over a portion of it. If two components of a complex signal are close in frequency, these displacements will overlap. There is thus a minimal resolution, called the critical band (Greenwood, 1961), inside of which the ear cannot separate two simultaneous frequencies...

The critical band also influences the perception of beats between two tones. Acoustically, the rate of the beats increases with their frequency difference. As such, as the two pure tones are mistuned from unison, we should hear beats that result from their interaction becoming progressively more rapid. This is in fact what happens at the beginning of the separation. But very soon, (after approximately 10 beats per second or a 10-Hz frequency difference) our perception changes from a slow fluctuation in amplitude toward an experience of more and more rapid fluctuations that produce roughness. Finally, if the separation becomes large with respect to the critical band, the strength of the sensation of beating diminishes, leaving us with the perception of two resolved pure tones. Three very different perceptual regions can therefore arise from the same acoustical stimulus (Pressnitzer & McAdams 2000, 44-46).

ENTROPY

The term “entropy” is a measure of disorder. An increase in entropy is associated with greater levels of disorder. An increase in entropy is associated with a forward time arrow, which is the basis of a theory of *time*, which is a consequence of the Big Bang and thus an expanding universe.

As any real change to a closed system tends towards higher entropy, and therefore higher disorder, it follows that the entropy of the universe (if it can be considered a closed system) is increasing and its available energy is decreasing” (Oxford Concise Science Dictionary, 1996).

FAMILY RESEMBLANCE

Family resemblance is a term Ludwig Wittgenstein uses in his discussion of language games in his book *Philosophical Investigations* to explain that groups of objects and concepts (such as numbers, words, and games) need not be connected by a single common trait. Rather, various objects may share some but not all traits with other members, which may not be the same set of traits shared between two other members of

the group.

FREQUENCY RATIO

Frequency ratios are often used to describe just intervals (intervals based on relations found within the harmonic series). This paper adopts the convention f_2 / f_1 where f_2 represents the higher of the two frequencies. For example, the interval that occurs between the fifth and fourth harmonics is a pure, or just, major third and is described by the ratio 5/4—whose numbers not only refer to harmonic components but also to the relative frequencies of two pitches. The term *frequency ratio*, *ratio*, and *interval* are interchangeable for the purpose of this paper.

Frequency ratios can be used to describe pitch classes as well as interval size. As a means for describing pitch class, all frequency ratios are reduced to within an octave (2/1), which is achieved by expressing the ratio in its simplest form and halving the numerator and/or doubling the denominator until the magnitude is less than 2/1:

e.g.
40/8 simplifies to 10/2,
numerator is halved to get 5/2
denominator is doubled to get 5/4

Using this language, the relative complexity of an interval, which is based approximately on the combined magnitude of the numerator and denominator of the ratio (and *limit* – see below), can be described by the terms *simple* or *complex*—which are at least roughly associated with historical concepts of consonance and dissonance.

To calculate the difference in size between two ratios, divide the larger by the smaller:

$$\begin{aligned} & \mathbf{5/4 - 10/9} \\ = & \mathbf{5/4 \div 10/9} \\ = & \mathbf{5/4 \times 9/10} \\ = & \mathbf{45/40} \\ = & \mathbf{9/8} \end{aligned}$$

To calculate the sum of two ratios, multiply:

$$\begin{aligned} & \mathbf{9/8 + 10/9} \\ = & \mathbf{9/8 \times 10/9} \\ = & \mathbf{90/72} \\ = & \mathbf{5/4} \end{aligned}$$

FUSION:

perceptual fusion

Perceptual (or spectral) fusion describes the effect that allows us to hear a fundamental tone and its harmonic makeup as a single pitch. That is, we do not hear a complex tone as an array of individual simple tones.

One of the first immediate effects of vertical organization is the grouping together in a same image of the multiple partials of a complex sound spectrum, as analyzed by the ear, into coherent parts. This kind of organization allows us to hear a note played by a violin as a *single* note rather than a collection of harmonic partials. The main object of vertical organization is therefore at each and every instant to group what is likely to come from the same acoustic source, and to separate it from what is coming from different acoustic sources. One of the characteristics of music, as we shall see, is to constantly try to break down this simple rule (Pressnitzer & McAdams 2000: 50).

tonal fusion

Tonal fusion occurs when the auditory system interprets certain frequency combinations as comprising partials of a single complex tone... Tonal fusion occurs both in the case of pure tones, and also where concurrent complex tones contain coincident or complimentary partials—consistent with the possible existence of a single complex tone (Huron 1991: 135).

Huron, (citing DeWitt & Crowder, 1987; Stumpf, 1890) provides a scale of intervals which are most likely to produce tonal fusion: The unison (1/1); the octave (2/1); the perfect fifth (3/2); and the perfect fourth (4/3) (Huron 1991: 136).

Tuning differences between equal temperament and just-tuned intervals may be significant in establishing what intervals do and do not tend toward tonal fusion. Tonal fusion is more likely to occur in just intonation, but tuning does not significantly affect the scaling order of intervals and tendencies toward tonal fusion (Huron 1991: 141).

HARMONIC SERIES

The *harmonic series* is a naturally occurring phenomenon that was first observed by Marin Mersenne (1588-1648) and independently explained by William Noble and Thomas Pigot in 1673.

In any complex, periodic sound⁸⁸ a series of tones is generated in exact⁸⁹ integer multiples from the frequency of the fundamental tone. For example, if an instrument sounds 'A' = 110Hz, its constituent elements will include the fundamental tone (the first harmonic or partial) at 110Hz (1 x 110Hz), a second harmonic or partial at 220Hz (2 x 110Hz), a third harmonic/partial at 330Hz (3 x 110Hz), and so on, theoretically to infinity and practically to the upper threshold of hearing (approx. 20,000Hz).

As the series progresses, the magnitude of the interval between adjacent partials becomes increasingly small. The interval between the first and second partials is an octave, between the second and third—a perfect fifth, between the third and fourth—a perfect fourth.



INTONATION CLUSTERS (MY OWN CONCEPT)

In *Fundamentals of Musical Acoustics*, Arthur Benade notes the important relationship that exists between pure intervals (specifically 5-limit intervals) and equal temperament. The intonation of intervals clusters around three points in reference to equal temperament.

⁸⁸ A complex sound is described as any sound with constituent elements beyond a single sine tone. A periodic tone is a tone with a waveform recurring at regular intervals. Any tone with a clearly identifiable pitch is considered periodic.

⁸⁹ (In a perfect world)

One group extends over a range of about 7 cents clustered at a point about 12 cents below the equally tempered setting; a similar group collects around a setting that is 12 cents above equal temperament, and a third collection of settings is found in the immediate neighbourhood of the equal-tempered note (Benade 1976/1990: 295).

Benade goes on to explain that this is why musicians are in the habit of “thinking” a note sharp or flat and that time is the important factor in intonation. Regardless, thinking a note sharp or flat usually results in a pitch shift of about 10 cents.

In *The Fifteen Cent Guitar*, I extend these points of collection to 15-cent increments to include most of the pure intervals found within the first 32 harmonics. (72tet achieves much the same result.) For example, many 7-limit intervals fall sharp or flat of equal temperament by about 30 cents, many 11-limit intervals and compounds of 7 and 5 ($7/4 + 5/4 = 35/32$; 46 cents flat of a major second) fall about 45 cents sharp or flat (11/8 is 49 cents flat). In intervals that are compounds or where the numerator and denominator are both higher primes, the intonation also tends to cluster around these same points. Thus, we find that the intonation of most just intervals derived from the first 16 harmonics, and for the most part to the 32nd harmonic, cluster around multiples of these 15 cent increments from 12tet (or around the 1/12th tone equal tempered steps of 72tet).

JUST INTONATION

The term *just intonation* refers to a subset of the vast/infinite microtonal umbrella based on the frequency ratios that occur naturally between components of the harmonic series. These intervals are smooth and often referred to as “pure” as they sound smooth when compared to the same interval classes occurring in various tempered systems. Some composers build scale systems using only whole number ratios. For example, a common just intonation major scale is described as:

Root	M2	M3	P4	P5	M6	M7	8va
1/1	9/8	5/4	4/3	3/2	5/3	15/8	2/1

where all the intervals are described in relation to the root.

LIMIT

The *limit* of a chord or scale refers to the largest prime number used in describing the scale degrees or intervals ratio form. Our traditional twelve-note system is typically thought of as an 5-limit system because all of the intervals can be described by ratios where the largest prime number is 5—including the augmented triad, for example, which has the interval 25/16, which is a compound of 5 ($5 \times 5 = 25$). The Greek modes are most often considered 3-limit systems.

MICROTONALITY

The term *microtonality* is problematic and is rarely used as the etymology of the word might suggest. For the purpose of this paper, it is defined as any tuning system other than twelve-tone equal temperament (12tet). 12tet is my theoretical reference point, regardless of the many anomalies inherent in precisely reproducing equal temperament to its theoretical ideal, and regardless of the cultural bias this may suggest. Microtonal systems are often described and measured with respect to 12tet. This is, in some cases, problematic but seems to be an efficient method for people coming new to the subject. Whether or not 12tet is a flawed system is beyond the scope of this paper; it is Western music's defacto cultural standard and the reference point readers will most easily understand.

OTONALITY

A tonality expressed by the over numbers of ratios having a Numerary Nexus;

in conventional musical theory, 'major tonality' (Partch 1974, 72).

ROUGHNESS

The roughness of beating tone pairs (measured thanks to experiments involving judgments by human listeners) has been found to depend not on the absolute frequency difference, but rather on the frequency difference related to the width of the critical band for a given center frequency (Fig. 10). Roughness should not therefore be thought of as an acoustic feature of sound, it definitely belongs to the world of perception. This has several consequences. As the width of the band varies (Fig. 8), a given pitch interval won't have the same roughness in different registers. Thirds, for example, are free of roughness in the upper register but can be quite rough in the lower one (Pressnitzer & McAdams 2000: 47).

UTONALITY

One of those tonalities expressed by the under numbers of ratios having a Numerary Nexus—in current musical theory, 'minor' tonality (Partch 1974, 74).

APPENDIX 2 - DATA FROM THE ANALYSIS OF *FIVE MOVEMENTS FOR STRING QUARTET - V, OP.5 (ANTON WEBERN)*

INTONATION OF MELODIC PASSAGE (BARS 1 AND 2) BY FOUR QUARTETS

Quartet	F#	B	G	G#	C	E	C#
Julliard	372Hz	497	397	416	522	333	280
A = 440	+10	+11	+22	+3	-4	+18	+18
corrected	0	+1	+12	-7	-14	+8	+8
Artis	369Hz	489	388	413	524.5	323	273
	-5	-17	-13	-10	+4	-35	-26
corrected	0	-12	-8	-5	+9	-30	-21
Emerson	373Hz	495	393	415	518	330	279
	+14	+4	+4	-1	-17	+2	+11
corrected	0	-10	-10	-15	-31	-12	-3
Kronos	371	494	392	419	520	328	279
	+5	0	0	+15	-11	-9	+11
corrected	0	-5	-5	+10	-16	-14	+6

INTONATION OF VERTICAL CHORDS (BARS 3 AND 4) BY THREE QUARTETS

Webern Chords – Emerson Quartet											
Chord 1			Chord 2			Chord 3			Chord 4		
D	1192Hz +25cents	1191 +24	B^b	930 -4	931 -2	C	1048 +2	1048 +2	G	784 0	
G	789 +11	789 +11	E^b	620 -6	621 -3	E	672? +33	664? +12	B	499 +18	
E^b	626 +10	625 +8	B	497 +11	491 -10	D^b	558 +11	557 *	A^b	416 +3	
B^b	464 -8	464 -8	F#	371 +5	370 0	A	444 +16	444 +16	E	330 +2	
E	332 +12	332 +12	C	259 -17	261 -4	E^b	312 +5	312 +5	B^b	231 -16	
E		83 +12	E		82 -9	E		84 +33			
C#	70 +17		C#	70 +17		C#	70 +17		C#	70 +17	

Webern Chords – Artis Quartet											
Chord 1			Chord 2			Chord 3			Chord 4		
D	1168Hz -10 cents	?	B^b	938 +10	938 +10	C	1044? -4	Stops before	G	781 -7	
G	789 +11	789 +11	E^b	626 +10	626 +10	E	661 +5	cello moves	B	497 +11	
E^b	620 -6	623 +2	B	491 -10	494? 0	D^b	557 +8	to E	A^b	413 -10	
B^b	467 +3	467 +3	F#	370 0	371 +5	A	441 +4		E	332 +12	
E	328 -9	327 -14	C	265 +22	265 +22	E^b	310 -6		B^b	233 -1	
E		163 -19	E		166 +12	E					
C#	136 -33		C#	136 -33		C#	137 -20		C#	137 -20	

Webern Chords – Kronos Quartet											
Chord 1			Chord 2			Chord 3			Chord 4		
D	1170Hz -7 cents	1168 -10	B^b	937 +9	935 +5	C	1048 +2	Stops before	G	781 -7	
G	782 -4	781 -7	E^b	623 +2	623 +2	E	661 +5	cello moves	B	497 +11	
E^b	620 -6	620 -6	B	489 -7	490 -14	D^b	557 +8	to E	A^b	416 +3	
B^b	468 +7	468 +7	F[#]	373 +14	374 +19	A	441 +4		E	330 +2	
E	332 +12	332	C	264 +16	264 +16	E^b	312 +5		B^b	234 +7	
E		166 +12	E		166 +12	E					
C[#]	140 +18		C[#]	140 +18		C[#]	139 +5		C[#]	139 +5	

APPENDIX 3 – DATA FROM ‘COMPANY’ ANALYSIS

Composer: Philip Glass

Performer: Kronos Quartet

Where left blank, data felt to be unreliable

sb = sidebands too strong to analyse

a = average used for cents calculation

section>	I.1.1	I.1.2	I.1.3	I.1.4	I.2.1	I.2.2	I.2.3	I.2.4
voice								
vl n I					E ^{655-659(-6a)}	E ^{same}	E ^{sb}	E ⁶⁵⁷⁽⁻⁶⁾
vl n II	A ⁴⁴⁰	A ⁴⁴⁰	A ⁴⁴⁰	A ⁴³⁹⁽⁻⁴⁾	A ⁴³⁹	A ⁴³³⁻⁴⁴⁰	A	A ⁴⁴¹
	E ³³¹⁽⁺⁷⁾	E	E	E	E ³³⁰	F	G	F ³⁴⁹⁽⁻²⁾
vla	A ²²⁰	A ²²¹⁽⁺⁸⁾	A ²¹⁹⁽⁻⁸⁾	(A)-G ¹⁹⁷⁽⁺⁹⁾	A ²¹⁷⁻²²²	A ²¹⁹	A ^{same}	(A)-G ¹⁹⁸⁽⁺¹⁸⁾
vlc	E ^{165.5(+7)}	F ^{175.5(+9)}	G ^{196.5(+4)}	(F-E- Bb) ¹¹⁷⁽⁺⁷⁾	E ^{163.5(-14)}	E ¹⁶⁶⁽⁺¹²⁾	E ^{same}	(E)-Bb ^{115.5(-16)}
	II.1.1	II.1.2	II.1.3	II.1.4	II.2.1	II.2.2	II.2.3	II.2.4
vl n I	C	C	C	C	E-A-C-E (repeated)			
vl n II	E	E	E	E				
vla			C	C				
vlc			A	A				

I.3.1	I.3.2	I.3.3	I.3.4	I.4.1	I.4.2	I.4.3	I.4.4
E661(+5)a	E	E	E	E	E-C	C	C-B
A	A	A	A	A	A	A	A
E	F	G	F	E	F#	G#	F#
C	C	C	C	C	C	C	C
A	A	A	A-G	A	A	A	A-G
E165	E-C	E	E-Bb	E	E	D	E-Bb
II.3.1	II.3.2	II.3.3	II.3.4	II.4.1	II.4.2	II.4.3	II.4.4
F-A-C-E-F (repeated)				E-A-D	E-A-E	E-A-D	E-A-C

APPENDIX 4 – GRISEY ANALYSIS DATA

	G#	E	B	F#	D -25	B
A	417Hz	331 – 334 Hz	248	370 – 371	297	246
	+7	+7 / +23 (vibrato)	+7	0 / +5	+20	-7
A'	417	336 (328 – 336)	495 – 494	370	295 – 296	246
	+7	+33 (don't hear it)	+4 / 0	0	+9 / +14	-7
A''	416	330	494 – 495	370	297	246
	+3	+2	0 / +4	0 / +5	+20	-7

Passage B

	G#	E	B	A \sharp	E
	41 6	332 – 334 – 335	250	467 – 468	334
	+3	+12 / +23 / +28	+21	+3 / +7	+22

Passage C

	E	B	A \sharp
1	330	(250)	470
	+2	(+20)	+14
2	E?	B?	A?
	336	250	462
	+33	+20	A \sharp -16
3	D \sharp	A \sharp	A
	?	485	478
	?	A \sharp -31	A+43
	D \sharp	A \sharp	A?
	310	476	(478)
	-6	+36	A +43
	D \sharp	A \sharp	G \sharp
	307 – 308	463	413
	-23	-12	G \sharp -10

Passage D						
G#	E	D?	C#	A#	F	F#
419	331	571	549	462 – 464	715 – 716	744
+15	+7	-49	C# -17	A# -16/-8	+40 / +43	+9
F#	D?	A#	E	B	F#	
745	577	489	329	498	374 – 376	
+11	-31	-17	-3	+14	+18 / +28	

**APPENDIX 5 - INTONATION OF PAGE 6 OF BEN JOHNSTON'S
QUARTET No. 4 PERFORMED BY THE KRONOS QUARTET.**

Page 6 – Bar 1											
Each Row >	Pitch In HZ	(ideal) actual									
Vln I	D ⁻ 295Hz	(-18) +8	G ⁻ 398	(-20) +26	A ⁻ 443	(-38) +12	B ⁻ 498	(-34) +14	G ⁻ 393	(-20) +4	
Vln II	G ⁻ 195	(-20) -9	E ⁻ 330	(-36) +2			D ⁻ 293	(-18) -4	E ⁻ 332	(-36) +12	
Vla	D ⁻ 293	(-18) -4	B ⁻ 248	(-34) +7	A ⁻ 221	(-38) +8	G ⁻ 197	(-20) +9	B ⁻ 250	(-34) +21	
Vc	G ⁻ 195	(-20) -9	E ⁻ 164	(-36) -9			G ⁻ 197	(-20) +9	A ⁻ 219	(-38) -8	

Page 6 – Bar 2												
Each Row >	Pitch In HZ	(ideal) actual										
Vln I	B ^{7b} 466	(-53) -1					A ⁻ 444	(-38) +16	G ⁻ 370	(-20) 0	E ⁻ 330	(-36) +2
Vln II	F7 354	(-51) +23	G ⁻ 393	(-20) +4	E ⁻ 332	(-36) +12			D ⁻ 293	(-18) -4	B ⁻ 250	(-34) +21
Vla	D ⁻ 293	(-18) -4			C ⁻ 262	(-22) +2			B ⁻ 249	(-34) +14		
Vc	G ⁻ 197	(-20) +9							F# 186	(-32) +9	C# 137	(-30) -20

Page 6 – Bar 3												
Each Row >	Pitch In HZ	(ideal) actual										
Vln I	D ⁻ 295	(-18) +8					D ⁻ 296	(-18) +14	F7 351	(-51) +9	E ⁻ 330	(-36) +2
Vln II	C7 262	(-49) +2	B ^{7b} 233	(-53) 0	C7 261	(-49) +2			D ⁻ 296	(-18) +14	G ⁻ 197	(-20) +9
Vla	A 222	(-16) +16	G ⁻ 197	(-20) +9	A 221	(-16) +8	B ^{7b} 236	(-53) +22	C7 264	(-49) +16	B ⁻ 234	(-34) +7
Vc	D ⁻ 147										C ⁻ 131	(-22) +2

Page 6 – Bar 4											
Each Row >	Pitch In HZ	(ideal) actual									
Vln I	D ⁻ 295	(-18) +8	C7 262	(-49) +2	D ⁻ 293	(-18) -4					
Vln II	A 222	(-16) +16							B ^{7b} 231	(-53) -16	
Vla	A 222	(-16) +16	F7 174	(-51) -6	D ⁻ 148	(-18) +14	F# 184	-32 -10	G ⁻ 197	(-20) +9	
Vc	D ⁻ 148	(-18) +14							G ⁻ 197	(-53) +9	

APPENDIX 6

SCALING DATA FOR *THE BEATEN PATH*

$f_p = p^{(x)} \times (f_1)$ where p = a ratio (or a partial number)							
$f_1 = 392\text{Hz}$ (G below A 440Hz)							
$x =$	3/4	15/16	1/1	9/8	6/5	4/3	3/2
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.1	-49.80	-11.17	0.00	20.39	31.56	49.80	70.20
0.2	-99.61	-22.35	0.00	40.78	63.13	99.61	140.39
0.3	-149.41	-33.52	0.00	61.17	94.69	149.41	210.59
0.4	-199.22	-44.69	0.00	81.56	126.26	199.22	280.78
0.5	-249.02	-55.87	0.00	101.96	157.82	249.02	350.98
0.6	-298.83	-67.04	0.00	122.35	189.38	298.82	421.17
0.7	-348.63	-78.21	0.00	142.74	220.95	348.63	491.37
0.8	-398.44	-89.39	0.00	163.13	252.51	398.43	561.56
0.9	-448.24	-100.56	0.00	183.52	284.08	448.24	631.76
$x = 1$	-498.04	-111.73	0.00	203.91	315.64	498.04	701.96
1.1	-547.85	-122.90	0.00	224.30	347.21	547.84	772.15
1.2	-597.65	-134.08	0.00	244.69	378.77	597.65	842.35
1.3	-647.46	-145.25	0.00	265.08	410.33	647.45	912.54
1.4	-697.26	-156.42	0.00	285.47	441.90	697.26	982.74
1.5	-747.07	-167.60	0.00	305.87	473.46	747.06	1052.93
1.6	-796.87	-178.77	0.00	326.26	505.03	796.87	1123.13
1.7	-846.68	-189.94	0.00	346.65	536.59	846.67	1193.32
1.8	-896.48	-201.12	0.00	367.04	568.15	896.47	1263.52
1.9	-946.29	-212.29	0.00	387.43	599.72	946.28	1333.71
2	-996.09	-223.46	0.00	407.82	631.28	996.08	1403.91
2.1	-1045.89	-234.64	0.00	428.21	662.85	1045.89	1474.11
2.2	-1095.70	-245.81	0.00	448.60	694.41	1095.69	1544.30
2.3	-1145.50	-256.98	0.00	468.99	725.97	1145.49	1614.50
2.4	-1195.31	-268.16	0.00	489.38	757.54	1195.30	1684.69
2.5	-1245.11	-279.33	0.00	509.78	789.10	1245.10	1754.89
2.6	-1294.92	-290.50	0.00	530.17	820.67	1294.91	1825.08
2.7	-1344.72	-301.67	0.00	550.56	852.23	1344.71	1895.28
2.8	-1394.53	-312.85	0.00	570.95	883.80	1394.51	1965.47
2.9	-1444.33	-324.02	0.00	591.34	915.36	1444.32	2035.67
3	-1494.13	-335.19	0.00	611.73	946.92	1494.12	2105.87
3.1	-1543.94	-346.37	0.00	632.12	978.49	1543.93	2176.06
3.2	-1593.74	-357.54	0.00	652.51	1010.05	1593.73	2246.26
3.3	-1643.55	-368.71	0.00	672.90	1041.62	1643.53	2316.45
3.4	-1693.35	-379.89	0.00	693.29	1073.18	1693.34	2386.65
3.5	-1743.16	-391.06	0.00	713.69	1104.74	1743.14	2456.84
3.6	-1792.96	-402.23	0.00	734.08	1136.31	1792.95	2527.04
3.7	-1842.77	-413.41	0.00	754.47	1167.87	1842.75	2597.23
3.8	-1892.57	-424.58	0.00	774.86	1199.44	1892.55	2667.43
3.9	-1942.38	-435.75	0.00	795.25	1231.00	1942.36	2737.62
4	-1992.18	-446.93	0.00	815.64	1262.57	1992.16	2807.82
4.1	-2041.98	-458.10	0.00	836.03	1294.13	2041.97	2878.02

4.2	-2091.79	-469.27	0.00	856.42	1325.69	2091.77	2948.21
4.3	-2141.59	-480.44	0.00	876.81	1357.26	2141.57	3018.41
4.4	-2191.40	-491.62	0.00	897.20	1388.82	2191.38	3088.60
4.5	-2241.20	-502.79	0.00	917.60	1420.39	2241.18	3158.80
4.6	-2291.01	-513.96	0.00	937.99	1451.95	2290.99	3228.99
4.7	-2340.81	-525.14	0.00	958.38	1483.51	2340.79	3299.19
4.8	-2390.62	-536.31	0.00	978.77	1515.08	2390.60	3369.38
4.9	-2440.42	-547.48	0.00	999.16	1546.64	2440.40	3439.58
5	-2490.22	-558.66	0.00	1019.55	1578.21	2490.20	3509.78
5.1	-2540.03	-569.83	0.00	1039.94	1609.77	2540.01	3579.97
5.2	-2589.83	-581.00	0.00	1060.33	1641.33	2589.81	3650.17
5.3	-2639.64	-592.18	0.00	1080.72	1672.90	2639.62	3720.36
5.4	-2689.44	-603.35	0.00	1101.11	1704.46	2689.42	3790.56
5.5	-2739.25	-614.52	0.00	1121.51	1736.03	2739.22	3860.75
5.6	-2789.05	-625.70	0.00	1141.90	1767.59	2789.03	3930.95
5.7	-2838.86	-636.87	0.00	1162.29	1799.16	2838.83	4001.14
5.8	-2888.66	-648.04	0.00	1182.68	1830.72	2888.64	4071.34
5.9	-2938.47	-659.21	0.00	1203.07	1862.28	2938.44	4141.53
6	-2988.27	-670.39	0.00	1223.46	1893.85	2988.24	4211.73
6.1	-3038.07	-681.56	0.00	1243.85	1925.41	3038.05	4281.93
6.2	-3087.88	-692.73	0.00	1264.24	1956.98	3087.85	4352.12
6.3	-3137.68	-703.91	0.00	1284.63	1988.54	3137.66	4422.32
6.4	-3187.49	-715.08	0.00	1305.02	2020.10	3187.46	4492.51
6.5	-3237.29	-726.25	0.00	1325.42	2051.67	3237.26	4562.71
6.6	-3287.10	-737.43	0.00	1345.81	2083.23	3287.07	4632.90
6.7	-3336.90	-748.60	0.00	1366.20	2114.80	3336.87	4703.10
6.8	-3386.71	-759.77	0.00	1386.59	2146.36	3386.68	4773.29
6.9	-3436.51	-770.95	0.00	1406.98	2177.92	3436.48	4843.49
7	-3486.31	-782.12	0.00	1427.37	2209.49	3486.28	4913.69

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