AN INVESTIGATION OF LOWER SECONDARY PUPILS' IMAGES OF MATHEMATICS AND MATHEMATICIANS

by

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ABSTRACT

This thesis reports on a three-part research project in which the images of mathematics and mathematicians held by lower secondary pupils were investigated.

A survey tool which asked pupils to *draw a picture of a mathematician at work*, and which included a Likert-type scale and open-ended writing prompts, was designed and developed for use in an international study of pupils in five countries (n = 476). The results indicate that while some pupils hold stereotypical images in common, all pupils appear to know very little about mathematicians and the work they do. Mathematicians' invisibility to pupils of this age appears to affect their images of mathematics.

The tool was refined and utilised again as part of two interventions in the United States: the first attempted to see if images would be affected by a unit in graph theory and discrete mathematics topics (n = 28); the second brought pupils (n = 174) together with a panel of mathematicians. Each intervention had different strengths, but both widened pupils' views of mathematics, enabling them to see it as more than just a study of numbers.

In a third small study, professionals in the mathematics field (n = *106*) from ten countries were asked in a short survey to comment on *Who is a mathematician?* and *Who may call oneself one?* Findings of this portion of the study indicate a lack of a unified vision among members of the mathematics community and some evidence of an elitism which would restrict who may define themselves as a mathematician.

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AUTHOR'S DECLARATION

At no time during the registration for the degree of Doctor of Philosophy has the author been registered for any other University award.

Publications and presentation of work:

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Berry, J. & Picker, S. H. (2000). Your pupils images of mathematicians and mathematics. Mathematics in School, 29(3), 24-26.

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Research Seminars:

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Chapter 1: INTRODUCTION

1.1 THE THESIS

This thesis reports on three interrelated studies designed to investigate lower secondary pupils' images of mathematics and mathematicians, using a qualitative research model. A survey tool was designed, developed and later refined for use in the first and third sections of the study.

There were two preliminary investigations, one of lower secondary pupils' ($n = 476$) images of mathematicians and mathematics in five countries (United Kingdom, United States, Finland, Sweden and Romania), and the second of professionals in the mathematics field ($n = 106$) mostly from the United States and United Kingdom, but including one or two respondents from eight other countries. In this aspect of the project professionals were asked their views on *Who is a mathematician?* And *Who may call oneself one?*

The third aspect of the project consisted of two interventions in New York City: in a small intervention ($n = 28$) a unit in graph theory and discrete mathematics topics was taught to a class of lower secondary pupils to ascertain if this could oppose negative stereotypes and misconceptions; in the larger intervention ($n = 174$) pupils met with a panel of eight mathematicians from the United States, England, Wales, Nigeria and Romania to question them and learn about their work and lives. Pupils were administered the survey tool once more to determine the success of such an intervention in opposing negative images and misconceptions of mathematics and mathematicians.

1.2 THE ORIGINS OF THE THESIS

I consider that a substantial part of my mathematical education began anew when in 1990 I participated in a summer program for teachers, the National Science Foundation funded, *Leadership Program in Discrete Mathematics* (Rosenstein & Debellis, 1997) at Rutgers University in New Jersey. Until that time I had very little understanding of the applications of mathematics and had never knowingly met a mathematician.

During that summer, I learned from researchers and professors of mathematics and computer science about a whole contemporary side of mathematics I never knew existed. The topics studied were graph theory, vertex colouring, algorithms and combinatorics (Roberts, 1984; Cozzens & Porter, 1987). In addition, I became aware of what was referred to as *the mathematics image problem* (Malkevitch, 1989; 1997)—and I began to think clearly about mathematicians and what they do for the first time.

The scope of mathematics and the fact that it is a field constantly being added to and in motion, was a revelation to me. I see now, only in retrospect, that a large portion of my belief system about mathematics had been altered through that experience, and it continued to change as I returned to Rutgers each summer, now a member of the program's staff.

These experiences encouraged me to be increasingly interested in my own professionalism as a teacher, including attending international conferences where I could see the field of mathematics education in a broader perspective.

I have written of what occurred when I returned to teaching at the New York City high school at which I then worked, after that first summer attending the *Leadership Program* (Picker, 1997). In that account, I mentioned in particular, a moment during one of the last classes, which still surprises me. My students had filled out a questionnaire early in the term, in which they had been asked, *Do you think you'll ever be a mathematician?* I reminded the students of this during the last week of the semester in which we had investigated topics in discrete mathematics.

A student in the class, José, had volunteered, "You know—we really like this, but we don't think we're going to be mathematicians!" And I had said, "Well, I have to tell you, I've learned that you *are* mathematicians because you've been doing mathematics. And a mathematician is a person who does mathematics." And at this, the students had suddenly and spontaneously burst into applause.

The students were very low achieving 15 to 17-year olds who had made no pretence of liking mathematics. And yet at the end of that term, it was clear to me that something had changed for these students—some prior image of mathematics, mathematicians, and themselves.

It was in those early responses that my interest in understanding pupils' images of mathematics and mathematicians began. Then, some years later, as I was beginning my research, I saw drawings that pupils in a New York City middle school had created of their perceptions of a mathematician (see **Figures 1.1** and **1.2**, next pages). They were so dramatic—at once funny and also disturbing, and I was determined to investigate the questions these images provoked.

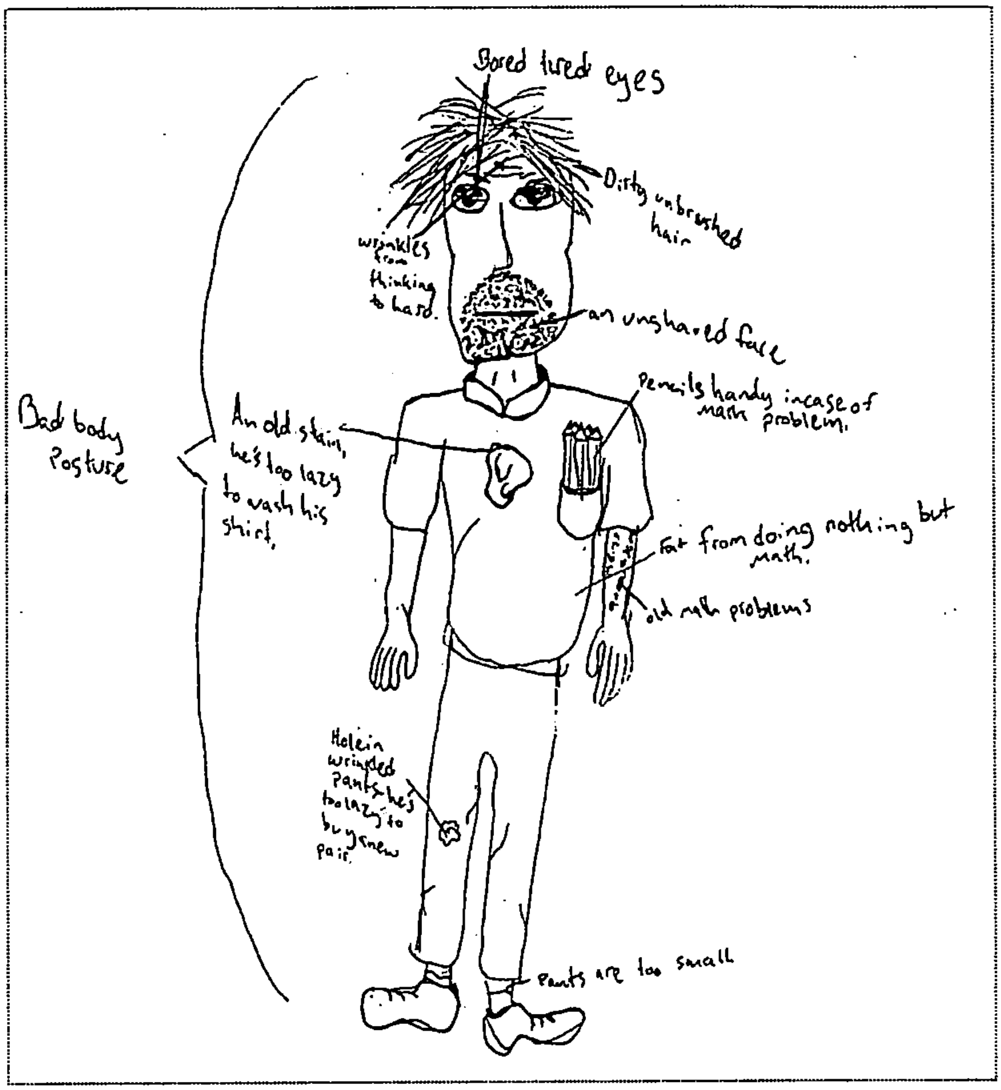

Figure 1.1 Drawing by male pupil (7th grade—12/13 years old)

"Mathematicians," the pupil-artist wrote in a list accompanying **Figure 1.1** "have:

-No friends. (Except other mathematicians)
-Not married or seeing anyone.
-Usually fat.
-Very unstylish.
-Wrinkles in forehead from thinking so hard.
-No social life whatsoever.
-30 years old.
-A very short temper."

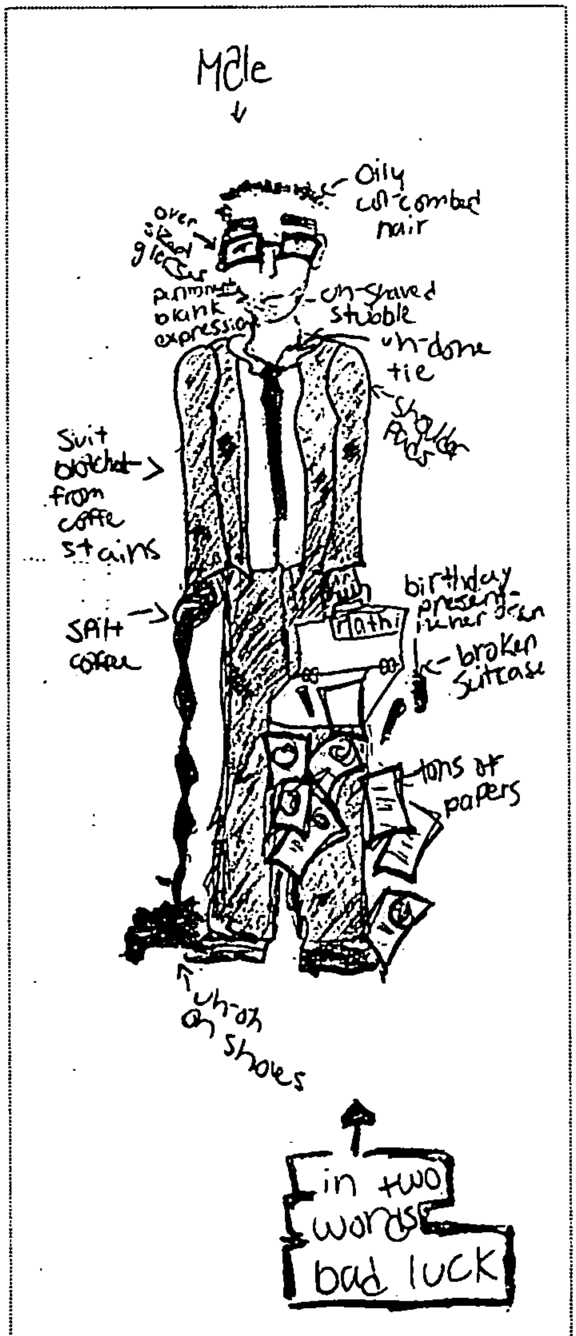

Figure 1.2 Drawing by female pupil (7th grade—12/13 years old)

1.3 THE OBJECTIVES AND RESEARCH QUESTIONS OF THE THESIS

There are many questions arising from the images in **Figures 1.1** and **1.2** and these form the basis for this thesis. They are:

- Where do these images come from?

• What do such images show about pupils' knowledge about what mathematicians actually do?

• What do these drawings show about these pupils' attitudes to and beliefs about mathematics?

• Are these drawings only common to pupils in New York City?

• Would a pupil who sees mathematicians in this way ever seriously consider studying mathematics?

. Do mathematicians themselves have anything to do with these images and with negative pupil attitudes?

• Is it possible for pupil attitudes to change?

• Can negative images of mathematicians be opposed and can they change?

. Would a change in attitude toward mathematicians affect a pupil's attitude toward mathematics?

And, because of my interest in graph theory and discrete mathematics and their applied nature:

. Could graph theory enable pupils to like the subject of mathematics more?

. Might graph theory and topics in discrete mathematics provide a means to a pupil understanding what a mathematician does?

. Will a pupil's greater understanding of what a mathematician does change stereotypical images?

At about the same time that I began formulating these questions, I discovered that there had been a series of studies conducted in which pupils were asked to draw a scientist, the Draw-A-Scientist-Test (DAST) (see Chambers, 1983; Finson, Beaver & Cramond,1995; Huber & Burton, 1995).

To date there have been studies of images of mathematics but there has been very little inquiry into pupils' images of mathematicians. Using a variation of the DAST with lower secondary pupils, this study uncovered images which were found to be held in common in the five different countries.

One of the findings arising from this project, is that pupils don't know enough about mathematicians and what they really do, to depict them fairly in a drawing. I came in time to conclude that for all practical purposes, mathematicians are invisible to school age pupils. Yet, if, as Jaworski (1994, p. 218) seems to imply, learning mathematics is related to *being* a mathematician, what she calls "being mathematical within a mathematical community," then mathematicians' invisibility to pupils needs to change.

And if pupils' images of mathematicians indicate, as do **Figures 1.1** and **1.2**, that they perceive mathematics to be an unattractive field of study (NSF, 1998), then the decline in enrolment of students in advanced mathematics courses (Garfunkel and Young, 1998) is sure to continue unabated, fulfilling predictions of an increasing shortage of mathematicians and teachers of the subject (National Research Council, 1989).

Rock and Shaw (2000, p. 550) suggest that understanding what children think about mathematicians and being able to alleviate children's misconceptions about them, may "facilitate and broaden children's thinking about their roles as future mathematicians."

That was a central goal of this study.

Yet canvassing persons in the mathematical field—school teachers, administrators, professors and mathematicians in industry, I found that there are divergent views as to what defines a mathematician and who may call themselves one. There appears to be a group who sees the title as inclusive and a group for whom the term mathematician is quite exclusive. This has an effect on society, on pupils and on their teachers.

Graph theory appealed to me as a place to begin to have pupils understand better how mathematicians spend their time. However, teaching a unit in graph theory with some topics in discrete mathematics, produced an effect, but it was a limited one. It did not seem clear that pupils had greater insight into the work of a mathematician. The largest change was that pupils now felt that mathematics was not merely the study of number, and they looked forward to studying mathematics further.

Eventually as the project proceeded, it seemed that I had uncovered negative images but pupils were still left with them. I felt that the ethical thing to do would be to try to leave pupils better off if possible. That is when I had the idea of inviting a panel of mathematicians to meet with pupils, answer their questions, and explain to them some of what a mathematician's work entails.

This intervention produced a larger positive effect on pupils' attitudes and significantly changed many of their drawings. In post-tests given to pupils who had been present at what came to be called the *Mathematicians Panel*, certain stereotypical prototypes that had been drawn in the pre-Panel surveys never reappeared in the post-Panel surveys, and pupils' own comments showed that they had come to critically reconsider images they had held about mathematicians and about mathematics itself.

Chapter 2: REVIEW OF THE LITERATURE

2.1 INTRODUCTION TO THE REVIEW

This review of the literature is in several sections. The first sections look at the conception of mathematical knowledge and the current state of mathematics education particularly with regard to confusions and dichotomies that have attended and affected them.

The next section looks at the literature around images and stereotypes of mathematics and mathematicians and what has been called the *mathematics image problem*, as a contributor to what is increasingly perceived as a crisis in mathematics. The section includes literature around what I am calling mathematics' and mathematicians' *invisibility factor*.

Section four deals with the affective domain in relation to pupils and teachers: their attitudes to mathematics, their belief systems and their images of mathematics. This section also includes a review of literature on critical thinking and the possibility of attitude change.

In section five the literature of the Draw-A-Scientist Test, or DAST is looked at as a preparation for using a variation of this test to a slightly different purpose in mathematics classes. Finally, section six deals with graph theory and discrete mathematics as a possible intervention for lower secondary school pupils' stereotypical images.

2.2 THE CONCEPTION OF MATHEMATICAL KNOWLEDGE

There is widespread agreement among scholars that mathematics arose over thousands of years to help societies cope with very practical everyday problems (Bell, 1945; Boyer, 1968; Borel, 1983; Eves, 1983; Katz, 1993; Stewart, 1995; Gullberg, 1997). Forms of counting arose—in many cases independently of each other—in nearly every ancient culture for which we have records (Ball, 1922; Conant, 1956). Yet, the knowledge of mathematics, especially as extensions were developed beyond practical necessity, was considered a "tool of power", and was generally passed on by oral tradition to a privileged few (Katz, 1993, p. 1).

Although written records are rare, the Rhind Papyrus of about 1650 B.C. which was probably based on an older document of 1841-1801 B.C., is the most comprehensive mathematical treatise extant from ancient Egypt (Boyer, 1968). On the first page, its title is stated in red:

> Correct method of reckoning, for grasping the meaning of things and knowing everything that is, obscurities...and all secrets. (Robins & Shute, 1987, p.11)

Bell (1945) believes that the same minds that created early number also helped invent or transmit number mysticism and even astrology. This parallel tradition of utility and secrecy, science and alchemy was continued in ancient Greece with the Pythagoreans, about 540 B.C., and their closed mathematical society, described as a cult or a closely-knit brotherhood, with secret rites and observances (Bergamini et al, 1963; Eves, 1983; Schecter, 1998).

The Pythagoreans were required to swear an oath not to reveal to anyone the teachings or secrets of the school. Pythagoras himself divided his lecture audiences into two separate classes, called probationers and Pythagoreans. While the majority in these audiences were probationers, his major discoveries were only revealed to the Pythagoreans (Ball,1922, p. 20).

When the Pythagoreans uncovered a group of numbers which they considered disturbing, but which today we know to be irrational numbers, they sought to suppress this knowledge because it didn't fit with their mystical philosophy of numbers as worthy of worship (Bergamini et al, 1963; Schecter, 1998). And when, insisting on the perfection of the number 10, they only found nine planets, Aristotle (1947, p. 255) in his *Metaphysics* states, "to meet this, they invent a tenth—the 'counter earth'."

In time, the Pythagoreans were perceived to hold so much power that feelings of resentment arose in the population and eventually gave way to fury. Pythagoras and a group of his followers were murdered. Yet in time a new group arose, calling themselves Pythagoreans and heirs to the school and they worked to continue Pythagoras' teachings (Ball, 1922).

Still, as mathematics developed over the centuries following the themes of number, magnitude and form, its general purpose continued to support practical needs. Kline (1962) asserts that unlike studies in the sciences, very little of the mathematics that has been created has been discarded, but Bell (1945, p. 12) has contradicted this, holding that if it "were true, mathematics would be the one perfect achievement of a race admittedly incapable of perfection." Instead, mathematics has continually moved forward discarding the "trivial, inadequate, or cumbersome...[and] definitely fallacious."

In the 17th century it was still difficult for one to make a living as a mathematician. Rather, those who were talented in mathematics found work as secret accountants on the finances of wealthy merchants (Hoffman, 1998).

Writing on Fermat, Hoffman (p. 185) asserts that the "tradition of clandestine work" was not only practised by those mathematicians "cooking the books of rich people" but the noted mathematician tended to keep his mathematical discoveries to himself, as well.

The tradition of science and alchemy which dates back to before the Pythagoreans, continued to have an intermingled history for a long time, and even Newton, whose name is nearly synonymous with physical mechanics is said to have frequently dabbled in alchemy (Jacobs, 2000).

A characteristic of the 18th century and the Age of Enlightenment was a tendency to apply all of the quantitative methods that had been successful in the physical sciences to aspects of society. It is not surprising then, that two of the greatest mathematicians of the time, Leonhard Euler (1707-1783) and Jean Le Rond d'Alembert (1717-1783) were writing on a wide variety of practical problems, including life expectancy, the value of annuities, lotteries, and other aspects of social science (Boyer, 1968).

When Euler subsequently wrote papers on pure number theory, he apparently felt that he had to justify it as being as important as his other work (Borel, 1983). This belief that pure mathematics deeply differs from applied mathematics, would, as time progressed, come to represent another duality in the history of mathematics (see Steen, 1978).

The roots of pure mathematics, however, go back at least as far as the Pythagoreans. Spurred by intellectual curiosity, they became interested in numbers as abstractions and figurate numbers in particular. This came in time to have a large effect on the contributions of such mathematicians as Fermat from the 18th through the 20th centuries (Bell, 1945; Schecter, 1998).

With the 19th century came great changes in the nature of mathematical thought and mathematics' initial concern with the material world began to shift, as "pure mathematics freed itself from limitations suggested by observations of nature." (Boyer, 1968, p. 1) Steen suggests that this swift growth proceeded in two opposite directions.

Although they came from common roots, one direction mathematics followed was "applied, concrete and externally influenced, the other theoretical, abstract and introspective." (Steen, 1978, p. 2) The effect of the latter direction, in particular, was profound. Kline (1959, p. viii) believed that it produced "an undue emphasis on abstraction, generality, rigor, and logically perfect deductive structures..." through which, he felt, mathematicians were losing sight of the meaning and value of mathematics in relation to nature, effectively isolating mathematics.

Kline (1959; 1980) described a schism that had developed in mathematics from the turn of the 20th century, between those who followed what he termed, honourable motivations, and those mathematicians "who, sailing with the wind, would investigate what strikes their fancy." Steen described a "schism and gulf" (p. 11) as well, but he felt it had come to be between mathematics and society. Both Kline and Steen felt the consequences to come would be enormous. Kline bemoaned one unfortunate consequence that had already occurred—that "the relationship of mathematics to the study of nature is not presented in our dry and technique-soaked textbooks." (p. viii) More ominously, Steen felt that society would eventually be unwilling to support a technology it could not comprehend.

The dichotomies which have arisen through the history of mathematics continue in the dichotomies present in the philosophy of mathematics, and these include absolutism, which argues that mathematics is infallible, certain, unambiguous, and fixed, and fallibilism, which views mathematical truth as imperfect, having contradictions and paradoxes, and subject to uncertainty. While fallibilism believes that mathematics involves human agency and is connected with and a part of all other areas of knowledge, absolutism holds that mathematics is a gradually uncovered truth existing independent of humanity (Kline, 1980; Ernest, 1998b; Davis & Hersh, 1981). These differing views will produce differing images of mathematics in their adherents. Included too, among ideas that affect the image of mathematics are those of Platonism and formalism. Davis and Hersh (1982) characterise Platonism as seeing mathematics as existing outside human existence and something, which can only be discovered.

This remoteness of mathematics to humanity is an image shared by formalism. Formalism states that mathematics consists only of axioms, definitions, and theorems—it's just formulas, and it is devoid of any meaning (Lange, 1987).

In time, this formalist style increasingly came to influence pedagogy at the university level, and eventually moved down into kindergarten in the name of the "new math" (Lange 1987).

But Hersh (1997, p.27) writes,

> A credible philosophy of mathematics must accord with the experience of *teaching and learning* mathematics. To a formalist or Platonist who presents an inhuman picture of mathematics, I ask, 'If this were so, how could anyone learn it?'

And he believes that,

> Platonism can justify a student's certainty that it's impossible for her/him to understand mathematics. Platonism can justify the belief that some people can't learn math. Elitism in education and Platonism in philosophy naturally fit together.

Continuing, Hersh makes a case for a philosophy, which differs greatly with Platonism and formalism: humanism.

> Humanist philosophy, on the other hand, links mathematics with people, with society, with history. It can't do damage the way formalism and Platonism can. It could even do good. It could narrow the gap between pupil and subject matter." (pp. 237-238)

According to Lange (1987, p. 98), it was Hans Freudenthal who first introduced the "slogan", "Mathematics is a *human* activity."

2.3 ON MATHEMATICS EDUCATION—WHAT *IS*

The characterisation of those philosophies that dominate mathematics education today—what is—and how or why this came about, can make clearer what the state is of mathematics education, which affects pupils today.

It is hard to find a more encompassing statement on education bearing as much meaning for educators today as Aristotle's opening sentence of the *Metaphysics*: "All men by nature desire to know." (Aristotle, 1947)

Aristotle believed that knowledge derived from the senses, and that in obtaining knowledge, the mind became in a sense all things, but without, as McKeon (1947, p. xvi) noted, "transforming things into changeless ideas, as Plato did." Nor did Aristotle separate logic from art, thought from feeling. He wrote in his *Metaphysics* (1947, p. 285):

> And thought thinks on itself because it shares the nature of the object of thought; for it becomes an object of thought in coming into contact with and thinking its objects, so that thought and object of thought are the same. For that which is capable of receiving the objects of thought, i.e. the essence, is thought. Therefore the possession rather than the receptivity is the divine element which thought seems to contain, and the act of contemplation is what is most pleasant and best.

What Aristotle is pointing to is a kinship between the thinker and the object thought about, "so that thought and object of thought are the same." This implies the making of meaning and the constructing of relationships (Dossey, 1992), with knowledge becoming a part of and enlarging oneself, and it is an active rather than passive process.

If we look at another writer on learning, in this case, Colburn on mathematics, there is a relation to Aristotle's philosophy—the idea of finding meaning, of discovering how new knowledge fits into one's existing knowledge, and then, through it inducing principles:

> Almost all who have ever fully understood arithmetic have been obliged to learn it over again in their own way. And it is not too bold an assertion to say, that no man ever actually learned mathematics in any other method than by analytic induction; that is, by learning the principles by the examples he performs; and not by learning principles first, and then discovering by them how the examples are to be performed.

The surprising thing is that this was first written in 1821! Colburn (1891, p. 208-212) shows that the "common method" of teaching arithmetic, however,

> ...entirely reverses the natural process; for the pupil is expected to learn general principles before he has obtained the particular ideas of which they are composed.

So why are we still struggling to implement teaching methods that agree with a philosophy expressed first in antiquity and re-expressed nearly two hundred years ago?

In *Everybody Counts*, a 1989 report on the future of mathematics education in the United States, there was a grim picture of the current state of mathematics education. A decade later, this is largely unchanged:

> Unfortunately, as children become socialised by school and society, they begin to view mathematics as a rigid system of externally dictated rules governed by standards of accuracy, speed and memory. Their view of mathematics shifts gradually from enthusiasm to apprehension, from confidence to fear. Eventually, most students leave mathematics under duress, convinced that only geniuses can learn it. (NRC, p. 43)

How did mathematics education come to this?

Resnick and Hall (1998) claim that the history of education in America is one of tinkering with institutional arrangements to make schools efficient, rather than implementing true educational reform. They cite two factors from the early 20th century that had a large impact on education in America and in

England as well—effects which continue today, keeping any philosophy of education akin to Colburn's from taking hold in what they call *sustainable education reform*: Thorndike's *associationist* theory of learning, which the authors refer to as a *core theory of learning;* and, a widely held assumption that a child's aptitude is the most important aspect of learning and that it is largely hereditary. This came out of theories of inherited intelligence and social Darwinism, and the authors refer to it as a *core theory of aptitude.*

Thorndike was a psychologist whose associationist theory of learning saw knowledge as bonds or links between pairs of mental entities or between an external stimulus and an internal mental response. The theory claimed that learning was a matter of changing the strengths of these bonds through practice in which rewards or punishment were used to facilitate this strengthening or weakening. Most of this theory grew out of laboratory research conducted for the most part on animals.

In time, Thorndike created an associationist theory of instruction, which he analysed particularly in relation to school arithmetic. The operations of arithmetic came to be decomposed into hundreds of separate bonds in Thorndike's *The Psychology of Arithmetic* (see also Kilpatrick, 1992). Speed and accuracy were paramount, along with frequent testing to determine which bonds had been or had not been mastered.

As Resnick and Hall point out (p. 94), Thorndike's theory "has been absorbed into the core pedagogy of American schools", and particularly in elementary schools, textbooks are still filled with practise on "minimally connected bits of information". Workbooks and drill-sheets continue this practise, as does the terse form of question and answer dialogue in most mathematics classes today.

"These," the authors conclude (p. 96), "are the familiar practises that teachers continue to use and that families and communities still recognise." And, they hold, the associationist classroom is valued more for the order and discipline it was designed to produce, than for intellectual engagement or autonomy of thought—neither of which it much calls nor allows for.

The idea of practising "minimally connected bits of information" has also been referred to as a "piece-meal approach" which strips all meaning and number sense from the mathematical activity. Brown (1981, p.31) has commented on the effect this has on pupils, and it is in keeping with the goals of the associationist theory:

> Schools are notorious for encouraging a "piece-meal" approach to virtually everything. Youngsters are given very little opportunity to reflect upon how the pieces fit together. Frequently, there is no rationale, and if there is one, it may be frightening—dealing more with conformity and authority than with the fostering of intelligence.

And Furinghetti & Somaglia (1998) who research in Europe, also write about what they call 'the fragmentation of knowledge' and its resultant affect on the image pupils have of mathematics:

> In particular, mathematics, which is difficult for many pupils, suffers from this situation and, more than other subjects, is considered to be separated from the cultural context. As a result, the image of mathematics held by pupils is very poor: pupils think that mathematics is a very boring subject, without any imagination, detached from real life.

As to the *core theory of aptitude*, this too, pervades our schools today, with the result, according to Resnick and Hall (p. 96), that "schools still function largely as if we believed that the 'bell curve' is a natural phenomenon that must necessarily be reproduced in all learning results, and that effort counts for little." Children are routinely tracked in less demanding courses according to some judgement of capability, often based in IQ or similar tests.

This system has come to be a self-sustaining one, where, the authors (p. 97) observe, "Hidden assumptions about aptitude are continually reinforced by the results of practises based on those assumptions."

What Resnick and Hall call for (p. 101), is a new learning theory, which they write "points to a position that can moderate the century-long polarity between passive drill pedagogies and child-centred discovery pedagogies." This new learning and social theory, called *knowledge-based constructivism* incorporates effort-based learning with socialising intelligence in what is designed as a nested learning community in which every member is respected as a life-long learner.

2.3.1 ON MATHEMATICS EDUCATION—WHAT SHOULD BE ITS PURPOSE?

In her coming-of-age novel, *A Tree Grows in Brooklyn*, (Smith, 1943, p.165) Betty Smith wrote of Francie Nolan, a poor girl, just starting school in 1908:

> She liked numbers and sums. She devised a game in which each number was a family member and the "answer" made a family grouping with a story to it. Naught was a babe in arms...
> Francie took the game with her up into algebra. X was the boy's sweetheart who came into the family and complicated it. Y was the boy friend who caused trouble. So arithmetic was a warm and human thing to Francie and occupied many lonely hours of her time.

There are many ways of finding meaning in mathematics, but for the most part, when it has occurred, it has been by lucky accident.

Hilton (1997, p. xx) writing on what he believes the goal of a mathematics education should be, notes that,

> ...the educated person must understand what mathematics is—but not in the sense of a dictionary definition. Such a person must have an appreciation of mathematical reasoning and of the role of mathematics in the evolution and development of human society. Such an

> appreciation requires one to understand something of what mathematicians *do*...

Devlin (1999, p.73) is in agreement with Hilton as he states that the goal of mathematics education should be to create an awareness of the nature of mathematics and the role it plays in contemporary society, and that to do this, mathematics must be taught as part of human culture. The goal, he argues, should be to create an educated citizen, "not a poor imitation of a $30 calculator." And he believes this educated citizen should be able to answer the following two questions: (1) What is mathematics? (2) Where and how is mathematics used? Devlin believes that at the present time, few people can answer these questions well.

2.4 IMAGES AND STEREOTYPES OF MATHEMATICIANS AND MATHEMATICS

One of the earliest references to mathematics as possessing a less than favourable image appears in a letter the poet Samuel Taylor Coleridge wrote at the age of seventeen to his brother in 1791:

> I have often been surprised that Mathematics, the quintessence of Truth, should have found admirers so few and so languid. Frequent consideration and minute scrutiny have at length unravelled the cause; viz. that though Reason is feasted, Imagination is starved; whilst reason is luxuriating in its proper Paradise, Imagination is wearily travelling on a dreary desert. (Cundy & Rollett, 1961, p.7).

A century later, in 1892, the following verse was found inscribed in a schoolboy's mathematics text:

> If there should be another flood,
> Hither for refuge fly,
> Were the whole world to be submerged
> This book would still be dry. (Furinghetti & Somaglia, 1998, p. 48)

The situation has hardly improved since. In 1999, two centuries after Coleridge expressed his youthful opinion, the American singer-songwriter Jimmy Buffet released an album which contained a song titled, "Math Suks":

> Then they asked the new Miss America
> Hey babe can you add up all those bucks?
> She looked puzzled, then just said
> 'Math Suks'. (Buffet, 1999)

For the past decade there has been increased discussion and research about images of mathematics and mathematicians (Furinghetti, 1993; Henrion, 1997; Lerman, 1998; Lim & Ernest, 1999; Berry & Picker, 2000; Picker & Berry, *in press*). Howson and Kahane (1990) and Malkevitch (1989; 1997) have referred to mathematics as having *an image problem*.

Furinghetti (1993, p. 34) points out that mathematics "is a discipline that enjoys a peculiar property: it may be loved or hated, understood or misunderstood, but everybody has some mental image of it." For many people this mental image is not a good one, which may be in turn why the public image in most developed countries is so poor (Howson & Kahane, 1990).

2.4.1 DEFINING 'IMAGE'

An *image* is defined in *The American Heritage College Dictionary* (Costello, 2000) to be "a mental picture of something not real or present," as well as "the concept of someone or something that is held by the public," and "the character projected by someone or something to the public, esp. as interpreted by the mass media." At the same time that it is a mental picture, an image can also be a "reproduction of a person or an object" (Costello, 2000)—so an image can be had in mind or in actuality.

A *concept* is defined (Costello, 2000) as "a general idea derived or inferred from specific instances or occurrences," and "a thought or notion." It is often used interchangeably with *conception* or *idea*.

Lim and Ernest (1999) point out that there is as yet no one agreed upon definition of *image of mathematics*. But they believe (1998, p.8) that

> ...the term 'image of mathematics' refers to a mental picture, view or attitude towards mathematics, ... a result of social experiences, through school, parents, peers, mass media or other influences.

Furinghetti, who has written a number of articles about images, also defines *image of mathematics* as the set of beliefs, generally not conscious, that a person holds about mathematics.

2.4.2 DEFINING 'STEREOTYPES'

The word *stereotype* comes from a French process of printing from fixed plates and has come to mean (Costello, 2000) a "conventional, formulaic, and oversimplified conception or image."

Stereotypes have been defined as a form of generalization that is fixed and unbending (Ruggiero, 1998b) and as beliefs shared about personality traits or behaviours of a group of people which rely on "naïve theories" for their formation. (Yzerbyt, Rocher & Schadron,1997, p. 27) The phrase "naïve theories" seems to imply that there is a lack of knowledge in the formation of these stereotypes.

Bem (1970) views stereotypes as based on a lack of sufficient knowledge as well, contending that beliefs lead to generalisations, and generalising from a limited set of experiences, leads to stereotypes.

2.4.3 RESEARCH ON IMAGES OF MATHEMATICS AND MATHEMATICIANS

Lim and Ernest (1999) appear to be among the few researchers who have studied images of mathematics—in this case images of mathematics held by the public. They point out that it is only through ascertaining how popular or unpopular mathematics is, that measures can be created to change and improve its public image. In their study, they canvassed 548 adults in the United Kingdom by having them respond to questionnaires that were subsequently followed up with telephone interviews.

Notable among Lim and Ernest's findings was that "most respondents did not seem to differentiate their images of mathematics from their images of learning mathematics." (p. 3) They conclude that it is likely that this lack of differentiation exists because respondents' images arose directly from their experiences of learning mathematics at school.

Lim and Ernest list as possible factors of influence on the formation of images of mathematics (in order of decreasing frequency of occurrence among respondents): school experiences; teachers' personalities and teaching styles; parental support and motivation (mostly from a father); and peer influence and support.

Lim also reports on a study in which she attempted to use metaphor analysis to investigate images adults hold of mathematics (Lim, 1999), but a large problem she found was that of multiple interpretations of the data. Because these were not respondents she had further interaction with, it was not possible to clarify their responses.

In an earlier study conducted by Gibson (1994), metaphors were also used, in that case with high school students. Gibson found that students' attitudes and conceptions were revealed through the metaphors, and a large finding was that many students viewed mathematics as something beyond their control. Because Gibson was able to discuss these metaphors with her students, she was able to find ways of getting them more active in their own learning processes to eliminate some of their negative conceptions.

As this thesis was being written, an article was published detailing a study of children's images of mathematicians. Except for Berry and Picker (2000) and Picker and Berry (in press), it is the only other published study I know of in this area.

Rock and Shaw (2000) conducted their study in the United States to explore children's thinking about mathematicians and their work. They placed a survey on the Internet and received responses from 215 children ranging from kindergarten through 8th grade (14-15 years old). In their survey they also asked for drawings of mathematicians at work, and received 132 drawings from children who ranged from kindergarten through 4th grade.

They found that at the younger ages, from kindergarten and first grade, where there were 93 respondents, and nearly equal numbers of males and females, that there were more female than male figures drawn. But for grades two through four, where there were 81 respondents, also about equally male and female, the drawings were also about equally male and female. Among the drawings there was only one "mad scientist" in a science laboratory drawn, with $E = mc^2$ written on the chalkboard.

Rock and Shaw also found that children tend to think that mathematicians do the same kind of mathematics in their work as the children do in the classroom. The authors concluded that children believe that mathematicians do the mathematics no one else wants to do—what the pupils referred to as "hard mathematics".

2.4.4 IMAGES OF MATHEMATICS AND MATHEMATICIANS IN THE MEDIA

In 1937, Bell (p. 8) noted that "[t]he mathematician is a much rarer character in fiction than his cousin the scientist, and when he does appear in the pages of a novel or on screen he is only too apt to be a slovenly dreamer totally devoid of common sense—comic relief."

In an essay on recent examples of mathematics as seen in the media, including film, novels and painting, Rockmore (1998, p. F4) defines what he terms some the worst stereotypes in mathematics, propagated, he adds, within as well as without the discipline. Writing first of the film *Good Will Hunting*, which he liked, Rockmore nevertheless argues that,

> the main messages are old and trite: You are either someone who can do math or you are not; mathematics is impossible to explain to others, even other mathematicians, and to be a mathematician and to think about mathematics is to separate yourself from society.

Papert (1993, p. 190-191) wrote that this belief that you are either someone who can do mathematics or you are not, along with the view that an appreciation of mathematics is something accessible only to a few in our society, is deeply embedded in our culture and given the "status of theoretical principle by Henri Poincaré", who, Papert writes, believed that an appreciation for mathematics is something with which one is born. A new work by Devlin (2000b) seeks to demolish this popularly held myth that there exists an actual *maths gene.*

The cult-film π, by Aronofsky, is another recent work in which stereotypes of a mathematician are put before the public. In it, a reclusive mathematician, who is the film's central character, goes mad while trying to secure a fortune in the stock market via decoding its chaotic patterns. As he is portrayed, the mathematician in the film "...is the furthest thing from an advertisement for the superiority of the intellect," according to one reviewer (Holden, 1998, p. E17). But then Holden concludes, "We live in a world where an obsessive irrationality may be a necessary prerequisite to penetrating the cool logic of numbers."

Rockmore (1998) asserts that despite this film's message, a deep understanding of mathematics need not lead to madness. In both *Good Will Hunting* and in π, he notes that the "core of mathematics" is neglected while there is "an arcane competition among arrogant men."

Both Rockmore and Saul (1998) contend that no mathematics is presented coherently in *Good Will Hunting*, about which Saul also notes, (p. 501) that "no one in the film is portrayed as taking particular pleasure in doing mathematics."

In a more recent work, a novel, *Uncle Petros and Goldbach's Conjecture,* (Doxiadis, 2000) are to be found these same stereotypes including a lack of pleasure in doing mathematics. This novel has caused a small stir on both sides of the Atlantic because both the American publisher, Bloomsbury, and Faber and Faber the British publisher, have announced a prize of $1 million dollars to the first person who can prove Goldbach's Conjecture by midnight, 15 March 2002.

Yet at the heart of this story is an ambitious man who has so dedicated himself to trying to prove the Conjecture that he has neglected every other aspect of his life. Mathematics has been a source of pain and the cause of bouts of madness. Devlin (2000a) writes of this aspect of the novel:

> Of course, for dramatic effect the obsession displayed by Uncle Petros is somewhat greater than is the case with any mathematicians I have met—and that includes Andrew Wiles—so, as with the hero in the movie *Pi*, it is not clear that nonmathematicians who read the book will view mathematics as an attractive pursuit, or mathematicians as completely sane. But most nonmathematicians probably think that already anyway.

Petros' nephew, the novel's narrator, loves and has studied mathematics, but he has rejected it as a profession, determined not to spend his life as his uncle has.

The stereotype that mathematics is impossible to explain to others (see Rockmore, 1998) can also be seen in the novel *Mazel* (Goldstein, 1995) in which a character listens to two mathematicians speaking to each other and observes:

> When these two talk their impenetrable language of symbols and equations, deducing one thing from another with the fleetness of the six-winged seraphim, then, my God, you feel like a complete child next to them. Not just a child. A *stupid* child.

In his essay, Rockmore also provided some recent examples in the media, which portray mathematicians, and mathematical ideas in a positive light. One of these examples is in the 1997 novel, *Larry's Party* by the Canadian author, Carol Shields. In it, the protagonist is drawn to botanical mazes and labyrinths and becomes a successful landscape architect. A notable sentence in the novel is, (p. 118): "Larry's brain sings, as though he has just worked out a long, difficult mathematical problem." There is more pleasure through mathematics shown here than in most works which exist in popular culture at this time.

In the spring of 2000, it was noted by a New York Times drama critic that a number of theatre works had opened on the American stage which had science and mathematics featured largely in their plots (Weber, 2000c). At least one of these, the Tony Award-winning play, *Copenhagen*, by Michael Frayn, is also running in London.

As preparation for one of the works, *Proof* (Weber, 2000b), which opened to excellent reviews, playwright David Auburn enlisted assistance from members of the faculty of New York University's mathematics department, who also attended rehearsals. This seemed to have had a large positive impact on the play (Weber, 2000c, E7) and contrasts with the excerpt from Goldstein's novel above:

> 'What the cast got from them,' Mr. Auburn said, 'more than any specific math information, was a feel for what the discourse is like. It isn't dry or analytical; they were having fun, argumentative discussions about their field, really scrapping with each other. It was surprising to the performers, I think.'

Weber seems to feel that in *Proof* and in Penny Penniston's *Now Then Again*, a play about two physicists that opened in Chicago, the characters transcend stereotypes. He also appears to consider that a rare occurrence:

> ...the main characters, all in their 20's, find the pursuit of difficult knowledge no less enlivening and nervous-making than things like their careers, their families, their social graces, their hormones and their heartaches. I can't recall having seen similar types on the stage before, and the two pairs of actors involved...make their characters revelatory. All four performances were thoroughly rounded and persuasive, illuminating an obviously extant kind of life experience I'd never before considered but was delighted to discover.

About six years ago a television commercial ran for a Vision Express-type optician's chain. In it, an algebra teacher drones on while scrawling symbols furiously on a chalkboard. As pupils' eyes glaze over, one young man falls asleep, his head pitching straight on into his desktop. The commercial was for unbreakable glasses. The message was that a mathematics class is boring and incomprehensible. Other commercials of this type have aired since, and it is possible that more than any other medium, television commercials get the message of prevailing public attitudes through to pupils most efficiently, since an average television viewer is expected to see more than 48,000 commercials annually (Ruggiero, 1998)

Ruggiero (2000, p.13) has expressed one of the largest effects of what he refers to as *mass culture*:

> In opposition to thinking, mass culture (particularly the advertising industry) plays on the public's needs and desires and prompts people to suspend critical judgement and accept biased testimony as fact.

On the front page of England's *The Daily Telegraph*, of September 30, 1999, the small cartoon in **Figure 2.1** accompanied an article in which it was announced that parents could face a £5,000 fine for their child's truancy.

'Dad, believe me, £5,000 to miss double maths is a bargain'

Figure 2.1 *Matt—The Daily Telegraph*, 30/9/99

The opinion of mathematics expressed by the boy is given credibility placed on the front page of a newspaper. And the attitude communicated is that it is acceptable to portray the subject of mathematics in this derisive way.

2.4.5 THE PREFERENCE FOR SEEING MATHEMATICIANS AS STRANGE

A number of writers have commented on what appears to be a preference in the media for seeing mathematicians as strange (Alexander, 1998; Brown, 1998; Hayes, 1998).

In the late 1990s, Theodore Kaczynski was convicted of being the *Unabomber*, a wild-looking recluse who, over many years, sent package bombs to individuals in the United States. When he was apprehended, it was discovered that he had been a mathematics major and had taught mathematics. Articles then appeared discussing the possible effect he would have on the image of mathematics and mathematicians (Durso, 1996; Devlin, 1996; Paulos, 1996; Brooks, 1996).

As Brooks (1996) noted, so few mathematicians make the news, that the Kaczynski story encouraged the public to link doing high-level mathematics and being a dangerous weirdo.

But it can be asked whether Kaczynski simply fitted perfectly into an existing image which was already rife in the media. This media image has been accounted for in Durso (1996) as the media's need for 'simple characterisation' which includes a growing trend to be less interested in matters of fact and instead to focus on controversy. Weber (2000c) calls this trend anti-intellectualism.

In his popularly received book, *A Tour of the Calculus*, Berlinski (1995, p. 218) included the following passage:

> The department of mathematics, on the other hand, was rather a malignant place, the atmosphere of intimidation characteristic of such institutions conveyed by posted homework assignments designed to stupefy even capable students and an enormous wall chart depicting famous mathematicians in poses suggesting (in pictures like those on post-office bulletin boards) ineliminable weirdness in each and every one...

Writing in this manner is a choice, the certain effect of which is to perpetuate a negative image.

Certainly Doxiadis (2000) exhibits this preference for depicting mathematicians as odd, even mad, throughout his novel. The author also enumerates a list of mathematicians who committed suicide or came to an unfortunate end. The implication is that this is somehow because of their mathematical abilities.

The inclusion of Alan Turing in the list however, makes it suspect, since Turing was hounded for years by the police because of his homosexuality. This had nothing to do with the extraordinary mathematical mind who made considerable contributions to the breaking of German codes, enabling the Allies to win the Battle of the Atlantic during World War II (Kahn, 1991). Turing was also forced to take the hormone oestrogen for a year, something which would be unthinkable now, but which affected his mental state considerably (Hodges, 2000).

In writing about the deaths of two other well-known mathematicians, Paul Erdős and André Weil, Brown (1998, p. 2) makes the point that after the deaths of important scientific figures stories build up that "often snowball into legends that hang only by the thinnest thread of truth." Brown adds that it is unfortunate that although these mathematicians were two of the 20th century's most original minds, "for better or worse, Erdős will be known widely for his peculiar character; Weil's name will live on only among connoisseurs."

In fact, 1998 saw the publication of two biographies of Erdős (Hoffman, 1998; Schecter, 1998). Hayes (1998) contends that these books were written in part because of the strangeness of the life they depict.

As Devlin (1996) notes about depictions of mathematics and mathematicians in general, "the truth...is hardly likely to stop a good story. No more than we can hope to stop journalists from describing the alleged Unabomber as 'a mathematician.'"

2.4.6 ANTI-INTELECTUALISM AND ITS EFFECTS

There is a history of anti-intellectualism, particularly in America, which has been traced to the early 1950s and the McCarthy era (Hofstadter, 1962). More recently it has been referred to both as a "dumbing down of America" and "a kind of celebration of ignorance" by Sagan (1996).

What this rise in anti-intellectualism has brought about is a disdain for intellect, which "is resented as a form of power or privilege." (Hofstadter, 1962, p. 34) Traced to 1952, and from the American presidential campaign in which Adlai Stevenson was viewed as an intellectual, is the introduction of the word *egghead* (Hofstadter, 1962) which seems to be a forerunner of today's more commonly used *nerd*.

What the media has been rather successful in doing has been to intensify and widen the effects of this anti-intellectualism movement so that the depiction of such intellectuals as mathematicians and scientists in stereotypical ways is now accepted with little questioning.

2.4.7 IMAGES THAT 'STICK'

The relation of communication and contagion (see also Gladwell, 2000) has been presented in Dawkin's conception of *memes*. A meme, (see Dawkins, 1989; Rothstein, 1998; Aunger, 1999) is a small bit of cultural information, a "catch phrase" according to Dawkins, who first described them, which is passed from person to person by imitation. Memes have been described as infectiously leaping from brain to brain, as a bit of a tune can suddenly permeate one's consciousness and seem unshakeable.

In the case of stereotypes of mathematics, memes can explain the preponderance of such phrases as 'math is for nerds,' or 'you either have a flair for mathematics or you don't', or the ubiquity of adults saying (with no shame), 'I was never very good in maths—it was my most hated subject!' And yet you will not hear people freely admit that they cannot read (Battista,1997).

The continued repetition of such phrases about mathematics ensures that they get transferred from person to person to the point where they are most often accepted without critical questioning, and Devlin (1997) believes that public perceptions about mathematics have been formed in this manner.

2.4.8 MATHEMATICS' 'INVISIBILITY' FACTOR

Recently, with a growing concern about mathematics' image, there has been discussion about mathematics in terms of the invisibility of many of its aspects. The effect of this invisibility is that the public knows less about mathematics and its uses than is the case with many other fields.

In a 1998 report, the National Science Foundation pointed out that particularly in industry, mathematical scientists rarely carry the title 'mathematician'. Rather they are known as 'engineers, or 'systems analysts', or by a number of other titles, which keeps the public from understanding that they are indeed part of the mathematical profession.

As Wain (1992, pp. 1-2) has observed, the true nature of the work mathematicians do:

> ...is shrouded in secrecy from the ordinary person and few know what mathematics contributes to industry, culture and society in general. The reason why so many people are so badly informed is partly due to the nature of school mathematics which has never attempted to put the subject into its social context and partly because most mathematicians have always seemed to be reluctant to communicate anything about their work presumably on the grounds that to do so in a way people could understand would trivialise the subject. (See also Wain, 1994.)

At the same time, part of the 'secrecy from the ordinary person' to which Wain refers, is really due to the fact that much of mathematical activity takes place in the background (Mankiewicz, 1998). As Cole (1998) concurs,

> ...like plumbing and electric wiring that lies unheralded behind the wall in the house, mathematics stays well out of sight. (Part A, col. 1)

Indeed, because of mathematics' wide usefulness, the cultural aspect of it is missed, rendering it, too, invisible (Gullberg, 1997).

Buerk (2000) comments on a related aspect of invisibility: a split between what she calls the private aspect of mathematics—the human exploration with its mathematical connections, and the public aspect in which mathematics is presented as polished and complete (p. 6; p. 8):

> By accepting the public image of mathematics, many thoughtful people find our discipline easy to reject, for it seems not to offer the opportunity for their own thought. Others find this image intimidating; they struggle to model someone else's thought process....By sharing only the tip of the mathematical iceberg, the public image of mathematics, we are encouraging a rote conception of mathematics... [and] are also reinforcing to many females the cultural stereotype that mathematics is a male domain.

This split between the polished finished process and the struggle that proceeds it makes for a further split: an insistence on seeing the product of mathematics rather than its process as the most important aspect. Indeed the process becomes so obscured as to be rendered invisible, further encouraging an image of mathematics as inhuman (Furinghetti, 1993).

Devlin is also interested in invisibilities in relation to mathematics (1997) but from a slightly different angle, explaining that mathematics has enabled us to 'see' such things as why a jet stays in the air, the roundness of the earth before photographs from space, the invisible patterns of sound which are music, and the future through increasingly accurate predictions. He believes that mathematics serves to make the invisible visible.

2.4.9 MATHEMATICIANS' INVISIBILITY—WHAT DO THEY DO?

Commenting on the place of mathematicians in society, Hammond (1978, p. 16) wrote, "Mathematicians are not a rare breed, simply an invisible one."

For most school children mathematicians are generally absent from the curriculum (Emmer, 1990) so that most students leave school knowing almost nothing about important mathematicians, or any mathematicians, for that matter. Alexander (1998), Emmer (1990) and Malkevitch (1989; 1997) note that for most people, it is unclear what mathematicians do.

"The bottom line for many students," writes Malkevitch (1997, p.93) "is that despite being exposed to mathematics continuously from Kindergarten through 10^{th} or 11^{th} grade, the typical high school graduate can not connect the value of the study of mathematics with what mathematicians really do. Put differently, students have learned when to 'call' or hire a doctor, electrician, geologist, or plumber, but not when to 'call' or hire a mathematician."

The fact that mathematicians are rarely discussed in schools may be part of the reason why.

Knowledge about what mathematicians do is also affected by the fact that in the minds of most of the public, mathematics is confused with arithmetic (Malkevitch, 1989; Wain, 1992; 1994), and so that is what most people believe mathematicians spend their time doing. Yet, as Hyde and Hyde (1991, p.25) write, "It may surprise and delight some students to learn that much of mathematics is not based on computation at all." And Dowker (1992, p. 54) points out in a study of the computational estimation strategies that mathematicians use, that mathematicians "do not in fact frequently perform calculations as part of their work."

2.4.10 DO MATHEMATICIANS PLAY A ROLE IN PERPETUATING NEGATIVE IMAGES?—EXCLUSION VERSUS INCLUSION

There are a number of writers who share the view that the mathematical community perpetuates the image that it is an exclusive group of insiders (Peterson, 1990; Emmer, 1990; Devlin, 1996; Henrion, 1997; Alexander, 1998; Eastaway, 1999).

Peterson (1990, p. 1) believes that mathematicians must shoulder much of the blame for the image of mathematics as a "private, almost magical game aimed toward mysterious, unworldly ends." He uses the phrases 'insiders' and 'outsiders'.

Devlin (1996) is critical of the mathematical community (of which he is a member) for an insularity it evinces—an insularity that he believes is as hurtful to mathematicians as it is to others.

Henrion (1997) writes at length about this separation, noting that the divide between the mathematical and non-mathematical world is another factor that promotes the image of mathematicians as loners (pp. 19-20):

> While many people make a concerted effort to create bridges between the two worlds, there is little institutional support for it. Expository writing, which can be an excellent vehicle for this 'cross-cultural' communication, is still typically denigrated as 'not real mathematics' or a lower form of professional activity.

This divide is present in the field of education as well, between mathematicians and those who teach mathematics on the pre-tertiary levels. Saul (1995, p. 980) equates this relationship to that in the Edwardian household in *Upstairs, Downstairs*: "...two social systems in proximity, struggling to reach a personal working relationship against the grain of the roles cast them by society." He notes that in the United States there are few programs linking schoolteachers with mathematicians, and none to bring together the mathematical community with pupils who do not have a special interest or ability in mathematics.

In stating that mathematicians are an invisible breed, Hammond (1987, p. 17) added that "it is worth asking whether mathematics is essentially remote, or merely poorly communicated."

It may be that much of the blame for this poor communication is due to the work of the mathematician G. H. Hardy. Malkevitch (1989, p. 4) describes Hardy's book, *A Mathematician's Apology*, as "overly influential," and adds that at the time of the mathematics community's greatest growth in the period after World War II, "Hardy's ideas were taught as gospel." Including, he notes, in Malkevitch's own schooling in the 1950s.

In *A Mathematician's Apology*, Hardy (1940, pp. 61-64) wrote:

> It is a melancholy experience for a professional mathematician to find himself writing about mathematics. The function of a mathematician is to do something, to prove new theorems, to add to mathematics, and

> not to talk about what he or other mathematicians have done...Exposition, criticism, appreciation, is work for second rate minds...The public does not have to be convinced that there is something in mathematics.

Gouvêa (1999, pp.2-3) refers to this view of mathematics and mathematicians as the *Hardy myth*, which states that,

> ...only the best mathematicians count, that only young men ever do creative and valuable mathematics, and that proving new and significant theorems is the only valuable thing a mathematician does. This is an influential view, both within the mathematics community and among well-informed outsiders...There are too many in people in the mathematics community who see themselves in this way, and it is too bad.

The *Hardy myth* can be contrasted with the belief of the mathematician Joseph-Louis Lagrange (1736-1813), that a mathematician has not completely understood his own work until he has made it so clear that he can fully explain it to the first person he meets on the street (Bell, 1937; Hoffman, 1998).

So there has been a divide between many of those who are in the mathematics field and those outside it, and those who teach at and below university level. Yet, as Saul (1995, p.982) writes, it would certainly make sense for mathematicians and teachers to come together for their common good.

> Teachers can learn from mathematicians about the dynamism of the subject they teach. Mathematicians can learn from teachers an expertise in transmission of knowledge which has been proven...

The feeling of inclusion or exclusion in relation to mathematics, the notion of a *real mathematician* versus a non-mathematician, of using mathematics to intimidate or welcome, can be felt from outside the field as well. As has been previously mentioned, in the spring of 2000, a number of dramatic works with science and mathematics as their main focus were running at the same time in the American theatre.

Two such works were reviewed in *The New York Times* by the same reviewer (Weber, 2000a; 2000b) when they opened off-Broadway. The criticism of the two pieces indicate that in the first, the reviewer felt distanced and excluded by the way mathematics was present and used in the work; in the second, the reviewer felt the opposite.

2.4.11 A SELF-FULFILLING PROPHECY?

In an interview about his play, *Proof*, which contains a character who is a mathematician with mental illness, playwright David Auburn said,

> I think there is some connection between extremely prodigious mathematical ability and craziness. I don't think that math drives people crazy, but those with edgy or slightly irrational personalities are drawn to it. (Gussow, 2000, p. E3)

Henrion (1997, p. xix) has written about an image held of mathematics and mathematicians which she believes can "*perpetuate* tradition." She insists that the dominant image of "a nerd engrossed in scribbles and equations, a calculator in one hand, chalk in the other, oblivious to anything but numbers and geometric shapes" (p. 5) along with the image of this person as a loner, has the potential to attract only those who would be drawn to that particular vision of the field of mathematics. Others who might be interested in a profession requiring them to work more closely with people would be discouraged from going into a field in which isolation appears to be prevalent.

2.4.12 A CRISIS IN MATHEMATICS

Fewer and fewer students now go into mathematical careers, creating what is sure to become an increasing shortage of mathematicians and teachers of mathematics (National Research Council, 1989; Mannix & Ross, 1995).

In 1995, the National Council of Teachers of Mathematics News Bulletin published the results of a survey showing that more than half of all students planned to drop mathematics at the first opportunity, and by 1998, this had begun. Garfunkel and Young (1998) noted that for the ten years between 1985 and 1995 there was a decline of 30 percent in the enrolment of students in advanced mathematics. Stating their belief that the mathematics profession is in desperate trouble, they wrote (pp. 256-257), "Simply put, we are losing our students..."

In 1998, as well, the National Science Foundation reported (p.1) that the current generation of students does not see careers in mathematics as attractive. And more recently, a decline has been noted in the number of women studying computer science, even as the total number of students has increased (Eisenberg, 2000). An article detailing the substantial attrition rate stated:

> It starts early. Women get turned off not only in college, but once they are at work. The image of the white, male, antisocial nerd who sits in front of a computer turns women off.

Rock and Shaw (2000, p. 550) suggest that understanding what children think about mathematicians and being able to alleviate children's misconceptions about them, may "facilitate and broaden children's thinking about their roles as future mathematicians."

2.4.13 WHO IS A MATHEMATICIAN?

There is very little literature on the question of who is, or who may call oneself a mathematician. Henrion (1997, pp. 5-6) believes that members of the mathematics community create what she calls a template of who a mathematician is supposed to be, and decisions about who is accepted into this community are based on this template. She thus sees this image as a "kind of gatekeeper, a way of defining who is an insider and who is an outsider (p. xix)."

Rosenstein & DeBellis (1997, p. 419) in writing about Rutgers University's *Leadership Program in Discrete Mathematics* explained that one of the goals of the program was to introduce teachers "to the idea of doing mathematics, and foster the idea that they themselves can function as mathematicians, as can their students." In a footnote to this statement they continued:

> One striking image of early LP institutes is Joe Malkevitch's asking the participants whether they thought of themselves as "mathematicians" and conveying to them that if they are doing mathematics, then it is entirely appropriate for them to refer to themselves, and their students, in that way.

2.5 THE AFFECTIVE DOMAIN

Reyes (1984) characterises the *affective domain* as referring to pupils" feelings about mathematics, aspects of the classroom, or pupils' feelings about themselves as learners. She sees two important reasons for studying affective factors: to find ways of helping pupils learn more mathematics; and because a positive attitude toward mathematics is an important educational outcome, regardless of achievement level. However, she makes clear that a positive attitude without knowledge is incomplete—knowledge and a positive attitude together, are essential.

According to McLeod (1992, p. 576), *affect* is a more general term that includes *beliefs, attitudes,* and *emotions*. These are "more specific descriptors of subsets of the affective domain."

Since the mid-1980s a number of researchers have designated affect as being the least investigated though most deserving of further study (Schoenfeld, 1983; Mandler, 1989; Hart, 1989). McLeod (1992, p. 578) lists three major factors as reasons why:

> First, students hold certain beliefs about mathematics and about themselves that play an important role in the development of their affective responses to mathematical situations. Second, since interruptions and blockages are an inevitable part of the learning of mathematics, students will experience both positive and negative emotions as they learn mathematics; these emotions are likely to be more noticeable when the tasks are novel. Third, students will develop positive or negative attitudes toward mathematics (or parts of the mathematics curriculum) as they encounter the same or similar mathematical situations repeatedly.

In mathematics education, there has been some inconsistency in defining aspects of the affective domain (Hart, 1989). Part of the difficulty is that the education community for a long time relied on the psychological community for these terms. Yet Hart asserts that mathematics educators have used terms like attitude, in a less clearly defined way than psychologists.

2.5.1 DEFINING 'ATTITUDE'

The word *attitude* originally came from aspects of posture (Ruffell, Mason & Allen, 1998, p. 2), or a way of carrying oneself, (Costello, 2000) which expressed an emotion. It is related to the word *disposition* (Costello, 2000), which is defined as "one's usual mood; temperament; a habitual tendency or inclination."

Aiken (1970) wrote that there is a reciprocal influence between attitudes and achievement, while Hart (1989) believes that there is no clear agreement on the relationship between affective variables and performance. Galbraith and Haines (1998) point out that it becomes a chicken/egg argument, as some argue that positive attitudes will improve the ability to learn while others take the position that the best way to promote positive attitudes is to provide a successful experience.

Aiken (1970) believes that a critical point in the determination of attitudes toward mathematics for students is the lower secondary age.

2.5.2 DEFINING 'BELIEF'

A *belief* is defined (Costello, 2000) as the "mental acceptance of or conviction in the truth or actuality of something."

Lindgren (1996) and Pehkonen and Törner (1996) agree that the *mathematical beliefs* of an individual are composed of that individual's subjective, experienced-based knowledge and their feelings about mathematics and its teaching and learning.

Defining *conceptions* as conscious beliefs, Lindgren sees these conscious and unconscious beliefs as forming a *belief system* for the individual. These are often so tangled with the individual's knowledge system, that it is difficult to distinguish between them. As a result, Lindgren writes (p. 53), "When the object of the belief system is mathematics, or mathematics teaching/learning I use the term *view of mathematics*."

Pehkonen and Törner (1996, p. 102) express the conception *view of mathematics* by dividing it into four main categories:

(1) beliefs about mathematics,

(2) beliefs about oneself within mathematics,
(3) beliefs about mathematics teaching,
(4) beliefs about mathematics learning,

Further, Pehkonen and Törner, see beliefs and learning as part of a circle, with pupils' mathematical learning experiences influencing and forming their beliefs. "The way mathematics are taught in the classroom will, step by step, form the pupils' view [sic] of mathematics."

2.5.3 PUPILS' BELIEFS ABOUT MATHEMATICS

Since the mid-1980s there has been an increase in the number of articles appearing which focus on research into student beliefs about mathematics. Much of this research has dealt with those student beliefs that arise from misconceptions about mathematics (see Cobb,1986; Schoenfeld, 1987; Garofalo, 1989a; 1989; Mtetwa & Garofalo, 1989; Underhill,1988; Frank, 1988; 1990; Buerk, 1994a). Underhill (1988, p. 56) characterises a misconception as "a belief held by the learner which is not shared by others who are more knowledgeable."

Beliefs are defined by Cobb (1986, p. 4), "as assumptions about the nature of reality that underlie goal-oriented activity." And he adds, "the goal, as an expression of beliefs, embodies implicit anticipations and expectations about how a situation will unfold....firmly held beliefs constitute for the believer, current knowledge about the world." (p. 4) Jussim et al (1995, p.5) point out that, by definition, to hold a belief, is to think it is true.

Schoenfeld (1987) and Garofalo (1989a) contend that those beliefs pupils hold are realistic conclusions based on what they have perceived of their classroom environments. One of the largest reasons why teachers need to become aware of students' beliefs is that they will affect pupils' behaviours in the classroom. (Schoenfeld, 1987; Spangler, 1992; Oaks, 1994)

Furinghetti (1996, p. 20) states that simply having attended school, any person has very particular beliefs about the subject of mathematics. In general, the beliefs held are not conscious, and when they are they tend to be "generic and fuzzy." Yet, she believes, this set of beliefs "constitutes the *image* of mathematics held by common people."

Researchers have been finding that when students' beliefs are fixed and negative, they become passive learners, relying primarily on memorisation as the safest course, rather than understanding (see Buerk, 1994a; Pehkonen & Törner, 1996).

Often this choice to be "safe" arises because students feel they're expected to be passive, but some students have explained having made this choice because they "have been intimidated or felt 'stupid' in mathematics classes in the past (Buerk, 1994a, p.3; see also Davis & Hersh, 1981). Indeed, Zinsser (1989, p.149) traced his math anxiety to a mathematics teacher who "...cowed me with his ownership of the right answer."

Research into pupils' beliefs about mathematics is relatively recent, stemming from the early 1980s. There are a number of researchers who agree that pupils tend to hold certain beliefs in common about mathematics. In particular, Schoenfeld (1983); Cobb (1986); Garofalo (1989a; 1989b); Mtetwa & Garofalo (1989); Frank (1988; 1990); Spangler (1992); Bock, 1994; Pehkonen and Törner, (1996); and Henrion (1997) have enumerated these beliefs, which include:

(a) *Mathematics is computation.* The implication is that in doing mathematics one always comes into contact with the four operations of arithmetic (Frank, 1988). This also reinforces that mathematics is about numbers.

(b) *Mathematics is a loose collection of facts, rules, procedures, and formulas.* This compels students to approach mathematics in a mechanical way, searching for key words and relying solely on memorisation (Mtetwa & Garofalo, 1989; Frank, 1988; Buerk, 1994a; Oaks, 1994; Pehkonen and Törner, 1996).

(c) *The goal of doing mathematics is to obtain 'right answers'.* Students see mathematics rigidly and narrowly; answers can only be completely right or wrong, and the only use for procedures is to get correct answers. In this situation there is no value seen in the process of mathematics, just its product. (Garofalo, 1989b; Frank, 1988; Bock, 1994).

(d) *The role of the mathematics student is to receive mathematics knowledge and demonstrate that it has been received.* Cobb (1986, p. 8) contends that students feel they have to show that they are adept at playing "the academic mathematics game." Success lies in convincing the teacher that the appropriate symbolic manipulations have been performed. A direct outcome of this belief is the "famous (or infamous) question 'Is this going to be on the test?'" (Garofalo, 1989a, p. 503; Oaks, 1994)

(e) *All of mathematics is known.* Schoenfeld (1983, p. 6), likens mathematics to Latin grammar in a student's mind, a dead subject, which "must be rehearsed until it is known."

(f) *Someone has to tell you what to do.* This belief places the teacher in a central role of authority (Bock, 1994), causing students to become dependent on their teachers in a way that gives the teacher, according to Buerk, (1994a, p.3), "a power that we neither want nor value."

The holding of these beliefs is seen as the cause of much of the difficulties students experience in mathematics, influencing how they learn and study the subject (see Schoenfeld, 1987; Mtetwa & Garofalo,1989; Spangler, 1992; and Oaks, 1994).

2.5.4 PUPILS' BELIEFS AND THE CLASSROOM

There is general agreement that pupils' experiences in the classroom, the curriculum and the methods of teaching to which they are often exposed, give rise over time and long experience to their beliefs (Schoenfeld, 1987; Frank, 1988; Cobb & Garofalo,1989a; Pehkonen & Törner, 1996; Rooney, 1998).

Indeed, Garofalo, Schoenfeld, Furinghetti (1993) and Henrion (1997), are overtly critical of the type of teaching which many students experience in the classroom—one which focuses on a product, rather than its process (see also Lax & Groat, 1981; Orton, 1994). This focus usually leads to a lecture style of teaching in which the teacher is an authority who is rarely questioned, "because mathematical knowledge is seen as certain, not something to be discussed or negotiated." (Henrion, p. 257, see also Davis & Hersh, 1981)

Furinghetti (p. 37) claims that this emphasis on just the product of mathematics creates a situation in which realistic problems, interesting ideas, and real life applications of mathematics are held back, while prominence is given to the student's acquisition of techniques and manipulative skills. "On these foundations," she notes, "the negative image of mathematics gels and mathematics becomes a discipline extraneous to the student's expectations and understandings."

The net effect is that students who come to view mathematics as a series of techniques to be memorized,

> ...see none of the beauty of mathematics. Moreover, because they are unable to construct mathematics in a meaningful way, they are unable to internalise mathematics as a body of knowledge. Mathematics is both threatening and boring...and they are likely to reject mathematics because they are unable to take ownership of it. (Oaks, 1994, p. 42)

Under a great deal of pressure, teachers develop strategies to deal with the increasing demands of their jobs, yet there is a paradoxical aspect to the efficiency of many teachers. The teacher's professionalism may actually work to their disadvantage, making difficult material appear as though it should be easy. Unfortunately, students may then feel incompetent when the material is not as easy for them (Schoenfeld, 1983).

The demands on teachers to cover expanded curriculum content, may also cause them to do students' thinking for them, to save time (Buerk, 1994a). Or rush through material (Davis and Hirsch, 1981, p. 282):

> Ideally, mathematical instruction says, 'Come, let us reason together.' But what comes from the mouth of the lecturer is often, 'Look, I tell you this is the way it is.' This is proof by coercion.

Finally, teachers may want to appear brilliant in front of their students, yet may really communicate the following to their students, as expressed by Davis and Hirsch (1981, p.282):

> What I'm telling you is pretty easy and obvious to me, and if you're not getting it, you must really be pretty stupid.

Henrion (1997, p. xix), suggests that imagery can provide insight into belief systems and that this imagery is important because "it reveals underlying beliefs, assumptions and expectations."

2.5.5 TEACHERS' BELIEFS ABOUT MATHEMATICS AND ITS TEACHING

For the majority of teachers of mathematics, their methods of teaching reflect the manner in which they themselves were taught, with the still prevailing belief that mathematics is a set of procedures for pupils to duplicate and that teaching it is a matter of telling pupils how to perform those procedures (Battista, 1994; see also Dossey, 1992; Nolan & Francis, 1992; Burton, 1999).

Researchers have found that teachers often view their beliefs as knowledge (Thompson, 1992), but a distinction between the two is that beliefs can "be held with varying degrees of conviction" (p. 129), and one holding a belief is aware that others may think differently. With knowledge, there is a general agreement, whereas a belief system is dynamic and undergoes change "as individuals evaluate their beliefs against their experiences." (p. 130)

Mura (1995) conducted a study of the images of mathematics held by university teachers of education and compared her findings to a similar study which she had conducted with university teachers of mathematics—referred to in the study as mathematicians. Her rationale was that there is a relationship between such conceptions and teaching practise. Mura found (p. 396) that both the educators and mathematicians held formalist views and that these views "contribute to and explain and justify their prevalence among school teachers."

Mura (p.398) also defined further the differences she sees between the conceptions of formalism and constructivism:

> ...formalism presents mathematics as a finished product...by contrast, constructivism portrays mathematics as an activity. It places in the foreground the thinking processes of those who 'do' mathematics (be they students or professional researchers) such as the finding of relations and the building of theories from real experiences.

In a study involving interviews with seventy mathematicians in the United Kingdom, many of whom also teach, Burton (1999) found that they only occasionally recognised that their teaching and their images of mathematics are related. She quotes a mathematician who had stated:

> ...it is quite easy to forget about context, what maths is, what its history is, where it is coming from, where it sits in society and people's attitudes to it.

When a teacher's beliefs are traditional, those beliefs include (Nolan and Francis, 1992, p. 46) seeing learning as accumulating bits of information and isolated skills; seeing teaching as meaning transferring knowledge directly to students; trying to change student behaviour as a primary goal; the focus of teaching and learning is chiefly on the interactions of the teacher and individual students. The result of these beliefs is a teacher centred conception of teaching in which the teacher "occupies the centre stage of the educational drama." (Nolan and Francis, 1992, p. 46)

2.5.6 CRITICAL THINKING AND ATTITUDE CHANGE

Hoskonen (1998, p. 51) alludes to critical thinking in writing that a,

> ...person is like a scientist who is constantly observing the world around him/her, testing, evaluating then implementing or rejecting. After that he/she begins the cycle once more. Every person is able to change or replace his/her interpretation of events by replacing them with an acceptable alternative.

Ruggiero (1998b) has written extensively on the subject of critical thinking, which he sees as crucial in changing attitudes. He defines critical thinking (p. 11) as,

> ...devoted to the evaluation of ideas. More specifically, it relates to testing the accuracy of statements and the soundness of the reasoning that links them. One of the keys to proficiency in critical thinking is skill in asking questions.

Using critical thinking as a tool, Ruggiero (1998a; 2000) proposes a three-step strategy for changing attitudes in children: (1) identify specific negative attitudes; (2) express the attitudes as beliefs; and (3) guide pupils in analysing the beliefs to reach conclusions that reflect both the principles of logic and pupils' own experiences. As Thompson (1992, p. 130) has shown, a belief system is dynamic and undergoes change "as individuals evaluate their beliefs against their experiences." I believe that evaluating one's beliefs against one's experience is a process of critical thinking.

2.6 THE DRAW-A-SCIENTIST TEST

The research into students' images of scientists which eventually gave rise to the Draw-A-Scientist Test (DAST), began in the mid-1950s, when the anthropologist Margaret Mead and psychologist Rhoda Métraux initiated a large pilot study in which they investigated the image of the scientist as held by thousands of American high school pupils (Mead & Métraux, 1957).

The researchers' rationale then was their desire to ascertain (p. 384),

> ...the state of mind of the students among whom the occasional future scientist must go to school and of the atmosphere within which the science teacher must teach. It gives us a basis for re-examining the way in which science and the life of the scientist are being presented in the United States today.

The major finding of the study was that the image of the scientist as held by students was "overwhelmingly negative." (p. 384) Although Mead and Métraux's study was conducted with essay questions, as a part of this project, collections were made of visual materials related to the image of the scientist, and these included "children's drawings made in response to the instruction 'Draw a scientist'." (p. 386)

From Mead and Métraux's initial study then, arose the Draw-A-Scientist Test (DAST) first used by Chambers (1983) in which students, were asked to draw a picture of a scientist. Chambers then identified seven key parts of the stereotypical images which were produced: white lab coat, eyeglasses, facial hair, symbols of research—scientific instruments and equipment, symbols of knowledge—books and filing cabinets, relevant captions such as formulae, and the 'Eureka!' syndrome. He found that the number of these indicators increased with the age of the child, so that by the 4th and 5th year of schooling, "the image as a rule, has fully emerged." (p. 260)

In time the DAST was modified to include a checklist of stereotypical characteristics—the DAST-C, (Finson, Beaver & Cramond, 1995) and, in a further modification, students were asked to draw a picture of a scientist *at work* (Huber & Burton, 1995).

Huber and Burton then created a second pilot test in which they felt the wisdom of their change in directions was confirmed, as most children in this latter study produced pictures which had more detail, creating a "clearer image of what 'scientist' meant to the students." (p. 372)

Barman (1999), in a small study comparing two groups of 5th grade students expected to find that asking students to draw two scientists instead of only one would give them a chance to draw a female or minority scientist in addition to the usual male response, but he found that the addition of another character in the drawing made no major difference in the results obtained.

Matthews and Davies (1999) did a study using the DAST and also had pupils draw two scientists. Their project had as its main goal an attempt to study primary children's changing images of the scientist. They found that as the children grew older the drawings became more male. They also found (p. 81) that "regardless of the percentage of black children in the class, our results appear to show that children have an overwhelming image of scientists as being white, no doubt gained largely from books and the media."

An interesting set of data from the Matthews and Davies study found that when children were asked in the instructions to specify whether their drawing was of a black or white scientist, there was a much higher percentage of black scientists drawn by the pupils. The authors note (p. 81) as one of their conclusions, "This indicates that if teachers draw attention to the variety of scientists' backgrounds, children may reflect more upon their own representations."

Matthews and Davies, who conducted their investigations with children in London, report that studies to date with the DAST,

> ...have indicated that the stereotypical scientist remains a powerful image in most children's minds. Most drawings are of a white male wearing a white coat, often with spectacles and balding. The percentage of female scientists drawn has been small. (p. 79)

Chambers (1983) who first utilised it felt that the DAST is probably more useful in identifying rather than in measuring attitudes. There is now extended literature in relation to the DAST and some criticism of its use, notably in Symington and Spurling (1990), who responded to an issue raised in a study by Ó Maoldomnaigh and Hunt (1988) in which it was suggested that pupils were not always sure how to respond to DAST instructions. If pupils thought that the instruction was to produce a drawing of a recognisable scientist, then they would be likely to include anything in the drawing that was part of the public stereotype. Ó Maoldomnaigh and Hunt reasoned that the drawing produced would therefore reflect pupils' knowledge of a public stereotype, rather than what the pupil knew of scientists.

Symington and Spurling changed their directions to *Do a drawing which tells me what you know about scientists and their work*, but they found no change in the drawing characteristics. Ó Maoldomnaigh and Hunt (1990, p.77) in response to Symington and Spurling's findings, still held to their argument, insisting that the drawn images be seen as those which the pupil "is willing to make public."

At this time one can find websites giving detailed directions in administering the DAST and in scoring student responses to it (see http://sweeneyhall.sjsu.edu/edit186/mod6h.html.) But although some mathematics teachers have heard of this test and although there is a suggestion in McIntosh and Draper (1997), to use a drawing to ascertain students' stereotypes and misconceptions about mathematicians, there appears to be little literature at this time of students in mathematics classes being asked to take a mathematics version of the DAST by drawing a picture of a mathematician at work, with the exceptions of Rock and Shaw (2000), discussed earlier, Berry and Picker (2000) and Picker and Berry (in press).

2.7 DISCRETE MATHEMATICS AND GRAPH THEORY

The mathematician Paul Erdős has been credited with founding the field of discrete mathematics (Kolata, 1996). Certainly he contributed the most to its growth in the last part of the 20th century (Hoffman, 1998; Schecter, 1998) by continually posing problems in graph theory and combinatorics, two large areas of discrete mathematics (Rosenstein, 1994).

Discrete mathematics has been defined to mean "...that collection of non-continuous mathematical ideas that have exploded in interest and study since World War II. In many cases these mathematical ideas had roots in much earlier times...but the invention of the digital computer served as a catalyst for the flowering of these ideas." (Malkevitch, 1997, p. 92)

Mathematics topics which are considered to be part of discrete mathematics include graph theory, graph and vertex colouring, combinatorics, ranking systems and social choice, fractal geometry and chaos theory, Markov chains, fair division strategies, discrete optimisation, iteration and recursion, matrices, (discrete) probability, and game theory (Hart, 1991; Rosenstein, 1994; Malkevitch, 1997).

Graph theory is believed to have originated with Leonhard Euler's 1736 proof showing the impossibility of solving the Königsberg Bridge Problem (see Harary, 1969; Eves, 1983; Wilson & Watkins, 1990; Gullberg, 1997; Schecter, 1998). In solving the problem, Euler seemed to be aware, that he was dealing with "part of a branch of geometry in which the relations depend on position alone and not at all on magnitudes." (Katz, 1993, p. 572) But it was not for another two centuries that the branch came to be systematically studied.

Discrete mathematics, which is also referred to as *decision mathematics* in the United Kingdom (see Berry, 1986), was recommended for inclusion in the high school (grades 9-12) curriculum among the 54 original standards of The *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989, p. 176). This document, which was a blueprint for reform and change in the teaching of mathematics and curriculum development in the United States, was recently revised (NCTM, 2000) and now, instead of separate treatment, the main topics of discrete mathematics are distributed across the *Standards* where they span the grades—pre-kindergarten up through grade 12.

The sections on graphs and graph theory (referred to as networks to avoid confusion with Cartesian and other graphs) begin with seventh grade (grade 8 in the U.K.).

Many mathematicians and educators agree that discrete mathematics holds unique aspects and strengths in the curriculum for a wide variety of student levels (see Niman, 1975; Berry, 1986; Hart et al, 1990; Hart, 1991; Dossey, 1991; Althoen et al, 1991; Holliday, 1991; Monaghan & Orton, 1994; Friedler, 1996; Picker, 1996; 1997; 1998; Rosenstein, 1994; and Rosenstein et al, 1997; NCTM, 2000). They agree that in discrete mathematics students find a change in emphasis away from memorising definitions and theorems, in which they often get to see the entirety of the mathematical process without the prerequisite of a sophisticated mathematical background.

Problems in discrete mathematics do get hugely complex. But it is easier for a novice to comprehend a problem in discrete mathematics and stand at the edge of what contemporary mathematicians know, than in many other topics in mathematics today. Kindt (1996, pp. v-vi) contends that the chief advantage discrete mathematics has over continuous mathematics is the conceptual simplicity which makes students feel comfortable. At the same time he points out that discrete mathematics is a very rich domain which has many interesting applications.

In a study on pupils' life-long perceptions of mathematics as influenced by teachers, Rooney (1998, p. 12) found that when a pupil had difficulties in one area of mathematics, it tended to demoralise them, make them feel inadequate in all areas of the subject, and caused them to develop negative perceptions of mathematics.

But discrete mathematics places pupils of varying backgrounds on a 'level playing field', that is, pupils generally all start learning together, not having previously seen the material. For discouraged pupils in particular, this can make the subject of mathematics suddenly come alive, and have them feel successful, perhaps for the first time in a mathematics class (see Biehl, 1997; Carney, 1997; Picker, 1997; Rosenstein, 1997; Doorman & Verhage, 1997).

Discrete mathematics topics can also serve to dispel the belief many pupils hold that mathematics is a dead subject (Schoenfeld, 1983) in which no new developments take place.

Finally, discrete mathematics topics can encourages pupils to see that mathematics is more than arithmetic (see Friedler, 1996), and about more than numbers (see Picker, 1998).

As Barbeau (1990, p.41) has noted:

> Most mathematicians at one time or another have probably found themselves in a position of trying to refute the notion that they are people with a 'head for figures', or that they 'know a lot of formulas'. At such times it may be convenient to have an illustration at hand to show that mathematics need not be concerned with figures, either numerical or geometrical.

Graph theory, in particular, would serve as an excellent illustration.

At this time there do not seem to be any studies of pupils using graph theory and discrete mathematics on the lower secondary level. The articles which exist are anecdotal in nature and only two address the lower secondary level (see Bush, 1972; Carney, 1997). Most of these articles are published in only two texts: Kenney & Hirsch (1991) and Rosenstein et al (1997).

Chapter 3: THE RESEARCH DESIGN

3.1 INTRODUCTION TO THE RESEARCH DESIGN

Research design utilises both research methodologies and research methods. Cohen and Manion (1994) and Ernest (1998) differentiate research methodology from methods and agree that methods refers to the techniques and procedures used in gathering data, while methodology has as its aim, not the product of an enquiry, but its process, what Ernest (1998, p. 23) calls "a general theoretical perspective on knowledge and research."

There are a number of general theoretical perspectives, but the two major ones are the normative and interpretive, or the quantitative and qualitative. There is increasing movement among researchers towards dissolving the tension existing between them (Cohen & Manion, 1994; Pirie, 1998; Prosser, 1998a). Pirie (p. 17) notes that it is time to put aside the debate that attempts to place quantitative methods as superior to qualitative methods, because, she notes,

> Neither has merit *in itself*. The appropriateness of methods and methodologies espoused by researchers can be considered only in the light of the intentions of the specific research being undertaken.

To that end, Yin (1994, p. 6), has proposed a matrix showing research questions and the different research strategies they require (see **Table 3.1.** below).

It is generally agreed that a well-designed study in the social sciences will use a combination of the strategies listed by Yin, in what is known as a triangulated study (see Merriam, 1988; Yin, 1994; Cohen & Manion, 1994).

This use of multiple investigations serves as a "cross-check" on the findings (Merriam, 1988; Yin, 1994; Cohen & Manion; 1994).

strategy	form of research question	requires control over behavioural events?	focuses on contemporary events?
experiment	how, why	yes	yes
survey	who, what, where, how many, how much	no	yes
archival analysis	who, what, where, how many, how much	no	yes/no
history	how, why	no	no
case study	how, why	no	yes

Table 3.1 Relevant Situations for Different Research Strategies (Yin, 1994)

3.2 COMPARING THE QUALITATIVE AND QUANTITATIVE METHODOLOGIES

Even as they move closer and are more often used together in one study, it is still useful to understand the difference in the characteristics of qualitative and quantitative research methodologies. The comparative chart below, from Merriam (1988) outlines these points of comparison:

POINT OF COMPARISON	QUALITATIVE RESEARCH	QUANTITATIVE RESEARCH
Focus of research	Quality (nature, essence)	Quantity (how much, how many?)
Philosophical roots	Phenomenology, symbolic interaction	Positivism, logical empiricism
Associated phrases	Fieldwork, ethnographic, naturalistic, grounded, subjective	Experimental, empirical, statistical
Goal of investigation	Understanding, description, discovery, hypothesis generating	Prediction, control, description, confirmation, hypothesis testing
Design characteristics	Flexible, evolving, emergent	Predetermined, structured
Setting	Natural, familiar	Unfamiliar, artificial
Sample	Small, non-random, theoretical	Large, random, representative
Data collection	Researcher as primary instrument, interviews, observations	Inanimate instruments (scales, tests, surveys, questionnaires, computers)
Mode of analysis	Inductive (by researcher)	Deductive (by statistical methods)
Findings	Comprehensive, holistic, expansive	Precise, narrow, reductionist

Table 3.2 Characteristics of Qualitative and Quantitative Research (Merriam, 1988, p. 18)

3.3 COMPONENTS OF THE RESEARCH DESIGN

This project utilised primarily qualitative techniques although some quantitative methods were used in the survey tool. Wiersma (1995) contends that the general components of qualitative research are: the *working design*, or preliminary plan for the research; the research questions, with foreshadowed problems; data collection; data analysis and interpretation.

3.3.1 THE WORKING DESIGN

After several small pilot surveys, I came to identify 12-13 year old (seventh grade in the U.S./eighth grade in Europe) pupils as the preferred age group of the subjects for the project. Classes of pupils were to be selected to be surveyed in schools in Europe and the United States in a purposeful, rather than random sampling.

A time frame was sequenced, with the expectation that the first (graph theory/discrete mathematics topics) intervention would take place in the school year following the one in which the international study took place.

3.3.2 RESEARCH QUESTIONS AND FORESHADOWED PROBLEMS

In qualitative research, there is no working hypothesis; rather hypotheses emerge as the research proceeds. Instead, there are the research questions. These had been posed at the outset and are enumerated in the introduction to this thesis, but they basically surrounded the overriding question, What *images of mathematics and mathematicians are held by lower secondary pupils?*

Foreshadowed problems identify factors for the researcher to observe during the research process (Wiersma, 1995). As the project proceeded, I felt that some of the foreshadowed problems included,

(1) The effect of the media on pupils' images.

(2) The influences of associationist and Platonist philosophies on teachers and their classrooms.

(3) Pupils' prior negative experiences in mathematics classes.

(4) A media culture that is hostile to mathematics.

(5) Anti-intellectualism.

(6) The invisibility of mathematicians and the mathematical process.

3.3.3 DATA COLLECTION

In deciding how to collect data, Wiersma (1995) states that the researcher also needs to decide if they will be a participant-observer or just an observer. In New York City, because of my role as a teacher-trainer/staff developer, I am more than just an observer when I am in a classroom, and so during the first intervention, as well as during the Mathematicians Panel, I was a limited participant-observer.

Pupils took the survey for the first intervention with little question as it was among the first days of the fall term when they often see questionnaires and surveys. Some two months later after the graph theory/discrete mathematics unit, and after the survey had been given again, I asked pupils for volunteers to be interviewed, and about a third of the class volunteered.

Pupils taking the survey for the second intervention were given no explanation for it, until immediately after the surveys were collected, when they were told about the Mathematicians Panel as an upcoming event they were to participate in. When pupils did ask questions, it was most often—*Is this a test?* For any further questions, I told pupils that we would discuss it at another time. Yet as I saw pupils in their schools after the surveys and after the Panel, they asked no further questions. They were involved in so many activities that these surveys did not appear to stand out for them.

The pupils of the district in which I work are used to being videotaped as part of a number of initiatives, and used to being observed by teams of adults. Permission slips for videotaping and interviews are routinely sent home with pupils for their parents' signatures. So the surveys and interviews were not seen as out of the ordinary.

The decision about the method of data collection was to create, field test, and refine a survey tool which could be used to identify images of mathematicians and mathematics held by lower secondary pupils. I wanted the tool to be user friendly—easy to use in a classroom, inexpensive to produce and therefore on a single page, require less time than a regular 45-minute class period to take, and provide quick and interesting feedback to teachers, so they would be motivated to use it.

A version of the *Draw a Scientist Test,* or DAST was designated for one side of the tool, with ten belief statements in a Likert-type scale on the other. In addition to the drawing being open-ended, the side with the Likert-type statements had two writing prompts: the first asked pupils to enumerate reasons for which a mathematician would be hired; the second asked for a written comment about the drawing.

Unlike the way the DAST was used by most researchers in the sciences, the hope in having pupils produce a drawing of a mathematician and then respond to the statements and prompts was to gain insight into pupils' images of mathematics as well as their images of mathematicians. I also did not want to use the tool to come to a total score for each pupil.

Although it had been done with the DAST, I did not want to create a means of evaluating pupils because it could be misused. I was also unsure that a score could provide an accurate picture of what the survey was meant to investigate. Rather, I saw the survey tool as an instrument to provide a window through which to view the pupils' images at a point in time.

During the international portion of the study, five pupils in New York City were interviewed using non-standard interviews. In this style of interview, questions are mixed and matched as information comes in from the subject, but all the basic questions are written down in advance.

The first intervention, in which the graph theory/discrete mathematics unit was taught, involved a refined version of the survey tool with a drawing, plus an observation of some classes and interviews with pupils. These also used a non-standard interview method.

The second intervention in which pupils met and interacted with mathematicians involved the same refined survey tool as the first intervention as well as an extra open-ended writing assignment in which pupils were asked to write about what had surprised them about the Mathematicians Panel.

The reliability of the Likert-type portion of the original survey tool was (Cronbach) alpha = .6144. The reliability of the refined survey tool ranged from (Cronbach) alpha = .7114 in the first intervention, to (Cronbach) alpha = .7981 after the Mathematicians Panel.

In the portion of the study involving the survey of professionals in the mathematics field, there was also a short survey tool in which respondents were given two open-ended writing prompts.

3.3.4 DATA ANALYSIS AND INTERPRETATION

Each set of written survey prompts was analysed for themes and then coded. These themes were discussed and compared with the findings of a colleague on the same data and then further refined.

After coding and data reduction, the survey tool data was analysed using SPSS. Further analysis, particularly of the images pupils created was analysed using an interpretive paradigm (Cohen & Manion, 1994) and techniques from Imaged-based Research (Prosser, 1998a).

3.4 TRIANGULATION

The problem of uncovering and measuring attitudes and changes in them is illuminated by Lowery (1966), who created an open-ended attitude-measuring instrument for science education. He found that using only one method in a questionnaire was not as reliable as creating an instrument with three interdependent techniques "coupled to serve as a check on one another." In his *Projective Tests of Attitudes*, one section was the "Sentence Completion Test". In this aspect of his test, the pupil was asked to complete sentences in which the first words were given.

Lowery hoped that since there could be no right or wrong answer, students would more freely and unconsciously reveal their opinions and attitudes. In fact he found that "this type of test was able to tap more specific positive and negative areas of the respondent's hidden feelings than the other tests." (p. 501) The one drawback Lowery noted, was that there were times when there was a need for follow-up inquiries to help clarify the data.

What Lowery had created in one instrument was a method of triangulation, which is described by Wiersma (1995) as a part of data collection that,

> ...cuts across two or more techniques or sources. Essentially it is qualitative cross-validation....It assesses the sufficiency of the data according to the convergence of multiple data sources or multiple data-collection procedures. (pp.263-264)

Wiersma contends that the use of multiple data-collection procedures, along with triangulation, tends to enhance internal validity.

Particularly in the international survey and the two interventions, in this study, I have used what Cohen and Manion (1994) call *methodological triangulation* in which different methods are used on the object of study. There is triangulation within the survey tools through the combination of pupils' drawings, writings, and responses to the Likert-type statements, and there is triangulation as well as through observations of pupils and interviews with them. The results of these varied methods as they have been used in this project appear to be consistent.

Cohen and Manion point out that the largest problem confronting researchers using triangulated methods, is that of validity. But that appears to be particularly the case when only qualitative methods are used. In this study however, the survey tool combines both quantitative and qualitative methods.

3.5 IMAGE-BASED RESEARCH

In the past three decades there has been increased interest in Image-based Research, which is an aspect of qualitative research. Prosser (1998a, p. 100), contends that images are still rarely used in qualitative research, "except as a representation of words and numbers," due to the continuing influence of quantitative paradigms which still influence qualitative approaches.

Image-based Research has its roots in the evolution of visual anthropology and sociology which were taken seriously as a sub discipline only in the late 1960s (Prosser, 1998b). The fields of anthropology and sociology were founded at approximately the same time as the early photographic processes were developing and so photographs were a part of early sociological journals.

As Prosser (1998b, p. 101), points out, there have been rapid changes in present day Image-based Research theory and visual sociologists are now drawing on a wider range of visual media. These include film, video, cartoons and drawings.

Much of the literature in the methodology of Image-based Research discusses photography (see Prosser & Schwartz, 1998), but two studies of particular interest that did not are a study that used cartoons, and one that used children's drawings and writings.

Warburton (1998) examined a study of cartoon representations of teachers and education in the British press. Warburton (p. 254) contends that cartoons "...communicate meaning which assumes a shared knowledge and understanding." Often this shared knowledge includes stereotypes that have a public recognisability. The technique suggested by Warburton (p. 257) in handling such images in research is to begin with a description that inventories the image and then to make a judgement about what the key signifiers are in it. The next step is to try to confirm the meaning of these signifiers "by identifying where messages are duplicated in supporting text of different forms."

Wetton and McWhirter (1998, p. 263), described their own research in the early 1980s in which they asked children to "draw and write," a methodology which they found to be "open ended and illuminative, and capable of yielding quantitative data." In their study, children's drawings and written comments were collected in order to develop an effective health curriculum in the United Kingdom. They believe that the wide replication of the investigation and the consistencies in the children's perceptions have established the technique as a valid and reliable research tool.

3.6 INTERNAL VALIDITY

The internal validity of research involves the ability to interpret results with confidence (Wiersma,1995), in order to feel that they match the reality of the investigator's experience (Merriam, 1988). LeCompte and Preissle (1993) list several threats to internal validity, including maturation, observer effects, mortality, and spurious conclusions.

Wiersma (1995, p. 273) contends that the "use of multiple data-collection procedures, along with triangulation, tends to enhance internal reliability." The extensive description in qualitative research, he argues, enables sources of disagreement to be identified within the description. Merriam (1988) also identifies peer examination and repeated observations at a site as other strategies to ensure internal validity.

3.7 EXTERNAL VALIDITY

External validity is concerned with the ability to replicate and generalise the results of a study (Wiersma, 1995, Merriam, 1988). LeCompte and Preissle (1993, p. 348) believe that threats to external validity of the findings of a qualitative study are whatever "obstructs or a reduces [its] comparability and translatability." They emphasise that a study and its results must be well enough described and defined so that researchers may use them in comparisons with other studies, and that the research methodologies and techniques must be accessible to and understood by researchers in the same and related disciplines.

Wiersma (1995, p. 277) notes that "as a procedural matter, external validity can be strengthened by multisite studies. If a phenomenon seems to be consistent across a number of studies, its generalisability is increased."

Chapter 4: THREE INTERRELATED STUDIES

4.1 INTRODUCTION TO THE INTERRELATED STUDIES

This project is comprised of three interrelated studies, two of which involved the design, development and refinement of a research survey tool with which to investigate pupils' images of mathematicians and mathematics:

In the first, an international study was conducted using the tool to compare those images held by 476 lower secondary pupils in five countries. The analysis of this first study served to inform the second.

In the second, two interventions were attempted in the United States. In the first intervention, one lower secondary class (n = 28; 7th grade, 12-13 year old pupils) was first given a modified version of the investigative tool.

Pupils were then taught a month long unit of graph theory and other topics in discrete mathematics to see if this could affect and change their images of mathematicians and mathematics. At the conclusion of the unit, pupils were again administered the survey tool.

This first class was then joined by pupils in six more classes from two more schools (for a total of n = 174). These pupils were also first administered the research tool. In a second intervention experienced by the seven classes comprising the 7th grade group, pupils gathered together to meet with a panel of eight mathematicians who discussed their careers and research interests and answered questions which the pupils posed to them. All pupils were given the survey tool once more after the panel.

In the third study, a short survey of open-ended questions was distributed to persons professionally involved in mathematics or its teaching.

The purpose was to ascertain what mathematicians themselves believe about being a mathematician and under what conditions one can consider oneself one.

4.2 AN INTERNATIONAL STUDY

In the international study of pupils' images of mathematicians and mathematics, the objective was to understand what kinds of images lower secondary pupils hold, and whether any of these images are held in common in the different countries participating in these surveys.

The pupils involved in the study were aged 12-13 years, who are designated as 7th graders in the United States, year 8 pupils in the United Kingdom. The participating pupils were from schools in the following countries: USA (*n*= 201), United Kingdom (*n*= 99), Finland (*n*= 94), Sweden (*n*= 49), Romania (*n*= 33).

The pupils in the United States were from five classes in schools in two neighboring northeastern states, New Jersey and New York; in the United Kingdom pupils were from two schools in the southwest of England, and one school each in Wales and Scotland; in Finland pupils were from schools in three cities; in Sweden pupils were from schools from two cities; and in Romania pupils were from two classes in one school near Bucharest.

To avoid difficulties in language for the pupils, where it was necessary, the survey tool was translated by their teachers into their mother tongue and then the pupils' responses were translated back into English for analysis.

This part of the study was conducted primarily via post, except in some schools in the United States.

4.2.1 DECIDING ON THE AGE OF PUPILS FOR THE STUDY

A number of small pilot studies were conducted both to get a general sense of pupils' images of mathematicians and mathematics and to assist in design decisions for the final survey tool and age group of the pupils.

The first small pilot involved giving an early version of the survey tool to two classes of upper secondary students, age 16-18 in a small New York City high school. I came to feel fairly quickly however, that there was a deep cynicism that had set in amongst the students in this group with a great deal of distaste expressed for mathematics. The students in this group appeared unwilling to spend thoughtful time on the questionnaire and I had to conclude that it would be better to conduct the study with younger pupils.

Five classes of 6^{th} grade (ages 11-12) pupils in two states showed some difficulty with reading the surveys and expressing themselves clearly in writing. Upon reflection, it appeared that they were just a grade too young.

Seventh grade pupils however, appeared to take the questionnaire seriously, were interested in its purpose and, after they asked and were reassured that no, this wasn't a test, appeared to have the right mix of a general earnestness with some sophistication. Adding to the decision to conduct the study with 7^{th} grade pupils was the fact that those first drawings that I had seen of pupils' perceptions of a mathematician (**Figures 1.1** and **1.2**) were also produced by pupils of this age.

4.2.2 GATHERING DATA

The project began with administration of the survey tool followed by non-scheduled standardized interviews conducted with five pupil volunteers from three classes in one of the participating American schools.

In each school, participating pupils were given the one page two-sided survey tool by their teachers, who had been instructed to have the pupils complete side one first. This side was primarily blank, but at the top *Activity 1* was indicated. The complete directions to the pupil read: Draw *a picture of a Mathematician at work.*

Side two of the survey had *Activity* 2 written at the top, followed by: *I am:* and two boxes for pupils to indicate their gender.

Side two, which provided the Likert-type scale, was designed to ascertain pupils' images of mathematicians and attitudes toward the subject of mathematics in general (See **Appendix—A**).

These ten statements asked pupils to respond by ticking the appropriate box. The very first statement, I *enjoy the school I attend* was intended as an ice breaker, although through it I hoped to obtain some understanding of how pupils felt about their school experience.

(2) *A mathematician's work looks like fun to me.* I hoped to ascertain whether pupils held a negative or positive image of what mathematicians do, whatever they perceived that to be.

(3) *I would never think of becoming a mathematician.* With this statement I hoped to understand whether pupils had any interest in what they believed the profession of mathematician to be.

(4) *Mathematicians seem like very patient people.* This statement arose from an earlier survey in which pupils most often indicated that they thought mathematicians were patient.

(5) *I would not want to marry a mathematician.* This statement was included because it had originally been used in the Mead and Métraux study.

(6) *I have met a mathematician.* The hope with this statement was to ascertain whether those images pupils held of mathematicians were based on any knowledge of an actual mathematician.

(7) *I don't enjoy my mathematics class.* I hoped through this statement to understand pupils' feelings about their mathematics classes.

(8) *I discuss what I am doing in mathematics class with my friends.* With this statement I thought I might see if pupils were sufficiently interested in their mathematics classes to discuss them outside of their classrooms.

(9) *Mathematicians usually work alone.* I hope to ascertain the extent to which pupils held to the stereotype of the mathematician working alone and in seclusion. Burton (1999) has shown that the majority of mathematicians now see mathematics as a collaborative activity.

(10) *I see myself as a mathematician.* I wanted to see to what extent pupils identified themselves with and as mathematicians.

In addition to the Likert-type scale, there also were the two open-ended writing prompts: With the first, I hoped to ascertain whether pupils knew what specific jobs mathematicians could actually be hired to do.

With the second prompt I hoped that pupils would provide a fuller picture of what they had tried to communicate in their drawing. The hope was that they might perhaps clarify the gender of their mathematician, or reveal something further about any beliefs they held about mathematicians.

The non-standard scheduled interviews involved asking the same core questions of each pupil, although they were not always asked in the same order (See **Appendix—B**). Pupils were also asked to comment further on some of their responses on the Likert-scale portion of their surveys.

The interviews were tape-recorded with the permission of the pupils and conducted during their lunch periods. After all the interviews had been completed, each pupil was given a copy of the game, *Uno*, for agreeing to be interviewed.

4.3 A FIRST ATTEMPT AT INTERVENTION: THE PUPILS AND THEIR SCHOOL

The class which participated in this first intervention (n = 26), was from the Gramercy Middle School, a small lower secondary school located in a quiet neighborhood near Gramercy Park in Manhattan. The school has existed for only the past four years, and is located on the top floor of a primary school. Approximately two hundred pupils attend grades six through eight (ages 11/12-14/15), as part of a growing movement in large cities in the United States to keep the transition from primary to secondary school as personal and informal as possible.

Since the fall of 1997, I have worked at Gramercy Middle one day a week as a staff developer for the mathematics teachers. I co-teach with them, serve as a resource, help them reflect on their teaching practice, and assist them in implementing a new curriculum. The other days of the week I do the same thing in other schools in my district, including Chelsea Middle School, whose pupils were part of the second intervention.

The pupils are selected at Gramercy Middle by interview. A strong interest in science is preferred as the school has a science theme that includes a cooperative relationship with a major university medical school in the area. Throughout the school year, pupils attend seminars at the medical school, which are geared for their age and level of scientific knowledge.

The population is as diverse racially and economically as New York City itself and the pupils come to Gramercy knowing their basic number facts and not requiring remediation.

The school is part of a district that has been described as "one of New York City's most adventurous school districts...long ranked near the top of the city school system." (Hartocollis, 2000) It has been the forerunner in many reforms and initiatives (see Resnick & Hall, 1998). Three years ago a new middle school mathematics curriculum was adopted, *Connected Mathematics* (Lappan et al, 1998).

Connected Mathematics was still being field tested at the time the district adopted it. It was developed at Michigan State University from 1991-7 with National Science Foundation funds and is among the first of the new curricula to be developed after, and therefore reflecting, the National Council of Teachers of Mathematics' *Curriculum and Evaluation Standards for School Mathematics* (1989).

The developers of the new program based it in four strands that spiral through grades 6, 7 and 8: number, geometry and measurement, algebra, statistics and probability. A large portion of the program involves hands-on investigations through which the mathematics is explored enabling pupils to construct much of their mathematical knowledge.

4.3.1 THE TEACHER

Marina Nierescu likes *Connected Mathematics,* although it is very different from the rigid and traditional schooling she experienced growing up in her native Romania. She has been the full-time mathematics teacher at Gramercy Middle for the past three years, and the other teachers in the department look to her for leadership. Before that, when she first arrived in the United States, she worked as a statistician for a year. Prior to that, Marina had been a mathematics teacher in Bucharest for four years, where she had earned a masters degree in mathematics. In her late twenties, Marina is dark haired and attractive, and always very smartly dressed.

In the summer of 1998, Marina attended a three week institute in Graph Theory at the Centre for Discrete Mathematics and Theoretical Computer Science (DIMACS) which is a collaborative project of Rutgers University, Princeton University, and a number of corporate technology research groups. It was created to bring together secondary teachers and an international invited group of graph theorists.

Some aspects of the program were shared; most ran parallel, with the researchers working on problems together while the secondary teachers studied graph theory and then worked together to prepare lessons through which to introduce the topic of graph theory to their students.

Marina Nierescu did well at the institute and she was accepted again to another three-week institute the following summer, in 1999. In the summer of 1999 as well, she was accepted to and attended the Rutgers *Leadership Program in Discrete Mathematics.* This two-week institute was held a few weeks before the graph theory institute.

The *Leadership Program* was created to take topics usually taught at the college level—if then, for many American mathematics majors have never studied any topics in discrete mathematics—and teach them to primary and lower secondary teachers. The hope and intent was that these teachers would then teach the topics in their own classrooms and create workshops with which to teach the topics to their colleagues.

Although the program spends a week studying graph theory, including graph colouring, other discrete topics are taught in the second week, including counting techniques, combinatorics and fractal geometry. Another difference between the DIMACS program and the *Leadership Program* is that in the *Leadership Program*, topics are taught with less of the terminology and in a more hands-on and constructivist manner than at the DIMACS program.

Marina was aware of my interest in seeing what the effects of graph theory might be on lower secondary pupils' images of mathematicians and mathematics. Since she would be teaching one class of the seventh grade in the new school year, she readily agreed to participate with her class in this study.

4.3.2 THE RESEARCH TOOL MODIFIED

In September, at the very beginning of the school year, pupils in Marina Nierescu's 7th grade class completed the research survey tool. As a result of analysis of the survey used in the International portion of the study, it had been modified. A number of the questions in the new survey had now been changed in an effort to produce a less ambiguous, more easily utilised, more useful tool (see **Appendix—C**).

Instead of having pupils respond to *I enjoy the school I attend,* the first statement on the refined tool was *I enjoy my mathematics class.* I felt that this was still an ice breaker, but one that now could more immediately give a sense of how the pupils felt about the experience of the class they were attending.

The next statement that differed from the survey used in the international study, was statement 4. Instead of *Mathematicians seem like very patient people,* which I felt pupils had little knowledge about, the statement was changed to, *I usually feel confident in math class.* I hoped this might provide a window into what the pupil felt about and during their classes in mathematics. I also felt it could give an indication of how empowered or autonomous the pupil felt during classes.

Question 5, which on the international tool had read, *I would not want to marry a mathematician,* was changed to *I plan to stop taking math as soon as I can.* This question seemed dated. While administering the international survey tool in New York, and in interviews with pupils, I had heard them say that they felt they were too young to know who they wanted to marry, so I did not feel that the original question yielded much useful information. But if a pupil agreed that they planned to stop taking mathematics, I felt it would indicate a dissatisfaction with, a lack of success in, or a lack of understanding of the usefulness of the subject.

Statement 6 was amended so that if a pupil agreed that they had met a mathematician, they now had to state who that person was. I had felt that many pupils had agreed with this statement without really thinking it through, for when questioned, they could not state who the mathematician was that they had met. This change appeared to make pupils more thoughtful about the statement, and fewer pupils agreed with it.

Statement 7 on the international survey, *I don't enjoy my mathematics class*, was now changed to *Mathematics is not a subject where I get to express my own opinions*. I hoped this statement might further indicate how empowered and independent pupils felt in their mathematics classes, especially considering the Standards-based curriculum they were using—one which encouraged them to find alternate strategies and solutions.

Statement 8 was also changed, from *I discuss what I am doing in mathematics class with my friends*, an assertion that had shown itself to have a number of interpretations, to *I look forward to taking more mathematics in school*, a positively worded variation of the negatively worded statement number 5.

Statement 9 on the international research tool was, *Mathematicians work alone*. But I was coming to see through preliminary analysis of the data in the international portion of the study how little pupils really knew about mathematicians. I therefore changed statement 9 to *Mathematics is the study of numbers*. I hoped through this statement to get a sense of whether pupils' perception of the field of mathematics was wide or narrow, for I felt that the seeing of mathematics as the study of numbers, negated pupils' experiences with geometry, and might indicate an unquestioning image of the subject.

The last thing on the survey that was changed was the addition of one more writing prompt, number 13, which asked pupils to complete the sentence, *To me, mathematics is...* With this last prompt I hoped to get one further opportunity to ascertain the image a pupil had of mathematics.

4.3.3 GRAPH THEORY AND DISCRETE MATHEMATICS IN A MIDDLE SCHOOL

Marina Nierescu began introducing graph theory to her pupils at Gramercy Middle the way that it is introduced to teachers who attend the Rutgers *Leadership Program*. She distributed large maps of the United States in which each of the individual states was outlined but not coloured in. Working in groups of three or four, pupils were given small coloured circles of paper and directed to place them on the map in such a way that no two states with a border contained the same colour.

After pupils accomplished a colouring of the fifty states, they were then challenged to eliminate colours and to find a minimum colouring. In the class discussion that followed, pupils were asked to try to prove that the four colours they had discovered was really the minimum. "Why couldn't three colours be used?" they were asked.

Exploring the map, pupils were able to discover and show that there were odd cycles, or simple closed paths, (see Roberts, 1984) around some states, as for example, there is an odd cycle of states with Utah as the hub (see **Figure 4.1**). Proceeding clockwise, Utah happens to be surrounded by five states that form a chain around it: Wyoming, Colorado, (not New Mexico since it meets Utah at a point only), Arizona, Nevada, and Idaho.

In that case, colouring the five states in the odd cycle requires three colours, since if you start colouring clockwise at Wyoming, when you get back to Idaho which touches Wyoming, you need a third colour. But then, since Utah, as hub, touches every state in the cycle, a fourth colour is necessary.

In this way, pupils found an informal proof that on the map of the United States four colours was the minimum number of colours necessary.

Figure 4.1 The Western States Map (from Microsoft® Encarta Encyclopedia 99)

From the colouring of maps, Marina introduced the class to the idea of shrinking the states down to points, or vertices, with the result that those vertices were joined by lines, or edges, if and only if the states the vertices represented were joined by a border. This led to vertex colouring and solving scheduling problems (see Malkevitch, 1981; Roberts, 1984; Cozzens & Porter, 1987).

With this knowledge of graph theoretic terminology, the class moved to the famous Königsberg Bridge Problem (see Wilson & Watkins, 1990;

Schechter, 1998), to Eulerian circuits and paths, then Hamiltonian circuits and paths and the Traveling Salesman Problem (see Cozzens & Porter, 1987; de Pomerai & Berry, 1998). Pupils also participated in a *Sprouts* Tournament (Eddins, 1998).

Sprouts, a game on graphs created by John Horton Conway and Michael Stewart Patterson at Cambridge in 1967 (Jacobs, 1994), can be used to introduce pupils to the mathematics inherent in games and their strategies, another topic in discrete mathematics.

In the following weeks—the unit lasted approximately a month—pupils also explored Steiner points (see Hoffman, 1998), fair division (see Brams & Taylor, 1996), and fractal geometry (see Peitgen et al, 1991).

Because of my own professional obligations in assisting new teachers in the early months of the fall, I was only able to be present sporadically in Marina's class for most of that month. When I was finally able to return to Gramercy Middle School, it was towards the end of the unit, and Marina was feeling very unsure of herself and her pupils. In a pre-class conference she told me, "I don't think they really remember any of the material."

As much to reassure her, as to ascertain whether her fears were accurate, we decided to present the pupils in her class with a variation of the *Königsberg Bridge Problem*, called the *Paris Bridge Sweepers Problem* (Picker, 1991). This problem is based on the fact that the city of Paris, like the old city of Königsberg, has two islands between two banks of a river (see **Figure 4.2**). In the case of the city of Paris, these assorted banks and islands in the Seine are joined by at least 15 bridges. Here is the problem:

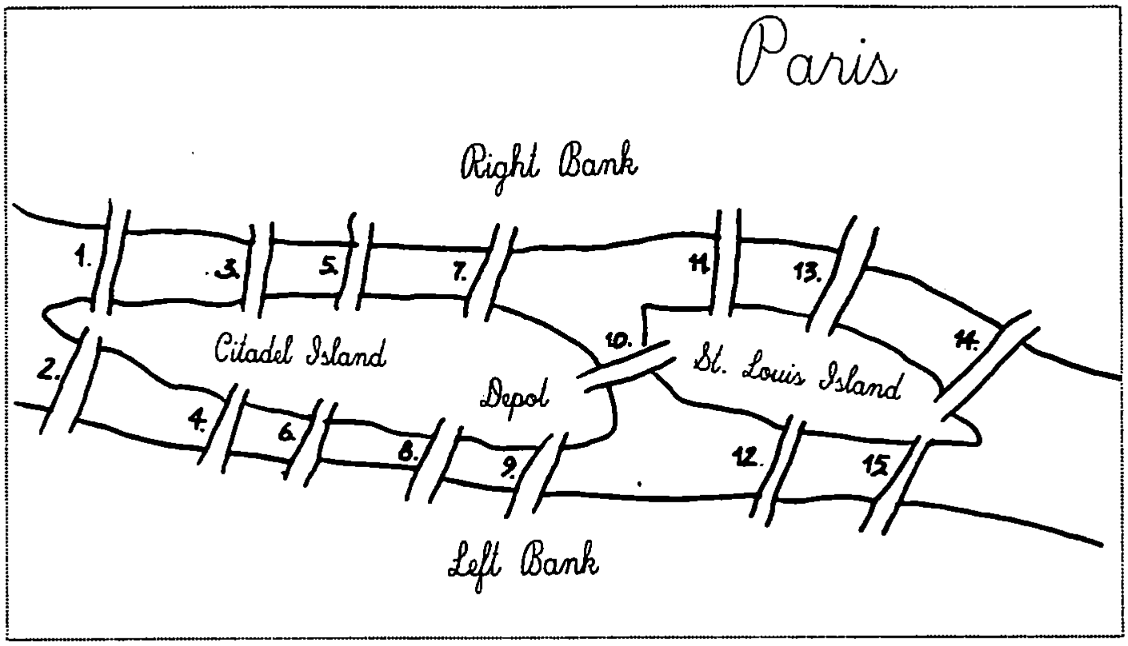

Figure 4.2 *Paris Bridge Sweepers* problem

> *Is it possible for the Paris bridge sweepers to leave their depot on Citadel Island (Ile de la Cité), sweep each bridge just once and return to their depot?*
>
> *A huge potato spill has resulted in the closing of bridges 1 and 2. Can the sweep team now start at the depot, sweep bridges 3-15 and return to their depot without repeating a bridge? Draw a graph and explain your answers.*

Although the pupils hadn't seen a problem like this since looking at the Königsberg problem three weeks previously, working in pairs they all were able first to change the map to a graph, and then to answer the questions posed by inspecting the parity of the vertices. The pupils enjoyed doing the problem, and Marina was greatly reassured.

At the conclusion of the unit—it was now November—pupils were retested using the same investigative survey tool. In all, there were n = 26 pupils who completed both surveys. In addition, eight pupils volunteered to be interviewed about their surveys and their experience with graph theory and discrete mathematics.

4.4 PREPARING FOR THE MATHEMATICIANS PANEL

The idea for a meeting between mathematicians and pupils which could serve as an intervention to overcome their lack of knowledge about and negative images of mathematicians, came from a similar panel I had attended some years ago at the Rutgers University *Leadership Program in Discrete Mathematics*. At that time, the audience consisted of teachers who were participants in the program, while the mathematicians, who worked mostly for AT&T and Bell Laboratories, were applied mathematicians working in the fields of graph theory and combinatorics—topics the teachers were studying.

This experience had given me a greater insight into the work that mathematicians do. Now I wished to see if a similar experience might serve as an intervention with 7th grade pupils. Marina Nierescu's class at Gramercy Middle was set to participate in the audience for the Panel. It was then necessary to choose more classes to take part in the event and intervention.

4.4.1 THE PARTICIPATING SCHOOLS AND THEIR TEACHERS

The two other schools that came to be part of the project were chosen for pragmatic reasons and indeed political reasons—unfortunately, politics is unavoidable in a district such mine. Since the event was to take place in Hudson Middle's auditorium, it was expected that the two 7th grade classes there would be invited.

And the inclusion of Chelsea Middle School came about for two reasons. The first is that it is located close by—just two subway stops away. Moving more than 90 lower secondary pupils through New York City's streets and subways involves chaperones, transportation permits, permission slips and a lot of careful planning. It was expected that I would make every effort to keep the potential chaos to a minimum.

The second reason is that I worked one day a week at Chelsea Middle, assisting and training the seventh grade mathematics teacher, Connie Sachs, and I had come to know her and her classes.

This was Connie's second year in our district, although she has been teaching middle school mathematics in other districts for the past eight years. She is in her mid-thirties, with short blonde hair and a strong background in mathematics. Last year, her first in the district, she struggled as a teacher.

Connie was teaching three of the seventh grade classes with the fourth taught by another teacher, Jodi Lin, who was extremely popular and known to the pupils. The pupils in Connie's class were disappointed that they didn't have Jodi Lin. And for some time, many of them didn't hide this disappointment.

For Connie Sachs, that year started off badly enough, with the deaths of two close relatives and with as having to teach a new curriculum, in a new and unfamiliar school, to pupils who were resentful. It was a very hard year. But now in the new academic year, her second at Chelsea Middle, Connie was teaching all four seventh grade classes. She was thriving, and she was eager to participate with her pupils in the Mathematicians Panel. "My classes are yours," she told me.

Chelsea Middle, as does Gramercy Middle, chooses its pupils selectively but in this case by an entrance examination when pupils are still in the 5th grade in their primary schools. The exam consists of writing, reading and mathematics skills and the pupils selected have the highest scores. This selectivity is the reason the school consistently ranks first in the city of New York on standardized tests in both reading and mathematics.

The pupils are primarily white although there are minorities, which include pupils of African, Latin and Asian (primarily Chinese) backgrounds.

Chelsea Middle School is another small school and it, too, shares a building, in this case, with two other small schools. The building dates from the early 1970's and Chelsea Middle has existed there for nearly ten years. It has a very stable teaching staff, a good sign in a system that is as large as New York City's.

At Hudson Middle School, Nancy Rockwell was the teacher of the two seventh grade classes. I knew her less well than I knew Marina Nierescu and Connie Sachs, as this was Nancy's first year teaching in the district. It was only her second year as a mathematics teacher. She is in her early twenties, and a recent university graduate. Fair-haired and tomboyish, she looks to be barely older than her pupils. Although she is working with a mathematics staff developer, she is struggling, as many new teachers do in their first year.

In a situation similar to the one encountered by Connie Sachs at Chelsea Middle, Nancy finds herself compared to the very popular mathematics teacher her students had last year, and compared unfavorably. Although her mathematics background is strong, her lack of sureness as a teacher seems to make for friction in the classes and some difficulties with discipline.

Hudson Middle shares an impressive-looking new building with a primary school, both of which are growing by adding a new grade each year. Since this is the second year the school has been open, Hudson Middle does not yet have an eighth grade. The school is in a new complex overlooking the

Hudson River on landfill created thirty years ago when the excavations were dug for what is now the World Trade Center.

Unlike both Chelsea Middle and Gramercy Middle, pupils are admitted to Hudson Middle by virtue of their living in a prescribed area within lower Manhattan. Since the school is only a few blocks from Wall Street and the financial district, many, though not all of the pupils are children of financial services professionals. Still, there is racial diversity among the pupil population, which is similar in composition to Gramercy Middle School.

4.4.2 PREPARING THE PUPILS FOR THE PANEL

Having established which schools and classes would participate and having received all the requisite permissions, I visited each of the seven classes to prepare them for the Mathematicians Panel. It was scheduled for a date in April during what is designated in the United States as "Math Awareness Month," and which is promoted by national organizations such as the National Council of Teachers of Mathematics (NCTM).

Except for Gramercy Middle, where pupils had completed the research survey tool in September, the first thing I did upon visiting a class was without any discussion, to administer the survey. This took about half an hour. After they were collected, the pupils were told of the event to take place, that is, that there would be a panel of mathematicians who would make brief presentations and then answer their questions.

Pupils were told the number of mathematicians on the panel—at that time I did not know if it would be seven or eight, so they were told 'around seven'—and that some members of the panel would be women. The pupils were also informed that the mathematicians would represent five countries.

In some classes they never inquired into what countries would be represented on the panel, but when they did, they were told: England, Nigeria, Romania, Wales, and the United States.

In each class we first discussed how pupils felt they should conduct themselves and represent their schools during the panel. In each class a number of pupils commented that they thought the mathematicians should be listened to respectfully and that they, the pupils, should be quiet during the presentations.

Since in each of the classes, pupils themselves came up with the guidelines for their own conduct, I then asked them to agree to abide by the statements made by their colleagues. All the pupils agreed to this in each class and they basically kept their word on the day of the Mathematicians Panel.

The pupils in each of the seven classes were then asked to come up with questions they would want to ask of the mathematicians. In the first school I visited, Gramercy Middle, a pupil stated that pupils shouldn't ask stupid questions. When I said that I wasn't worried because I didn't think stupid questions would be asked, the next pupil who spoke said he wanted to ask the mathematicians, *What is the exact number of* π *?*

Other pupils in the class immediately told him in a surprisingly friendly way, that it was a stupid question. The boy shrugged; the discussion went on. The pupils for the most part regulated their own questions.

The only influence I had on any of the questions pupils posed was at Chelsea Middle, when I pointed out that we weren't going to be playing stump the mathematicians—this was after a pupil proposed a question that was similar to the pi question. And when pupils said they wanted to ask the question, *How much do you make?* I suggested they might ask for a ballpark figure, as it was more polite.

In each class, the teacher recorded the questions for me, while I wrote them down for the pupils on the chalkboard or overhead. The pupils copied the questions and were told to bring them with them on the day of the panel. When I left each class I took with me the set of questions the teacher had recorded for typing up and sending to the mathematicians in advance of the panel.

After meeting with the seven classes there were 144 questions. Some were asked in common by different classes; some were unique to a class. But there was one question that was asked in every one of the seven classes:

What does a mathematician do? (See **Appendix—D** for all of the pupils' questions.)

4.4.3 THE MATHEMATICIANS

The mathematicians who agreed to appear on the panel accepted the invitations enthusiastically, expressing pleasure at the idea. They were told in the email communication inviting them to participate, that I had been finding that for pupils (and their teachers!) mathematicians are, for all practical purposes invisible. I added that I had come to the idea for the panel in an effort to change this, and to challenge first hand pupils' misconceptions of mathematicians.

I knew Dana Frank from the summer *Leadership Program in Discrete Mathematics*. She was also co-editor of the book, *Discrete Mathematics in the Schools* of which I am a contributing author. I have spoken to her on a regular basis both about my research and about her preparation of primary school teachers as a university professor. And also I knew that she did mathematical research in the area of graph theory in addition to her teaching. She is a petite woman in her late thirties, who grew up in Northern California.

When I told Dana that I wanted to have as diverse a panel as possible, but didn't know any mathematicians of African heritage living close enough to take part, it was she who suggested Drs. Juliet Grey and Noemi Anjolo. Both are on the faculty of a university that happens to be located across the street from the school auditorium where the panel was to be held.

Dr. Grey, a tall attractive woman of African-American heritage who nearly went into modelling, is in her late forties. Dr. Anjolo is from Nigeria and spent a part of her life in Sierra Leone, before coming the United States to attend graduate schools. She is a petite woman with a wonderful laugh. It is hard to tell her age—she may perhaps be in her late forties. Both women were quite different from each other and added immensely to the diversity that I wanted the panel to present to the pupils.

Wendy Mills was the second mathematician invited to participate, and the date for the panel was actually set to coincide with her plans to visit New York City. Attractive and thoughtful, and in her early forties, she is an engineer who was writing her dissertation at Plymouth when I began as a research student.

I invited Marina Nierescu to take part as a panelist for two reasons—she is a worked as a statistician, and she was born and trained as a mathematician in Romania. Her class was to attend the panel. She is described further in the section above.

It was harder to find male mathematicians to take part in this panel. Two emails I sent were never answered; one had only asked for suggestions of mathematicians to invite since the person I was writing to was at a university in the south western United States. I emailed an invitation to a mathematician of Indian heritage at the Courant Institute at New York University whom I knew from the Rutgers program, and he replied that he was now working for a financial services company on Wall Street, and unfortunately he had no time to take part in the panel.

When I learned that the son of a professor I knew in Cardiff was now working at Princeton's Institute for Advanced Studies in combinatorial biology, I emailed an invitation to him to join the panel. He accepted, and although I had never met him, I knew Alun Llandaff was very young—just in his mid-twenties—which I considered to be a real plus. I had overheard a pupil tell his classmates—*I bet they're all over forty*—when speaking about the upcoming mathematicians panel.

Having never met him, I now found myself hoping that Alun Llandaff didn't have a beard, because I knew that both Michael Mann and James Montauk did. It turned out that he didn't, and in fact with his blond good looks and ponytail he got very loud applause for everything he said. He was popular with the girls, but his youth and science background seemed to make him appealing to the boys as well, one of whom later referred to him as *the computer dude.*

I knew Michael Mann from my first year in the Rutgers program. Although that had been nearly ten years ago, I was very fortunate to be able to locate him via the Internet. He is the first mathematician I had ever really met, and it seemed fitting to me that he be invited. I thought his area of computer security would be interesting to the pupils, and although at first he had a conflict for the April 6th date, he very quickly got back to me saying that he had resolved it. He was the first panelist who asked if I wanted a brief bio, which encouraged me to ask for one of all the panelists. He is in his very early forties, with a short neatly trimmed beard and graying hair.

As the date got closer and I still only had two men to the four women on the panel, I invited my colleague James Montauk to take part. He is in his mid-thirties with short sandy-coloured hair and beard and a very quirky sense of humor. He considers himself a mathematician, has an M.A. in mathematics, and is thinking seriously about going back to graduate school. James Montauk turned out to be a good choice, as his earlier interests in heavy metal music and surfing provided another layer of diversity amongst the panel and one which was later cited by the pupils in the audience.

4.4.4 BRIEF BIOGRAPHIES OF THE MATHEMATICIANS

A week before the panel, each of the seven mathematicians was asked to submit a brief introduction. For five of them, it led into a short presentation to the pupils; the other mathematicians, James Montauk and Wendy Mills, had no planned presentations and so were introduced as looking forward to answering pupils' questions. These introductions are quoted below in the order in which the mathematicians were introduced to the pupils. Since Dana Frank had to leave by 10:30 a.m. in order to return to her university for office hours, she was first:

Dana Frank is currently a mathematics professor at the CUNY campus on Staten Island. She works in an area called discrete mathematics, which is the study of patterns and arrangements. One problem she is working on now arises in the design of computer chips. She has a PhD from MIT, which is one of the main research centres in the U.S. for discrete math. She teaches a variety of classes, including courses in discrete mathematics and geometry for teachers. She considered many different careers, but, while in college, finally decided to stay in mathematics.

Michael Mann is head of the Specification and Algorithm Research Department at AT&T Labs. He is a recognized expert in computer security and distributed computing. He has a Ph.D. in Information and Computer Science, has published more than thirty-five research articles and holds four patents, three in the area of computer security. In 1995, he co-authored the book, "Atomic Transactions."

Dr. Juliet Grey is a mathematics educator who has taught the subject at every level from middle school through college; she is currently teaching at the college level. Dr. Grey has also been involved in many professional organizations, including the Mathematical Association of America, the National Council of Teachers of Mathematics, the Association of Women in Mathematics, The New York State Mathematical Association of Two-Year Colleges, and the American Mathematical Association of Two-Year Colleges.

James Montauk was born and raised on Long Island. While trying to decide what to major in at Hunter College, he worked at a variety of jobs, including as a construction labourer and a lifeguard. He also performed as a bass player in several awful heavy metal/blues bands. In time, he decided that majoring in mathematics was a natural next step in the progression. After beginning his teaching career at Martin Luther King High School, near Lincoln Centre, he earned an M.A. in mathematics from Brooklyn College and is now thinking about a further degree. He is an adjunct lecturer at Lehman College, teaching mathematics to teachers. This is his second year in District

2, and his third as a staff developer. He believes that his interest in and love of mathematics is a natural complement to his love of puzzles and games.

Wendy Mills has an MA in engineering from Cambridge University and a Ph.D. in mathematics education from the University of Plymouth. Her first employment was as a design engineer working on turbo machinery for the British Ministry of Defence. Her research interests are mathematical thinking in engineering undergraduates and multimedia authoring. She also collects interesting oddities in statistics. She resides in Exeter in the south west of England where she and her husband are raising three sons.

Dr. Noemi Anjolo is an assistant professor of Mathematics at Borough of Manhattan Community College and a Co-Editor of "Mathematics in College," a refereed journal of the CUNY Mathematics Discussion Group. She is listed in *Who's Who Among American Teachers (1998)*, and is on the *Project Kaleidoscope Faculty for the Twenty-First Century* (1997-2000).

Marina Nierescu is Romanian born and trained in mathematics. She worked for a year as a statistician in the United States before coming to teach in District 2 at Gramercy Middle School three years ago. She has been learning from and working with international researchers in Graph Theory for the past two summers at Rutgers University.

Alun Llandaff is from Wales, and is a mathematical biologist. He studied math at Cambridge University and then did a doctorate in the Department of Biology in Oxford. He uses mathematical ideas and techniques to study problems in medicine and biology and is particularly interested in infectious diseases. He is currently at the Institute for Advanced Studies at Princeton University, working in the Department of Theoretical Biology.

4.5 THE MATHEMATICIANS PANEL

On the morning of 6 April 2000, pupils from three schools and mathematicians from five different countries began arriving in the auditorium of Hudson Middle School. We were slated to begin at 9 a.m. but were delayed fifteen minutes waiting for the class from Gramercy Middle School to go both across town and down town on the subway. When they hadn't arrived by 9:15 we had to start without them. They did not arrive until 9:45.

I was the moderator for the panel, which involved welcoming everyone, reading the introductions, welcoming the students to come up to ask their questions, and then thanking everyone. At no time did I answer any questions or become involved in the discussions, except to note the time or enlist the assistance of the teachers in the audience to quiet pupils (something that wasn't often necessary) or distribute a reaction questionnaire as the panel ended. The nearly two hours were video recorded by a colleague and from it a transcript was produced (see **Appendix—E** for the complete transcript.)

As the panel began, pupils had their notebooks out and open, lists of questions out, ready to tick off questions the mathematicians might answer as they spoke, and ready to ask questions when the time came for the mathematicians to take them.

Dana Frank was introduced and spoke first. Relating both her early interest in art and a present enjoyment in creating origami polyhedra to the fact that many mathematicians spend a lot of time procrastinating, she showed origami polyhedra of a cube and a dodecahedron. She then went on to speak about a problem of finding Hamilton circuits on the planar graphs of such polyhedra, and the question of whether there was a nice way to construct such walks as the polyhedra grew increasingly complicated.

Michael Mann spoke next about his work as a researcher at AT&T. He noted that I had sent pupils' questions to the panel, commented that the questions were quite good, and that they particularly asked what a mathematician did and what a day was like. He said that it might surprise pupils to find out how social mathematics really is, and how much he enjoyed working with colleagues who were sometimes on the other side of the globe.

Dr. Mann took up the question the classes had posed regarding how much money a mathematician earns. When there was some laughter at the mention of this question, he said quite seriously that students had a right to know if they could make a good living working at mathematics. You can make a good living, he advised them, as he told of a friend who had made trillions starting his own company although, Dr. Mann noted, in a recent stock market dip, the friend had lost millions, too.

Juliet Grey was the third mathematician to address the audience. She had started a career in business in order to make a lot of money, she explained, but when she found herself in a situation where she was forced to interview workers and then design computer systems which would cause them to lose their jobs, she decided she'd had enough of the business world. She entered teaching and found she really liked it. She emphasized to pupils that one of the most important things she had learned is that mathematics is a language and science of patterns.

James Montauk was being introduced as the pupils and teacher of Gramercy Middle School finally arrived. At that point Marina Nierescu, their teacher, took her place on the panel. Then there was applause from the pupils upon hearing that James Montauk had been with a heavy-metal blues band before deciding to study mathematics.

Wendy Mills was also introduced at that time and there was audible reaction among the pupils when they heard that she had designed turbo-machinery for the British Ministry of Defence.

Noemi Anjolo was now introduced. When she took the microphone to speak she first asked the students if they knew where Nigeria was. She was pleased when almost all the hands went up. She identified herself as of Igbo ethnicity.

She brought up one of the questions pupils had posed, asking how she had become interested in entering her profession. It was easy for her, she noted, because her mother was a mathematician. And it was from her mother, Dr. Anjolo said, that she came to see mathematics as fun.

As she began to discuss the research in which she was currently engaged, Dr. Anjolo asked the students if they played games. When they indicated that they did, she asked if they knew some of the oldest board games. Besides chess, Mancala was mentioned by a number of pupils in the audience.

Saying that she was glad to hear Mancala mentioned, Dr. Anjolo told the audience that she was particularly interested in ethnomathematics, which she defined as mathematical ideas inherent in cultures. By cultures, she added, she didn't necessarily mean ethnic groups, but something wider, including the culture of a classroom, or a culture of work. Her research was now focused on studying the strategies of Mancala, particularly the version found in the Igbo culture.

Dr. Anjolo showed an overhead of a Mancala game board, and she enumerated the mathematical ideas found in playing the game, including counting, combinatorics, probability and geometry. In the Igbo culture, she noted, this Mancala version was one of the ways in which pupils are taught basic number skills. Finally, Dr. Anjolo pointed out that the game is very easy to construct—an egg carton, and maybe pebbles or counters, and you've got it—and you can design the carton anyway you please.

Marina Nierescu was the next panelist to speak. Briefly describing her schooling and background, which has been noted previously above, Marina began by saying that she was surprised that she had been invited to be part of the panel since, she said, "You know lots of people don't consider mathematics teachers as being mathematicians." She said to the pupils in the audience, "I would really like to open this discussion for you—what do you guys think? How many of you think your math teacher is a mathematician?"

At this point many hands went up among pupils from Gramercy Middle and Chelsea Middle; fewer from Hudson Middle. Marina Nierescu asked one pupil who had her hand up, a girl from Chelsea Middle, to give some reasons why she thought her teacher, in this case Connie Sachs, was a mathematician.

"Our teacher uses patterns and equations to solve problems," the pupil stated.

Marina now continued, "Do you think that's enough? —Somebody works with patterns and they can be considered a mathematician?" A pupil in the audience now said—"No."

"So why do you consider your teacher as a mathematician?"

The pupil replied, "Because she's good in math."

At this point, Marina changed the subject, and spoke briefly about her interest in discrete mathematics. Dana Frank had been introduced with a brief mention and definition of discrete mathematics, but the Gramercy Middle School class had missed this in arriving late.

The last mathematician to speak before the pupils were to ask their questions was Alun Llandaff. His field, mathematical biology is really on the cutting edge of mathematical knowledge, and he began by acknowledging that pupils had probably not heard of it. Although he was using a microphone, he has a very quiet voice, and it became completely quiet in the auditorium as pupils strained to hear him. He was clearly the closest in age to them and he was dressed in a manner the pupils would describe as trendy—similar to the way they would dress for an occasion like this—a dark purple shirt out over dark jeans with work boots.

Dr. Llandaff explained that in his field they used mathematical ideas to help understand disease processes, or questions of oncology, or evolution. He placed up an overhead slide of three graphs showing the number of cases of measles in three countries—Britain, Denmark, and Iceland, and he spoke about the kinds of questions mathematical biologists asked and looked to answer: What is going on here? —Why do we see different patterns? What are the similarities and differences in the patterns we see?

It was clear as he spoke and used the term *we*, that like Michael Mann, Alun Llandaff worked with others in a research group. Showing peaks on the graph which represented epidemics, he concluded by explaining that in the field of mathematical biology, they used "mathematical models as physical techniques to try to understand" the questions raised. There was very loud and sustained applause as he concluded.

4.5.1 THE PUPILS QUESTION THE PANEL

It was now time for the pupils to ask their questions of the mathematicians and they basically asked the questions they had brainstormed together. Although at first they were hesitant, eventually many pupils came forward to pose a question, and at the time we had to conclude the panel, there were still at least fifteen pupils who never got their chance to speak.

The first question came from a girl at Chelsea Middle, who asked, "What is your goal as a mathematician?"

Four mathematicians provided answers to this question: Noemi Anjolo said that when people heard about mathematical ideas in Africa, they mainly thought of Egypt, but that she wanted to have the mathematics of other areas in Africa better known; Michael Mann said that his personal goals lately had been to "understand problems related to networking and the internet and to do mathematics that is relevant to the efficient management of resources."

Juliet Grey spoke of her work with the mathematical preparation of elementary teachers. "So my goal," she said, "is to raise a new generation of mathematicians among all of you. By working with your teachers." Wendy Mills, referring to Dr. Anjolo's description of ethnomathematics, commented that she'd just learned that she'd been studying the ethnomathematics of engineers. "And one of my goals," she asserted, "is to try to work out mathematics training programs for engineers."

The next question also came from a pupil from Chelsea Middle—a boy who told the panel, "I guess you all met each other before now." At this, the members of the panel emphatically shook their heads no. "Or you've met other mathematicians before," the pupil continued. "So my question is, from meeting other mathematicians, and from yourselves, do you find that people with exceptional mathematical skills also have exceptional skills in like different topics as well, that usually a lot of mathematicians have in common?

Nearly every mathematician spoke about an interest they had in art or music, and Dr. Grey cited research that showed that "people good in mathematics also happen to be good in music." James Montauk pointed out that "there's an art to mathematics as well, in terms of finding patterns and looking at interesting ways to fill space; whether that space is composed of time, as in music, or a piece of paper, as in art."

Dr. Mann mentioned a colleague who had left AT&T Labs to write scripts for *Futurama*, a science fiction television show all the pupils knew about. "So people with very diverse interests go into and out of mathematics," he concluded. Marina Nierescu mentioned that in Romania and in the United States, many mathematicians, besides being involved with music and art, are also very interested in philosophy.

4.5.2 THE PUBLIC'S IMAGE OF MATHEMATICIANS AND MATHEMATICS COMES UP

From Hudson Middle, a pupil named Annie next asked, "I was wondering, what are the downsides of being a mathematician—if any?"

Being given the dinner check to figure out when dining with friends was mentioned, as was the perseverance necessary in putting up with the frustrations one encountered in solving a problem before getting to the exciting part.

Dr. Grey then said, "One of the most frustrating things about being a math person, is the reaction of other people, when you tell them what you do.

I have sometimes resorted to telling people that I'm a clerk at Bloomingdale's because I just hate the reaction that I see when I tell people what I do. And when I begin to talk about mathematics, then I get an even worse reaction sometimes, and it's all because everyone has had, or very many people have had very difficult experiences with mathematics. And so they connect you with part of that very difficult experience, which is unfortunate.

Dr. Anjolo added to Juliet Grey's answer by stating, "I think that one of the down sides is that people think you're a nerd—you can't have fun." There was a lot of laughter both in the audience and on the panel at this. She continued, "But mathematics is fun! You just have to see the fun that is in it!"

Two girls came up together to the microphone, and introduced themselves as being from Hudson Middle: "Okay, now we were wondering—and this is only for the women—do you ever get discouraged in mathematics simply because of your gender?"

Four of the five women on the panel responded to this question, and said that although there were times had felt isolated they hadn't really felt discouraged. Marina Nierescu said, "Don't even think of this as a problem." Noemi Anjolo said she was always part of a tiny group of women, but as a black woman, she was often the only one. Loving what one does and finding a support system were paramount, as was the fact that Dr. Anjolo's mother was also a mathematician. "I could always hang on to that fact to keep me focused whenever the isolation or the feeling of isolation started to seep in," she said.

At this juncture, we realized that if so many panel members responded to each question, we would get through very few more, and so I had to ask the panel members to allow one answer in order to entertain as many questions as we could in the time remaining.

The next question was quickly answered. A pupil named Peter from Hudson Middle asked, "Would you recommend us to be mathematicians?

"Certainly," replied Noemi Anjolo.

"What?" Peter asked.

"*Certainly,*" repeated Noemi Anjolo.

The audience applauded.

Gwendolyn, a pupil from Hudson Middle next asked, "What was your school experience?" Wendy Mills answered that she went to an all girls school in England where nobody told her that girls don't become engineers. She also said that probably at the same age of the pupils in the audience, she had found a textbook at school with pictures that made her feel that mathematics was interesting and cool.

The next question came from a Chelsea Middle School pupil, James, who asked, "What is your definition of a mathematician?"

There were two related answers to his question, from Michael Mann and Noemi Anjolo: Dr. Mann pointed out that it was hard to pin down one definition of mathematician at this time, because of the wide variety of applications in such diverse fields. He noted that when he was in high school, he looked into becoming a mathematician but the only information he could find "was about teaching more mathematicians, which is important, but I was wondering why we needed them."

Dr. Anjolo now added, "I would say, someone who does research in mathematics; someone who applies mathematics to solve challenging problems that we have in real life—social problems that we have in life, as well as scientific problems. And also someone who teaches mathematics to get others to think mathematically."

A pupil named Matt from Gramercy Middle now asked, "What is like the newest field in mathematics?"

At this, all heads in the panel turned, all fingers pointed, to Alun Llandaff. He laughed. "Well, I'd say mathematical biology, of course, but I'm a bit biased. But there are a whole load of new fields; computational areas have opened up with the advent of large-scale computers in the last few years, and I think through biology particularly, you're going to see a real explosion of mathematical ideas opened up both by opportunities offered by computers, but also genetic data becoming available through the genome project."

At this point two girls from Chelsea Middle came up to the microphone to ask, "If you wanted to pursue a career in mathematics, where would you start?"

They were advised that a way to begin is to join a math team, as Dana Frank had, and in a couple of years to attend a summer math program for high school pupils where they may be able to participate in a variety of projects, including research. "Put yourself in situations where there're a lot of people who are excited about mathematics," Dr. Frank told them.

Tony, a pupil at Chelsea Middle, now asked, "How do you think math will affect our future?"

Dr. Anjolo laughed. "How does it affect your present? I think if you have the answer to that, you'll know how it's going to affect your future!"

Perhaps because his question had been answered so quickly, Tony started scanning his questions. "Well, I have another one—What is your favorite kind of math?" There was a pause, so he pointed to James Montauk. "Mr. Montauk, what is your favorite kind of math?"

James Montauk replied that liking all aspects of mathematics, he didn't think he had a favorite. When solving a problem, he pointed out, you never know which part of math you might bring in to solve it. But he did admit he had some trouble with combinatorics—counting techniques.

A girl from Gramercy Middle named Angelina, asked Dr. Anjolo if she had heard of a particular mathematical game, which perhaps she would be willing to teach to Angelina's teacher. Noemi Anjolo replied that she hadn't but that she would be interested in looking for it.

Sam, from Chelsea Middle now asked, "Is there someone who like teaches you math?"

Always, he was told. The mathematicians said they learned from going to conferences, by talking to other mathematicians, and from their students, who "often come up with interesting ways of solving problems that you had not considered."

A boy named Remi from Hudson Middle asked the next question: "Do any of you have a mentor?"

Dr. Grey spoke of a woman who had been very influential during her career.

Another pupil from Hudson Middle, Matthew, now asked, "What job would you like if you weren't a mathematician?" Noemi Anjolo replied that she thought she would have been an artist; Wendy Mills said she would have liked to write science fiction; Dr. Mann answered that he would love to teach philosophy; and Dr. Grey admitted that with her love of *Nancy Drew* mysteries, she might have been a private detective.

Keith, a pupil at Gramercy Middle, now asked, "Have you ever had second thoughts about becoming mathematicians?"

Dr. Mann went back to something mentioned previously—the periods of frustration one can encounter, with the questioning of oneself, Can I do it? And the times when he wondered if he would ever manage to get a Ph.D. But he also pointed out that friends, peers and mentors get one through those times.

4.5.3 A PUPIL ASKS ABOUT STEREOTYPES

James, a pupil at Hudson Middle, now asked the panel, "I was wondering what you thought about the stereotype of mathematicians—like with the hat, like they're a wizard or something?" There was a lot of laughter in the panel and in the audience at this.

Dr. Anjolo said, "Like all stereotypes, never believe it!" To which Dr. Grey added, "I think that a lot of different—" At this point Michael Mann had disappeared momentarily backstage and now came back with a hat on—a black cowboy-type hat. There was more laughter. Juliet Grey continued, "—I think that a lot of careers have their—some individuals stereotyped, but I think that in mathematics you have to remember that there are a lot of people who do mathematics—we have examples here on our panel—(she touched Alun Llandaff's shoulder at this point)—of people who integrate other careers with mathematics.

"But I think that there's one area of mathematics that is a universal area that really prohibits us from stereotyping the person in any one particular way. And that area is problem solving. Problem solving is a major area of mathematics, but it's a part of every single career that exists. So it's impossible to stereotype, you know, any one person who is a problem solver."

At this point I had to tell the audience that we had time for only two more questions.

A pupil named Jack, from Hudson Middle now asked, "What did you want to be when you were twelve or thirteen years old?" Wendy Mills said, a biochemist; Dr. Mann said when that he was a year older then they, he decided to become a mathematician; Juliet Grey said, first a writer, then a scientist, then a model; Dr. Llandaff wanted to work with computers; and James Montauk wanted to be a professional surfer, although he soon realized it wouldn't pay much.

The last question of the morning came from Naomi, a pupil from Chelsea Middle, who asked, "I want to know from those of you who are from foreign countries, or who are from other parts of the world, is there any type of math that you feel is done like differently in that country or done a better way or a worse way, in your opinion?"

Dr. Anjolo answered that in Nigeria, they don't think of mathematics as separate from society: "It's integrated in what the people do or actually in how they work. Also, with technology, as you know, and the underdevelopment in Africa, there's bound to be a difference, because a lot of areas in Africa are not as technologically equipped as areas in the developed world, and so there's certainly going to be differences in how mathematics is done."

Dr. Mills spoke about English schools, saying that they are beginning to move away from the exploratory side of mathematics in the classroom. This was something about which she said she was quite sad.

4.5.4 PUPILS WRITE ABOUT THEIR REACTIONS

The questions portion of the panel was now concluded. The time had passed quite quickly. Pupils were now given a half sheet of paper on which to record a written reaction to the panel. The sheet read,

> *Please write some sentences about what you thought about today's Mathematicians Panel, including what you learned that most surprised you, what questions you still might have, and what you might tell a reporter who interviewed you about today's event. (You may use the front and back of this sheet.)*

Pupils were given about ten minutes to write, and when they were finished there was applause to thank the members of the panel.

4.6 SURVEYING PROFESSIONALS IN THE MATHEMATICS FIELD

In this small study conducted during 1998 and 1999, a short survey (see **Appendix—F**) was given to professionals working in the field of mathematics. The respondents included professors, supervisors, research mathematicians, teachers and teacher trainers of mathematics from elementary school through the secondary grades.

The respondents were approached in England at the University of Plymouth and at Exmouth during seminars; by post to Romania (with translation to Romanian and back again into English by a colleague); in Community District 2, where I work in New York City; at summer programs at Rutgers University at which there were teachers from all over the United States and visiting professors from universities on the east coast; at a Texas Instruments T3 Conference in Chicago early in 1999 and at a technology conference held at the University of Plymouth in the summer of 1999.

In all, there were (n = 106) respondents to the survey. The professionals were from the U. S. (n = 68); the U. K. (n = 28); Sweden (n = 2);

Romania (n = 2); and one each from Norway, the Netherlands, Lebanon, France, Belgium, and Israel.

The survey opened with four general information questions, which were followed by an open-ended writing prompt designed to elicit an explanation for one of the answers respondents had given, and a second open-ended question, different from but related to the first.

The purpose of the survey was to get a sense of the prevailing opinions of professionals working in the field, about whether they considered themselves a mathematician, and who they thought may call themselves one.

Chapter 5: DATA

5.1 INTRODUCTION TO THE DATA SECTION

This chapter contains data for two of the three parts of the study, presenting the international study data first, and then the data of the survey of professionals in the mathematics field. The analysis follows in Chapter 6 and the intervention data will be presented in Chapter 7.

5.2 DATA FOR THE INTERNATIONAL STUDY

This section begins first with data pertaining to pupils' drawings, then looks at the data from the Likert-type survey tool and finally at pupils' written responses to the first writing prompt. The second writing prompt, in which pupils were asked to describe their drawing, was used in more clearly categorising the drawings, i.e., ascertaining more surely the gender of the character in the drawing, or whether the pupil intended to depict a teacher.

5.2.1 THE CODING SYSTEM FOR CATEGORISING THE DRAWINGS

Examination of pupils' drawings yielded two distinct categories of drawings and two themes. The drawings appeared to be either neutral in tone, or they depicted a character that was stereotypical and/or weird in some fashion (see **Table 5.1** below). The two themes that emerged in the drawings were depicted in characters engaged in either teaching or in performing the duties pupils believed a *mathematician at work* would perform. The latter mainly consisted of characters at blackboards or at desks with pen and paper or computers.

In deciding whether to characterise a drawing as stereotypical, markers were looked for and a drawing had to have at least two of these markers. Alternatively, the pupil may have indicated in writing disdain for mathematicians and/or a written indication that this was a stereotypical drawing. Markers included:

Bald or balding figure; weird hair on figure; facial hair; spectacles; weird clothing; laboratory instruments; weird teeth; pocket protectors with pens and pencils; pencil behind ear; instruments of mathematics; mathematical notations on clothing; lab coat; laboratory flasks or notations of "potions"; "mad scientist" indications; steam coming out of the figure's ears; thought balloons; light bulbs; "eureka!" indications; wizard hat or cloak.

5.2.2 THE TWO CATEGORIES OF DRAWINGS

	DRAWING CHARACTERISTICS			TOTAL
	Neutral/Non-descript	Stereotypical/Weird	missing	
USA	154/76.6%	47/23.4%		201
UK	78/78.8%	21/21.2%		99
Finland	56/59.6%	24/24.5%	14/14.9%	94
Sweden	13/26.5%	36/73.5%		49
Romania	24/72.7%	9/27.3%		33
	325/68.3%	137/28.8%	14	476

Table 5.1 By country: Frequency and percentage of neutral or stereotypical drawing characteristics.

5.2.3 TWO THEMES

Pupils' drawings appeared to separate into two distinct themes: those in which pupils drew their idea of what they believed a *mathematician at work* would look like, and those in which pupils drew a teaching figure. In **Table 5.2** is a comparison by country of the drawings:

						TOTAL
		no drawing	Teacher	Mathematician	Indeterminate	
	USA		41/20.4%	158/78.6%	2/1%	201
	UK		21/21.2%	78/78.8%		99
	Finland	14/14.9%	27/28.7%	51/54.3%	2/2.1%	94
	Sweden		9/18.4%	40/81.6%		49
	Romania		4/12.1%	28/84.8%	1/3%	33
Total		14	102/21.4%	355/74.6%	5/1%	476

Table 5.2 By country: Frequency and percentage of drawings representing a mathematician drawn at work or a figure teaching

A figure was only counted as a teacher if it was made very clear in the drawing or accompanying written explanation that this was indeed a teacher. Thus, even if the figure was at a chalkboard, unless it was explicitly referred to as someone teaching, the figure was classified as being the child's representation of a mathematician at work.

5.2.4 THE GENDER OF THE MATHEMATICIANS IN THE DRAWINGS

Within **Table 5.3** are the percentages in each country of the gender of the mathematicians in the drawings, where the drawing gender was discernable. In some cases, particularly in the United States, a small percentage of drawings were intentionally drawn ambiguously in order to show, in the words of the pupils, that *a mathematician can be anyone.*

Country	Males drawing males/%	Males drawing females/%	Females drawing males/%	Females drawing females/%
USA	93.8	3.1	61	30.5
UK	93.8	6.3	41.2	56.9
Finland	86.4	0	52.0	20.0
Sweden	100	0	79.1	20.1
Romania	100	0	75.0	16.7

Table 5.3 By country: Gender in pupils' drawings, by percent

5.2.5 INTIMATIONS OF EINSTEIN

There were a number of drawings which contained a reference to Albert Einstein, either by name or by inclusion of the equation $E = mc^2$ someplace in the drawing or its explanation. Below are the frequencies and percentages of these references:

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	***(a)***	435	91.4	91.4	91.4
	(b)	41	8.6	8.6	100.0
	Total	476	100.0	100.0	

Table 5.4 Frequency and percentage of drawings which contained (***a***) no reference to Einstein; (***b***) reference to Einstein

In the table below, are the frequencies and percentages of the Einstein references, by country:

COUNTRY	EINSTEIN REFERENCE frequency/percentage
USA	33/16.4%
UK	2/2.0%
Finland	3/3.2%
Sweden	2/4.0%
Romania	1/3.0%

Table 5.5 By country: Frequency and percentage of references to Einstein

5.2.6 THE SURVEY TOOL

The survey tool had been tested for reliability, with the result that alpha (Cronbach) was .6144, indicating internal consistency. The survey tool data was examined and separated into two groups: those with *neutral drawings,* and those that contained *stereotypical drawings,* in order to analyse the similarities and differences of these two groups of pupils. The survey response data for each group for each of the ten statements of the Likert-type survey tool appears in the tables that follow.

For the group with *neutral drawings, n* = 325; for the group with *stereotypical drawings, n* = 137. 14 surveys were suppressed just for this set of data because they contained no drawings. These were from Finland and many contained the written comment, *I won't* (or *can't) draw!*

Those tables designated with an ***a*** represent the group with neutral drawings, those with a ***b*** represent the surveys containing stereotypical drawings:

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly disagree	11	3.4	3.4	3.4
	-1 disagree	10	3.1	3.1	6.5
	0 not sure	54	16.6	16.6	23.1
	1 agree	168	51.7	51.7	74.8
	2 strongly agree	82	25.2	25.2	100.0
	Total	325	100.0	100.0	

Table 5.6a Statement 1: *I enjoy the school I attend.*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly disagree	5	3.6	3.6	3.6
	-1 disagree	9	6.6	6.6	10.2
	0 not sure	38	27.7	27.7	38.0
	1 agree	61	44.5	44.5	82.5
	2 strongly agree	24	17.5	17.5	100.0
	Total	137	100.0	100.0	

Table 5.6b Statement 1: *I enjoy the school I attend.*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly disagree	37	11.4	11.4	11.4
	-1 disagree	79	24.3	24.3	35.7
	0 not sure	106	32.6	32.6	68.3
	1 agree	77	23.7	23.7	92.0
	2 strongly agree	26	8.0	8.0	100.0
	Total	325	100.0	100.0	

Table 5.7a Survey statement 2: *A mathematician's work looks like fun to me.*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly disagree	38	27.7	27.7	27.7
	-1 disagree	42	30.7	30.7	58.4
	0 not sure	30	21.9	21.9	80.3
	1 agree	22	16.1	16.1	96.4
	2 strongly agree	5	3.6	3.6	100.0
	Total	137	100.0	100.0	

Table 5.7b Survey statement 2: *A mathematician's work looks like fun to me.*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly agree	74	22.8	22.8	22.8
	-1 agree	64	19.7	19.7	42.5
	0 not sure	121	37.2	37.2	79.7
	1 disagree	47	14.5	14.5	94.2
	2 strongly disagree	19	5.8	5.8	100.0
	Total	325	100.0	100.0	

Table 5.8a Survey statement 3: *I would never think of becoming a mathematician*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly agree	54	39.4	39.4	39.4
	-1 agree	15	10.9	10.9	50.4
	0 not sure	30	21.9	21.9	72.3
	1 disagree	21	15.3	15.3	87.6
	2 strongly disagree	17	12.4	12.4	100.0
	Total	137	100.0	100.0	

Table 5.8b Survey statement 3: *I would never think of becoming a mathematician*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly disagree	20	6.2	6.2	6.2
	-1 disagree	35	10.8	10.8	16.9
	0 not sure	120	36.9	36.9	53.8
	1 agree	97	29.8	29.8	83.7
	2 strongly agree	53	16.3	16.3	100.0
	Total	325	100.0	100.0	

Table 5.9a Survey statement 4: *Mathematicians seem like very patient people*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly disagree	18	13.1	13.1	13.1
	-1 disagree	17	12.4	12.4	25.5
	0 not sure	53	38.7	38.7	64.2
	1 agree	31	22.6	22.6	86.9
	2 strongly agree	18	13.1	13.1	100.0
	Total	137	100.0	100.0	

Table 5.9b Survey statement 4: *Mathematicians seem like very patient people*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly agree	60	18.5	18.5	18.5
	-1 agree	37	11.4	11.4	29.8
	0 not sure	161	49.5	49.5	79.4
	1 disagree	43	13.2	13.2	92.6
	2 strongly disagree	24	7.4	7.4	100.0
	Total	325	100.0	100.0	

Table 5.10a Survey statement 5: *I would not want to marry a mathematician*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly agree	33	24.1	24.1	24.1
	-1 agree	18	13.1	13.1	37.2
	0 not sure	54	39.4	39.4	76.6
	1 disagree	25	18.2	18.2	94.9
	2 strongly disagree	7	5.1	5.1	100.0
	Total	137	100.0	100.0	

Table 5.10b Survey statement 5: *I would not want to marry a mathematician*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly disagree	38	11.7	11.7	11.7
	-1 disagree	24	7.4	7.4	19.1
	0 not sure	71	21.8	21.8	40.9
	1 agree	71	21.8	21.8	62.8
	2 strongly agree	121	37.2	37.2	100.0
	Total	325	100.0	100.0	

Table 5.11a Survey statement 6: *I have met a mathematician*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly disagree	21	15.3	15.3	15.3
	-1 disagree	9	6.6	6.6	21.9
	0 not sure	43	31.4	31.4	53.3
	1 agree	28	20.4	20.4	73.7
	2 strongly agree	36	26.3	26.3	100.0
	Total	137	100.0	100.0	

Table 5.11b Survey statement 6: *I have met a mathematician*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly agree	16	4.9	4.9	4.9
	-1 agree	44	13.5	13.5	18.5
	0 not sure	67	20.6	20.6	39.1
	1 disagree	134	41.2	41.2	80.3
	2 strongly disagree	64	19.7	19.7	100.0
	Total	325	100.0	100.0	

Table 5.12a Survey statement 7: *I don't enjoy my mathematics class*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly agree	16	11.7	11.7	11.7
	-1 agree	22	16.1	16.1	27.7
	0 not sure	30	21.9	21.9	49.6
	1 disagree	49	35.8	35.8	85.4
	2 strongly disagree	20	14.6	14.6	100.0
	Total	137	100.0	100.0	

Table 5.12b Survey statement 7: *I don't enjoy my mathematics class*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly disagree	66	20.3	20.3	20.3
	-1 disagree	64	19.7	19.7	40.0
	0 not sure	51	15.7	15.7	55.7
	1 agree	99	30.5	30.5	86.2
	2 strongly agree	45	13.8	13.8	100.0
	Total	325	100.0	100.0	

Table 5.13a Survey statement 8: *I discuss my mathematics class with friends*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
	-2 strongly disagree	43	31.4	31.4	31.4
	-1 disagree	41	29.9	29.9	61.3
	0 not sure	13	9.5	9.5	70.8
	1 agree	29	21.2	21.2	92.0
	2 strongly agree	11	8.0	8.0	100.0
	Total	137	100.0	100.0	

Table 5.13b Survey statement 8: *I discuss my mathematics class with friends*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly agree	25	7.7	7.7	7.7
	-1 agree	52	16.0	16.0	23.7
	0 not sure	142	43.7	43.7	67.4
	1 disagree	68	20.9	20.9	88.3
	2 strongly disagree	38	11.7	11.7	100.0
	Total	325	100.0	100.0	

Table 5.14a Survey statement 9: *Mathematicians work alone*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly agree	20	14.6	14.6	14.6
	-1 agree	22	16.1	16.1	30.7
	0 not sure	70	51.1	51.1	81.8
	1 disagree	18	13.1	13.1	94.9
	2 strongly disagree	7	5.1	5.1	100.0
	Total	137	100.0	100.0	

Table 5.14b Survey statement 9: *Mathematicians work alone*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly disagree	127	39.1	39.1	39.1
	-1 disagree	53	16.3	16.3	55.4
	0 not sure	104	32.0	32.0	87.4
	1 agree	30	9.2	9.2	96.6
	2 strongly agree	11	3.4	3.4	100.0
	Total	325	100.0	100.0	

Table 5.15a Survey statement 10: *I see myself as a mathematician*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly disagree	70	51.1	51.1	51.1
	-1 disagree	22	16.1	16.1	67.2
	0 not sure	35	25.5	25.5	92.7
	1 agree	7	5.1	5.1	97.8
	2 strongly agree	3	2.2	2.2	100.0
	Total	137	100.0	100.0	

Table 5.15b Survey statement 10: *I see myself as a mathematician*

The data was cross tabulated for the stereotypical and non-stereotypical drawings groups to see if survey responses were independent of or dependent on country of origin using Pearson's Chi-Square (2-tailed). Results appear in the tables below:

STATEMENT	PEARSON CHI-SQUARE VALUE	DF	P-VALUE (2-TAILED)
Enjoy school I attend	25.373	16	.064
Mathematician's work looks like fun	47.747	16	.000
Never think of becoming a mathematician	18.883	16	.275
Mathematicians are patient	34.019	16	.005
Would not want to marry a mathematician	67.536	16	.000
Have met a mathematician	64.248	16	.000
Don't enjoy maths class	13.833	16	.611
Discuss maths class with friends	46.623	16	.000
Mathematicians work alone	63.802	16	.000
See myself as a mathematician	19.492	16	.244

Table 5.16 *Neutral* or Non-Stereotypical drawings survey responses correlated to country

STATEMENT	PEARSON CHI-SQUARE VALUE	DF	P-VALUE (2-TAILED)
Enjoy school I attend	19.605	16	.239
Mathematician's work looks like fun	63.674	16	.000
Never think of becoming a mathematician	23.355	16	.105
Mathematicians are patient	25.050	16	.069
Would not want to marry a mathematician	32.144	16	.010
Have met a mathematician	29.199	16	.023
Don't enjoy maths class	22.849	16	.118
Discuss maths class with friends	35.140	16	.004
Mathematicians work alone	33.615	16	.006
See myself as a mathematician	30.489	16	.016

Table 5.17 *Stereotypical* drawings survey responses correlated to country

With the surveys separated into two groupings, the data for drawing gender and Einstein references was re-examined. The tables below present the data in the *neutral drawings* and *stereotypical drawings* categories, by frequency and percentage. For the *neutral drawings* group, $n = 304$ due to ambiguous drawings.

	Males drawing males	Males drawing females	Females drawing males	Females drawing females
Neutral $n = 304$ males $n = 132$ females $n = 172$	126/95.5%	6/4.5%	94/54.7%	78/45.3%
Stereotypical $n = 137$ males $n = 81$ females $n = 54$	81/100%	0	54/96.42%	2/3.7%

Table 5.18 Gender of mathematicians in pupils' drawings

	EINSTEIN REFERENCES
Neutral $n = 325$	19/5.9%
Stereotypical $n = 137$	22/ 16.1%

Table 5.19 Number of references to Einstein in pupils' drawings

Presented in the tables below are frequencies and percentages for each of the survey tool statements.

								TOTAL
			-2 strongly disagree	-1 disagree	0 not sure	1 agree	2 strongly agree	
	USA	Count	8	12	49	98	34	201
		% within Country	4.0%	6.0%	24.4%	48.8%	16.9%	100.0%
	UK	Count	4	2	9	52	32	99
		% within Country	4.0%	2.0%	9.1%	52.5%	32.3%	100.0%
	Finland	Count	5	3	24	43	19	94
		% within Country	5.3%	3.2%	25.5%	45.7%	20.2%	100.0%
	Sweden	Count		3	12	25	9	49
		% within Country		6.1%	24.5%	51.0%	18.4%	100.0%
	Romania	Count	1		1	18	13	33
		% within Country	3.0%		3.0%	54.5%	39.4%	100.0%
Total		Count	18	20	95	236	107	476
		% within Country	3.8%	4.2%	20.0%	49.6%	22.5%	100.0%

Table 5.20 By country, frequency and percentage of responses for survey tool statement 1, *I enjoy the school I attend*

								TOTAL
			-2 strongly disagree	-1 disagree	0 not sure	1 agree	2 strongly agree	
	USA	Count	39	66	51	33	12	201
		% within Country	19.4%	32.8%	25.4%	16.4%	6.0%	100.0%
	UK	Count	13	20	39	23	4	99
		% within Country	13.1%	20.2%	39.4%	23.2%	4.0%	100.0%
	Finland	Count	10	24	35	20	5	94
		% within Country	10.6%	25.5%	37.2%	21.3%	5.3%	100.0%
	Sweden	Count	15	12	14	7	1	49
		% within Country	30.6%	24.5%	28.6%	14.3%	2.0%	100.0%
	Romania	Count		1	4	18	10	33
		% within Country		3.0%	12.1%	54.5%	30.3%	100.0%
Total		Count	77	123	143	101	32	476
		% within Country	16.2%	25.8%	30.0%	21.2%	6.7%	100.0%

Table 5.21 By country, frequency and percentage of responses for survey tool statement 2, *A mathematician's work looks like fun to me*

								TOTAL
			-2 strongly agree	-1 agree	0 not sure	1 disagree	2 strongly disagree	
	USA	Count	57	35	69	24	16	201
		% within Country	28.4%	17.4%	34.3%	11.9%	8.0%	100.0%
	UK	Count	22	20	31	19	7	99
		% within Country	22.2%	20.2%	31.3%	19.2%	7.1%	100.0%
	Finland	Count	38	16	27	11	2	94
		% within Country	40.4%	17.0%	28.7%	11.7%	2.1%	100.0%
	Sweden	Count	15	4	11	13	6	49
		% within Country	30.6%	8.2%	22.4%	26.5%	12.2%	100.0%
	Romania	Count	3	5	16	4	5	33
		% within Country	9.1%	15.2%	48.5%	12.1%	15.2%	100.0%
Total		Count	135	80	154	71	36	476
		% within Country	28.4%	16.8%	32.4%	14.9%	7.6%	100.0%

Table 5.22 By country, frequency and percentage of responses for survey tool statement 3, *I would never think of becoming a mathematician*

								TOTAL
			-2 strongly disagree	-1 disagree	0 not sure	1 agree	2 strongly agree	
	USA	Count	23	21	81	43	33	201
		% within Country	11.4%	10.4%	40.3%	21.4%	16.4%	100.0%
	UK	Count	9	11	26	40	13	99
		% within Country	9.1%	11.1%	26.3%	40.4%	13.1%	100.0%
	Finland	Count	2	8	44	31	9	94
		% within Country	2.1%	8.5%	46.8%	33.0%	9.6%	100.0%
	Sweden	Count	3	6	17	12	11	49
		% within Country	6.1%	12.2%	34.7%	24.5%	22.4%	100.0%
	Romania	Count	2	7	13	4	7	33
		% within Country	6.1%	21.2%	39.4%	12.1%	21.2%	100.0%
Total		Count	39	53	181	130	73	476
		% within Country	8.2%	11.1%	38.0%	27.3%	15.3%	100.0%

Table 5.23 By country, frequency and percentage of responses for survey tool statement 4, *Mathematicians seem like very patient people.*

								TOTAL
			-2 strongly agree	-1 agree	0 not sure	1 disagree	2 strongly disagree	
	USA	Count	25	20	115	30	11	201
		% within Country	12.4%	10.0%	57.2%	14.9%	5.5%	100.0%
	UK	Count	21	12	44	13	9	99
		% within Country	21.2%	12.1%	44.4%	13.1%	9.1%	100.0%
	Finland	Count	20	9	40	16	9	94
		% within Country	21.3%	9.6%	42.6%	17.0%	9.6%	100.0%
	Sweden	Count	5	9	20	10	5	49
		% within Country	10.2%	18.4%	40.8%	20.4%	10.2%	100.0%
	Romania	Count	25	6	2			33
		% within Country	75.8%	18.2%	6.1%			100.0%
Total		Count	96	56	221	69	34	476
		% within Country	20.2%	11.8%	46.4%	14.5%	7.1%	100.0%

Table 5.24 By country, frequency and percentage of responses for survey tool statement 5, *I would not want to marry a mathematician*

								TOTAL
			-2 strongly disagree	-1 disagree	0 not sure	1 agree	2 strongly agree	
	USA	Count	21	17	53	49	61	201
		% within Country	10.4%	8.5%	26.4%	24.4%	30.3%	100.0%
	UK	Count	6	2	12	20	59	99
		% within Country	6.1%	2.0%	12.1%	20.2%	59.6%	100.0%
	Finland	Count	20	13	27	13	21	94
		% within Country	21.3%	13.8%	28.7%	13.8%	22.3%	100.0%
	Sweden	Count	11	3	19	11	5	49
		% within Country	22.4%	6.1%	38.8%	22.4%	10.2%	100.0%
	Romania	Count	2	1	6	7	17	33
		% within Country	6.1%	3.0%	18.2%	21.2%	51.5%	100.0%
Total		Count	60	36	117	100	163	476
		% within Country	12.6%	7.6%	24.6%	21.0%	34.2%	100.0%

Table 5.25 By country, frequency and percentage of responses for survey tool statement 6, *I have met a mathematician*

								TOTAL
			-2 strongly agree	-1 agree	0 not sure	1 disagree	2 strongly disagree	
	USA	Count	13	28	48	80	32	201
		% within Country	6.5%	13.9%	23.9%	39.8%	15.9%	100.0%
	UK	Count	9	12	16	42	20	99
		% within Country	9.1%	12.1%	16.2%	42.4%	20.2%	100.0%
	Finland	Count	9	18	26	26	15	94
		% within Country	9.6%	19.1%	27.7%	27.7%	16.0%	100.0%
	Sweden	Count	3	8	7	19	12	49
		% within Country	6.1%	16.3%	14.3%	38.8%	24.5%	100.0%
	Romania	Count	1	4	3	17	8	33
		% within Country	3.0%	12.1%	9.1%	51.5%	24.2%	100.0%
Total		Count	35	70	100	184	87	476
		% within Country	7.4%	14.7%	21.0%	38.7%	18.3%	100.0%

Table 5.26 By country, frequency and percentage of responses for survey tool statement 7, *I don't enjoy my mathematics class*

								TOTAL
			-2 strongly disagree	-1 disagree	0 not sure	1 agree	2 strongly agree	
	USA	Count	54	44	30	55	18	201
		% within Country	26.9%	21.9%	14.9%	27.4%	9.0%	100.0%
	UK	Count	10	16	15	31	27	99
		% within Country	10.1%	16.2%	15.2%	31.3%	27.3%	100.0%
	Finland	Count	21	27	18	23	5	94
		% within Country	22.3%	28.7%	19.1%	24.5%	5.3%	100.0%
	Sweden	Count	19	21	1	7	1	49
		% within Country	38.8%	42.9%	2.0%	14.3%	2.0%	100.0%
	Romania	Count	8	2	2	14	7	33
		% within Country	24.2%	6.1%	6.1%	42.4%	21.2%	100.0%
Total		Count	112	110	66	130	58	476
		% within Country	23.5%	23.1%	13.9%	27.3%	12.2%	100.0%

Table 5.27 By country, frequency and percentage of responses for survey tool statement 8, *I discuss my mathematics class with friends*

								TOTAL
			-2 strongly agree	-1 agree	0 not sure	1 disagree	2 strongly disagree	
	USA	Count	15	28	86	42	30	201
		% within Country	7.5%	13.9%	42.8%	20.9%	14.9%	100.0%
	UK	Count	4	7	40	34	14	99
		% within Country	4.0%	7.1%	40.4%	34.3%	14.1%	100.0%
	Finland	Count	14	20	53	5	2	94
		% within Country	14.9%	21.3%	56.4%	5.3%	2.1%	100.0%
	Sweden	Count	7	12	28	2		49
		% within Country	14.3%	24.5%	57.1%	4.1%		100.0%
	Romania	Count	6	10	13	4		33
		% within Country	18.2%	30.3%	39.4%	12.1%		100.0%
Total		Count	46	77	220	87	46	476
		% within Country	9.7%	16.2%	46.2%	18.3%	9.7%	100.0%

5.28 By country, frequency and percentage of responses for survey tool statement 9, *Mathematicians usually work alone*

								TOTAL
			-2 strongly disagree	-1 disagree	0 not sure	1 agree	2 strongly agree	
	USA	Count	88	32	57	16	8	201
		% within Country	43.8%	15.9%	28.4%	8.0%	4.0%	100.0%
	UK	Count	43	17	31	6	2	99
		% within Country	43.4%	17.2%	31.3%	6.1%	2.0%	100.0%
	Finland	Count	44	15	27	8		94
		% within Country	46.8%	16.0%	28.7%	8.5%		100.0%
	Sweden	Count	19	13	14	1	2	49
		% within Country	38.8%	26.5%	28.6%	2.0%	4.1%	100.0%
	Romania	Count	9	1	15	6	2	33
		% within Country	27.3%	3.0%	45.5%	18.2%	6.1%	100.0%
Total		Count	203	78	144	37	14	476
		% within Country	42.6%	16.4%	30.3%	7.8%	2.9%	100.0%

5.29 By country, frequency and percentage of responses to survey tool statement 10, *I see myself as a mathematician*

Each statement in the survey tool was correlated to see if the responses were independent of or dependent on country of origin. **Table 5.30** contains data for the Pearson Chi-Square tests.

STATEMENT	PEARSON CHI-SQUARE VALUE	DF	P-VALUE (2-TAILED)
Enjoy school I attend	34.206	16	.005
Mathematician's work looks like fun	87.676	16	.000
Never think of becoming a mathematician	33.644	16	.006
Mathematicians are patient	34.710	16	.004
Would not want to marry a mathematician	91.929	16	.000
Have met a mathematician	74.463	16	.000
Don't enjoy maths class	18.933	16	.272
Discuss maths class with friends	74.766	16	.000
Mathematicians work alone	79.499	16	.000
See myself as a mathematician	24.332	16	.082

Table 5.30 Chi-Square tests on each survey statement correlated with country

The survey tool was also correlated to see if the drawings created were independent of or dependent on country of origin. The Pearson chi-square test was utilized and the Chi-Square and P-values appear in **Table 5.31** below:

	VALUE	DF	ASYMP. SIG. (2-SIDED)
Pearson chi-square	52.361	4	.000
Likelihood Ratio	47.643	4	.000
Linear-by-Linear Association	17.955	1	.000
N of Valid Cases	462		

Table 5.31 Pearson chi-square and P-value for drawings correlated to country

5.2.7 THE WRITING PROMPT

The first writing prompt asked pupils to enumerate reasons for which a mathematician would be hired. In **Table 5.32**, below, are the top four reasons given in each country, with the frequency of that response.

USA	UK	Finland	Sweden	Romania
Teaching (57)	Accounting (31)	Teaching (20)	Teaching (8)	Teaching (16)
Accounting (26)	Teaching (26)	Building (8)	Building (4)	Problem Solving (4)
Architecture (24)	Banking (22)	Banking (4)	Navigation (4)	
Area/Perimeter (17)	Programming (10)	Programming (2)	To Do Bills (3)	

Table 5.32 By country: top four reasons given by pupils for why a mathematician would be hired

As was noted previously, the second writing prompt, which asked pupils to look back at and comment on their drawings, was used to analyse the drawings, including to more surely characterise the figure and its gender.

5.3 DATA FOR THE SURVEY OF MATHEMATICS PROFESSIONALS

With this section of the study, the majority of respondents were anonymous and many submitted their surveys by post. The tables which follow contain data for these surveys of professionals in the mathematics field:

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	**US**	68	64.2	64.2	64.2
	UK	28	26.4	26.4	90.6
	Norway	1	.9	.9	91.5
	Sweden	2	1.9	1.9	93.4
	Romania	2	1.9	1.9	95.3
	Netherlands	1	.9	.9	96.2
	Lebanon	1	.9	.9	97.2
	France	1	.9	.9	98.1
	Belgium	1	.9	.9	99.1

	Israel	1	.9	.9	100.0
	Total	106	100.0	100.0	

Table 5.33 Countries represented in the survey of mathematics professionals

Table 5.34 below, contains data of the working level of the respondents. It should be noted that some of the university professionals also do some research with industry.

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	**University**	50	47.2	47.2	47.2
	High School	37	34.9	34.9	82.1
	Middle School	10	9.4	9.4	91.5
	Elementary School	8	7.5	7.5	99.1
	Industry	1	.9	.9	100.0
	Total	106	100.0	100.0	

Table 5.34 Working level of the mathematics professionals

Table 5.35 below, displays the gender of the respondents in this survey.

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	**Female**	46	43.4	43.4	43.4
	Male	60	56.6	56.6	100.0
	Total	106	100.0	100.0	

Table 5.35 Gender of respondents

In **Table 5.36** and **Figure 5.1** below are the frequencies and percentages for the survey question, *Do you consider yourself a mathematician?* This is followed by **Table 5.37** and **Figure 5.2** which show the professional level of the respondents on this question.

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	**Yes**	69	65.1	65.1	65.1
	No	24	22.6	22.6	87.7
	Unsure	13	12.3	12.3	100.0
	Total	106	100.0	100.0	

Table 5.36 Frequencies and percentages for the question, *Do you consider yourself a mathematician?*

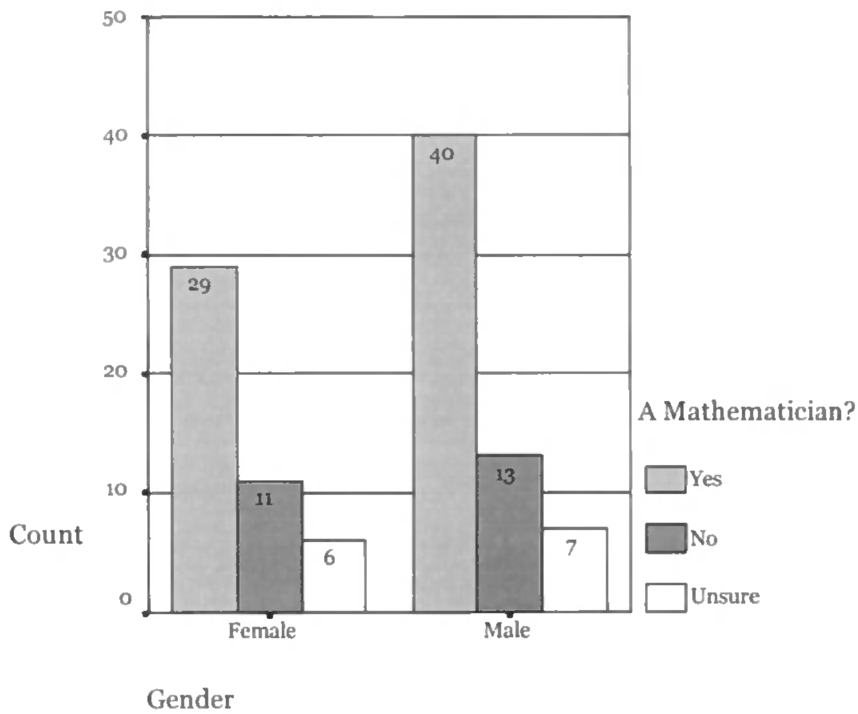

Figure 5.1 By gender, frequencies of responses to: *Do you consider yourself a mathematician?*

						TOTAL
			Yes	No	Unsure	
	University	Count	38	8	4	50
		% within Working Level	76.0%	16.0%	8.0%	100.0%
	High School	Count	23	9	5	37
		% within Working Level	62.2%	24.3%	13.5%	100.0%
	Middle School	Count	6	2	2	10
		% within Working Level	60.0%	20.0%	20.0%	100.0%
	Elementary School	Count	1	5	2	8
		% within Working Level	12.5%	62.5%	25.0%	100.0%
	Industry	Count	1			1
		% within Working Level	100.0%			100.0%
Total		Count	69	24	13	106
		% within Working Level	65.1%	22.6%	12.3%	100.0%

Table 5.37 Frequency and percentage by professional level: *Do you consider yourself a mathematician?*

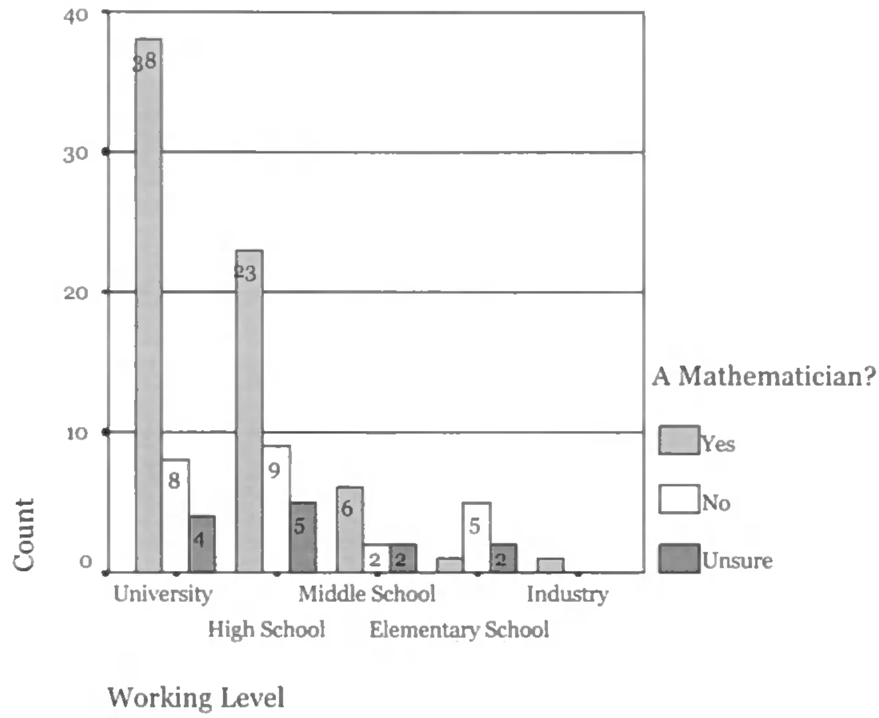

Figure 5. 2 Frequency by professional level

5.3.1 THE CODING SYSTEM FOR ANALYSING WRITTEN SECTIONS OF THE SURVEY

The written sections of the survey asked the respondents (1) to explain why they answered as they did the question of whether or not they considered themselves a mathematician; (2) under what circumstance(s) they believed one could consider oneself a mathematician.

After examining the responses, they were categorised into three sections: *Why I consider myself to be a mathematician*; *Why I do not consider myself to be a mathematician*; *Who may call oneself a mathematician*.

The coding system for *Why I consider myself to be a mathematician* was based on six themes which appeared in the written responses. The listed theme is followed by actual written comment (s):

1 = **Thinking:** *I think mathematically; I bring mathematical logic to solving*

problems.

2 = **Use:** *I use mathematics in my job; I am a professional mathematician.*

3 = **Study:** *I have a mathematics degree; I still learn new mathematics; I studied mathematics.*

4 = **Teaching:** *I teach mathematics.*

5 = **Enjoyment:** *I enjoy doing mathematics; I love mathematics.*

6 = **Research:** *I am a researcher; I do theoretical research.*

In **Table 5.38,** below, are the frequencies and percentages for each written comment. One respondent did not furnish an explanation.

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	1 Thinking	31	44.9	45.6	45.6
	2 Use	10	14.5	14.7	60.3
	3 Study	13	18.8	19.1	79.4
	4 Teaching	5	7.2	7.4	86.8
	5 Enjoyment	7	10.1	10.3	97.1
	6 Research	2	2.9	2.9	100.0
	Total	68	98.6	100.0	
Missing	System	1	1.4		
Total		69	100.0		

Table 5.38 *Why I consider myself to be a mathematician*

The coding system for the written comment *Why I do not consider myself to be a mathematician/Unsure* appears below and is based on five themes which appeared in the written responses. The listed theme is followed by actual written comment (s):

1 = **No research:** *I don't create new mathematics; I don't do research.*

2 = **Only a teacher:** *I see myself as only a teacher.*

3 = **Other title:** *I don't consider myself a mathematician as I'm a statistician/engineer.*

4 = **Unsureness:** *I'm unsure of my mathematical background/I know less mathematics than others.*

5 = **Don't use:** *I don't "do" mathematics.*

In **Table 5.39**, below, are the frequencies and percentages for each coded response. The missing notation indicates respondents who don't consider themselves mathematicians but gave no reason:

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	1 No research	6	16.2	17.6	17.6
	2 Only a teacher	8	21.6	23.5	41.2
	3 Other title	2	5.4	5.9	47.1
	4 Unsureness	15	40.5	44.1	91.2
	5 Don't use	3	8.1	8.8	100.0
	Total	34	91.9	100.0	
Missing	System	3	8.1		
Total		37	100.0		

Table 5.39 *Why I don't consider myself a mathematician*

The coding system for the written comment, *Who may call oneself a mathematician* is based on ten themes which were observed in the written responses. The listed theme is followed by actual written comment (s):

1 = **Thinking:** *One who uses mathematical thinking and brings mathematical logic to solving problems.*

2 = **Personal choice:** *Anyone who chooses to call oneself one.*

3 = **Teaching:** *A teacher.*

4 = **Research:** *One who creates new mathematics; one who does research.*

5 = **Study:** *One still learning new mathematics; one with a degree in mathematics.*

6 = **Use:** *Anyone who "does" mathematics.*

7 = **Higher maths:** *One doing or learning "upper level" mathematics.*

8 = **Not a personal choice:** *No one may call themselves one—it's for others to say one is a mathematician.*

9 = **Not sure:** *I'm unsure.*

10 = **Enjoyment:** *When mathematics thrills their soul.*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	1 Thinking	32	30.2	30.2	30.2
	2 Personal choice	14	13.2	13.2	43.4
	3 Teaching	3	2.8	2.8	46.2
	4 Research	20	18.9	18.9	65.1
	5 Study	11	10.4	10.4	75.5
	6 Use	9	8.5	8.5	84.0
	7 Higher maths	12	11.3	11.3	95.3
	8 Not a personal choice	2	1.9	1.9	97.2
	9 Not sure	2	1.9	1.9	99.1
	10 Enjoyment	1	.9	.9	100.0
	Total	106	100.0	100.0	

Table 5.40 *Who may call oneself a mathematician?*

		WORKING LEVEL					TOTAL
		University	High School	Middle School	Elementary School	Industry	
	1 Thinking	11	15	5	1		32
	2 Personal choice	11	3				14
	3 Teaching	3					3
	4 Research	9	8	1	2		20
	5 Study	5	1	2	2	1	11
	6 Use	2	4	1	2		9
	7 Higher maths	7	3	1	1		12
	8 Not a personal choice	1	1				2
	9 Not sure	1	1				2
	10 Enjoyment		1				1
Total		50	37	10	8	1	106

Table 5.41 *Who may call oneself a mathematician?*—By working level

	VALUE	DF	P-VALUE (2-SIDED)
Pearson Chi-Square	35.446	36	.495
Likelihood Ratio	35.038	36	.514
Linear-by-Linear Association	.610	1	.435
N of Valid Cases	106		

Table 5.42 Pearson Chi-Square for correlation of working level with *Who may call oneself a mathematician?*

Table 5.43 and **Table 5.44** below, contain data of the chi-square test correlating professional working level with respondents' answers to why they don't consider themselves a mathematician.

								TOTAL
			1 don't "create maths" /do research	2 I'm only a teacher	3 I'm a statistician/ engineer	4 unsure of my mathematics/ know less than others	5 not "doing" maths	
Working Level	University	Count	2		2	4	2	10
		% within Working Level	20.0%		20.0%	40.0%	20.0%	100.0%
	High School	Count	3	6		5		14
		% within Working Level	21.4%	42.9%		35.7%		100.0%
	Middle School	Count		2		2		4
		% within Working Level		50.0%		50.0%		100.0%
	Elementary School	Count	1			5	1	7
		% within Working Level	14.3%			71.4%	14.3%	100.0%
Total		Count	6	8	2	16	3	35
		% within Working Level	17.1%	22.9%	5.7%	45.7%	8.6%	100.0%

Table 5.43 Cross tabulation of working level and *Why I don't consider myself a mathematician*

	VALUE	DF	P-VALUE (2-SIDED)
Pearson Chi-Square	18.281	12	.107
Likelihood Ratio	22.972	12	.028
Linear-by-Linear Association	.434	1	.510
N of Valid Cases	35		

Table 5.44 Chi-square p-value of working level and *Why I don't consider myself a mathematician*

Chapter 6: ANALYSIS OF THE DATA

6.1 INTRODUCTION TO ANALYSIS OF THE DATA

This chapter analyses the data from two of the three studies of this project: The international study and the survey of professionals in the mathematics field. The analysis of the two interventions appears in Chapter 8.

6.2 THE INTERNATIONAL STUDY

This section begins with analysis of the data pertaining to pupils' drawings and analysis of the drawings themselves, using an interpretive design. The analysis of the data from the Likert-type survey tool follows.

6.2.1 THE TWO CATEGORIES OF DRAWINGS

As mentioned previously, pupils' drawings appear to fall into two distinct categories: those in which pupils drew a mathematician as a teacher, and those in which they drew their perception of what a (non-teaching) mathematician at work would look like.

In this study, 21.4% of the drawings depicted a teacher, with 74.6%, nearly three-quarters depicting pupils' ideas of what a mathematician at work would look like. As **Table 5.2** shows, by country, this percentage is fairly close for the United States, the United Kingdom and Sweden, with only Romania having a lower percentage of teachers depicted and with Finland showing a higher percentage of teachers depicted. Finland also had the highest percentage of missing drawings as a number of pupils, nearly 15%, insisted they could or would not draw.

						Total
		no drawing	Teacher	Mathematician	Indeterminate	
	USA		41/20.4%	158/78.6%	2/1%	201
	UK		21/21.2%	78/78.8%		99
	Finland	14/14.9%	27/28.7%	51/54.3%	2/2.1%	94
	Sweden		9/18.4%	40/81.6%		49
	Romania		4/12.1%	28/84.8%	1/3%	33
TOTAL		14	102/21.4%	355/74.6%	5/1%	476

Table 5.2 By country: Frequency and percentage of drawings representing a mathematician drawn at work or a figure teaching

Pupils' drawings depicting mathematicians as teachers appear to be directly related to their written responses to the open-ended prompt on the survey that asked pupils to list reasons why a mathematician might be hired. From these responses, it appears that pupils lack an understanding of what it is that mathematicians do.

In every country except Romania there were pupils who left this section blank or left a question mark in it. In Sweden, five pupils wrote *I don't know* (respondents ***S9, S11, S13, S32, S35***), as another pupil (respondent ***S19***) expressed, *I am not sure of what a mathematician actually do*[es]. In Finland four pupils wrote *I don't know* (respondents ***F63, F71, F90, F93***) as a fifth (respondent ***F62***) wrote, *No one is so stupid as to hire a mathematician.*

In the U.S., one pupil wrote *I don't know* (respondent ***US155***) as another wrote *I have no idea why anyone would need a mathematician* (***US173***). A third pupil expressed, *I don't think you would need one* (***US4***).

In the U.K., as well, there was one pupil (***UK61***) who wrote, *Don't know,* while another wrote (***UK57***), *We do not need a mathematician because our maths teacher teaches us to become a mathematician.*

It is interesting to note that in Romania, pupils' written responses were more self-involved. While many pupils in each country expressed the need for a mathematician as a tutor, the Romanian pupils wrote about this in even more personal terms and with more detail. These pupils' concerns appear to stem from the intensity of their studies as well as their fears for succeeding on the very rigorous examinations they are required to take (Nicolescu, 1999).

A pupil named Lavinia (***R29***) wrote:

> *I can only use a mathematician in the 8th grade just to put him in my bookbag and take him to the exam with me. ATÂT! (Nothing else!)*

Another female Romanian pupil (***R5***) wrote: *He will do my homework and go to school in my place.* A number of the Romanian pupils expressed this idea of a mathematician as a sort of sorcerer's apprentice, or personal wizard, taking exams, doing homework (*I will never have to do my homework again* [***R1***]), even according to one boy (***R32***), to *teach me math tricks.*

In the table of the top four reasons listed by pupils in each country as to why someone would need to hire a mathematician (**Table 5.32**, reproduced below), the reasons expressed by the Romanian pupils other than personal reasons such as the ones mentioned above, fell only in the two categories listed.

It can be seen that with the exception of the United Kingdom, where teaching comes in a close second, in each country where pupils expressed reasons for hiring a mathematician, teaching was mentioned most often. It is interesting to contrast this with the drawings pupils created in which only 21.4% of the drawings were of a teacher. This points to some confusion in pupils' minds: the top reason for hiring a mathematician is to teach, yet less than a quarter of the drawings depicted a teacher. Is a teacher a mathematician? There were indications that pupils are not sure.

Jason, a pupil who was interviewed in New York City said he was unsure if he had met a mathematician:

> Jason: I'm not sure if a math teacher is a mathematician or not, so...If a math teacher is a mathematician I guess I have. If not, I'm unsure.

As I looked at the other reasons listed by pupils, with the exception of computer programming and solving hard mathematical problems (problem solving), the reasons enumerated show a general lack of awareness about mathematicians' work, confusing it with other professions which appear to require computation and/or measurement.

And even as pupils listed *solving hard problems* there was no evidence that they had any concrete notions of what these hard problems might include. In fact, notations on the blackboards in pupils' drawings depicted either trivial arithmetic, or complicated-looking nonsense. Only in Romania was there a higher proportion of non-trivial computations depicted on the chalkboards drawn by pupils.

USA	UK	Finland	Sweden	Romania
Teaching (57)	Accounting (31)	Teaching (20)	Teaching (8)	Teaching (16)
Accounting (26)	Teaching (26)	Building (8)	Building (4)	Problem Solving (4)
Architecture (24)	Banking (22)	Banking (4)	Navigation (4)	
Area/Perimeter (17)	Programming (10)	Programming (2)	To Do Bills (3)	

Table 5.32 By country: top four reasons given by pupils for why a mathematician would be hired

As has been previously discussed, mathematicians are essentially invisible to the public, so it is no surprise that few know what the work of a mathematician entails (Hammond, 1978; Emmer, 1990; Malkevitch, 1989; 1997; and Alexander, 1998). Further, this lack of knowledge about the work of mathematicians includes confusing arithmetic computation with mathematics (Malkevitch, 1989; Eastaway, 1999). I believe that this is the reason why pupils assume that the work a mathematician does is related to such computational jobs as accounting, banking, and, as many pupils put it, *doing hard sums*.

In order to draw *someone*, many pupils appear to rely on the person closest to their experience whom they have seen doing hard sums—a mathematics teacher. At the same time, since I know what most of the teachers of the classes participating in this study look like, when the pictures are drawings of teachers, they are not the pupils' current teachers. And when in the very few cases they are, the drawings are fairly sedate and the teachers identified by name.

In trying to express the tasks for which a mathematician would be hired, pupils also appear to go to the very limited experiences with applications they have had in their own mathematics classes. A similar finding was observed by Rock and Shaw (2000, p. 551) which led them to conclude that "children tend to think that mathematicians do the same kind of mathematics that they themselves do in the classroom."

This of course makes sense, for particularly in the case of Rock and Shaw's study, which involved mostly younger children, the word *mathematics* can only conjure up an image of what pupils know from their classrooms.

Rock and Shaw (p. 553) also concluded that children associate mathematicians with doing the kind of "mathematics that no one else wants to do." They found this to be related in some children's minds to extremely large numbers, which they suggested the children may link to 'hard mathematics'.

I believe that this taking on by mathematicians of the "mathematics that no one else wants to do", is implied in the *hard sums* expressed by pupils in this study. And it may also explain why 46% of pupils in the survey believe that *Mathematicians seem like very patient people.* They would have to be, to do tasks that pupils see as difficult and unattractive.

It appears that one of the few real-world applications lower secondary school pupils have seen thus far in their education, at least in the U.K. and the U.S., involves area and perimeter. Two pupils in the U.K. (***UK76***; ***UK82***) referred to applications of area or perimeter in expressing reasons for hiring a mathematician.

The first wrote: *To work out how many tiles you would need to do your bathroom,* while the second pupil expressed, *To find the volume of a room. The perimeter if you wanted to paint it.* I know that in the United States mathematics teachers complain that pupils constantly mix up area and perimeter, which is the reason they give for teaching these topics over and over. Therefore it is not surprising that among the U.S. pupils, there were 17 references to area and/or perimeter in reasons expressed for hiring a mathematician.

6.2.2 A GENDER GAP

Table 5.3, reproduced below, shows that with the exception of the U. K. and the U. S., the mathematicians depicted by the pupils are primarily male, even amongst those drawn by female pupils. In Finland, Sweden and Romania, not one of the boys drew a female mathematician.

Certainly in Romania many of the mathematics teachers are females who consider themselves to be mathematicians (Nicolescu, 1999), yet this appears not to have affected the boys' drawings.

This data seems to complement the findings of Rock and Shaw (2000), whose recent study of children drawing mathematicians suggests that as pupils grow older, the numbers of male mathematicians drawn increases.

And in a study in London schools in which children drew scientists at work, Matthews and Davies (1999), found a clear trend in which pupil's images of scientists were increasingly male as the children grew older.

Country	Males drawing males/%	Males drawing females/%	Females drawing males/%	Females drawing females/%
USA	93.8	3.1	61	30.5
UK	93.8	6.3	41.2	56.9
Finland	86.4	0	52.0	20.0
Sweden	100	0	79.1	20.1
Romania	100	0	75.0	16.7

Table 5.3 By country: Gender in pupils' drawings, by percent

In the United Kingdom, the television program, *Countdown,* appears to have had a large effect on pupils, since the character they most often drew, particularly pupils in Wales, was Carol Vorderman, a former engineer who has been the show's star. The program's popularity seems to have encouraged many of these pupils to understand that mathematics is not a completely male domain, with the result that boys in the study from the U.K. drew twice as many female mathematicians as their U.S. counterparts.

There is no equivalent program to *Countdown* on television in the United States, but there has been an increasingly well-funded gender equity movement, which initially arose from such reports as *A Nation at Risk* (1983) and the National Council of Teachers of Mathematics (NCTM) *Standards* (1989).

Such projects as Operation SMART™ which endeavoured to interest girls in mathematics and science, have been initiated (Ross, 1999), and there are also an increasing number of publications devoted to the topic of gender equity in the classroom and the encouragement of girls in the sciences, with some effort being made to erase stereotypical views (see, for example, Karp et al, 1998).

Yet for all the programs and money being spent in the United States, it appears that one television program in the United Kingdom, *Countdown*, has been able to accomplish more, showing the effect the media tends to have on children and society.

And while some progress has been made in gender equity in the United States, it appears that girls in the U. S. still lack role models in the field of mathematics. One such girl in New Jersey ***(US38)*** wrote wistfully of her drawing, *I drew a woman mathematician because there seems to be only men mathematicians and I wanted to depict a woman doing the work a man usually does. My drawing is of no particular person.* There is no way of knowing how many other girls may have drawn female mathematicians out of wishful thinking, as did this pupil.

6.2.3 IMAGES IN THE DRAWINGS

The drawings from the 476 surveys in this international study show many similarities from the different cultures. Examination for commonalities among the five countries identified these seven sub themes:

Mathematics as coercion, in which pupils drew mathematicians as teachers who use intimidation, violence, or threats of violence to make their pupils learn material. This was a completely unexpected theme that emerged from the drawings;

The foolish mathematician, in which mathematicians were depicted as lacking common sense, fashion sense, or computational abilities;

The overwrought mathematician, in which mathematicians were depicted as being *overstrained*, to quote a female pupil from Sweden (respondent ***S5***);

The mathematician who can't teach, in which a classroom is drawn which the mathematician cannot control, or in which he doesn't know the material;

Disparagement of mathematicians who are depicted by pupils as being too clever or in some other way contemptible;

The Einstein effect; and

The mathematician with special powers, which may include wizardry and special potions.

While some of the drawings could easily fall under more than one of the sub themes, the hope and intent is to highlight international commonalties amongst them.

6.2.4 MATHEMATICS AS COERCION

The first sub theme can be seen in two drawings from a school in Finland, and in drawings from Sweden, the United Stated and the United Kingdom. In each, the pupil has drawn a situation in which a large authority figure tries to coerce someone smaller, sometimes with violence or threats of it.

In the first drawing, **Figure 6.1** from Finland (***F57***), a Svengali-like figure who is the teacher, prompts a trembling pupil in the first panel, then, in the second panel, a devil's tail peeks out from his coat as laughing maniacally, he beats the pupil for not knowing the answer to a simple arithmetic problem. The difference in their stature is accented by the pupil's having to stand on a stool.

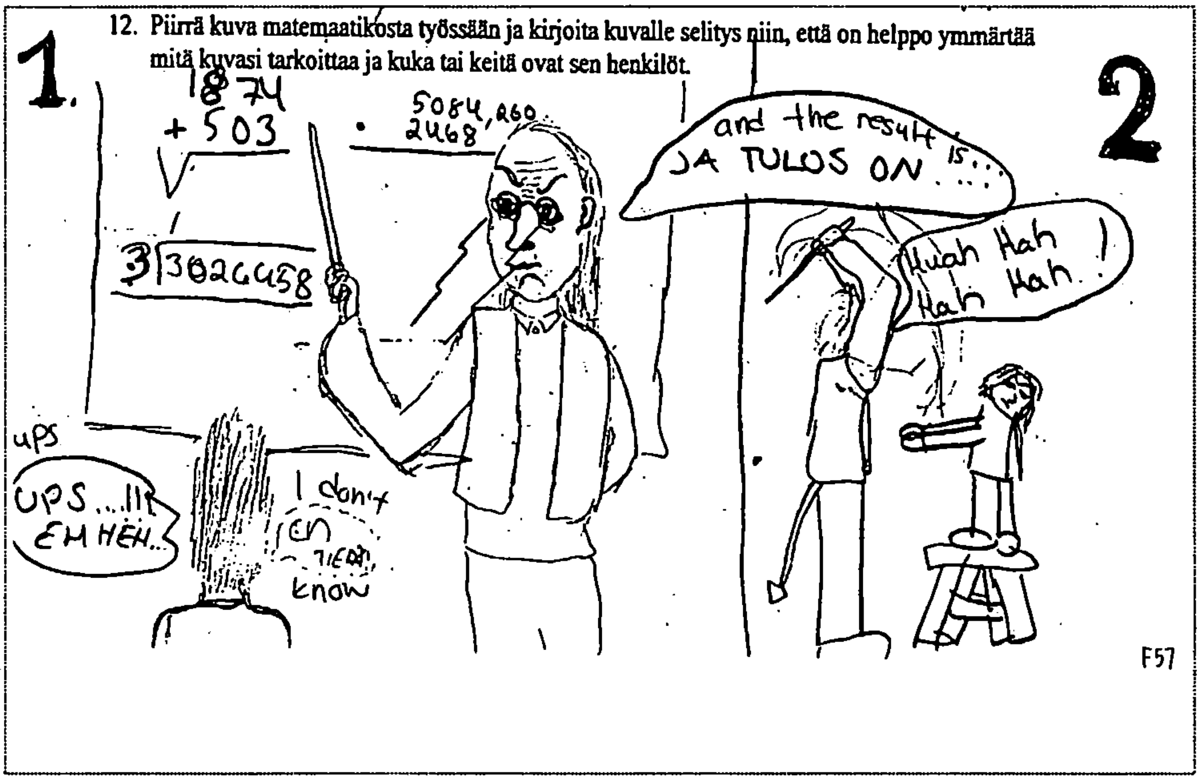

Figure 6.1 Finland—Female pupil

In the next drawing, also from Finland (***F61***), in **Figure 6.2**, a figure sitting in a pupil's place, who is actually a balding and bearded mathematics teacher, is told: *Learn, or...* as a rifle is pointed at him. The pupil wrote of his drawing, *Teachers can't do calculations, problems so a mathematician has been hired to teach teachers.*

Since the figure holding the rifle appears younger, this may be a situation where the pupil has turned the tables and is treating a teacher the way he may have felt himself treated in the past. Here again, the coerced figure doesn't know the simplest arithmetic as he muses: *7 + 7 = ?*

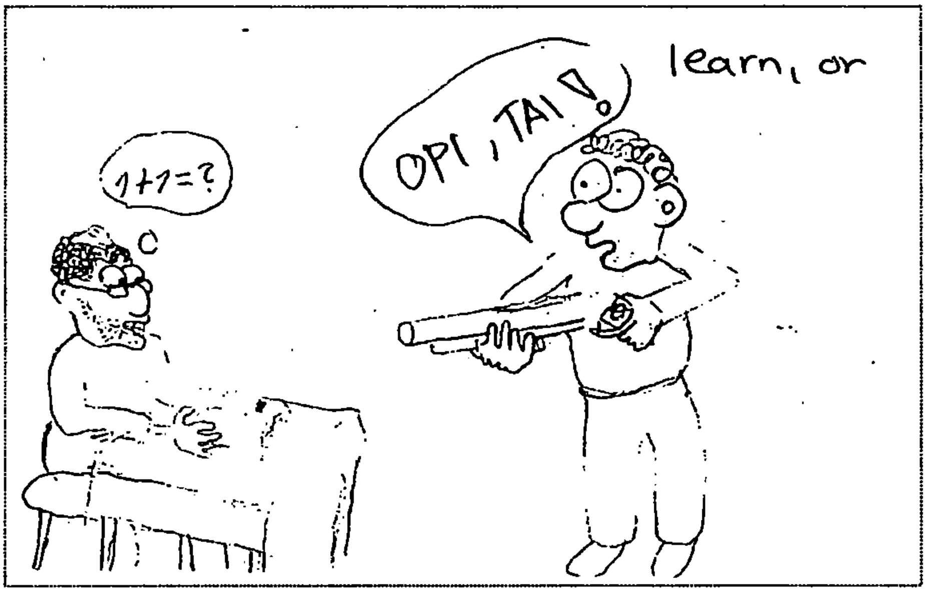

Figure 6.2 Finland—male pupil

A rifle appears in a second drawing, from Sweden, (***S6***) in **Figure 6.3**, in which a pupil is also being asked to do simple arithmetic. *Can you answer this*, an imposing figure with a large rifle asks. *Hope so*, the small figure at the desk replies.

The pupil wrote about his drawing: *He is a strong mathematician. If you answer wrong he* [will] *KILL you.*

It is quite jarring to see these images of guns and violence from the countries of Finland and Sweden, neither of whose societies are known for this type of behaviour, within schools or without. Similar drawings from the

United States, which however has a regrettable history of violence in its schools, nevertheless contained intimidation but in a different form, with no such threats of violence coming from a teacher.

Figure 6.3 Sweden—male pupil

In **Figures 6.4** and **6.5**, which are from the U.S. and U.K. respectively (***US184*** and ***UK6***), there are again large authority-figures.

In the first drawing (**Figure 6.4**), a bearded, bespectacled man is berating a small person, who is asking, *What's an x?* While the teacher yells, *You should all know this??!!*

The girl who drew it wrote:

A white cocaision [sic] *male saying complicated things to a class of small children (only 1 child represented).*

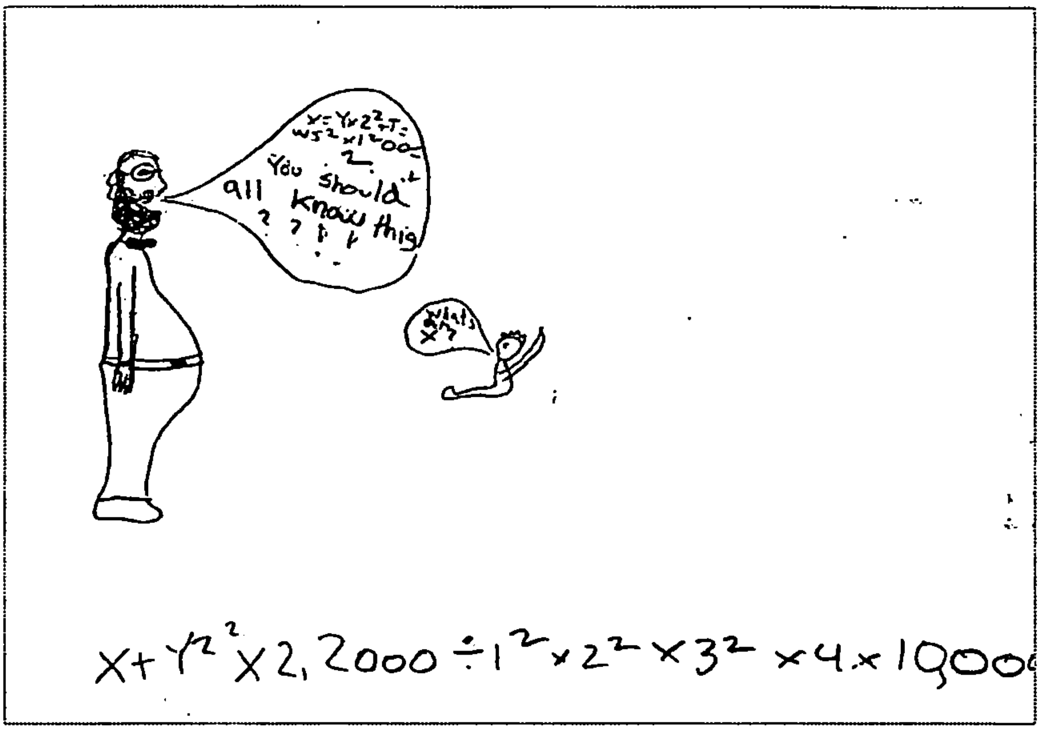

Figure 6.4 U.S.—female pupil

In **Figure 6.5**, the teacher is drawn on a stage ordering punishment—*D7 for the class*, evincing what Nolan and Francis (1992, p.46) call a teacher centred conception of teaching in which the teacher "occupies the centre stage of the educational drama."

In these drawings again, the differences in statures between the authority-figures and the pupils is notable. And it is worth noting, too, that pupils have chosen to draw *small children* although the pupils creating these drawings are no longer small children, but in their early teens.

It is possible that for the pupils creating this type of drawing, the experiences that have produced such images come from a time when they were much younger and felt more keenly their own lack of power. But these images are now carried into the present, with the result that the image of mathematics represented in each of these drawings is that of a bewildering and intimidating subject, placing pupils in a situation over which they have no control; of being excluded from the world the teacher inhabits—the teacher on a *stage* is one example of this remove; of sitting powerlessly in a class while a large adult says *complicated things*.

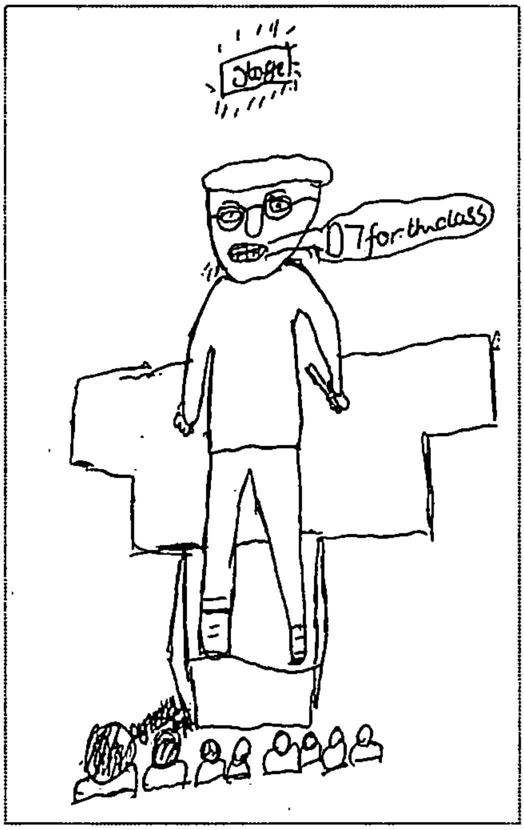

Figure 6.5 U.K.—male pupil

Davis & Hersh (1981, p. 282) have illuminated the origin of this perception of powerlessness in the minds of students:

> Mathematical presentations, whether in books or in the classrooms, are often perceived as authoritarian and this may arouse resentment on the part of the student. Ideally, mathematical instruction says, 'Come, let us reason together.' But what comes from the mouth of the lecturer is often, 'Look, I tell you this is the way it is.' This is proof by coercion. There are several reasons for this to happen. First of all, there is a shortage of time...we cannot afford to linger lovingly over any of the difficulties but must rush breathlessly through our set piece.
>
> Then there is the desire on the part of some teachers to appear brilliant. (What I'm telling you is pretty easy and obvious to me, and if you're not getting it, you must really be pretty stupid.)

The theme of power is a large one in children's literature, often including secret and supernatural powers. It may also be a large factor in what is being referred to as "the Harry Potter phenomenon" (Jacobs, 2000), on both sides of the Atlantic, for as one reviewer of the fourth Rowling book (Acocella, 2000, p.77) observed: "The subject of the Harry Potter series is power, an important matter for children, since they have so little of it."

In an interview, Rowling (Fraser, 2000, pp. 5-6; 8) spoke about her own schooldays and the experience of feeling intimidated in school:

> We were seated according to the teacher's perception of our brightness, and after ten minutes she put me in the 'dim' row. There are a number of people who influenced the character of Snape in my books, and that teacher was definitely one of them. I found it extremely scary. We used to have The Daily Ten—mental arithmetic—and on my first day I got half a point. Well, I'd never done fractions before! I think that I grew on that teacher in time, but I had to work hard at it. And at the fractions.
>
> ...My least favourite teacher was just a bully. I've met quite a few teachers now, both when I was teaching and when I've been visiting schools, and the bullies really do stand out. I understand from the teacher's point of view that it's very easy to be a bully, but it's also the worst, shabbiest thing you can do. We're back to Snape here.

As Brown (1981) has noted,

> Schools are notorious for encouraging a 'piece-meal' approach to virtually everything. Youngsters are given very little opportunity to reflect upon how the pieces fit together. Frequently, there is no rationale, and if there is one, it may be frightening—dealing more with conformity and authority than with the fostering of intelligence.

The large size of the teachers in these drawings would seem to indicate pupils' perceptions of having more often had to deal with this conformity and authority in the classroom; with a philosophy that says, *Look, I tell you this is the way it is*, as Davis and Hersh expressed it, rather than one which invites, *Come, let us reason together*.

It should be noted that there appear to be no drawings of scientists akin to these, showing coercion and threats of violence, which have resulted from the DAST.

6.2.5 THE FOOLISH MATHEMATICIAN

The theme of power is also, I believe, behind pupils' depictions of mathematicians as foolish. The impetus behind this section of drawings is, I believe, an attempt on the part of pupils to try to redress what many may perceive to be an unfair balance of power. And the experience of being made to feel stupid in a mathematics class may be a large part of their perception of this unfair balance.

In **Figure 6.6** from Sweden we see a man in a lab coat with silly hair, glasses, a pocket protector and mismatched socks and shoes.

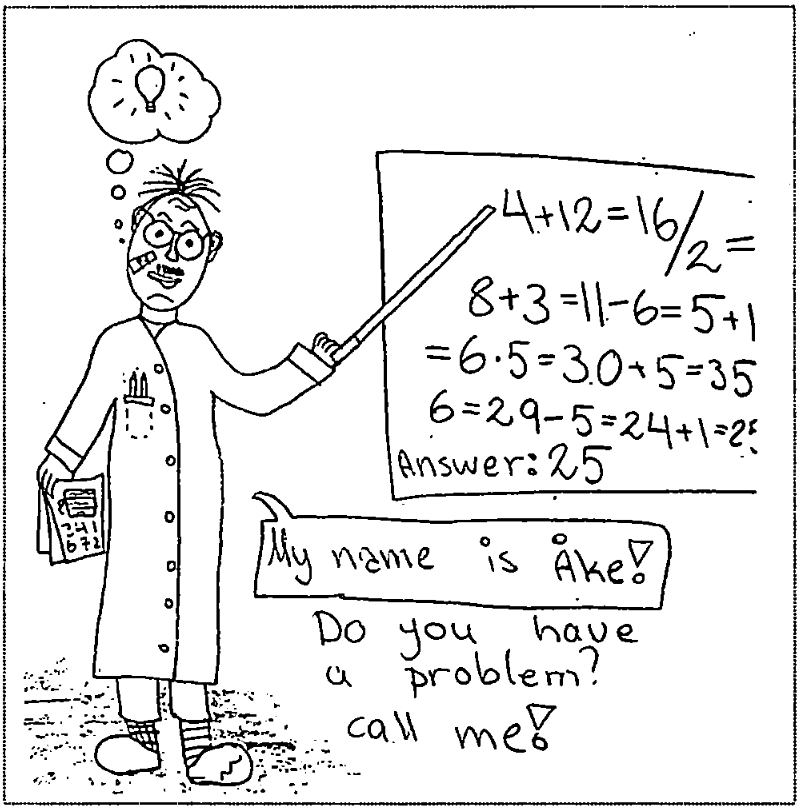

Figure 6.6 Sweden—female pupil

The girl who drew him (***S8***) has written:

I think a mathematician is a man and he is very crazy. The man's name is Åke because I think Åke is a crazy name.

The light bulb over Åke's head seems to indicate the idea that has taken him from *4 + 12* to *Answer: 25.*

Figure 6.7 from the United States, below, also depicts a man in what looks to be a lab coat. He is holding a document on which is written, *2 + 2 = 8 I'm not smart.* The pupil (***US27***) who drew it wrote in his explanation that mathematicians have *no friends,* and wear *high water pants and thick black glasses.* This drawing bore no resemblance to the young female mathematics teacher he had at the time of the survey.

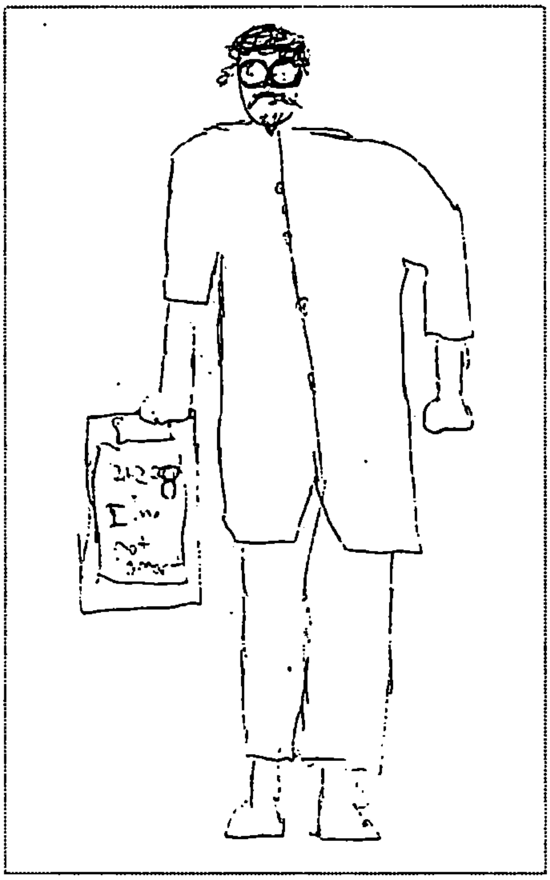

Figure 6.7 U.S.—male pupil

In a third drawing, in **Figure 6.8** drawn by a boy (***UK11***) in the United Kingdom, a man with thick black glasses and a large question mark over his head sits at a desk next to a tall stack of tests.

The pupil's explanation is that the figure is *a maths teacher who does not know how to mark fast track tests.*

I believe that a further aspect of the power imbalance pupils feel, and why they would depict mathematicians in this foolish manner, lacking ability and sense, is because they have observed adults involved with mathematics present themselves as authorities and present mathematics as absolute knowledge in an intimidating manner (Zinsser, 1989).

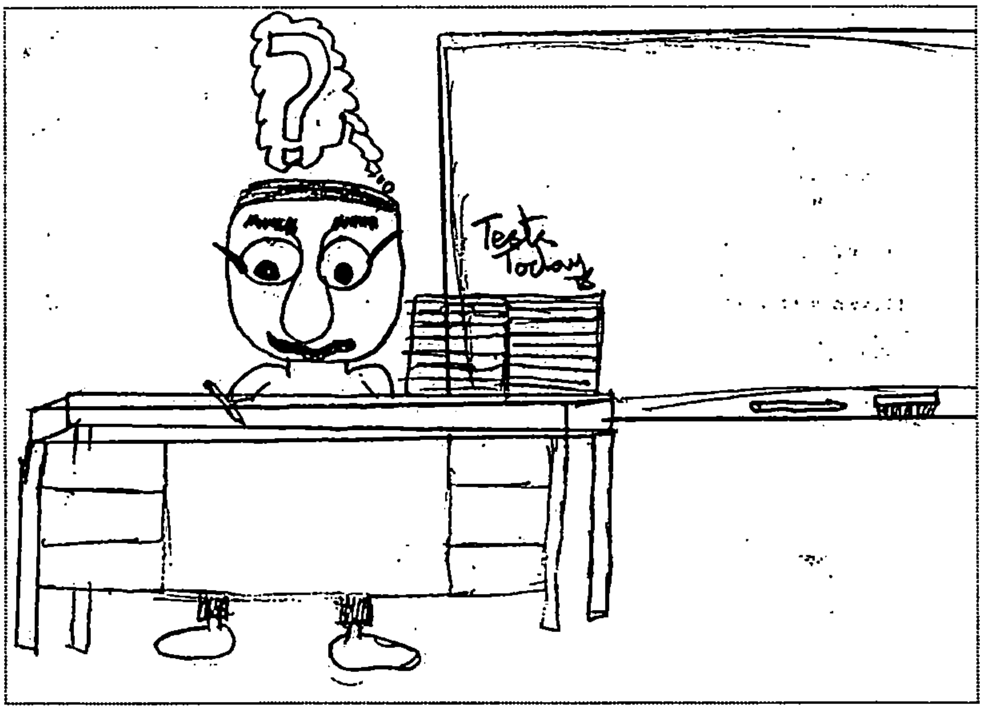

Figure 6.8 U.K.—male pupil

When pupils are made to feel that there is no room for their own independent thought and creativity (Buerk, 1994a) and instead come to see mathematics as something to "rehearse" (Schoenfeld, 1983b, p. 6; 1994), they will also likely have an image of mathematics as the "arbitrary dictates of an authority..." (Cobb, 1986, p.7) I believe these drawings are an attempt to put the "authority" in its place, for holding up adults to ridicule is a way, I believe, of asserting some power back over these adults.

Figure 6.9 is a drawing in which a mathematician in Finland is belittled. The pupil (***F69***) has drawn a bespectacled mathematician who has suffered some sort of demotion so that he has to teach pre-school. On the chalkboard very elementary arithmetic is depicted.

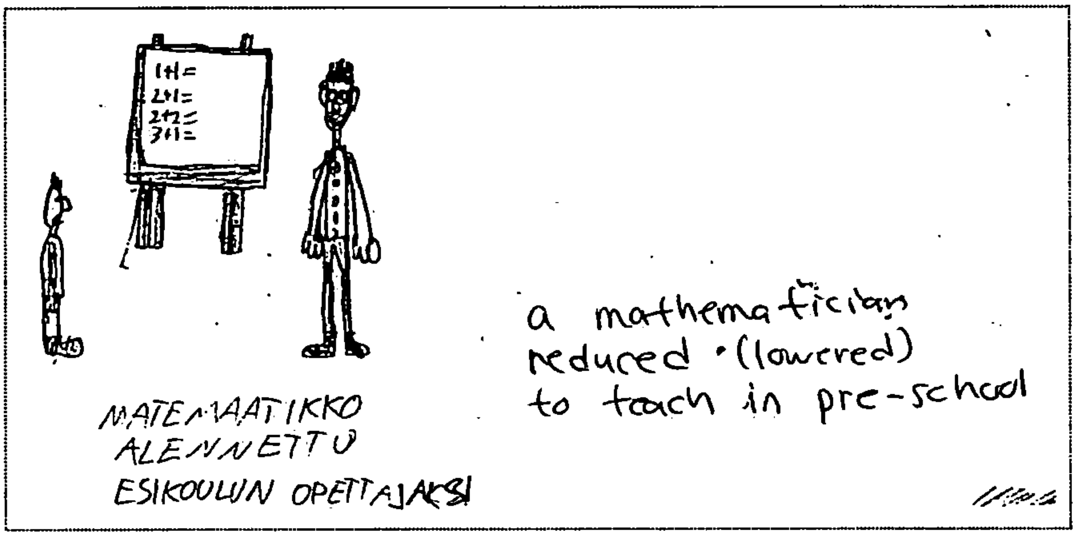

Figure 6.9 Finland—male pupil (***F69***)

And in **Figure 6.10**, in a careful drawing by a female pupil in Romania (***R23***), a man wearing spectacles with a pencil behind his ear is standing before both a computer on a desk and a writing desk, each of which has a pencil and a pad filled with notations. But on the chalkboard behind him are eight arithmetic equations, each of which is incorrect. The pupil has written:

> *This is a very intelligent mathematician. He knows complicated computations but sometimes he makes small mistakes...His clothes are out of fashion because for a long time he only stayed in his office.*

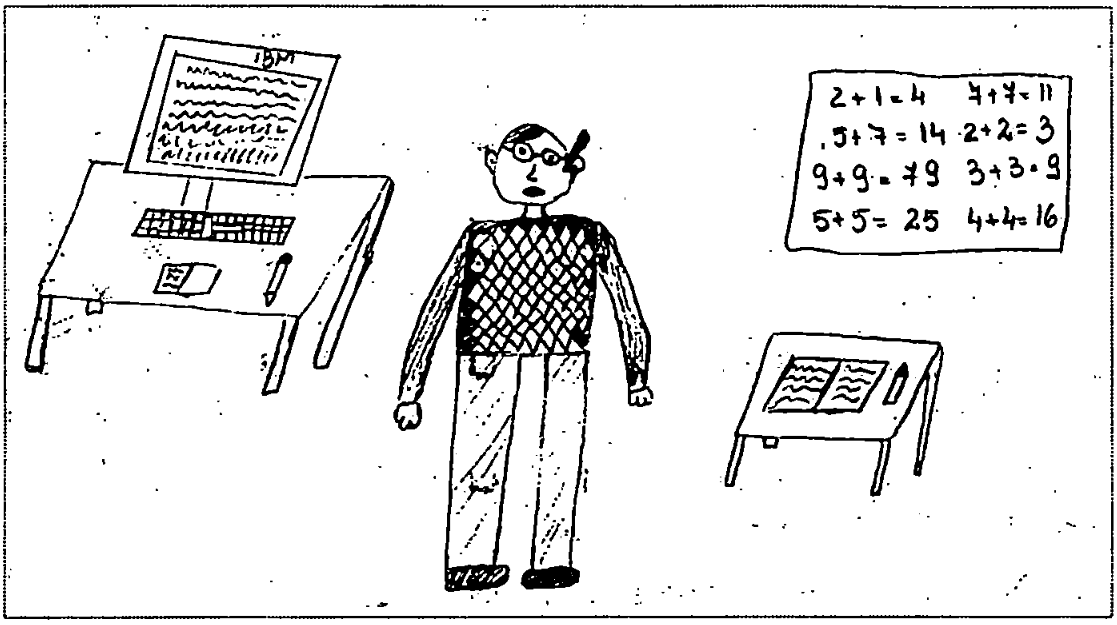

Figure 6.10 Romania—female pupil

To show a mathematician as incapable of simple arithmetic can serve to assuage some of a pupil's own insecurities. And it is also probable that pupils have seen teachers make careless mistakes in class. If a teacher has set himself or herself up as the expert in the room, these mistakes will have even greater resonance.

There is, too, in the statement, *His clothes are out of fashion because for a long time he only stayed in his office*, the image of a mathematician alone and separate from the rest of humanity.

The theme of presenting mathematicians as silly and even stupid, appears to be a universal one with pupils, along with the appearance in these drawings of mathematicians doing, or having difficulty doing, the simplest arithmetic calculations.

I believe it has been shown through the literature that the emphasis in the drawings on arithmetic comes primarily from a lack of knowledge about the width of the subject of mathematics. Most pupils of this age (12-13 years) have begun learning at least pre-algebra with simple equations, if not algebra itself, and yet there is an insistence on depicting arithmetic computations. It has been noted that one of the drawbacks to the *Countdown* program is that since she is viewed as a mathematician, Carol Vorderman has helped reinforce the impression in most people's minds that mathematics and arithmetic are the same thing (Eastaway, 1999).

The resulting confusion is most likely the reason why a female pupil (***UK50***) wrote about her drawing of a male mathematician, ...*a mathematician to me is someone who can work out difficult sums in their heads in front of people.* But it appears that even in countries without *Countdown* this impression is widespread.

And along with the accent on arithmetic in these drawings, is the unmistakable message that the most important thing in a mathematics class is getting the right answer. The emphasis is on a product, not its process; on tests, not on reasoning. Over and over throughout pupils' drawings question marks abound along with such questions as, *What's the answer? What's the result? What's going on here?* There are equal signs facing question marks in drawing after drawing.

6.2.6 THE OVERWROUGHT MATHEMATICIAN

In **Figure 6.11**, a boy in Finland has drawn a bespectacled man working at a desk wearing an *I ♥ Maths* t-shirt. Numbers swirling around his head seem to be making him cross-eyed.

The pupil's written explanation of his drawing is that *He is paying his bills.* In explaining why someone would need to hire a mathematician, the pupil wrote:

If you are too busy to count your bills you need a mathematician. So it appears that the mathematician is doing something someone else is *too busy* to do—but it is making his head swim, and overtaxing his computer as well.

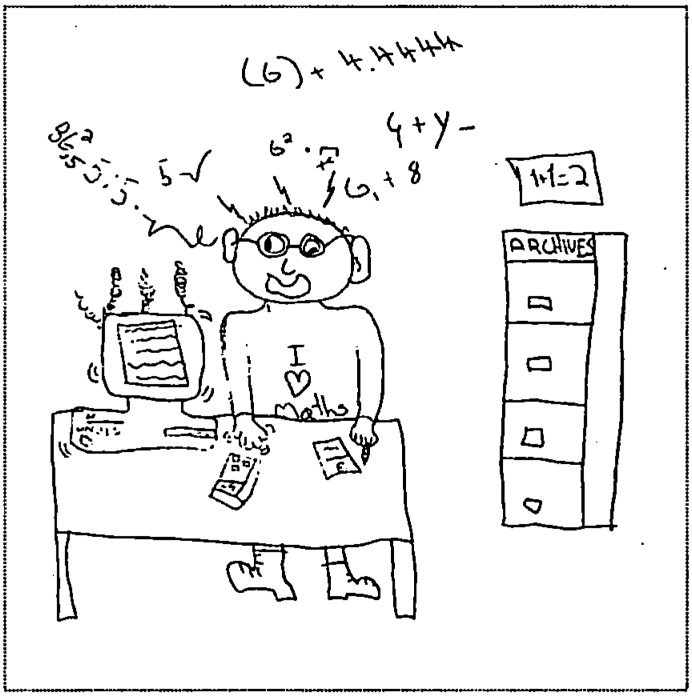

Figure 6.11 Finland—male pupil (***F3***)

Figure 6.12 is from the United States and depicts, *a psycho crazy guy* according to the pupil who drew it. She wrote:

> *When I think of a mathematician, I think of a psycho crazy guy that stays in his room all day and does math work. This is a picture of a crazy guy doing math work.*

This drawing and the others in this section seem to imply a relationship between doing mathematics and looking, if not going crazy, as each character's brain swirls or appears over-taxed. Since the images in fiction and in the media appear to support this relationship of mathematics and insanity, and since mathematics is an abstract and cerebral subject, it is not so surprising that these images appear amongst pupils' drawings. It may also be that the discomfort and stress some pupils have experienced in their mathematics classes are now projected onto the experiences of a mathematician.

There is, too, in what this pupil wrote, *a psycho crazy guy that stays in his room all day and does math work,* the image of the mathematician as a loner, separate from the rest of humanity.

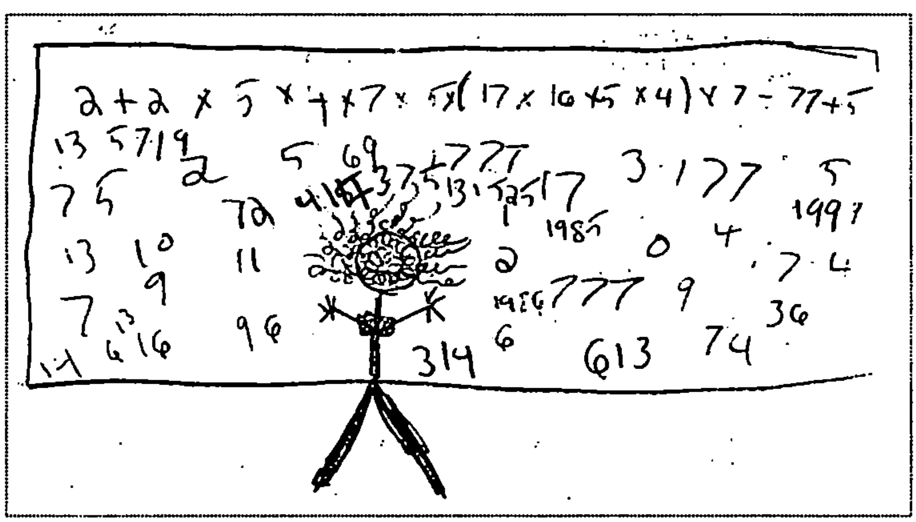

Figure 6.12 U.S.—female pupil (***US39***)

Drawn in full colour by a Romanian pupil, **Figure 6.13** depicts another mathematician gone cross-eyed, with steam emerging from his ears. He seems to be squeezing the ink from his pen, as surrounded by calculations, he repeats, *I want to pass the class, I want to pass the class...*

The pupil who created this drawing wrote, *The name of this guy is Radicalis Geometricus, a guy who was held over because of mathematics....* And she wrote of hiring a mathematician, *(1) I will never have to do my homework again (2) They will take the exams in my place.* The pupil's comment and mantra about passing the class appear to support the possibility that the pupil's own stress about the subject has affected her image of mathematicians and therefore the content of her drawing.

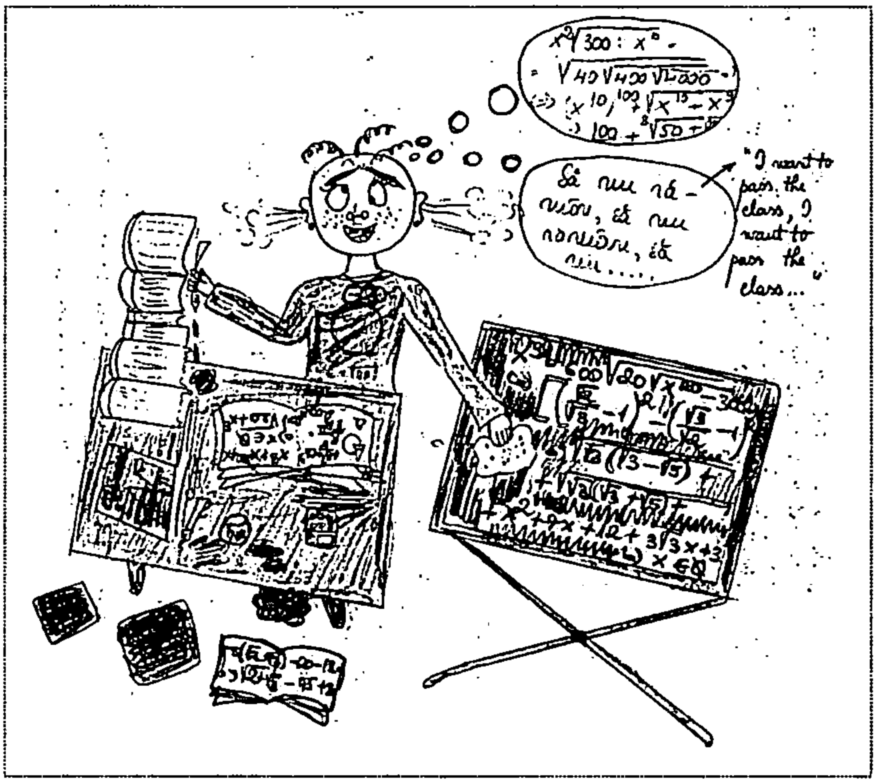

Figure 6.13 Romania—female pupil (***R1***)

In the last drawing of this section, in **Figure 6.14**, a pupil from Sweden has also depicted a figure with smoke, numbers, and what look like three bolts, coming out of his ears and swirling around his head.

About her drawing, the pupil has written, *I think that a mathematician man seems to be a little bit of overstrained! Verry overstrained!! But it is only a joke-picture!!*

This pupil may have meant **Figure 6.14** to be a *joke-picture* but I think it is significant that such similar images were created by pupils in four different countries.

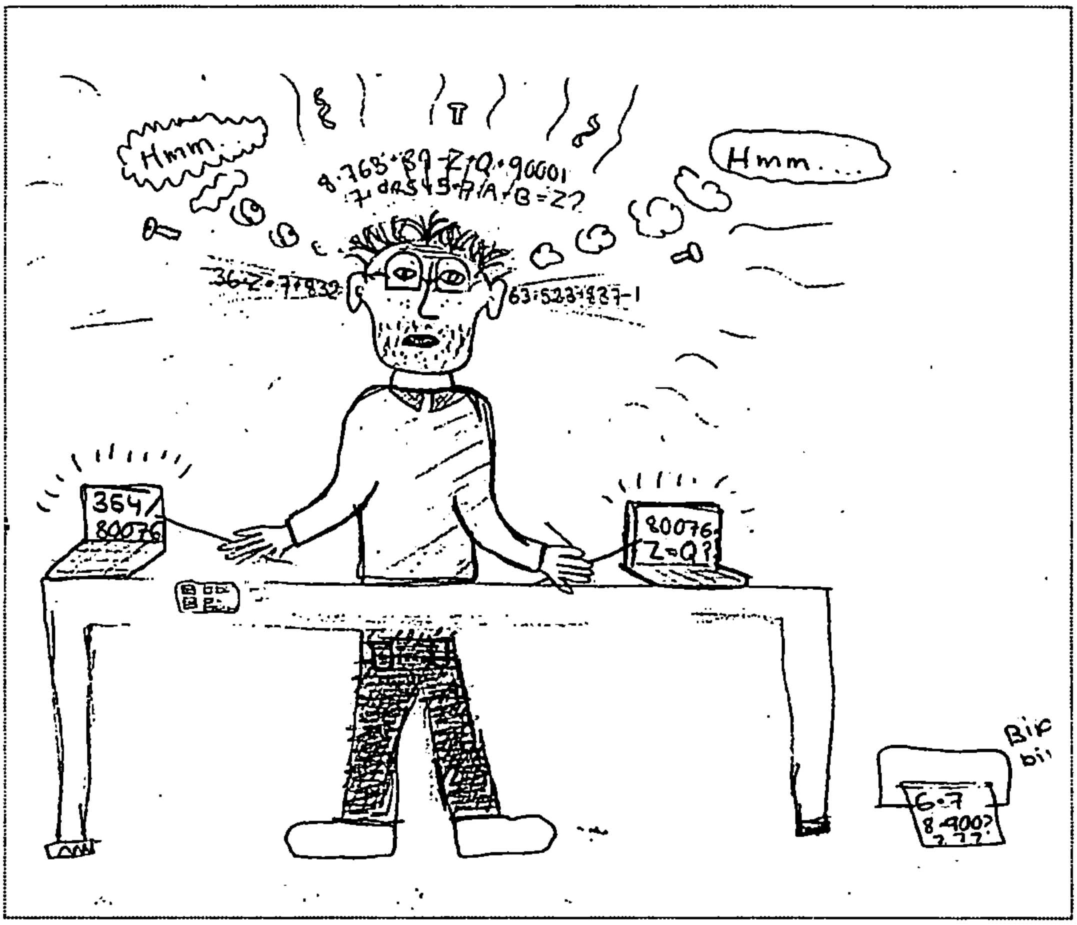

Figure 6.14 Sweden—female pupil (***S5***)

6.2.7 THE MATHEMATICIAN WHO CAN'T TEACH

A chaotic classroom in which all the pupils are talking to each other is depicted in **Figure 6.15**. As in most of the previous drawings, the teacher has been drawn as the largest figure, and he is trying to get the class to some order: *Now class! SETTLE DOWN! QUIET! I can't get there* [sic] *attention!* Down below, the words *Blah blah* are written over and over above what appears to represent a large group of children. The teacher then says, *Do I know the answer?* All this while on the side, a cross-looking figure is musing, *Should he be hired? Nah He doesn't know the answer.* The pupil has written as his explanation:

> *From what I've seen of math teachers they usually don't know the answer to the question they're asking. Also they can never get the kids' attention.*

Again we see this emphasis on correct answers in a mathematics classroom, used here as a hiring criterion. And here, if only in the drawing, the pupil seems to be playing a part in the hiring process.

Figure 6.15 U.S.—male pupil (***US111***)

It is interesting that at the top of the survey form, the pupil has scratched out his name. Perhaps he discovered afterward that his name was optional, or that he felt his drawing was too critical and he didn't want to be identified with it. Nevertheless, here again, a pupil has not depicted anyone physically resembling his current, in this case female, mathematics teacher.

It should be noted too, that this is the only drawing amongst the 476 respondents where the character depicted may not be Caucasian. Considering the fact that among the 201 United States respondents, nearly one-half of the pupils were non-white (includes pupils of African, Caribbean, Latin, Chinese, and Indian heritage) this further suggests that for pupils of this age, the dominant image of the mathematician is white as well as male.

Matthews and Davies (1999) found in their DAST study that even in a class with the percentage of black children as high as 43%, a similar white image among the pupils emerged, which image they concluded was gained mostly from books and the media.

Figure 6.16 is a drawing of three small stick figures. The centre figure labelled *teacher* has a question mark over its head, while the stick figure labelled *a mathematician* appears to be holding forth with a mathematical equation. The pupil has written:

> *Teachers can't do calculations. So a mathematician has been hired to do books for teachers where you can see the answers.*

Again we see a teacher criticised for not knowing answers, which then have to be supplied by a mathematician. And from what he has written, the pupil does not appear to see this teacher as a mathematician. In the drawing, the mathematician almost appears to be whispering the answer to the querulous teacher.

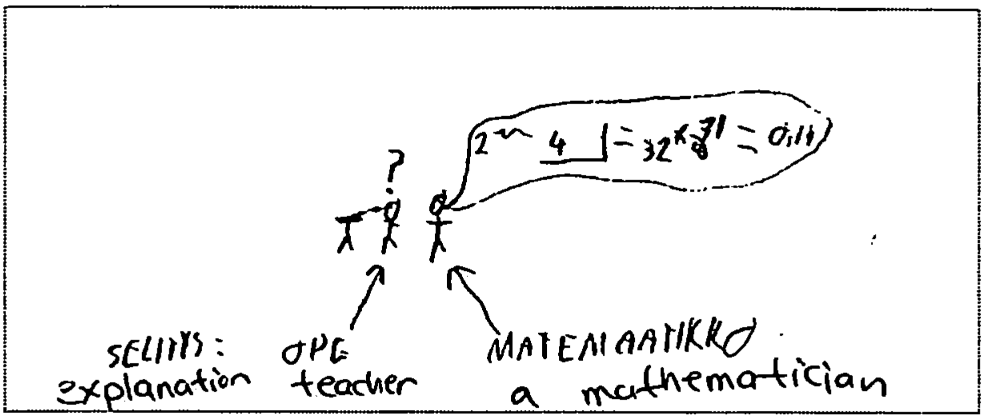

Figure 6.16 Finland—male pupil (***F67***)

In the third drawing, in **Figure 6.17**, the only character depicted is meant to represent the teacher. He is bearded and wearing sunglasses and a strange high hat.

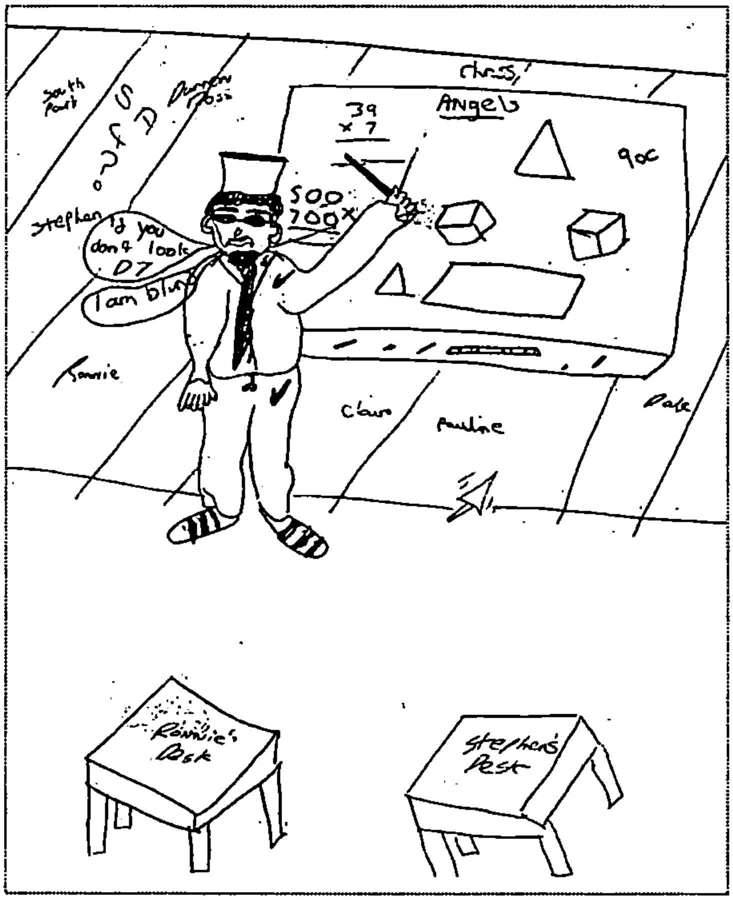

Figure 6.17 U.K.—male pupil (***UK20***)

The teacher is gesturing toward the chalkboard and saying *If you don't look D7.* He also says *I am blind.* There are no pupils depicted in the drawing but they are represented by the two empty desks and perhaps by the names of pupils on the wall around the chalkboard.

There also appears to be a paper aeroplane on the floor under the chalkboard, which may be an indication that the teacher cannot control his classroom. The pupil has written at the top of his work, *This maths teacher is teaching nobody.* Unfortunately, the writing prompts have been ignored.

The phrase *I am blind* is interesting because often teachers who are martinets have absolutely no idea of what is going on in their classrooms and pupils learn very quickly how to take advantage of this.

Skemp (1986) comments that teachers have to try to reconcile two different types of authority and two meanings of the word discipline—that which comprises the discipline of sometimes unruly pupils, and the discipline or subject of mathematics itself, which knowledge makes the teacher an authority. Yet the other type of authority, is related to that which a teacher commands as a result of status or function. The confusion or conflict between these, lies in their difference.

6.2.8 DISPARAGING THE MATHEMATICIANS

In this section, mathematicians are depicted as sneaky and clever and taking advantage in some fashion. In **Figure 6.18**, a mathematician is presented laughing to himself while using a calculator. He is standing in front of a sign that says *No Calculators.*

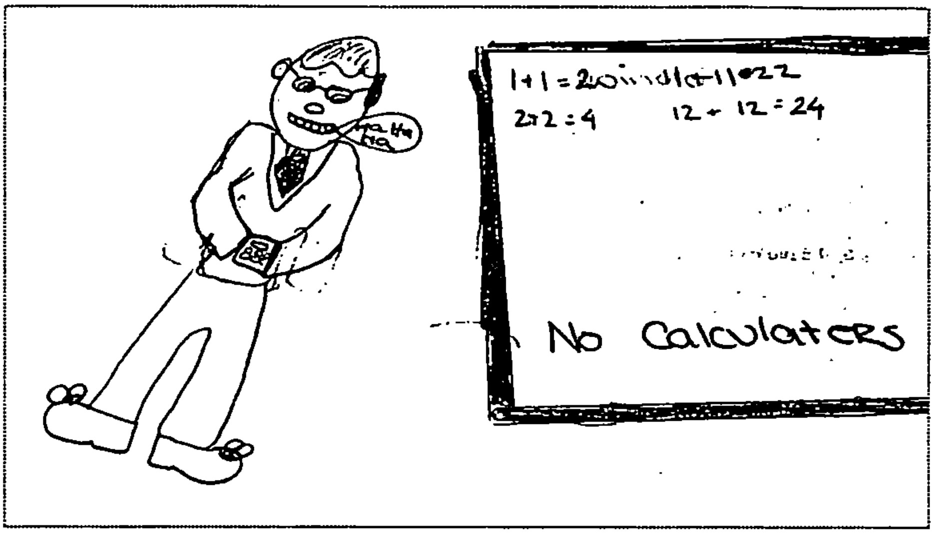

Figure 6.18 U.K.—female pupil (***UK16***)

The pupil who made the drawing wrote: *He is cheating. He is using a calculator when it says not to.* At the same time, the examples on the board do not really demand a calculator, but I think the pupil who drew this is trying to make a point about hypocrisy, most probably as a result of observations made at school.

The next drawing, **Figure 6.19**, also seems to be about perceived hypocrisy by a mathematician. The largest figure in the drawing is bearded with wild hair, and appears to be thinking both about mathematical calculations and about money. He is writing with one hand, and carries a stack of dollars in the other. Two figures on the left in the drawing appear to be looking expectantly at the mathematician, while four dejected-looking figures hang their heads and are heading away from him.

The pupil wrote of his drawing:

> *The mathematician is working and thinking and people are looking at him thinking he's honest, but he is only doing his deed for fame and fortune.*

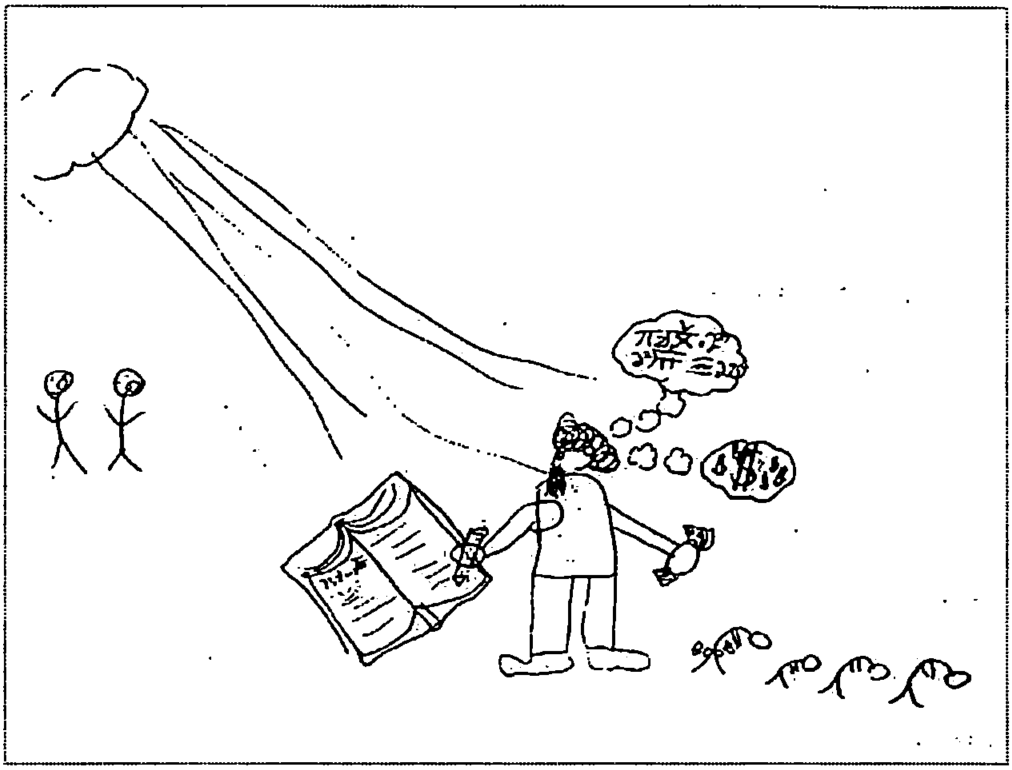

Figure 6.19 U.S.—male pupil (***US109***)

There may be some relation here of the assertion (Howson & Kahane, 1990) that the public does not associate mathematicians with the trappings of success, because the disappointment of the small figures seems to indicate the expectation that a mathematician will work altruistically, rather than for *fame and fortune*. Here the mathematician is not an honest person and he is obtaining his wealth by fooling people. This idea of fooling people may also be related in this pupil's mind to the power mathematicians have in possessing their knowledge.

Although the character in the drawing in **Figure 6.20** is smiling and appears cheerful, the question he is asking, *Can anyone but me find the answer*, is one the pupil-artist appears to have heard in its various forms, for the implication of this question to the class is that the teacher is the only intelligent person in the room. This drawing shows the conflict about authority that Skemp states can be in teachers.

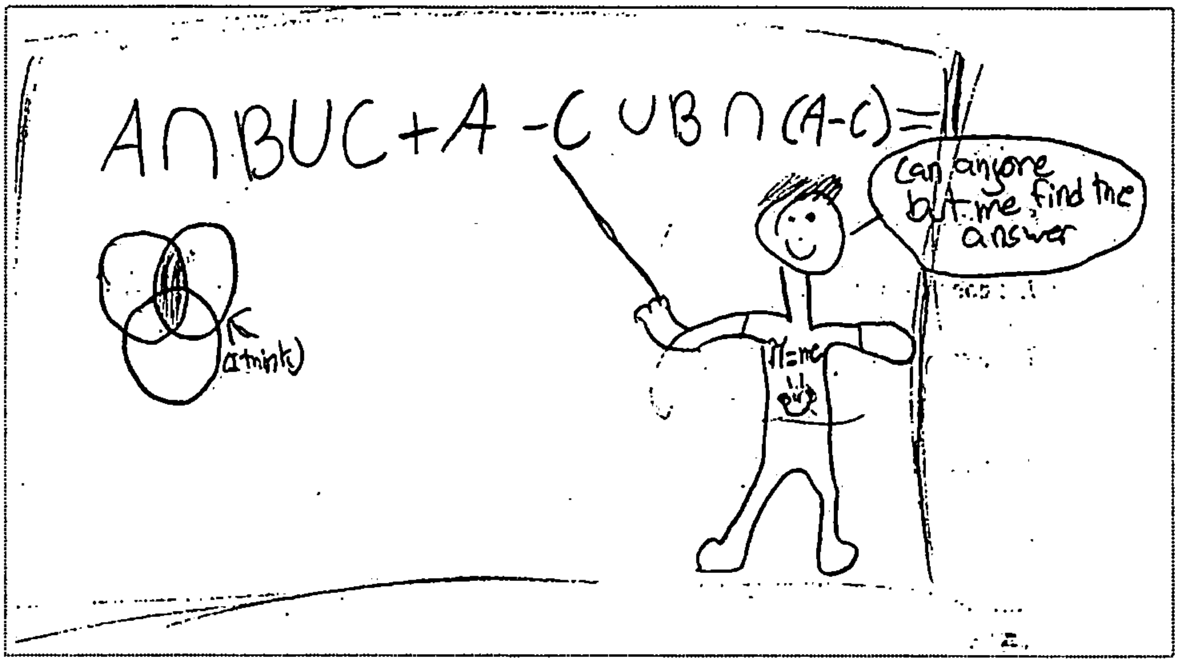

Figure 6.20 U.S.—male pupil (***US110***)

The pupil has written, in one of the angriest declarations I have seen in these surveys:

> *I think that mathematicians always think of complicated things as obvious. This drawing has a snotty person saying "Can anyone but me find the answer," and I dislike mathematicians.*

In this situation, I believe that the pupil has taken a well-founded criticism of teachers he may have had, and projected it onto a mathematician. It should be noted that the teacher he had at the time of the survey was female. Yet it may also be possible that the pupil feared presenting a character too close to his teacher with the criticism he wanted to express.

The criticism of the use of the word 'obvious' by one more knowledgeable is an important one, and it is discussed as it appears in a study of mathematicians' writings by Burton and Morgan (2000).

In the final drawing in this section, in **Figure 6.21**, the mouse is well acquainted with Einstein's equation, but the *miserable mathematician* who is drawn as balding and bespectacled, appears puzzled. *No one is so stupid as to hire a mathematician,* the pupil who made this drawing has written.

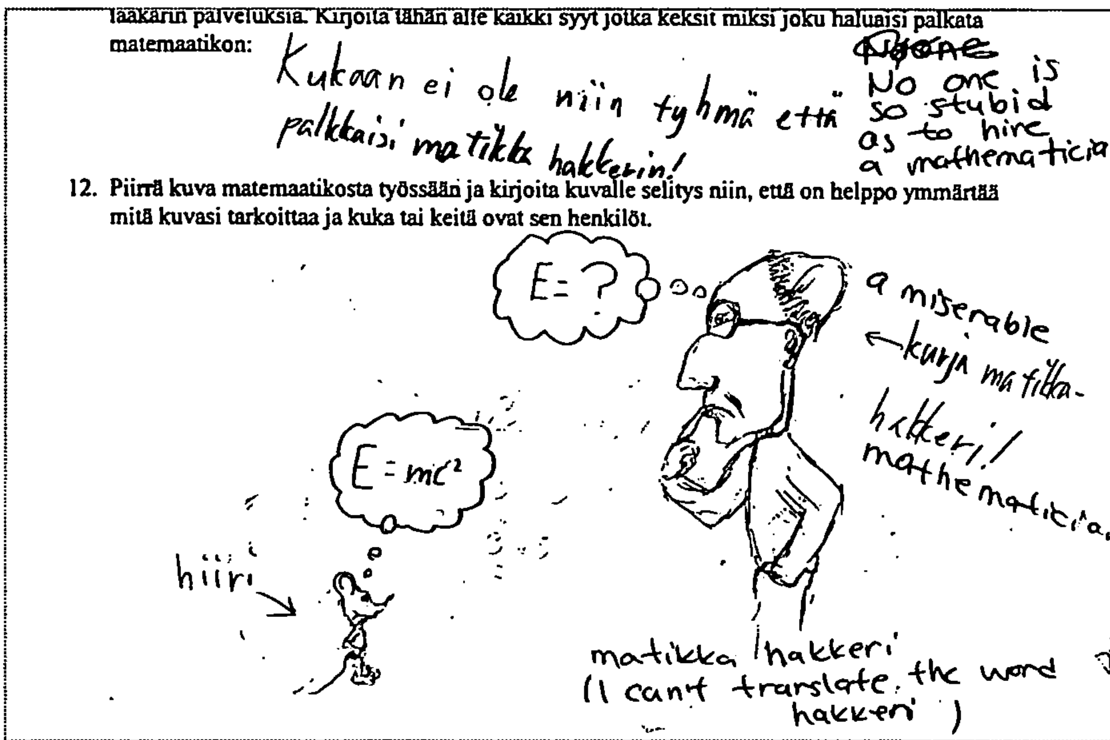

Figure 6.21 Finland—male pupil (***F62***)

The outright contempt for mathematicians in these drawings appears to come from a number of sources, including an uninformed view of mathematicians fostered by images in the media, unfortunate experiences in a mathematics class which appear to cause mathematics teachers and mathematicians to become interchangeable in some pupils' minds, and even possibly a lingering post-World War II/Cold War suspicion of a profession that was responsible for creating such devastating technology as the atom bomb (Malkevitch, 1989).

A male pupil in New Jersey (***US151***) wrote in response to the prompt about why a mathematician would be hired:

> *To make nuclear warheads and figure out stuff for NASA. It means that becoming a mathematician is* [a] *stupid idea, I think.*

Howson and Kahane (1990) have enumerated the ways mathematicians are generally thought of by society, including,

> arrogant, élitist, middle class, eccentric, male social misfits. They lack social antennae, common sense, and a sense of humour.

We can see evidence of this in pupils' drawings and in their written comments from each country.

6.2.9 THE EINSTEIN EFFECT

The appearance of 41 references to Albert Einstein (8.6%) indicates that for some pupils he has come to represent mathematics and mathematicians. At least one drawing representing Einstein or referring to his famous equation (see **Figure 6.21** above) appeared in every country, with the most coming from the U.S.—33/16.4% (see **Table 5.2.6**).

Since Einstein spent the latter part of his life in the United States it is not surprising that he would appear most frequently on U.S. surveys. His image often appears in mathematics classroom posters.

In an interview with Alex (***US132***), a pupil in New York City (see **Appendix—B** for the complete transcript) who had drawn an Albert Einstein figure on his questionnaire, another possible source for the Einstein image was revealed—one which may well have affected pupils in Europe, too—cartoons.

> Alex*:* Warner Brothers [cartoons]—the early ones—they played a lot off of Albert Einstein because well, at the time that they created all of these cartoons and such he was a major scientist.

SP: Now can you, if you remember, can you describe some scenario from one of the cartoons and what he's doing in them?

Alex: I think he was in like a laboratory—well it was with a Bugs Bunny cartoon. Bugs Bunny was the test rabbit and they were trying to actually change the brain of a rabbit to a chicken. (Laughs) And the main mad scientist was sort of modelled off of Einstein because of his dialect and hair—funny hair.

SP: And was there any way of thinking, watching this, that he was also a mathematician?

Alex: Uh, no, I actually think that all the ways he has been portrayed with, uh, as in cartoons and comic strips, it seems as if he's been a scientist.

SP: So what do you think is the reason why people tend to see him as a mathematician as well?

Alex: I guess because of the equation that he used...it seems as if it's like an algebraic equation.

SP: But you've never discussed this equation in your math class?

Alex: No, I haven't.

It is surprising to have Einstein representing a subject in which he never really appears. Students do not study him in mathematics classes, nor do they study his famous equation. But Einstein does figure in the stereotyping of mathematics:

> What image does the public have of mathematicians (and scientists)? When the public is asked to name a prominent mathematician, one of the more common responses is Albert Einstein. Although he was an extremely great physicist, Einstein was not distinguished as a mathematician (also see Halmos, 1968). He was, however, distinguishable for his hairdo. The Einsteinian image has done much to paint mathematicians in people's minds. (Malkevitch, 1989, p. 10)

A drawing by a girl from the U. S. (***US34***) seems to bear this out. She depicted a female mathematician with tall wild hair and wrote:

> *The woman in the drawing is like an Albert Einstein. The hair represents Albert Einstein.*

Another interview with a pupil in the United States, adds more to the understanding of where stereotypical images may come from. The pupil, a boy named Owen (***US112***), had created a careful drawing of what looked like a very normal young person who could have been Owen himself (see **Appendix—B** for the complete interview transcript). He wrote of the figure in his drawing:

> *This person is thinking about math in a new way, which I think, makes him a mathematician. Even if he isn't good at math. I think even I'm a mathematician.*

The figure in the drawing is saying: *Intervals of 6 with a reciprocal of...*and Owen was asked about this.

> Owen: I was just kind of writing like a kind of nonsense because I just wanted to portray the point that this guy is thinking about math; he's thinking about how "anyone who thinks differently about math, that's how the greats began." Just like, 'cause I umm, there's a quote by Albert Einstein, let's see, what was it? Uhh, "Great spirits have always encountered great opposition from mediocre minds." ...Well I think that people like this can still think about things, like I know I think about math all the time and I think that that probably makes me a mathematician 'cause I'm trying to find something here. And just like this person I've got to make him like totally normal. Maybe let's say that he's a—what can he be—maybe he's a stockbroker, no—that's a bad example—let's say maybe he fixes clocks, and maybe he's just thinking about math while he's doing it although he's no mathematician. Just like the movie "Good Will Hunting". You see that guy and he was a custodian and he thought about math though—that would make him a mathematician. Not because—he wasn't a mathematician like as a profession, but he was a mathematician.

Owen spoke further about where he felt the stereotypical images came from and why he had never thought, as he puts it, *in that stereotypical way.*

> SP: You said that you put this sort of nonsense in just to give the idea that he's thinking about mathematics.

> Owen: Yeah.

SP: Do you think that most people think of mathematicians as being normal guys looking like this?

Owen: I doubt it. I mean there's always stereotypical...

SP: So where do you think the stereotypical stuff comes from?

Owen: Uhh, I think the stereotypical stuff just comes from...personally, I think it's TV. That kind of people that never have any reaction but buy just like junk, and so when people think of mathematicians, the first thing they have to have in their head is somebody that works in this big lab with all this mathematical equipment trying to find things out. And that's what they think of as the people who look like the people who make the stuff that you're doing in school and stuff. But really, the people like that...

SP: When do you think you decided that you weren't going to think in that stereotypical way?

Owen: I never really thought in that stereotypical way.

SP: You didn't?

Owen: Yeah, before that I just didn't really know.

Here, Owen, who is very thoughtful, seems to be saying that he made a choice when he *just didn't really know* not to fill that void with a stereotype.

Finally, there is also a confusion in some pupils' minds of mathematicians and scientists, probably due to the images, which recur in cartoons and comics. Owen himself shows this when he says,

> *...and so when people think of mathematicians, the first thing they have to have in their head is somebody that works in this big lab with all this mathematical equipment trying to find things out.*

A pupil (***US128***) who had drawn a mad scientist-type on her survey wrote, *Mathematicians seem like mad scientists in what they do, which is why he looks like that. Though my aunt is very nice.*

This pupil appears to have an aunt who is a mathematician, and yet she still chose to draw a male mathematician who looks like a mad scientist.

6.2.10 MATHEMATICIANS WITH SPECIAL POWERS

The drawings in this theme contain references to special powers or depict mathematicians who appear to have some special power.

The mathematician depicted in **Figure 6.22** has a big Superman-like ***S*** on his chest and is working on an algebraic equation. The window, with its day and night motif, and the clock, which reads 11, indicate that the mathematician works all the time, because, as the pupil writes, the *mathematician loves mathematics*. And if you think of a mathematician as someone who works all the time doing something that to most people looks difficult and at times incomprehensible, it's only natural to assume some supernatural powers make this work possible.

The Superman image and the large S on the chest is certainly an image based in comics and the media, inferring otherworldly powers beyond that of mere humans.

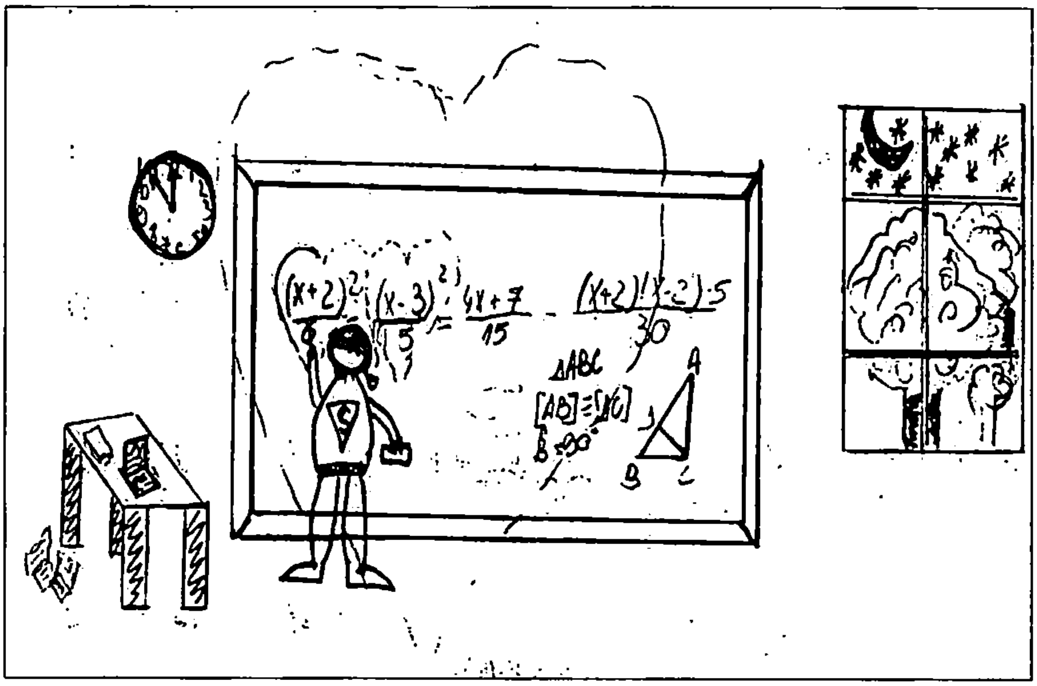

Figure 6.22 Romania—male pupil (***R30***)

In **Figure 6.23**, a mathematician who says, *Maths is brill,* is depicted in a laboratory setting and as *making a maths potion.*

The very idea of a *maths potion* seems to imply that some extraordinary chemistry is necessary in order to do mathematics. As with the super powers image, something extra is needed to enable a person to take part in mathematical activity. Here again, there is some mix-up between mathematics and the activities of a scientific laboratory as the potion bubbles, and numbers swirl through the mathematician's head.

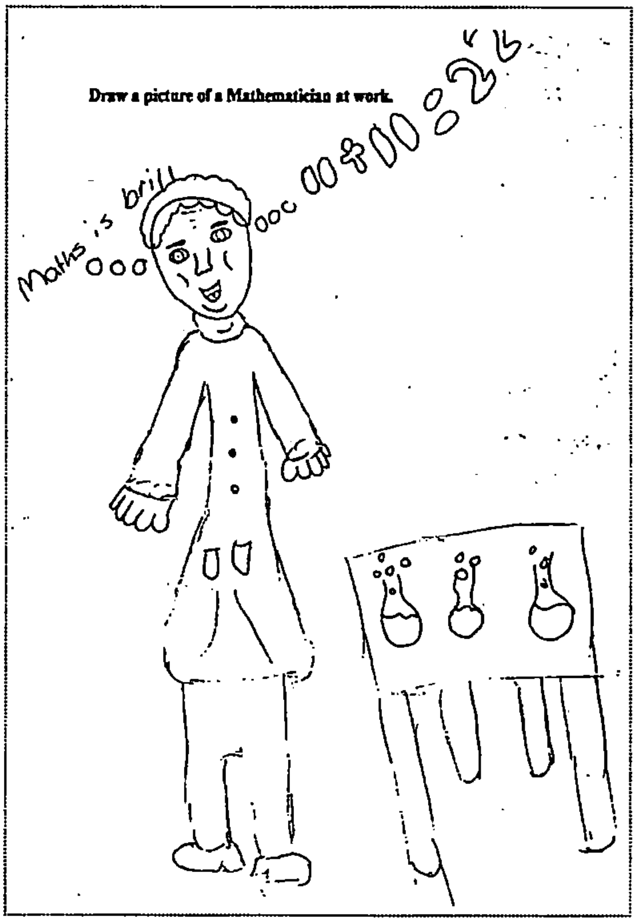

Figure 6.23 U.K.—female pupil (***UK55***)

Wizards appear in **Figures 6.24** and **6.25, which** are from the United States and Finland respectively. In the United States, at least, the image of a wizard whose cloak is covered with numbers and symbols probably comes from the character of the *Mathemagician* in the adolescent novel *The Phantom Tollbooth* (Juster, 1961), but it is not certain if this is the case in Finland. The book has been translated into Danish, but not Finnish, and it is not available there in English according to the large Finnish publisher, WSOY.

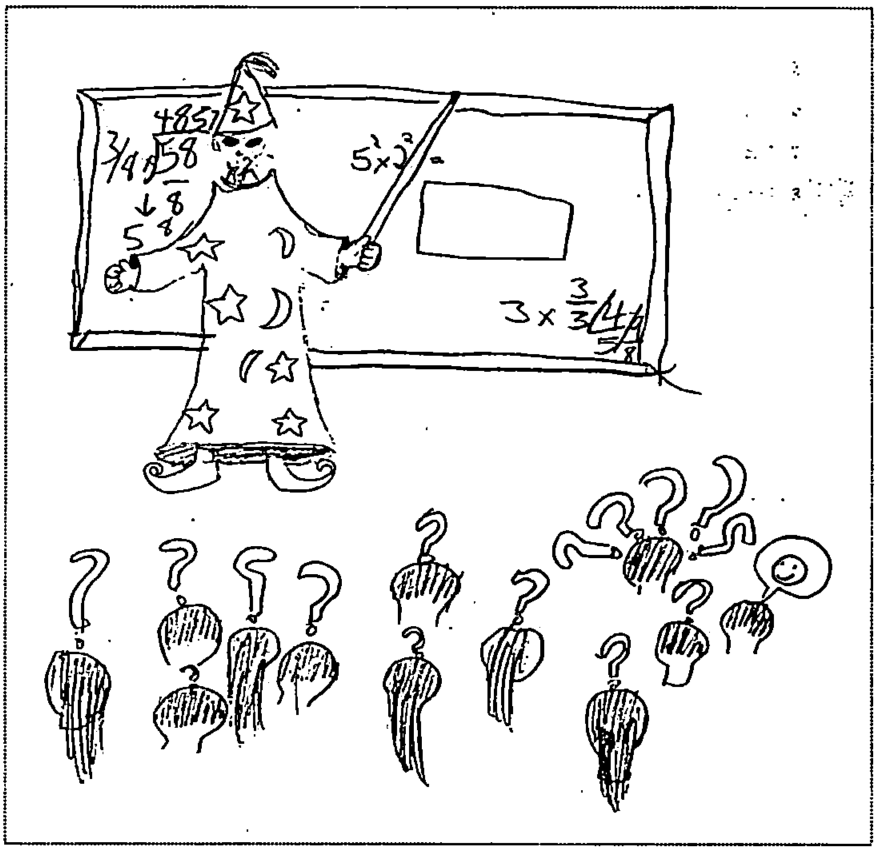

Figure 6.24 U.S.—male pupil (***US171***)

The two wizards are slightly different from each other, although in both drawings they are the largest figures. The American boy who drew the first wizard wrote, *When I thought of a 'mathematician at work' I just thought of about a wizard so I drew a wizard.* He left the section blank, which asked why a mathematician would be hired. It is difficult to tell if this bearded wizard looks benign or not as his blackened eyes make him appear remote.

He stands in front of a group of children only one of whom lacks a question mark over his head. That pupil, on the far fight in the drawing, has a smiley face over his head instead, which seems to indicate some understanding of what the wizard is trying to impart. The pupil to his left however, has five question marks over his head, which seems to indicate an even greater amount of bewilderment than the other pupils. For all of his supposed special power, this wizard does not seem to be providing much help to the assembled pupils.

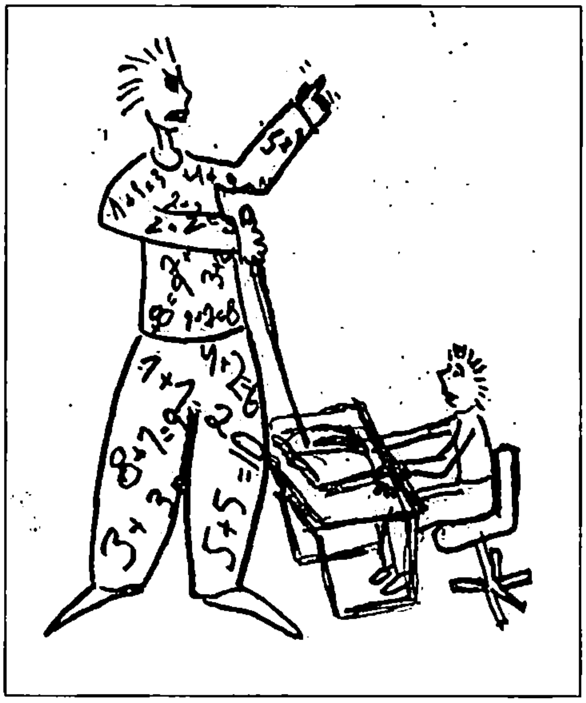

Figure 6.25 Finland—male pupil (***F56***)

In the Finnish drawing, the pupil wrote, *A mathematician as a family tutor*. One way in which his wizard looks different from the American wizard, is that this wizard looks rather cross and he is wagging his finger. His pointer is on the small pupil's open book, and the pupil looks worried or scared.

For the wizard to be a *family tutor* already seems to indicate that the pupil is having some difficulties with mathematics. This drawing does not seem to indicate things going any better for the pupil with the wizard's presence.

Both wizards seem to have some knowledge apart from the pupils and they are poised in a stance of superiority in each drawing.

The idea of giving magical powers to a mathematician, who is viewed as someone who can do something better than one may feel oneself can, is present in these drawings as well as in the comments of the Romanian pupils who would use a mathematician as a personal sorcerer's apprentice.

It is also related to the general invisibility of the mathematical process, for with the process hidden, mathematical facility looks more like a power than an ability, which anyone has the possibility to learn. It looks more like something you might have to create a potion for, or something, which requires a *super power*, rather than study.

And unfortunately teachers can, in their haste to impart difficult material, make the process of studying mathematics which in actuality can be hard and messy, look so smooth and easy—like magic—that, as Schoenfeld (1983) has noted, pupils are made to feel incompetent when it isn't as easy for them.

Although these are not the same wizards as in Harry Potter—the books had not caught on at the time of this survey as they have since, comments on the meaning of the magic in relation to the books are still, I believe, relevant. For as Jacobs (2000) indicates, anything that is "sufficiently inscrutable" might as well be the product of wizardry. And he quotes Arthur C. Clarke's observation (and you can see Schoenfeld's point) that, "Any smoothly functioning technology gives the appearance of magic."

At the same time there is also in pupils' tendencies to give magical powers to mathematicians something to criticise. Reiss (2000, p.3), also writing on Harry Potter, expresses the criticism in this way: "People have seen magic as a way of being able speedily to manipulate the world—of not having to work to understand it."

In preferring an image of the mathematician as someone having magical powers, the pupil has accepted the Platonist view of mathematics as existing outside and apart from humanity; has created a separation between the knowledge mathematicians have and the study with which the pupil could also have this ability. Why invest time and effort in the study of mathematics, if a mathematician will take the test or do the homework for you? It is the opposite of the kinship between knowledge and the self that Aristotle believed was crucial to learning, and it is an opting as well, for an attitude of passivity rather than activity.

6.2.11 THE SURVEY TOOL—*NEUTRAL* VS. *STEREOTYPICAL* DRAWINGS GROUPS

In this section data from the Likert portion of the survey tool is analysed. The first set of data compares those surveys which contained what were designated as *neutral* drawings with those that contained what were designated as *stereotypical* drawings.

For the purposes of comparison, the choices *agree* and *strongly agree* and the choices *disagree* and *strongly disagree* were each looked at as if they were one choice.

Although survey statement 1: *I enjoy the school I attend,* was intended as an icebreaker, the survey data show that there is a difference in the two groups in their feelings of enjoying the school they attend, with the pupils producing the stereotypical drawings more unsure (27.7%--compared with 16.6% for the *neutral* drawings group—see **Tables 5.6a** and **5.6b**) about whether they enjoy their school, and with fewer of them (62% compared with 76.9% in the *neutral* drawings group) agreeing that they enjoyed their school. On average however, pupils do appear to enjoy the schools they attend.

Looking at the chi-square p-values of the two groups (**Tables 5.16** and **5.17**), which are large, the data supports the null hypothesis that enjoying one's school and country are independent, suggesting that the individual school experiences of pupils may not contribute significantly to their perception of whether their school is enjoyable or not.

Survey statement 2 was *A mathematician's work looks like fun to me.* Although it is not clear that pupils really know what a mathematician's work looks like, many more pupils in the group that produced the *stereotypical* drawings disagreed with the statement (58.4% compared with 35.7% for the *neutral* drawings group—See **Tables 5.7a** and **5.7b**). In the *neutral* drawings group, the three categories are about evenly divided which makes sense if it is true that pupils really know very little about what mathematicians do (31.7% agree, 32.6% unsure, 35.7% disagree). The majority of pupils by only a small margin, disagreed with the statement.

Examining the chi-square p-values of the two groups (**Tables 5.16** and **5.17**), both of which are .000, the data supports the alternate hypothesis that pupils' opinions are associated with their country.

The two groups are close on survey statement 3, *I would never think of becoming a mathematician,* and agree with the statement (42.5% in the *neutral* drawings group compared with 45.9% in the *stereotypical* drawings group—see **Tables 5.8a** and **5.8b**). Interestingly, more pupils in the *stereotypical* drawings group disagreed with the statement (27.7% compared with 20.3% for the *neutral* drawings group). Looking at the p-values of this statement for both groups (**Tables 5.16** and **5.17**), the measure of fit is large and the independence of the statement and country is supported.

On survey statement 4, *Mathematicians seem like very patient people,* the two groups were close in their unsureness—36.9% for the *neutral* drawings group, to 38.7% for the *stereotypical* drawings group (**Tables 5.9a** and **5.9b**). But more pupils in the *stereotypical* drawings group (25.5% to 17% for the *neutral* drawings group) felt that mathematicians were not patient. Since pupils do not seem to have real experience with mathematicians, that approximately one-third of pupils is unsure about this statement makes sense. The fact that more pupils in the *stereotypical* drawings group believed that mathematicians were not patient, may have arisen from pupils' experiences in mathematics classes in which they felt their teachers were not patient, or from media images which depict mathematicians as odd and aloof.

The two groups have differing p-values in the chi-square tests (See **Tables 5.16** and **5.17**). For the *neutral* drawings group, the data support the alternate hypothesis that the statement and country are associated; for the *stereotypical* drawings group, the data support the reverse: the factors country are independent.

Survey statement 5, *I would not want to marry a mathematician,* (**Tables 5.10a** and **5.10b**) was included because it had originally been used in the Mead and Métraux (1957) study but I came to feel early that it was not a particularly useful statement. However, there was a telling instance in New York City when administering the survey, where one young man raised his hand and asked, "*But how are the boys supposed to answer this one?*"

Examining the chi-square p-values for both groups (**Tables 5.16** and **5.17**), the measures of fit are small and indicate that this statement, with which the majority of pupils agree, is associated with country.

For survey statement 6, *I have met a mathematician,* 59% of pupils in the *neutral* drawings group believe that they have, compared to 46.7% of the *stereotypical* drawings group (See **Tables 5.11a** and **5.11b**). At the same time, more pupils in the *stereotypical* drawings group (31.4%) are unsure if they have met a mathematician, compared to 21.8% for the *neutral* drawings group. This again seems to indicate that where pupils lack knowledge of mathematicians, stereotypical images of them have filled this void.

It is possible that where pupils indicate that they have met a mathematician, they are thinking of their mathematics teacher as a mathematician. But I encountered a good deal of the unsureness on this statement. When administering the survey to a class of pupils in England, a boy began asking of his teacher, *Sir, are you a mathematician? Are you a mathematician, sir?*

If pupils believed a mathematician is a person who *does hard sums,* and observed their mathematics teacher doing calculations, they may have settled on their mathematics teacher as being a mathematician in order to respond positively to this statement.

For both groups, the chi-square p-values are small and support the rejection of the null hypothesis: pupils' beliefs that they have met a mathematician are associated with country.

For survey statement 7, *I don't enjoy my mathematics class,* the groups are close in their unsureness, 20.6% for the *neutral* drawings group compared with 21.9% for the *stereotypical* drawings group. But 27.8% of pupils in the *stereotypical* drawings group agree and are sure that they don't enjoy their mathematics class, compared to 18.4% in the *neutral* drawings group. On average, pupils in both groups indicate they enjoy their mathematics classes, although a larger percentage of pupils in the *neutral* drawings group do (60.9% as compared to 50.4% for the *stereotypical* drawings group).

The comparison of the p-values for this statement (**Tables 5.16** and **5.17**) show that measures of fit for both groups are large, indicating the acceptance of the null hypothesis: pupils' feelings about enjoying their mathematics classes are independent of their country.

Survey statement 8, *I discuss my mathematics class with friends,* produced very different results in the two groups as well. Only 29.2% of the *stereotypical* drawings group agreed, while 44.3% of the pupils in the *neutral* drawings group agreed with the statement. This may indicate that pupils in the *neutral* drawings group are more inclined to associate mathematics with social activity than the pupils in the *stereotypical* drawings group. It may also indicate that pupils in the *neutral* drawings group are more involved in, or concerned about their mathematics class and so are more apt discuss it than the pupils in the *stereotypical* drawings group.

The p-values for both groups' responses to the statement are very low, and indicate that the statement is associated with country.

For survey statement 9, *Mathematicians work alone,* 30.7% of pupils in the *stereotypical* drawings group perceive mathematicians as working alone, compared with 23.7% of the pupils in the *neutral* drawings group. This too, would further indicate that for the *stereotypical* drawings group, doing mathematics is not perceived as a social activity involving other people.

The chi-square p-values are small and reject the null hypothesis: the pupils' beliefs about this statement and their country are associated.

Finally, in survey statement 10, *I see myself as a mathematician,* 7.3% of pupils in the *stereotypical* drawings group agreed, compared to 12.6% of pupils in the *neutral* drawings group. 32% of pupils in the *neutral* drawings group were unsure, compared to 25.5% in the *stereotypical* drawings group.

On this statement the chi-square p-values differ. For the *neutral* drawings group, the statement and country are independent; for the *stereotypical* drawings group, the statement and country are dependent.

Interestingly, the pupils in the *stereotypical* drawings group were less unsure than their *neutral* drawings group counterparts on six of the ten statements (1, 2, 3, 5, 8 and 10), and therefore may be seen as more adamant in some of their beliefs.

These data indicate that pupils who drew stereotypical drawings of mathematicians have a more firmly held and more negative set of beliefs about mathematicians and mathematics, and appear to enjoy both their school and mathematics classes less than their classmates in the *neutral* drawings group.

6.2.12 THE SURVEY TOOL—INTERNATIONAL COMPARISONS

The survey tool was also analysed for international comparisons and similarities. This analysis appears in this section. For the purposes of comparison, the choices *strongly agree* and *agree* and *strongly disagree* and *disagree* were each looked at as if they were one choice.

The percentages for pupils agreeing with survey tool statement 1, *I enjoy the school I attend,* appear fairly close among the countries (see **Table 5.20**), with only the United States and Finland below the 49.6% average of pupils agreeing.

Pupils in Romania seem most sure that they enjoy their school, but this may be due to the fact that the school they attend is a specialised one attracting a population of pupils who already excel in science and mathematics. The pupils in Finland participating in this study appear to enjoy their school the least, and be most unsure (25.5%) about whether they enjoy their schools.

On survey statement 2, *A mathematician's work looks like fun to me,* (see **Table 5.21**) there is a large difference between the response of pupils in Romania and the responses in other countries. Only one child (3%) from Romania disagreed with the statement, compared with the highest percentage of disagreement in the United States (32.8%).

On the surveys from Romania, however, pupils made references to mathematics Olympiads and contests in which many of them participate. This may be a factor in disposing pupils toward feeling more favourably about the work of a mathematician even if they are not so sure what mathematicians do.

On survey statement 3, *I would never think of becoming a mathematician* (see **Table 5.22**), 32.4% of pupils were unsure, compared to 45.2% of pupils who agreed with the statement. It may have been the strong wording of the statement that contributed to nearly one-third of pupils being unsure, but it also may be that pupils cannot clearly imagine what a mathematician does. 22.5% of pupils disagreed. Most pupils would not like to do the work of a mathematician, and whatever they perceive it to be, it is clearly not attractive to them.

For survey statement 4, *Mathematicians seem like very patient people* (see **Table 5.23**), more than one-third of pupils are unsure (38%) compared to 42.9% of pupils who agreed. But pupils in Romania appear to disagree most with this statement (27.3% as compared with an average of 19.3%) which may reflect the type of pressure they experience in mathematics classes that led them to hope for a mathematician who could do their homework and take their tests for them.

Most pupils, (46.4%) are unsure about the statement, *I would not want to marry a mathematician* (see **Table 5.24**), although a larger percentage of them agreed (32%) with it, than disagreed (21.6%). This was not really a useful question. Pupils feel that marriage is a long way off, and can't really speak for whom they may or may not marry. And again many pupils have a lack of clarity about mathematicians, or have an image that is unattractive, and so these responses are not surprising.

55.2% of pupils believe that they have met a mathematician (see **Table 5.25**), and yet when questioned in interviews they became very vague and could not name a mathematician they had met. Owen, a pupil in New York City was questioned about this (see **Appendix—B** for the full transcript):

SP: And, now you said that you agreed with the statement, "I have met a mathematician." And I'm just curious as to who that might be.

Owen: Well, let me see, I don't really know why I put that. I think I put that probably to correspond with my drawing.

SP: As you look at it know, would you say you really have met a mathematician?

Owen: Yes.

SP: And who would that be?

Owen: There's a lot of people. I think it's just people.

S: That you've met.

Owen: I don't think there's anybody that you'd *label* a mathematician. I mean probably there's a lot, a lot, lots and lots and lots and lots and lots of people that are that nobody knows—sometimes it's neat to find out.

S: And have you met someone who you know made their living as a mathematician?

Owen: Umm, no.

In another interview, Rebecca, who was also a New York City pupil was asked about her agreement with the statement (see **Appendix—B** for the full transcript).

SP: Can you think of who it was you were thinking about?

R: Umm, I don't know. Maybe I was thinking of us as mathematicians, or the teachers; I can't quite remember.

SP: Do you see your teacher as a mathematician?

R: Well, umm, kinda, (laughs). I don't know.

SP: Okay, so you don't know.

R: I don't know.

It is possible that pupils felt they *should* know a mathematician, and so they answered in the affirmative. 24.6% of pupils indicated that they were unsure if they had met a mathematician.

It is interesting that more pupils in Finland (35.1%) disagreed with the statement than any other—the average was only 20.2% of pupils disagreeing. This may explain why Finland also had the highest number of pupils refusing to draw a mathematician on their surveys.

More pupils disagreed (57%) with survey statement 7, *I don't enjoy my mathematics class*, than both agreed and were unsure (22.1% and 21% respectively). The p-value for the chi-square test for this statement was .272 (**Table 5.30**), which appears to support the null hypothesis that country of pupil and this statement are independent. We might then expect a pupil to have this opinion in any country we surveyed at this time.

Romania (75.7%) and Sweden (63.3%) appear to enjoy their classes most and are well above the average, followed by the U.K. (62.6%), U.S. (55.7%) and Finland (55.7%). What is interesting about Romania and Sweden leading this group, is that Sweden and Romania respectively had the highest percentages of stereotypical drawings.

A larger percentage of pupils (46.6%) do not see themselves as discussing their mathematics classes with friends, as compared with 39.5% of pupils who do. I believe this may in part be related to how much discussion goes on within mathematics classes. In the two countries which had the lowest percentages of pupils agreeing with this statement, Sweden (16.3%) and Finland (29.8%) pupils sit in rows in their classrooms, rather than in groups, as they do in the U.K. and the U.S. In Romania, which had 63% of pupils agreeing, pupils sit in rows, but they work together on Olympiads and, I have been told, during exams, where a surprising amount of cheating goes on.

On survey statement 9, *Mathematicians work alone,* the majority of pupils in each country were unsure, showing, I believe their general unsureness about what they know about mathematicians. Certainly, mathematicians are generally portrayed in comics, cartoons, and the media as working alone, but many pupils are now working together in their mathematics classrooms. The countries where pupils are sitting in rows agreed in the highest percentages—Romania (48.5%), Sweden (38.8%), and Finland (36.1%) compared to the U.S. (21.4%) and U.K. (11.1%) where the pupils who took part in this survey sit together and work in co-operative groups.

The chi-square test relating this statement and country was .000 supporting the alternate hypothesis that the statement and country are associated.

On survey statement 10, *I see myself as a mathematician,* **(Table 5.29)** the U.S., U.K., Finland and Sweden were closely matched in disagreeing with the statement—59.7%, 60.6%, 62.8%, 65.3% respectively, as compared to only 27.3% of the pupils in Romania disagreeing with the statement. The difference may be due to the fact that the Romanian pupils were from a specialised school in which mathematics is a subject many of them would be expected to excel in.

On the chi-square test, the high p-value indicates that the statement and country of pupils are independent, and we might expect pupils in any country we surveyed at this time to hold this opinion.

6.3 THE MATHEMATICS PROFESSIONALS SURVEY

The surveys for this section of the project were, for the most part, returned anonymously or by post. But at one conference there were also some verbal responses by some university-level instructors who indicated that these were questions requiring more thought than they could give at the moment and they took the survey away for a while. In one case, a Belgian professor said of the last question, "What a difficult question." Then he added, "This is a deep question."

The data in the survey indicate the lack of a unified vision among members of the professional mathematics community and along with it some confusion and disagreement within its membership. Certainly the understanding of who is a mathematician and who may call oneself one is rather varied. The question is, what message do these various opinions give to undergraduates, to teachers in training, and eventually to young pupils? For as Mura (1995) has emphasised, mathematics' professors views of mathematics have a potential influence on school teachers' views.

Table 6.1 below, is a partial reprint of **Table 5.40** which contains the frequencies for responses to the question, *Who may call oneself a mathematician?* Included on this version of the table is the order of the frequency of the responses.

As can be seen in the table, the idea that *someone who reasons mathematically* can be considered a mathematician was highest in frequency (30.2%), with the opinion that one must create new mathematics and do theoretical research appearing second (18.9%) in frequency. I believe that there is a difference of opinion between these two, for those who hold that one must do *theoretical research* to consider oneself a mathematician would not, I believe, agree that *someone who reasons mathematically* is a mathematician, nor *anyone who wants to* (13.2%).

		FREQUENCY	PERCENT	VALID PERCENT	ORDER OF RESPONSE
Valid	1 Thinking	32	30.2	30.2	1
	2 Personal choice	14	13.2	13.2	3
	3 Teaching	3	2.8	2.8	7
	4 Research	20	18.9	18.9	2
	5 Study	11	10.4	10.4	5
	6 Use	9	8.5	8.5	6
	7 Higher maths	12	11.3	11.3	4
	8 Not a personal choice	2	1.9	1.9	8
	9 Not sure	2	1.9	1.9	8
	10 Enjoyment	1	.9	.9	9
	Total	106	100.0	100.0	

Table 6.1 *Who may call oneself a mathematician?*

Sixth in frequency (8.5%) was the opinion that anyone doing mathematics may consider themselves a mathematician. It is interesting that so few hold this opinion, because we would not, I think, challenge the notion of someone who painted on weekends referring to themselves as a painter, or if they sang or played an instrument, a musician.

Yet the standards for calling oneself a mathematician appear to be different, including for the two respondents (one at university the other at high school level) who indicated that calling oneself a mathematician is not for one to say, but rather for others to confer.

Some responses were contradicted by others, indicating the wide variety of opinions held. An American respondent stated that she was not a mathematician because,

> *I just graduated with a B.A. and I do not know enough to consider myself a mathematician.*

Yet another respondent, a professor of mathematics, wrote:

Good question! Someone who only has a bachelor's degree in math can consider themselves a mathematician.

A female high school mathematics teacher wrote:

I am now a math teacher. I used to work as a mathematician when I applied my math knowledge to industry.

A professor of mathematics agreed with this. She wrote:

...teaching elementary maths (calculus and lower) does not make one a mathematician.

But a British University lecturer in mathematics disagreed. He wrote:

I teach maths. I do maths research. I love maths!—Any one of the above.

An interesting question is what do respondents mean by *creating new mathematics?* Of the 20 professionals holding this opinion, 9 were on the university level, 8 were on the high school level, 1 on the middle school level and 2 on the primary school level and I think that there are differing conceptions among them as to what constitutes new mathematics.

The word *new* has a number of distinct meanings, including (Costello, 2000): "recent; still fresh; just found, discovered, or learned, not previously experienced or encountered; novel or unfamiliar; recently obtained or acquired".

In relation to mathematics, is having just learned something in the subject the same as having discovered something not previously experienced or encountered? Because the next question is, encountered by whom? By oneself, or by the whole of the history of mathematics? I contend that for some professionals it is the latter—what is discovered must be completely new in mathematics, while for others, including those who hold a constructivist view of mathematics, it is the former.

One would not expect Dame Kiri Te Kanawa, as an example, to create the songs she sings in order to be considered a musician. What she creates instead is a fresh perspective and a sound different from that of another singer, so that a new hearing of an aria comes to be in the mind of a listener. This interpretation of creativity does not appear to be what some mathematicians mean when they hold that creating new mathematics is necessary to be considered a mathematician, yet for others it is.

Owen, one of the pupils quoted earlier, defined himself as a mathematician:

> *This person is thinking about math in a new way which I think makes him a mathematician...I think I'm even a mathematician.*

And he said in his interview:

> *...like I know I think about math all the time and I think that probably makes me a mathematician 'cause I'm trying to find something here...You see that guy* [Will Hunting] *and he was a custodian and he thought about math though—that would make him a mathematician. Not because—he wasn't a mathematician like a profession, but he was a mathematician.*

And this directly contradicts many of the responses from professionals in the mathematics field. Does Owen have the right to call himself a mathematician? Some respondents would say he doesn't. Owen thinks he is a mathematician because he thinks about mathematics *all the time* and he is trying to think about it in new ways: *I'm trying to find something here.* To Owen, being paid to do mathematics, is not the deciding factor. But to some respondents it is.

Looking at **Table 5.41** which shows responses to *Who may call oneself a mathematician?*—by working level, there were 2 respondents on the elementary level who felt that doing research or new mathematics enabled one to call oneself a mathematician, yet only one elementary school level professional identified themselves as a mathematician. The other felt that they couldn't because they didn't create new mathematics (see **Table 5.43**).

So there is either a difference of opinion about what creating new mathematics means and/or one of these persons has felt disqualified by their interpretation of the meaning of their own statement. This points to a lack of a uniform vision in the mathematics community and this includes confusions in teachers' minds as to how to see themselves and whether they can place themselves among mathematicians. Furinghetti (1993, p. 33) believes that we should not be surprised by this:

> The image of mathematics among professional mathematics [sic] is tortuous and controversial; it should not surprise us, therefore, that for mathematics teachers, deciding what image to transmit to their pupils is a source of doubt.

Papert (1972, pp. 249-250) writes cogently about these differing points of view:

> ...being a mathematician, again like being a poet, or a composer or an engineer, means *doing*, rather than knowing or understanding...It is generally assumed in our society that every child should, and can, have experience of creative work in language and plastic arts. It is equally generally assumed that very few people can work creatively in mathematics. The author believes that there has been an unwitting conspiracy of psychologists and mathematicians in maintaining this assumption. The psychologists contribute to it out of genuine ignorance of what creative mathematical work might be like. The mathematicians, very often, do so out of elitism, in the form of a deep conviction that mathematical creativity is the privilege of a tiny minority.

So Papert feels that elitism contributes to an attitude of exclusivity in mathematics. And there is reference to this elitism and exclusivity in some of the written responses. An American university professional who identified herself as a mathematician wrote:

I do not think of myself as a research mathematician in the narrowest traditional 'ivory tower' sense of the term, however, every person <u>*is!*</u>

And another respondent, a research student and University-level teacher from Sweden had this comment:

> *Has to do with how confidant you are in mathematics, and also what kind of work you do. I think a lot of maths teachers really could call themselves mathematicians but don't do this because of some kind of 'big brother complex' towards university mathematicians.*

It does appear that there is a feeling among members of the mathematical profession that there is some divide among them. The Swedish respondent also believes that this keeps teachers from feeling that they may call themselves mathematicians.

A secondary teacher who held that he did not consider himself a mathematician wrote:

> *I know some math, I teach some math, I learn new math, but I don't create new math, which is my definition of a mathematician.*

But Brown (1981, p. 28) writes that the act of understanding is already an act of creativity. He is critical of an assertion by Poincaré that they are different:

> In contrasting mere *understanding* and *creating*, he [Poincaré] assumes that they are different states of mind or different kinds of activities. Understanding mathematics is one thing—creating is another! What is it that leads us to believe that 'mere understanding' is so simple a construct and so divorced from an act of creation?
> We have been misdirected partly by a technological input/output view of the world to conclude that 'coming to understand' is a relatively straightforward matter. The viewpoint is connected to a commonly held myth regarding good teaching. Good teachers are supposed primarily to be able to explain things well and to be able to 'get us' to understand things that we could not do well on our own! I would like to explore a more dynamic model of understanding mathematics.

When Owen says, *I know I think about math all the time and I think that probably makes me a mathematician 'cause I'm trying to find something here,* I think that he is implying the creativity of coming to understand something. *Trying to find something* appears to be a part of Owen's *thinking about math in a new way.*

The chi-square correlation of working level and *Who may call oneself a mathematician* had a p-value of .495 (see **Table 5.42**) indicating that the statement and working level were independent. At the same time, it appears that as the working level decreases from tertiary to secondary through primary school, the sureness of the respondent in considering themselves mathematicians also decreases. This can be noted in **Table 5.37.** Generally, the mathematical knowledge of teachers is lower as they move down these levels, but the requirements for high school teachers is certainly high, and it is not uncommon for middle school teachers to have a high level of mathematical training as well. And yet there is this hesitation. Perhaps the respondent from Sweden was right in asserting that there is a certain permission needed. He also implied that there was an inferiority felt by teachers not on the tertiary level. But it needs to be asked how much those at the university level contribute to this unsureness, however unintentionally.

In reasons professionals gave for why they did not consider themselves to be a mathematician (see **Table 5.39**) the one given most often indicates an unsureness about their mathematical background; some feeling that they don't know as much mathematics as others. For these respondents, it appears they believe one can only consider oneself to be a mathematician based on having some quantifiable amount of knowledge, rather than as Papert states, *doing*, or on how one thinks and reasons.

For some respondents to this survey, the seeing of oneself as a mathematician is based on thinking and reasoning mathematically, and this points to another philosophical disagreement; one which is deeply related to the question of what (or who) is a mathematician.

Is it someone with a degree and a quantifiable amount of knowledge, or is being a mathematician a habit of mind—a way of thinking and reasoning?

A cross tabulation of working level and *Why I don't consider myself a mathematician* is displayed in **Table 5.43**. The chi-square p-value is in **Table 5.44** and is .107. The data support the null hypothesis: the working level of a professional and not considering oneself a mathematician appear to be independent factors.

There were 2 university-level professionals who returned this survey and contended that they were not mathematicians because they had another title—statistician and engineer. In a study she conducted of mathematicians, Burton (1999) also noted respondents who considered themselves by other titles. This distinction, which in another country, or at another university might not be made, highlights an aspect of the invisibility of mathematicians, making it that much harder for the general public and therefore young pupils, to understand just what it is that mathematicians do.

Chapter 7: INTERVENTIONS DATA

7.1 INTRODUCTION TO THE INTERVENTIONS DATA

This section contains the data for the two interventions: the graph theory and discrete mathematics unit, followed by the data arising from the intervention of the Mathematicians Panel.

7.2 DATA FOR THE GRAPH THEORY AND DISCRETE MATHEMATICS INTERVENTION

In **Table 7.1** below, is the gender of the pupils in the Gramercy Middle School class:

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	**female**	10	35.7	35.7	35.7
	male	18	64.3	64.3	100.0
	Total	28	100.0	100.0	

Table 7.1 Gender of pupils in Gramercy Middle School class (n = 28)

Below, in **Table 7.2**, is data for the gender of the mathematicians pupils drew before the intervention.

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	**ambiguous**	5	17.9	17.9	17.9
	female	4	14.3	14.3	32.1
	male	19	67.9	67.9	100.0
	Total	28	100.0	100.0	

Table 7.2 Drawing gender before the intervention

Table 7.3, below, contains percentages of pupils drawing male and female mathematicians by gender. Five drawings (2 by males; 3 by females) contained no figure or were ambiguous.

Males drawing males by %	Males drawing females by %	Females drawing males by %	Females drawing females by %
83.3	5.6	40	30

Table 7.3 Male and female drawings by gender of pupils before intervention

The coding system for characterising pupils' drawings arose from the six themes observed among these and subsequent drawings in the second intervention surveys. They were:

1 = Wizard

2 = Weirdo

3 = Teaching

4 = Thinking or calculating

5 = No figure or non-descript

6 = Einsteinian reference, i.e., $E = mc^2$

Drawing characteristics before and after the intervention are displayed in **Tables 7.4a** and **7.4b,** and **Figures 7.1a** and **7.1b,** below:

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	**Teaching**	3	10.7	10.7	10.7
	Thinking/Calculating	21	75.0	75.0	85.7
	No figure/Non-descript	3	10.7	10.7	96.4
	$E = mc^2$	1	3.6	3.6	100.0
	Total	28	100.0	100.0	

Table 7.4a Drawing characteristics before the intervention

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	**Teaching**	5	17.9	17.9	17.9
	Thinking/calculating	21	75.0	75.0	92.9
	$E = mc^2$	2	7.1	7.1	100.0
	Total	28	100.0	100.0	

Table 7.4b Drawing characteristics after the intervention

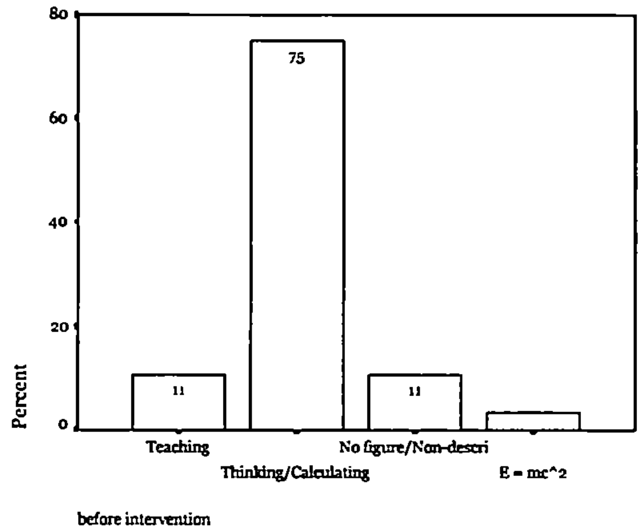

Figure 7.1a Drawing characteristics before the intervention

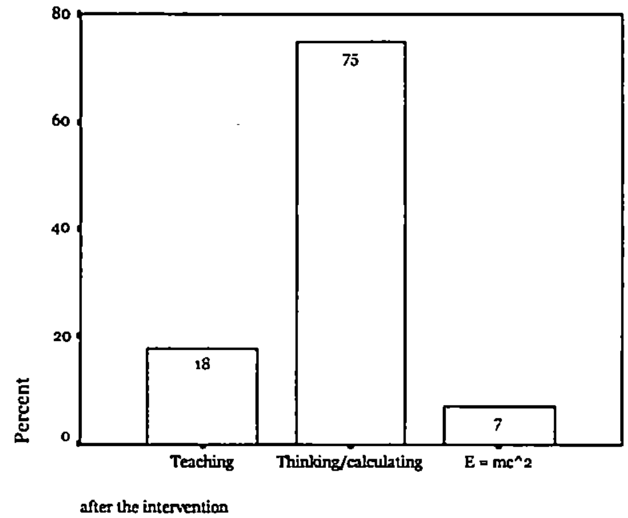

Figure 7.1b Drawing characteristics after the intervention

In addition, seven of the drawings after the intervention contained references to the discrete mathematics unit.

Table 7.5 below, contains the data for the gender of pupils' drawings after the intervention; **Table 7.5a** the data for Wilcoxon signed ranks test:

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	**ambiguous**	4	14.3	14.3	14.3
	female	6	21.4	21.4	35.7
	male	18	64.3	64.3	100.0
	Total	28	100.0	100.0	

Table 7.5 Gender of drawings after the intervention

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly disagree	2	7.1	7.1	7.1
	-1 disagree	3	10.7	10.7	17.9
	0 not sure	6	21.4	21.4	39.3
	1 agree	15	53.6	53.6	92.9
	2 strongly agree	2	7.1	7.1	100.0
	Total	28	100.0	100.0	

Table 7.7a Survey statement 1, *I enjoy my mathematics classes*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly disagree	1	3.6	3.6	3.6
	0 not sure	5	17.9	17.9	21.4
	1 agree	14	50.0	50.0	71.4
	2 strongly agree	8	28.6	28.6	100.0
	Total	28	100.0	100.0	

Table 7.7b Survey statement 1, *I enjoy my mathematics classes*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly disagree	6	21.4	21.4	21.4
	-1 disagree	4	14.3	14.3	35.7
	0 not sure	9	32.1	32.1	67.9
	1 agree	7	25.0	25.0	92.9
	2 strongly agree	2	7.1	7.1	100.0
	Total	28	100.0	100.0	

Table 7.8a Survey statement 2, *A mathematician's work looks like fun to me*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly disagree	5	17.9	17.9	17.9
	-1 disagree	4	14.3	14.3	32.1
	0 not sure	9	32.1	32.1	64.3
	1 agree	8	28.6	28.6	92.9
	2 strongly agree	2	7.1	7.1	100.0
	Total	28	100.0	100.0	

Table 7.8b Survey statement 2, *A mathematician's work looks like fun to me*

	DRAWING GENDER - DRAWING GENDER
Z-value	-.431
P-value (2-tailed)	.666

Table 7.5a Wilcoxon signed ranks test—drawing gender and drawing characteristics before and after Panel

Table 7.6, below, contains percentages of pupils drawing male and female mathematicians by gender. Four drawings (2 by males; 2 by females) contained no figure or were ambiguous.

Males drawing males by %	Males drawing females by %	Females drawing males by %	Females drawing females by %
83.3	5.6	30	50

Table 7.6 Male and female drawings by gender of pupils after Intervention

	DRAWING CHARACTERISTICS - DRAWING CHARACTERISTICS
Z-value	-.832
P-value (2-tailed)	.405

Table 7.6a Wilcoxon signed ranks test for drawing characteristics before and after Panel

The tables below contain data for each statement of the survey for before (designated by ***a***) and after (designated by ***b***) the first (graph theory) intervention:

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly agree	7	25.0	25.0	25.0
	-1 agree	6	21.4	21.4	46.4
	0 not sure	10	35.7	35.7	82.1
	1 disagree	3	10.7	10.7	92.9
	2 strongly disagree	2	7.1	7.1	100.0
	Total	28	100.0	100.0	

Table 7.9a Survey statement 3, *I would never think of becoming a mathematician*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly agree	3	10.7	10.7	10.7
	-1 agree	9	32.1	32.1	42.9
	0 not sure	10	35.7	35.7	78.6
	1 disagree	2	7.1	7.1	85.7
	2 strongly disagree	4	14.3	14.3	100.0
	Total	28	100.0	100.0	

Table 7.9b Survey statement 3, *I would never think of becoming a mathematician*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly disagree	2	7.1	7.1	7.1
	0 not sure	6	21.4	21.4	28.6
	1 agree	16	57.1	57.1	85.7
	2 strongly agree	4	14.3	14.3	100.0
	Total	28	100.0	100.0	

Table 7.10a Survey statement 4, *I usually feel confident in math class*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-1 disagree	2	7.1	7.1	7.1
	0 not sure	6	21.4	21.4	28.6
	1 agree	16	57.1	57.1	85.7
	2 strongly agree	4	14.3	14.3	100.0
	Total	28	100.0	100.0	

Table 7.10b Survey statement 4, *I usually feel confident in math class*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly agree	4	14.3	14.3	14.3
	-1 agree	2	7.1	7.1	21.4
	0 not sure	4	14.3	14.3	35.7
	1 disagree	12	42.9	42.9	78.6
	2 strongly disagree	6	21.4	21.4	100.0
	Total	28	100.0	100.0	

Table 7.11a Survey statement 5, *I plan to stop taking math as soon as I can*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly agree	2	7.1	7.1	7.1
	-1 agree	1	3.6	3.6	10.7
	0 not sure	3	10.7	10.7	21.4
	1 disagree	13	46.4	46.4	67.9
	2 strongly disagree	9	32.1	32.1	100.0
	Total	28	100.0	100.0	

Table 7.11b Survey statement 5, *I plan to stop taking math as soon as I can*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly disagree	8	28.6	28.6	28.6
	-1 disagree	3	10.7	10.7	39.3
	0 not sure	10	35.7	35.7	75.0
	1 agree	3	10.7	10.7	85.7
	2 strongly agree	4	14.3	14.3	100.0
	Total	28	100.0	100.0	

Table 7.12a Survey statement 6, *I have met a mathematician*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly disagree	5	17.9	17.9	17.9
	-1 disagree	1	3.6	3.6	21.4
	0 not sure	11	39.3	39.3	60.7
	1 agree	5	17.9	17.9	78.6
	2 strongly agree	6	21.4	21.4	100.0
	Total	28	100.0	100.0	

Table 7.12b Survey statement 6, *I have met a mathematician*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly agree	5	17.9	17.9	17.9
	-1 agree	6	21.4	21.4	39.3
	0 not sure	8	28.6	28.6	67.9
	1 disagree	5	17.9	17.9	85.7
	2 strongly disagree	4	14.3	14.3	100.0
	Total	28	100.0	100.0	

Table 7.13a Survey statement 7, *Mathematics is not a subject where I get to express my own opinions*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-1 agree	7	25.0	25.0	25.0
	0 not sure	10	35.7	35.7	60.7
	1 disagree	5	17.9	17.9	78.6
	2 strongly disagree	6	21.4	21.4	100.0
	Total	28	100.0	100.0	

Table 7.13b Survey statement 7, *Mathematics is not a subject where I get to express my own opinions*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly disagree	2	7.1	7.1	7.1
	-1 disagree	4	14.3	14.3	21.4
	0 not sure	12	42.9	42.9	64.3
	1 agree	9	32.1	32.1	96.4
	2 strongly agree	1	3.6	3.6	100.0
	Total	28	100.0	100.0	

Table 7.14a Survey statement 8, *I look forward to taking more math in school*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly disagree	1	3.6	3.6	3.6
	-1 disagree	6	21.4	21.4	25.0
	0 not sure	3	10.7	10.7	35.7
	1 agree	12	42.9	42.9	78.6
	2 strongly agree	6	21.4	21.4	100.0
	Total	28	100.0	100.0	

Table 7.14b Survey statement 8, *I look forward to taking more math in School*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly agree	12	42.9	42.9	42.9
	-1 agree	11	39.3	39.3	82.1
	0 not sure	4	14.3	14.3	96.4
	2 strongly disagree	1	3.6	3.6	100.0
	Total	28	100.0	100.0	

Table 7.15a Survey statement 9, *Mathematics is the study of numbers*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly agree	3	10.7	10.7	10.7
	-1 agree	2	7.1	7.1	17.9
	0 not sure	5	17.9	17.9	35.7
	1 disagree	10	35.7	35.7	71.4
	2 strongly disagree	8	28.6	28.6	100.0
	Total	28	100.0	100.0	

Table 7.15b Survey statement 9, *Mathematics is the study of numbers*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly disagree	10	35.7	35.7	35.7
	-1 disagree	7	25.0	25.0	60.7
	0 not sure	10	35.7	35.7	96.4
	2 strongly agree	1	3.6	3.6	100.0
	Total	28	100.0	100.0	

Table 7.16a Survey statement 10, *I see myself as a mathematician*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	-2 strongly disagree	7	25.0	25.0	25.0
	-1 disagree	8	28.6	28.6	53.6
	0 not sure	11	39.3	39.3	92.9
	1 agree	1	3.6	3.6	96.4
	2 strongly agree	1	3.6	3.6	100.0
	Total	28	100.0	100.0	

Table 7.16b Survey statement 10, *I see myself as a mathematician*

Table 7.17 below, contains results of the Wilcoxon 2-related samples test for the survey tool, before and after the first (graph theory) intervention:

QUESTION	Z-VALUE	P-VALUE
1—Enjoy maths classes	-2.757	.006
2—Mathematician's work looks like fun	-.692	.489
3—Never think of becoming a mathematician	-1.327	.185
4—Usually feel confident in math class	-.034	.973
5—Plan to stop taking math as soon as I can	-1.633	.102
6—Have met a mathematician	-1.835	.066
7—Maths not a subject to express my opinions	-1.719	.086
8—Look forward to taking more maths	-2.055	.040
9—Maths is the study of numbers	-4.143	.000
10—See myself as a mathematician	-1.500	.134

Table 7.17 Wilcoxon test for the survey tool (1st intervention)

In **Table 7.18** below are the frequencies for the writing prompt before and after the intervention, in which pupils were asked to list all the reasons for which a mathematician would be hired:

BEFORE DISCRETE UNIT	AFTER DISCRETE UNIT
Taxes & Bills (13)	Taxes & Bills (10)
Teaching (7)	Teaching (7)
Architecture (5)	Architecture (7)
"Don't Know"/Blank (3)	

Table 7.18 Writing prompt responses before and after the first intervention

7.3 THE MATHEMATICIANS PANEL

Immediately at the conclusion of the Mathematicians Panel, pupils were given a half sheet of paper on which to record their reactions. The directions asked for the pupil's name and school and then directed: *Please write some sentences about what you thought about today's Mathematicians Panel including what you learned that most surprised you, what questions you still* *may have, and what you might tell a reporter who interviewed you about today's event. (You may use the front and back of this sheet.)*

179 pupil responses were returned. There were three other comments that couldn't be traced; either the name was unreadable or the pupils had been absent when the first survey was given and so weren't in the database. There were also pupils who either hadn't taken the survey before the panel or after it, and so were removed from the before and after section of the study, although their responses to the panel were retained for that section of the study. In all, 174 pupils completed the two surveys and the reaction to the panel.

7.3.1 THE CODING SYSTEM FOR ANALYSING PUPILS' WRITTEN REACTIONS TO THE PANEL

Analysis of pupils' written reactions to the mathematicians panel yielded five major themes. Pupils' responses were coded into the following five categories:

(1)= Some previously held belief was changed through the panel.

(2)= I was affected by the diversity of the panel.

(3)= I didn't like the panel, and/or I was bored.

(4) =The panel was interesting, and/or I liked it.

(5)= I learned what mathematicians do, and/or more about mathematics.

The responses were coded by using the following rubric: Response (1) will take precedence if there are other comments; response (4) will be coded only if that is the only response with no other detail; response (5) will take precedence over (3).

Table 7.19 and **Figure 7.2** below, show the breakdown in frequency and percent, of the five general pupil responses to the panel.

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	Change Indicated	50	27.9	27.9	27.9
	Diversity Noted	17	9.5	9.5	37.4
	Negative Comment	9	5.0	5.0	42.5
	Interesting	41	22.9	22.9	65.4
	Mathematicians	62	34.6	34.6	100.0
	Total	179	100.0	100.0	

Table 7.19 Pupils' responses to the Mathematicians Panel

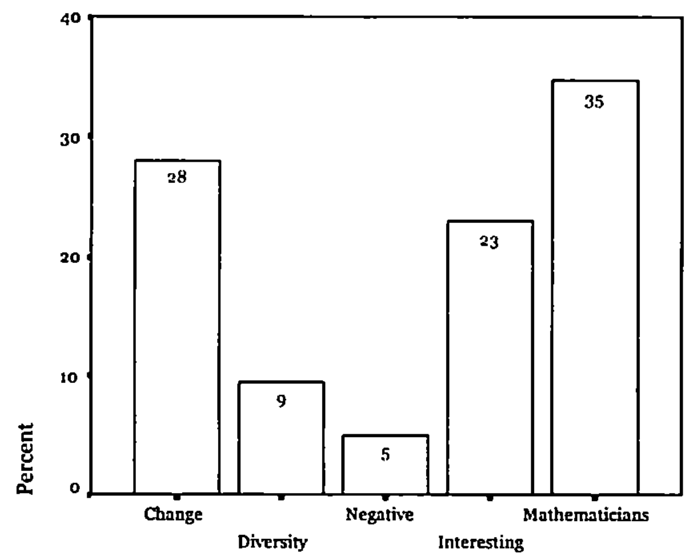

Figure 7.2 Pupils' responses to the Mathematicians Panel, by percent

7.3.2 THE SURVEY TOOL BEFORE AND AFTER THE PANEL

The refined survey tool was tested for reliability. The previous survey tool utilized in the international potion of the study had an alpha (Cronbach) of .6144; the refined tool had an alpha (Cronbach) of .7981 after the post-test, indicating increased internal consistency.

The tables below contain the data by frequency and percent for the three New York City schools responding to the survey tool before and after the Mathematicians Panel on significant statements. A letter ***a*** after the table designation indicates before the Panel intervention; ***b*** indicates after.

								TOTAL
			-2 strongly disagree	-1 disagree	0 not sure	1 agree	2 strongly agree	
School	Hudson	Count	1	8	20	16	5	50
		% within School	2.0%	16.0%	40.0%	32.0%	10.0%	100.0%
	Gramercy	Count	2	1	5	12	1	21
		% within School	9.5%	4.8%	23.8%	57.1%	4.8%	100.0%
	Chelsea	Count	5	13	27	47	11	103
		% within School	4.9%	12.6%	26.2%	45.6%	10.7%	100.0%
Total		Count	8	22	52	75	17	174
		% within School	4.6%	12.6%	29.9%	43.1%	9.8%	100.0%

Table 7.20a Survey statement 1, *I enjoy my mathematics classes*

								TOTAL
			-2 strongly disagree	-1 disagree	0 not sure	1 agree	2 strongly agree	
School	Hudson	Count	9	6	18	14	3	50
		% within School	18.0%	12.0%	36.0%	28.0%	6.0%	100.0%
	Gramercy	Count	1		3	7	10	21
		% within School	4.8%		14.3%	33.3%	47.6%	100.0%
	Chelsea	Count	6	20	23	44	10	103
		% within School	5.8%	19.4%	22.3%	42.7%	9.7%	100.0%
Total		Count	16	26	44	65	23	174
		% within School	9.2%	14.9%	25.3%	37.4%	13.2%	100.0%

Table 7.20b Survey statement 1, *I enjoy my mathematics classes*

								TOTAL
			-2 strongly disagree	-1 disagree	0 not sure	1 agree	2 strongly agree	
School	Hudson	Count	7	6	25	9	3	50
		% within School	14.0%	12.0%	50.0%	18.0%	6.0%	100.0%
	Gramercy	Count	5	1	8	6	1	21
		% within School	23.8%	4.8%	38.1%	28.6%	4.8%	100.0%
	Chelsea	Count	10	30	37	20	6	103
		% within School	9.7%	29.1%	35.9%	19.4%	5.8%	100.0%
Total		Count	22	37	70	35	10	174
		% within School	12.6%	21.3%	40.2%	20.1%	5.7%	100.0%

Table 7.21a Survey statement 2, *A mathematician's work looks like fun*

								TOTAL
			-2 strongly disagree	-1 disagree	0 not sure	1 agree	2 strongly agree	
School	Hudson	Count	9	20	11	7	3	50
		% within School	18.0%	40.0%	22.0%	14.0%	6.0%	100.0%
	Gramercy	Count	2	2	9	6	2	21
		% within School	9.5%	9.5%	42.9%	28.6%	9.5%	100.0%
	Chelsea	Count	10	28	34	29	2	103
		% within School	9.7%	27.2%	33.0%	28.2%	1.9%	100.0%
Total		Count	21	50	54	42	7	174
		% within School	12.1%	28.7%	31.0%	24.1%	4.0%	100.0%

Table 7.21b Survey statement 2, *A mathematician's work looks like fun*

								TOTAL
			-2 strongly disagree	-1 disagree	0 not sure	1 agree	2 strongly agree	
School	Hudson	Count	1	7	14	16	12	50
		% within School	2.0%	14.0%	28.0%	32.0%	24.0%	100.0%
	Gramercy	Count	1		5	11	4	21
		% within School	4.8%		23.8%	52.4%	19.0%	100.0%
	Chelsea	Count	2	10	18	45	28	103
		% within School	1.9%	9.7%	17.5%	43.7%	27.2%	100.0%
Total		Count	4	17	37	72	44	174
		% within School	2.3%	9.8%	21.3%	41.4%	25.3%	100.0%

Table 7.22a Survey statement 4, *I usually feel confident in math class*

								TOTAL
			-2 strongly disagree	-1 disagree	0 not sure	1 agree	2 strongly agree	
School	Hudson	Count		4	14	17	15	50
		% within School		8.0%	28.0%	34.0%	30.0%	100.0%
	Gramercy	Count		2	4	7	8	21
		% within School		9.5%	19.0%	33.3%	38.1%	100.0%
	Chelsea	Count	6	7	21	46	23	103
		% within School	5.8%	6.8%	20.4%	44.7%	22.3%	100.0%
Total		Count	6	13	39	70	46	174
		% within School	3.4%	7.5%	22.4%	40.2%	26.4%	100.0%

Table 7.22b Survey statement 4, *I usually feel confident in math class*

								TOTAL
			-2 strongly disagree	-1 disagree	0 not sure	1 agree	2 strongly agree	
School	Hudson	Count	4	8	14	18	6	50
		% within School	8.0%	16.0%	28.0%	36.0%	12.0%	100.0%
	Gramercy	Count	2	2	11	6		21
		% within School	9.5%	9.5%	52.4%	28.6%		100.0%
	Chelsea	Count	6	18	36	32	11	103
		% within School	5.8%	17.5%	35.0%	31.1%	10.7%	100.0%
Total		Count	12	28	61	56	17	174
		% within School	6.9%	16.1%	35.1%	32.2%	9.8%	100.0%

Table 7.23a Survey statement 8, *I look forward to taking more mathematics*

								TOTAL
			-2 strongly disagree	-1 disagree	0 not sure	1 agree	2 strongly agree	
School	Hudson	Count	4	10	15	16	5	50
		% within School	8.0%	20.0%	30.0%	32.0%	10.0%	100.0%
	Gramercy	Count	2	1	5	6	7	21
		% within School	9.5%	4.8%	23.8%	28.6%	33.3%	100.0%
	Chelsea	Count	6	14	34	35	14	103
		% within School	5.8%	13.6%	33.0%	34.0%	13.6%	100.0%
Total		Count	12	25	54	57	26	174
		% within School	6.9%	14.4%	31.0%	32.8%	14.9%	100.0%

Table 7.23b Survey statement 8, *I look forward to taking more mathematics*

			-2 strongly agree	-1 agree	0 not sure	1 disagree	2 strongly disagree	TOTAL
School	**Hudson**	Count	7	22	13	8		50
		% within School	14.0%	44.0%	26.0%	16.0%		100.0%
	Gramercy	Count	9	8	4			21
		% within School	42.9%	38.1%	19.0%			100.0%
	Chelsea	Count	25	33	18	17	10	103
		% within School	24.3%	32.0%	17.5%	16.5%	9.7%	100.0%
Total		Count	41	63	35	25	10	174
		% within School	23.6%	36.2%	20.1%	14.4%	5.7%	100.0%

Table 7.24a Survey statement 9, *Mathematics is the study of numbers*

			-2 strongly agree	-1 agree	0 not sure	1 disagree	2 strongly disagree	TOTAL
School	**Hudson**	Count	4	19	9	15	3	50
		% within School	8.0%	38.0%	18.0%	30.0%	6.0%	100.0%
	Gramercy	Count	3		5	5	8	21
		% within School	14.3%		23.8%	23.8%	38.1%	100.0%
	Chelsea	Count	18	27	18	22	18	103
		% within School	17.5%	26.2%	17.5%	21.4%	17.5%	100.0%
Total		Count	25	46	32	42	29	174
		% within School	14.4%	26.4%	18.4%	24.1%	16.7%	100.0%

Table 7.24b Survey statement 9, *Mathematics is the study of numbers*

			-2 strongly disagree	-1 disagree	0 not sure	1 agree	2 strongly agree	TOTAL
School	**Hudson**	Count	14	12	20	4		50
		% within School	28.0%	24.0%	40.0%	8.0%		100.0%
	Gramercy	Count	7	5	9			21
		% within School	33.3%	23.8%	42.9%			100.0%
	Chelsea	Count	31	23	35	11	3	103
		% within School	30.1%	22.3%	34.0%	10.7%	2.9%	100.0%
Total		Count	52	40	64	15	3	174
		% within School	29.9%	23.0%	36.8%	8.6%	1.7%	100.0%

Table 7.25a Survey statement 10, *I see myself as a mathematician*

			-2 strongly disagree	-1 disagree	0 not sure	1 agree	2 strongly agree	TOTAL
School	**Hudson**	Count	18	17	12	3		50
		% within School	36.0%	34.0%	24.0%	6.0%		100.0%
	Gramercy	Count	4	4	9	1	3	21
		% within School	19.0%	19.0%	42.9%	4.8%	14.3%	100.0%
	Chelsea	Count	30	22	36	12	3	103
		% within School	29.1%	21.4%	35.0%	11.7%	2.9%	100.0%
Total		Count	52	43	57	16	6	174
		% within School	29.9%	24.7%	32.8%	9.2%	3.4%	100.0%

Table 7.25b Survey statement 10, *I see myself as a mathematician*

Table 7.26 below, contains Z- and P-values results of the Wilcoxon 2-related samples test for the survey tool statements before and after the Mathematicians Panel:

STATEMENT	Z-VALUE	P-VALUE (2-TAILED)
1—I enjoy my mathematics classes	-1.566	.117
2—Mathematician's work looks like fun	-.841	.400
3—Never think of becoming a mathematician	-.462	.644
4—Usually feel confident in math class	-.007	.995
5—Plan to stop taking math as soon as I can	-1.488	.137
6—Have met a mathematician	-8.852	.000
7—Math not a subject to express my opinions	-.409	.682
8—Look forward to taking more math	-1.734	.083
9—Math is the study of numbers	-5.443	.000
10—See myself as a mathematician	-.380	.704

Table 7.26 Wilcoxon test for survey tool statements

The tables below contain Wilcoxon tests for each school on statements with significant differences after the Panel:

STATEMENT	Z-VALUE	P-VALUE (2-TAILED)
1-Enjoy maths class	-3.029	.002
2- Mathematician's work looks like fun	-2.608	.009
6- Met a mathematician	-4.847	.000
9-Maths study of number	-2.491	.013
10-See myself as mathematician	-2.406	.016

Table 7.27 Hudson Middle after the Panel—Wilcoxon test

STATEMENT	Z-VALUE	P-VALUE (2-TAILED)
1-Enjoy maths class	-2.980	.003
6- Met a mathematician	-3.771	.000
8-Look forward to taking more maths	-2.456	.014
9-Maths study of number	-3.452	.001
10-See myself as mathematician	-2.397	.017

Table 7.28 Gramercy Middle after the Panel—Wilcoxon Test

STATEMENT	Z-VALUE	P-VALUE (2-TAILED)
4—Usually feel confident	-2.333	.020
6- Met a mathematician	-6.422	.000
8—Look forward to more maths	-3.213	.001

Table 7.29 Chelsea Middle after the Panel—Wilcoxon test

Table 7.30 below, contains the data of the gender of pupils in this portion of the study (n = 174):

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	female	92	52.9	52.9	52.9
	male	82	47.1	47.1	100.0
	Total	174	100.0	100.0	

Table 7.30 Gender of pupils in Mathematicians Panel intervention

Tables 7.31a and 7.31b below, contain data for the gender of pupils' drawings before and after the Mathematicians Panel. 23 drawings, (14 female, 9 male) were ambiguous, one survey (female) was missing a figure:

Males drawing males by %	Males drawing females by %	Females drawing males by %	Females drawing females by %
85.4	3.7	46.7	37.0

Table 7.31a Male and female drawings by gender of pupils, before Panel

In the table below, is the data for the gender of pupils' drawings after the Mathematicians Panel. 17 drawings (13 female, 4 male) were ambiguous.

Males drawing males by %	Males drawing females by %	Females drawing males by %	Females drawing females by %
86.6	8.5	34.8	51.1

Table 7.31b Male and female drawings by gender of pupils, after Panel

	DRAWING GENDER - DRAWING GENDER	DRAWING CHARACTERISTICS - DRAWING CHARACTERISTICS
Z-value	-.431	-3.817
P-value (2-tailed)	.666	.000

Table 7.32 Wilcoxon signed ranks test for drawing gender and drawing characteristics before and after Panel

Table 7.33a and **Table 7.33b** below, each contain the data for Chi-square tests for the survey tool statements before and then after the Mathematicians Panel:

STATEMENT	VALUE	DF	P-VALUE (2-TAILED)
1-Enjoy math classes	8.942	8	.347
2-Mathematician's work looks like fun	13.157	8	.107
3-Never think of becoming a mathematician	6.067	8	.640
4-Usually feel confident	7.737	8	.460
5-Plan to stop taking maths	9.091	8	.335
6-Have met a mathematician	22.261	8	.004
7-Can't express my opinions	4.317	8	.827
8-Look forward to taking more maths	6.409	8	.602
9-Maths is the study of numbers	18.194	8	.020
10-See myself as a mathematician	5.171	8	.739

Table 7.33a Chi square tests for survey statements before Panel

STATEMENT	VALUE	DF	P-VALUE (2-TAILED)
1-Enjoy math classes	38.162	8	.000
2-Mathematician's work looks like fun	15.590	8	.049
3-Never think of becoming a mathematician	7.298	8	.505
4-Usually feel confident	8.537	8	.383
5-Plan to stop taking maths	3.048	8	.931
6-Have met a mathematician	32.235	8	.000
7-Can't express my opinions	7.308	8	.504
8-Look forward to taking more maths	9.330	8	.315
9-Maths is the study of numbers	20.854	8	.008
10-See myself as a mathematician	16.575	8	.035

Table 7.33b Chi square tests for survey statements after Panel

The tables below contain drawing characteristics data for before and after the Mathematicians Panel:

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	**1 Wizard**	9	5.2	5.2	5.2
	2 Weirdo	15	8.6	8.6	13.8
	3 Teaching	32	18.4	18.4	32.2
	4 Thinking/Calculating	99	56.9	56.9	89.1
	5 No Figure/Non-descript	7	4.0	4.0	93.1
	6 E=mc^2	12	6.9	6.9	100.0
	Total	174	100.0	100.0	

Table 7.34a Drawing characteristics before the Panel

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	**1 Wizard**	2	1.1	1.1	1.1
	2 Weirdo	4	2.3	2.3	3.4
	3 Teaching	44	25.3	25.3	28.7
	4 Thinking/Calculating	90	51.7	51.7	80.5
	5 No figure/Non-descript	2	1.1	1.1	81.6
	6 E=mc^2	5	2.9	2.9	84.5
	7 Reference to/Drawing of Panel	27	15.5	15.5	100.0
	Total	174	100.0	100.0	

Table 7.34b Drawing characteristics after the Panel

The tables below contain the frequencies of responses to the writing Prompt asking pupils to list reasons for hiring a mathematician:

BEFORE DISCRETE UNIT (*n*= 28)	AFTER DISCRETE UNIT (*n* = 28)	*AFTER PANEL* (*n* = 21)
Taxes & Bills (13)	Taxes & Bills (10)	Teaching (9)
Teaching (7)	Teaching (8)	Taxes & Bills (6)
Architecture (5)	Architecture (7)	Math Calculations (5)
Math Calculations (4)	Math Calculations (7)	Games Design (4)
		Architecture (1)
"Don't Know"/Blank (3)		

Table 7.35 Gramercy Middle School only: Top reasons given for why one would hire a mathematician

BEFORE PANEL	AFTER PANEL
Teaching (50)	Teaching (77)
Taxes & Bills (65)	Computer Applications (54)
Banking (28)	Taxes & Bills (43)
Architecture (24)	Scientific Uses (30)
	Architecture (24)
"Don't know"/left blank (14)	"Don't know"/left blank (4)

Table 7.36 Top four reasons why a mathematician would be hired: Before Mathematicians Panel and after (n = 174)

Chapter 8: ANALYSIS—THE INTERVENTIONS DATA

8.1 INTRODUCTION TO ANALYSIS OF THE INTERVENTIONS DATA

This chapter contains the analysis of the data for the two interventions: the graph theory/discrete mathematics topics unit, and the data pertaining to the Mathematicians Panel.

8.2 THE FIRST INTERVENTION—GRAPH THEORY AND DISCRETE MATHEMATICS

Although the sample size for this portion of the intervention was small, (n = 28), I believe this intervention was an important step leading to the Mathematicians Panel and useful data arose from it.

This section begins with analysis of before and after drawings of three pupils in the Gramercy Middle School class and then moves to analysis of the data from the Likert-type survey tool.

The drawings of two of these pupils, are also referred to later in this chapter, along with the third drawing each of them created after the Panel. The other pupil whose drawings are in this section, Kate, was absent from school the day of the Panel, and so produced no third drawing for comparison.

Most of the drawings from the Gramercy Middle class were rather unremarkable, not especially stereographic, and neither particularly interesting as drawings nor particularly interesting as before-and-after drawings. This may be because these pupils were self-selected, having opted to be in a school for pupils who are interested in the sciences, and were perhaps already more disposed to see mathematics in a more positive fashion than their peers.

Pupils were also beginning a second year of direct contact with scientists at a nearby medical school; an experience denied most pupils of their age group, and one that may have already had an effect on some stereotypical views of scientists and mathematicians. And although almost all the pupils in the class did well in mathematics, nevertheless there were still a number of pupils who had difficulties with the subject, who found it confusing at times, and did not look forward to studying it further.

There was change within the class during the early part of the term—two pupils moved and left the school. There were also personality difficulties between two of the boys, so one was transferred to a different class. And there were pupils absent from the retaking of the survey or for the Panel. Consequently, while 28 pupils were initially in the project, the number of completed surveys after the Mathematicians Panel had dropped to 21.

Upon the completion of the graph theory/discrete mathematics unit, and after pupils had retaken the survey, I interviewed eight pupil volunteers, nearly a third of the class, upon subsequent visits to the school.

8.2.1 PUPILS DRAWINGS—KATE

Kate has a mature quality and is quiet and thoughtful in class. A few pupils in the class worked well in groups but didn't speak very often and at this point in the term Kate seemed to be one of them. Yet she volunteered to be interviewed, and the interview took place early in November 1999.

Her first survey drawing appears in **Figure 8.1a**. One of the interesting things about it is that it looks as if she cannot draw and so is using stick figures, which nevertheless can be quite revealing and useful in gauging a pupil's images of mathematics and mathematicians.

Yet if one looks ahead to her second drawing, after the unit, in **Figure 8.1b**, she has chosen to draw a full rather than stick figure, showing she can actually draw. She explained this choice saying it was because she didn't exactly know what a mathematician was at the time of her first drawing:

> Kate: This one I think, umm, I didn't, I didn't really think—I didn't exactly know what it was, so I just drew stick figures to do it faster. And this one, I think I sort of had a better idea—it's just hard to explain about what they do. I have a better idea about it so I didn't want to make it so simple-looking.

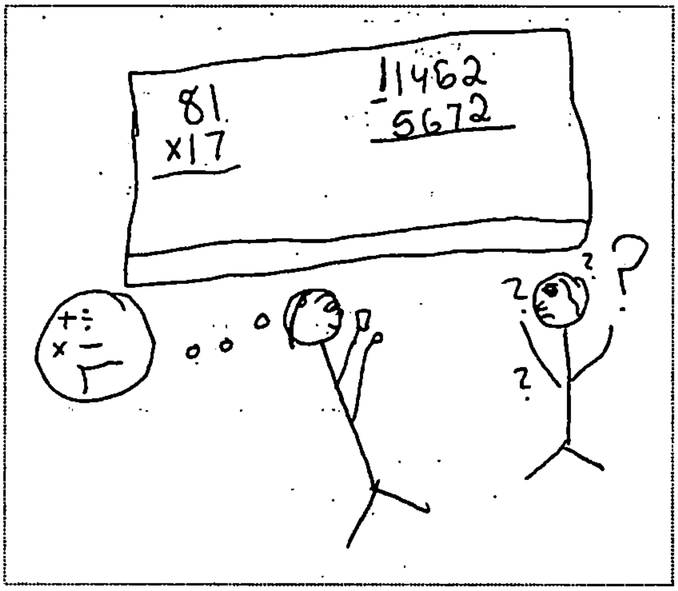

Figure 8.1a Kate's drawing (before discrete unit)

In her first drawing Kate has two figures in front of a blackboard, with one thinking in mathematical symbols, and the other with question marks over its head. She has written, *The mathematician is figuring out a problem. The other person doesn't understand.*

In the interview, Kate talked about the drawing in **Figure 8.1a.** (The preliminary surveys were on white paper, while the post-discrete surveys were on yellow paper, and are sometimes referred to by the colours of the paper.)

Kate: Well, I wasn't really sure of what a mathematician does and I'm still not sure, but I think they like figure out different sorts—they think of different sorts of like math problems and like they do different things with not only like numbers but with like nature and things like that.

SP: So do you remember what you were thinking when you drew the white page—the first one?

Kate: I thought it was sort of like a math teacher...

SP: And which one was the math teacher?

Kate: This one.

SP: Okay—on the left.

Kate: And this person was supposed to be like a puzzled person like not understanding what they're doing.

Kate at times may see herself as this puzzled person, for she wrote in response to the prompt, "To me, mathematics is:" *sometimes easy, but if you don't understand something it could get confusing.*

I asked Kate about the relationship in her mind between mathematicians and teachers, which led to her explaining how she knows whether or not a teacher is a mathematician.

SP: So when you think of mathematicians do you mostly think of teachers?

Kate: Umm—yes and no. Because a lot of, like some math teachers are mathematicians and some aren't. Like I think Miss Nierescu is a mathematician.

SP: And how would you know that—that some are and some aren't?

Kate: Umm—well sometimes they tell you (laughs). But, and like sometimes if they, if they really like math and like they really like their job, and like you could just sometimes just tell. In all the things they do, and like sometimes just the approach they take to math.

SP: And did Miss Nierescu tell you that she's a mathematician?

Kate: Umm, she never really specified it, she just always, like she always says she's part of these little groups like with mathematicians and things and like I think she is and even if she isn't I think she'd make a great mathematician.

SP: So when someone is sort of clearly not a mathematician, what hints do you get about that—when they've been your teacher?

Kate: Well, like if they can't really explain very well—because mathematicians, they study things and they can explain all their answers really well. And like if a teacher—even if she's saying something and the children don't really understand the problem, then she shouldn't—you can just sort of tell. (Laughs.)

Kate seems to be referring to Marina Nierescu's descriptions of her experiences at Rutgers University. Perhaps because they hear about them so rarely, references to mathematicians appear to make a impression on pupils.

I believe that Kate is also saying that she hopes to have confidence in a teacher and judges them by their clarity—although she is struggling to put it into words, and so she ends, *you can just sort of tell.*

Later in the interview, Kate explains what she likes about the way Marina Nierescu teaches. And it appears that at some point in school she has seen a teacher become angry if a child hasn't comprehended something:

> ... I like how Miss Nicolescu teaches. If she always—if she like teaches a new subject she's always—she always asks if this makes sense and she won't get mad if you don't understand. And she explains everything really well so if it comes to answering a problem I can answer.

In Kate's second drawing, in **Figure 8.1b,** after the discrete unit, the lone character is standing in front of a *Stage 1* fractal and seems to be female, although the gender of the figures in the first drawing was not at all clear. She was asked about that in the interview.

SP: Is this male or female?

Kate: It wasn't really meant to be anything. I think mathematicians can be both.

SP: Well this on the yellow paper, it looks like a female.

Kate: Yeah.

SP: Although maybe it's a guy with long hair...

Kate: Noo. (Laughs.)

SP: So that was meant to be female.

Kate: Yeah.

SP: And is this a mathematician, on the yellow page?

Kate: Yeah.

SP: And can you just tell me a little more about what I'm looking at here?

Kate: Well, this person is studying fractals, and she's studying, um, the how, if...she's—I can't explain it really well...she's just studying how the different kinds of stages you get—she's trying to make it as small as possible.

Four of the ten girls in this class started out with female figures in their drawings, and Kate's was one of the two drawings where there was a change to a clear female character in the second drawing. The other girls in the class retained the gender of the first drawing in their second.

I asked Kate whether she had liked fractals best of the topics covered in the unit, to which she replied,

Kate: Yeah...It wasn't like the best. I like everything—I think it was very interesting.

SP: And the graph theory?

Kate: Umm—which one was...

SP: With the vertices and the edges...

Kate: Yeah. That one—I liked that. I liked the Hamiltonian circuit and the paths--I think that was cool. And I also liked the graphs without picking your pencil—that was cool, too.

SP: Did it give you any further insight into what a mathematician does?

Kate: Well that's why I sort of drew that, with the fractal. And I think that they try—mathematicians try to interpret all the different kinds of things that—uh different kinds of like—they don't—it's really hard to explain—I can't really—but in my mind I know what it is but it's really hard to explain. Like I know they don't only study numbers and shapes and you know, lines. I know they use—like we watched a movie that this man he was, umm, not a mathematician but he was some other word.

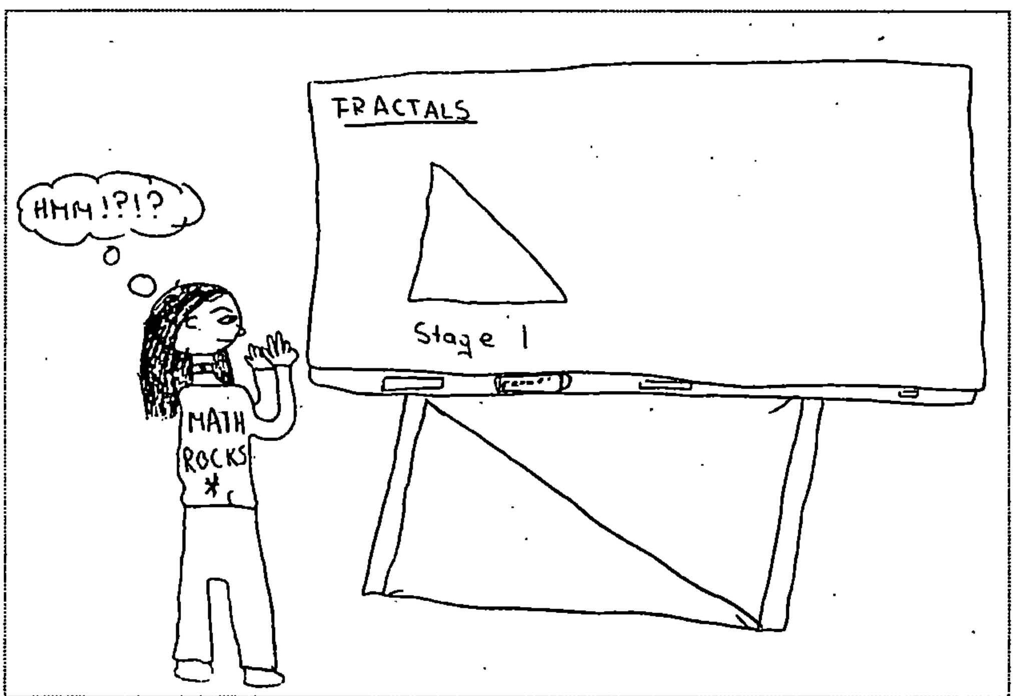

Figure 8.1b Kate's drawing (post-discrete unit)

There is a lack of clarity about fractals, and some prompting is needed for Kate to remember the graph theory, although once she does, she supplies specific details that shows she does remember. But she is not clear as to whether she really has more insight into the work of a mathematician.

However, Kate does say, *Like I know they don't only study numbers and shapes and you know, lines.* She was asked further about this:

SP: Now this idea of yours that math is more than numbers, is that something that you've come to recently or you knew last year and the year before?

Kate: No, recently—in this year.

SP: And can you recall what made that change?

Kate: Umm, because when Miss Nierescu used to say that when because math isn't only the study of numbers—it's a lot more than that. And I think that when you actually think about it—it is.

SP: Did she say that?

Kate: Well, she said it's umm, more—it's not only the study of numbers, and when we watched the movie—that also kind of changed my mind because this man is studying nature from math. And that was cool.

It is interesting that Kate mentions the film she saw in which, *this man he was, umm, not a mathematician but he was some other word.* That mathematicians can be known by other titles (National Science Foundation, 1998) appears further to pronounce their invisibility to the general public.

When questioned about her responses to the writing prompts on the surveys, Kate seems to indicate that she holds the belief that all mathematics problems can be solved, especially if those problems appear in textbooks.

Although she has now seen some problems in graph theory for which there are many solutions, and seen the Königsberg Bridge problem for which there is no solution at all, Kate still holds the belief that all mathematics problems have an answer, provided one finds the right authority. She says, *because if a problem is impossible, which it probably isn't...*

Kate then refers to a hard problem, but when pressed to state a specific hard problem, talks about the possibility of not understanding something in a high school textbook and one's parents not being able to help. For Kate, high school is fully two years away at this point, yet she appears to anticipate mathematics difficulties awaiting her.

The *Dial-A-Teacher* to which she refers, is a service provided by the teachers union, to which pupils can phone for assistance in any subject.

SP: In terms of listing reasons to hire a mathematician you said here: 'to figure out a complicated math problem, to find new things in math'; and here, you said 'if you have a problem which you think a mathematician should solve; to figure out more about mathematics using the random problems in the world.' Do you see those as similar or different?

Kate: I think they're sort of different. I think the first one is sort of, umm, sort of like there's—'cause this one means like a complicated math problem and this one is a problem which you think a mathematician should solve. That's pretty much the same thing because if a problem is impossible, which it probably isn't, umm, some people might call a mathematician, or like see a mathematician to see what they get, even though that's probably not an exact reason (laughs) what you would hire a mathematician for.

SP: So when you said 'to figure out a complicated math problem', did you have anything specific in mind?

Kate: Umm, well like probably not. Like I actually sort of did, but it's probably kind of dumb. When like 'cause sometimes when there's problems in like the high school math books like children ask their parents and the parents don't even know what the answer is. And like usually a lot of people call the "Dial A Teacher" and maybe there's mathematicians that work there.

Kate also shows her dependence on finding right answers when she comments on what makes Marina Nierescu a good teacher, saying:

> *And she explains everything really well so if it comes to answering a problem I can answer.*

Kate has indicated that she does not see herself as a mathematician, however this may be due in part to her interpretation of the question:

Kate: I like math a lot and it's just not something that I would like to do. I want to be—I want to study science, not math... I think if I ever had a chance, if I couldn't study science, then I don't think it would be so horrible. I mean because in math sometimes you do things with science, and a lot of things in science are also part of math.

SP: So, tell me if I'm hearing correctly, you're thinking that this question means in the future—what about now?

Kate: Now, as a mathematician? Umm, no. I still want to continue learning science.

SP: But do you see yourself as being a mathematician?

Kate: Umm, I can't imagine myself, but I don't think it's impossible.

It seems to be hard for Kate to simply imagine herself as being a mathematician, even as she loves science so much. *I can't imagine myself* she says. Perhaps she adds *...but I don't think it's impossible,* because she knows she is being interviewed about mathematics, but seeing mathematics as attractive enough to imagine herself among mathematicians, is something that she does not feel at this point.

So for Kate, the graph theory and discrete mathematics topics have somewhat widened her view of mathematics and provided material which she has enjoyed. The largest change appears to be in her view that mathematics is not just about numbers, but for her, that may also be due in part to Marina Nierescu's having stated as much in class.

In another interview, Kate's classmate Olivia was also questioned about a change from seeing mathematics as the study of numbers to disagreeing with that statement. Olivia had jumped right into a fairly detailed explanation for the change (see **Appendix—B** for the complete transcript):

SP: Originally you agreed that math is the study of numbers. Now, you disagree. You want to say something—

Olivia: Because, look at the discrete math. Most kids around my block, and me myself, think that math was just about numbers. And now that we've learned the discrete math you know it's shapes and sizes and things like that. You look at the vertex and that had nothing to do with numbers. It had to do with like ways of the path and stuff. We didn't do numbers all that week.

Jason, another classmate, answered similarly when he was interviewed:

Jason: Oh, yeah, in the beginning of the year we didn't do numbers at all—we did discrete math and it was like logic and stuff. So a lot of math is numbers and plenty enough of it isn't.

Kate indicated that the discrete unit did not give her further insight into the work of a mathematician, and I found that to be the case with other pupils I interviewed. Robert, another pupil was asked about this, too:

SP: So in the beginning of the term you weren't sure if you liked your class then you felt you agreed, you did like it.
R: Uh-huh.
SP: You want to say anything more about that?
R: Umm, yeah. Well, we started to have pretty much some fun with what we were doing.
SP: And what were you doing at that point that you had fun?
R: Umm, like finding like one thing had to be one colour but the other thing had to be another colour; no two colours could touch.
SP: And what did you like about doing that?
R: Oh, it was so like a brain teaser and it was sort of like a design thing.
SP: Did it give you any more insight into what a mathematician does?
R: Umm, no, not really.

And this was the case with Jason, as well:

SP: Earlier in the year with Miss Nierescu you looked at discrete math and you were doing graphs and colouring and shortest paths—what did you think about that?
Jason: It was cool. That was fun. Umm, I liked the map colouring the best.
SP: Any reason?
Jason: Umm, I don't really know—there was a lot of like estimating going on, and I like that. And it was fun like sorting things; like sorting different colours for different objects and states—and when we were doing the country.
SP: Did it give you any insight into what a mathematician does for a living?
Jason: No.

So it appears that pupils had not yet made a relationship with what they had been learning and the work of a mathematician.

8.2.2 PUPILS' DRAWINGS—BEN

Ben is a small quiet boy in the Gramercy Middle class who does well in mathematics. Like many pupils he seemed to be adjusting to the increased discussion and interaction which Marina Nierescu was implementing. When called on, he always had something to say, but I didn't see him speaking in class as much as he might have at the beginning of the year. Nor was he one of the pupils who volunteered to be interviewed.

Ben's first drawing, before the discrete unit is in **Figure 8.2**. In it, a character that Ben identifies as a mathematician, *is try*[ing] *to do his homework.*

Since the character is doing homework, one might think he is meant to be a pupil, or a depiction of Ben himself, but Ben has indicated on the Likert-type portion of the survey, that he is not sure he sees himself as a mathematician.

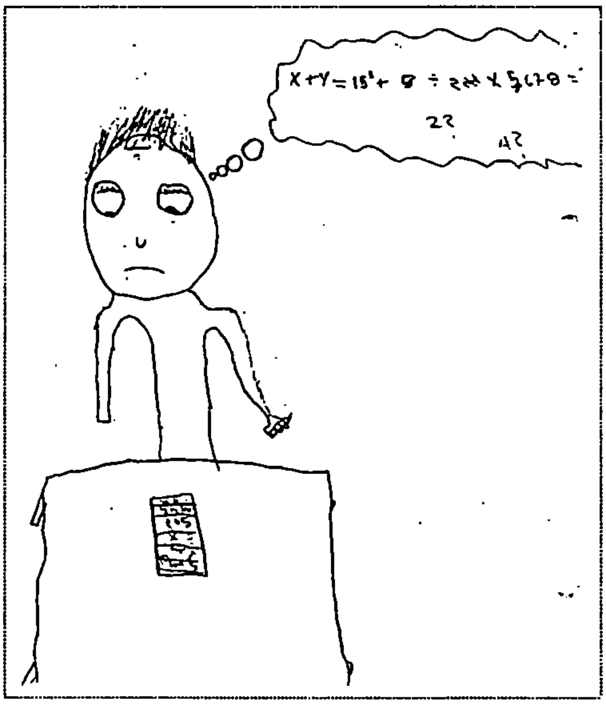

Figure 8.2a Ben's drawing (before Discrete Unit)

Ben's second drawing, in **Figure 8.2b,** after the discrete unit, also involves homework. There is a sheet of paper labelled as such in the lower right. The character's hair in the second drawing is tamer, but this may be Ben's drawing style, rather than anything else—it is hard to tell.

The character in this second drawing is thinking,

> *Why can you, by adding 5 lines make this a hexagon, why not a rectangle? I need to think of a rule.*

The question posed seems to be an imaginary one, but it is similar to problems I have seen Marina Nierescu give pupils on a board she has set up with challenges to enhance logical thinking and problem solving skills.

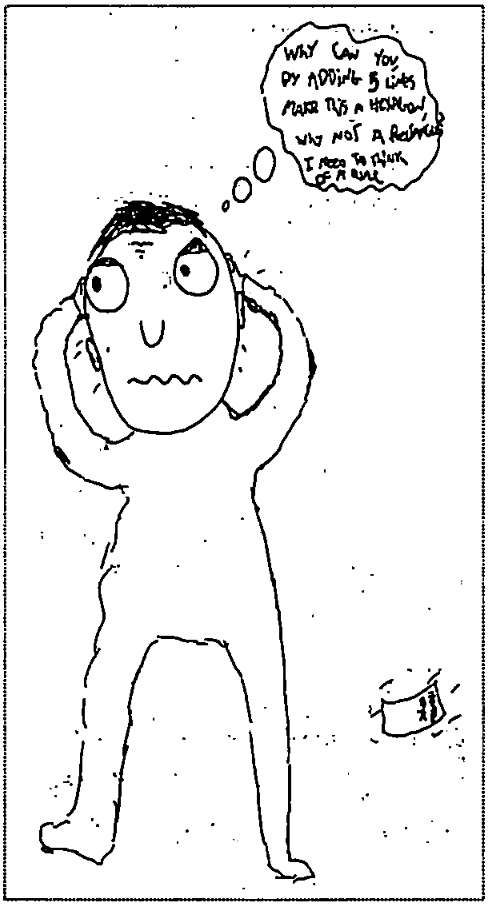

Figure 8.2b Ben's drawing (post-Discrete Unit)

The quality of the thought in Ben's new drawing is deeper and truer to what mathematical thinking is than in most of the other drawings in this sample. It is one of the best representations of the mathematical process. Ben appears to feel confident enough to be able to leave the problem without a solution. In contrast, in most drawings in which a pupil has posed a problem the pupil has included many question marks, or presented the answer. Instead Ben reflects, *I need to think of a rule.*

In this second drawing, Ben has identified the mathematician in the drawing as himself:

> *The person is me, trying to come up with a rule or explanation for my homework.*

And now, to the statement, "I see myself as a mathematician," he has checked *agree.*

8.2.3 PUPILS' DRAWINGS—MATT

Matt was one of the pupils in Marina Nierescu's class who was cheerful and dependable, participating actively in discussions, and having a studious air about him. Although he did not volunteer to be interviewed, he was one of the pupils from Gramercy Middle who asked a question during the Mathematicians Panel.

Matt's drawing in **Figure 8.3** was carefully worked out and fully realized when it appears that he changed his mind and crossed it out. Yet he didn't obliterate it so it couldn't be seen, as many pupils tend to do when they are unhappy with their work.

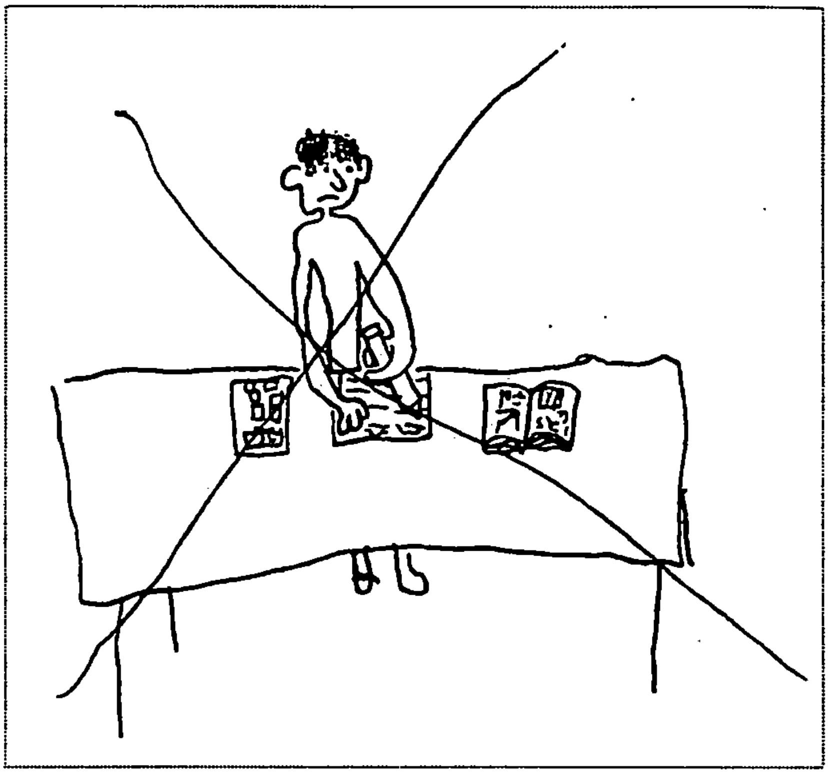

Figure 8.3 Matt's discarded drawing (before discrete unit)

The drawing Matt chose instead to represent his idea of a mathematician at work before the discrete unit (**Figure 8.3a**) , was much smaller than the first, and was completed high up on the page near the instructions, where there was less room for a drawing. Yet it appears to have been created with the same care. The main difference appears to be that the figure in the second drawing is working at a computer. Matt wrote, *He is using a computer to do someone's taxes.*

There were three reasons Matt listed for why a mathematician would be hired: *for astronomy, an accountant, doing taxes.* It appears that he has misinterpreted the writing prompt, "To me, mathematics is:" for he wrote, *Someone who works with numbers.* And he agreed with the statement that mathematics is the study of numbers.

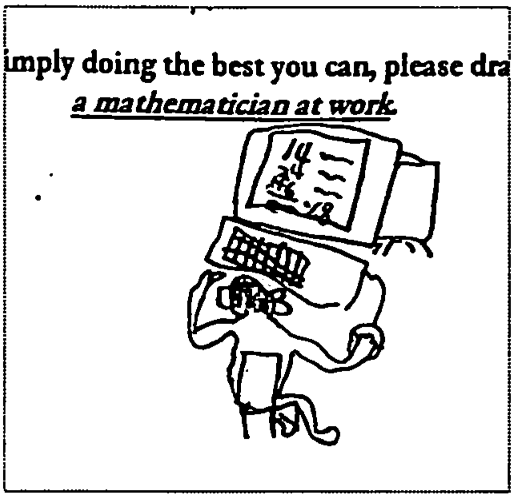

Figure 8.3a Matt's preferred drawing (before discrete unit)

In his drawing after the discrete unit, in **Figure 8.3b**, we see a carefully detailed picture of a figure again at a computer, about which Matt wrote: *The mathematician is doing someone's taxes.*

This image of a mathematician as a kind of *super-accountant* (Stewart, 1987) appears to be deeply ingrained. At the same time, there are changes in Matt's thinking.

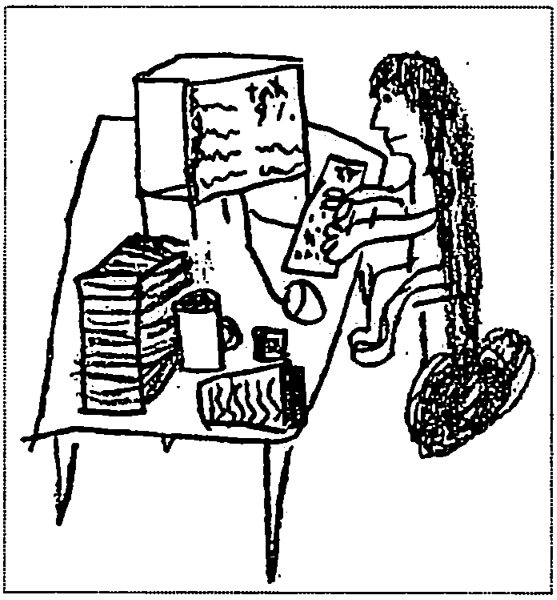

Figure 8.3b Matt's drawing (after Discrete Unit)

Now Matt disagrees that mathematics is the study of numbers, and he writes that mathematics to him is, *problem solving*.

Both Ben and Matt are discussed further in Section **8.3.3** where their drawings made after the Mathematicians Panel appear.

8.2.4 PUPILS' SURVEYS

Analysis of the before and after data of the Likert-portion of the survey tool for the first intervention indicates that there was a significant change on three statements: *(1) I enjoy my mathematics classes*; *(8) I look forward to taking more mathematics*; and *(9) Mathematics is the study of numbers*. This can be seen in the data from the Wilcoxon test in **Table 7.17**.

For the purposes of comparison, the two choices *agree* and *strongly agree* and the two choices *disagree* and *strongly disagree* are each looked at as one choice in the discussion that follows.

For survey statement 1, after the intervention, 78.6% of pupils agreed that they enjoyed their mathematics classes, compared with 70.7%; and where 17.8% of pupils had disagreed with the statement, after the intervention, disagreement was reduced to 3.6%, an indication that pupils had responded positively to the discrete unit.

On statement 8, the percentage of pupils agreeing that they looked forward to taking more mathematics increased from 35.7% before the intervention, to 64.3% after. Interestingly, the percentage of pupils disagreeing, slightly increased, from 21.4% to 25%, yet the percentage of pupils unsure of this statement, dropped from 42.9% to 10.7%. Pupils after the intervention appear to believe that mathematics contains something more attractive or interesting than they had previously thought.

Before the intervention, on survey statement 9, 82.2% of pupils agreed that mathematics is the study of numbers; after the intervention, only 17.8% of pupils agreed. Now 64.3% disagreed, while there was a slight increase in pupils who were unsure.

For a pupil to see mathematics as primarily the study of numbers indicates a more static and rigid view of the subject. With the discrete mathematics intervention this changed for the majority of the pupils, perhaps because, as Olivia had put it: *We didn't do numbers all that week.* Leading them to conclude, as Jason noted:

So a lot of math is numbers and plenty enough of it isn't.

It is interesting that as pupils recounted their experience with the unit, what appeared to stand out for many of them was that they had studied "shapes". Since many pupils were used to doing one thing with closed shapes—finding area and perimeter, there came to be some confusion with graphs and area and perimeter. One explanation for this may be found in Langer (1997) in which studies showed that what pupils learn by rote they usually cannot use imaginatively and with flexibility in fresh contexts.

One thing did not change: after the unit, pupils kept the opinion that the largest reason for hiring a mathematician is to do taxes and bills. Even so, I believe there are hopeful signs of the effects of graph theory and discrete mathematics on some misconceptions of mathematics held by pupils.

8.3 THE MATHEMATICIANS PANEL

This section begins with analysis of the pupils' questions for the second intervention, the Mathematicians Panel, followed by their reaction to the Panel, and then analysis of the survey tool (before and after the Panel).

8.3.1 PUPILS' QUESTIONS FOR THE MATHEMATICIANS

After meeting with each of the seven classes that were to attend the Mathematicians Panel, pupils had generated 144 questions (see **Appendix—D**). The questions posed in common among the classes afford further insight into the knowledge pupils had about mathematicians prior to the Panel.

The questions which appeared with the greatest frequency among the seven classes, were:

What does a mathematician do? This question was the only one asked in every single class and I believe it is a clear indication of the invisibility of mathematicians for pupils. For pupils to have reached their eighth year of schooling and yet have little idea of the work a mathematician does indicates that mathematicians are, as Emmer (1990) has noted, absent from the curriculum. In six classes two questions related to the first were also asked, *Who hires mathematicians?* And, *Where do mathematicians work?*

Pupils were curious, I believe, about what encouraged members of the panel to enter the field. In six classes pupils asked: *Who was your idol* or *who or what inspired you?*

Pupils were also curious as to whether becoming a mathematician was a lifelong dream. In five classes the question was asked, *Did you always want to be a mathematician?* I think both the question about inspiration and the question about being a mathematician as a life-long goal, may be related to the question of how single-minded mathematicians are, since in drawings and open-ended writing, pupils portrayed mathematicians as working day and night to the exclusion of all other pursuits.

I think these questions also reflect some disbelief on the part of many pupils that anyone would want to be a mathematician. This attitude was also evinced in pupils' comments.

A question that indicated pupils' interest in the mathematics used by mathematicians was asked in three classes, *What is your favorite area in mathematics?* With, *What kind of math do you do?* asked in a fourth.

Pupils also asked in four classes what a mathematician's actual day looked like, because, I think, they have no way of visualizing this.

Finally, pupils in four classes wanted to know how much money mathematicians make, again, I believe, because they have no basis for knowing this, and because the prevailing image, as Howson and Kahane (1990, p.4) have noted, includes "the absence of the association of 'success' in some publicly acceptable form—acclaim, wealth, knowledge, medals, etc... with mathematics or mathematicians..." Most pupils appeared to assume that mathematicians make very little money.

8.3.2 PUPILS' REACTIONS TO THE PANEL

179 pupil responses to the Panel were collected, indicating that nearly every pupil present turned their forms in (see **Table 7.3.1**, reprinted below.) Three of the 179 responses were anonymous, but they contained serious comments. One of these stated, *This gave a human face to mathematicians.*

		FREQUENCY	PERCENT	VALID PERCENT	CUMULATIVE PERCENT
Valid	Change Indicated	50	27.9	27.9	27.9
	Diversity Noted	17	9.5	9.5	37.4
	Negative Comment	9	5.0	5.0	42.5
	Interesting	41	22.9	22.9	65.4
	Mathematicians	62	34.6	34.6	100.0
	Total	179	100.0	100.0	

Table 7.19 Pupils' written reactions to the Mathematicians Panel

The smallest grouping (3) contained nine negative comments. These ranged from expressions of boredom, although with praise for the Panel, *They seemed very nice*, to the expression that the Panel was too long and the pupil hated mathematics anyway, and so saw no point in the time spent.

Seventeen pupils commented on the diversity of the Panel (2) and mentioned the fact that the mathematicians were from many countries.

Forty-one said the panel was interesting (4) and that they learned something from it but were no more specific than that. Had they been more specific, they would probably have been coded in a different category.

Fifty pupils, nearly a third (28%) had a comment that indicated a change in some previously held belief (1). Pupils showed via their writing that some image of a mathematician had changed through the experience of meeting the members of the Panel.

The largest group of responses, sixty-two, or more than a third, indicated that the pupils had learned more about what mathematicians really do and that they had learned more about mathematics itself.

The Gramercy Middle class had unfortunately come late to the Panel and missed particular references to graph theory and discrete mathematics in Dana Frank's presentation, and they also sat in the back of the auditorium where the acoustics were poor. They also seemed to be almost distracted by Marina Nierescu's inclusion in the Panel, mentioning it to the exclusion of all else in 8 comments out of 25.

At the same time, seeing their teacher on the Panel recognised as a mathematician may also have been a distraction to some pupils because they had not thought of her as one before, and thus had to reconsider a previous image of her. And in the end, the Gramercy Middle School Class may well have been suffering from test fatigue, since this was the third time in eight months that they had seen the survey tool.

I have organised representative comments by themes, in order to analyse them further.

Changing stereotypical ideas about mathematicians:

The original meaning of the word *stereotype,* from a French process of printing from fixed plates has come to mean (Costello, 2000) a "conventional, formulaic, and oversimplified conception or image." There is the implication here that mathematicians, if viewed in a *formulaic* and *oversimplified* way, would be seen as undifferentiated, all the same, and as having lives vastly different from pupils'.

In their many comments pupils indicated that this is, indeed, how they saw mathematicians, until the Panel presented an image of mathematicians that enabled pupils to see them otherwise—as individuals and as persons not so different from the pupils themselves. This further indicates that such an oversimplification comes directly from a lack of knowledge.

Since they were asked to write what surprised them about the Panel, it is interesting that many pupils chose to write about changing their beliefs. There was no cue to write about this. Pupils' comments below indicate critical thinking leading to a rejection of a number of their previously held images. Pupils' schools are indicated by the initials after their first names.

> *I learned that mathematicians are normal people with normal lives, and they don't all do the same thing.* (Eboni—HM)

> *I enjoyed this presentation because I thought it was interesting the way that all of them had different skills, opinions and reactions. It made me keep an open mind to all the differences...I am glad that I came.* (Sarah—CM)

...I realized that mathematicians really aren't that nerdy, they're regular people with regular lives. I see that lots of people who are mathematicians like art. I think that I might become a mathematician because I love art and I joined the math team. This might lead me to becoming one, even thought I want to be a lawyer. I liked how the panel compared games to mathematics. I also liked hearing about the careers the mathematicians were interested in when they were our ages. (Jimei Louise—CM)

I really enjoyed learning about the life of a mathematician. I always thought mathematicians were nerds who sat in front of the computer all day. Now I realized how fun it can be. (Danny—CM)

Today I learned that mathematicians are just normal people. They have normal lives and I also learned what they do. (Genevieve—CM)

I was surprised because they were more interesting and less stereotypical than I thought they would be. I didn't realize that there were so many fields of math. I also didn't realize how many games involved math. (Jonathan—CM)

I was a little surprised to hear that mathematicians can make a lot of money because I thought that mathematicians didn't make a lot of money. I also found it interesting that mathematicians could do a lot of different jobs. (Stephen—HM)

I thought math was boring. They proved it wasn't. They have interesting lives and jobs. (Sam—HM)

I think that I've learned that math can be fun, when I used to think that it sucked. I agree with my friend Sam as well, that mathematicians have interesting lives. Peace! (Conor—HM)

I thought it was great that some of the mathematicians had so many different skills and were so multi-talented. It made me rethink my conviction that being a mathematician is not for me. I liked the fact that they were very artistic and had talents <u>*outside*</u> *mathematics. It showed me that they are a lot like me in some ways.* (Naomi—CM)

But I was surprised that they didn't wear pointy hats. (Kaitlyn—CM)

What surprised me the most was that these people are normal. They aren't weird. (Samuel—CM)

I used to think that mathematicians were very serious and their life was all about math, but they do have a life outside of math. (Colleen—CM)

I think that some of the mathematicians were really cool. All of the mathematicians came off as real people, which made it easier to ask questions (and less boring). (Sara—HM)

I learned that the stereotypes about mathematicians aren't true! (Cristina—HM)

...you see a side of a mathematician's mind that you never knew was there... (Tammy—CM)

Mathematicians have lives that are the same as anyone else's. (Marcelo—CM)

I thought it was very interesting!! Now, I have a whole new outlook on mathematicians. (Annie—HM)

...I would tell a reporter that anyone can become a mathematician. (Jon—GM)

...I learned about what a mathematician does which I was confused about before. (Margot—CM)

I found this panel really interesting. I noticed that there was one mathematician that said that there is a stereotype that mathematicians are nerds. I think that it is true and I think that this panel helped to break that stereotype. I really enjoyed this panel. (Harry—CM)

Changing stereotypical ideas about mathematics: Making the invisible visible:

Along with a change in stereotypical images, pupils indicated that some of the invisibility of mathematicians and mathematics had begun to change. Yvonne, a pupil at Hudson Middle School points to the invisibility of mathematicians:

I learned that there are a lot of mathematicians but not everyone will know that they are one. (Yvonne—HM)

This, too, stems from a lack of knowledge and is related to the earlier comment by Kate at Gramercy Middle, who mentions a mathematician in a film she saw who *...was...not a mathematician but he was some other word.*

The comment by another Hudson Middle pupil, Brittney, below, shows evidence for mathematics' invisibility, when she says, *...I learned that* *everything has math, you may know it or not.* She was surprised to learn that what mathematicians do is very different from what she had thought:

> *I learned that mathematicians are normal people with normal lives. What surprised me is that what they do is totally different than what I expected. It was actually kind of interesting, and I learned that everything has math, you may know it or not. But lots of things revolve around math.* (Brittney—HM)

It appears that another large part of pupils' stereotypical views of mathematics, is the image of it as narrow and monolithic. It is interesting that in the United States, mathematics is shortened to *math*, making it appear even more singular and further obscuring the fact that it is a very wide field with many different areas. As Emmer (1990, p. 91) points out:

> ...very few people realize that there is not just one subject *mathematics* but a whole series of different specializations that flow into the wide river of mathematics.

This appears to have become clearer to pupils as a result of the Panel, for pupils' comments indicate that some of the "wide river" of mathematics has become more visible to them.

Another aspect of the narrowness of pupils' images of mathematics is the belief that it cannot be associated with anything pupils might consider to be fun. I think it is why they were so surprised to find through Dr. Anjolo's presentation, that the strategies of board games are a part of mathematics.

And there is also an image pupils have of mathematics which defines it as a school subject, rather than an occupation (Howson & Kahane, 1990). This is yet another area where the Panel encouraged pupils to reexamine their beliefs, as the comments by David and Ruth, below, indicate.

> *What I enjoyed most...is listening to the different experiences with math from a variety of different people. I always thought mathematics only existed in places like classrooms but now I know it's everywhere.* (David—CM)

I learned that there is a lot more to math than what we have learned so far from Elementary school to now. I always thought that math was just some regular calculations but there is more to it. Things that I never thought deal with math actually deal with math like playing games, Mancala, and engineering. (Ruth—HM)

I think this was a productive activity in learning the different types of mathematics and how different jobs use them. I learned about the new fields of math that are not known about by most of the population. This was an inspirational and informative panel. (Gabriel—CM)

...It helps us understand more of a mathematician's work. How they study what they do. What actually amazes me is that mathematicians study games, diseases, and graphs. (May—GM)

...I don't think I'll become one, but it sounds interesting. I didn't know so many things were related to math either. (Allison—HM)

I was surprised how much mathematics works in virology. (Matt—GM)

I thought the people on the panel were very open, and gave us a very wide view of the world of math. (Allie—CM)

I thought that it was very funny and very educational. The mathematicians didn't fake things like saying that math is great. They told the truth. It was also a great chance to find out about how math is applied in the real world. (Rachel—CM)

...I was also surprised that so many things use and/or require mathematics. (James—GM)

One of the most interesting things I learned is that math isn't just numbers it's many things. Biology, computers, and even games like Mancala. (Remi—HM)

I never considered math was such a big part of so many jobs. (Rorie—CM)

Today I learned a lot of new things. For example, I didn't know about all the different kinds of math and that so many cultures were into math. I learned that a large part of math is solving <u>*many*</u> *different types of problems. Puzzles, equations and many other things.* (Jamie—HM)

The thing that most surprised me was the concept of ethno-mathematics. It's really interesting that Mancala has to do with mathematics. I always thought it was just luck. Also I was interested that a biologist was a mathematician. I would tell a reporter that it *was fun and I learned that mathematics is more important than I thought.* (Ben—GM)

I learned that mathematicians are not only involved with numbers, [they're] *involved with shapes and computers. I learned that the mathematician with the wizards hat is just a stereotype.* (Matthew—HM)

I also thought it was interesting how each mathematician was interested in a different kind of math. (Theresa—HM)

I thought it was cool to see mathematicians from all over the world with different ethnic backgrounds. It expanded our mathematical thinking and understanding. (Wendy—GM)

I thought that today's mathematicians panel had a good amount of diversity, both in sex, race and work. We learned a lot about what they do as mathematicians. What I learned that was most surprising was that games such as Mancala include math. Another thing I found surprising was that mathematicians work in the field of contagious diseases. (Shannon—CM)

Changing about the subject and being inspired by the Panel:

I think math seems more fun. I don't think I would do it but it is more interesting to me after the meeting. (Jeremiah—HM)

I really liked the mathematics panel and what they said. It in a way inspired me more to pursue my goal which is to be a physicist. They said it's always important to have a mentor but I have not chosen one yet. (Adam—CM)

...I learned a lot about mathematics around the world and I think this inspired me to work harder with my math. One thing that surprised me was how much the mathematicians work with computers. It showed how much the technology has affected math. (Nathan—CM)

I really found the mathematicians very interesting. And was also a little shocked when the female mathematicians said that they were not discouraged because of their gender. I might think about going into the field of mathematics because of this day. Thanks!! (Eve—CM)

...Because of this experience I am willing to persevere and be more interested in mathematics. (Celeste—CM)

I think that this experience will help me to enjoy math more. I now think of it less as a class and more as fun. (Matthew—CM)

...I was able to learn about what it is like to be a mathematician. Some of the mathematicians are very inspiring, and I might consider becoming a mathematician when I'm older. (Thomas—CM)

It appears that for many pupils, a gulf they originally felt between themselves, mathematics and mathematicians was bridged.

8.3.3 GRAMERCY MIDDLE—DRAWINGS AFTER THE PANEL

In the figures below are the third drawings created by Gramercy Middle pupils, Ben and Matt after the Mathematicians Panel. Kate was absent the day of the Panel and so there is no third drawing by her for a comparison.

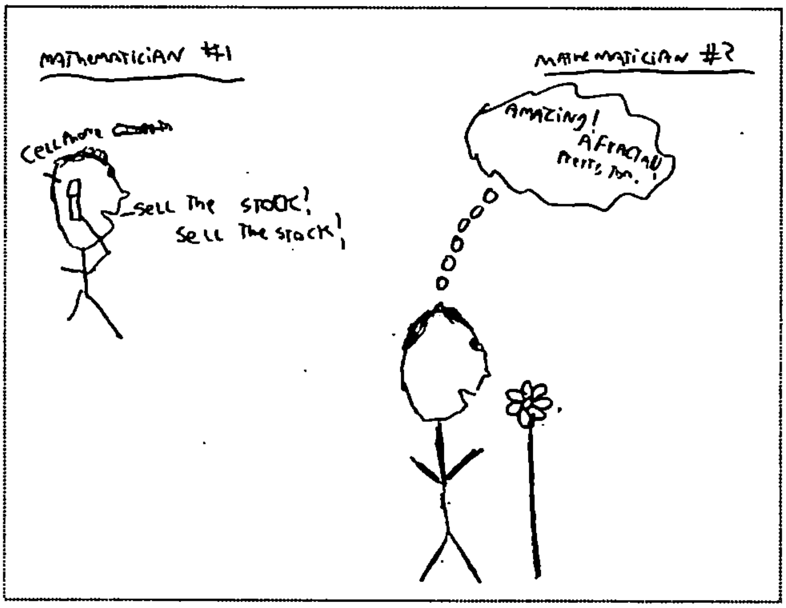

Figure 8.2c Ben's drawing (post-Panel)

It is hard to be sure that Ben meant his first two drawings (**Figures 8.2a** and **8.2b**) to contain a character that looked strange, especially since the second drawing was supposed to depict him. It is important to leave some room to account for a pupil's particular drawing style. Yet Ben's first two drawings did contain a somewhat odd-looking wall-eyed character whilst in his third drawing, in **Figure 8.2c**, after the Panel, the character looks less strange and happier at the same time.

Where Ben's first two drawing's each contained a figure concerned about homework, now Ben has drawn two figures that he has labelled *Mathematician #1* and *Mathematician #2*. Mathematician #1 is yelling into his phone *Sell the stock! Sell the stock!* While Mathematician #2 is standing before what looks like a flower, saying: *Amazing! A fractal! Pretty, too.*

Ben wrote: *Mathematician # 1 is playing the stock market and wants to sell the stock, #2 is studying fractals in nature (and smelling the flowers.)* He had listed the following reasons for why a mathematician might be hired: *Stock consultant, analyse data in an experiment, building a house (taking measurements), engineers, business, teacher, designer.*

Two of these, *analyse data in an experiment* and *engineer* appear to have come out of the presentations of Alun Llandaff and Wendy Mills.

Matt's third drawing appears in **Figure 8.3c**, and although the character in it looks to be the same long-hired character who appeared in **Figure 8.3b**, the figure is not engaged in doing someone's taxes, as in Matt's previous drawings. The description reads: *The mathematician is studying information about the Challenger disaster.* It is interesting to note that Matt was probably born the year of the Challenger disaster, which occurred in January 1986.

During the questions portion of the Mathematicians Panel Matt had asked: *What is like the newest field in mathematics?* To which Alun Llandaff had replied:

> Well, I'd say, mathematical biology, of course, but I'm a bit biased. But there are a whole load of new fields; computational areas have opened up with the advent of large scale computers in the last few years, and I think throughout biology particularly, you're going to see a real explosion of mathematical ideas opened up both by—opportunities offered by computers, but also genetic data becoming available through the genome project.

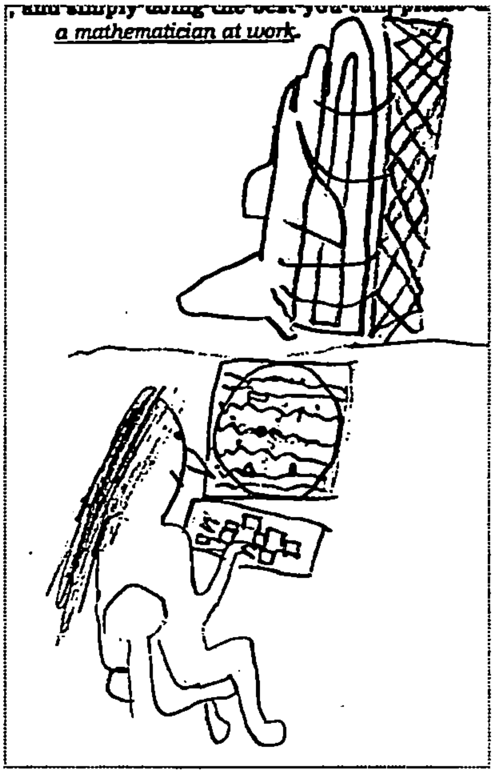

Figure 8.3c Matt's drawing (post-Panel)

About the Mathematicians Panel, Matt wrote *I was surprised how much mathematics works in virology.* And yet his drawing does not appear to reference that. It is quite possible that pupils have things they are most comfortable depicting and that Matt just didn't quite know how to put what he heard from Alun Llandaff into a drawing.

Nevertheless unlike his two previous drawings, Matt has his mathematician using a computer in analysing data, which was a large part of the work Alun Llandaff had described himself as doing.

Matt listed the following reasons for why a mathematician might be hired: *Make a computer game, launch a satellite, file taxes, make calculations, do surveys.* He has not completely given up the idea that a mathematician does taxes, but it has moved down his list, possibly because he heard no reinforcement of that idea during the Panel.

Matt shows a further change in his image of mathematics in what he wrote to finish the survey writing prompt, "To me, mathematics is." Matt wrote: *Everywhere,* showing, I believe, the effect of the answer to his question about the newest areas of mathematics, and a resulting deeper and wider understanding of the uses of mathematics.

8.3.4 SURVEY DRAWINGS BEFORE AND AFTER THE PANEL

Figures 8.4a and **8.4b** and **Figures 8.5a** and **8.5b** show two pupils' drawings before and after the Mathematicians Panel. In each of their first drawings they have created Einstein-influenced characters, but after the Panel both pupils' drawings and their written comments have changed.

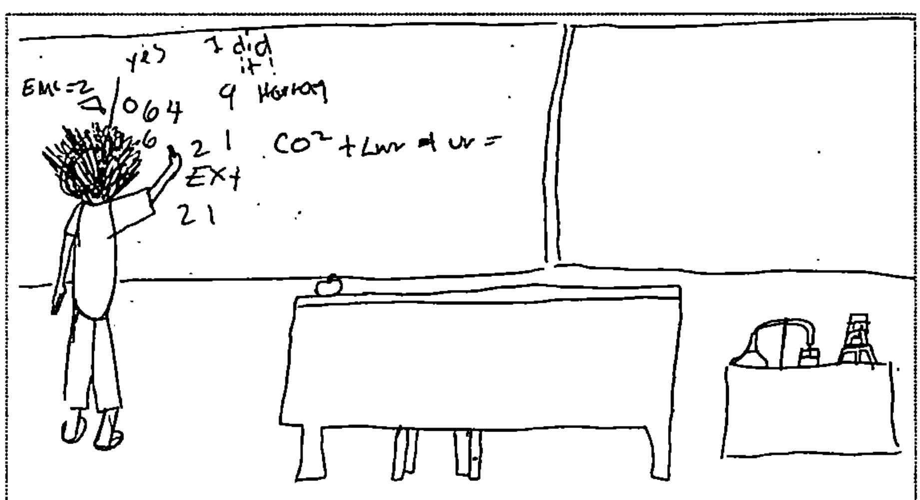

Figure 8.4a Remi's drawing (pre-Panel)

In **Figure 8.4a**, a character with wild Einsteinian hair, who has written his own version of Einstein's famous equation, has also inscribed on the chalkboard, *Yes I did it! Hoorray!* On a nearby table are a Bunsen burner and other laboratory equipment. Here again, there is an intermingling of images of scientists with those of mathematicians. Einstein appears to bring the two together for many pupils.

In his description of the drawing, Remi, a boy at Hudson Middle, has written:

> *This is a picture of a man or a woman making an equation on a chalkboard. He/she is making it for a science experiment that the class is doing.*

In response to the first writing prompt asking why a mathematician might be hired, Remi expressed:

> *I'm not really sure, but maybe to figure something out that has to do w/the army if the general or anyone else has no time? Also for architecture and teaching. ?? I really have no idea?? Maybe for scientific reasons.*

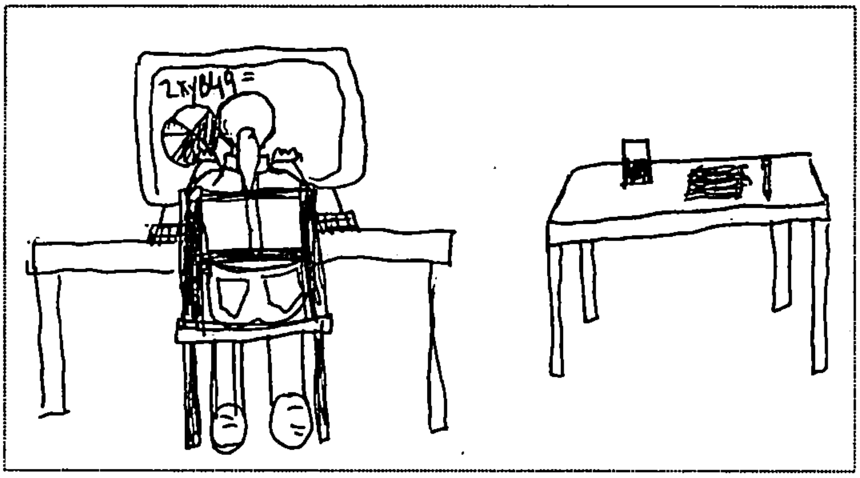

Figure 8.4b Remi's drawing (post-Panel)

Now, in his post-Panel drawing, in **Figure 8.4b**, Remi appears to have drawn Alun Llandaff down to his ponytail and jeans. He wrote:

> My *drawing is of a mathematician working in a lab at a computer figuring out an equation. (Biology.)*

As to why a mathematician would be hired, Remi explained:

> *I think that a mathematician could be hired to do just about anything related to science or math. It depends what the person studies.*

So it appears that Remi, too, has a wider, clearer, and more specific image of mathematicians and mathematics after the Panel.

At the Mathematicians Panel Remi had asked: *Do any of you have a mentor?* Both the question and its answer were important in enabling pupils to see that the work of a mathematician involves a process that includes the encouragement and assistance of others.

The prevailing image of a mathematician alone and cut off from other people (see also **Figure 8.10a**) was challenged as Juliet Grey spoke of her mentor; a female mathematician who guided and encouraged her career.

About the Mathematicians Panel, Remi later expressed:

> *I learned a lot today. One of the most interesting things I learned is that math isn't just numbers, it's many things. Biology, computers, and even games like Mancala. I think that being a mathematician could be fun and exciting if you dedicated your time to it.*

So here, without it being pointed out to him, as may have been done for the Gramercy Middle class by Marina Nierescu, Remi has been able to reach the conclusion that mathematics *isn't just numbers, it's many things.*

In **Figure 8.5a**, there is an even wilder-haired figure drawn by Cara, a pupil at Hudson Middle School, who wrote:

> *This is Albert Einstein. When I think of the word mathematician I automatically think of Albert Einstein. He is frantically writing on the board, that is what I think if him doing.*

Writing frantically on a board is a part of the one-dimensional image many pupils have of mathematicians—that they are constantly involved in endless calculations.

But in her drawing after the panel, in **Figure 8.5b,** Cara has not only abandoned the Einstein and *frantic writing* themes, but she has changed the gender of the mathematician, as well.

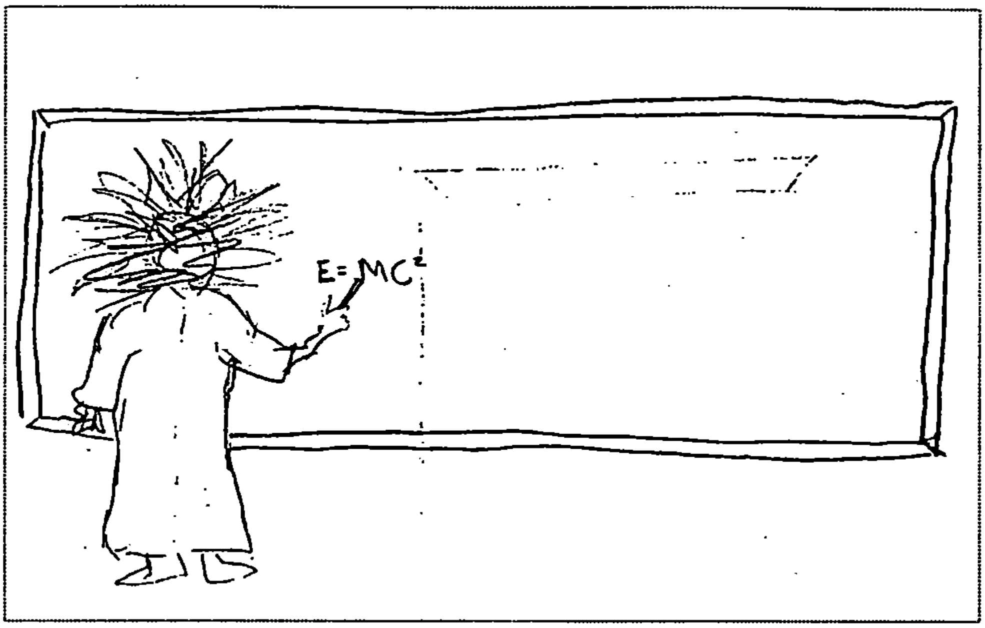

Figure 8.5a Cara's drawing (pre-Panel)

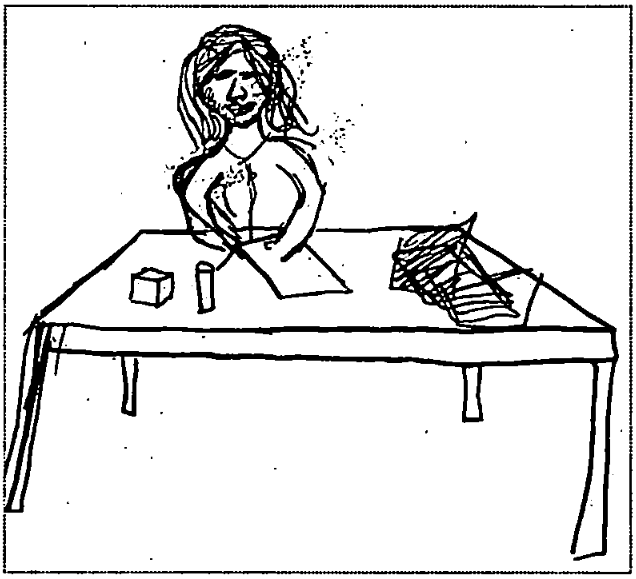

Figure 8.5b Cara's drawing (post-Panel)

Cara wrote about her second drawing:

This is a normal every day person. This picture also happens to be a woman. I don't mean to imply that only women can be mathematicians, however I mean to imply that women are mathematicians just as much as men are. I also noticed at the panel that many people had lots of papers with evidence on it. Also they had 3-D shapes with them.

The papers and 3-D shapes are represented in **Figure 8.5b** on the desk next to the attractive looking woman. Cara wrote after the Panel,

> *I was surprised that they were SO INTERESTED in their field, and things like music. I don't mean that it is bad to be interested, but they were very devoted. They find ways to work 24 hours. This got me to understand what a mathematician really does and how they go about doing problems. It interested me that they took a lot of time to find a problem.*

And when Cara wrote, *I also noticed at the panel,* after writing about female mathematicians, she seems to indicate that the gender of the Panel members was something that she first noticed.

In each of the three pairs of drawings which follows there appears a pre-Panel drawing of a wizard, who does not reappear in the post-Panel drawing.

In the first pair, in **Figures 8.6a** and **8.6b**, drawn by Mickey, a pupil at Chelsea Middle School, a sleeping wizard in the first drawing becomes a clean-cut young man pointing to an equation on a chalkboard, in the second.

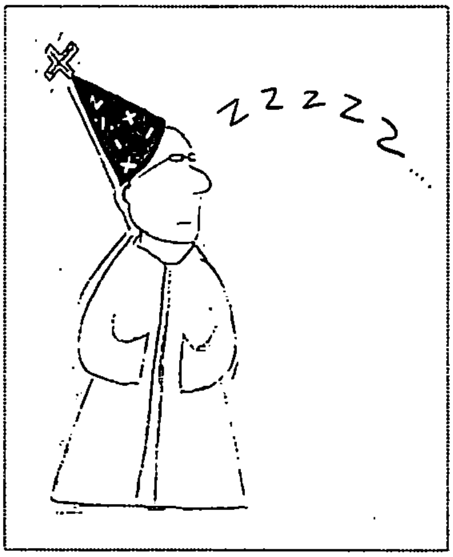

Figure 8.6a Mickey's drawing (pre-Panel)

Of the first, Mickey wrote, *The mathematician is sleeping.* As to why a mathematician might be hired, he wrote: *to learn math; to gain knowledge.* On his later drawing Mickey has repeated this list but added two more reasons: as *a tutor*, and *to see the world from a different perspective.* About his second drawing, he wrote, *The drawing is of a mathematician teaching.*

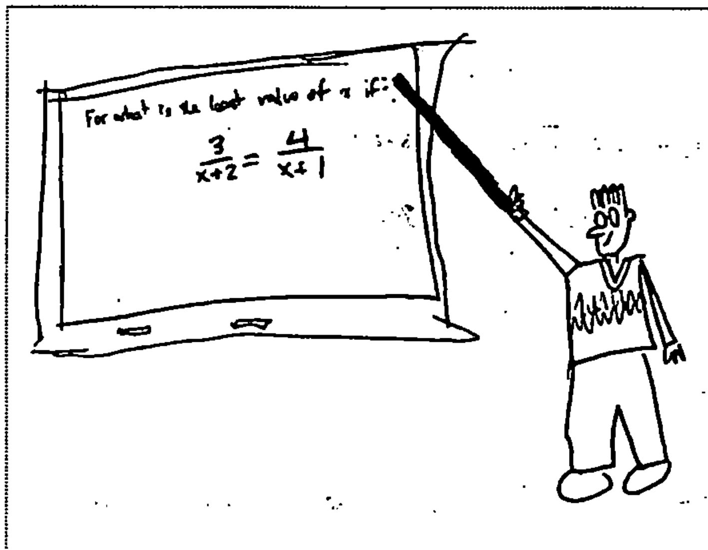

Figure 8.6B Mickey's drawing (post-Panel)

Commenting on the Mathematicians Panel, Mickey wrote:

> *Today's Math Panel was very fun and educational. It made me think about what math is really about.*

Maria is the pupil at Chelsea Middle School who drew **Figures 8.7a** and **8.7b**. Her first drawing depicted a bearded wizard with a pointy hat. In her description she wrote: My *drawing is of a mathemagician, he is making math equations, he has them on his robe.* As to why a mathematician might be hired, she has written, *To teach math.*

There is a sort of aura or halo around the number the wizard is pointing to, implying some supernatural power.

Figure 8.7a Maria's drawing (pre-Panel)

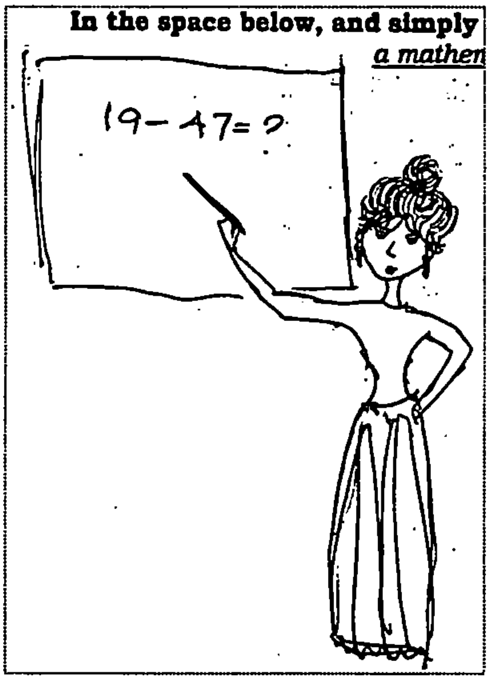

Figure 8.7b Maria's drawing (post-Panel)

In her drawing after the Panel, Maria now has drawn an attractive woman, whom she described as ...*an average math teacher, because they are mathematicians.* She has written about why a mathematician might be hired: *A computer company might need the help of a mathematician.*

In explaining in survey statement 6, who the mathematician was that she had met, she wrote, *Ms. Sachs, my math teacher.* Yet her drawing did not look at all like her teacher, Connie Sachs, who is blonde and wears glasses.

Looking at the two drawings together, the basic faces and stances of the figures are nearly identical. Both are using pointers and gesturing toward numbers. Both even involve negative numbers. Yet in the second drawing, Maria has made the choice to depict a person who could be real, about whom she uses the word *average*, rather than some otherworldly figure with supposed magical powers.

In the third pair of drawings that includes a wizard, a Chelsea Middle School pupil, Milagros, has carefully labelled her wizard, but has called him a mathematician, not a *mathemagician* as Maria did. Nevertheless, the idea for this wizard character, appears to have come from *The Phantom Tollbooth* (Juster, 1961) which has been reprinted many times. Milagros wrote, *The mathematician is solving problems and writing the answer.*

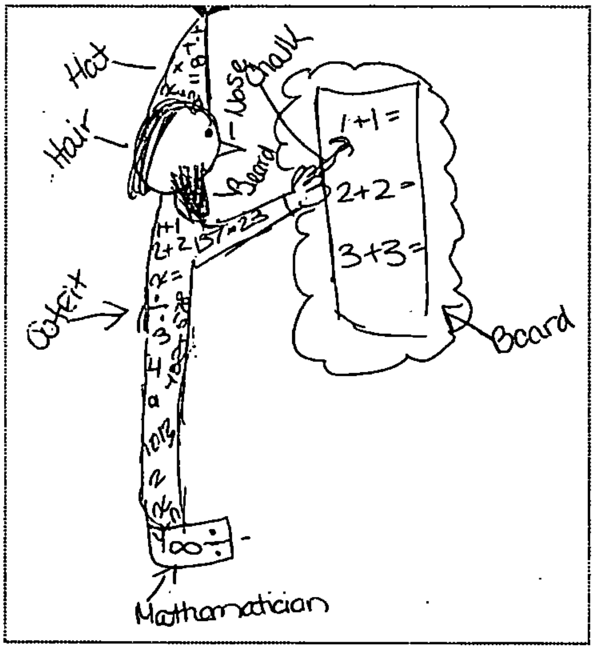

Figure 8.8a Milagros' drawing (pre-Panel)

Milagros' second drawing, after the Panel, in **Figure 8.8b**, is a more abstract one. There is a smiling stick figure, above which whirls a spiral with mathematical words going from *arithmetic* on the outside, to *math* on the inside, and including along the way, *calculus, algebra, trigonometry, Pythagoras, multiplication, division,* and *integers*; indicators of the "wide river" that is mathematics, even as the choice of words is limited by Milagros' own experiences with mathematics.

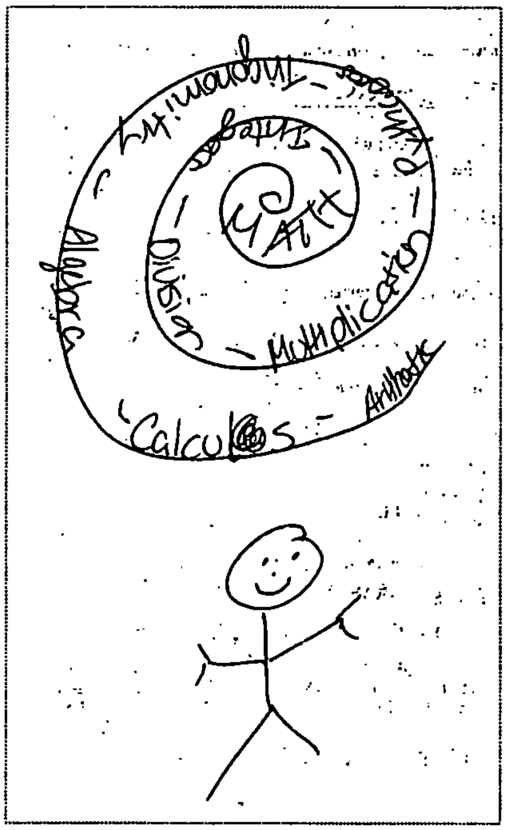

Figure 8.8b Milagros' drawing (post-Panel)

The drawings below created by Annie, a pupil at Hudson Middle, in **Figures 8.9a** and **8.9b**, show a similarity to Maria's drawings above. Both the pre- and post-Panel characters have almost identical faces and stances.

But as with Maria's post-Panel drawing, Annie has changed the gender of the mathematician to female in her second drawing. And the small details in the drawing are also, I believe, quite significant.

In Annie's first drawing, the male mathematician is balding with glasses. He is sitting at a desk with just a paper in front of him on which there are three fairly simple division problems. Annie's explanation of the drawing is: *The mathematician is hard at work figuring out difficult number problems at his desk.* The word *difficult* has been inserted.

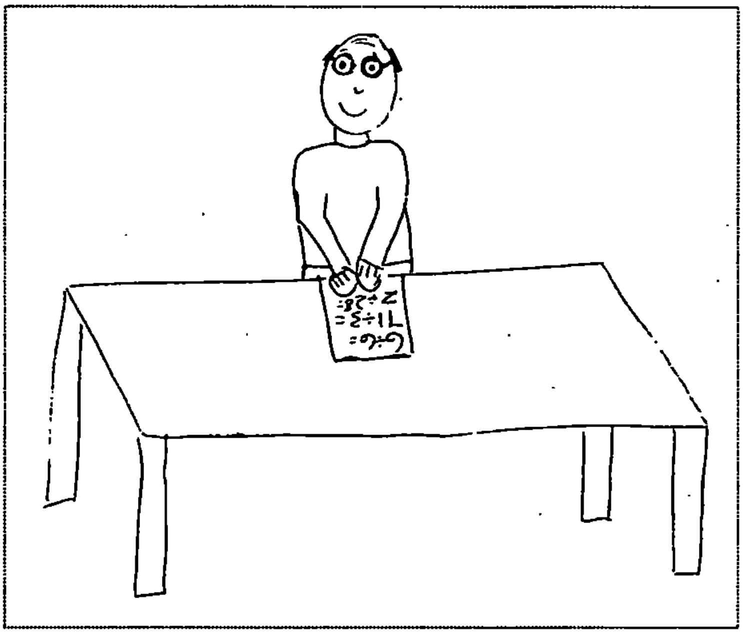

Figure 8.9a Annie's drawing (pre-Panel)

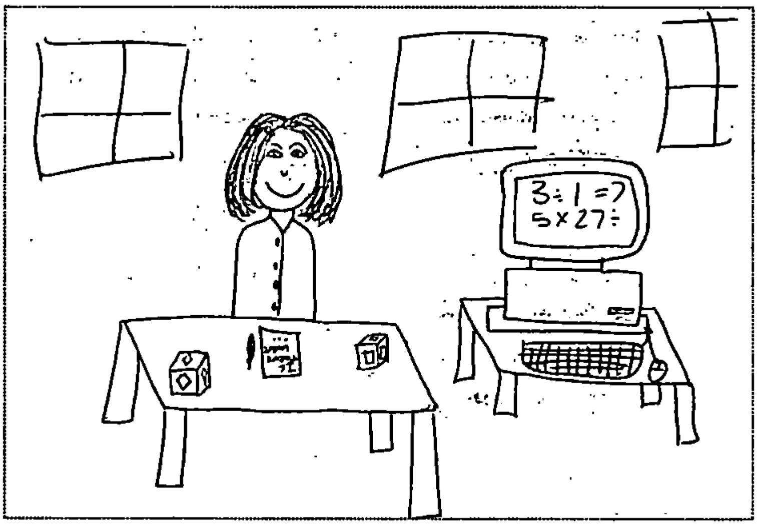

Figure 8.9b Annie's drawing (post-Panel)

Although he is looking out, he looks as if he is also engaged in writing the equations. The walls are bare, so there is no frame of reference or context for the character, which is, I believe, because pupils before the Panel had very little sense of context for mathematicians.

The details in Annie's post-Panel depiction are different. The female mathematician is attractive looking and without glasses. (Of the five women on the Panel, two did not wear glasses; of the three men, one did not.) The figure is situated in a more realistic setting, with mathematical objects on her desk and a computer nearby. There are now three windows in the picture, which seems to place her in the world rather than apart from it.

In this drawing there are also simple division problems, although they appear here on the computer screen. And the items on the desk, which include two geometric shapes that appear to come from Dana Frank's talk, indicate that a mathematician is involved with more than just calculations. There is a pen and paper as well, but instead of calculations, the paper has on it, *If there were...*

About this second drawing, Annie has written *It is of a mathematician in a nice room with lots of different kinds of math around her.* As Milagros has, Annie now seems to see math as something plural, as having *...lots of different kinds...*

In her pre-Panel survey Annie has listed two reasons for hiring a mathematician: *If you need to find how many of something you need,* and *for the census.* But in the post-Panel survey she has four more specific reasons, which have clearly come out of what she heard at the Panel: *To help stop epidemics, to secure computers, to make them better,* and *engineering.* While Annie still agrees that mathematics as the study of numbers, she completed the post-Panel survey writing prompt, "To me, mathematics is:" *Exciting and interesting. It's the study of numbers and patterns.*

Annie has changed on four statements on the survey tool. Now where she had initially disagreed with the statement that a mathematician's work looks like fun, she agrees. Where she had agreed that she would never think of becoming a mathematician, she now disagrees. I believe that her drawing the second mathematician as a woman indicates that Annie can now see some relationship of herself to mathematicians.

Where she had been unsure as to whether she planned to stop taking mathematics as soon as she could, Annie now disagrees. And where she had disagreed about seeing herself as a mathematician, she is now unsure.

David, a pupil from Chelsea Middle School drew the mathematician in **Figure 8.10a**, and wrote: *Why would you hire a mathematician?* In response to the first writing prompt.

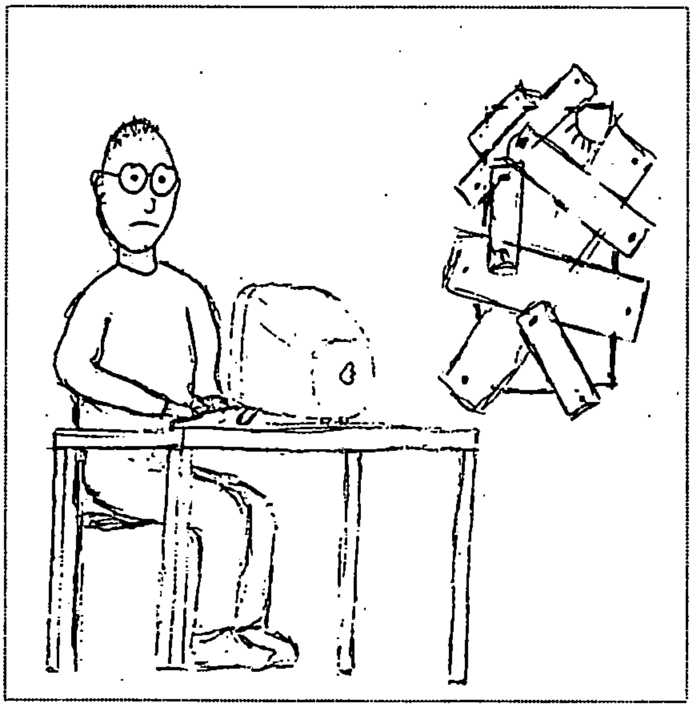

Figure 8.10a David's drawing (pre-Panel)

David's comment on his drawing explains why the window is boarded up blocking the sun:

> *I think that mathematicians are nerds that work on their computer all day and don't see sunlight for weeks on end.*

His drawing is reminiscent of Annie's first drawing but in David's drawing, the world is completely shut out, because that is what he thinks mathematicians do.

I laughed out loud when I saw David's drawing after the Panel, in **Figure 8.10b**. The main character looks like Dana Frank, the first speaker on the panel, who is using an overhead projector and pointing to a figure on the screen. The other panellists are asleep and snoring away.

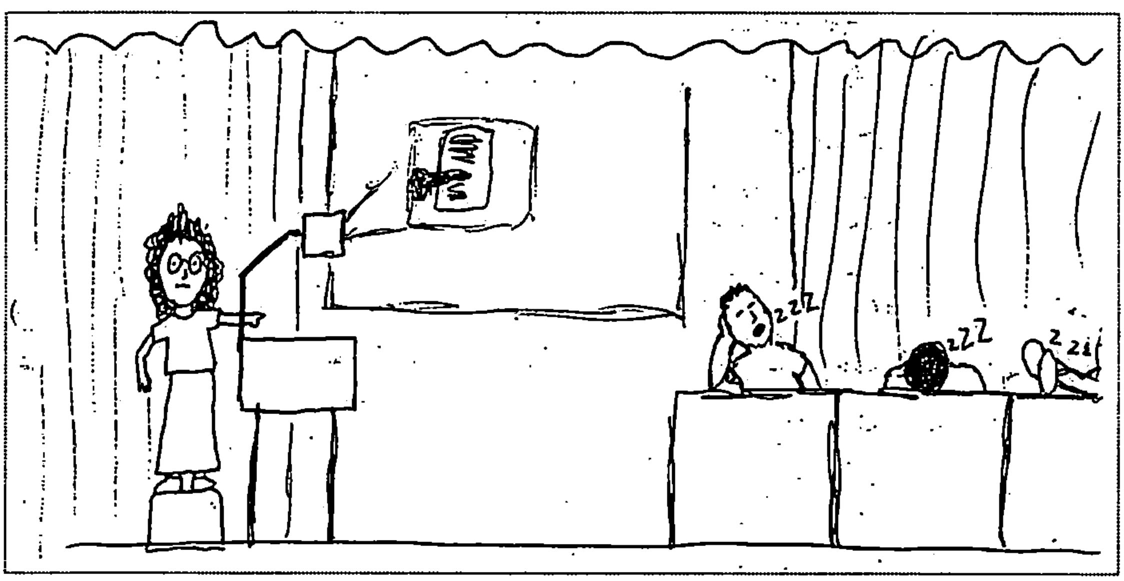

Figure 8.10b David's drawing (post-Panel)

Nevertheless, the drawing has changed from a lone figure preferring the sun blocked out, to a female mathematician surrounded by others. David has not repeated his earlier question, *Why would you hire a mathematician?* Instead, he now explains why one would pursue a career in mathematics:

I think that people would become mathematicians because they like math.

While he still does not look forward to taking more mathematics, nor see mathematics as more than the study of numbers, there is no mention of nerds now, and David appears to have examined some of his beliefs. While one might think from the snoring figures in his drawing that David hated the Mathematicians Panel, this is what he wrote directly afterwards:

I thought this activity helped me understand what mathematicians do. At first, I thought they would be nerds, but some of them weren't. I liked it.

8.3.5 PUPILS' SURVEYS BEFORE AND AFTER THE PANEL

There were two questions on the survey tool in which there was a significant difference after the Mathematicians Panel. For the purposes of comparison and discussion, the choices *agree* and *strongly agree* and the choices *disagree* and *strongly disagree* are looked at as if they are one choice.

Looking at **Table 7.30** with data from the Wilcoxon 2-related samples test, the two statements are: *(6) I have met a mathematician;* and *(9) Mathematics is the study of numbers.* Of course it makes sense that pupils would feel they had met a mathematician, because they had. 87% of pupils now agreed that they had met a mathematician and only 6.9% were unsure, as compared with 37% agreeing before the panel and 31% unsure.

As in the first intervention, pupils have reconsidered their belief that mathematics is the study of numbers. Twice as many pupils, 40.8% as compared to 20.1% before the Panel, now disagreed with this statement. It appears that learning about and from mathematicians has widened pupils' understanding of what mathematics is. But unlike the first intervention, there is no significant change in pupils liking their mathematics classes, nor a significant change in their looking forward to taking more mathematics.

There is a small change in the gender of the drawings after the Panel with more females drawn, including by males. Males drawing females increased from 3.7% to 8.5% after the Panel; females drawing females increased from 37% to 51.1%. This appears to be the effect of the Panel and the five women on it who at one point who were asked whether they had been discouraged because of their gender. They all answered that they hadn't.

One pupil wrote that she was 'a little shocked' by that answer, adding, *I might think about going into the field of mathematics because of this day.*

Wilcoxon signed ranks test data in **Table 7.32** indicates that the change in the drawing characteristics is significant. Wizards, weirdos, missing figures and Einstein references were reduced from 24.7% to 7.4% after the Panel. Pupils now had real images in mind, rather than the images they had received uncritically from the media.

It appears that the graph theory unit had strengths different from the Mathematicians Panel: pupils enjoy mathematics more and looked forward to studying mathematics further. But the Panel had its strengths in combating stereotypical images, and in enabling pupils to know what mathematicians do. And both appear to have encouraged pupils to view mathematics as a study beyond mere calculation and number.

The data in **Table 7.35** also indicates that Gramercy Middle School's pupils shifted after the Panel from seeing taxes and bills as the top reason for hiring a mathematician, to teaching.

When the schools are separated, it appears that their survey responses are each slightly different after the Panel. The Chelsea Middle class was retested with the survey tool two days after the panel, while the Gramercy Middle class was retested a week later. Due to a variety of conflicting situations that arose at the school, the Hudson Middle class wasn't retested until a month later.

In **Tables 7.27, 7.28**, and **7.29** are results of the Wilcoxon tests for each school's survey responses. For Chelsea Middle after the Panel, three statements had significant changes: (4) *I usually feel confident in my mathematics class*; (6) *I have met a mathematician*; and *(8) I look forward to taking more mathematics.*

For the statement *I usually feel confident in my mathematics class,* pupil's confidence had decreased, from 70.9% to 67% after the Panel, with more pupils unsure (20.4% as compared with 17.5% before the Panel). However, this may be the result of something happening in the classroom, rather than this being an effect of the Panel.

On (8) *I look forward to taking more mathematics,* the Chelsea pupils responded more positively after the Panel, from a 41.8% to 47.6% agreement.

For Gramercy Middle, after the Panel there is one more change from their post-discrete intervention. It is on statement (10) *I see myself as a mathematician.* After the first intervention, two pupils indicated that they saw themselves as mathematicians. After the Panel two more pupils agreed, representing 19.8% of the class.

Hudson Middle showed the most changes indicated as significant on statements (1) *I enjoy my mathematics* class; (2) *A mathematician's work looks like fun to me;* 6; 9 and 10 . But only on 9 was the change positive.

On the retaking of the survey tool, pupils enjoyed their mathematics class less (42% agreed before the Panel but only 34% agreed after); and disagreed more strongly that a mathematician's work looked like fun (26% disagreement before the Panel, compared with 58% after); and more vehemently denied seeing themselves as mathematicians: 52% disagreed before the Panel while 70% disagreed after.

It is possible that this was an indication of occurrences in the classroom, where the friction between Nancy Rockwell and her classes had not improved. And it should be noted that the particular statements are not really beliefs one could expect to change in one meeting with mathematicians, but rather through an intervention more like the first.

In their top four reasons for hiring a mathematician prior to the Panel (see **Table 7.36**), pupils had listed *teaching*, *taxes & bills*, *banking*, and *architecture*. After the Panel, teaching was still first, but now 77 pupils (compared with 50) listed it. Where 65 pupils had listed *taxes & bills*, now only 43 did, and it actually came in third on the list. Second on the list after the Panel was *computer applications* which was listed by 54 pupils. This hadn't even registered on the surveys before the Panel.

Fourth on the list was another new listing, *scientific uses* which appeared on 30 surveys. *Architecture,* which before the Panel had been listed fourth by 24 pupils, still was mentioned on 24 surveys, but after the Panel it came in fifth. And where 14 pupils had either left the section blank or written *Don't know,* now there were only 4 such responses. These changes indicate that the Panel enabled pupils to more surely answer their own question, *What do mathematicians do?*

Chapter 9: CONCLUSIONS AND IMPLICATIONS

9.1 INTRODUCTION TO CONCLUSIONS

This section is written to correspond to the three sections of the study and reports conclusions in relation to each. The first section reports conclusions of the international portion of the study; the second on the survey of mathematics professionals; the third on both interventions: the graph theory/discrete mathematics unit and the Mathematicians Panel.

9.2 CONCLUSIONS OF THE INTERNATIONAL STUDY—THE INVISIBILITY OF MATHEMATICIANS

The first and largest overall finding of this section of the study is that mathematicians are for all practical purposes invisible to pupils. Although 55.2% of pupils claimed to have met a mathematician, there is much evidence to the contrary. The reason most often given by pupils for why one would need to hire a mathematician is to teach, yet paradoxically, only 21.4%, fewer than one-quarter of the pupils, depicted a mathematician as a teacher.

Those reasons enumerated by pupils for which one would hire a mathematician show a general lack of knowledge about the work they do, confusing it with other professions that involve computation and/or measurement.

Pupils believe that mathematicians do applications similar to those they have seen in their own mathematics classes, including arithmetic computation, area and perimeter, and measurement. They also believe that a mathematician's work involves accounting, doing taxes and bills, and banking;

work which they contend includes doing *hard sums* or *hard problems*; yet pupils can supply no specifics about what such problems entail.

9.2.1 IMAGES THAT FILL THE VOID

The appearance of 41 surveys containing references to Albert Einstein with at least one from each country, underscores the main source of pupils' images as arising from the media, including comic books and cartoons. These media images appear to become mixed with images from pupils' experiences in their mathematics classes to produce the most stereotypical images.

Those pupils who drew stereotypical drawings in this portion of the study appear to have a more firmly held and more negative set of beliefs about mathematicians and mathematics, and enjoy both their school and mathematics classes less than their classmates who drew more neutral looking drawings of mathematicians.

One of the most surprising and startling images pupils drew in almost every country is one of small children powerless before mathematicians who were drawn as authoritarian and threatening. There were also images in every country depicting mathematicians as foolish, crazy, incompetent, and contemptible. Yet none of these images appears to arise from any real contact with mathematicians, as pupils in their own writings and in interviews admitted that they had not met any.

The experiences reflected in pupils' images of classrooms may come from a time when these pupils were younger and smaller. It is also conceivable that the experience of feeling coerced in a classroom encouraged pupils to view themselves as younger and smaller. But pupils appear to use experiences of having been intimidated in mathematics classes (*You should* *know this!*; *Can anyone but me find the answer?*) and their criticisms of their teachers for doing this, to depict mathematicians at times in their drawings in a vengeful manner and in this they are aided by images they have seen of mathematicians in the media.

The projection of supernatural powers onto mathematicians appeared in drawings by pupils in each country, and is related to the general invisibility to pupils of the mathematical process, for with the process hidden, mathematical facility looks more like a power than an ability which anyone has the possibility to learn. When Arthur C. Clarke (in Jacobs, 2000) observed that, "Any smoothly functioning technology gives the appearance of magic," he could as easily been commenting on what pupils perceive as a "smoothly functioning" ease most teachers exhibit with mathematics, a facility that to many pupils also looks like magic.

But in preferring an image of the mathematician as someone having magical powers, the pupil has accepted and is perpetuating the Platonist view of mathematics as existing outside and apart from humanity.

Drawings in all five countries revealed a number of consistencies in pupils' images showing that the experiences of pupils and the atmospheres of their mathematics classrooms have many similarities. One of the few differences I noted, is that in classrooms drawn by the Romanian pupils fewer of the computations depicted on chalkboards tended to be as trivial as those depicted in the drawings of the other four countries.

9.2.2 THE DOMINANT IMAGE

The dominant image of a mathematician that emerges from this study is consistent with Rock and Shaw's (2000) study of images of mathematicians and the DAST images of scientists—that of a white, middle aged, balding or wild-haired man. In surveying pupils who were much younger than those in this study, Rock and Shaw found the images becoming increasingly male as pupils moved up the grades in school. Images of mathematicians which were entirely female in the early primary grades had become half male and half female by the middle primary grades. The present study appears to continue Rock and Shaw's findings in revealing that at the lower secondary age, the images become overwhelmingly male.

This points to both a gender and a racial gap in pupils' images of mathematicians, which is consistent with those findings with the DAST.

9.2.3 THE SURVEY TOOL

On the Likert-type portion of the survey tool, pupils in all countries agreed that they enjoyed the schools they attended, and liked their mathematics classes. Yet most pupils indicated that they did not discuss their mathematics classes with their friends. The highest percentages of pupils agreeing that they discussed their mathematics classes came from schools in the United Kingdom and the United States where pupils sit and work together in groups. This would seem to indicate that those pupils who do some cooperative work in their classrooms tend to see mathematics as a more social endeavour than those pupils who experience their mathematics classes sitting in rows.

The exception was in Romania, where pupils sit in rows, but where they work collaboratively in teams as they prepare for mathematics Olympiads. Those pupils who do some collaborative work were also more likely to disagree that mathematicians work alone.

Although it was refined further after this portion of the study, the survey tool appears to be user-friendly and easy for teachers to administer and interpret, with the prospect of looking at pupils' drawings interesting enough to encourage them to want to do so.

The consistency of the images uncovered in this study with other published studies, including Rock and Shaw (2000) and those images of scientists in the DAST studies demonstrates the validity and reliability of such a survey tool. Indeed, after a short article was published (Berry & Picker, 2000) with preliminary findings of this portion of the project in which teachers were encouraged to try the survey tool in their own classrooms, the head of mathematics in a school in Phuket, Thailand wrote of her pupils' drawings:

> Although the majority of students are Thai their images appear to be the same as talked about in your article.

The mathematicians depicted were western-looking, white and male, showing further evidence of a consistency in the image of a mathematician which can be found among pupils at this time.

9.3 CONCLUSIONS OF THE MATHEMATICS PROFESSIONALS SURVEY

There appears to be a lack of a unified vision among members of the professional mathematics community and a wide divergence in their beliefs about their profession. In particular, the understanding of who is a mathematician and who may call oneself one is quite varied. Respondents indicated with greatest frequency (30%) that they believed *someone who reasons mathematically* can be considered a mathematician, but the opinion with the second highest frequency (18.9%) was a contradictory one: one who *creates new mathematics* or *does theoretical research*.

There is a large question about what respondents mean by "creating new mathematics," because while some professionals defined it as *seeing mathematics in a new way*, others expressed that it means doing *important research* or *theoretical research*.

The beliefs of some survey respondents appear to contradict the views of Owen, a pupil who has defined himself as a mathematician. Two respondents hold that he doesn't have the right to do this, claiming instead, that it is for someone else to say if one is a mathematician. This only reinforces a belief held by respondents and other mathematicians (Papert, 1972; Peterson, 1990; Devlin, 1996; Henrion, 1997) that within the field of mathematics there is an elitism and exclusivity which affects and restricts who enters the field of mathematics.

The survey data support an association between the working level of a professional and not considering oneself a mathematician. It appears that as the working level of respondents decreases from tertiary to secondary through primary school, the sureness of the respondent in considering themselves a mathematician also decreases.

There were also some further disagreements among the respondents. Two of them didn't consider themselves mathematicians, considering themselves instead, to be an engineer and a statistician. To be known by a title other than mathematician is sometimes the result of the culture of a university or country. Burton (1999), found this to be the case among the respondents to her study. Unfortunately, it serves to further obscure mathematicians from the public's view.

9.4 CONCLUSIONS OF THE INTERVENTIONS—GRAPH THEORY AND DISCRETE MATHEMATICS

When pupils were interviewed as part of this portion of the project, they admitted that they were not sure what mathematicians do. With the insertion of the prompt for pupils to name the mathematician they had met, only 25% of pupils agreed that they had met a mathematician, compared with 55.2% of pupils who agreed in the international study where no name was requested.

The graph theory/discrete mathematics intervention, despite its applied nature did not appear to provide pupils with insight into the work of a mathematician and pupils said as much when interviewed. Nor did it change pupils perceptions of mathematicians as "super-accountants," doing taxes and bills. What it did enable pupils to do was to see mathematics in a wider way, as being about more than just the study of numbers, and it encouraged pupils to enjoy their mathematics classes more and to indicate that they looked forward to taking more mathematics.

At a time when students are turning off to mathematics and looking forward to dropping it as soon as they can (NCTM, 1995), discrete mathematics appears to be able to provide pupils with more positive and novel mathematics experiences than they have had thus far in their schooling. I believe that this merits further study in a larger project.

9.4.1 CONCLUSIONS OF THE MATHEMATICIANS PANEL

The questions posed by pupils in preparation for the Mathematicians Panel further showed how little pupils really know about mathematicians, which was consistent with findings from the international portion of this study. In each of the seven classes pupils wanted to ask the mathematicians: *What do you do?* They also wanted to know *Who hires mathematicians?* and *Where do they work?*

Pupils wanted to know about the mathematics the mathematicians used and who the persons were who had inspired them. And they appeared to believe that mathematicians probably made very little money. All these questions are consistent with the finding that for pupils, mathematicians are invisible. Yet once pupils were given the opportunity to think about and formulate questions, they became curious about mathematicians and interested in meeting them.

The comments written by pupils after the Panel showed that they had viewed mathematicians in a formulaic, oversimplified and therefore stereotypical way, as an undifferentiated group, rather than as individuals and real persons. This too, is consistent with the invisibility of mathematicians to pupils before the Panel. After the Panel, pupils had real knowledge about mathematicians and their work, with the result that their image of them shifted to an image of *normal people with normal lives.*

The change in the characteristics of pupils' drawings after the Panel was significant. Wizards, weirdos, missing figures and Einstein references were reduced from 24.7% to 7.4% after the Panel. As can occur with a belief system (Thompson, 1992) pupils appear to have evaluated their previously held beliefs against the experience of meeting mathematicians via the Panel. With real images in mind, pupils could now reject those images they had received uncritically from the media.

Pupils also came to see mathematics as a much wider study—they were surprised at the diversity of its uses and applications. While the graph theory and discrete mathematics intervention did not affect pupils as positively in this area, yet for the pupils at Gramercy Middle School who were so interested in science, the chance to hear Alun Llandaff, who is barely ten years older than they, explain how he uses mathematics and biology together in his work seems to have had a positive effect.

It appears that the graph theory unit had strengths different from the Mathematicians Panel: pupils enjoyed mathematics more and looked forward to studying mathematics further. But the Panel had its strengths in combating stereotypical images, in showing the *human face* of mathematicians, as one pupil expressed it, and in enabling pupils to know what mathematicians do. And both interventions appear to have encouraged pupils to view mathematics as a study beyond mere calculation and number.

9.5 IMPLICATIONS FOR PEDAGOGY

At the panel I attended at Rutgers University some years ago, in which mathematicians spoke to mathematics teachers about their lives, careers, and schooling, a number of the mathematicians traced their middle school years to the time when they first realized that mathematics was more than just a school subject, but an interesting and exciting profession. This points to the importance and sensitivity of this particular age in a pupil's life and the large and positive affect teachers can have.

But teachers are largely unaware of the role they can play both in shaping and in changing their pupils' images about mathematicians and mathematics. That is why I believe that the findings of this study have significant implications for teachers and their practice. As the DAST has been a means for sensitising teachers and pupils to the images they hold of science and scientists (Mason, Kahle & Gardner, 1991), so too can teachers become more fully aware of the existence of images of mathematics and mathematicians through this study. Although it is often assumed that teachers have no stereotypical images themselves, yet Furinghetti (1993, p.33) reminds us:

> The image of mathematics among professional mathematic[ians] is tortuous and controversial; it should not surprise us, therefore, that for mathematics teachers, deciding what image to transmit to their pupils is a source of doubt.

Teachers are not yet aware that there is an element missing from their mathematics classes: mathematicians. And unfortunately those assignments to write reports on mathematicians in history generally serve to reinforce an image of mathematicians as mostly male and mostly of the past.

At the present time it is very rare to be in a mathematics class and hear the word mathematician used during a lesson. But teachers refer to and address their pupils as scientists in a science class; poets, novelists, or writers in an English class (see Pinsky, 1998) and yet it is very unusual to hear pupils addressed in a mathematics class as mathematicians. There seems to be no other subject studied in school where pupils are placed at such a distance from the discipline than occurs with mathematics. Yet if we want pupils to find the subject attractive and interesting, we need to be the means of this changing.

As I look at the drawings pupils created for this project, I find myself thinking that a course for teachers could be created with the pupils' drawings forming the outline of study. The two forms of authority and the two forms of discipline which can fight for supremacy in a teacher can be seen and discussed through the drawings, as can the two differing philosophical stances from which teachers can approach their pupils: It can be—"Look I tell you, this is the way it is," or the approach can be—"Come let us reason together." Neither is generally discussed with directness as teachers are being readied for the classroom.

How pupils can be made to feel in a classroom by teachers, appears centrally in many of the drawings, from feeling intimidated at not knowing something, to being dazzled by a teacher's polish and ability, to exploiting a teacher's inability to control a classroom, and I think seeing all these possibilities portrayed in the drawings would provide rich discussions and significantly raise teachers' consciousnesses.

As I looked again at the drawings of teachers holding rifles and wondered again where the images came from, I happened to be in a classroom in New York City where I heard a teacher say to her middle school mathematics class: "Look—I don't have a gun; I'm not going to shoot you if you get it wrong." And I realized, there is often a great gap between what teachers say and what pupils hear. But for the teacher to say that—where did *that* come from? Why put it *that* way? What are guns even doing in a classroom? But I think now that it is part of that struggle about authority that exists in teachers. Guns signify an ultimate authority. The teacher thought she was being reassuring! But what did the pupils hear?

9.6 PROPOSED SCHEMATIC OF STEREOTYPE PERPETUATION

In **Figure 9.1**, below is a proposed schematic outlining the perpetuation of stereotypical image formation as a cycle and ways of breaking into the cycle:

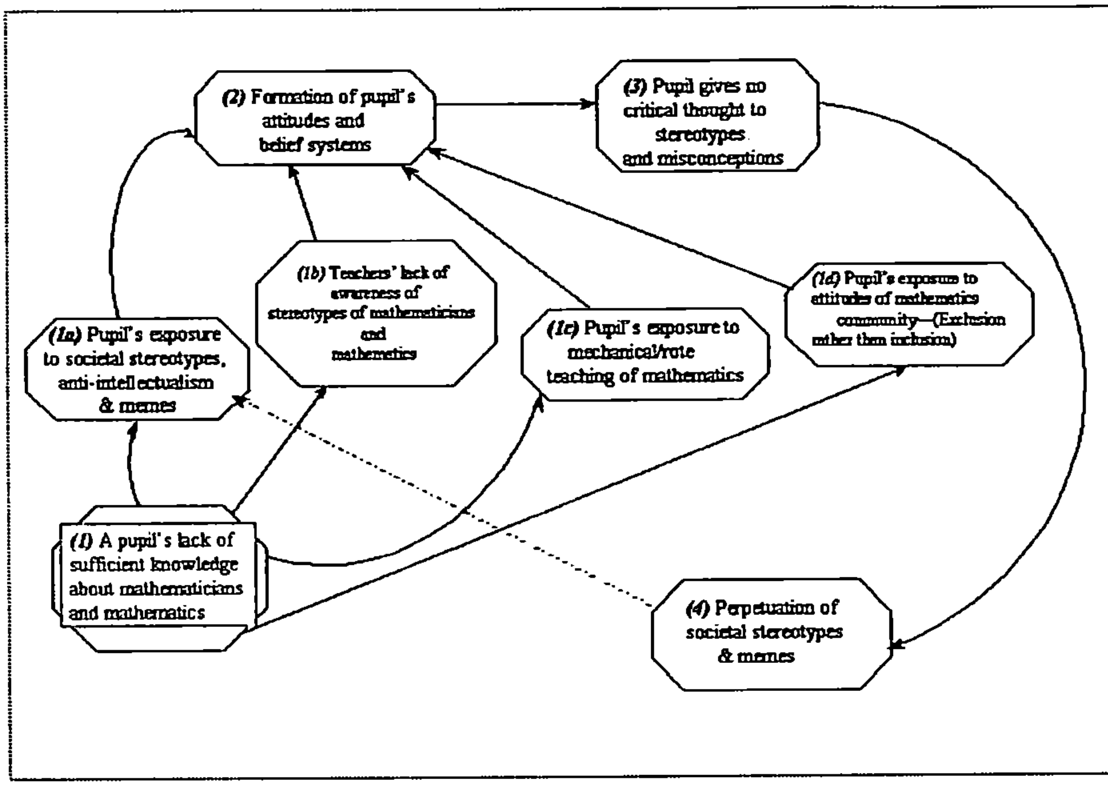

Figure 9.1 The Proposed Picker-Berry Cycle of the Perpetuation of Stereotypical Images of Mathematicians and Mathematics

In ***(1)*** a pupil starts at the beginning of their school experience knowing very little about mathematicians and mathematics, but generally, in the first few years of schooling likes the subject (National Research Council (NRC), 1989, p.43). In ***(1a)*** the pupil is exposed to a variety of cultural and societal stereotypes via television, comic books, cartoons and other media (see for example, Buffet, 1999), as well as through peers and adults repeating phrases (memes) they've heard through the media and popular culture.

A meme, (see Dawkins, 1989; Rothstein, 1998; Aunger, 1999) is a small bit of cultural information, a "catch phrase" according to Dawkins, who first described memes, which is passed from person to person by imitation. Memes have been described as infectiously leaping from brain to brain, as a bit of a tune can suddenly permeate one's consciousness and seem unshakable.

In the case of stereotypes of mathematics, memes can explain the repetition of such phrases as *math is for nerds,* or *you either have a flair for mathematics or you don't*, or adults saying (with no shame), *I was never very good in maths—it was my most hated subject at school!* Yet in our society you will not hear people freely admit that they cannot read (Battista,1997).

As children become socialized by their school experiences and the influences of society, they begin to perceive the subject of mathematics as "a rigid system of externally dictated rules governed by standards of accuracy, speed, and memory." (NRC, 1989, p. 43)

In ***(1b)*** the pupil meets in their teacher a lack of awareness of stereotypes of mathematics and mathematicians, which arises out of and includes a lack of knowledge about mathematicians and what they do. It may be that the teacher holds certain stereotypes, too, or that among the many subjects the teacher must teach, mathematics is the one the teacher least likes and is least comfortable with. So the beginning exposure to these stereotypes goes unchallenged in the mind of the young pupil.

In ***(1c)***, often during the primary years, the pupil experiences a mechanical and rote "broadcast" method (Dossey, 1992) of teaching of mathematics fostered by an associationist philosophy in the culture of the school, in which pupils are expected to "practise minimally connected bits of information" and in which the pursuit of right answers rather than the process of mathematical thinking is emphasised (Resnick & Hall, 1998). During this time the pupil sees very little application of the mathematics being taught. In being taught mathematics with virtually no context, the pupil never gets to experience firsthand a sense of what mathematics is really about, nor do they get beyond a belief that mathematics is a cold dehumanised subject passed down through the generations as though on tablets.

In ***(1d)***, the pupil is very subtly being affected by attitudes of the mathematics community through teachers and through the media: a philosophy that includes the message that it is not important for mathematicians to have a clear and ongoing dialogue with the general public about what they do; that mathematics is just for a privileged few who have the innate ability to do it; that mathematics is a special closed language (see Hammond, 1978; Howson & Kahane, 1990; Peterson, 1991; Henrion, 1997); that a pupil must be quick at it to be good at mathematics (NRC, 1989, p. 43), and more. All these messages begin formation of an image of mathematicians and mathematics in the mind of the pupil that is authoritarian and distant.

Through ***(1a) (1b) (1c)*** and ***(1d)***, having been exposed to some of these or all of them, the pupil in ***(2)*** formulates first, an attitude to mathematics, then a belief system (Aiken, 1970). By ***(3)***, these belief systems are well formed and if unquestioned, the promotion of stereotypes begun. As Bem (1970, p. 9), contends, beliefs lead to generalizations, and generalizing from a limited set of experiences, leads to stereotypes.

Nowhere along this long path has an alternative view been presented and a more positive picture of mathematicians interceded strongly enough to combat the pupil's lack of knowledge. As far as the pupil is concerned, mathematicians are invisible. Stereotypes have filled this void.

In ***(4)*** the pupil exchanges their stereotypical views with other pupils and perhaps with younger siblings. In 1970, Bem wrote (p.75), that even more than the various media, the "major influence upon people is people. Even in our technologically advanced society, there appears to be no substitute for direct personal contact." Since the mid-1970s, the concept of memes in which ideas become catch-phrases accepted uncritically and passed on to others, has been increasingly discussed.

In exchanging stereotypical views with others, the pupil now perpetuates such memes as *maths is boring, I'll never use this maths* (Boaler, 1999), *math was the most difficult subject I ever...* (Bogomolny, 1996), and many more. And so, as a 'representative of society', the pupil now contributes to others' views of mathematics. The cycle is complete.

Yet, if in ***(1b)*** the teacher is aware of stereotypes and can direct her pupils to a different view of mathematics and mathematicians than appears in the media, or if in ***(2)*** or ***(3)*** the pupil can be directed to critically look at and reconsider their beliefs, this cycle can be broken and kept from perpetuating itself.

9.7 QUESTIONS ARISING FROM THE RESEARCH

I believe a larger study on the effects of graph theory and discrete mathematics is justified, as are further studies involving pupils meeting with mathematicians. I also believe that a comparison study of the beliefs of pupils who consider themselves mathematicians, and mathematics professionals' beliefs about who is a mathematician would be valuable to pursue.

The following further questions also arise from this study:

. Since images originate from past experiences, from how far back in their pasts do the images in pupil's drawings come?

. What would be the effect on a class of using pupil's drawings and the results of the survey tool for an ongoing term-long dialogue?

. Would inviting a single mathematician to a classroom have as much of an impact as pupils meeting with a larger panel?

. What is the cause and effect of the confusion many schoolteachers feel about whether or not to consider themselves mathematicians? Can an intervention be designed to enable them to feel more confident?

In addition, I believe that the survey tool can be further refined and tested. It appears to have been an effective beginning means for ascertaining pupils' images of mathematicians and mathematics, and a means to opening the dialogue necessary to dispel misconceptions and challenge stereotypes. It is a user friendly, inexpensive and quick way for a teacher to learn about some of the images her pupils hold—images which may impede their present and future study of mathematics.

9.9 SOME FINAL THOUGHTS

As this study has proceeded, I have seen a great deal of thoughtful dialogue begin. The teachers who helped facilitate the pilot studies and surveys that were part of this project were amazed at many of the drawings their pupils initially produced. Almost all of them came to realise that they themselves knew very little about the work of mathematicians.

One of them, Connie Sachs, asked me recently if there would be another Mathematicians Panel, because she said might have a panellist to recommend—a mathematician was coming into her classroom that afternoon to look at probability problems with her pupils. She would never have considered doing such a thing before the experience of the Panel.

In trying to explain to friends and colleagues just what this project is about, I have found that among the non-mathematics public, as well as among teachers of mathematics and mathematicians, stereotypes are acknowledged, but not very clearly thought about. Yet I have come to understand through this study that it is only through critical thinking about them that these stereotypes of mathematicians and misconceptions about mathematics can be challenged and changed.

The mathematicians who came to participate in this study as part of the *Mathematicians Panel* were very interested in the idea of changing currently held stereotypes and were happy to meet with pupils to answer their questions. It seems very hopeful that a much closer link between pupils in schools and mathematicians can be forged. It is clear that there are many mathematicians who are in favour of this. This bodes well for the field of mathematics which would benefit greatly from finding itself populated with the greatest variety of personalities; the better to appeal to a wider public audience and to happier and more appreciative pupils.

Appendix—A

I am: Female ❑ Male ❑

	strongly agree	agree	not sure	disagree	strongly disagree	
1. I enjoy the school I attend.	1.❑	❑	❑	❑	❑	
2. A mathematician's work looks like fun to me.	2.❑	❑	❑	❑	❑	
3. I would never think of becoming a mathematician.	3.❑	❑	❑	❑	❑	
4. Mathematicians seem like very patient people.	4.❑	❑	❑	❑	❑	
5. I would not want to marry a mathematician.	5.❑	❑	❑	❑	❑	
6. I have met a mathematician.	6.❑	❑	❑	❑	❑	
7. I don't enjoy my mathematics class.	7.❑	❑	❑	❑	❑	
8. I discuss what I am doing in mathematics class with my friends.		8.❑	❑	❑	❑	❑
9. Mathematicians usually work alone.	9❑	❑	❑	❑	❑	
10. I see myself as a mathematician.	10❑	❑	❑	❑	❑	

11. If you have a leaky faucet, you need to hire a plumber; if you break your leg, you need the services of a doctor. List below all the reasons you can think of for which someone would need to hire a mathematician:

12. Look back at the drawing you made of a mathematician at work and write an explanation of the drawing so that anyone looking at it will understand what your drawing means, and who the persons are in it.

Appendix—B

Non-Standard Interview Questions

1. What do you like most about mathematics?
2. What do you like least about mathematics?
3. Do you find you do better, worse, or the same if you like your teacher?
4. What do you talk about when you discuss your mathematics class with your friends?
5. Can you think of a specific time when you felt you were really enjoying your mathematics class? What do you think was the reason? Was it the teacher in any way?
6. On the questionnaire you stated (#2/3/5/6/10): Could you please say more about that?
7. Could you please tell me more about your picture?

Pupil Interviews—International Study

Interview with Alex
New York City Middle School
Spring 1999

SP: I was very interested in your "Integer Booklet" because I was interested particularly in your choice. So I wanted to know if you could say a little about the picture and your choice of using that for this project.
Alex: Um, I didn't really have a major motive for actually using that person. I just thought it would add a little pep or interest into the whole thing and so I just thought of smart person who probably everyone knows, so...
SP: And in your mind you associate him with mathematics in some way?
Alex: Yeah. And science—mathematics and science.
SP: Okay, could you just say more about what you think of when you think of him?
Alex: Umm, I just actually, like, realize how much of an impact he's had, like, his mathematical achievements and his scientific achievements have impacted—even the present, even though he has discovered these things over 70 years ago, so, that's something that comes to my mind.
SP: Do you remember where in school you first learned about Albert Einstein?
Alex: Uh, you mean, umm, a major thing, or...
SP: When did you first become aware of him?
Alex: Oh.
SP: If you remember...
Alex: Third grade, or second.
SP: How did it come about?
Alex: Umm, I didn't really know much about him or his achievements, I just knew that he was a great physicist and mathematician, and, umm, I wasn't really sure about—and I knew that he had created the equation

$e=mc^2$ although I didn't really know what that meant either. (laughs) Umm...

SP: Do you know what it means now?
Alex: Yeah!
SP: And where did you learn that?
Alex: What, the meaning of it?
SP: Yes.
Alex: I believe it was last year. We were watching—in sixth grade—a movie about physics and, uh, one segment of the video explained Albert Einstein and his theories.
SP: So have you ever really learned about him in a math class?

Alex: Umm, I'm taking this extra program and I was told to do a report on a scientist or mathematician and so I actually learned an extensive amount on him there.
SP: What program is that?
Alex: It's called "The Math-Science Institute".
SP: And where is that?
Alex: It's branched all over New York. I go to the Stuyvesant High School one. It's on Wednesday afternoons, Saturday and during the summer.
SP: And so in your mind when you think of a famous mathematician is he...
Alex: Yeah, he's a primary person who comes to mind...when I think of mathematics and science, and physics and such.

Looking at the drawing of a mathematician at work on his questionnaire:

SP: Could you just talk a little about the drawing itself?
Alex: Umm. I actually now realize that this doesn't look much like him—(laughs)—like Albert Einstein—cause...
SP: In what way doesn't it look like him?
Alex: Umm, I think this one looks more like him [in his new project] because of the hair. The hair and uh, his—like his tired eyes.
SP: Did you get this picture from like, have you seen it somewhere else?
Alex: Umm. I think I've seen a picture of him working and writing on a chalkboard. But it's not in this exact format.
SP: Now could you read what you wrote here on the "Integer Booklet"?
Alex: Okay, it's supposed to be in a sort of accent.
SP: So what does it say?
Alex: (Reads) "Yesh, integersh are very intereshting."
SP: Okay, so I'm just curious as to whether you saw that in a cartoon or TV...
Alex: (Laughs) I think, yeah, it possibly was in a cartoon or there was an exhibit on him, uh I'm not sure...yeah, I think it was the Natural History...there was either an exhibit on him somewhere or there was like a documentary on TV and, uh, I just heard that he had such a strong German accent.
SP: Do you remember seeing any cartoon character that was also kind of based on Einstein?
Alex: Umm, yeah—Warner Brothers,--the early ones—they played a lot off of Albert Einstein because, well, at the time that they created all of these cartoons and such he was a major scientist.
SP: So you're saying that these were from the time when he was alive?
Alex: Yeah.

SP: You've done a study of Warner Bros. Cartoons?
Alex: Yeah, I've been interested in them for like five years now.
SP: Now can you, if you remember, can you describe some scenario from one of the cartoons and what he's doing in them?
Alex: Umm...let's see if I can actually recall...uh, I think he was in like a laboratory, umm, well it was supposed to be this, uh this test, uh, yeah, it was with a Bugs Bunny cartoon. Bugs Bunny was like the test rabbit and they were trying to actually change the brain of a rabbit to a chicken.

SP: Of course.
Alex: (Laughs) And uh the main mad scientist was sort of modelled off of Einstein because of his dialect and hair—funny hair.
SP: And was there any way of thinking, watching this, that he was also a mathematician?
Alex: Uh, no, I actually think that all the ways he has been portrayed with, uh, as in cartoons and comic strips, etc., umm it seems as if he's been a mathematician—
SP: What gives you...
Alex: —I'm sorry—a scientist.
SP: So what do you think is the reason why people tend to see him as a mathematician as well?
Alex: Umm, I guess because of the equation that he used...it seems as if it's like a, an algebraic equation.
SP: But, you've never discussed this equation in your math class, then you—
Alex: No, no, yeah...I mean...
SP: Just curious.
Alex: No, I haven't.
SP: But you have discussed it in your science class.
Alex: To a certain extent...not really...uh, yeah.
SP: Just to say what each of the letters represented, perhaps?
Alex: Well, there was some mathematics involved with it well, not a major thing, but, uh, squared part, the c-squared and the m times uh, the speed of light squared, that portion.

Interview with Rebecca
New York City Middle School
Spring 1999

SP: You said you weren't sure how to answer the statement, "A mathematician's work looks like fun to me.

R: I don't really, I mean, like a mathematician, like, are we considered mathematicians?—Not really...

SP: Well, that's a good question. What do you think?

R: Umm, it seems like, more of like, a mathematician seems more people who, like their jobs are being mathematicians. We're students. I mean we do math...but I don't think I would choose it as a career when I'm older, I think.

SP: In answer to the statement about becoming a mathematician, you said you were unsure. Now you just said, well, I probably wouldn't choose it.

R: Well, I mean, it's a possibility but I don't really know; I haven't really thought about what I want to be, so it's not, it's not out of the question.

SP: In answer to "I would not want to marry a mathematician," you disagreed. So you...

R: I don't think it matters...

SP: What they do, you mean.

R: Right.

SP: Okay. And then in answer to "I have met a mathematician," you said yes, you agreed. Can you think of who it was you were thinking about?

R: Umm, I don't know. Maybe I was thinking of us as mathematicians, or the teachers; I can't quite remember.

SP: Do you see your teacher as a mathematician?

R: Well, umm, kinda, (laughs). I don't know.

SP: Okay, so you don't know.

R: I don't know.

SP: In answer to "I see myself as a mathematician," you said you were not sure.

R: Umm, I don't know.

SP: Okay. And I just wanted to ask you, I think you obviously care for drawing and for art, I just wanted to know if you could say a little more about your picture. You remember this, right?

R: Umm.

SP: It was awhile ago.

R: Umm, yeah. This is supposed to be [points to woman with chart] umm, I guess this is someone at work using mathematics at their job; I guess I did consider that as a mathematician. Umm, this is a contractor, something like that...so, umm, I think this is a teacher or something like that. And I know you use math when you're doing science...you like, how many cups of things you're using...

SP: Have you discussed that in your science class?

R: No. --I don't know.

SP: So, can you think of place where you might have learned that?

R: No. (laughs)

Interview with Owen
New York City Middle School
Spring 1999

SP: You disagreed with the statement "I would *not* want to marry a mathematician."

Owen: Well, I *would* want to marry a mathematician because I'd have someone to talk to about something interesting a lot.

SP: And, now you said that you agreed with the statement, "I have met a mathematician." And I'm just curious as to who that might be.

Owen: Well, let me see, I don't really know why I put that. I think I put that probably to correspond with my drawing.

SP: As you look at it know, would you say you really have met a mathematician?

Owen: Yes.

SP: And who would that be?

Owen: There's a lot of people. I think it's just people.

SP: That you've met.

Owen: I don't think there's anybody that you'd *label* a mathematician. I mean probably there's a lot, a lot, lots and lots and lots and lots and lots of people that are that nobody knows—sometimes it's neat to find out.

SP: And have you met someone who you know made their living as a mathematician?

Owen: Umm, no.

SP: You haven't.

Owen: No.

SP: Okay.

Owen: Well, I mean as I say again, just a lot of people's livings are using mathematics and probably during their livings they think about new ways to do things, so probably my answer will be yes. Because let's say somebody's down in a shop and they're making, uhh, lunchboxes, and somebody says. "If we increase this angle, maybe it might be easier to, umm, carry around. And if we create a new kind of angle, that has never been seen before, then maybe we'll find it out. Just like, -- what's his name -- Henry Ford did, in 19...09, I think it was. He thought about how the car could be used in an assembly line, each adding on a different thing, and that was, if you look at it in a certain way, that could be mathematical.

SP: Would you characterize Henry Ford as a mathematician?

Owen: Yes, I would.

SP: I just would like to know if you could tell me a little bit more about your picture. If you could talk about it.

Owen: Well, first of all, I don't know how I could have drawn a person (laughs)—I can't do that anymore...anyway, okay.

SP: Would you do something different now, if I were to ask you to do it again?

Owen: No, because what this "intervals of 6 with reciprocal of..." I was just kind of writing like a kind of nonsense because I just wanted to portray the point that this guy is thinking about math; he's thinking about how "anyone who thinks differently about math, that's how the greats began." Just like, 'cause I umm, there's a quote by Albert Einstein, let's see, what was it? Uhh, "Great spirits have always encountered great opposition from mediocre minds." ...Well I think that people like this can still think about things, like I know I think about math all the time and I think that that probably makes me a mathematician 'cause I'm trying to find something here. And just like this person I've got to make him like totally normal. Maybe let's say that he's a—what can he be—maybe he's a stockbroker, no—that's a bad example—let's say maybe he fixes clocks, and maybe he's just thinking about math while he's doing it although he's no mathematician. Just like the movie "Good Will Hunting". You see that guy and he was a custodian and he thought about math though—that would make him a mathematician. Not because—he wasn't a mathematician like as a profession, but he was a mathematician.

SP: Well, of course at the end of the movie he did enter the profession.

Owen: Yeah, well, yeah, but I mean in the beginning.

SP: Did you like that movie?

Owen: Yeah.

SP: What was the thing you liked best about it.

Owen: Umm, well, uh, he's my role model because he reads all the time and he can read fast and I could never read that fast. (laughs) But anyway, he finds the ways to do these things even though he's not in a lab—I guess he's in a school, but he's mopping up the school and he does those puzzles on the board, and it's his most interaction with them.

SP: You said that you put this sort of nonsense in just to give the idea that he's thinking about mathematics.

Owen: Yeah.

SP: Do you think that most people think of mathematicians as being normal guys looking like this?

Owen: I doubt it. I mean there's always stereotypical...

SP: So where do you think the stereotypical stuff comes from?

Owen: Uhh, I think the stereotypical stuff just comes from...personally, I think it's TV. That kind of people that never have any reaction but buy just like junk, and so when people think of mathematicians, the first thing they have to have in their head is somebody that works in this big lab with all this mathematical equipment trying to find things out. And that's what they think of as the people who look like the people who make the stuff that you're doing in school and stuff. But really, the people like that...

SP: When do you think you decided that you weren't going to think in that stereotypical way?

Owen: I never really thought in that stereotypical way.

SP: You didn't?

Owen: Yeah, before that I just didn't really know.

....
.....

SP: Anything else that comes to mind about the picture?

Owen: I just wanted to portray that labels aren't as important as what you're thinking. Like some people are labeled and this person's not labeled but he's maybe thinking the exact thoughts of somebody in a huge laboratory that has *numbers* all around them.

Pupil Interviews—Interventions

Interview with Kate
Gramercy Middle School, New York City
8 November 1999

SP: Could you talk a little more about these two pictures?
Kate: Well, I wasn't really sure of what a mathematician does and I'm still not sure, but I think they like figure out different sorts—they think of different sorts of like math problems and like they do different things with not only like numbers but with like nature and things like that.
SP: So do you remember what you were thinking when you drew the white page—the first one?
Kate: I thought it was sort of like a math teacher...
SP: And which one was the math teacher?
Kate: This one.
SP: Okay—on the left.
Kate: And this person was supposed to be like a puzzled person like not understanding what they're doing.
SP: So when you think of mathematicians do you mostly think of teachers?
Kate: Umm—yes and no. Because a lot of, like some math teachers are mathematicians and some aren't. Like I think Miss Nierescu is a mathematician.
SP: And how would you know that—that some are and some aren't?
Kate: Umm—well sometimes they tell you (laughs). But, and like sometimes if they, if they really like math and like they really like their job, and like you could just sometimes just tell. In all the things they do, and like sometimes just the approach they take to math.
SP: And did Miss Nierescu tell you that she's a mathematician?
Kate: Umm, she never really specified it, she just always, like she always says she's part of these little groups like with mathematicians and things and like I think she is and even if she isn't I think she'd make a great mathematician.
SP: So when someone is sort of clearly not a mathematician, what hints do you get about that—when they've been your teacher?
Kate: Well, like if they can't really explain very well—because mathematicians, they study things and they can explain all their answers really well. And like if a teacher—even if she's saying something and the children don't really understand the problem, then she shouldn't—you can just sort of tell. (Laughs.)
SP: Is this male or female?
Kate: It wasn't really meant to be anything. I think mathematicians can be both.
SP: Well this on the yellow paper, it looks like a female.
Kate: Yeah.
SP: Although maybe it's a guy with long hair...
Kate: Noo. (Laughs.)
SP: So that was meant to be female.
Kate: Yeah.
SP: And is this a mathematician, on the yellow page?

Kate: Yeah.
SP: And can you just tell me a little more about what I'm looking at here?
Kate: Well, this person is studying fractals, and she's studying, um, the how, if...she's—I can't explain it really well...she's just studying how the different kinds of stages you get—she's trying to make it as small as possible.
SP: And, anything you want to say about this?
Kate: Well, (laughs) I put that there because if a mathematician—
SP: If you just want to say—it says, 'Math Rocks.'
Kate: (Laughs.) Exactly. If they really like math and if it's a mathematician, they like math.
SP: Now, when you drew this, you drew stick figures, but here this isn't a stick figure. Were you less interested in drawing this or was there something else going on, or were you rushed when you drew this—that this one is a little more sophisticated. Is there a reason for that?
Kate: Uhh...This one I think, umm, I didn't, I didn't really think—I didn't exactly know what it was, so just drew stick figures to do it faster. And this one, I think I sort of had a better idea—it's just hard to explain about what they do. I have a better idea about it so I didn't want to make it so simple-looking.
SP: Let's take a look at what you did on the other side, but first I'll just ask you since you have the fractals here and I know it's from the unit that Miss Nierescu had done on discrete math.
Kate: Yeah.
SP: And you did work with networks, graphs...
Kate: Yeah.
SP: Umm, what did you think of that?
Kate: I think that was really cool. I mean I have never seen anything like that because it was cool how one big shape turns into this huge shape that keeps on expanding. And it's strange because that one shape is still in that really big shape so it's like all the same thing. And it's really cool. I never really thought of it.
SP: So did you like fractals the best of all that she did with the unit—I mean you remember the vertices and the edges—
Kate: Yeah...It wasn't like the best. I like everything—I think it was very interesting.
SP: And the graph theory?
Kate: Umm—which one was...
SP: With the vertices and the edges...
Kate: Yeah. That one—I liked that. I liked the Hamiltonian circuit and the paths--I think that was cool. And I also liked the graphs without picking your pencil—that was cool, too.
SP: Did it give you any further insight into what a mathematician does?
Kate: Well, that's why I sort of drew that, with the fractal. And I think that they try—mathematicians try to interpret all the different kinds of things that—uh different kinds of like—they don't –it's really hard to explain—I can't really—but in my mind I know what it is but it's really had to explain. Like I know they don't only study numbers and shapes and you know, lines. I know they use—like we watched a movie that this man he was, umm, not a mathematician but he was some other word. Umm, he studied nature for fractals—he studied nature because

when you find fractals like in leaves. Like you have the shape and it's like sort of like getting bigger.

SP: Was this person a geometer?

Kate: I don't remember—it was—we watched the movie a while ago...

SP: Now this idea of yours that math is more than numbers, is that something that you've come to recently or you knew last year and the year before?

Kate: No, recently—in this year.

SP: And can you recall what made that change?

Kate: Umm, because when Miss Nierescu used to say that when because math isn't only the study of numbers—it's a lot more than that. And I think that when you actually think about it—it is.

SP: Did she say that?

Kate: Well, she said it's umm, more –it's not only the study of numbers, and when we watched the movie—that also kind of changed my mind because this man is studying nature from math. And that was cool.

SP: Let's take a look here and basically in September you said that you enjoyed your math class and there's no change—you feel that you enjoy your math class. At that point you said that you disagreed that a mathematician's work looked like fun to you; now you're not sure. So does that indicate a shift in any way?

Kate: Because umm, first I thought it was kind of boring because, who wants to do math all day? And then I thought, well, maybe it's not so bad; maybe it's kind of fun because if you like math then you'll like being a mathematician. It all just depends and I think it wouldn't be so bad, now.

SP: Well, is it because of things that you've seen more recently?

Kate: Yeah.

SP: And you said that you weren't sure about the statement 'I would never think of becoming a mathematician.' And now you agree: You wouldn't think of becoming a mathematician.

Kate: No because, I like math a lot and it's just not something that I would like to do. I want to be—I want to study science, not math. I mean I like math at first and I know you have to take math in school but I wouldn't want to be a mathematician.

SP: Now originally this space was left blank; and it may have been an oversight, but...

Kate: Uh huh.

SP: So you didn't respond so there's nothing to compare it, so I'll just ask you, Do you believe that in September that you agreed –would have agreed, or do you feel more confident more recently.

Kate: I would have put—then I would have put not sure.

SP: And now you feel that you do agree that you feel more confident?

Kate: Yeah.

SP: So could you explain why?

Kate: I feel—I feel better because I like how Miss Nierescu teaches. If she always—if she like teaches a new subject she's always—she always asks if this makes sense and she won't get mad if you don't understand. And she explains everything really well so if it comes to answering a problem I can answer.

SP: And how did you feel last year?

Kate: Umm, last year I was in elementary school, math was simple.
SP: What school were you in last year?
Kate: P.S.110, that was in Brooklyn.
SP: Now originally you said you really plan to stop taking math as soon as you can. Now you've changed from strongly agree to strongly disagree.
Kate: Oh, I guess—that was a mistake. Because I don't plan to stop taking math 'cause I know I need it.
SP: Now here you said you're not sure if you've met a mathematician; now you feel you have. And you did mention Miss Nierescu.
Kate: In here, I wasn't sure if I met a mathematician because like I've met a lot of people and who knows if they're mathematicians or not. And here I think Miss Nierescu is a mathematician.
SP: Now here you weren't sure if math is a subject where you get to express your own opinions. And here, basically you still feel—you're not sure if you get to express your own opinions in math class. So do you feel that it's different from, say humanities class?
Kate: Yeah, sort of because it's a whole different subject and in math, I can't really explain, but when you really, --I can't really explain exactly what I'm trying to say sometimes 'cause it's confusing and like in humanities, it's easier because you have critical thinking questions and these are math problems and, I mean sometimes you can express your own opinions because that's true, you can. But sometimes you can't.
SP: Originally, you said that you agreed that you look forward to taking more math in school, and you feel the same way,--
Kate: Yeah.
SP: Now here, you weren't sure about the statement, 'Mathematics is the study of numbers.' Here you say you disagree with that. And I think you said some things—
Kate: Yeah.
SP: Would you like to add anything to what you said before?
Kate: Umm, no. I pretty much agree with everything I said...
SP: Now here, originally you said you strongly disagreed that you see yourself as a mathematician. Now here you say you disagree. You have to say whether you see there's a difference between strongly, in your mind, a difference between strongly—
Kate: Yeah—
SP: It is—
Kate: Strongly is more like, that yes, I definitely want to be it and just disagree is like no, simply, no. And umm, I think if I ever had a chance, if I couldn't study science, then I don't think it would be so horrible. I mean because math sometimes you do things with science, and a lot of things in science are also part of math.
SP: So, tell me if I'm hearing correctly, you're thinking that this question means in the future—what about now?
Kate: Now, as a mathematician? Umm, no. I still want to continue learning science.
SP: But do you see yourself as being a mathematician?
Kate: Umm, I can't imagine myself, but I don't think it's impossible.
SP: So let's see what you wrote here. In terms of listing reasons to hire a mathematician you said here: to figure out a complicated math problem, to find new things in math; and here, you said 'if you have a

problem which you think a mathematician should solve, to figure out more about mathematics using the random problems in the world.' Do you see those as similar or different?

Kate: I think they're sort of different. I think the first one is sort of, umm, sort of like there's 'cause this one means like a complicated math problem and this one is a problem which you think a mathematician should solve. That's pretty much the same thing because if a problem is impossible, which it probably isn't, umm, some people might call a mathematician, or like see a mathematician to see what they get, even though that's probably not an exact reason (laughs) what you would hire a mathematician for.

SP: So when you said 'to figure out a complicated math problem', did you have anything specific in mind?

Kate: Umm, well like probably not. Like I actually sort of did, but it's probably kind of dumb. When like 'cause sometimes when there's problems in like the high school like math books like children ask their parents and the parents don't even know what the answer is. And like usually sometimes a lot of people call the "Dial A Teacher" and maybe there's mathematicians that work there.

SP: Did you have that in mind here or is there something different here—on the yellow page?

Kate: Umm, that's pretty much the same.

SP: Now in terms of your speaking about this, I'm not sure we really need to say so much—you're just talking about the two pictures. Do you want to say any reason why you like fractions but don't feel you enjoy decimals?

Kate: I don't like decimals because they were always just so confusing. I can never understand all the zeroes here and there're so many numbers and then they go on forever. And from the whole numbers and to the decimals and it just always confused me. I could do the simple things, like I could do the regular tenths, hundredths, thousandths, but once you go past millionths, umm I'm out of there. (laughs.) It's really confusing.

SP: What do you like most about mathematics?

Kate: Like my—what I like to do in math?

SP: Yes.

Kate: Umm...

SP: Interpret it the way you like.

Kate: I like a lot of things in math. I like fractions. I like, I also like doing the geometry. I like geometry a lot. That's my favourite. Because I like studying the shapes and the fractions--parts of it and I think that's cool. I mean I know decimals are pretty—you could change the fractions into decimals, but it's always just more confusing—to multiply it and divide it.

SP: And, what do you like least about math?

Kate: Umm, probably—Okay, this is probably, I know--I don't like decimals, but, I also don't like multiplying and dividing by like, when you have, by like five-digit numbers and stuff. When you have like 468 divided by like 2,800 or something. And I don't—'cause it's just too hard. It's not hard, but it's just that it, it's really confusing 'cause you have these big numbers. Multiplying—I like multiplication but division, it's—I did a lot of that in my old school and I know how to do it—I could do it—it's just that it takes a long time to find the answer because you have to subtract everything and then find the remainders...

Interview with Olivia
Gramercy Middle School, New York City
4 November 1999

SP: What do you like most about math?
O: The challenge. It's like finding out; like even if you're wrong you can still try to prove yourself right. It's like finding out how to do it.
SP: And what do you like least?
O: Fractions. I hate fractions.
SP: What did you feel about doing the discrete math earlier in the year with Miss Nierescu?
O: Oh, I thought that it was fun; I liked it, really. Like with the fractals, and stuff you know I thought we was going to be learning like fractions again and like the perimeter and all—that's what we're doing now. When we was doing discrete math, I was like "I never heard of this before, this is like something new." And it was fun with the shapes and all that.
SP: Did you like doing the work with the graphs?
O: Yeah.
SP: And the shortest paths?
O: Yeah, I like graphs, especially like umm, like the most part I like about it is like how—I don't know I just liked it a lot. And it was so fun—it's like just new and not learning the fractions like how we do like every year we learn the same things...And with this discrete math we learned like—I use it in life, really. I don't know—I just--
SP: Can you give an example?
O: Like umm, with the trains I found myself lost one day and I was like, "Okay, the 2 or the 3 stops at Chambers" –that's where I was trying to get—and so there's the 1 and the 9, and I was like, "Okay, the 1 and the 9 stops at Christopher and the 2 or the 3 doesn't and so I'll go get me there faster," and just found myself making these little dots on my paper, and I was like, "Oh..."
SP: So did it help you to read the subway map?—Because of studying the vertices and the edges?
O: Right—yeah.
SP: Did it give you any better insight into what a mathematician does for a living?
O: Yeah, I usually like thought of a mathematician as like a every day person like you could see a teacher a mathematician, you could see a store clerk a mathematician. And I was finding out like umm the people that found out how to do this discrete math was mathematicians, too. Like everyday people just found out how to this and got credit for it. Like a new type of math.
SP: Let's take a look at your drawings. This is the one you did earlier—on the white page and I see some fractions here. And do you just want to talk about what's happening here?
O: Uh, here, I was trying to draw like a person like how I just explain—like a person finding out like how to do fractions and stuff.
SP: So is this a mathematician or a student?
O: A mathematician.
SP: And he's learning how to do...

O: Learning how to do—He's ummm, learning how—I don't know who invented fractions or something—I was thinking of that person making fractions, writing down, and finding an easier way for people so it wouldn't be so complicated for them.
SP: And in this one? (post questionnaire.)
O: In the yellow page, on the yellow paper, I drew a store clerk sort of, uh down here, I drew a store clerk at a computer, 'cause I remember we just finished buying our computer, so it's like okay, at COMP-USA I seen a lot of mathematicians.
SP: So what would be a definition that you could give for a mathematician?
O: A mathematician has to be like a person that—they don't necessarily have to be doing math or something—it can be like finding a way about math. They don't necessarily have to make math easy—I don't know how to explain a mathematician...
SP: Well, do you see yourself as a mathematician?
O: Yes—It can be a student, cause a student, they are doing math and at the same time they're learning and teaching it to other people.
SP: At first you said you weren't sure if you enjoyed your math class.
O: Yeah.
SP: Now you agree.
O: Yeah.
SP: What do you think made for the change?
O: Because at first I was like, --usually I don't like math. This was like my first year ever really liking math—except for the third grade. And we were doing like the vertices and the vertexes and I wasn't really understanding. Then I had to call "Dial-A-Teacher" –she gave me some number for after-school help and then I started understanding it and realized that it was fun. And I put "I agree." That's it.
SP: Now you haven't changed-you still feel that a mathematician's work does not look like fun to you. Do you want to say anything more about that?
O: I was like thinking about myself becoming a mathematician—it's fun, but then it's—it's fun really, but then when you see a person doing math—sitting at a computer and stuff, to a kid's point of view, it looks boring.
SP: "I would never think of becoming a mathematician."—originally you said you disagreed, but now you feel that you do agree that you would never think of becoming a mathematician.
O: Back then when I did this, I didn't really know anything about mathematicians and all, so now I think about it I would probably put "not sure." Because lawyers do math, too and stockbrokers do too.
SP: "I usually feel confident in math class"--now originally you said you weren't sure, but now you say that you disagree. So do you feel less confident now in math class?
O: A little it.
SP: Is it because of what you're covering right now?
O: No, it's just because I understand it and everything, I know—people tell me that yes, you do it good and stuff but me personally, I don't feel that I understand it. It's like you can understand it; write it down; do the problems and everything; talk about it—but personally I don't feel that I really understand it.

SP: Now interestingly, although you feel that way, originally you said you did plan to stop taking math as soon as you can, but now you disagree. Now you disagree—now you don't think it's a good idea.

O: Because I—in college I want to take math class because the jobs that I have for my future they include math and I know that I, personally I feel that I need more math help.

SP: And now you had said you have met a mathematician, but you forgot the person's name. Now you say there's someone named Mr. Lewis, who works around your neighbourhood and you believe he's a mathematician.

O: Yes.

SP: How do you know that?

O: Because, like, first he told us. 'Cause, I was doing my homework and I remembered this paper and I said, "Are you a mathematician?" And he said yes. And second of all, he likes to make block parties around my block and stuff, and I was looking at him doing his work and he was like, "Okay, our budget cuts say we can have that—that we can't," and that's math.

SP: Originally, you said that you strongly agreed that math was not a class where you got to express your opinions. Now you've shifted a little. So do you feel in class that it's still really not a place where you get to express your own opinions?

O: No, because like if you're wrong in a math problem, you can't just stop the whole class to talk about why you think you're right and stuff. You have to like stay after school with the teacher. And in class you can't express your opinions.

SP: So you feel that for all of your years of schooling, or more recently.

O: That you really can't express you own opinion. You can, but not for a long period of time—as long as you want. You have to wait until--like you're wasting like other people's time. They might just say okay you're wrong there—that's it. You say, like why am I wrong? You can't like do that in math class. You have to wait and she would consider that as like needing help with your problems.

SP: You originally said you weren't sure that you look forward to taking more math in school, but now you don't feel that you look forward to taking more math in school.

O: I don't know, because not more math, because, yeah I don't really want to take more math in school because I know I get enough of it; one, I love expressing my opinions and like challenging, and disagreeing and asking like why. And you just don't get to do that.

SP: How about your shift here: Originally you agreed that math is the study of numbers. Now, you disagree. You want to say something—

O: Because, look at the discrete math. Most kids around my block, and me myself, think that math was just about numbers. And now that we've learned the discrete math you know it's shapes and sizes and things of that. You look at the vertex and that had nothing to do with numbers. It had to do like ways of the path and stuff. We didn't do numbers all that week. We learned just discrete math.

SP: And you originally said, that you strongly disagreed that you see yourself as a mathematician—now you've moved over a little.

O: Yeah.

SP: And then I asked you a little before and you said you did.

O: Because every job that I found myself in the future with, had everything to do with math. A lawyer, a stockbroker, a teacher. A plumber. Even if you want to become that, it has to do with math.

SP: Originally you had said, "It's useful in life; it's not exactly the profession I would choose but all through life we will need it. And then, I don't know, maybe you didn't get a chance to say anything here?

O: Right.

Interview with Robert
Gramercy Middle School, New York City
4 November 1999

SP: I wanted to ask you first, before I ask you about what you drew and what you wrote, what's the thing you like most about mathematics?
R: I don't know—the fact that it's pretty much essential...
SP: And, what would you say you liked least?
R: Well, that it, you know, it's school work, you know...you have a lot of stuff to do and you know...
SP: So are you saying it's a subject?
R: Uh-huh.
SP: You were here last year?
R: Yeah.
SP: Could you talk more about them [drawings]?
R: Well, this one's pretty much just a professor teaching at a college...
SP: This is the yellow [post] sheet.
R: And this one's just a guy trying to complete a formula in his head and he's been working on it for days and days.
SP: What is that?
R: Professor hat. [looks like a mortarboard]
SP: Let me ask about this? [point to $\mathbf{e = mc^2}$] Ever study that in math class?
R: Nope.
SP: So, could you say why that's there?
R: Well, because it's a pretty much a mathematical, or, scientific equation and I thought it was like a little bit appropriate for the guy.
SP: And basically, this is just to represent what?
R: He's dividing and subtracting and doing decimals and stuff.
SP: Do you want to say anything about the way that you drew the face? Does one seem to be a little more cartoon?
R: Yeah, this one is just, he's pretty much enjoying what he's doing.
SP: The professor.
R: Uh-hmm, and he's just got very aggravated because he keeps on failing and doing the same thing over and over again.
SP: So is this a student or a mathematician?—I just want to get clear.
R: Uh, I'm not sure, really, I think it might be a mathematician.
SP: Okay, but he's failing...
R: Uh-hmm.
SP: Either of these a character that you've ever seen before, or did you make them up?
R: Make them up.
SP: So in the beginning of the term you weren't sure if you liked your class then you felt you agreed, you did like it.
R: Uh-huh.
SP: You want to say anything more about that?
R: Umm, yeah. Well, we started to have pretty much some fun with what we were doing.
SP: And what were you doing at that point that you had fun?
R: Umm, like finding like one thing had to be one colour but the other thing

had to be another colour; no two colours could touch.

SP: And what did you like about doing that?

R: Oh, it was so like a brain teaser and it was sort of like a design thing.

SP: Did it give you any more insight into what a mathematician does?

R: Umm, no, not really.

SP: Okay. Here, originally you said that you disagreed that a mathematician's work looked like fun, and you pretty much felt the same way a few weeks ago when you took this. Any change?

R: Nope.

SP: And pretty much the same: you would never think of becoming a mathematician. Is that because of the work they do or because you just haven't made up your mind what you want to do?

R: Umm, I think it's because of both.

SP: And here, you didn't change: usually you feel pretty confident in math class.

R: Yes.

SP: In the last few years, or recently?

R: Last few years.

SP: Is that in this school or also out of this school, too.

R: Out of the school, too.

SP: Now here, originally you said that you plan to drop math as soon as you could, but now you're not sure. Anything you want to comment there?

R: Umm, well because that it's essential, it isn't like it's just something you have to do, something that it's absolutely essential.

SP: And I see no change in terms of that you've met a mathematician. Here you pretty much disagree—you feel you do get to express your opinions in math class.

R: Yeah.

SP: So has that been for the time before you came to the school?

R: Even before.

SP: And you're not sure that you look forward to taking any more math.

R: No.

SP: So, no change there. Here, it said "Mathematics is the study of numbers," and you agreed, but now you feel that you strongly disagree, so--

R: Yeah, because, it doesn't only have to do with numbers; it has to do with, you know, like, angles, degrees, like numbers, and colours, you know, and design, and everything...

SP: And finally, in terms of seeing yourself as a mathematician, that you don't feel that you agree.

R: Yeah.

SP: Do you want to say something about the graph theory and the discrete math that Miss Nierescu was doing earlier in the year?

R: Well, it was fun because it was pretty much discrete—and it didn't have to do with just study, study, study; it had to do with umm, working on like shapes and designs and stuff.

SP: So that was the part that you liked the best?—the shapes and the designs?

R: Uh-hmm.

SP: Did you think that the type of problems were different from what you're doing in CMP? Or were they similar?

R: What?

SP: The type of problem...

R: Umm. Pretty much—they were sort of the same...

SP: Did they seem like the kind of problems you would like use in your real life?

R: Yeah.

SP: And the CMP does as well?

R: Yeah...

SP: Just want to know if you want to add anything or talk about any change in terms of your understanding of why you would want to hire a mathematician?

R: Well, I forgot about tutoring on this one—that's a good—umm, I can't really think of anything really.

Interview with Jason
Gramercy Middle School, New York City
4 November 1999

SP: I want to just start by asking you what you like most about mathematics.
J: I guess I like statistics, 'cause you know I like—every day I read the sports section and there's a lot of statistics. Like you know, football, baseball—I like that a lot, I guess.
SP: What do you like least?
J: I don't like stuff least. I just guess it gets boring like if we do stuff like—for like forever. It seems like that I sort of lose interest in stuff—I mean, that doesn't happen a lot but like we were doing the perimeter thing and area for parallelograms and that really lost my interest but now we're doing something else so, yeah—I don't really have a least favourite thing.
SP: Earlier in the year with Miss Nierescu you looked at discrete math and you were doing graphs and colouring and shortest paths—what did you think about that?
J: It was cool. That was fun. Umm, I liked the map colouring the best.
SP: Any reason?
J: Umm, I don't really know—there was a lot of like estimating going on, and I like that. And it was fun like sorting things; like sorting different colours for different objects and states—and when we were doing the country.
SP: Did it give you any insight into what a mathematician does for a living?
J: No.
SP: So you were asked to draw a mathematician at work and these were drawn at different times and I was wondering if you just wanted to comment. One thing I wanted to ask you about is—
J: Oh, that? Oh, I don't know—I was just drawing math stuff—I guess that's more like science. But you know...
SP: Did you ever study that in a math class?
J: No. I haven't studied that at all—I'm not really sure what it is. I know it's the theory of relativity or something, but I don't know if that's math or science.
SP: So is there any reason why you decided to put that there?
J: I just wanted to put a lot of math stuff on there—like you know I did this and that—dividing and adding and multiplying here. I just put a lot of math stuff on this.
SP: And so this is the mathematician?
J: No. That's me. The mathematician isn't the first person—so like the mathematician had called on me like he was teaching a class or something and I went up so he's watching me. Or she.
SP: Is it--which is it?
J: He or she—it doesn't matter.
SP: So the mathematician is actually somewhere here off this side of the paper.
J: Right.
SP: How about here?

J: Okay—this is a mathematician and then these are students—this one's me. And the mathematician is teaching stuff. And I guess this probably happens before this because then the mathematician calls on someone.
SP: Although this was done a couple of months actually after this one.
J: Yes.
SP: Basically you felt that you agreed that you enjoyed your math classes—no change there. And you also felt that you really weren't sure if a mathematician's work looked like fun to you.
J: Uh-huh.
SP: And here, where you originally agreed that you would never think of becoming a mathematician, now you agree, but it's not as strong.
J: Yeah. I mean, I really don't ever think that I would ever be a mathematician. I can't tell right now.
SP: Now here you said that you usually feel confident in your math class—you strongly agreed, but now you only agree, rather than strongly.
J: Yeah. I don't know. Well, I don't know, I don't really think there's like a big difference between strongly agree and agree....but yeah, I feel confident in math class. I did at the beginning of the year and I do now.
SP: And has that been true for the last few years?
J: Yeah.
SP: And basically you disagreed with the idea of stopping taking math as soon as you can.
J: Right, if I go to college and need to take math, I'll take math.
SP: And you're not sure if you've met a mathematician.
J: Yeah—I'm not sure if a math teacher is a mathematician or not, so... If a math teacher is a mathematician, I guess I have. If not, I'm not sure.
SP: Here there was a change: You said it's not a subject where you get to express your opinions. Originally you disagreed but now you agree.
J: Yeah 'cause I thought in like, I don't know, I thought there was like formulas for everything and like everything had this answer now I know that there's lots of different answers, like particularly with the discrete math. So there wasn't like one right answer or one wrong answer. It was a lot of different ones. So there's more opinion.
SP: So do you feel that you actually *do* get to express your opinions more?
J: Yeah.
SP: Okay, so these are sort of a little reversed. And you didn't change about looking forward to doing more math—you still feel the same way. Here, "Mathematics is the study of numbers," --originally you said that you agreed with that, but now you're not sure.
J: Oh, yeah, in the beginning of the year we didn't do numbers at all—we did discrete math and it was like logic and stuff. So a lot of math is numbers and plenty enough of it isn't.
SP: So have you shifted a little there?
J: Yeah.
SP: And last, originally you said that you weren't sure if you saw yourself as a mathematician—now you don't think you do.
J: I don't. I see myself as a student.
SP: So originally you said math was a large part of science, now you say it's the study of numbers, logic and shapes. So do you think that—
J: I think that both is right. It is a large part of science and it is the study of numbers logic and shapes and stuff like that.

SP: And originally you'd said, and you said again, "I didn't know you could hire a mathematician." So as you've heard the word mathematician all these years, basically what have you had in your mind as a picture of a mathematician?

J: I really don't know, I thought it was just someone studying math by themselves. I guess maybe, a mathematician could teach people but that's the only idea I have—I'm not really sure.

SP: Have you seen any mathematicians like in movies or anything like that, or cartoons or comics?

J: No.

Appendix—C

SIDE 2: PLEASE COMPLETE *AFTER* SIDE 1

I am: Female ❑ Male ❑

		strongly agree	agree	not sure	disagree	strongly disagree
1. I enjoy my mathematics classes.	1.	❑	❑	❑	❑	❑
2. A mathematician's work looks like fun to me.	2.	❑	❑	❑	❑	❑
3. I would never think of becoming a mathematician.	3.	❑	❑	❑	❑	❑
4. I usually feel confident in math class.	4.	❑	❑	❑	❑	❑
5. I plan to stop taking math as soon as I can.	5.	❑	❑	❑	❑	❑
6. I have met a mathematician.	6.	❑	❑	❑	❑	❑

(if you answered **strongly agree** or **agree** to #6, who is the mathematician you have met?

____.)

7. Mathematics is not a subject where I get to express my own opinions.	7.	❑	❑	❑	❑	❑
8. I look forward to taking more math in school.	8.	❑	❑	❑	❑	❑
9. Mathematics is the study of numbers.	9.	❑	❑	❑	❑	❑
10. I see myself as a mathematician.	10.	❑	❑	❑	❑	❑

11. If you have a leaky faucet, you need to hire a plumber; if you break your leg, you need the services of a doctor. List below all the reasons you can think of for which someone would need to hire a mathematician:

12. Look back at the drawing you made of a mathematician at work and write an explanation of the drawing so that anyone looking at it will understand what your drawing means, and who the persons are in it.

13. (***Please complete this sentence:***) To me, mathematics is:

Appendix—D

Pupils' Questions for the Mathematicians Panel (144)

Gramercy Middle School (1 class—13 March 2000)

1. How much money does a mathematician make?
2. As a mathematician, what does your every-day life look like?
3. Who hires mathematicians?
4. Did you always want to be a mathematician?
5. What do you consider a 'mathematician at work'?
6. What are you working on right now?
7. What does a mathematician do?
8. How long does it take to become a mathematician?
9. How long do you go to school?
10. Did you always like math?
11. What makes a mathematician?
12. What does it take to be a mathematician?
13. Where do they (mathematicians) work?
14. Why did you go into mathematics?
15. Who was your idol?
16. What is your favorite area in mathematics?
17. Does the job differ around the world?
18. Do you like what you do?

Hudson Middle School (2 classes—22 March 2000)

1. What do mathematicians do?
2. What kind of jobs have you been hired for?
3. How do you become a mathematician?
4. When did you decide you wanted to become a mathematician?
5. How much do you get paid?
6. What was your education?
7. What are the downsides to being a mathematician?
8. Where do you work?
9. Why do you do mathematics?
10. Have you always been interested in mathematics?
11. What kind of person goes into mathematics?
12. What kind of math do you do?
13. Do you do math differently in your home country?
14. Was there someone in your family who was a mathematician who inspired you?
15. What did you want to be when you were 12-13 years old?
16. How long have you been a mathematician?
17. Who is/was your idol?
18. Do you like being a mathematician?
19. Do you intend to stay a mathematician?
20. Is being a mathematician fun or exciting?
21. Have you ever seriously considered another job?
22. Does it bother you that some people have a stereotype about mathematicians?
23. Have you ever taught about mathematicians?

24. Have you ever had a different job?
25. What job might you like if you weren't a mathematician?
26. If you were offered a higher paying job out of math would you take it?

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27. What got you interested in math?
28. What do you do as a mathematician?
29. What kinds of people do you consider mathematicians?
30. How long did it take you to become what you consider to be a mathematician?
31. How much do you get paid?
32. What did you major in?
33. Do you have a mentor?
34. Who teaches you?
35. Who do you work for?
36. When did you really get interested in math?
37. At what age did you want to be a mathematician?
38. Were you always good at math?
39. Do you also do math for fun?
40. What is the most interesting part of your job?
41. What is the hardest part of your job?
42. As a woman, is it hard to work in this field?
43. Were there any doubts of you because you are a woman?
44. What university did you attend?
45. Did you get good grades in math?
46. Did you go to grad school?
47. Were your parents mathematicians or did their work influence you?
48. Why is math important to you?
49. What do you do when you're not doing math?

Chelsea Middle School (4 classes—24 March 2000)

1. Who do you work for?
2. What do you do?
3. What is different in math in other countries?
4. What inspired you to be a mathematician?
5. What is your definition of mathematics?
6. Are there different kinds of mathematicians?
7. How many hours/days do you work?
8. Do you enjoy your job?
9. Would you recommend a job in math to others?
10. Do you think there will ever be another mathematician like Archimedes or Pythagoras?
11. Do you work in an office or at home?
12. What kind of schooling enabled you to become a mathematician?
13. How do you feel about calculators?
14. If you wanted to pursue a career in math where would you start?
15. How do you use math in your daily life?
16. Were you good in math when you were in school?
17. What was your school experience like?

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18. What is your typical day?
19. How did you do in math when you were a child?

20. Is the math you learned in your country different from the U.S.?
21. What is your goal as a mathematician?
22. What inspired you to be a mathematician?
23. Did you feel as strongly about math [when younger] as you do now?
24. What kinds of [math] problems do you come across?
25. How did your career start?—where did you go to become a mathematician?
26. Which area of math do you like best and what do you specialize in?
27. What is your definition of mathematics?
28. What do you feel is most important to you about math?
29. What theories have you worked on?
30. Have you met any child prodigies?
31. re there any areas in math that you're not as strong in?
32. What type of job is mathematician—I know they do math, but...
33. What do you think the world would be like today without math?
34. Is there a certain mathematician you'd like to be like?
35. Who is your role model?
36. Do you have any other interests besides math?—What are they?
37. What are the benefits of becoming a mathematician?
38. What were your interests as a child?

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39. What exactly do you do?
40. Why do you like mathematics?
41. Where did you learn to do what you do?
42. What type of math did you study?
43. Is there any type of math that gives you problems?
44. Who or what inspired you to become a mathematician?
45. Who do you work for?
46. What are you working on currently?
47. What types of things do you use your skills on?
48. Do you ever get frustrated?
49. When did you get really focused on math?
50. Was it harder to become a mathematician as a woman?
51. As a woman, were you intimidated in becoming a mathematician?
52. What are some situations in which mathematicians are needed?
53. What tools do you use?
54. Do you have a role model in mathematics?
55. Does anyone treat you as a role model?
56. Are there any other topics you are interested in that are, or are not math-related?
57. Do you enjoy your job?
58. What is your daily schedule like?
59. What is your definition of a mathematician?

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60. What do they (you) do?
61. Are you hired by companies?—What kind?
62. What is a typical day at work for you?
63. How does math vary around the world?
64. Do you enjoy your work?
65. How do you think math will affect our lives in the future?
66. What is your favorite kind of math?

67. Do you get paid well?
68. What is your definition of a mathematician?
69. Is the field of math as open for women as it is for men?
70. How does your job affect the world?
71. How complex does the math you work with become?
72. Have you always wanted to be a mathematician?
73. Is modern technology lowering the number of mathematicians?
74. Is technology altering mathematics?
75. What made you decide to become a mathematician?
76. Why do we need mathematicians?
77. Would we be able to live without math?

Appendix—E

THE MATHEMATICIANS PANEL

6 April 2000 9am-11am

ABBREVIATIONS KEY

Panelists	Students
AL: Alun Llandaff	**FH**: Female—Hudson Middle
DF: Dana Frank	**MH**: Male—Hudson Middle
JG: Juliet Grey	**FC**: Female—Chelsea Middle
JM: James Montauk	**MC**: Male—Chelsea Middle
MM: Michael Mann	**FG**: Female—Gramercy Middle
MN: Marina Nierescu	**MG**: Male—Gramercy Middle
NA: Noemi Anjolo	
SP: Susan Picker	
WM: Wendy Mills	

SP: Good morning. We're still waiting for Gramercy Middle School, so when they come, wherever we are, we'll stop for a minute and allow them to sit down and take their places, but, apparently, going across town and downtown has proven to be a problem for them.

I want to welcome you here. You'll be meeting each of the mathematicians shortly, but I want them to know who is in the audience as well. So I want to first thank Ellen Foote and Ronni Najjar, who are the directors of this beautiful school, for hosting this occasion, and the entire Hudson Elementary & Middle School community. And I want to let the mathematicians know that the seventh graders of Hudson Middle School are sitting here—so if you raise your hands they'll know who and where you are. And I've been in every one of your classes and I know that you all have questions and you're going to get a chance to ask them in a little while. I also want to welcome the entire seventh grade from Chelsea Middle School, I want to welcome your teacher Connie Sachs, and your chaperones, and all of the seventh grade teachers who I understand are here today. And also to Nancy Rockwell, teacher of the seventh grade at Hudson Middle. And hopefully Gramercy Middle will be here soon. I also want to especially thank Barbara and Sean for helping us to set everything up here today. So. I'm going to introduce you to the first mathematician, and what will happen is, with each mathematician who is introduced, they may say a few words or show you something, or they may wait for question later; but I see that most of you have your notebooks out and pens out and be ready to take down questions, and also know if they've answered any of the questions that you've already come to.

So the first mathematician that we're going to hear from this morning is Dana Frank, who is sitting right there. She is currently a mathematics professor at the CUNY campus on Staten Island. And she works in an area called discrete mathematics, which is the study of patterns and arrangements. One problem that she is working on right now arises in the design of computer chips. She has a PhD from MIT, which is one of the main research centers in the U.S. for discrete math. She teaches a variety of classes, including courses in discrete mathematics and geometry for teachers. She considered many different careers, but, while in college, finally decided to stay in mathematics. Please welcome Dana Frank.

DF: Can you hear me? Yes, can you hear me? Okay. I said I was interested in many different careers, and one of them was in art, I think when I was around your age and in high school. It's something that I've still kept an interest in and one of the things I do—you asked what is a typical day like—well, one part of the day you often spend procrastinating—when you're supposed to be doing your work. And one of the things I do while I'm procrastinating, is make things that are somewhat mathematical and somewhat artistic. So here's a cube; this is made out of paper—out of origami. And I can actually pass these around if they come back. I would like them back. That's a cube. Does anyone—I don't know—in the front row, can you see this? What is it? (*Students say it's a dodecahedron.*) Yes, good—it's a dodecahedron. I'll pass this around also. This was designed by a friend of mine. And I'm going to tell you just a little bit about a problem—a math problem that I worked on, related to these shapes, that was—it was a fun problem for me. This is not the problem that's related to computer chips—I decided to tell you about a different problem that I worked on. So let me go over to the overhead and show you.

Okay, is this on—can you hear me? Okay, this is the problem that I worked on. Hamiltonian cycles on polyhedra. So polyhedra are things like cubes and dodecahedron. Umm, Hamiltonian cycles—what's that? Ah!—well, start on a corner of a cube, or your favorite polyhedron, and try to visit all of the corners—all of the vertices, by walking along the edges. And you only want to visit each one once, and return to where you started. So this was a problem that a friend of mine asked. He said, "Is there a—for simple polyhedras—is there a nice way to construct these?" So believe it or not, this is a cube. (*A planar graph of a cube is placed on the overhead.*) Okay, can you tell that's a cube?—Good. Okay—you want to visit all the corners--only once--and return to where you started. So try to think how to do that for, uh, two seconds. Here's my answer. So walk around the top; switch down to the bottom and come back around and then come back to the top. Okay—that's a cube. Here's a dodecahedron. (*Places up a planar graph of a dodecahedron.*) Can you tell that's a dodecahedron? No?—okay, if you look at a dodecahedron in the right way—this is it. And it turns out you can do a similar kind of walk. You can spiral around—just like on the cube. (*Shows a solution—there is a minute break in the tape here.*)

Okay, so here's a crazy polyhedron (*Places up a more complicated planar graph similar to the dodecahedron.*) that I did not make out of origami. Okay, and this is a—it's got lots of layers. And in fact, when I—the problem that I solved, is to show that no matter how many layers I added, no matter how complicated it got, I could always find one of these paths. Okay, I'm sure, that you can look at this picture and instantly see that there is a Hamiltonian path here. Right? Okay. Here is one, again, another spiral believe it or not, so I can go around the top; switch over to the outside; make these two arms—spiral arms, like a galaxy, (*The acetate slips.*) oops, get back on there; switch down, then spiral around to the top. Okay, so I can do that no matter how many layers I have, and not matter how complicated. Okay—Thank you.

SP: The next speaker I'd like to introduce you to is Michael Mann, who is head of the Specification and Algorithm Research Department at AT&T Labs. He is a recognized expert in computer security and distributed computing. He has a Ph.D. in Information and Computer Science, and has published more than thirty-five research articles and holds four patents, three in the area of computer security. In 1995, he co-authored the book, *Atomic Transactions*. Please welcome Michael Mann.

MM: Good morning. I really appreciate you taking the time out—sacrificing your class time to come here this morning, and join us. I know that's a big burden for you. Susan sent the questions that you had asked to us, and you asked some really fantastic questions, so—more than we could certainly answer in the time we have, but I'll try and address some of them. A lot of your questions are, to the question of what do mathematicians do—what happens during the day and when a mathematician is doing work. Well, the kind of mathematics that I do as Susan said, is involved with computers. And a lot what we do is to figure out ways that computers can count things or manipulate things and counting doesn't sound very interesting, but it's actually very fascinating, and one of the things I think that you might find surprising is how much fun we have working together. That mathematics, when you're doing it, is actually a very social activity. A lot of it is like working on word problems. How many of you, in your math classes, have struggled to work on word problems, as opposed to just problems that are already just with numbers? (*Many hands go up.*) Okay...So doing mathematics is like doing word problems, except that the word problems you're given, you probably know that there is an answer and that you can find the answer using the techniques earlier in the book. So the kinds of word problems that we work on, we don't have the luxury of knowing all the time that there is an answer. But we work together, to first of all come up with the word problems—the mathematician who can generate interesting problems, that's a friend you want to have if you're a mathematician. Finding the right problems is really, really an important skill. And then, working together to explore different possible ways of attacking a problem and the kind of fun you have communicating, and then when you actually solve a problem, and you're able to communicate that solution to somebody else—'cause that's basically what a proof is, as a way of showing somebody else what the answer to a problem is, and convincing them. And then the next step, after you've convinced them with a blackboard, after you've found a solution, is to write it up in a paper, and have that paper published. So, I have a viewgraph of my own, (*goes to overhead projector; puts up an algorithm.*) So, Susan asked us to tell you what we're working on now—now I don't expect you to read and understand this in the few seconds it'll be up here, but a problem I'm working on which is not very applied—it's a theoretical problem to show the limits of what you might do with computers—is to figure out how a group of computers can elect a leader, sort of like we're about to do in November, as a president. Where you don't know which computers are actually going to come out say, in the internet to be involved, and decide which of them is going to be a leader. And we found an algorithm which we're trying to write down here, on this text, in which these computers can elect a leader when all they can do is read and write in the same place. Each one reading and then writing, and reading and writing, and over writing whatever another computer might have done. So it's a very abstract problem without important applications. That's one kind of problem that I'm working on. Another kind is, I'm working with some colleagues in Israel on ways to route a packet through the internet so that when there's a failure, the packets can be rerouted efficiently around that failure. And it's a lot of fun working with people remotely, using the internet to study problems like this. Especially, one of the advantages of working with somebody as far away as Israel, is that I can work on it during the day, and then I send them the results of how far I've gotten so they can—they wake up way before I do, while I'm still asleep—they can work on it and send it back to me. That's very important when you're working on deadlines—to be able to be working 24 hours a day and still get a little sleep because you're in different time zones. And so again, there's this social interaction that I think is a surprising part of mathematics. Another question you asked is, "How much money do mathematicians make?" (*Some laughter.*) And that's a reasonable question if you're thinking about a career—is this a career you can make a good living at? You can certainly make a good living, at the limits—there are very big extremes. I have a colleague, a friend at MIT, who's in the math department, who founded a company that is working on storing information and delivering it to the internet. This company has been doing very well, and the stock he owns has made him a billionaire—more than a thousand million dollars. On the other hand, the other day, the stock market had a dip, and on paper at least, he lost a million dollars. So, you can make a lot of money, or lose a lot of money doing mathematics. (*Applause.*)

SP: I'd like now to introduce you to Dr. Juliet Grey and we're very happy that she and Dr. Anjolo are here today, because they're our next door neighbours—they work across the street at BMCC. She is a mathematics educator who has taught the subject at every level from middle school through college; she is currently teaching at the college level. Dr. Grey has also been involved in many professional organizations, including the Mathematical Association of America, the National Council of Teachers of Mathematics, the Association of Women in Mathematics, The New York State Mathematical Association of Two-Year Colleges, and the American Mathematical Association of Two-Year Colleges. So please welcome Dr. Juliet Grey.

JG: Thank you for inviting me. And what I would like to do is take a little time to tell you about my own background and how I came to become a teacher of mathematics. I think you're going to find it very unusual. My first love was not mathematics, like many of my colleagues here on the panel. I do not really consider myself a mathematician. I consider myself a mathematics educator. When I was young, about your own age, I came into a very unusual situation. I was a good student; I could do just about every subject equally well—my first love was really English—I liked to write; I was very picky about the way that my sentences were composed, and I wanted my paragraphs to flow very smoothly so I was a very picky writer. I would take hours choosing just the right word or words to use in a sentence. My adjectives, my adverbs had to be just right to describe whatever it is I was writing about. At one point, my godmother—who was almost like a fairy godmother—she was having some difficulties with her son. Her son was struggling with mathematics so she contacted me and she said, "Can you help us out? Would you please tutor him?" And she said, "I'll pay you." Which were the magic words. Okay, those three words: "I'll pay you." Well okay, you know I'm really busy here, I've got a lot of work of my own, but, I'll try it. And so, I helped him out, and you know, for awhile he was doing better, and then, you know, he went along and he passed, and was promoted, and so this I considered a major accomplishment, but I thought to myself, my God, if I had to do this every day, oh! What a way to earn a living. Okay, it was not my first love. Eventually, I was in high school, and I thought to myself, you know, life is really hard and I'd like to live out the rest of my life a very wealthy person, so I'm going to go into business. And, yes I can do math and clearly there's a need for mathematics and obviously there's more of a need for math people than there is for people who can read and write English. All right, English wasn't the career to go into if you wanted to make any money. Okay, because the only place for people who were good in English was journalism, or teaching, and it didn't seem to offer a lot of money—at least not for me. So I actually wound up in college, as a business major; I have a bachelors in business administration with a major in mathematics—all right, two majors—business and math. Towards the end of my career in college and in business, I was asked to do something that I found very uncomfortable. I was in the world of computers in the early days, 2nd generation equipment I'm sorry to admit—that really dates me. And what I had to do when I was working for a very, very large bank, was I had to survey people in a department and find out what that department was doing. My very ambiguous goal, was to actually replace those people with computers. It was a very, very uncomfortable situation to be in. I had to interview the people who were the managers of the department, had to find out exactly what that department was doing and I had to design a computer system that would replace those people in that department. And it was at that point in my life that I decided that the business world was not for me, and that I was going to have to find something else to do with my mathematics. And I had a sister-in-law who was a teacher, now she's an administrator, and she said, "Well, you know, you should really consider teaching." And I said to myself, "Oh my God, you know now I'm going to have to teach math all day long." Eventually I found, that this was not a terribly difficult thing to do, and after awhile I found out that it was a lot of fun. And I'm going to explain to you exactly why I began to see it as more fun.

(*Goes to overhead projector. Places a yellow overhead transparency which has a very complicated, busy map on it called "Mathland" with place names sounding like mathematical terms.*)

When I was a student of mathematics, this is the way I saw math. It's basically like a land that you kind of weave your way through and it turns out that the land is really filled with very, very difficult areas. There's *Compumania*, there's *Statland*, there's *Analytica*; all these different areas in mathematics seemed so disconnected to me. There's *Setheoryoria*, which is basically a theorem place; *Algebria*, *Conjecture Wilderness*; *Topologia*—these areas didn't make a lot of sense. I could navigate my way through them in one way or another. And so this is the way that I really saw mathematics, and unfortunately, in the early days of my teaching career, it was the way that I thought was appropriate, to teach my students. After a while I began to see that mathematics was not really all these separate topics. That was not what the subject was about at all. And I'm going to answer one of the questions that you wrote, okay? For me now, mathematics is something that's totally different from original way that I used to see it, and maybe the way that some of you see it, now. I see mathematics as a science and a language of patterns—a science and a language of patterns. Not something that has to do with all these different areas—not necessarily limited to algebra, and statistics, and calculus, and trigonometry, and so on. But a science and a language of patterns. It's also an art—an art; it's characterized by consistency and order. It's a tool. It's a tool for problem-solving; and it's that simple. It's that simple. And when you begin to see mathematics as a science and a language of patterns, it is then that everything begins to fit together. You can see all the connectedness between all these different areas. And that's what I hope we'll be taking more about today. Thank you.

SP: I want to introduce you to James Montauk. He was born and raised on Long Island, and while trying to decide what to major in at Hunter College, he worked in a variety of jobs, including as a construction laborer and a life-guard. He also performed as a bass player in several really awful heavy metal blues bands. But in time, he decided that majoring in mathematics was a natural next step in the progression.

(The class from The Gramercy Middle School has arrived, about 25 minutes late.)

So we'll let the Gramercy Middle students come in: Welcome.

(Marina Nierescu, their teacher, takes her place on the panel.)

So, I want to just welcome the Gramercy Middle School, and their teacher, Marina Nierescu, and their students felt that she should be on this panel today, and she is. I'm just introducing James Montauk, who after working in several awful heavy-metal blues bands decided that the next progression would be to become a math major. He began his teaching career at Martin Luther King High School near Lincoln Center, and earned an M.A. in mathematics from Brooklyn College and he's now thinking about pursuing a further degree. He's an adjunct lecturer at Lehman College, teaching mathematics to teachers. This is his second year in District 2, his third as a staff developer. He believes that his interest in, and love of mathematics is a natural complement to his love of puzzles and games. James will be answering questions, he's not going to speak right now, but he's going to answer questions shortly. So please welcome him.

And I would like to introduce Wendy Mills to you. When she speaks later, when she starts to answer questions you'll realize that she's come from a few thousand miles. She has an M.A. in engineering from Cambridge University and a PhD in mathematics education from the University of Plymouth. Her first employment was as a design engineer working on turbo machinery for the British Ministry of Defence. (*There's a murmur among the students at this.*) Her research interests are mathematical thinking in engineering undergraduates and multimedia authoring. She also collects interesting oddities in statistics. She lives in Exeter, which is in the southwest of England where she and her husband are raising three sons. Please Welcome Dr. Wendy Mills.

And now I'm going to introduce you to our other neighbor, Dr. Noemi Anjolo. She's an assistant professor of mathematics at Borough of

Manhattan Community College. The co-editor of *Mathematics in College*, a refereed journal of the CUNY Mathematics Discussion Group. She's in *Who's Who Among American Teachers 1998*, and she's worked on *Project Kaleidoscope*; she's part of the *Faculty for the 21st Century*. So please welcome her.

NA: Hi, all of you—how are you doing? That's great. I'll say a little bit about myself and then I'll tell you about what I'm working on. I'm a Nigerian. How many of you know where Nigeria is? (*Almost all hands go up.*) That's good! I'm of Igbo ethnicity. I was born in Nigeria but I didn't necessarily spend all my life there. My early childhood was spent in Sierra Leone. How many of you know where Sierra Leone is? (*Many hands again go up.*) Okay. That's also in West Africa. And I went to primary school there, secondary school; actually I went to one of the oldest girls schools in Sierra Leone, the Anna Walsh Memorial School. I went back to Nigeria for the early part of my higher education—I got my B.S. in mathematics from the oldest university in Nigeria. I worked in Nigeria, both as a statistician, and also as a math professor, and then I came to the U.S. for my higher education; my masters in mathematics from the University of Connecticut and I have a PhD in mathematics education from Syracuse University, where I also minored in cultural foundations. One of the questions you folks asked, is what—how I came to make a choice of my profession. For me it was always easy. Right from the time I was small, I always wanted to be in the profession that I am. And that has to do with the most significant influence in my life, which is my mother. My mother is a mathematician, and she's also a teacher and even though she did not specifically tutor me in mathematics or specifically put mathematical ideas in my head, or push me to do mathematics—in fact she didn't want me to do mathematics—I think I got thinking about the subject and finding the subject attractive from watching her work, and observing what she does. And that's how I got to start thinking about mathematics and start getting to like the subject and seeing it as fun. How many of you see math as fun? That's nice.

I'll tell you a little bit about what I do. How many of you play games? (*Many hands go up.*) Okay. How many of you play strategy games? (*Many hands up.*) Which ones do you play? Chess...what else? Okay. Some of you said board games. How many of you know some of the oldest games in the world? Chess, right, is one of them. (*A student says, "Mancala".*) Mancala—I'm glad somebody said Mancala. (*She puts up an overhead of the game board for Mancala.*) Mancala is one of the things that I am looking at from a research perspective. I consider myself both a mathematician and a math educator. A math educator because I teach mathematics to students, at the college there, and a mathematician because I do research in mathematics. However my area of interest happens to be what they call ethnomathematics. Who knows what ethnomathematics is? (*A boy is called on who has raised his hand, but then he says he doesn't know.*) Ethnomathematics involves looking at the mathematical ideas that are inherent in a particular culture. And when I use the word culture, I'm not necessarily referring to ethnic groups. It could be a culture of work; it could be a culture of what you do as students in a classroom. So culture is used in a more diverse category. Well I'm looking right now at

Mancala and the mathematical ideas that are existing in this game. And in particular I'm looking at the version of Mancala that relates to my own cultural group, the Igbo cultural group in south eastern Nigeria. There are several versions of the Mancala game but the particular version that we played is the version that I have on the board and usually the game board is not a simple board—the game board is also designed in a way that also in itself exhibits mathematical ideas. What type of mathematical ideas can you see from this game board I have just designed for you? Geometry, right? What aspects of geometry are you seeing there? Triangles—so you see some polygons there. So these are some of the things I'm looking at, and some of the mathematical ideas, if you know how to play Mancala, some of the mathematical ideas that are inherent in playing the game, involves counting; is one if the ways in which we teach students at an early age how to count in my cultural group; how to add, subtract, multiply, divide and understand the concepts of one-to-one correspondence. There are geometrical aspects of the game in the game board itself through symmetry; groups of strict design classes like tessellations. You can also use the game to help students enhance their understanding of probability through drawing a Mancala tree diagram for the various strategy choices of conditional probability. And also in the area of combinatorics, because you can use color-coded counters and things like that which will help you to think about counting principles. So these will just give you an idea of some of the things that I do in my area of mathematics. And one thing I like very deeply about Mancala, is that you can easily construct the game. At least this version of the game. All you need is an egg carton—a one-dozen egg carton and maybe pebbles or counters and you've got a Mancala game and you can design the egg box any way you want to design it. Thank you.

SP: I want to introduce you to Marina Nierescu who was born and mathematically educated in her native Romania. She worked for a year as a statistician, before coming here to teach in Community School District 2. Since we did not get a chance to speak earlier, I'm going to ask her right now...(*She indicates that she is prepared to speak.*) Great.

MN: Hi. My name is Marina Nierescu; I come from Romania, as Miss Picker says, and my background is I have a masters in mathematics, from the University of Bucharest, Romania. I worked as she said, as a statistician, and right now I'm teaching at the Gramercy School. I'm teaching middle school. And I was half surprised by Miss Picker's invitation for me to be on this panel. Since, you know lots of people don't consider mathematics teachers as being mathematicians. And I would really like to open this discussion for you—what do you guys think? How many of you think your math teacher *is* a mathematician? (*Many hands go up, but not all and not as many in Hudson Middle as Gramercy Middle and Chelsea Middle Schools.*) --And why? You think they are?—Why? So maybe somebody would like to give a couple of reasons why. Why do you think they're mathematicians? Because people usually think about mathematicians being lots of different ways. Go ahead.

SP: Could she go up to the microphone? There's a microphone over there.

MN: Oh, there's a microphone that's right there. Go ahead.

FC: Our teacher uses patterns and equations to solve problems.

MN: What do you guys think? Do you think that is a sufficient reason why somebody would be a mathematician—to be considered a mathematician? Why not? I know, I'm asking you? Sure. So do you think that's enough?—Somebody works with patterns—and they can be considered a mathematician? (*Student in the audience says, "No."*) So why do you consider your teacher as a mathematician? (*She's good in math.*) Because she's good in math. Okay. I don't know—should I talk a little more about myself?

SP: Well, maybe say something about also Rutgers.

MN: Okay. One other thing that I'm doing right now and I consider very exciting, is I take over the summers, I take courses at the University of Rutgers and these courses are in discrete mathematics. Did you ever hear about discrete mathematics?—How many of you have heard? (*Hands go up, possibly because Dana Frank has mentioned it, but this was before the Gramercy Middle class arrived.*) About this branch—what does discrete mathematics do? I mean we know what geometry means, or algebra and stuff like that. What do you know about discrete math?

MC: Um, I think, I'm not really sure, that it's working with patterns; finding patterns and things-I'm not really quite sure about it.

MN: Okay, I think I saw another hand going up. Uh, discrete mathematics is the branch of mathematics that works with graphs—and not the type of graphs that you guys do in school, like bar graphs or line graphs, and stuff like that. Graphs that are considered—graphs with vertices and edges—you might have heard these words before. Or whenever you do combinatorics you also do the discrete mathematics—it's really interesting and very new. Not a lot of people are going in this direction, so if you ever want to be famous, and you're interested in mathematics, you might want to touch this area because it's, not a lot of people are working on this. Thank you.

SP: The last mathematician we'll hear from before we take your questions is Alun Llandaff who is from Wales. He's a mathematical biologist who studied mathematics at Cambridge University, and then he got his doctorate in the department of biology at Oxford. He uses mathematical ideas and techniques to study problems in medicine and biology, and he's particularly interested in infectious diseases. So please welcome Dr. Alun Llandaff.

AL: So I guess mathematical biology is something you wouldn't necessarily have heard about much at school. But it's quite a new, and very interesting, very exciting area of maths. Where essentially we use mathematical ideas to help understand, for example, disease processes, or questions like oncology, or evolution. And as an example I have an overhead myself. So, I hope you can see that pretty well. Here we have three graphs showing the number of cases of a childhood disease called measles, in three different countries. At the top in Britain, in the middle we have Denmark, which is a smaller European country, and at the bottom Iceland, which is a much smaller country. And the sort of the question we ask is, in the presentation of this sort of data, we look at cases that happened in the past and we want to understand what is going on—why do we see different patterns? What are the similarities and differences in the patterns we see. And one thing is quite striking immediately, it's that we have these repeating patterns—these epidemics. (*He takes out and begins using a laser pointer. The students react to this as students are banned from bringing laser pointers to school.*) One of the perks of being a mathematician you get the chance to have these fancy pointers. You get these epidemics to recur after a few years and once we notice for example in the big countries, these epidemics occur very regularly, and in the smaller countries the epidemics occur regularly but less frequently. And so we use mathematical models as physical techniques to try and understand the similarities and differences between these countries' (*unintelligible.*) (*Applause.*)

SP: So this is what I'd like to do in order to make sure that the questions that you have that you didn't hear answered, and the questions that you've now thought of in the last period as the mathematicians have spoken—what I'd like to do is I'd like to have no more than three people waiting near the mikes. You can take the seats near the mikes—and as you see the people sit down, then you can get up. We'll take as much time as we need to so that your questions are answered. So don't worry about running up or anything. I know that many of you have questions, so why don't we start with three people going over close to the mike and being ready to ask your questions. Okay—you can ask a question of a specific mathematician or you can ask a question in general. Who would like to be second? Great. Why don't you just go up to the mike. And please say what school you're from.

FC: Hi, I'm Cassie; I'm from the Chelsea Middle School. (*There is a lot of applauding and cheering.*)

SP: Okay, what's going to happen is, you're going to slow us down. So we've already applauded for each school so why don't you just let people ask their questions, okay.

FC: Okay, my question is—what is your goal as a mathematician?

NA: I can tell you that my—I think that in my presentation I kind of emphasized one of my goals as a mathematician. We all know that other than the mathematics that exists in Egypt, there's not much said about the mathematical ideas that are inherent in other parts of Africa. And one of my most significant goals, I think, as a mathematician, is to make people aware of those mathematical ideas that are inherent in my cultural group—the Ibo culture. Thank you.

MM: So Cassie, that's one of the questions that stood out for me as well, because I guess I haven't thought recently about what are my goals as a mathematician. And so it was really helpful for me to be confronted with that. I've been doing a lot of management lately—I run a department with—I'm privileged to have eight wonderful mathematicians who are from all over the world—two from Denmark, one from South Africa, one from Israel, one from Canada. And I can spend a lot of time thinking about coaching them, and the work that they're doing. But my personal goals as a mathematician most recently have been to understand problems related to networking and the internet and to do mathematics that is relevant to the efficient management of resources in the internet; and to help us understand that space.

JG: Hold on Cassie; because of my background, as a mathematics educator, I really consider that my goals are much closer to home. I am a person who was born and raised in the City of New York, and I have watched many curriculum changes take place. Uh, from the 60's, the 70's, the 80's, the 90's—I've been teaching for a very, very long time. My goal is to help New York City teachers to teach mathematics more effectively. I am very involved with the mathematical preparation of elementary teachers. I work with undergraduate students; graduate students; and with people who are already teaching in the classroom. So my goal is to raise a new generation of mathematicians among all of you. By working with your teachers. Thank you.

WM: Well I have learned something today, because I have learned that what I am doing is studying the ethnomathematics of engineers. Now I really believe that when Ibo children, Ibo adults play Mancala, they don't think they're doing mathematics. They think they're playing Mancala. And when engineers are doing engineering, they don't think they're doing mathematics—they think they're doing engineering. Okay, I don't know how many of you—how many of you do sports? (*Many hands go up.*) Okay, to help you play these sports better, you do exercises, don't you? That don't look like football; they don't look like baseball; they don't look like basketball, they're just—they're exercises to help you. And one of my goals, is to try to work out mathematics training programs for engineers, so that these are like the exercises that they do to help them to be engineers. And try and stop them thinking they're doing mathematics—try and help them to think that they are doing a training program for their engineering.

SP: (*To student.*) Please come up.

MC: Hello, I'm Jordan from the Chelsea Middle School and I guess you've all met each other before now, (*Mathematicians shake their heads, no.*) or you've met other mathematicians before. So my question is, from meeting other mathematicians, and from yourselves, do you find that people with exceptional mathematical skills also have other exceptional skills in like different topics as well, that usually a lot of mathematicians have in common?

DF: So actually, something that I have noticed, in part because of my own interest in music, is that there are a lot of mathematicians who are involved in music someway. So my own interests are in—especially in dance of various kinds. And I'm currently trying to learn Salsa. But there are a lot of mathematicians who are either musicians or who are involved with other kinds of musical interests.

JG: I'd like to second that. The research has shown that people who are good in mathematics also happen to be good in music. But a long time ago, there was a question that was posed to a very, very intelligent women, Marilyn Vos Savant. Anybody here ever heard of her before?— Marilyn Vos Savant. Okay, she has a reputation for being one of the most intelligent people in the world—a very high I.Q. And people would often write in and ask her different kinds of questions and so she had her own newspaper column. And at one point, someone wrote and said, "You know, I have trouble with mathematics, and I'd like to really know what I can do to improve my performance in mathematics and on tests." And what she did was, she not only pointed to the fact that everyone has their own unique individual talents, but she also said that people who do well in mathematics, tend to be very picky people. They tend to be those people who would notice the minutia—that's the little things. And I think that that's one of the reasons why I did well in mathematics, and remember that my first love was really English—I also loved science. But I was very picky—always very picky. And so I think that it also points towards the different talents or characteristics of the individual who will do well in mathematics.

NA: Just to piggy-back on what Dr. Grey said, I think that part of the training in becoming a mathematician, is it involves you to do a lot of observation, so you find out that people who make good mathematicians usually have very good observational skills in terms of being able to see the patterns that exist in whatever they're doing. In addition to that, usually, those who do a lot of work in geometry are also people who have a lot of artistic skills from the fact that geometry in itself is an art. So I think that's another area that you find mathematicians, especially geometers are having a lot of skills.

JM: I think in terms of other skills, art and geometry and music, there's an art to mathematics as well, in terms of finding patterns and looking at interesting ways to fill space; whether that space is composed of time, as in music, or a piece of paper, as in art. I've found a lot of the mathematicians that I know, are fascinated by puzzles—puzzles and often-times games--in terms of figuring out strategies to win. Or persevering at a puzzle. And I think most people have trouble with mathematics, but mathematicians are the ones who persevere and overcome them—in a lot of ways.

MM: I just wanted to give you a concrete example of an outside interest. A member of my department, a brilliant mathematician who had taught at Yale before joining us, very recently moved on to a new job. How many of you have seen the show, *Futurama*? (*Many hands go up.*) Well, he moved from doing research in Computer Science, at AT&T Labs, he's now writing scripts for *Futurama*. So people with very diverse interests and skills go into and out of mathematics.

MN: I know you guys didn't expect that many people to answer your question. I'd like just to point out first of all to second the opinion about music. Lots of mathematicians are involved with music and with art. And I found, especially in my country, and also here, that they're also interested in philosophy, as a science. Because mathematics pushes you towards purer reasoning but at one point you reach a level where you need something more—so you need another type of reasoning and philosophy will give you that.

FH: Hi, I'm Annie Caruso from Hudson Middle, and I was wondering, what are the down sides of being a mathematician?—If any.

JM: I think one down side that I have experienced an awful lot is when you go out to dinner with friends and you're figuring out the check, and you come up with a bizarre number, and they look at you and go—what's wrong with you—you're a mathematician you should be able to do this easily.

DF: To follow up on what someone else said, that doing mathematics take a lot of perseverance—you have to stick to something. There's a lot of frustration involved, so you have to be able to get through the frustration to the part that's exciting. But you have to be willing to put up with a lot of frustration.

JG: I'd like to answer also. I think that one of the most frustrating things about being a math person, is the reaction of other people, when you tell them what you do. I have sometimes resorted to telling people that I'm a clerk at Bloomingdale's because I just hate the reaction that I see when I tell people what I do. And when I begin to talk about mathematics, then I get an even worse reaction sometimes and it's all because everyone has had, or very many people have had very difficult experiences with mathematics. And so they connect you with part of that very difficult experience, which is unfortunate.

NA: To add to what Dr. Grey has said, I think that one of the down sides is that people think you're a nerd—you can't have fun. But mathematics is fun! You just have to see the fun that is in it.

WM: My personal down side in mathematics is that I get into a problem, and while I'm busy doing something else, I can put the problem out of my head, but when I'm going off to sleep at night, it comes back. And it just keeps niggling away—it keeps me awake, so that's my down side.

FH: Okay, I'm Allegra...

FH: And I'm Gaby, and we're from Hudson Middle. Okay, now we were wondering—and this is only for the women—do you ever get discouraged in mathematics simply because of your gender?

JG: I cannot say that I've been discouraged in mathematics because of my gender, however, when I was a high school student, there was a period of time when I really had as my goal to become an engineer. I have an older brother who went into engineering and who was really part of the reason why I looked at engineering as a career. And at that point I was discouraged—my classmates, my peers—actually looked at me and said, "Are you crazy?--The engineering classes are all full of guys."--and at that point in time it seemed to be very intimidating. Alright, so I was kind of diverted from pursuing that career because of my gender. In terms of the current time, I would say that it's actually more exciting to be a woman in mathematics today. It's certainly still a challenge, however, because of the new vistas that are opening up in technology, it's a very, very exciting time to be a woman who can do mathematics.

MN: If I can second what was said. I never felt that I'm discouraged by my gender. Because I never thought of it as a problem. I'm always proud to be a woman and proud to a mathematician and I've always thought that people who will ever think about this being a problem, should not consider mathematics because mathematics is equally open to boys and girls. And I strongly encourage you to do the same thing—don't even think about this as a problem.

DF: I'd like to respond also. So there definitely were times when I felt discouraged, but it really has to do—it has to do not with ability, but with the social make-up of sciences, and also mathematics. Especially when I first began as a student and would go to a meeting and was surrounded as a 28-year-old by lots of 40-year-old men. It felt very uncomfortable. So it has a lot more to do with who's there. It's sort of like there's a club and it used to be mostly men, and so if you were a woman you felt like an outsider—just the same way you'd feel like an outsider if you came to a new school. Okay—it's that kind of feeling—it's very hard to break in; you're not sure if people are going to accept you. Again, if you hang around long enough, you start to become more accepted—I think it's—and I found now that there are a lot more women so that I feel much more that I'm part of the club. And I think again, it's, there are times when you feel like an outsider, but you have to really be there enough to try to make a difference.

NA: I'll just like to add a few words to that. It wasn't discouragement as such but there was sometimes that sense of isolation and I can say that for my part because when I did my bachelor's degree, I think I was one of three female math students; when I did my master's degree, I was one of a few females, and I think I was the only black female; when I went to do my PhD, I was one of a few and again I was the only black female. And so there's the sense of isolation and what you really need to do is to be able to get a support system. I think one of my greatest strengths was the fact that I really loved what I do; I like the subject; I hung in there and I had a very significant influence in my life, which was my mother being a mathematician and I could always hang onto that fact to keep me focused when ever the isolation or the feeling of isolation started to seep in.

SP: Okay, we still, I see have a number of students who want to ask questions, so what I'll do is, I'll see if we, ourselves, the mathematicians, if we can just give one answer, maybe two, so that we can get as many questions answered as we can in the time remaining, okay.

MH: Well, I'm Peter from Hudson Middle and my question is, would you recommend us to be mathematicians?

NA: Certainly.

MH: What?

NA: Certainly.

SP: It's a good answer.

(*Students applaud.*)

FH: I'm Gwendolyn and I'm from Hudson Middle and I want to know, what was your school experience?

WM: I was at an all girls school so nobody told me that girls don't become engineers; girls don't become mathematicians. The thing at school that really made me think mathematics was interesting, was that probably about your age, I got a math book that had pictures in it. And that was the thing that made me think that mathematics was interesting. It had pictures on set theory; pictures of networks, that sort of pictures, and that was what made me think that mathematics was cool—was interesting.

MC: I'm James, from the Chelsea Middle School and my question is, what is your definition of a mathematician?

MM: Well, I think one of the exciting things about mathematics today, is how many new areas are opening up that use mathematics and involve discovering patterns searching for the underlying structure, and then finding answers and proving those answers, so that you're not just solving one problem, you're able to demonstrate a solution that goes across all possible instances of a problem. And, so my degree is in computer science, a degree that didn't exist thirty years ago, but I do theorems, I work as a mathematician. Biologists now are using mathematics; it's appearing in more and more engineering disciplines. And different kinds of mathematics. In computer security, number theory—one of the greatest number theoreticians ever, delighted in the fact that his work would never be applied, and now fundamental work on prime numbers is what makes e-commerce possible on the internet. So he's probably spinning at some prime rate in his grave. But it's very exciting to see mathematics get out of the schools and universities and into applications in a lot of different disciplines. When I was in high school I looked into becoming a mathematician and all I could find information about was teaching more mathematicians, which is important, but I was wondering why we needed them; or working for an insurance company to figure out at what rate people would die so you'd know what to charge them for life insurance. And there's a lot more applications today, and a lot more opportunities for different kinds of math so it's very hard to pin down a single definition for mathematician.

NA: I would say, someone who does research in mathematics; someone who applies mathematics to solve challenging problems that we have in real life—social problems that we have in life, as well as scientific problems. And also someone who teaches mathematics to get others to think mathematically.

MG: Hi, I'm Matt Renner and I'm from the Gramercy Middle School. What is like the newest field in mathematics?

(*Panelists all look and some point to Alun Llandaff.*)

AL: Well, I'd say, mathematical biology, of course, but I'm a bit biased. But there are a whole load of new fields, computational areas have opened up with the advent of large-scale computers in the last few years, and I think throughout biology particularly, you're going to see a real explosion of mathematical ideas opened up both by—opportunities offered by computers, but also genetic data becoming available through the genome project. And all the opportunities that offers.

FC: I'm Andie—

FC: And I'm Danielle, from the Chelsea Middle School. And we were wondering, if you wanted to pursue a career in mathematics where would you start?

DF: I think one of the things that actually got me involved was a teacher suggesting that I join a math team, and we would get together with other schools and solve problems. It was a competition but it was fun so I would say, start now, pursuing anything in mathematics that looks interesting to you; and in high school, think about doing a summer program where you're with a lot of other students who are interested in mathematics and where you might be doing some research and where you might be getting involved in some projects. But to put yourself in situations where there're a lot of people who are excited about mathematics.

MC: My name is Tony and I'm from the Chelsea Middle School. And I want to know—how do you think math will affect our future?

NA: How does it affect your present? And I think if you have the answer to that, you'll know how it's going to affect your future.

SP: Okay, we don't have a lot of time, in fact we have very little time, so we want to get the questions being asked as quickly as we can.

MC: Well I have another one—what is your favorite kind of math? (*Points to James Montauk.*) Mr. Montauk, what is your favorite kind of math?

JM: I don't think I really have a favorite, because I like all aspects of mathematics. And the thing is, when you're looking at a problem or you're working on something, you looking at it as a ... you never quite know which part of mathematics you're going to bring in to solve that problem. I do have to say though, that I have a bit of trouble at times with combinatorics—counting.

FG: My name is Angelina and I come from the Gramercy Middle School and I have a question for you (*Points to Dr. Anjolo.*) Since you said earlier that you do something with strategies and games, well when I was in fifth grade at PS103, my math teacher had taught us a game called "Tibbie". And I want to know if you ever heard of it? And if so, are you willing to teach it to my teacher to teach it to us?

NA: Actually, I haven't heard of the game, but I think I'm going to go and look and check it out, now that you've mentioned it.

MC: My name is Sam; I'm from the Chelsea Middle School. Is there someone who like teaches you math?

NA: Always, always, always. You learn when you go to conferences; you learn by talking to other mathematicians doing similar things that you're doing; so there's always somebody who teaches you something.

DF: And you also learn from your students. They often come up with interesting ways of solving problems that you had not considered.

MH: I'm Remi and I'm from Hudson Middle. I just wanted to know if any of you have a mentor.

JG: When I first began working at Borough of Manhattan Community College, my first day of work there, the chairperson of the department introduced me to a professor who had been working there for quite some time. And her name was Dr. Sallie Brock. She was introduced to me as my "Big Sister". I'm a very tall person and when I looked at my big sister, she stood only about maybe five foot two—and I'm five-nine. So I couldn't understand how this person could possibly mentor me, or teach me anything. Because I was so much bigger. It turned out that she has been one of the most influential persons in my career. Alright-- Dr. Sallie Brock is well-known throughout the country in her work in mathematics. She is the current president of the American Mathematics Association of Two-Year Colleges and she's also involved nationally and internationally with mathematics. She teaches me.

MH: I'm Matthew from Hudson Middle and I would like to ask—What job would you like if you weren't a mathematician?

NA: That's for all of us? I think I would have been an artist.

WM: I would love to be Terry Pratchett and write science fiction.

MM: I think I would like to teach philosophy; I think if I couldn't do mathematics, I would love to teach philosophy.

JG: I am really a person who's very into mysteries. As a child I loved *Nancy Drew*—maybe some of your grandparents remember *Nancy Drew*. But I think that maybe I would have loved to have been a private detective.

MG: I'm Keith Brock from the Gramercy Middle School and I want to know if you ever have second thoughts about becoming mathematicians.

MM: I think for me, part of becoming a mathematician was trying to figure out whether I could do it. And being in graduate school there were lots of times when I wondered if I would ever manage to get a PhD. There were long dry periods of frustration when I didn't seem to be making much progress. And somehow that frustration—I've seen it happen with other students—sort of builds up until finally you just get so frustrated with not making progress, you go back to some idea that you rejected because it wasn't rich enough or important enough, and you realize when you look at it more carefully, there's a lot more to do and suddenly you're past that point and then it's smooth sailing. So there are times when you question yourself—whether this is going to ever work out, and having friends, and peers, and mentors, is a crucial part of getting over that kind of frustration.

MH: I'm James, from Hudson Middle and I was just wondering what you thought about the stereotype of mathematicians—like with the hat, like they're a wizard or something? (*Laughter in audience.*)

NA: Like all stereotypes—never believe it.

JG: I think that a lot of different (*MM goes backstage to get* his *hat, a black western type, which he puts on his head. More laughter.*) I think that a lot of careers have their—some individuals stereotyped, but I think that in mathematics you have to remember that there are a lot of people who do mathematics--we have examples here on our panel--of people who integrate other careers with mathematics but I think that there's one area of mathematics that is a universal area that really prohibits us from stereotyping the person in any one particular way. And that area is problem solving. Problem-solving is a major area of mathematics but it's a part of every single career that exists. So it's impossible to stereotype, you know, any one person who is a problem solver—think of it that way.

SP: Okay, I'm told that we have time for only a couple more questions.

MH: My name's Jack and I'm from Hudson Middle, and I was wondering—what did you want to be when you were twelve or thirteen years old?

WM: I wanted to be a biochemist.

MM: When I was six I wanted to be a dog. And then when they explained to me that this was unlikely to work out, my second choice was to be a mommy. So I had a frustrating time as a toddler. But when I was twelve or thirteen—I was actually a little bit older than you—I was doing very well in math and science and I had good teachers who reinforced that. And I was only in eighth grade when I first thought that what I would like to do is become a mathematician.

JG: I wanted to be a lot of things. When I was very, very young I liked to write, and I loved science, so I thought maybe I'll be a writer—maybe I'll be a scientist. When I got a little bit older and became more aware of the world around me, I decided that I had pretty decent looks so I wanted to be a model. And then after that I decided that good looks don't last long enough so I wanted to have a more substantial career, and that's how I began to look more at business and more at mathematics, and more at teaching.

AL: I think also by about age twelve or thirteen, I realized roughly what I wanted to do, although at that time I was very into computers. I really wanted to work with those. Maybe I'd be a bit more rich by now if I had stuck with that, but I' happy doing what I'm doing.

MN: I don't think I knew exactly what career I'm going to choose but I was interested in mathematics when I was twelve.

JM: When I was twelve or thirteen I spent a lot of time on the beach, and I wanted to be a professional surfer but that didn't seem like it was going to pay much.

SP: We have time for one more question...

FC: I'm Naomi from Chelsea Middle, and I want to know from those of you who are from foreign countries or who are from other parts of the world, is there any type of math that you feel is done like differently in that country or done a better way or a worse way, in your opinion?

NA: Well, I'm from Nigeria, which is in Africa, and I think that the way they do mathematics, or think about mathematics, they don't think of it as separate from the society. It's integrated in what the people do or actually in how they work. Also, with technology, as you know, and the underdevelopment in Africa, there's bound to be a difference, because a lot of areas in Africa are not as technologically equipped as areas in the developed world and so there's certainly going to be differences in how mathematics is done.

WM: I don't know enough about American mathematics teaching, but I've seen some very exploratory sort of work on squares and rectangles and the areas of them and I think that the English schools are starting to lose the exploratory side of mathematics. Which is something I'm very sad about.

SP: We have one more thing that we need you to do this morning. So if you would take your seats, we need to get some small written reaction from you, so if your teachers could come up and give these out. We'll give you a little time; please just have a pen out. (*Students are given a half sheet on which is written: "Please write some sentences about what you thought about today's Mathematicians panel including what you learned that most surprised you, what questions you still might have, and what you might tell a reporter who interviewed you about today's event. (You may use the front and back of this sheet.)"*

Please join me in thanking each of the mathematicians, and thank you very much.

Appendix—F

UNIVERSITY OF PLYMOUTH

Questionnaire—Research Project
University of Plymouth, England

Thank you for taking the time to answer these questions.

1. Job title:____

2. Job level: University❑ High School❑ Middle School❑ Elementary❑ Industry❑

3. Sex: F❑ M❑

4. Do you think of yourself as a mathematician? Yes❑ No❑ Unsure❑

5. Please use this space to explain why you answered question #4 as you did:

6. Under what circumstance(s) do you think *someone else* is a mathematician?

Appendix—G

[IN PRESS—*Teaching Mathematics in the Middle School*]

Your Students' Images of Mathematicians and Mathematics

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"Mathematics is a discipline that enjoys a peculiar property: everybody has some mental image of it." (Furinghetti, 1993)

What images do your students have of mathematicians and mathematics? Finding out more about your students' images of mathematics can be a way of better understanding their attitudes, misconceptions and opinions of the subject. One way to find these out is to ask your students to create a drawing of mathematician. You may be surprised at the results!

When a group of 7^{th} graders in a New York City middle school was given the assignment, to 'draw your perceptions of a mathematician,' they produced images of which **Figures 1.** and **2.** are examples:

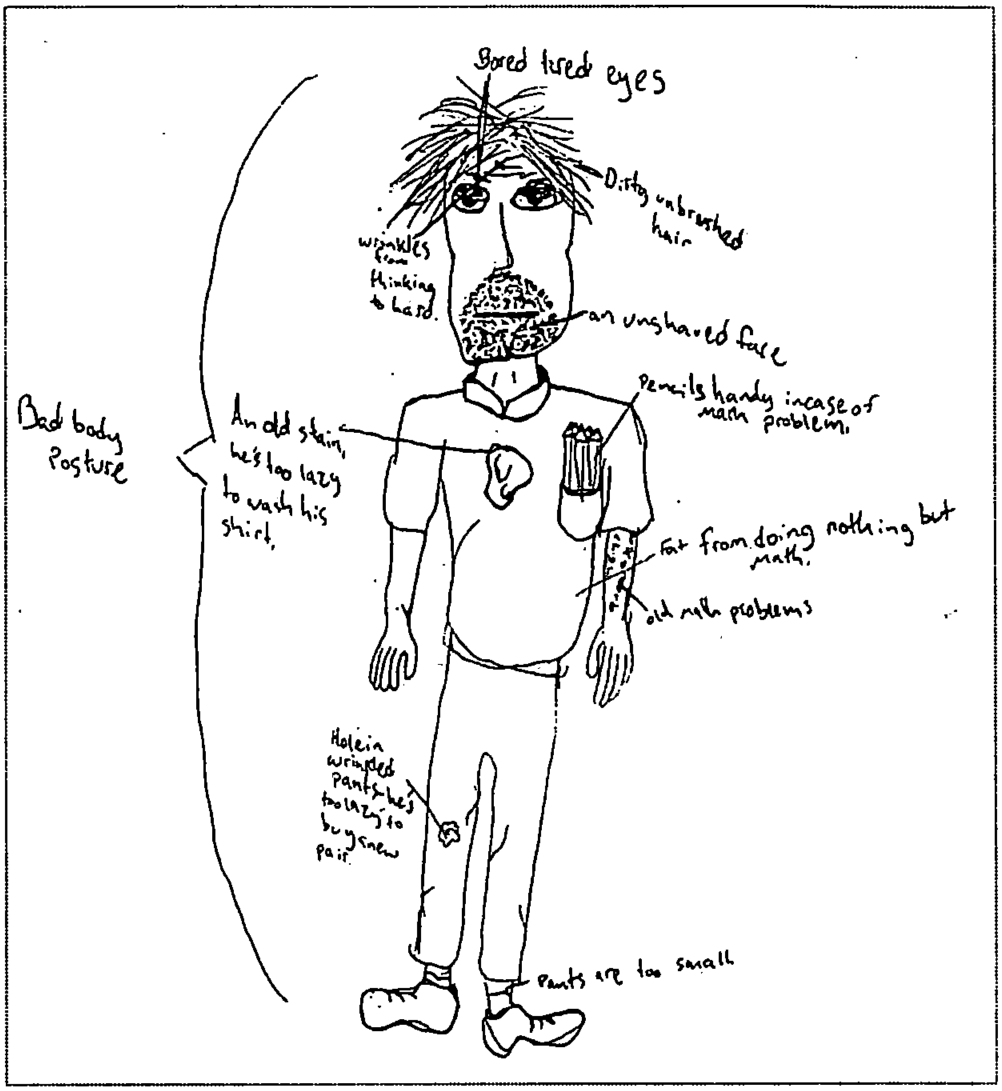

Figure 1. Drawing by male pupil (7th grade—12/13 years old)

Elaborating on his drawing, the student-artist of **Figure 1.** appended the following in a list:

-No friends. (Except other mathematicians)
-Not married or seeing anyone.
-Usually fat.
-Very unstylish.
-Wrinkles in forehead from thinking so hard.
-No social life whatsoever.
-30 years old.
-A very short temper."

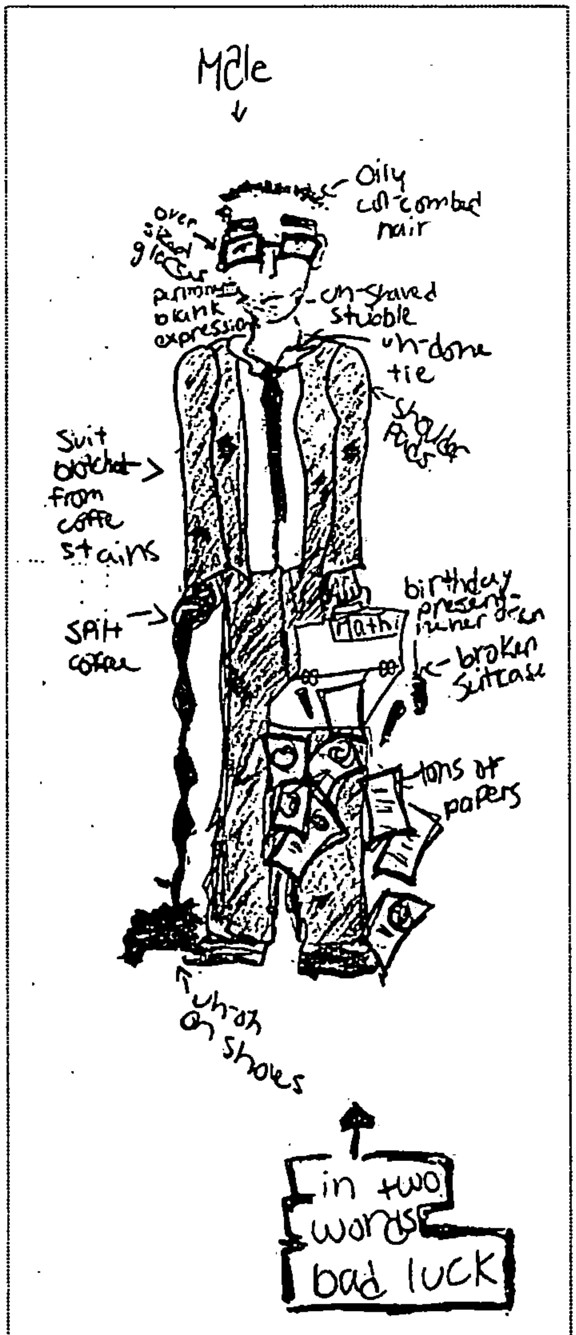

Figure 2. Drawing by female pupil (7th grade—12/13 years old)

We who teach mathematics need to be concerned about these images and what they may indicate about the attitudes of the students having them. It is important too, that we be interested in asking: What do these drawings say about the students who have them? Where do these images come from? What do they show about students' knowledge about what mathematicians really do? Would a student who sees mathematicians in this way ever consider seriously studying mathematics? And most important, how can we oppose and change these views?

Teacher concern about negative images

As teachers we need to be aware of negative images of mathematicians and mathematics held by our students because they tend to have a negative effect on their attitudes to mathematics education:

> "It is a matter of concern that...negative images of mathematics might be one of the factors that has led to the decrease in student enrolment in mathematics and science at institutions of higher education, in the past decade or two....the term 'image of mathematics' refers to a mental picture, view or attitude towards mathematics, presumably developed as a result of social experiences, through school, parents, peers, mass media or other influences." (Lim & Ernest,1998, pp.7-8)

Students (and their teachers!) are as affected by society and the media's views of mathematics as anyone, and the image generally portrayed of mathematics and mathematicians is not a good one. As an often quoted example, explaining to someone that one teaches mathematics seldom elicits any other response than,

> "'I was never good at math,' as if displaying a badge of courage for enduring what for them was a painful and useless experience. In contrast, people do not freely admit that they can't read." (Battista, 1999, p. 426)

Two recent negative examples in the media include the 1998 film, π, in which the protagonist is a mathematician who is psychotic, and the singer Jimmy Buffet's new album, *Beach House on the Moon* which contains a song with the unfortunate title, *Math Suks*. These examples are part of an atmosphere surrounding students in which mathematics looks neither 'cool' nor attractive. As Lim and Ernest (1998) point out, "We need to ascertain how popular or unpopular mathematics is, before we can design measures to improve or promote better public images." (p. 7) We believe this is true for students as well—we need to understand how they see mathematics and mathematicians before we can create interventions.

A small study

A small study was conducted in seven 7th grade classrooms in three schools; one in New York City and two in suburban New Jersey (n = 201). The study included a student questionnaire/survey and interviews with five student volunteers in New York City. The questionnaire/survey consisted of a two-sided sheet on the front of which students were first asked to *draw a picture of a mathematician at work.*

On the reverse side students were asked to indicate their gender, and then asked to respond to a ten statement Likert-type scale dealing with their attitudes to mathematicians and mathematics. This was followed by two open-ended writing prompts: Students were asked to list as many reasons they could as to why someone would need to hire a mathematician; then they were asked to explain their drawing.

The idea of drawing *a mathematician at work*, as opposed to having students draw their perceptions of a mathematician arose after we saw **Figures 1.** and **2.** and then read about studies conducted in which students were asked to draw a scientist, the Draw-A-Scientist-Test (DAST) (see Chambers, 1983; Finson, Beaver & Cramond,1995; Huber & Burton,1995). These early studies with the DAST led to the change in the direction to students to *draw a scientist at work* (Huber & Burton, 1995) which led to a slightly more studious drawing and also gave further insight into how much students knew of the work of a scientist.

The DAST originally arose partly as a result of a pilot study in the 1950's conducted by the anthropologist Margaret Mead and psychologist Rhoda Métraux which explored high school students' images of the scientist (Mead & Métraux, 1957). The researchers' rationale then was their desire to ascertain "...the state of mind of the students among whom the occasional future scientist must go to school and of the atmosphere within which the science teacher must teach. It gives us a basis for reexamining the way in which science and the life of the scientist are being presented in the United States today." (p. 384)

The major finding of their study was that the image of the scientist was "overwhelmingly negative." (p. 384) Arising from the Mead and Métraux study, the DAST has been used in studies which have turned up images of scientists which are highly stereotypical. Matthews and Davies (1999), report that studies to date "have indicated that the stereotypical scientist remains a powerful image in most children's minds." (p. 79). Thus the images of the mathematicians in **Figures 1.** and **2.** led to the idea of looking further to see how widespread these stereotypical images of mathematicians are among middle school students.

Images of mathematicians

In the 201 questionnaire/surveys returned, the images of mathematicians were primarily male, all were white, the majority with glasses and/or a beard, balding or with weird hair, invariably at a blackboard or computer. This created a certain "prototype" that recurred among students in the different classes and different schools.

While the 104 female students created many drawings of mathematicians who were female, the majority of their drawings were of male mathematicians. Of the 93 male students' drawings, only four contained a mathematician who was female. (A figure was missing in some drawings.) (See **Table 1.**) The disparity between the boys and girls in envisioning mathematicians of their own sex is disturbing and shows that even with advances in gender equity reform, girls still lack role models in mathematics.

We don't know how many of the drawings by girls of female mathematicians indicate "wishful thinking," but one girl, a student in New Jersey, wrote the following in explaining her drawing:

"*I drew a woman mathematician because there seems to be only men mathematicians and I wanted to depict a woman doing the work a man usually does. My drawing is of no particular person.*"

Writing in *Education Week*, Patricia B. Campbell and Beatriz Chu Clewell commented on similar results including the small number of females drawn when students are asked to draw a scientist:

> "Nurturing girls' passion for science and mathematics is not easy in our current society…Lurking behind these drawings is the disturbing myth of the math 'gene.' This is the erroneous, but strongly held, perception that there is a genetic or biological basis for gender difference in math." (p. 53)

And while the ethnic backgrounds of the students included in this study were diverse—as diverse as the New York/New Jersey area itself, it is striking that no drawings emerged which represented that diversity. Indeed, in the more than 500 drawings by students we have now looked at in pilot studies, only two drawings have been created in which the character drawn was not Caucasian (see **Figures 3. & 4.**)

The male mathematician in **Figure 3.** was drawn by a young man of African-American descent and the mathematician in the drawing appears to be of African-

American descent as well; the rather beautiful drawing of a male Chinese mathematician in **Figure 4.** was created by a young Asian woman.

This lack of ethnic and female drawings of mathematicians is an indication of a separation between students' views of themselves and views of members of the mathematics community—students' lack of role models. This difficulty in being able to identify with mathematicians makes it far easier for students to continue their stereotypical perceptions of mathematicians than to change them.

As we looked at students' drawings, we noted that when the drawing included a blackboard, one of two types of writings was generally on it: trivial arithmetic, such as 1+1=2; or a meaningless gibberish of mathematical symbols and formulas. Often among these symbols could be discerned $E=mc^2$, and an Einstein-like character or some reference to either Einstein or his famous equation appeared in 50 of the drawings (see **Figure 5.**) When questioned in interviews, however, students did not remember ever discussing either $E=mc^2$ or Einstein in a mathematics class, but they did remember seeing this character in cartoons on television.

On many of the questionnaires, the section which asked students to list all the reasons for which one would hire a mathematician was left blank. When students did write comments they revealed that they didn't really know how to respond. Three students expressed, "*I have no idea why anyone would hire a mathematician,*" as another student confessed, "*I can't think of any reasons.*" Still another student wrote: "*I don't think that you would need one.*"

The top four reasons given by students for hiring a mathematician were, teaching (57); accounting (26); architecture and building (24); and 'to solve hard problems' (12), although no specific type of 'hard problem' was ever mentioned.

By and large it does not seem to be at all clear to students what it is that mathematicians do and what types of problems they can solve:

> "The bottom line for many students is that despite being exposed to mathematics continuously from Kindergarten...the typical [student] cannot connect the value of the study of mathematics with what mathematicians really do. Put differently, students have learned when to "call" or hire a doctor, electrician, geologist, or plumber, but not when to "call" or hire a mathematician." (Malkevitch, 1997, p.93.)

And although the majority of students indicated 'teaching' as the primary reason to hire a mathematician, they appear to see no clear connection between their own teachers and mathematicians. The teacher of the students who drew **Figures 1.** and **2.**, for example, is female, Asian, and at the time of the drawing, younger than 30. The teacher of the student who drew **Figure 5.** is also female. In fact, teachers have reported that they have been asked by their students as they were administering our questionnaire/survey, "Are you a mathematician?"

The Likert-Scale Portion of the Questionnaire/Survey

The ten statements in the Likert-scale portion of our questionnaire/survey (see **Appendix-A)** asked students to respond with *strongly agree, agree, unsure, disagree* or *strongly disagree*. The statements came after students had drawn their picture of a mathematician at work. Of the ten statements we decided to concentrate on five in particular. The other five statements we saw more as distracters. Although the very first statement, *I enjoy the school I attend* was meant as an ice breaker, the overall majority of students in each school indicated that they agreed, showing us that at this age students generally like their schools.

The four other statements we concentrated on were:

(2) *A mathematician's work looks like fun to me.* We hoped to ascertain whether students had a negative or positive view of what mathematicians do, whatever they perceived that work to be. But we found that only 23.8% of the students agreed with this statement and the median response was *disagree.* We believe that this response though, may indicate a general confusion felt by students rather than a dislike, since students could not clearly enumerate what a mathematician might be hired to do.

(3) *I would never think of becoming a mathematician.* With this statement we hoped to understand whether students had any interest in what they perceived the profession of mathematician to be. 45.2% of students indicated that they *agree* with this statement, with 35.3% indicating *not sure.*

We believe that this *not sure* response indicates that students both have too little information about mathematicians and see the idea of choosing a profession as too distant to make a clear choice. The statement was also worded very strongly, "*I would never think...*" and in interviews, students said that they just weren't sure enough yet about what they might want to do in the future.

(7) *I don't enjoy my mathematics class.* We wanted to know what students felt overall about their mathematics classes. 57.5% of students disagreed with this statement indicating that in the mean, students did enjoy their math classes.

(10) *I see myself as a mathematician.* We wanted to see to what extent students identified themselves as mathematicians. 60.2% of students indicated that they *disagree*; 26.3% indicated that they were *not sure*; 12.4% of students indicated that they *agree* with this statement.

We also included the statement, (5) *I would not want to marry a mathematician*, because it had originally been used in the Mead and Métraux study.

We eventually felt that it was not that useful a statement, but we did have a telling instance in New York City where one young man raised his and asked, "*But how are the boys supposed to answer this one?*"

Implications for Pedagogy

The preliminary findings of this study have significant implications for teachers and their practice. Teachers need to be much more aware of the role they can play both in shaping and in changing their students' misconceptions about mathematicians and mathematics.

In one of the few instances we have seen where teachers are encouraged to ascertain their students' stereotypes and misconceptions about mathematicians (McIntosh & Draper, 1997) there is yet no suggestion beyond a possible classroom discussion for opposing and beginning to change these stereotypes. It is furthermore assumed that teachers have none of these stereotypical images themselves. Yet as Furinghetti (1993) points out, "The image of mathematics among professional mathematic[ians] is tortuous and controversial; it should not surprise us, therefore, that for mathematics teachers, deciding what image to transmit to their pupils is a source of doubt." (pg. 33)

At the present time it is very rare to be in a mathematics class and hear the word mathematician used during a lesson. And while we have heard teachers refer to and address their students as scientists in a science class; poets, novelists, or writers in an English class; historians in a social studies class; musicians and singers in a music class, yet it is very unusual to hear students addressed in a mathematics class as mathematicians. Students need to be placed in that role by teachers who have some insight into what it means. There seems to be no other subject studied in school where pupils are placed at such a distance from the discipline than occurs with mathematics.

The ramifications of this distance is explained by Henrion (1997), "When mathematics becomes dehumanized and decontextualized, it is more likely to be seen as irrelevant to students' lives, further discouraging them from pursuing mathematics." (p. 257)

Take time for action

It is important for teachers to become more aware of their own knowledge about what mathematicians do and there is no reason why we cannot learn alongside our students. We may assign students to write reports on famous mathematicians of the past but overlook the fact that this may only reinforce an image of mathematicians as mostly male and of the past.

Instead, consider the following suggestions for uncovering and changing students' stereotypes of mathematicians:

- Give your students a "Draw-a-Mathematician-at-Work" assignment which can then be discussed as an ongoing topic. Once one is clearly aware of it, it is increasingly easy to see that many of the prevailing stereotypes of mathematicians and negative images of mathematics come from the media, including the cartoons and comics which are so popular with students.
- Create a class bulletin board which can begin to educate students in your entire school, called "Mathematicians: The Stereotype vs. The Reality". Student drawings can comprise one side of the display; refutations of the stereotypes based on facts and knowledge can be on the other. Eventually this bulletin board can display pictures of "Young Mathematicians At Work"—your students.

- Invite a mathematician to your classroom to talk to your students and answer questions about the work they do. Teachers are often themselves intimidated by mathematicians, but many mathematicians are concerned about the way they are perceived and would welcome the chance to talk to students about the work they do. Students hearing a mathematician explain how she creates the schedules for college basketball teams would have to be positively and profoundly affected by the experience. Finding a mathematician whose background can enable them to be a role model for your students can pay rich dividends.
- Two recommended websites: A website students can contact to get a clearer view of what mathematicians do is the Society of Industrial and Applied Mathematicians (SIAM) http://www.siam.org whose members have been very interested in reaching out to educators. Another excellent site for students, and particularly young women is NASA's 'Women in Science & Mathematics' site: http://quest.arc.nasa.gov/women . At this website women, including women representing a wide variety of diverse ethnic backgrounds, talk about their work as scientists and mathematicians.
- Two books which can serve as resources: (1) *Women In Mathematics: The Addition of Difference* by Claudia Henrion, 1997, Indiana University Press. Accounts by women mathematicians of their lives, education and professional experiences. The author also writes at length about stereotypical perceptions of mathematicians and how they are perpetuated. (2) *Feisty Females: Inspiring Girls to Think*

Mathematically by Karen Karp, E. Todd Brown, Linda Allen & Candy Allen, 1998, Heinemann. An account of an action research project in which a literature-based approach to mathematics instruction was created in which books which had strong female role models—feisty females—were utilized to successfully affect how students perceived females..

- Finally, consider teaching some non-traditional topics in mathematics such as graph theory, vertex coloring, cryptography, voting theory, tessellations, or fractal geometry. These topics, which are often grouped under the heading of *discrete mathematics* and which can sometimes be found in the "enrichment" section of students' textbooks, are more applied than are many of the topics in the current curricula and as such may also serve to give students a window on what it is that a mathematician does.

Many mathematicians trace their middle school years to the time when they first realized that mathematics was more than just a school subject, but an interesting and exciting profession. Changing your students' negative images may enable the same thing to happen to them.

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