Accounting for multivariate probabilities of failure in vertical seawall reliability assessments

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Summary

The aim of this paper is to appraise the current knowledge on seawall performance and reliability, and to make the case for improved reliability assessments of vertical seawalls, which are used here as a representative for coastal flood defences. In order to achieve this aim, a brief introduction to flood risk management is first given. Then, vertical seawalls are introduced, and their most prominent failure modes are discussed. Reliability analysis is introduced within the context of flood risk management. More specifically, the fragility curve approach that is currently in use in industry is described, and its limitations are discussed. Finally, it is argued that recent advances in multivariate extreme value models would enable improvements to the approaches currently applied in practice. It is stressed that future risk assessment models of coastal flood defences ought to include multiple failure modes and their interactions, a thorough analysis of the model uncertainties, and potential computational costs, in view of providing practitioners with an improved and functional risk assessment tool.

Introduction

A significant proportion of the UK's coastline is protected by seawalls of various types and a broad spectrum of ages and conditions. These seawalls provide defence from coastal flooding and protection against coastal erosion. However, in some cases these structures may be damaged, may become unstable, and may fail. The reliability of coastal flood defences may be influenced by a range of hydraulic forcing parameters, beach levels, scour at the toe of the structure, as well as the condition of the defence. State-of-the-art reliability methods recognise the existence of this wide variety of parameters influencing seawall reliability, and usually involve a multi-dimensional integral to evaluate failure probability and performance. However, state-of-the-art systems approaches still have severe limitations, in particular in relation to a) interactions between the different performance and structural failure modes and their impact on the overall reliability of coastal flood defences; and b) appropriate methodological systems tools for multidimensional, probabilistic fragility risk assessment, and associated uncertainties. Such improvements will have a critical influence on the decisions made by coastal engineers and managers on coastal flood protection management, and on the design of new coastal structures and systems of defences.

Flood Risk Management

Over 5% of the UK population live in the 12200km^2 that is at risk from flooding by rivers and the sea (HR Wallingford, 2000). Many find it desirable to live by the coast, often leading to new development on coastal floodplains. Risk is a central consideration in providing appropriate defences for these types of locations. For example a coastal town would expect to receive higher levels of protection than rural sparsely populated areas (Sayers *et al.*, 2002).

There are many definitions of 'flood risk'. In the UK, when considering coastal flooding, risk is associated with the likelihood of extreme flood events and failure of coastal defence structures, and the damage the events cause to the surrounding area. So, the failure could be related to hydraulic performance, for instance when the overtopping rate exceeds the maximum design overtopping rate, or to structural performance, which occurs when the defence suffers structural damage. In the context of flooding, risk is defined as the likelihood of an event occurring and the impact the event would cause if it occurred:

$$Risk = Probability \times Consequence, \tag{1}$$

where *Probability* is the probability of failure, and *Consequence* is the impact of the event. The consequence can either be desirable or undesirable, but generally with flood and coastal defence schemes the undesirable consequence of loss of life and economic loss are considered. Flood risks can therefore be managed by a variety of structural and non-structural measures. Flood events occur across the world, and lessons are generally learnt from each event. This leads to developments in research, on how risk can be reduced. It is important to understand the level of risk, distinguishing between rare and catastrophic events, and the more frequent but less severe events, even though estimates of risk may be similar.

Over the past decade there has been a move to systems based modelling which looks at the system as a whole. Sayers *et al.* (2002) developed the source-pathway-receptor model for flood risk and this has been an integral part of systems modelling. This approach has been implemented within the national flood risk assessment of England and Wales for a number of years, Hall *et al.* (2003), with improvements to enable catchment-scale analyses implemented by Gouldby *et al.* (2008). Figure 1 illustrates the schematics of the model.

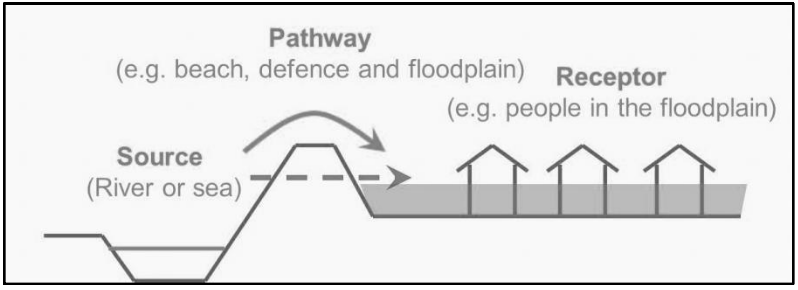

Figure 1 - Schematic of the Source-Pathway-Receptor model (S-P-R) for defining flood risk (HR Wallingford, 2001).

With recent increases in storm frequency and intensity induced by climate change, flood risk management has become a key research area. With a greater understanding, improvements could be made which would help to reduce the effects of extreme flood events on loss of life, damage to properties and infrastructure, and pollution.

As opposed to flood risk management, traditional standards-based design requires structures to be developed with reference to specific structural performance criteria. For example, flood and coastal structures are designed to allow for a maximum overtopping rate of 0.03 l/s/m only, so if the overtopping rate exceeds this value, the defence has suffered a hydraulic performance failure because it did not to meet its performance target. How well a structure performs is dependent on the design criteria for that particular defence. The reliability of a structure depends on the probability of the structure performing its intended function for a specified period of time under stated conditions. The level of service is defined by designers who often incorporate commissioning bodies required limits (Reeve, 2009). Defences may suffer performance or structural failures. Structural failures of vertical seawalls will be discussed in more detail below.

Vertical Seawalls and Structural Failure Modes

Vertical Seawalls: Definition and Characteristics

Seawalls are constructed from a variety of materials including concrete, steel, masonry, rock and timber. Seawalls are generally classified according to their seaward profile as either sloping or vertical. Vertical-type seawalls include vertical, battered and recurved walls. Vertical seawalls can be designed as either gravity or embedded structures. A gravity structure relies on its own weight to achieve stability, whereas embedded structures derive their stability from the passive resistance of the soil in front of the embedded length, and sometimes with external support such as tie rods. A number of different types of vertical seawalls can be seen in Fig. 2.

There are many examples of a range of coastal seawalls, which offer protection against rising sea levels, both in the UK and abroad. For example, approximately 40% of Japan’s 35,500km coastline is lined by concrete seawalls or similar, designed to protect against high wave attack (Onishi, 2011). The world’s largest seawall in Japan failed during the occurrence of the 2011 tsunami event. The National Taiwan Ocean University, on the North-east coast of Taiwan, is protected by vertical concrete seawalls. Given that typhoons occur frequently in this region, several researchers have assessed the damage and failure of these walls due to typhoon wave loading (Chen *et al.*, 2010; Tsai *et al.*,2006).Increasing the understanding of the behaviour and performance of different vertical seawalls is therefore important for flood risk management in areas where these seawalls are used for flood defence.

Figure 2–Examples of Vertical Seawalls

Definition of Failure

Coastal structures can be designed to perform one or more functions at a specified performance level. Failure occurs if the structure does not fulfil one or more of its intended functions. Failure might concern structural failure, i.e. structural collapse, or failure to provide a service for which the structure was designed, i.e. performance failure. Structural failure is known as the structure's ultimate limit state, whilst performance failure is known as its serviceability limit state (EA, 2007). This terminology derives from structural engineering (Thoft-Christensen and Baker, 1982), where failure is categorised into ultimate limit state (ULS), serviceability limit state (SLS), and fatigue limit state (FLS).ULS and SLS are the most common methods of assessing failure of a structure and are used for reliability analysis. The third limit state, the FLS, is not generally considered in flood or coastal engineering reliability analysis. FLS is related to the loss of strength of the structure under repeated loads; this is akin to deterioration (Melchers, 1999).

To assess the reliability of a coastal or flood defence, a limit state equation is applied. This has become a central concept in reliability-based assessment and provides a representation of the strength of the defence, R, and the load on the defence, S. The limit state equation relates to the ULS and SLS of a structure's performance targets. It is the basis for the reliability analysis, and provides providing a model of the reliability of certain failure mode of a structure. The limit state equation is $Z = R - S$, or

$$Z = R(x_1, x_2, \ldots, x_m) - S(x_{m+1}, \ldots, x_n), \tag{2}$$

where Z is the response variable. R is a vector comprising random variables associated with the structure, e.g. crest level, or size of revetment armour unit. $S$is a vector comprising the random variables of the hydraulic loads, i.e. sea conditions typically comprising significant wave height, wave period and sea level (Melchers 1999; Reeve, 2009).

Structural Failure Modes of Vertical Seawalls

Failure modes refer to the different ways in which a structure may fail. However, when failure does occur, it may not always be clear that there is a single mode of failure, it could be a combination of factors. Some structural failure factors, such as the use of novel construction materials that turn out to be not as good as more traditional ones, are controlled by human intervention; others, such as sliding or overturning, are a result of natural coastal processes. This section introduces some of the more prominent structural failure modes affecting vertical seawalls, and the processes affecting these failure modes.

Crest erosion and scour behind structure

Crest erosion and scour behind the structure can lead to seawall collapse. These can occur as a result of excessive wave overtopping, causing erosion of the hinterland and the crest. This failure mode has been incorporated in reliability analyses for coastal and flood defences in the performance curves used in the Environment Agency's national flood risk assessment(EA, 2009).In this analysis, the predominant load parameter causing this failure mode is the overtopping discharge, which fits with the current reliability analysis methodology used by the Environment Agency, that is, the fragility curves. Fragility curves will be detailed in a following section. Figure 3 illustrates this failure mode.

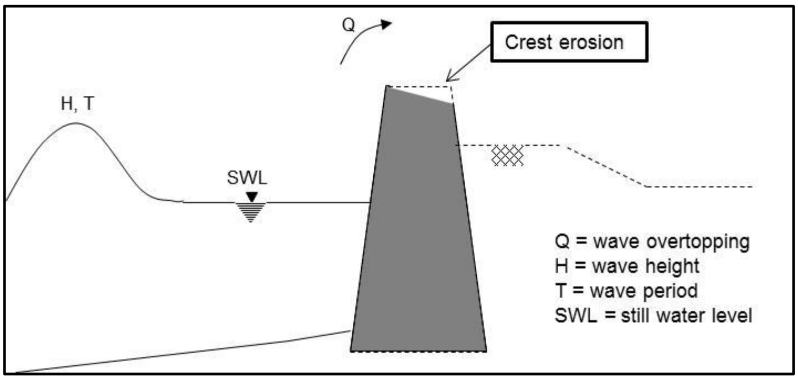

Figure 3 – Scour behind structure diagram

Toe Scour

Scour is the erosion of material by wave and current action at the waterside toe of the of the structure. It was identified in CIRIA(1986) as one of the leading contributors to seawall failure. Toe scour occurs by wave action processes lowering the beach level to below the design level at the toe of the structure. This can lead to destabilisation of the structure, to overturning or collapse, as the foundations are compromised. Toe scour can lead to another failure mode called ‘fill washout’.

Fill Wash Out

Beach lowering and toe scour may lead to voids at the base of the structure. These voids increase in size as more and more material is washed out by wave and current action. Unless scour holes are refilled, eventually soft fill materials within or behind the structure may then be washed out too. Over time, this can lead to large voids within the structure, which in turn may cause the deck of the structure to collapse, or the structure itself to overturn. Fill wash out is a hidden danger, which often goes undetected before collapse occurs. Although not generally noted as a prominent failure mode, it has been observed to be a common cause of failure of structures around the UK.

Overturning and Settlement

Seaward overturning and settlement occurs when the beach level seawards of the structure drops, and the toe of the structure becomes increasingly vulnerable to scour, which leads to a reduction in the passive resistance and the bearing capacity of the foundation soil. This leads to a load from the active backfill pressure, the high groundwater table and the self-weight of the structure, all three causing a bearing capacity failure. This results in overturning of the wall, usually combined with structural settlement (USACE, 2005). Settlement may also occur over a period of time, especially in new construction. It is either due to consolidation of the foundation soil or when the foundation load exceeds the bearing capacity of the soil, a soil mechanics failure (USACE, 2005). Settlement can cause the level of the structure to change or it can begin to tilt seawards.

Excessive overtopping, on the other hand, can lead to scour on the landward side of the structure, and to landward overturning. A possible side effect of this is a reduction in passive resistance from the backfill. The structure may also tilt landwards as a result of impacting wave loads and varying water levels on the front of the structure (USACE, 2005). In the case of block work structures, blocks may be dislodged in to the void created behind the structure, due to loss of passive resistance of the soil.

Removal of Block or Slab Units

Vertical block work can experience removal of their individual block or slab units, due to uplift and seaward pressures that propagate through any cracks in the structure. The pressure forces generated exceed gravity and friction forces, and hence succeed in displacing the units(USACE, 2005). Removal of any unit affects the integrity of the structure. It may lead to a reduced crest height, leading to increased overtopping, or breach, or collapse of the structure.

Reliability Analysis

Overview

Reliability theory provides a means for assessing the performance of existing structures, and quantifying the uncertainties associated with new ones. The aim of a reliability analysis is to quantify either the probability of failure or probability of survival of the structure. In flood risk assessments the probability of failure approach is more commonly applied. Equation (2) introduced the simple limit state equation that defined the reliability function Z (Thoft-Christensen and Baker, 1982). However, there is generally always uncertainty in the strength and/or load variables, therefore R and S are usually assumed to be random variables, characterised by their probability distributions: $f_R(r)$, and $f_S(s)$, respectively. The probability of failure is, hence, calculated from a joint probability density function for resistance and load:

$$p_f = p(Z \leq 0) = \iint_{R \leq S} f_{RS}(r,s) dr ds \tag{3}$$

where $\mathrm{f_{RS}(r,s)}$ is the joint probability density function (Schultz et al., 2010); if strength and load are independent, then

$$f_{RS}(r,s) = f_R(r)\, f_S(s). \tag{4}$$

Most design or risk assessment problems will involve many variables, which increases the complexity. This means that the integration in (3) will need to be carried out over a volume in many dimensions. Another issue is dependence between strength and load variables, as is the case with the effect of beach levels on wave conditions at the toe of a structure (Reeve, 2009). However it is often assumed that R and S are independent, and hence that Eq. (4) holds.

Assuming independence between R and S may simplify the problem, but can introduce significant errors. Dependence of variables is an important aspect to consider even if the reliability method applied is basic, as if independence is assumed the methodology will lead to conclusions that may not be valid for dependent variables (Melchers, 1999). Hence, caution is advised, and measures of uncertainty that take into account the errors likely to result from methodological assumptions need to be carefully considered. Recent improvements in statistical methods have, however, expanded the range of applicability of joint probability methods that capture dependence. These methods are discussed in more detail below.

The Fragility Curve Approach

The probability of failure of a structure, conditional on a specific load, describes the reliability of a structure over a range of loading conditions, through the use of a function rather than a point estimate, providing a more comprehensive perspective on system reliability(Casciati and Faravelli, 1991). The probability of failure has to be manipulated to be conditional on the load, and it can then be plotted to give the identifiable sigmoidal-type curve, such as that shown in Fig. 4 (Schultz *et al.*, 2010). From Fig. 4, it is clear that fragility curves depend heavily on a condition grade (CG), with the probability of failure at a given load increasing with increasing CG. Conditions grades are assigned to a structure using expert judgement, and are associated with several failure modes (Redaelli, 2012). Hence, there are many uncertainties in the estimation of the conditional probability of failure, therefore lower and upper bounds are generally applied (Simm *et al.*, 2009).

The concept of fragility has been widely used in reliability analysis to characterise structural performance over a range of loads. It has been used in flood risk analyses for well over a decade (USACE, 1996). Other examples of system based flood risk analysis models that use fragility include, Vorogushyn *et al.* (2009), and de Moel and Aerts (2011). A comprehensive review of fragility was given in a report by Schultz *et al.* (2010).

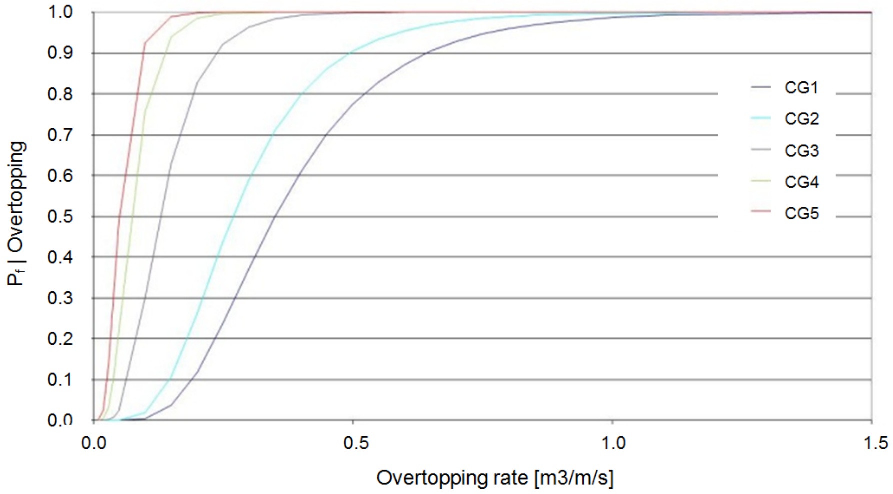

Figure 4 – Example of a fragility curve for a vertical wall using the RELIABLE tool

Fragility curves can be classified into four broad categories: judgemental, empirical, analytical and hybrid. The judgemental approach to fragility curves is based on expert opinion and/or engineering judgement. Generally this method is used as a last resort where data is limited. The judgemental approach appeared in the early development of the fragility curves, for existing levee reliability, to estimate the cost and benefit of flood protection (USACE, 1993). The empirical approach to fragility curves is based on experimental and field observational data. Controlled experiments, where tests can be duplicated, can be undertaken to obtain a systematic set of failure data of a structure at varying loads. The analytical approach to fragility curves is based on structural reliability methods that utilise the limit state functions (Simm et al., 2009). The approach covers analytical and numerical solution methods, such as: first-order and second order reliability methods, Monte Carlo methods, and response surface methods (Schultz *et al.*, 2010). These methods have become the most adopted, both in industry and research. Finally, the hybrid approach to fragility curves combines two or more of the methods discussed.

Multivariate-load Methods

In coastal systems, the *Source* of flood risk is typically represented by a range of variables, significant wave height, period, and sea level, for example. For the analysis of flood risk it is necessary to consider extremes of these variables and also their spatial variation. Assumptions of full dependence or independence between the variables in the joint upper tails are rarely satisfactory and hence multivariate extreme value methods that capture this dependence are often employed. These methods are well established, see Coles and Tawn (1994), for example. These approaches have been applied in the context of coastal flooding for many years, see for example, Tawn (1992), or Hawkes *et al.* (2002). These earlier applications suffered, however, from limitations relating to the dependence structure. More specifically, in the joint tail region, one variable is assumed to be independent of or asymptotically dependent on the other variables. However, a more recent advance in the underlying statistical methods, described in Heffernan and Tawn (2004), has overcome a number of these limitations.

As with many other methods, the Heffernan and Tawn (2004) method proceeds by separating the assessment of the marginal distributions from the dependence analysis, the so-called Copula approach. For the marginal extremes analysis, the standard peaks-over-threshold approach of Davison and Smith (1990), is used: cluster maxima are identified from the time series and the excesses above a suitably high threshold are fitted to the generalised Pareto distribution (GPD). This defines a probability model for large values of each (i) individual variable X_i:

$$P\{X_i > x \mid X_i > u_i\} = \left[1 + \xi_i \frac{(x - u_i)}{\beta_i}\right]_+^{-1/\xi_i} \quad \text{for } x > u_i, \qquad (5)$$

Where β_i, ξ_i are the GPD parameters and u_i is a threshold that is selected. The GPD fit above the threshold is then combined with the empirical distribution of the X_i values to give the semi-parametric function defined by Coles and Tawn (1991). The two-part marginal distribution for each variable is then transformed to Gumbel scales. The vector of transformed multivariate data $\boldsymbol{Y} = (Y_1, \ldots, Y_n)$ are then used for the dependence analysis. The dependence between extreme values of the conditioning variable and each other variable are then analysed using a multivariate non-linear regression model, typically:

$$\boldsymbol{Y}_{-i} \mid Y_i = \boldsymbol{a} Y_i + Y_i^{\boldsymbol{b}} \boldsymbol{Z} \quad \text{for } Y_i > v \qquad (6)$$

where v is a high threshold on Y_i, $\boldsymbol{a} \in [0, 1]$ and $\boldsymbol{b} < 1$ are vectors of parameters and $\boldsymbol{Z}$ is a vector of residuals. To apply the method in practice, once the various parameters are obtained, a simulation procedure is used, whereby a sample of the conditioning variable is combined with the parameter estimates and a sample from the residuals to generate estimates from the other variables. This process is repeated to generate large samples of synthesised data that are used in subsequent analyses.

The method increases flexibility in the dependence structure thereby allowing extension to more variables and also extension to larger spatial scales. These attributes have been explored on a number of relevant studies. Jonathan *et al.* (2013) have applied the method to sea condition variables and Keef *et al.* (2009), Environment Agency (EA, 2011), and Lamb *et al.* (2010) used the method to explore spatial characteristics of fluvial floods. Wyncoll and Gouldby (2013) have applied the method to improve assumptions currently used within the national flood risk assessment of England and Wales, The Environment Agency (EA, 2011) describe how the method has been applied to offshore wave conditions that have then been input to the SWAN wave transformation model. The wave overtopping model used comprises a neural network fitted to data from physical model experiments(Kingston *et al.*, 2008). The output of the analysis can be used to derive estimates of extreme coastal overtopping rates for coastal flood risk analysis. This approach could be used to overcome the simplifying assumption of full dependence of hydraulic loads, currently used within the national flood risk assessment (Gouldby *et al.*, 2008).

Discussion and Conclusions

Flood risk management has emerged as the dominant approach within England and Wales over the past decade. A primary component of this emergence has been the use of quantitative systems models of flood risk. Reliability analysis techniques, used within these systems models, have played an important role in defining the performance of coastal structures. To date however, the risk analysis models have been limited in terms of the number of failure mechanisms used to define fragility, and assumptions relating to the dependence of the hydraulic loads. Methods are emerging that enable multiple failure mechanisms, as well as dependence between multiple variables, to be considered. However, these emerging methods still have many limitations. For example, many failure modes are poorly understood, and hence their associated probabilities of failure may be inaccurately modelled (Redaelli, 2012). It is recognised that good measures of uncertainty are lacking (Gouldby *et al.*, 2008), and such measures can only be quantified with a full probabilistic analysis of both the strength and the load, an analysis of the impact of limited data availability on the probability of failure estimates, as well as the effect of other factors including the level of human intervention. Human intervention could be incorporated into a fatigue limit state component, where the probability of failure associated to the structure's condition is quantified more accurately than with the condition grades currently used. This could then lead to the incorporation of time-dependent structural deterioration in flood risk assessments. Last but not least, multidimensional probabilistic assessments are computationally expensive, as highlighted by Harvey et al (2012), so care must be taken that the tools developed take into consideration the constraints faced by practitioners. However, with cloud computing, novel multidimensional risk assessment techniques and uncertainty analyses will become easier to perform, and thus likely to become more widespread in flood risk practice in future.

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