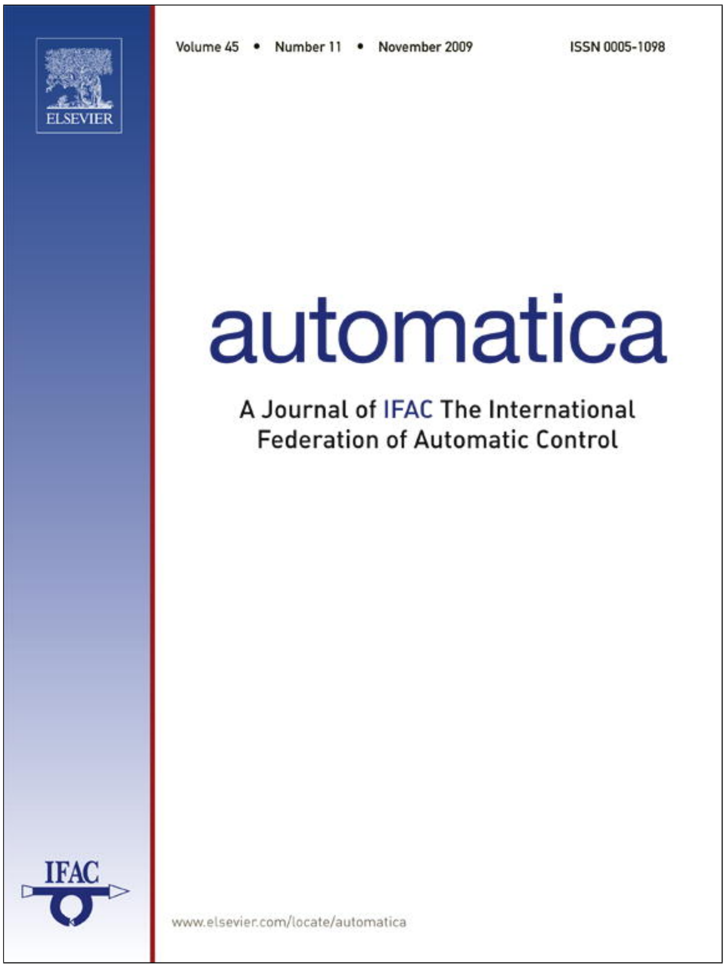
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Robust adaptive control of a class of nonlinear strict-feedback discrete-time systems with exact output tracking☆

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ABSTRACT

In this paper, adaptive control is studied for a class of single-input–single-output (SISO) nonlinear discrete-time systems in strict-feedback form with nonparametric nonlinear uncertainties of the Lipschitz type. To eliminate the effect of the nonparametric uncertainties in an unmatched manner, a novel future states prediction is designed using states information at previous steps to compensate for the effect of uncertainties at the current step. Utilizing the predicted future states, constructive adaptive control is developed to compensate for the effects of both parametric and nonparametric uncertainties such that global stability and asymptotical output tracking is achieved. The effectiveness of the proposed control law is demonstrated in the simulation.

1. Introduction

Robustness in adaptive control has been the subject of much research in both continuous-time and discrete-time, because modeling uncertainties may result in poor performance and even instability of the closed-loop system as observed by Egardt (1979) and Tao (2003). To enhance the robustness of the adaptive control system, many update law modifications were proposed, such as normalization (Goodwin & Mayne, 1987; Tao, 2003) where a normalization term is employed; deadzone method (Egardt, 1979; Peterson & Narendra, 1982) which stops the adaptation when the error signal is smaller than a threshold; projection method (Khalil, 1996; Zhang, Wen, & Soh, 1999, 2001) which projects the parameter estimates into a limited range; σ-modification (Ioannou & Kokotovic, 1983) which incorporates an additional term; and e-modification (Narendra & Annaswamy, 1989) where the constant σ in the σ-modification is replaced by the absolute value of the output tracking error. These methods make the adaptive closed-loop system robust in the presence of an external disturbance or model uncertainties but sacrifice the tracking performance.

On the other hand, adaptive control using the sliding mode has been extensively studied in continuous-time to deal with modeling uncertainty or external disturbance. Recently, many research results of adaptive sliding mode control have also been reported in the discrete-time (Chen, 2006; Chen, Fukuda, & Young, 2001; Lee & Oh, 1998). In contrast to continuous-time systems for which a sliding mode control can be constructed to eliminate the effect of the general uncertain model nonlinearity, in discrete-time the uncertain nonlinearity is required to be of a small growth rate or globally bounded, but sliding mode control is not able to completely compensate for the effect of nonlinear uncertainties in discrete-time.

As a matter of fact, adaptive control design for discrete-time systems is much more difficult than for continuous-time systems. As indicated in Xie and Guo (2000), when the growth rate of the uncertain nonlinearity is larger than a certain number, even a simple first-order discrete-time system cannot be globally stabilized. In an early work (Lee, 1996) on time-varying systems, it is also pointed out that when the parameter time-variation is large, it may be impossible to construct a global stable control even for a first order system. On the other hand, the main stability analysis tool in discrete-time adaptive control, the Key Technical Lemma in Goodwin and Sin (1984), becomes not applicable for the unknown parameters multiplying nonlinearities that are of growth rates faster than linear.

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Due to these difficulties, discrete-time counterparts of continuous-time systems remain largely unexplored. In most existing robust adaptive control results for systems with nonlinear uncertainties, asymptotical tracking performance cannot be achieved though global stability can be guaranteed. It is interesting and challenging in discrete-time adaptive control to fully compensate for the nonparametric uncertainties and to achieve asymptotic tracking. Some recent successful attempts to completely eliminate a class of nonparametric nonlinear uncertainty were made in Sokolov (2003) and Ma, Lum, and Ge (2007), but the designs were limited in the first order system with unknown scalar parameter. To explore adaptive control with full compensation of the nonlinear uncertainties for a general class of minimum phase linear system, a novel adaptive control design using gradient update law has been developed in Yang, Zhai, Ge, Chai, and Lee (2008).

Recently, nonlinear systems in the lower triangular form have attracted great interest in discrete-time adaptive control area. Adaptive backstepping design in discrete-time has been proposed in Yeh and Kokotovic (1995) for a class of parameter-strict-feedback systems. Later, robust adaptive control has been studied for parameter-strict-feedback systems in Zhang et al. (1999, 2001) using projection in parameter estimates update law. In Zhao and Kanellakopoulos (2002), a novel parameter estimator is proposed for parameter-strict-feedback systems in the absence of any disturbance and model uncertainties and it guarantees the convergence of estimates to the real values in finite steps. However, it is noted in Ge, Yang, and Lee (2008b) that these results on parameter-strict-feedback systems are not directly applicable to more general strict-feedback systems with unknown control gains. Therefore, following the concept of system transformation in Ge, Li, and Lee (2003) and Ge, Yang, and Lee (2008a), future states prediction based adaptive control has been developed in Ge et al. (2008b). The prediction method has also been extended to output prediction in Yang, Ge, and Lee (2009). For a class of strict-feedback systems with partially unknown control gains and nonlinear uncertainty in the control range (matched uncertainty), adaptive control with uncertainty compensation has been studied in Yang, Dai, Ge, and Lee (2009), in which asymptotic tracking is guaranteed.

In this paper, we further study adaptive control of strict-feedback systems with both matched and unmatched uncertainties. Continuous-time adaptive control for this class of systems has been developed in Polycarpou and Ioannou (1996) and Jiang and Praly (1998). However, the nonlinear damping method used in the these works to counteract the nonparametric uncertainties is not applicable to discrete-time systems, even when the nonparametric uncertainties only appear in the control range. One reason is the difference of a quadratic Lyapunov function in discrete-time does not inherit linearity property of differential of counterpart Lyapunov in continuous-time, the other reason is that in the discrete-time system formulation, the current input only affects future states which are not available for feedback at current step. In this paper, future states prediction approach is developed which extends the prediction methods in Ge et al. (2008b) by introducing auxiliary states and their estimates, based on which prediction can be proceeded with compensation for the effect of unmatched uncertain nonlinearities. In addition, a novel deadzone method is proposed to guarantee boundedness of closed-loop signals. By sorting growth orders of closed-loop signals, it is finally proved rigourously that asymptotical tracking is achieved.

Throughout this paper, the following notations are used.

- $\|\cdot\|$ denotes the Euclidean norm of vectors and induced norm of matrices.
- $A := B$ means that B is defined as A.
- $(\,)^{\mathrm{T}}$ represents the transpose of vector.
- $\mathbf{0}_{[p]}$ stands for p-dimension zero vector.
- Z_t^+ represents the set of all integers which are not less than a given integer t.

2. Problem formulation and preliminaries

2.1. System representation

Consider a class of SISO nonlinear discrete-time systems with both parametric and nonparametric uncertainties in the following strict-feedback form:

$$\begin{cases}\xi_i(k+1) = \Theta_i^{\mathrm{T}}\Phi_i(\bar{\xi}_i(k)) + g_i\xi_{i+1}(k) + \upsilon_i(\bar{\xi}_i(k)) \\ \qquad i = 1, 2, \ldots, n-1 \\ \xi_n(k+1) = \Theta_n^{\mathrm{T}}\Phi_n(\bar{\xi}_n(k)) + g_n u(k) + \upsilon_n(\bar{\xi}_n(k)) \\ y(k) = \xi_1(k)\end{cases} \tag{1}$$

where $\bar{\xi}_j(k) = [\xi_1(k), \xi_2(k), \ldots, \xi_j(k)]^{\mathrm{T}}$ are measurable system states, $\forall k \in Z_{-n}^+$, $\Theta_j \in R^{p_j}$, $g_j \in R$, $j = 1, 2, \ldots, n$, are unknown parameters (p_j's are positive integers), $\Phi_j(\bar{\xi}_j(k)) : R^j \to R^{p_j}$ are known vector-valued functions, $\upsilon_i(\bar{\xi}_j(k))$ are nonparametric nonlinear certainties, $k \in Z_{-n}^+$, which can be regarded as nonlinear model uncertainties, $u(k)$ and $y(k)$ are system input and output, respectively. The control objective is to make the output $y(k)$ exactly track a bounded reference trajectory $y_d(k)$ and to guarantee the boundedness of all the closed-loop signals. It is noted that the nonparametric nonlinear uncertainties $\upsilon_i(\cdot)$ are unmatched (out of the control range). Though matched uncertainties (in the control range) have been extensively studied in the robust control literature (Chan, 1994; Chen, 2006; Chen et al., 2001; Myszkorowski, 1994), which guarantee global stability but not asymptotical tracking performance, there are few results on studying compensation of unmatched uncertainties.

Assumption 1. The nonparametric uncertain functions $\upsilon_i(\cdot)$, are Lipschitz functions with Lipschitz coefficients L_{υ_i}, i.e., $|\upsilon_i(\varepsilon_1) - \upsilon_i(\varepsilon_2)| \le L_{\upsilon_i}\|\varepsilon_1 - \varepsilon_2\|$, $\forall \varepsilon_1, \varepsilon_2 \in R^n$, where $\max_{1\le i\le n} L_{\upsilon_i} < \lambda^*$ and λ^* is a small number defined in (49). The system functions, $\Phi_i(\cdot)$, $i = 1, 2, \ldots, n$, are also Lipschitz functions with Lipschitz coefficients L_i.

Assumption 2. The signs of control gains g_i, $(i = 1, 2, \ldots, n)$ are known. Without loss of generality, it is assumed that g_i are positive with known lower bounds $\underline{g}_i > 0$, i.e., $g_i \ge \underline{g}_i > 0$.

Remark 1. As pointed in Xie and Guo (2000), it is impossible to obtain global stability results for discrete-time controlled system when the nonlinear uncertainties are of large growth rates. Thus, it is usual to assume that the nonparametric nonlinear uncertainties are of small growth rates (Chen, 2006; Chen et al., 2001; Myszkorowski, 1994; Zhang et al., 1999, 2001) or even globally bounded (Chen & Narendra, 2001; Tao, 2003) and their growth rates can be guaranteed to be smaller than a specified constant. In the case that the discrete-time model is derived from a continuous-time model, the growth rates of the nonlinear uncertainties can be made small enough by choosing a sufficiently small sampling time T.

Remark 2. The counterpart of system (1) in continuous-time has been studied in Polycarpou and Ioannou (1996) and Jiang and Praly (1998) by combining backstepping design and nonlinear damping method. However, like high gain control, nonlinear damping is not applicable to discrete-time for complete nonlinear uncertainties compensation. In this paper, a novel design is proposed to utilize states information at previous steps to compensate uncertainties at current step.

2.2. Useful definitions and lemmas

Definition 1 (*Chen & Narendra, 2001*). Let $x_1(k)$ and $x_2(k)$ be two discrete-time scalar or vector signals, $\forall k \in Z_t^+$, for any t.

- We denote $x_1(k) = O[x_2(k)]$, if there exist positive constants m_1, m_2 and k_0 such that $\|x_1(k)\| \le m_1 \max_{k' \le k} \|x_2(k')\| + m_2$, $\forall k > k_0$.
- We denote $x_1(k) = o[x_2(k)]$, if there exists a discrete-time function $\alpha(k)$ satisfying $\lim_{k\to\infty} \alpha(k) \to 0$ and a constant k_0 such that $\|x_1(k)\| \le \alpha(k) \max_{k' \le k} \|x_2(k')\|$, $\forall k > k_0$.
- We denote $x_1(k) \sim x_2(k)$ if they satisfy $x_1(k) = O[x_2(k)]$ and $x_2(k) = O[x_1(k)]$.

Definition 2 (*Ge et al., 2008a*). The future state variables of a discrete-time system are said to be semi-determined future states (SDFS) at the time instant k, if they can be determined based on the available system information up to time instant k, and control inputs up to time instant $k-1$ under the assumption that the dynamics of the plant and the disturbance are known.

Let us consider a class of general lower-triangular nonlinear systems described as

$$\begin{cases} \xi_i(k+1) = f_i(\bar{\xi}_i(k), \xi_{i+1}(k)), \quad i = 1, 2, \dots, n-1 \\ \xi_n(k+1) = f_n(\bar{\xi}_n(k), u(k), d(k)) \\ y(k) = \xi_1(k) \end{cases} \tag{2}$$

with Lipschitz functions $f_i(\cdot)$, $i = 1, 2, \dots, n$, $(n \ge 2)$ and bounded external disturbance $d(k) \in R$. Assuming that there exist constants $\bar{g}_j > \underline{g}_j > 0$ such that the control gain functions, $g_{1,i}(\cdot) = \frac{\partial f_i(\bar{\xi}_i(k), \xi_{i+1}(k))}{\partial \xi_{i+1}(k)}$, $i = 1, 2, \dots, n-1$, and $g_{1,n}(\cdot) = \frac{\partial f_n(\bar{\xi}_n(k), u(k), d(k))}{\partial u(k)}$, satisfy $\underline{g}_j \le |g_{1,j}(\cdot)| \le \bar{g}_j$, $j = 1, 2, \dots, n$, then, we have the following lemmas:

Lemma 1 (*Ge et al., 2008a*). *In system (2), the future states $\bar{\xi}_i(k+j)$, $i = 1, 2, \dots, n-1$, $j = 1, 2, \dots, n-i$, are SDFS, and there exist prediction functions $P_{j,i}(\cdot)$ such that $\bar{\xi}_i(k+j) = P_{j,i}(\bar{\xi}_{i+j}(k))$. In addition, the prediction functions $P_{j,i}(\cdot)$ are also Lipschitz functions.*

Lemma 2. *In system (2), the states and input of the system satisfy $\xi_i(k) \sim y(k+i-1)$, $i = 1, 2, \dots, n$, $u(k) = O[y(k+n)]$.*

Proof. See Appendix A. ■

Lemma 3. *Given a bounded sequence $X(k) \in R^m$. Define $l_k = \arg\min_{l \le k-n} \|X(k) - X(l)\|$. Then, we have $\lim_{k\to\infty} \|X(k) - X(l_k)\| = 0$*

Proof. The proof has been given in Xie and Guo (2000) for $m = 1$ and $n = 1$ and it is easy to extend the proof when m and n are larger than one. ■

3. Future states prediction

According to Lemma 1, there exist prediction functions $P_{n-i,i}(\cdot)$ for system (1) with Lipschitz coefficients L_{pi} such that $\bar{\xi}_i(k) = P_{n-i,i}(\bar{\xi}_n(k-n+i))$. Then, system (1) can be rewritten as follows:

$$\begin{cases} \xi_i(k+1) = \Theta_i^{\mathrm{T}} \Phi_i(\bar{\xi}_i(k)) + g_i \xi_{i+1}(k) \\ \quad + \nu_i(\bar{\xi}_n(k-n+i)), \quad i = 1, 2, \dots, n-1 \\ \xi_n(k+1) = \Theta_n^{\mathrm{T}} \Phi_n(\bar{\xi}_n(k)) + g_n u(k) + \nu_n(\bar{\xi}_n(k)) \\ y(k) = \xi_1(k) \end{cases} \tag{3}$$

where

$$\begin{aligned} \nu_i(\bar{\xi}_n(k-n+i)) &= \upsilon_i(P_{n-i,i}(\bar{\xi}_n(k-n+i))) \\ &= \upsilon_i(\bar{\xi}_i(k)) \end{aligned} \tag{4}$$

are unknown composite functions satisfying the Lipschitz condition.

According to Lemma 3, we define

$$l_k = \arg\min_{l \le k-n} \|\bar{\xi}_n(k) - \bar{\xi}_n(l)\| \tag{5}$$

from which, it is obvious that $l_k \le k-n$. Further, let us define

$$\Delta\bar{\xi}_n(k) = \bar{\xi}_n(k) - \bar{\xi}_n(l_k). \tag{6}$$

To facilitate the adaptive control design, let us consider predicting future states $\xi_i(k+j)$, $i = 1, 2, \dots, n-1$, $j = 1, 2, \dots, n-i$, in the following manner.

First, we define auxiliary states $\xi_i^a(k)$, $i = 1, 2, \dots, n-1$, as follows:

$$\xi_i^a(k) = \Theta_i^{\mathrm{T}} \Phi_i(\bar{\xi}_i(k)) + \nu_i(\bar{\xi}_n(k-n+i)) \tag{7}$$

which include both uncertain parameters Θ_i and uncertain nonlinearities $\nu_i(\cdot)$. From (3) and (7), we have

$$\xi_i(k+1) = \xi_i^a(k) + g_i \xi_{i+1}(k), \quad i = 1, 2, \dots, n-1 \tag{8}$$

and it is easy to derive that

$$\begin{aligned} \xi_i^a(k) &= \xi_i^a(k) + \xi_i^a(l_{k-n+i}+n-i) - \xi_i^a(l_{k-n+i}+n-i) \\ &= \Theta_i^{\mathrm{T}}[\Phi_i(\bar{\xi}_i(k)) - \Phi_i(\bar{\xi}_i(l_{k-n+i}+n-i))] \\ &\quad + \xi_i(l_{k-n+i}+n-i+1) - g_i \xi_{i+1}(l_{k-n+i}+n-i) \\ &\quad + \nu_i(\bar{\xi}_n(k-n+i)) - \nu_i(\bar{\xi}_n(l_{k-n+i})) \end{aligned} \tag{9}$$

where l_{k-n+i} is defined in (5) and it satisfies $l_{k-n+i}+n-i+1 \le k-n+1$.

Let $\hat{\Theta}_i(k)$ and $\hat{g}_i(k)$ be the estimates of Θ_i and g_i at the kth step, respectively. Now, let us define

$$\begin{aligned} \hat{\xi}_i^a(k) &= \hat{\Theta}_i^{\mathrm{T}}(k-n+2)[\Phi_i(\bar{\xi}_i(k)) - \Phi_i(\bar{\xi}_i(l_{k-n+i}+n-i))] \\ &\quad + \xi_i(l_{k-n+i}+n-i+1) \\ &\quad - \hat{g}_i(k-n+2)\xi_{i+1}(l_{k-n+i}+n-i) \end{aligned} \tag{10}$$

as the estimate of the auxiliary state $\xi_i^a(k)$ defined in (7).

According to (8), we define one-step ahead prediction $\hat{\xi}_i(k+1|k)$, $i = 1, 2, \dots, n-1$, as the estimate of one-step future states $\xi_i(k+1)$ as follows:

$$\hat{\xi}_i(k+1|k) = \hat{\xi}_i^a(k) + \hat{g}_i(k-n+2)\xi_{i+1}(k). \tag{11}$$

Similar to (10), the $(j-1)$-step future auxiliary state $\xi_i^a(k+j)$, $i = 1, 2, \dots, n-1$, $j = 2, 3, \dots, n-i$, can be predicted as

$$\begin{aligned} \hat{\xi}_i^a(k+j-1|k) &= \hat{\Theta}_i^{\mathrm{T}}(k-n+j+1)[\Phi_i(\bar{\hat{\xi}}_i(k+j-1|k)) \\ &\quad - \Phi_i(\bar{\xi}_i(l_{k-n+i+j-1}+n-i))] \\ &\quad + \xi_i(l_{k-n+i+j-1}+n-i+1) \\ &\quad - \hat{g}_i(k-n+j+1)\xi_{i+1}(l_{k-n+i+j-1}+n-i) \end{aligned} \tag{12}$$

where $l_{k-n+i+j-1}+n-i+1 \le k-n+j$ holds according to (5) and $\bar{\hat{\xi}}_i(k+j-1|k) = [\hat{\xi}_1(k+j-1|k), \hat{\xi}_2(k+j-1|k), \dots, \hat{\xi}_i(k+j-1|k)]^{\mathrm{T}}$ are predicted states at previous steps.

Then, let us define j-step ahead prediction $\hat{\xi}_i(k+j|k)$, $i = 1, 2, \dots, n-1$, $j = 2, 3, \dots, n-j$, as the estimate of j-step ahead future states $\xi_i(k+j)$

$$\begin{aligned} \hat{\xi}_i(k+j|k) &= \hat{\xi}_i^a(k+j-1|k) \\ &\quad + \hat{g}_i(k-n+j+1)\hat{\xi}_{i+1}(k+j-1|k). \end{aligned} \tag{13}$$

Remark 3. Similar to the future states prediction developed in Ge et al. (2008b), it is noted in the prediction developed above (refer to equations (12) and (13)), the j-step ahead predictions $\hat{\xi}_i(k+j|k)$ are based on the $(j-1)$-step ahead predictions $\hat{\xi}_i(k+j-1|k)$. Additionally, in this paper we have introduced auxiliary states and their predictions, in which the states information at previous steps has been utilized to compensate for the effect of nonparametric uncertainties at the current step as shown in (9) and (10). The effect of nonparametric uncertainties in the parameter update law for future states prediction will be dealt with by a novel deadzone in the parameter update law.

According to the definition of $\nu_i(\bar{\xi}_n(k-n+i))$ in (4), Assumption 1, Lemma 1 and the definition of $\Delta\bar{\xi}_n(k)$ in (6), we have

$$|\nu_i(\bar{\xi}_n(k-n+i)) - \nu_i(\bar{\xi}_n(l_{k-n+i}))| \leq L_{pi}L_{\upsilon_i}\|\Delta\bar{\xi}_n(k-n+i)\| \quad (14)$$

where L_{pi} and L_{υ_i} are Lipschitz coefficients of prediction functions $P_{n-i,i}(\cdot)$ and nonparametric uncertainty functions $\upsilon_i(\cdot)$, respectively.

Let us denote $\hat{c}_i(k)$ as the estimate of L_{pi}. The update laws for $\hat{\Theta}_i(k), \hat{g}_i(k), \hat{c}_i(k), i = 1, 2, \ldots, n-1$, are given as follows:

$$\begin{aligned}
\hat{\Theta}_i(k+1) &= \hat{\Theta}_i(k-n+2) - \frac{a_i(k)\gamma[\Phi_i(\bar{\xi}_i(k)) - \Phi_i(\bar{\xi}_i(l_{k-n+i}+n-i))]\tilde{\xi}_i(k+1|k)}{D_i(k)} \\
\hat{g}_i(k+1) &= \hat{g}_i(k-n+2) - \frac{a_i(k)\gamma[\xi_{i+1}(k) - \xi_{i+1}(l_{k-n+i}+n-i)]\tilde{\xi}_i(k+1|k)}{D_i(k)} \\
\hat{c}_i(k+1) &= \hat{c}_i(k-n+2) + \frac{a_i(k)\gamma\lambda|\tilde{\xi}_i(k+1|k)|\|\Delta\bar{\xi}_n(k-n+i)\|}{D_i(k)}
\end{aligned} \quad (15)$$

with

$$\tilde{\xi}_i(k+1|k) = \hat{\xi}_i(k+1|k) - \xi_i(k+1)$$

$$\begin{aligned}
D_i(k) = {} & 1 + \|\Phi_i(\bar{\xi}_i(k)) - \Phi_i(\bar{\xi}_i(l_{k-n+i}+n-i))\|^2 \\
& + |\xi_{i+1}(k) - \xi_{i+1}(l_{k-n+i}+n-i)|^2 \\
& + \lambda^2\|\Delta\bar{\xi}_n(k-n+i)\|^2
\end{aligned} \quad (16)$$

$$a_i(k) = \begin{cases} 1 - \dfrac{\lambda\hat{c}_i(k-n+2)\|\Delta\bar{\xi}_n(k-n+i)\|}{|\tilde{\xi}_i(k+1|k)|}, & \text{if } |\tilde{\xi}_i(k+1|k)| \\ \quad > \lambda\hat{c}_i(k-n+2)\|\Delta\bar{\xi}_n(k-n+i)\| \\ 0, \quad \text{otherwise} \end{cases} \quad (17)$$

$$\hat{\Theta}_i(0) = \mathbf{0}_{[n]}, \qquad \hat{g}_i(0) = 0, \qquad \hat{c}_i(0) = 0$$

where $0 < \gamma < 2$ and λ can be chosen as any constant satisfying $\max_{1\leq i\leq n} L_{\upsilon_i} \leq \lambda < \lambda^*$, with λ^* defined later in (49). The deadzone defined in (17) is inspired by the work in Chen et al. (2001) and Chen (2006). According to the deadzone (17), we have

$$-a_i^2(k)\tilde{\xi}_i^2(k+1|k) = -a_i(k)\tilde{\xi}_i^2(k+1|k) + \lambda a_i(k) \times \hat{c}_i(k-n+2)|\tilde{\xi}_i(k+1|k)|\|\Delta\bar{\xi}_n(k-n+i)\|. \quad (18)$$

Lemma 4. *Consider the future states prediction laws defined in (11) and (13), in which the estimated parameters are calculated from the update law (15). The estimated parameters $\hat{\Theta}_i(k)$, $\hat{g}_i(k)$ and $\hat{c}_i(k)$, $i = 1, 2, \ldots, n-1$, are bounded and there exist constants $\bar{c}_{n-i}$ such that the future prediction errors satisfy*

$$\|\bar{\tilde{\xi}}_i(k+n-i|k)\| \leq o[O[y(k+n-1)]] + \lambda\bar{c}_{n-i}\Delta_s(k, n-1) \quad (19)$$

where

$$\bar{\tilde{\xi}}_i(k+n-i|k) = [\tilde{\xi}_1(k+n-i|k), \ldots, \tilde{\xi}_i(k+n-i|k)]^{\mathrm{T}}$$

$$\Delta_s(k, m) = \max_{1\leq j\leq m}\{\|\Delta\bar{\xi}_n(k-n+j)\|\} \quad (20)$$

with $\tilde{\xi}_i(k+n-i|k) = \hat{\xi}_i(k+n-i|k) - \xi_i(k+n-i)$ and $\Delta\bar{\xi}_n(k)$ defined in (6).

Proof. See Appendix B. ∎

4. System transformation and adaptive control

In this section, the adaptive control will be synthesized using predicted future states obtained in Section 3. To begin with, let us rewrite system (3) as follows:

$$\begin{cases} \xi_i(k+n-i+1) = \Theta_i^{\mathrm{T}}\Phi_i(\bar{\xi}_i(k+n-i)) + \nu_i(\bar{\xi}_n(k)) \\ \quad + g_i\xi_{i+1}(k+n-i), \quad i = 1, 2, \ldots, n-1 \\ \xi_n(k+1) = \Theta_n^{\mathrm{T}}\Phi_n(\bar{\xi}_n(k)) + g_n u(k) + \nu_n(\bar{\xi}_n(k)) \\ y(k) = \xi_1(k). \end{cases} \quad (21)$$

By iterative substitution, we obtain the following equation from (21)

$$y(k+n) = \Theta_f^{\mathrm{T}}\Phi(k+n-1) + gu(k) + \Theta_g^{\mathrm{T}}\bar{\nu}(k) \quad (22)$$

where

$$\Theta_f = [\Theta_{f1}^{\mathrm{T}}, \ldots, \Theta_{fn}^{\mathrm{T}}]^{\mathrm{T}} \in R^p, \qquad \Theta_{f_1} = \Theta_1, \qquad g_{f1} = 1,$$

$$g = \prod_{j=1}^{n} g_j, \qquad \Theta_{f_i} = \Theta_i\prod_{j=1}^{i-1} g_j, \qquad g_{fi} = \prod_{j=1}^{i-1} g_j, \quad i = 2, \ldots, n,$$

$$\Theta_g = [g_{f1}, \ldots, g_{fn}]^{\mathrm{T}} \in R^n, \qquad \Phi(k+n-1) = [\Phi_1^{\mathrm{T}}(\xi_1(k+n-1)), \ldots, \Phi_n^{\mathrm{T}}(\bar{\xi}_n(k))]^{\mathrm{T}} \in R^p, \quad (23)$$

$$\bar{\nu}(k) = [\nu_1(\bar{\xi}_n(k)), \ldots, \nu_n(\bar{\xi}_n(k))]^{\mathrm{T}} \in R^n \quad (24)$$

where $p = \sum_{i=1}^{n} p_i$ and it is easy to check that $g \geq \prod_{j=1}^{n} \underline{g}_j := \underline{g}$. It is noted that in system (22), function $\Phi(k+n-1)$ involves future states. This is the reason why future state prediction has been carried out in Section 3.

Remark 4. Based on (22), the adaptive control design will be carried out using predicted future states. The future states prediction based adaptive control in this paper extend the authors' previous work (Ge et al., 2008b) by incorporating compensations of nonparametric uncertainties in both stages of future states prediction and controller design. An auxiliary output will be introduced and in the prediction of the auxiliary output, the effect of $\bar{\nu}(k)$ will be compensated for using the similar idea used in Section 3 for future state prediction.

Let us introduce an auxiliary output $y_a(k)$ as

$$y_a(k+n-1) = \Theta_f^{\mathrm{T}}\Phi(k+n-1) + \Theta_g^{\mathrm{T}}\bar{\nu}(k). \quad (25)$$

Then, equation (22) can be rewritten as

$$y(k+n) = y_a(k+n-1) + gu(k). \quad (26)$$

From (25) and (26), it is easy to derive that

$$\begin{aligned}
y_a(k+n-1) &= y_a(k+n-1) - y_a(l_k+n-1) + y_a(l_k+n-1) \\
&= \Theta_f^{\mathrm{T}}[\Phi(k+n-1) - \Phi(l_k+n-1)] \\
&\quad + \Theta_g^{\mathrm{T}}[\bar{\nu}(k) - \bar{\nu}(l_k)] + y(l_k+n) - gu(l_k).
\end{aligned} \quad (27)$$

Denote $\hat{\Theta}_f(k)$ and $\hat{g}(k)$ as the estimates of unknown parameters Θ_f and g defined in (23). The parameter estimates will be calculated from (35). Define the estimate of $y_a(k+n-1)$ as follows:

$$\hat{y}_a(k+n-1|k) = \hat{\Theta}_f^{\mathrm{T}}(k)[\hat{\Phi}(k+n-1|k) - \Phi(l_k+n-1)] + y(l_k+n) - \hat{g}(k)u(l_k) \quad (28)$$

where l_k is defined in (5) satisfying $l_k + n \leq k$, and

$$\hat{\Phi}(k+n-1|k) = [\Phi_1^{\mathrm{T}}(\hat{\xi}_1(k+n-1|k)), \ldots, \Phi_n^{\mathrm{T}}(\bar{\xi}_n(k))]^{\mathrm{T}} \quad (29)$$

with $\bar{\hat{\xi}}_i(k+n-i|k) = [\hat{\xi}_1(k+n-i|k), \ldots, \hat{\xi}_i(k+n-i|k)]^{\mathrm{T}}$, $i = 1, 2, \ldots, n-1$, defined in Section 3.

Define parameter estimate errors $\tilde{\Theta}_f(k) = \hat{\Theta}_f(k) - \Theta_f$ and $\tilde{g}(k) = \hat{g}(k) - g$, and then from (27) and (28), we have the estimation error of the auxiliary output as

$$\begin{aligned}\tilde{y}_a(k+n-1|k) &= \hat{y}_a(k+n-1|k) - y_a(k+n-1)\\ &= \tilde{\Theta}_f^{\mathrm{T}}(k)[\Phi(k+n-1) - \Phi(l_k+n-1)]\\ &\quad - \Theta_g^{\mathrm{T}}[\bar{\nu}(k) - \bar{\nu}(l_k)] + \beta(k+n-1) - \tilde{g}(k)u(l_k)\end{aligned} \tag{30}$$

where

$$\beta(k+n-1) = \hat{\Theta}_f^{\mathrm{T}}(k)[\hat{\Phi}(k+n-1|k) - \Phi(k+n-1)]. \tag{31}$$

Using the estimated auxiliary output, the adaptive control law is designed as

$$u(k) = -\frac{1}{\hat{g}(k)}(\hat{y}_a(k+n-1|k) - y_d(k+n)) \tag{32}$$

where the parameter estimate $\hat{g}(k)$ will be guaranteed to be bounded away from zero such that the above control law (32) is well defined. Define the output tracking error as $e(k) = y(k) - y_d(k)$. Considering adaptive control law in (32), the estimation error of auxiliary output in (30), and system (26), we obtain the closed-loop error dynamics as

$$\begin{aligned}e(k) &= y_a(k-1) + \hat{g}(k-n)u(k-n)\\ &\quad - \tilde{g}(k-n)u(k-n) - y_d(k)\\ &= -\tilde{y}_a(k-1|k-n) - \tilde{g}(k-n)u(k-n)\\ &= -\tilde{\Theta}_f^{\mathrm{T}}(k-n)[\Phi(k-1) - \Phi(l_{k-n}+n-1)]\\ &\quad - \tilde{g}(k-n)[u(k-n) - u(l_{k-n})] - \beta(k-1)\\ &\quad + \Theta_g^{\mathrm{T}}[\bar{\nu}(k-n) - \bar{\nu}(l_{k-n})].\end{aligned} \tag{33}$$

According to the definition of $\bar{\nu}(k)$ in (24) and Eq. (14), we have

$$|\Theta_g^{\mathrm{T}}[\bar{\nu}(k-n) - \bar{\nu}(l_{k-n})]| \le \lambda\theta_g\|\Delta\bar{\xi}_n(k-n)\| \tag{34}$$

where $\theta_g = \sum_{i=1}^n g_{fi}L_{p_i}$ is an unknown constant and λ can be any constant satisfying $\max_{1\le i\le n} L_{\upsilon_i} \le \lambda < \lambda^*$, with λ^* defined later in (49).

Denote $\hat{\theta}_g(k)$ as the estimate of θ_g and define the estimate error as $\tilde{\theta}_g(k) = \hat{\theta}_g(k) - \theta_g$. The parameter estimates used in control law (32) are calculated by the following update law

$$\begin{aligned}\hat{\Theta}_f(k) &= \hat{\Theta}_f(k-n)\\ &\quad + \gamma\frac{a(k)e(k)[\Phi(k-1) - \Phi(l_{k-n}+n-1)]}{D(k-n)}\\ \hat{g}(k) &= \begin{cases}\hat{g}'(k), & \text{if } \hat{g}'(k) > \underline{g}\\ \underline{g}, & \text{otherwise}\end{cases}\\ \hat{g}'(k) &= \hat{g}(k-n) + \frac{\gamma a(k)e(k)}{D(k-n)}[u(k-n) - u(l_{k-n})]\\ \hat{\theta}_g(k) &= \hat{\theta}_g(k-n) + \frac{a(k)\gamma\lambda|e(k)|\|\Delta\bar{\xi}_n(k-n)\|}{D(k-n)}\\ D(k-n) &= 1 + \|\Phi(k-1) - \Phi(l_{k-n}+n-1)\|^2\\ &\quad + [u(k-n) - u(l_{k-n})]^2 + \lambda^2\|\Delta\bar{\xi}_n(k-n)\|^2\end{aligned} \tag{35}$$

where $0 < \gamma < 2$ and $\max_{1\le i\le n} L_{\upsilon_i} \le \lambda < \lambda^*$ can be chosen as the same value as used in (15)–(17), and the deadzone indicator $a(k)$ is defined as

$$a(k) = \begin{cases}1 - \dfrac{\lambda\hat{\theta}_g(k-n)\|\Delta\bar{\xi}_n(k-n)\| + |\beta(k-1)|}{|e(k)|}, & \text{if } |e(k)| \\ \quad > \lambda\hat{\theta}_g(k-n)\|\Delta\bar{\xi}_n(k-n)\| + |\beta(k-1)| & \\ 0, \quad \text{otherwise} & \end{cases} \tag{36}$$

and from the definition of $a(k)$ above, it is guaranteed that

$$\begin{aligned}a(k)|e(k)| &\ge |e(k)| - \lambda\hat{\theta}_g(k-n)\|\Delta\bar{\xi}_n(k-n)\|\\ &\quad - |\beta(k-1)|.\end{aligned} \tag{37}$$

Remark 5. The value of parameter λ satisfying $\max_{1\le i\le n} L_{\upsilon_i} \le \lambda < \lambda^*$ used in (35)–(36) and (15)–(17) can be obtained by a practical engineer in a trial and error process. The existence of λ^* will be established in (49). As mentioned in Remark 1, it is reasonable in discrete-time to assume the nonlinear uncertainties are of growth rates L_{υ_i} smaller than a given constant λ^*.

4.1. Stability analysis and asymptotic tracking performance

The main result of the control performance is summarized in the following theorem.

Theorem 1. *Consider the adaptive closed-loop system consisting of system (1), states prediction laws defined in (11) and (13) using parameter update law (15), control law (32) using parameter update law (35). All the signals in the closed-loop system are bounded and furthermore, the tracking error $e(k)$ converges to zero.*

Proof. Choose a Lyapunov function candidate as

$$V(k) = \sum_{j=k-n+1}^{k}[\|\tilde{\Theta}_f(j)\|^2 + \tilde{g}^2(j) + \tilde{\theta}_g^2(j)].$$

It follows that the difference of $V(k)$ is

$$\begin{aligned}\Delta V(k) &= V(k) - V(k-1)\\ &\le \tilde{\Theta}_f^{\mathrm{T}}(k)\tilde{\Theta}_f(k) - \tilde{\Theta}_f^{\mathrm{T}}(k-n)\tilde{\Theta}_f(k-n)\\ &\quad + \tilde{g}'^2(k) - \tilde{g}^2(k-n) + \tilde{\theta}_g^2(k) - \tilde{\theta}_g^2(k-n)\end{aligned} \tag{38}$$

where the inequality $\tilde{g}^2(k) \le \tilde{g}'^2(k)$ is used, which can be easily verified from (35).

Consider the error equation (33), the inequality (34), the update laws in (35), and the deadzone $a(k)$ defined in (36). Then, applying the similar techniques in the proof of Lemma 4 in Appendix B, one can easily show that $\Delta V(k)$ is non-positive and thus $V(k)$, $\hat{\Theta}_f(k)$, $\hat{g}(k)$, and $\hat{\theta}_g(k)$ are bounded. Furthermore, we have

$$\lim_{k\to\infty}\frac{a^2(k)e^2(k)}{D(k-n)} = 0 \tag{39}$$

$$|e(k)| - \lambda\bar{\theta}_g\|\Delta\bar{\xi}_n(k-n)\| - |\beta(k-1)| \le a(k)|e(k)| \tag{40}$$

where (40) is obtained from (37) with a positive constant $\bar{\theta}_g$ satisfying $\hat{\theta}_g(k) \le \bar{\theta}_g, \forall k \in Z^+_{-n}$.

Further, according to the definition of $\beta(k+n-1)$ in (31), Lemma 4 and Assumption 1, there exists a constant c_β such that

$$|\beta(k+n-1)| \le o[O[y(k+n-1)]] + \lambda c_\beta\Delta_s(k, n-1). \tag{41}$$

Considering $\Delta_s(k, n-1)$ defined in (20) and $\Delta\bar{\xi}_n(k)$ defined in (6) and noting the fact that $l_k \le k-n$, it follows

$$\begin{aligned}\Delta_s(k, n-1) &= \max_{1\le j\le n-1}\{\|\bar{\xi}_n(k-n+j)\\ &\quad - \bar{\xi}_n(l_k-n+j)\|\} \le 2\max_{k'\le k}\{\|\bar{\xi}_n(k')\|\}, \quad \forall k \in Z^+_{-n}\end{aligned} \tag{42}$$

$$\Delta\bar{\xi}_n(k) \le 2\max_{k'\le k}\{\|\bar{\xi}_n(k')\|\}. \tag{43}$$

From Lemma 2, the definition of $o[\cdot]$ in Definition 1, and inequality (42), it is clear that

$$\begin{aligned}|\beta(k+n-1)| &\le o[O[\bar{\xi}_n(k)]] + \lambda c_\beta\Delta_s(k, n-1)\\ &\le (\alpha(k) + \lambda)c_{\beta,1}\max_{k'\le k}\{\|\bar{\xi}_n(k')\|\}\\ &\quad + \alpha(k)c_{\beta,2}, \quad \forall k \in Z^+_{-n}\end{aligned} \tag{44}$$

where $\alpha(k)$ is a sequence that converges to zero, and $c_{\beta,1}$ and $c_{\beta,2}$ are finite constants. Since $\lim_{k\to\infty}\alpha(k) \to 0$, for any given arbitrary small positive constant ϵ_1, there exists a k_1 such that $\alpha(k) \le \epsilon_1, \forall k > k_1$. Thus, it is clear that

$$|\beta(k+n-1)| \le (\epsilon_1+\lambda)c_{\beta,1}\max_{k'\le k}\{\|\bar{\xi}_n(k')\|\} + \epsilon_1 c_{\beta,2} \quad \forall k > k_1. \tag{45}$$

From Lemma 2, we have $\bar{\xi}_n(k-n+1) = O[y(k)]$, which yields

$$\|\bar{\xi}_n(k-n+1)\| \le C_1 \max_{k'\le k}\{|e(k')|\} + C_2, \quad \forall k \in Z^+_{-n} \tag{46}$$

where $y(k) \sim e(k)$ is used and C_1 and C_2 are finite constants. Using (40), thus inequality (46) can be expressed as

$$\begin{aligned}
&\|\bar{\xi}_n(k-n+1)\| \\
&\le C_1 \max_{k'\le k}\{|e(k')| - \lambda\bar{\theta}_g\|\Delta\bar{\xi}_n(k'-n)\| - |\beta(k'-1)| \\
&\quad + \lambda\bar{\theta}_g\|\Delta\bar{\xi}_n(k'-n)\| + |\beta(k'-1)|\} + C_2 \\
&\le C_1 \max_{k'\le k}\{a(k')|e(k')|\} + \lambda\bar{\theta}_g C_1 \max_{k'\le k-n}\{\|\Delta\bar{\xi}_n(k')\|\} \\
&\quad + C_1 \max_{k'\le k-n}\{|\beta(k'+n-1)|\} + C_2, \quad \forall k \in Z^+_{-n}.
\end{aligned} \tag{47}$$

From inequalities (43), (45) and (47), we have $C_3 = (2\bar{\theta}_g + c_{\beta,1})C_1$, $\epsilon_2 = c_{\beta,1}\epsilon_1 C_1$ and $C_4 = C_2 + \epsilon_1 c_{\beta,2} C_1$ such that

$$\begin{aligned}
\max_{k'\le k-n+1}\{\|\bar{\xi}_n(k')\|\} &\le C_1 \max_{k'\le k}\{a(k')|e(k')|\} \\
&\quad + (\lambda C_3 + \epsilon_2)\max_{k'\le k-n+1}\{\|\bar{\xi}_n(k')\|\} + C_4, \quad k > k_1
\end{aligned} \tag{48}$$

which implies the existence of a small positive constant

$$\lambda^* = \frac{1-\epsilon_2}{C_3} \tag{49}$$

where ϵ_2 can be arbitrarily small. It further implies that

$$\begin{aligned}
\max_{k'\le k-n+1}\{\|\bar{\xi}_n(k')\|\} &\le \frac{C_1}{1-\lambda C_3-\epsilon_2}\max_{k'\le k}\{a(k')|e(k')|\} \\
&\quad + \frac{C_4}{1-\lambda C_3-\epsilon_2}, \quad k > k_1, \forall \lambda < \lambda^*.
\end{aligned} \tag{50}$$

Note that inequality (50) implies $\bar{\xi}_n(k-n+1) = O[a(k)e(k)]$. From $\Phi(k+n-1)$ in defined (23), Lemma 2, and Assumption 1, it can be seen that $\Phi(k-1) = O[\bar{\xi}_n(k-n)]$ and $u(k-n) = O[y(k)] = O[\bar{\xi}_n(k-n+1)]$. According to the definition of $D(k-n)$ in (35) and inequality (43), we have

$$\begin{aligned}
D^{\frac{1}{2}}(k-n) &\le 1 + \|\Phi(k-1) - \Phi(l_{k-n}+n-1)\| \\
&\quad + |u(k-n) - u(l_{k-n})| + \lambda\|\Delta\bar{\xi}_n(k-n)\| \\
&= O[\bar{\xi}_n(k-n+1)] = O[a(k)e(k)].
\end{aligned}$$

Then, applying the well-known Key Technical Lemma (Goodwin & Sin, 1984) to (39) yields

$$\lim_{k\to\infty} a(k)e(k) = 0. \tag{51}$$

From inequality (50), we see that the boundedness of $\bar{\xi}_n(k)$ is guaranteed. It follows that the output $y(k)$ and tracking error $e(k)$ are bounded, as well as the the control input $u(k)$, according to Lemma 2. Next, from Lemma 3, we have

$$\lim_{k\to\infty} \|\Delta\bar{\xi}_n(k)\| = 0 \tag{52}$$

which further leads to

$$\lim_{k\to\infty} \|\Delta_s(k, n-1)\| = 0. \tag{53}$$

Additionally, considering (41) and noting that $y(k) \sim e(k)$, it follows

$$|\beta(k-1)| \le o[O[e(k)]] + \lambda c_\beta \Delta_s(k-n, n-1) \tag{54}$$

which yields

$$\begin{aligned}
&|e(k)| - |\beta(k-1)| + \lambda c_\beta \Delta_s(k-n, n-1) \ge |e(k)| \\
&\quad - o[O[e(k)]] \ge (1-\alpha(k)m_1)|e(k)| - \alpha(k)m_2
\end{aligned} \tag{55}$$

where m_1, m_2 are positive constants, and $\lim_{k\to\infty}\alpha(k) \to 0$, according to Definition 1. Because of $\lim_{k\to\infty}\alpha(k) \to 0$, there exists constant k_3 such that $\alpha(k) \le 1/m_1, \forall k > k_3$. Therefore, it can be seen from (55) that

$$\begin{aligned}
&|e(k)| - |\beta(k-1)| + \lambda c_\beta \Delta_s(k-n, n-1) + \alpha(k)m_2 \\
&\quad \ge (1-\alpha(k)m_1)|e(k)| \ge 0, \quad \forall k > k_3.
\end{aligned} \tag{56}$$

On the other hand, note that (40) implies

$$\begin{aligned}
&|e(k)| - |\beta(k-1)| + \lambda c_\beta \Delta_s(k-n, n-1) + \alpha(k)m_2 \\
&\quad \le a(k)|e(k)| + \lambda c_\beta \Delta_s(k-n, n-1) \\
&\qquad + \lambda\bar{\theta}_g\|\Delta\bar{\xi}_n(k-n)\| + \alpha(k)m_2.
\end{aligned} \tag{57}$$

From (56) and (57), we have $\forall k > k_3$

$$\begin{aligned}
0 &\le (1-\alpha(k)m_1)|e(k)| \\
&\le a(k)|e(k)| + \lambda c_\beta \Delta_s(k-n, n-1) \\
&\quad + \lambda\bar{\theta}_g\|\Delta\bar{\xi}_n(k-n)\| + \alpha(k)m_2
\end{aligned} \tag{58}$$

which implies that $\lim_{k\to\infty} e(k) = 0$ according to (51)–(53), and $\lim_{k\to\infty}\alpha(k) \to 0$. This completes the proof. □

Remark 6. From (34) and (54), it can be seen that the last two terms in (33), $\beta(k)$ caused by prediction error and $\bar{\nu}(k)$ caused by nonlinear model uncertainties will ultimately vanish due to $\|\Delta\bar{\xi}_n(k-n)\| \to 0$. This illustrates the underlying mechanism of our control design: to use states information at previous steps to compensate for the uncertainties at current step. It is a great contrast to the continuous-time counterpart results presented in Polycarpou and Ioannou (1996) and Jiang and Praly (1998), where nonlinear damping is used to compensate for the effect of nonlinear uncertainties.

Remark 7. The update law (35) and (15) requires the computation of l_k defined in (5) and the computation may cost infinite memory as time increase. In practice however, finite memory control can be obtained by computing l_k not from range $[0, k-n]$ but from $[k-M-n, k-n]$, where $M > 0$ can be chosen as a large integer. In this way, the stability will not be affected and the magnitude of ultimate tracking error can be made sufficiently small by increasing M.

5. Simulation studies

The following second order nonlinear plant is used for simulation.

$$\begin{cases}
\xi_1(k+1) = 0.2\xi_1(k)\cos(\xi_1(k)) + 0.3\xi_1(k)\sin(\xi_1(k)) \\
\quad + 0.4\xi_2(k) + \upsilon_1(\xi_1(k)) \\
\xi_2(k+1) = 0.5\xi_2(k)\dfrac{\xi_1(k)}{1+\xi_1^2(k)} + 0.5\dfrac{\xi_2^3(k)}{2+\xi_2^2(k)} + 0.8u(k) \\
\quad + \upsilon_2(\bar{\xi}_2(k)) \\
y(k) = \xi_1(k)
\end{cases}$$

where $\upsilon_1(\xi_1(k)) = 0.04(\sin(0.05k))\xi_1(k)$ and $\upsilon_2(\bar{\xi}_2(k)) = 0.04(\cos(0.05k))(\xi_1(k)+\xi_2(k))$. The control objective is to make the output $y(k)$ track the desired reference trajectory $y_d(k) = 1.5\sin(\frac{\pi}{5}kT) + 1.5\cos(\frac{\pi}{10}kT)$, where $T = 0.1$. The initial system

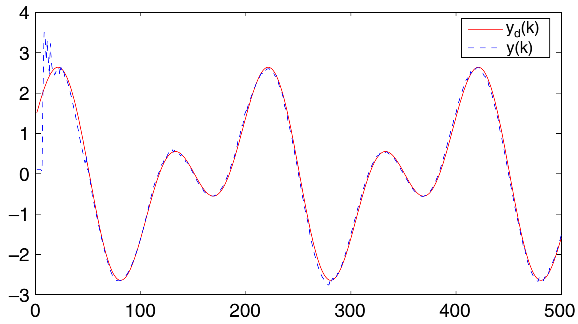

Fig. 1. Reference signal and system output.

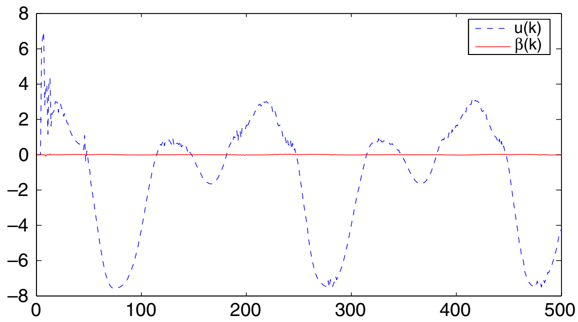

Fig. 2. Control signal and signal $\beta(k)$.

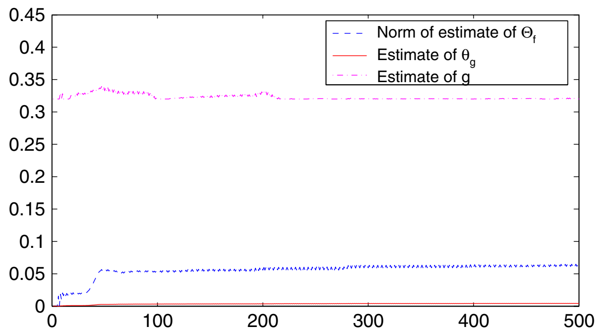

Fig. 3. Boundedness of parameter estimates in control law.

states are $\bar{\xi}_2(0) = [0.1,\ 0.1]^{\mathrm{T}}$. The control parameters are chosen as $\underline{g} = 0.32$, $\gamma = 0.1$, and $\lambda = 0.05$. The simulation results are presented in Figs. 1–4. Fig. 1 shows the reference signal y_d and system output $y(k)$. Fig. 2 illustrates the control input $u(k)$ and signal $\beta(k)$ caused by prediction error. Fig. 3 shows the boundedness of parameter estimates in the adaptive control law. Fig. 4 demonstrates the boundedness of parameter estimates in the future states prediction law. It can be seen from Fig. 1 that the system output $y(k)$ asymptotically tracks the reference signal $y_d(k)$. From Fig. 2, it is seen that signal $\beta(k)$ caused by prediction error converges to zero.

6. Conclusion

In this paper, adaptive control with complete compensation of nonparametric nonlinear uncertainty has been studied for a class of SISO strict-feedback system, in which the nonlinear uncertainties appear in both matched and unmatched manner. Under the proposed adaptive control, the boundedness of all the closed-loop signals are guaranteed and the effect of the nonparametric model uncertainties has been eliminated such that the tracking error converges to zero ultimately.

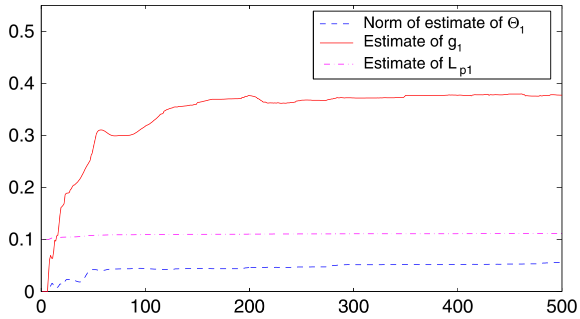

Fig. 4. Boundedness of parameter estimates in prediction law.

Appendix A. Proof of Lemma 2

The first equation of system (2) can be written as follows according to the Mean Value Theorem

$$\begin{aligned} y(k+1) &= f_1(\xi_1(k), \xi_2(k)) \\ &= f_1(y(k), 0) + g_{1,1}(y(k), \xi_2^c(k))\xi_2(k) \end{aligned} \tag{A.1}$$

where $\xi_2^c(k) \in [\min\{0, \xi_2(k)\}, \max\{0, \xi_2(k)\}]$ and the control gain functions $g_{1,1}(\cdot) = \frac{\partial f_1(\xi_1(k),\xi_2(k))}{\partial \xi_2(k)}$ have been assumed to be bounded. Due to function $f_1(\cdot)$ satisfying the Lipschitz condition, we have

$$\bar{\xi}_2(k) = O[y(k+1)], \qquad y(k+1) = O[\bar{\xi}_2(k)]. \tag{A.2}$$

Similarly, the second equation of system (2) can be written as

$$\begin{aligned} \xi_2(k+1) &= f_2(y(k), \xi_2(k), \xi_3(k)) = f_2(y(k), \xi_2(k), 0) \\ &\quad + g_{1,2}(y(k), \xi_2(k), \xi_3^c(k))\xi_3(k) \end{aligned} \tag{A.3}$$

where $\xi_3^c(k) \in [\min\{0, \xi_3(k)\}, \max\{0, \xi_3(k)\}]$ and $g_{1,2}(\cdot) = \frac{\partial f_2(y(k),\xi_2(k),\xi_3(k))}{\partial \xi_3(k)}$ has also been assumed to be bounded. Substituting equation (A.3) into (A.1) yields

$$\begin{aligned} &y(k+2) = f_1(y(k+1), 0) + g_{1,1}(y(k+1), \xi_2^c(k+1)) \\ &\times [f_2(y(k), \xi_2(k), 0) + g_{1,2}(y(k), \xi_2(k), \xi_3^c(k))\xi_3(k)]. \end{aligned} \tag{A.4}$$

Noting the boundedness of $g_{1,1}(\cdot)$ and $g_{1,2}(\cdot)$, the Lipschitz condition of functions $f_1(\cdot)$ and $f_2(\cdot)$, equations (A.2) and (A.4), we have

$$\bar{\xi}_3(k) = O[y(k+2)], \qquad y(k+2) = O[\bar{\xi}_3(k)]. \tag{A.5}$$

Continuing the above procedure, we have

$$\bar{\xi}_i(k) = O[y(k+i-1)], \qquad y(k+i-1) = O[\bar{\xi}_i(k)] \tag{A.6}$$

which results in $\bar{\xi}_i(k) \sim y(k+i-1)$.

The result $u(k) = O[y(k+n)]$ has been established in Ge et al. (2008a). This completes the proof. □

Appendix B. Proof of Lemma 4

Define $\tilde{\Theta}_i(k) = \hat{\Theta}_i(k) - \Theta_i$, $\tilde{g}_i(k) = \hat{g}_i(k) - g_i$, and $\tilde{c}_i(k) = \hat{c}_i(k) - L_{pi}$. It follows from (8)–(11) that

$$\begin{aligned} \tilde{\xi}_i(k+1|k) &= \hat{\xi}_i(k+1|k) - \xi_i(k+1) \\ &= \hat{\xi}_i^a(k) - \xi_i^a(k) + \tilde{g}_i(k-n+2)\xi_{i+1}(k) \\ &= \tilde{\Theta}_i^{\mathrm{T}}(k-n+2)[\Phi_i(\bar{\xi}_i(k)) - \Phi_i(\bar{\xi}_i(l_{k-n+i}+n-i))] \\ &\quad + \tilde{g}_i(k-n+2)[\xi_{i+1}(k) - \xi_{i+1}(l_{k-n+i}+n-i)] \\ &\quad - [\nu_i(\bar{\xi}_n(k-n+i)) - \nu_i(\bar{\xi}_n(l_{k-n+i}))] \end{aligned} \tag{B.1}$$

which yields

$$\begin{aligned}
&-\{\tilde{\Theta}_i^{\mathrm{T}}(k-n+2)[\Phi_i(\bar{\xi}_i(k))-\Phi_i(\bar{\xi}_i(l_{k-n+i}+n-i))] \\
&\quad+\tilde{g}_i(k-n+2)[\xi_{i+1}(k)-\xi_{i+1}(l_{k-n+i}+n-i)]\} \\
&\quad\times\tilde{\xi}_i(k+1|k) \\
&= -\tilde{\xi}_i^2(k+1|k)-[\nu_i(\bar{\xi}_n(k-n+i))-\nu_i(\bar{\xi}_n(l_{k-n+i}))] \\
&\quad\times\tilde{\xi}_i(k+1|k) \\
&\le -\tilde{\xi}_i^2(k+1|k)+\lambda L_{pi}|\tilde{\xi}_i(k+1|k)|\|\Delta\bar{\xi}_n(k-n+i)\| \qquad \text{(B.2)}
\end{aligned}$$

where the last inequality is established by (14) and $\max_{1\le i\le n} L_{\nu_i}\le\lambda$.

To prove the boundedness of all the estimated parameters, we choose the Lyapunov function candidate as follows:

$$V_i(k)=\sum_{j=k-n+2}^{k}[\|\tilde{\Theta}_i(j)\|^2+\tilde{g}_i^2(j)+\tilde{c}_i^2(j)]. \qquad \text{(B.3)}$$

From (15), the difference of $V_i(k)$ is given by

$$\begin{aligned}
\Delta V_i(k) &= V_i(k+1)-V_i(k) \\
&= \tilde{\Theta}_i^{\mathrm{T}}(k+1)\tilde{\Theta}_i(k+1)-\tilde{\Theta}_i^{\mathrm{T}}(k-n+2)\tilde{\Theta}_i(k-n+2) \\
&\quad+\tilde{g}_i^2(k+1)-\tilde{g}_i^2(k-n+2) \\
&\quad+\tilde{c}_i^2(k+1)-\tilde{c}_i^2(k-n+2) \\
&= \{\|\Phi_i(\bar{\xi}_i(k))-\Phi_i(\bar{\xi}_i(l_{k-n+i}+n-i))\|^2 \\
&\quad+|\xi_{i+1}(k)-\xi_{i+1}(l_{k-n+i}+n-i)|^2 \\
&\quad+\lambda^2\|\Delta\bar{\xi}_n(k-n+i)\|^2\}\frac{a_i^2(k)\gamma^2\tilde{\xi}_i^2(k+1|k)}{D_i^2(k)} \\
&\quad-\{\tilde{\Theta}_i^{\mathrm{T}}(k-n+2)[\Phi_i(\bar{\xi}_i(k))-\Phi_i(\bar{\xi}_i(l_{k-n+i}+n-i))] \\
&\quad+\tilde{g}_i(k-n+2)[\xi_{i+1}(k)-\xi_{i+1}(l_{k-n+i}+n-i)]\} \\
&\quad\times\tilde{\xi}_i(k+1|k)\frac{2a_i(k)\gamma}{D_i(k)}+\lambda\tilde{c}_i(k-n+2)|\tilde{\xi}_i(k+1|k)| \\
&\quad\times\|\Delta\bar{\xi}_n(k-n+i)\|\frac{2a_i(k)\gamma}{D_i(k)}. \qquad \text{(B.4)}
\end{aligned}$$

According to the definition of $D_i(k)$ in (16) and inequality (B.2), the difference of $V_i(k)$ in (B.4) can be written as

$$\begin{aligned}
\Delta V_i(k) &\le \frac{a_i^2(k)\gamma^2\tilde{\xi}_i^2(k+1|k)}{D_i(k)}-\frac{2a_i(k)\gamma\tilde{\xi}_i^2(k+1|k)}{D_i(k)} \\
&\quad+\frac{2a_i(k)\gamma\lambda\hat{c}_i(k-n+2)|\tilde{\xi}_i(k+1|k)|\|\Delta\bar{\xi}_n(k-n+i)\|}{D_i(k)} \\
&= \frac{a_i^2(k)\gamma^2\tilde{\xi}_i^2(k+1|k)}{D_i(k)}-\frac{2a_i^2(k)\gamma\tilde{\xi}_i^2(k+1|k)}{D_i(k)} \\
&= -\frac{a_i^2(k)\gamma(2-\gamma)\tilde{\xi}_i^2(k+1|k)}{D_i(k)} \qquad \text{(B.5)}
\end{aligned}$$

where $L_{pi}+\tilde{c}_i(k-n+2)=\hat{c}_i(k-n+2)$ and equality (18) are used.

Noting that $0<\gamma<2$, we can see from (B.5) that the difference of Lyapunov function $V_i(k)$, $\Delta V_i(k)$, is nonpositive and thus, the boundedness of $V_i(k)$ is guaranteed. It further implies the boundedness of $\hat{\Theta}_i(k)$, $\hat{g}_i(k)$, and $\hat{c}_i(k)$. Thus, there exist finite constants $\bar{\Theta}$, $\bar{g}$, and $\bar{c}$, such that

$$\|\hat{\Theta}_i(k)\|\le\bar{\Theta},\qquad \hat{g}_i(k)\le\bar{g},\qquad \hat{c}_i(k)\le\bar{c},\quad i=1,2,\ldots,n-1. \qquad \text{(B.6)}$$

Taking summation on both hand sides of (B.5), we obtain

$$\sum_{k=0}^{\infty}\frac{a_i^2(k)\gamma(2-\gamma)\tilde{\xi}_i^2(k+1|k)}{D_i(k)}\le V_i(0)-V_i(\infty)$$

which together with the boundedness of $V_i(k)$ implies

$$\frac{a_i^2(k)\tilde{\xi}_i^2(k+1|k)}{D_i(k)}:=\alpha_i(k)\to 0,\quad i=1,2,\ldots,n-1. \qquad \text{(B.7)}$$

From Assumption 1, Lemma 2, and the definition of $D_i(k)$ in (16), it can been seen that

$$\begin{aligned}
D_i^{\frac{1}{2}}(k) &\le 1+\|\Phi_i(\bar{\xi}_i(k))-\Phi_i(\bar{\xi}_i(l_{k-n+i}+n-i))\| \\
&\quad+|\xi_{i+1}(k)-\xi_{i+1}(l_{k-n+i}+n-i)| \\
&\quad+\lambda\|\Delta\bar{\xi}_n(k-n+i)\| \\
&= O[y(k+i)],\quad i=1,2,\ldots,n-1. \qquad \text{(B.8)}
\end{aligned}$$

From equation (B.7) we have

$$\begin{aligned}
a_i(k)|\tilde{\xi}_i(k+1|k)| &= \alpha_i^{\frac{1}{2}}(k)D_i^{\frac{1}{2}}(k)=o[D_i^{\frac{1}{2}}(k)] \\
&= o[O[y(k+i)]],\quad i=1,2,\ldots,n-1. \qquad \text{(B.9)}
\end{aligned}$$

Further, we have

$$\begin{aligned}
a_i(k)\|\bar{\tilde{\xi}}_i(k+1|k)\| &\sim a_i(k)|\tilde{\xi}_i(k+1|k)| \\
&= o[O[y(k+i)]],\quad i=1,2,\ldots,n-1. \qquad \text{(B.10)}
\end{aligned}$$

From the definition of deadzone in (17), we have

$$\begin{aligned}
|\tilde{\xi}_i(k+1|k)| &\le a_i(k)|\tilde{\xi}_i(k+1|k)|+\lambda\hat{c}_i(k-n+2) \\
&\quad\times\|\Delta\bar{\xi}_n(k-n+i)\| \qquad \text{(B.11)}
\end{aligned}$$

which together with (B.6) and (B.9) and the definition of $\Delta_s(k,i)$ in (20) yields

$$|\tilde{\xi}_i(k+1|k)|\le o[O[y(k+i)]]+\lambda c_1\Delta_s(k,i) \qquad \text{(B.12)}$$

where $c_1=\bar{c}$. Denote $\bar{c}_1=nc_1$, we further have

$$\begin{aligned}
\|\bar{\tilde{\xi}}_i(k+1|k)\| &\le \sum_{j=1}^{i}|\tilde{\xi}_j(k+1|k)| \\
&\le o[O[y(k+i)]]+\lambda\bar{c}_1\Delta_s(k,i). \qquad \text{(B.13)}
\end{aligned}$$

Continuing the analysis above, for the j-step estimation error $\tilde{\xi}_i(k+j|k)$, $i=1,2,\ldots,n-1$, $j=2,3,\ldots,n-i$, we have

$$\tilde{\xi}_i(k+j|k)=\check{\xi}_i(k+j|k)+\tilde{\xi}_i(k+j|k+1) \qquad \text{(B.14)}$$

where

$$\begin{aligned}
\tilde{\xi}_i(k+j|k+1) &= \hat{\xi}_i(k+j|k+1)-\xi_i(k+j) \\
\check{\xi}_i(k+j|k) &= \hat{\xi}_i(k+j|k)-\hat{\xi}_i(k+j|k+1). \qquad \text{(B.15)}
\end{aligned}$$

Based on the results in previous steps, for j-step estimation error $\tilde{\xi}_i(k+j|k)$, $j=2,3,\ldots,n-i$, $i=1,2,\ldots,n-1$, we see that there exist constants c_{j-1} and $\check{c}_{j-1}$ such that

$$\begin{aligned}
|\tilde{\xi}_i(k+j-1|k)| &\le o[O[y(k+i+j-2)]] \\
&\quad+\lambda c_{j-1}\Delta_s(k,i+j-2) \\
|\check{\xi}_i(k+j-1|k)| &\le o[O[y(k+i+j-2)]] \\
&\quad+\lambda\check{c}_{j-1}\Delta_s(k,i+j-2). \qquad \text{(B.16)}
\end{aligned}$$

From (12) and (13), we have

$$\begin{aligned}
\check{\xi}_i(k+j|k) &= \hat{\xi}_i(k+j|k)-\hat{\xi}_i(k+j|k+1) \\
&= \hat{\xi}_i^a(k+j-1|k)+\hat{g}_i(k-n+j+1) \\
&\quad\times\hat{\xi}_{i+1}(k+j-1|k)-\hat{\xi}_i^a(k+j-1|k+1) \\
&\quad-\hat{g}_i(k-n+j+1)\hat{\xi}_{i+1}(k+j-1|k+1) \\
&= \hat{\Theta}_i^{\mathrm{T}}(k-n+j+1)[\Phi_i(\bar{\hat{\xi}}_i(k+j-1|k)) \\
&\quad-\Phi_i(\bar{\hat{\xi}}_i(k+j-1|k+1))] \\
&\quad+\hat{g}_i(k-n+j+1)\check{\xi}_{i+1}(k+j-1|k). \qquad \text{(B.17)}
\end{aligned}$$

According to the Lipschitz condition of $\Phi_i(\cdot)$ and (B.15), the following equality holds:

$$\begin{aligned}&\|\Phi_i(\bar{\tilde{\xi}}_i(k+j-1|k)) - \Phi_i(\bar{\tilde{\xi}}_i(k+j-1|k+1))\|\\ &\quad \le L_i\|\bar{\tilde{\xi}}_i(k+j-1|k)\|.\end{aligned} \tag{B.18}$$

According to (B.14)–(B.18), there exist constants c_j such that

$$|\tilde{\xi}_i(k+j|k)| \le o[O[y(k+i+j-1)]] + \lambda c_j \Delta_s(k, i+j-1).$$

Denote $\bar{c}_j = nc_j$, then we have

$$\begin{aligned}\|\bar{\tilde{\xi}}_i(k+j|k)\| &\le \sum_{j=1}^{i} |\tilde{\xi}_j(k+j|k)|\\ &\le o[O[y(k+i+j-1)]] + \lambda \bar{c}_j \Delta_s(k, i+j-1).\end{aligned} \tag{B.19}$$

Let $j = n - i$, $i = 1, 2, \ldots, n-1$, then we see that (B.19) leads to (19) and it completes the proof. □

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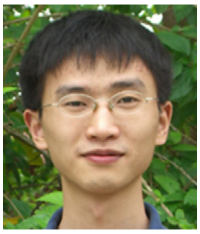

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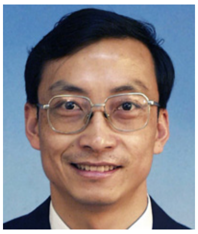

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