MATHEMATICS VERSUS THE ARTS: A COMPARATIVE LOOK AT STUDENTS' ATTITUDES AND BELIEFS

by

MARY ANN HARASYMOWYCZ

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Faculty of Technology

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ABSTRACT

Mary Ann Harasymowycz

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The words that students use to paint a picture of mathematics are very different from those which they use to describe their experiences in art and music. In the views of students, mathematics is *pointless* and *repetitive* while the arts are *creative, relaxing* and an *expression of themselves*.

This thesis reports on the findings of a two part research project designed to investigate the attitudes of high school students when learning mathematics, art and music. The focus of this study was a comparative look at their confidence and enjoyment in learning these subjects.

A questionnaire was designed and developed for use in a study of students in the United States and England (n = 1226). The intent of this questionnaire, which contained seven Likert-type questions and one open-ended response, was an in-depth look at the existence of confidence and enjoyment in learning mathematics, art and music. The results indicated that, the highest frequency of students in the mathematics group were confident in their ability in mathematics but did not enjoy learning it. This study also found, however, that there were very low percentages of students that were confident in art and music but did not enjoy learning them. Additionally there was a high frequency of students who had no confidence in art but did enjoy learning it compared to a low frequency of students in mathematics who were not confident but enjoyed learning it.

To further explore these findings, repertory grid interviews were conducted on a selection of questionnaire participants from the United States (n = 42). Honey's method of content analysis was used to analyze the data. Among the differences found between students' confidence and enjoyment in learning mathematics compared to the arts were their perceptions of the routine nature of their daily lessons in mathematics versus their active, creative, personally engaging experiences while learning art and music.

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ACKNOWLEDGEMENTS

Only through education does one come to be dissatisfied with his own knowledge, and only through teaching does one come to realize the uncomfortable inadequacy of his knowledge. Being dissatisfied with his own knowledge, one realizes that the trouble lies with himself, and realizing the uncomfortable inadequacy of his knowledge one then feels stimulated to improve himself. Therefore, it is said, "the process of teaching and learning stimulate one another."

--Confucius, circa 500 BC

Many have asked in the time since I began the doctoral program at the University of Plymouth's Centre for Teaching Mathematics why I chose to travel so far from my home in the United States to pursue my lifelong dream of a PhD. My unequivocal response has always been the unselfish assistance, guidance and friendship that I have found on this journey.

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With eternal gratitude

AUTHOR'S DECLARATION

At no time during the registration for the degree of Doctor of Philosophy has the author been registered for any other University awards without prior agreement of the Graduate Committee.

A programme of advanced study was undertaken which included semi-annual seminars and presentations at The Centre for Teaching Mathematics at the University of Plymouth.

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Chapter 1 Introduction

1.1 The Origins of This Thesis

The 2006 book by mathematician Ian Stewart, Letters to a Young Mathematician, is an attempt to give insights into the experiences of mathematicians. Stewart updates the impressions left by G.H. Hardy, in his often quoted essay on mathematics as an art, A Mathematician's Apology (1967), for those who might be interested in mathematics regardless of their background. Hardy (1967, pp. 84 - 85) wrote,

> *"A mathematician, like a painter or poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with ideas ... beauty is the first test: there is no permanent place in the world for ugly mathematics."*

Ian Stewart (2006, p. 7) says that mathematics gives him a sense of the world in a way that he never understood or knew before by opening his *"eyes to nature's laws and patterns."* He believes (2006, p. 9) that mathematics, *"done for its own sake can be exquisitely beautiful and elegant. Not the 'sums' we all do at school; as individuals those are most ugly and formless."*

The mathematics that students learn at school creates their first impressions of it. In an interview with Hadley Hooper written by Christopher Tilghman (2003, p. 14) in "Hemispheres" magazine, Hooper is quoted as saying, *"I just endured high school so I could get on with it and go to art school."* I am certain that there are countless other students, like Hadley Hooper, who sadly leave their school years with a sense of ugliness from their first encounters with mathematics. The first impression a person gets by looking at a painting or listening to a symphony is not the same. John Dewey (1980, p. 145) says that, *"the impression directly made by an harmonious ensemble in any art is often described as the musical quality of that art."*

As a student myself, learning mathematics always provided me with great deal of pleasure, so much so that, I can't remember a time when I wanted to do anything other than become a mathematics teacher and convey to students the beauty, joy and excitement that I had experienced. Over the thirty years that I have been teaching, I have been disappointed that students have not always shared that view with me.

As a mathematics teacher, I have noticed the doodles and drawings of my students and have been amazed at the artistry that they exhibited. Many of the students that I have encountered struggle to learn mathematics but have different experiences in the arts. The cartoon in Figure 1, below, conveys a prevalent belief that many students possess:

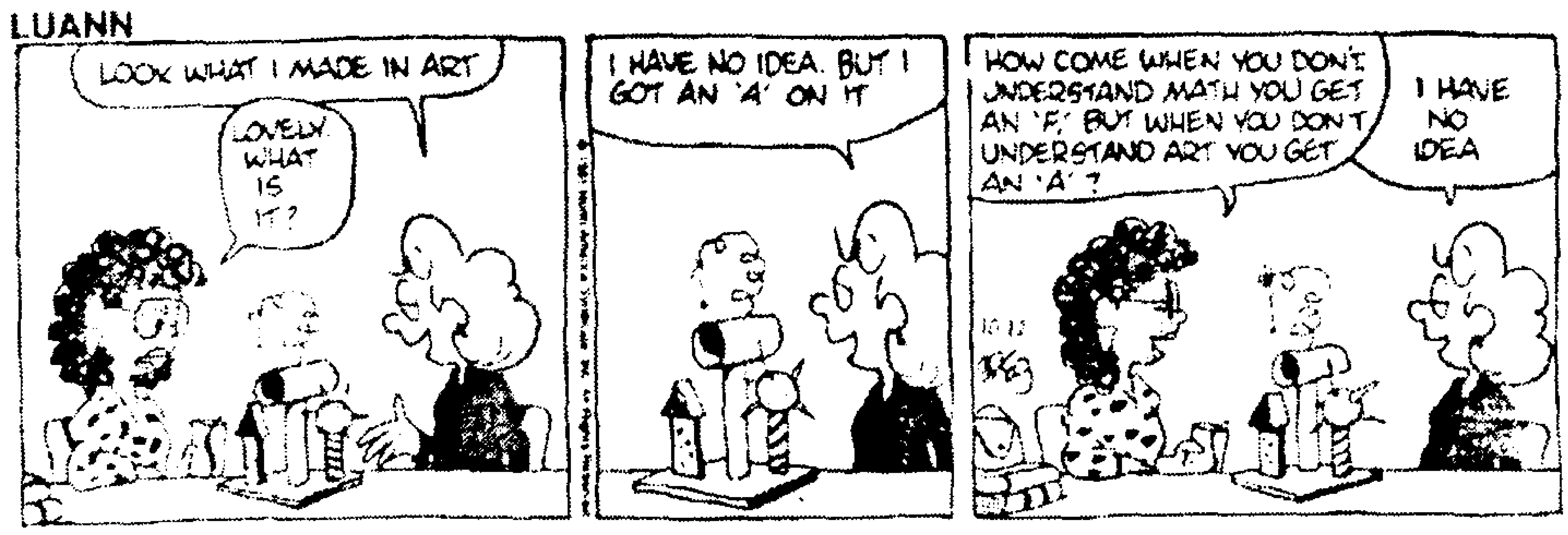

Figure 1.1 [Reprinted with the permission of the cartoonist, Greg Evans]

I have often wondered what I could do to improve the learning of my students and envied the results that an art colleague achieved. Her students had positive attitudes toward art but they did not have a positive attitude toward mathematics. It was an experience with such one student, who was barely passing my class but had his art work hanging on the bulletin board of the art room, which became the catalyst for my exploration into finding ways to make connections between mathematics and the visual arts for my students.

After a casual conversation about our mutual interest in the artist M.C. Escher, this same art colleague and I began years of successful collaboration to integrate our two diverse disciplines in a variety of mathematics and art projects. In our joint endeavours with lower secondary students, the mathematical foundations, instructions and creation of their designs were completed in mathematics class and then taken to art class for the colouring and any extensions. Our interdisciplinary projects included the 'tessellations' of M.C. Escher, the 'op art' of Larry Poons and, after a 1994 fellowship that we received to do research into other ideas for connections between mathematics and art, 'anamorphic art.' Student reflections on their experiences in these mathematics and art projects were overwhelmingly positive. After many years of working together, I took a position in a different school district and our collaboration ended. Determined to provide my students with these integrated mathematics and art experiences, I still assign such projects to my classes. My goal has always been to strengthen student understanding of mathematics and improve their attitudes about mathematics by using it as a tool to guide them in the creation of their own works of art.

In his 1992 book, The Art of Mathematics, mathematics professor Jerry King writes about the way that mathematics was taught to him:

> *"All of us have endured a certain amount of classroom mathematics. We lasted, not because we believed mathematics was worthwhile, nor because ... we found the environment favourable. We had no other choice."* (King, 1992, p. 15)

King (1992, pp. 16 - 17) defined three groups of people to be found in the mathematics classroom: the *'scientists'* who showed interest and aptitude for the subject, the *'humanists'* who were the exact opposite and the *'teacher.'* In King's view (1992, p.16), the teacher neither liked, understood or thought that mathematics was important. King continues,

> *"The 'scientists' were told they had to continue with mathematics ... while the subject may never be understood, it could at least be learned. As the curriculum advanced, the "humanists" became more and more ignored ... and were allowed – even encouraged - to drop out of the mathematics sequence."* (1992, p. 19)
>
> *Nothing in our backgrounds had prepared us for the aesthetics of mathematics. ... The moment was worth the wait but we would be forever aware that we had come to it entirely by chance. And along the way there had been many dropouts. ... Suddenly we understood that mathematics has an aesthetic value as clearly defined as that of music or poetry."* (1992, pp. 20 - 23)

King's words describing his discovery of the beauty of mathematics only at the college level are a sad commentary on the state of the secondary mathematics teaching and learning that he experienced. My concern is that there are too many *"dropouts"* along the way and that our current system of teaching and learning has not changed since Jerry King was a student.

Mathematician, Helaman Ferguson (NCTM News Bulletin, 1994, p. 7) and artist, M.C. Escher (1989, p. 21) both credit a teacher as providing them with the encouragement and inspiration to pursue their passions. As a result, both Ferguson and Escher are recognized for their fascinating pieces of mathematical art. Ironically, Escher was considered a poor student of mathematics and if his abilities had not been nurtured, we may not have had the opportunity to experience the Moebius band (Escher, 1989, pp. 63 – 64) in his *"Bond of Union,"* the five platonic solids (Escher, 1989, p. 47) represented in his *"Reptiles"* and so many other pieces of his mathematical art! Reading about the experiences of Escher and Ferguson encouraged me to continue integrating art into the curriculum, with the dream of perhaps influencing even one future mathematician/artist who could benefit from linking the teaching of mathematics to art.

According to the 2000 *Connection Standard* of the National Council of Teachers of Mathematics of the United States (pp. 64 – 66), instructional programs should enable all students to recognize, understand and apply the interconnectedness of mathematical ideas within the study of mathematics and in contexts outside of mathematics. Jamison (1997, p. 205) and Sinclair (2001, p. 25) both believe that by linking mathematics with the arts, students will develop an aesthetic sense of mathematics and in turn, the subject will become more personally relevant and spark their curiosity.

The impact of my endeavours to make interdisciplinary connections for my students using mathematics and art has exceeded my expectations. Two years ago, I received a note from an honours pre-calculus student at the end of the school year (Harasymowycz, 2007b, p. 25) which thanked me for making the course more interesting and enjoyable because of the inclusion of an anamorphic art project even though sometimes she felt *"frustrated"* because of the difficulty and the tedious nature of the mathematics. Other students have equally positive comments about their mathematics and art experiences (Harasymowycz, 2007a, p. 13):

> *"I thought that math was rather ... not 'useless' per se, but just not incredibly necessary to those whose jobs would not involve math. But here's where I was wrong: math, and especially in this project, can get you to think in a way you never have before much like art."*

G.H. Hardy (1967, p. 61) states, *"the function of a mathematician is to do something."* Students often wonder *"I don't know why I need to learn all this math"* (Hopkins and Dorsey, 1992, p. 10) and *"what do mathematicians do with mathematics?"* (Harasymowycz, 2007b, p. 22) In the spring of 2008, the anamorphic art design of one of my honours pre-calculus students was selected to be published in our school's annual arts and letters journal of student work. She is a very artistic person and a good mathematics student. Her project was the first completed in the class and I could easily recognize her enjoyment and enthusiasm in being able to *do something* using both disciplines. Providing students with the opportunity to see that some mathematicians *do* art and to see that mathematics is more than the dull, rote, drill and practice of irrelevant calculation is very valuable. After visiting the Alhambra, in Granada, Spain in 1936, Escher wrote about his work on tessellations:

> *"The ideas that are basic to them often bear witness to my amazement and wonder at the laws of nature which operate in the world around us ... I ended up in the domain of mathematics ... I often seem to have more in common with mathematicians than my*

> *fellow artists."* (as quoted in Bool, Kist, Locher and Wierda, 1992, p. 55)

Confident that the connections of mathematics and art do indeed improve students' attitudes from my own experiences, I began to read what others had done and commented on regarding the integration of mathematics and the arts. Hopkins and Dorsey (1992, p. 11) support connecting mathematics and other disciplines because it results in a *"more mathematically relevant curriculum."* Cynthia Bickley-Green (1995, p. 17), Phillips and Bickley-Green (1998, p. 49), Schramm (1997, p. 9), Morgan (1998, p. 25), Woodman (2001, pp. 28 - 29) and Forseth (1980, p. 26) found that both art and mathematics learning is enhanced by the use of interdisciplinary activities. Jamison however, cautions:

> *"the goal is not just to look for applications of mathematics in art. Nor is it to use art as a sugar coating for the bitter pill of mathematics. Rather I hope to capture the artistic spirit in mathematics as something beautiful and creative."* (1997, p. 205)

As I continued to read and search more deeply, I began to understand that what I intuitively knew from working with my students was in agreement with what others had already investigated.

1.2 The Objectives and Research Questions of This Thesis

When I began my research my original questions were:

- Does the use of interdisciplinary projects/connections truly increase students' positive attitudes toward mathematics and its value as a necessary and useful tool?
- What factors, when mathematics is taught in context, have the most impact on building student confidence?

- Is there a significant increase in student "mathematical power" when they can make connections?

- Do students continue to study mathematics because of the links to other subjects?

Since several other researchers had studied the impact of interdisciplinary education and its affect on students' attitudes in learning mathematics, I wanted to look at the issues from a different angle.

I reread Sonia Forseth's 1980 article "Art Activities, Attitudes, and Achievement in Elementary Mathematics" and was struck by her quote (p. 22) of Frances E. Anderson (1971, p. 54) who commented on the open-mindedness and positive attitude toward art of students even if they "*have an inferiority complex ... about their own personal art behaviours.*"

Anderson's conclusion caused me to wonder whether students can still have a positive attitude toward the subject of mathematics, despite having an inferiority complex about their own personal mathematics behaviour. This idea gave birth to my research project. Through pilot data and the survey of students in the first part of this research project, I determined that indeed it was possible for students not confident in mathematics to enjoy learning the subject. An unexpected result indicating the lack of enjoyment found in students who were confident in mathematics intrigued me and the quest for an understanding of this result became my new focus.

Kloosterman (1988, p. 345) states that "*students who are confident of their ability to learn mathematics are more likely to take math in school* *when it becomes optional."* But, if being confident makes students take more mathematics that they don't enjoy, doesn't that take us back to Jerry King's view of his mathematics education? *"We had no other choice"* (1992, p. 15). We need to discover ways to increase student enjoyment of mathematics not only for the non-confident students, but to prevent the loss of the apparently large number of confident *"dropouts"* so that the aesthetics of mathematics are understood early on and not *"by chance."*

Gregory J. Chaitin wrote his 2002 book, Conversations with a Mathematician: Math, Art, Science and the Limits of Reason as a series of interviews that he was asked to give. He claims that, despite being a very logical discipline, creating mathematics requires the same components as creating art does: *"passion, intuition, imagination and inspiration."* (2002, p. v) Chaitin (2002, p. 137) believes that mathematics is *"a music that not all of us can hear"* and it is *"definitely an art form."*

In attempting to look for connections between mathematics and art, it seemed both reasonable and appropriate to extend my investigation into students' confidence and enjoyment in mathematics and art to include the subject of music also. In the end, I decided that this research project would compare students' attitudes in all three subjects. As I embarked on this journey of exploration into students' attitudes toward the learning of mathematics versus the arts, from my initial ideas through part one of this project, a series of questions emerged for this thesis:

- Why don't students like learning mathematics when they are good at it?

- What makes students who don't perform well in mathematics still like learning it?

- How do these compare with students' attitudes and beliefs in the same categories above for art and music?

- What can we learn from students' attitudes in art and music that can improve the teaching and learning of mathematics and help foster a positive attitude toward learning mathematics in all students?

In order for students, both the scientists and the humanists, to persevere in the study of mathematics they must understand its beauty, value and usefulness. In the words of Jerry King,

> *"... the aesthetic experience of mathematics both excites and soothes mathematicians ... the excitement brings them to the subject ... the soothing becomes addictive and draws them back again and again ... the aesthetic pleasures are unavailable to the humanists ... because the right view of mathematics has been hidden from them."* (1992, p. 3)

1.3 An Overview of the Study

I had long questioned whether an understanding of students' perceptions of the arts could have a positive impact on their perceptions of mathematics. In looking to answer this question, I undertook an investigation into the attitudes of high school students when learning mathematics, art and music – most especially, their confidence in and enjoyment of learning these subjects. The results of my findings are contained in this thesis.

The first part of this research project was a questionnaire designed for use in an international study with students from the United States and England. The questionnaire contained seven Likert-type responses and one open-ended response (n=1226). The intriguing results of the analysis of my questionnaire data prompted me to further examine students' attitudes towards the learning of mathematics as related to their confidence and enjoyment. In looking to probe more deeply into these findings, I used the repertory grid technique, developed by George Kelly in 1955, to conduct interviews (n=42) for the second part of this research project. Students were selected for the repertory grid interviews from the questionnaire participants based upon their confidence and enjoyment categories.

Chapter 2 A Review of the Literature

2.1 Introduction

> *"Arithmetic is where the answer is right and everything is nice and you can look out of the window and see the blue sky - or the answer is wrong and you have to start all over and try again and see how it comes out this time."* (Sandburg, 1960, p. 115)

Students often react to their experiences while learning mathematics in much the same way as Carl Sandburg expressed in his poem, *"Arithmetic."* They believe that mathematics is all about the answers being right or wrong as opposed to the arts where they perceive that there is no right or wrong. What else do students experience while they are learning mathematics? What motivates them to learn? What roles do confidence and enjoyment play in the process? How do learning experiences of students in the arts compare?

To set the stage for this research, my review of the literature has been organized into five sections. In the first section, different perspectives on the underlying views on the teaching of mathematics are explored. The second section of this chapter is concerned with the issues of motivation on student learning. In the third and fourth sections, the three subjects at the core of this investigation, mathematics, art and music, are looked at in two different ways: first, as discrete entities with their own unique characteristics, and second, in conjunction with each other with respect to their commonalities. The last section of this chapter is focused on the confidence and enjoyment of students in learning mathematics.

2.2 The Philosophy of Mathematics Education

The mathematician, logician and teacher, Augustus De Morgan explained (1943, p. 7) that the benefits of studying mathematics are *"as a discipline of the mind and a key to the attainment of other sciences."* He (1943, pp. 7 - 10) believed that one must learn to reason as a habit of mind just as one must learn to swim because these are not inherent abilities.

What is it that we want our teachers and schools to do in order to educate our students in mathematics? Frasier and Armentrout (1927, p. 135 -148) wrote that the method chosen for teaching in general should be based on, the aims of education, the nature and needs of society, and the nature of the child. With regard to the nature of the child, they state (1927, pp. 139 - 140),

> *"It is not what is presented to him that educates him, but rather the responses that he himself makes to what is presented ... we learn to do by doing ... in other words, it is the self activity which educates him."*

As part of the learning of mathematics, Frasier and Armentrout (1927, pp. 139 - 140), Mann (2006, p. 243), De Morgan (1943, pp. 7 - 10) and Boyer (1945, p. vi) believe that mathematics lessons should not just focus on the sharpening of skills in computation, but also make students aware of the essential need to study mathematics because of its use and value in many, various and universally accepted applications. To Jo Boaler (1993, p. 15), what is important is the use of contexts for applications of mathematics to make the transfer of knowledge possible: *"appreciation can only come from an examination and reflection of the underlying structures and processes which connect experiences"*. Boaler (1993, p. 17) emphasizes the importance that, in order for students to become personally involved in the mathematics of the task and make connections to further their mathematical knowledge and understanding, the activities in which students are engaged must allow them to *"follow routes which are their own."*

In 1945, the Canadian Mathematical Congress encouraged the study of mathematics because of its significant value and use in a variety of professions that may be of interest to many students. The Congress stressed the importance of mathematics beyond the basics for an appreciation of the scientific nature of the world in which, even those not interested in such careers, live. They stated (1945, pp. 23 - 24) that high school mathematics courses are *"ideal clinical training in reasoning ... and a proving ground for logic in real-life situations"* although they showed that others may disagree that what is provided is not enough for the transfer of reasoning to non-mathematical situations unless the students are made aware of specific settings.

> *"It is hard for the young to understand why they are shut up in their classrooms and taught skills such as trigonometry ... they may resent it bitterly. They cannot be told directly or convincingly how learning trigonometry will fit into their future existence ... partly because they cannot realize the value of mathematical thinking ... they should be given to understand in as many ways as possible that the two worlds are closely and necessarily connected ..."* (Highet, 1950, p. 50)

The relevance of mathematics to the world of students is clearly not a new idea. *Problem solving* and *applying mathematics in contexts outside of mathematics* continue to be valued by the United States' National Council of Teachers of Mathematics as noted in their Principles and Standards for School Mathematics (2000, p.402) for all levels of students.

Philosopher, historian, logician and mathematician, Bertrand Russell (1957, p.56) claimed that, when asked, many would say that the purpose of teaching and learning mathematics is one of practicality, for example, for the manufacture of products, for travel, to win a war and to develop reasoning ability in students. To Russell (1957, p. 57) however,

> *"Mathematics, rightly viewed, possesses not only truth, but supreme beauty – a beauty cold and austere, like that of a sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show."*

Santayana writes in his book, The Sense of Beauty (1955, p. 51), *"beauty is value positive, intrinsic, and objectified ... beauty is pleasure regarded as the quality of a thing."* Rothstein (1995, p. 178) states that beauty in the arts can have the same origins as beauty in mathematics, that is, suddenly finding oneself confronted by an unexpected pleasurable event, situation or place. According to Russell (1957, p. 58), mathematicians have not made us consciously aware of the beauty that underlies the study of mathematics. He suggests that the teaching of mathematics *"rightly... awakens the learner's belief in reason, his confidence in the truth of what has been demonstrated, and in the value of the demonstration."*

Russell also asserts that students are not generally provided with such experiences and, as a result, in the views of students truth or falsity occurs mainly on the whim of the teacher. In what still holds true for today's classrooms full of students, Russell (1957, p. 60) chided: rules supplied to students are followed blindly without explanation and, in the minds of students, mastery is only achieved when the teachers' answers are obtained with no understanding of the foundations behind the processes.

Polya (1962, p. 100) would appear to agree since he states that the purpose of teaching mathematics is to teach students *"to think"* not just to convey information but enable students to use what they have learned. To Russell (1957, p. 66), it is not the accuracy of the theorems that should be emphasized but the beauty in the ways the theorems can be used that should be conveyed to students. The goals of teaching and learning mathematics according to Russell (1957, p. 68) are not just the subject itself but all to promote in students, *"the means of creating and sustaining a lofty habit of mind."* And Harold Jacobs writes in his beloved book, Mathematics A Human Endeavor (1982, xii), *"unfortunately, even though mathematics is a very broad subject, many people leave school with a very narrow view of it."* In the words of Davis and Hersh (1981, p. 169), the lack of awareness of the aesthetic nature of mathematics is what leads many to feel that mathematics is *"as dry as dust."*

In his 1999 article, "The Mathematical Miseducation of America's Youth," Battista (p. 426), claims that the traditional approach to teaching mathematics is *"taking its toll on the nation and on individuals."* Indeed, Battista (1999, p. 426), Mtetwa and Garofalo (1989, p. 613), the 2002 report of the National Research Council (p.16) and Garofalo (1989, p. 503) all state that for many students, the nature of doing school mathematics *"is simply a matter of memorizing and reproducing facts, rules, procedures, and formulas at the appropriate time."* It isn't surprising then that an advertisement for a textbook series (Understanding Maths, 2005) claimed that success in mathematics exams hinges on three criteria: *understanding*, *memorizing*, and *applying* theorems, formulae, and techniques. This opinion prevails in the community as a whole, including students and their parents for whom a good exam grade is the primary focus. It is common for students only to value mathematics if it is 'on a test' Garofalo (1989, p. 503)!

Hoffman (1988, p.2) argues against the view that all that is necessary is repeated computation in order to learn mathematics. Battista (1999, pp. 426 - 427) appears to agree with Garofalo (1989, p. 504) who advises that we must create classroom environments that will help students understand the real nature of mathematics instead of the daily routine: *"here's the procedure, here's a few examples, now here's some for practice."* Davis and Hersh (1981, p. 282) calls this the *"authoritarian"* approach in which instead of *"come let us reason together"* we find teachers communicating the message, *"look, I tell you this is the way it is."*

"The young child has learned to play the academic mathematics game," says Paul Cobb (1986, p. 7), which is to *"satisfy the demands of the authority* (teacher) *rather than to learn academic mathematics per se."* To improve the situation, he claims that mathematics placed in context provides students with meanings for what they are learning in the classroom is a possible solution (1986, p. 9). Boaler, William and Zevenbergen (2000, p. 5) and Boaler (1998, p. 47) express similar concerns. Boaler (1998, p. 47) noticed that during observation of mathematics lessons at one school, students based *"their mathematical thinking on what they thought was expected of them rather than on the mathematics within a question."* These students who were *"motivated and hard working,"* found that mathematics was *"tedious and boring"* (1998, p.

49). Wanting to improve student performance in mathematics at her school, principal Cathryn Busch (1995, pp. 90 - 113) observed mathematics lessons and talked to students and they expressed sentiments similar to what Boaler found. Busch (1995, p. 100) was puzzled by the lack of enjoyment in learning mathematics articulated by students, even the high achieving students in the class. Through an intervention with this class, which eliminated both the use of a textbook (for a time) and eliminated an authoritative step-by-step procedural approach to the lessons, and instead utilized a strategy which encouraged thinking, both interest and enjoyment in learning mathematics improved.

This verifies the view of the popular mathematics and science writer Martin Gardner (1982, p. xi), whose specialty is recreational mathematics, that the dislike of mathematics is not instinctive but rather the result of the way mathematics is taught and that this negative attitude can be changed.

Nardi and Steward (2003, p. 356) also agree that many feel that mathematics only involves a mindless series of task completions. Langer (1997, p. 14) discusses the pitfalls because *"learning the basics in a rote unthinking manner almost ensures mediocrity ... mindless practice keeps the activity from becoming our own."* For learning to be meaningful, Oddleifson (1994, p. 447) believes that students must make their own sense of what they are learning. They do not acquire knowledge passively. Students achieve meaning when they can relate concepts with *"active use of our perceptive abilities"* and an *"analytic framework."* In Oddleifson's opinion (1994, p. 447), analysis and reason alone are not sufficient for learning to take place: *"talking at the students"* does not do the job; *"intellect and the senses must work together as coequal partners."*

There have been three inter-related components of the *"progressive educator ideology in mathematics education"* over the last one hundred years claims Ernest (1995, p. 188):

- *"The provision of an appropriately structured environment and experiences for the learning of mathematics"*
- *"The fostering of active and autonomous inquiry in mathematics, by the child"*
- *"A concern with the child's feelings, motivation and attitudes and the shielding from negative aspects"*

Ernest (1995, p. 191) explains that this philosophical approach places emphasis on *"mathematics as a language, and on the creative and human side of mathematical experience."* One of its aims is to promote student confidence in and positive attitudes toward learning mathematics. While I am not sure that this ideology is universally evident in all schools, Pehkonen and Torner (1996, p. 101) claim that the teaching of mathematics in most schools across the world has been changing and continues to do so, from a view of mathematics as a static system to one in which mathematics is dynamic. Garofalo (1989, p. 504), Oddleifson (1994, p. 447), Pehkonen and Torner (1996, p. 101), and Mc Leod (1987, pp. 134 - 135) all believe students learn best when they are actively involved, not *"passive listeners and practicers,"* exploring, reflecting, discussing, seeking patterns and generalizations, connections, applications, etc. rather than performing exercises requiring only rote memorization. Boaler (2002, p. 44) claims that while many students are disaffected by learning mathematics in a passive way, some *"did not need to think about 'how and why' mathematics worked and they seemed to appreciate the passive positions."* It seemed to her that it is mostly the more verbal students who prefer the active approach. A statement made by a participant in Hoyles' work (1982, p. 369) might summarize student feelings regarding a good experience in learning mathematics: *"I really enjoyed that, miss, you sitting there and listening to me – makes a change somehow, doesn't it?"*

Anderson (2007, pp. 8 – 11) describes four different characteristics of the identities that learners assume to define themselves and their place in the mathematics classroom.

- *Engagement* is related to students' capabilities as mathematical learners whether it is to respond quickly and correctly to a question or to creatively develop their own methods and meanings to solve problems.
- *Imagination* allows students to envision the use of mathematics in the broader picture of their lives at the moment and in the future.
- *Alignment* enables students to recognize the mathematical requirements for their goals, for example, to attend college.
- *Nature* is the recognition of students of those factors over which they have no control as in their innate abilities to do mathematics.

Anderson (2007, p. 10 - 13) claims there may be overlaps in the identities and makes suggestions for mathematics educators to develop students' positive identities as mathematical learners, such as using a variety of problems to encourage creativity of expression and communication of ideas in expressing solutions to the problems as well as making students aware of ways that they can *"create and use mathematics in their work and play"* (2007, p. 13).

Mathematician and writer, Keith Devlin (2000, p. 16), asserts that a great deal of time in the mathematics curriculum is spent on the first face of his 'four faces of mathematics' as the world sees it, which is 'computation, formal reasoning, and problem solving.' Devlin acknowledges that there is some reference to the fourth face, 'applications,' in the curriculum but, very little, if any, attention to the other two faces, mathematics as a 'way of knowing' and a 'creative medium.' He states that all four faces are important in mathematics education.

A study conducted by Boaler (2002, p. 43) found differences in the mathematics learned in two schools that used different approaches to teaching it. Students in School A learned mathematics by working through textbook exercises and those in School B learned mathematics through open, group-based projects. Boaler found that the knowledge and understanding of mathematics that students in each of these schools had attained was directly related to the manner in which mathematics was taught to them. Students in School A were significantly out-performed by those from School B in the open-ended, applied or discussion based assessments because they had *"learned to watch and faithfully reproduce the procedures"* of their teachers and the textbooks. Students from School B had a more flexible use of the knowledge about mathematics that they had acquired which proved more useful. Due to different pedagogical strategies, students from School A did not learn less mathematics than those from School B, they learned different mathematics. Boaler's findings seem to agree with the views of Skemp (1987, p. 204), who notes that there are differences in the learning of students based on whether that learning has come through memorization versus understanding.

Dreyfus and Eisenberg (1996, pp. 264 – 265) remind us that structure and flexibility of thought in mathematics are its most important aspects because relationships are much more important than facts.

In their reference book, Tilings and Patterns, Grunbaum and Shephard state,

> *"It is curious that almost all aspects of geometry relevant to the 'man on the street' are ignored by our educational systems ... to people who wish to apply geometric ideas in their work – engineers, scientists, architects, artists and the like."* (1989, p. vii)

Grunbaum and Shephard believe that a reason for this sad situation is that the study of geometry has been mainly considered as a foundation for logic and reasoning without much recognition of the beauty and visual nature of its content at the high school level. This they feel is because the *"visual appeal has been completely submerged in technicalities and abstractions."*

Williams (1983, pp. 86-87) would appear to agree with their approach to education as she explains that there is a visual perception involved in all subjects, including mathematics. In her view, mathematics *"involves perceptions of relationships which can often be represented visually."* Williams notes that by mere observation of students in art classes or science laboratories, one can become aware of their wide range of perceptions. She notes that *"one of the purposes of school is to give students experience through which to develop and refine their observational abilities."* Williams states that when the focus of a lesson is to demonstrate a rule or principle from a book, students are not taught how to notice information on their own.

In his book, Teenagers, Teachers, and Mathematics, David Thomas (1992) expounds on the notion that the passive attainment of information means nothing all by itself. Instead, Thomas (1992, p. 63) states, *"the learner's role is more involved with linking up previously learned concepts in order to create new, more efficient insights than the rote memorization of isolated facts"*

The 1945 Canadian Mathematical Congress (p. 27) encouraged students and teachers to see that mathematics can and should be studied for no particular reason other than *"to obtain pleasure, stimulation and insight into the world around us"* because *"it is a free flight of the imagination"* even though that view is not entirely obvious. They suggested some simple ways to expose students to novelty in mathematics: using recreational mathematics to utilize the imagination and promote interest; creating an unexpected result as a source of pleasure; and promoting the understanding of difficult and vague concepts and situations to yield satisfaction.

The 1945 Canadian Mathematical Congress (p. 29) believed that, with regard to mathematics, *"the most effective method is to study it for its own sake,"* for in their view, the emphasis on calculation and skill sharpening is not the goal of mathematics any more than working on scales is the goal of learning music. Using this strategy, through the mastery of mathematics, *"a student will often experience a thrill of imagination and an elevation of spirit"* similar to the feelings experienced when a student masters the study of music.

Rademacher and Toeplitz (1957, p.7) make a similar parallel between the experiences of learning mathematics and those of music when they explain that looking for themes in the development and motives of problems in order to appreciate mathematical reasoning is much like looking for themes while listening in order to appreciate a musical composition. Jacobs (1982, p. xiii) answers the question of what good the study of mathematics is in this way, *"mathematics has its own kind of beauty and appeal to those who are willing to look."* Stewart (2006, p. 9) would agree with this statement. Cuoco, Goldenberg and Mark (1995, p. 183) believe that the beauty of mathematics emanates from the interrelatedness of its ideas and that, as students develop an appreciation of these connections, they will be able to recognize this beauty for themselves. Davis and Hersh (1981, p. 169) remind us that, while the beauty of mathematics is more intangible, it *"is of importance, can be cultivated, can be passed from generation to generation, from teacher to student."*

In the view of the National Council of Teachers of Mathematics (2000, p. 18),

> *"Teaching mathematics well involves creating, enriching, maintaining, and adapting instruction to move toward mathematical goals, capture and sustain interest, and engage students in building mathematical understanding."*

In Boaler's study (1999, p. 134), none of the students' responses indicated that they were persuaded to realize the beauty and creativity of the study of mathematics and problem solving, factors that would certainly inspire their desire to learn mathematics. Rickart (1996, p. 294) states that creative mathematical experiences are hard to describe to non-mathematicians and they often require a great deal of hard work.

Cornell (1999, pp. 225 - 230) asked pre-service elementary school teachers to list the factors that caused to their negative attitudes toward mathematics. Among those factors were: teachers' lack of patience and understanding of students' difficulties in learning mathematics, obscure vocabulary frequently used in textbooks, incomplete instructions for computational algorithms which provided the 'one way' to solve a problem, too many exercises for drilling of skills, endless repetitive homework problems, a great emphasis on rote memorization, and skills taught in isolation as an end in themselves. Suggestions for improving mathematics instruction and teacher training included finding means to *"increase both content and affective factors ... to give meaning, purpose and relevance to math instruction."* (Cornell, 1999, p. 230)

Polya (1973, p. ix) was disturbed by the view of mathematics as the least favourite school subject and the notion that prospective teachers go through their own elementary school experience learning to hate mathematics and then return to the elementary classroom to enable a new generation of students to reach the same conclusion. In his rules for teachers, Polya (1973, p. 173) states, *"a teacher wishing to impart the right attitude of mind toward problems to his students should have acquired that* *attitude himself."* As for the training of teachers in the abstract disciplines such as mathematics, Conklin (1970, p. 36) advises:

> *"A professor should strive to make his class aesthetically enjoyable and pedagogically exemplary, since the methods he uses teach themselves as they convey the subject matter to the prospective teachers who are his students."*

2.3 Motivation to Learn

The noted philosopher, psychologist and educational reformer, John Dewey (1913, p. 60) defined motive as *"the end or aim in respect to its hold on action, its power to move."* Motivation to Skemp (1991, p. 96) is *"a description we apply to behaviour which is directed towards satisfaction of some need."* Skemp (1987, p. 92) states that it is difficult to assess motivation because as it is internal to an individual, it is not easily observable.

According to Edward Deci's (1975, p. 23) definition of intrinsic motivation, by engaging in activities for their own sake *"rather than as the means to an end,"* the individual receives no reward from the activity but does derive a sense of enjoyment. According to Deci (1975, p. 61) and Deci and Ryan (1985, p. 32), an individual engages in intrinsically motivating behaviours because of a need for competence and self-determination oftentimes seeking out challenges with pleasurable results. Characteristic of intrinsic motivation, Deci and Ryan (1985, p. 245) state, is the energizing of students' learning through their natural curiosity and interest. Deci (1975, pp. 210 - 212) and others have wondered what happens to change children, who begin life intrinsically motivated to learn, into students who are bored and not interested in learning. They hypothesize that it may be because earlier challenges sought out by students have become too easy or that the learned habits of engaging in activities merely for rewards (e.g. memorizing well) have taken away students' abilities to think creatively through discovery thereby decreasing their competence motivation.

McDaniel (1985, p. 19) states in his "*Ten Commandments of Motivation*" that "*boredom is the arch enemy of motivation*" (p. 19) and that "*one of the strongest intrinsic motivators for all of us is* ourselves" (p. 22). According to Skemp (1987, p, 96) the learning of mathematics is mostly affected by factors outside (extrinsic) of the learner, such as the value as a tool in our culture and reward or punishment. Skemp (1987, p. 97) also claims that the need to be satisfied can also be an intrinsic motivator for some. He asserts (1987, p. 91) that it is widely accepted that the level of motivation decreases as the level of complexity in the task assigned increases. Collins and Amabile (1999, p. 300) state however, that "*creative people are energized by challenging tasks, a sign of high intrinsic motivation.*" They (1999, pp. 305 - 306) claim that in the classroom, freedom of choice to engage in "*personally exciting ideas*" allows for a greater level of intrinsic motivation in students.

It appears that in the learning of mathematics there needs to be a balance between the simplicity and the complexity of a task for a student to like it and be motivated to become engaged in the task. Levitin (2006, p. 229) claims this balance must also be true in music and art.

According to Amabile (1989, p. 51), there are four components to intrinsic motivation: "*love, dedication, a combination of work and play, and a* *concentration on the activity itself."* Middleton (1995, p. 254), who uses 'intrinsic motivation' as synonymous with 'fun', states that while it seems not to matter whether a student is intrinsically motivated to engage in an activity or that the motivation comes from the activity itself in some disciplines, it does seem to matter in learning mathematics. Middleton and Spanias (1999, p. 66) more specifically state that with regard to mathematics, if students enjoy a task they become intrinsically motivated in it. In the view of Skemp (1987, p. 97) the intrinsic motivation to do mathematics for some comes from a *"need to grow"* mentally and the enjoyment an individual feels for taking part in intellectually enhancing activities. It is the opinion of Csikszentmihalyi, Rathunde and Whalen (1993, p. 13) that individuals are motivated to do something that they enjoy and they will enjoy something that they can do well. Amabile (1989, p.54) indicates that one is intrinsically motivated, by doing something for its own sake because it is *"interesting, enjoyable, satisfying or personally challenging,"* and when these factors are in place is the time that a person is at their most creative. This intrinsic motivation varies from person to person depending on their level of interest and the level of social context.

As Amabile (1989, p. 54 - 55) sees it, an activity is *"only intrinsically interesting for a particular person at a particular point in time."* She notes another key feature of intrinsic motivation is the feeling that a person has that the activity that they are doing is their choice and something done for their own reasons and not someone else's. A student may not be interested in doing mathematics in the classroom, but those same concepts when connected to a hobby or pastime outside of the classroom could be quite motivating. In Skemp's view (1987, p. 97) the best type of learning experience is one in which an individual is both intrinsically (*"by enjoying the learning and doing of mathematics"*) and extrinsically (*"a practical or academic goal to be achieved with the help of a knowledge of mathematics"*) motivated.

In John Dewey's (1913, p. 1) view, *"interest is the sole guarantee of attention"* and if a student is interested in a subject, it is certain that the student will make every attempt to master the skills and concepts involved. Dewey (1913, p. 16) states that psychologically, to *"take interest"* in any matter is to have an active and objective concern with it and that concern must be directly, personally and emotionally related to an individual. Dewey (1913, p. 21) considers interest as *"a self-expressive activity"* through which growth is a result of emerging inclinations.

According to Middleton (1995, p. 255), who used the repertory grid technique, student constructs with regard to motivation came in three clusters: *'arousal'* - the cognitive incentive of an activity; *'personal control'* - freedom of choice or appropriate difficulty; *'interests'* - how much students liked the activity, felt it was important or matched their abilities. Agreeing with Amabile about individuals possessing different degrees of motivation in mathematics, Middleton (1995, p. 255) sees this as the reason for the varying levels of perceptions of mathematics as motivating. *"Individuals with differing interests define motivating activities in different ways."* (1995, p. 257)

Dewey (1913, p. 61) asserts that all too often teachers look *"for a motive for the study or the lesson, instead of a motive in it."* Middleton (1995, p. 275) and Skemp (1987, p. 98) relating Dewey's view to the study of mathematics, believe that teachers do not know and understand the importance of intrinsic student motivation in the learning of mathematics and therefore they focus their lessons on extrinsic motivators, such as applications that they thought students would be interested in such as, grouping and hands-on activities. Csikszentmihalyi, Rathunde and Whalen (1993, p. 195) assert that *"students will learn only if they are motivated"* – whether intrinsically or extrinsically. Middleton also found that while students believed in the value and importance of mathematics, this alone was not enough to motivate them to achieve. Skemp (1987, p. 98) states that until the significance of intrinsic motivation in the learning of mathematics is understood and utilized, *"mathematics will remain for many a subject to be endured, not enjoyed, and to be dropped as soon as the necessary exam results have been achieved."*

Hoyles (1982, p. 369) found that students did not view mathematics as having any interest to them in itself, but just as something that they had to do. Amabile (1989, p. 62) asserts that those intrinsically motivated in a task are willing to explore pathways and solutions to accomplish the task utilizing a certain level of creativity in order to do so. Those whose motivation is strictly extrinsic or outside of the activity itself, for example to win a prize, to meet a deadline, to avoid punishment, will complete the task in the simplest path or solution possible with little reliance on creativity in their endeavours. In Amabile's opinion (1989, p.87) motivation is enhanced in students when the teacher is less controlling and the classroom environment allows more autonomy. *"Rules that control, rather than inform, can kill creativity."* (1989, p. 69) Dreyfus and Eisenberg (1996, p. 258) assert that passive learning by just sitting back and absorbing the curriculum becomes for students a more *"digestive process than a creative one."* Students will be able to think more creatively about numbers if they have been exposed to connections and linkages during learning experiences and not just rote memorization of the facts. Others like Mann (2006, p. 240) agree. Sternberg (1995/ 1996, p. 80) explains that creativity is *"an attitude toward life"* and has been curbed in students in educational environments which insist on uniformity. In Gardner's (1982, p. ix) opinion, the most certain way to stifle a student's zeal for mathematics is to overwhelm them with boring exercises and problems unrelated and meaningless to their interests. Boaler (1993, p. 14) agrees but cautions that the random inclusion of applications in order to motivate and create interest for students in mathematics may *"act as distracters or even barriers to understanding"* because the context in which they appear is not in the everyday life of students but a forced real life school version for their lives.

Donald B. Eperson, himself a mathematician and teacher (2000 and 1933, pp. 92 - 100), writes about another mathematician and teacher Charles L. Dodgson, who under the penname of Lewis Carroll penned Alice in Wonderland which is full of mathematical references. Dodgson was clearly aware of the importance of recreational mathematical to stimulate student interest in a subject which many found to be difficult. Dodgson wrote books of puzzles and problems, some of which he invented, and others on the subject of mathematics, such as Symbolic Logic, hoping to share mathematics with others, to encourage them to study it, to enhance students' views about it and their understanding of it. Furinghetti (1993, p. 35) also makes note of Dodgson's style as a good approach to teaching mathematics.

Oddleifson (1994, pp. 446 - 447) believes that, in addition to our *"standard, subject-matter-driven curricula in schools,"* the motivation of students to learn is the *"hidden curriculum."* He adds that, most believe that the purpose of education is the attainment of a certain body of facts and many people feel that what is taught in art is *"irrelevant."* Oddleifson found that when the arts are studied as a serious subject involving all students, not just those talented in art, both academic achievement, even in *"subjects such as math and science,"* and motivation increase. It is the opinion of the well known advocate for an integrated curriculum, James Beane (1995, p. 5), that *"only when the curriculum engages students' hearts and minds does learning have meaning for them."*

Schiefele and Csikszentmihalyi (1995, p. 173) found that *"interest was the strongest predictor of quality of experience in mathematics class."* Wang, Haertel and Walberg (1993/ 1994, p. 75) believe that ability has the most effect on student learning and motivation leads to student effort and perseverance. According to them, these are the *"key attributes necessary for developing self-controlled and self-regulated learners."* Kloosterman's (1988, p. 349) finding makes at least a partial connection between students' motivation and their thinking about their ability and self-confidence.

Mailhoit and Kobasigawa (1997, pp. 644 - 648), acknowledging that the motivating factors involved in the study of many subjects involve an end product, investigated students' (ages 9 years and 11 years) perceptions of the purposes of learning certain school subjects both in the long and short term. From student responses, they found that the top ranking reasons were for:

- mathematics – *subject specific*: computation, graphs and estimation and *cross-curricular*: do well now and in the future

- music - *subject specific*: learning different rhythms, melodies and songs and *cross-curricular*: do well now and in the future, prepare for after school, improve life and learn to work with other people

- art - *subject specific*: use creativity and imagination, express ideas through paintings and learn different kinds and shapes of materials and understand others' ideas in their paintings and *cross-curricular*: do well now and in the future, prepare for after school, improve life and learn to work with other people

The students who participated in this study of Mailhoit and Kobasigawa obviously have different outlooks on the purposes for and values of learning mathematics, art and music.

Boerkaerts (1994, p. 8) claims that there are many factors involved in the learning of mathematics. She asserts that even students who have ability to learn and do mathematics are not motivated and do not view mathematics to be useful to them because of the nature of the traditional textbooks with their heavy emphasis on computational skills. Teachers' heavy reliance on textbook problems, Mtetwa and Garofalo (1989, p. 615) tell us is another factor that contributes to students' difficulties and negative attitudes in mathematics is the sense that in mathematics there is just one right answer. Many others share this view including Ginsburg (1996, pp. 184 - 185) who reminds us that we, as teachers, reinforce this belief in children who are *"overly sensitive to being wrong"* and when we should be communicating more to our students that the heart of mathematics is not just getting right answers but being able to think in a creative way.

Another suggestion for mathematics educators is to find ways to utilize students' errors in mathematics to enhance instruction. Borasi (1996, pp. 3 - 4) believes that focussing on student errors would promote exploration and problem solving in the classroom instead of as a source of the negative attitudes. In Sternberg's (1995/ 1996, p. 83) view if students are afraid to make mistakes they will not take risks to think independently. Boaler (1999, p. 134) asserts that students were motivated in mathematics only to get the correct answers.

Boaler (1999, p. 134) also found that students did not mind working in subjects other than mathematics because they possessed an interest in the subject. Boaler (2002, p. 44) states that despite success in traditional classrooms, many students expressed a dislike for mathematics and experienced a decrease in motivation to continue studying it because they no longer wished to perceive themselves as passive receivers of knowledge with no room for interpretation or input of their own. They were rejecting mathematics not because of the subject itself, but the way in which it was taught to them. According to Polya (1973, p. v), if a teacher spends lesson time *"drilling his students in routine operations he kills their interest, hampers their intellectual development, and misuses his opportunity."* Polya (1962, p. 103) says that active learning, where student interest is high and pleasure in the activity is great is the best motivator for the learning of mathematics.

Berkas and Pattison (2006, p. 4) believe that the most valuable impact that teachers can offer to their students is motivation and opportunity to learn important mathematics. According to the National Council of Teachers of Mathematics (2000, p. 18),

> *"there is no one 'right way' to teach ... effective teachers ... shape students' mathematical dispositions and can create rich settings for learning ... more than just a physical setting with desks, bulletin boards, and posters, the classroom environment communicates subtle messages about what is valued in learning and doing mathematics."*

In 2000, the National Council of Teachers of Mathematics stated (p. 374), that experiences that involve students in the mathematics of the workplace can be useful in motivating students. Williams (1983, p. 187) suggests the importance that going beyond a reliance on the text and encouraging students to have and ask questions and think independently might make class more motivating. *"A subject which may have become 'old hat' and rather dull can come alive and stimulate new learning."* Langer (1997, pp. 38 - 43) explains that by providing novelty in the way a lesson or the curriculum is presented enables students to pay attention and not have their minds wander.

In order for students to be motivated to learn mathematics as an end in itself, we must ensure that as teachers we not only impart knowledge to them but, in the words of Csikszentmihalyi, Rathunde and Whalen (1993, p. 195), also be able to *"spark the joy of learning"* in them. In mathematics, Hoffman (1988, p. 3) and Gardner (1982, p. x) state that, it is important to convey to students that mathematics is full of magic, mystery and surprises and that just like the arts there is still so much more out there to learn and discover about mathematics.

2.4 Three Disciplines: Mathematics, Art, and Music

> *"In nearly all of the important subjects ... music and the arts ... there is a constant upsurge of discussions on new problems and points of view ... only in mathematics and the inorganic sciences, perhaps, do the foundations of the first three or four storeys of the subject stand firm and unaltered year by year."* (Highet, 1950, pp. 81 - 82)

There are no school systems in which the study of mathematics is considered irrelevant to the culture and education of its students and so, mathematics is a requirement of learning for all students. That is not the case in the arts. Perrin (1994, p. 452) states that too often the arts are considered a *"frill."* Many art educators, claims Oddleifson (1994, p. 448), have a somewhat elitist attitude toward their subject, only hoping to find and work with the most talented among their students. In order to create a balance in cognitive development, Jones (1993, p. 2) asserts that the arts are as important to students as the academics explaining it this way: *"the mission is not to create more artists (any more than math is designed to create more mathematicians)."*

Most of us have been taught to believe, Hoffman (1988, p. 3) says, that mathematics is all about logic and deductive reasoning. In his words, *"if it* *were only that simple!"* Oddleifson (1994, p. 448) tells us that, while the *"Greeks created logic to make sense of perception,"* since then all of the efforts of education have been focused on the *"logic of reason rather than the logic of perception."* Perception was left to the arts, while logic and reason were considered to be part of the domain of mathematics et al. Although, Conklin (1970, p. 27) explains, the view of some is that mathematical intuition is similar to sense perception. Oddleifson (1994, p. 449) believes that, the perceptive actualities of students are seen as much more real than the *"school-based"* realities, which focus only on *"words, reason, logic, and analysis,"* and much more important to success in life.

Psychologist Howard Gardner (1994, p. 33) differentiates mathematics from the arts in the nature of their bodies of knowledge. The principles of the sciences build up based upon newly discovered theories that can modify and get assimilated into the old theories. In the arts, old works are not supplanted by new ones. In addition, Gardner recognizes that the arts are a sensory medium, pleasing our organs of sight and sound. *"It is in the lack of immediate sensory appeal, and not in the elegance or subtlety of its conception, that a mathematical formula departs from the characteristics of the arts."* (Gardner, 1994, p. 34) He also asserts that mathematics and the arts are different in their goals. Science seeks to explain the world in attempting to provide a rationale for certain events and the arts seek to re-create, remark and respond to those events. Gardner (1994, p. 311) claims that, as a means of communication, the arts and mathematics are different in the messages that they convey. The artist wishes to represent an understanding of the feelings of living objects in the world in which the artist is a part. The work of the scientist involves objects of the world and has nothing to do with the scientist himself/ herself. The artist gives us a part of themselves personally and the scientist intends no such involvement. Problem solving exists in the realm of both the scientist and the artist says Gardner (1994, p. 314). For the scientist, the problem emanates from an already familiar field, perhaps being worked upon by others and its solution depends on agreement with other scientists. For the artist, the source of a problem and its solution are determined by the artist alone without the approval of others. Both the artist and scientist, however, do share a similarity in convincing others of the solution to the problem they have tackled. Gardner (1994, p. 315) uses multi-faceted Leonardo DaVinci and others, considered both artists and scientists, as examples. They *"would not consider replacing their works by some kind of more conclusive and inclusive formula, and remain relatively uninterested in the audience's 'agreement' with their views."* My students are amazed to learn that it was DaVinci who said: *"Let no one who is not a mathematician read my works."* (quoted in Kline, 1976, p. 133).

Another key difference between mathematics and the arts according to Gardner (1994, p. 310) is in the difference in the demonstration of the body of work. For the artist, creator and perceiver, this relies on the *"execution"* where understanding and interpretation of the work can vary depending on the observer and how well the perception translates to that intended by the artist. In the case of the scientist, understanding and interpretation of the work relies on *"conception"* which can immediately be determined because of its use of formal operations to present the work.

Aesthetics, which may be defined as the idea of something that is perceived to be beautiful, is usually used when referring to the arts. Gardner (1994, p.8) shares the view of the mathematician Birkhoff, who believed that the aesthetics of any art could be determined by *"a mathematical formula relating degree of order to degree of complexity."* According to Gardner (1994, p. 11), in some views, the aesthetics of a musical work can be assessed through the *"quantification of uncertainty"* because it both *"creates and resolves expectations."*

Conklin (1970, p.26) describes that on a continuum of aesthetics, most people would believe that the arts would be at one end and, an abstract discipline such as mathematics, would be at the opposite end. Conklin (1970, p.25) writes that despite a debate on its existence, mathematics also possesses aesthetic aspects. Conklin explains a view that some have that, while usually considered to be related to the five senses, aesthetic experiences also include *"meanings, associations and emotions, whether these come through the senses or otherwise."* Conklin (1970, p. 26) asserts that since the arts are taught to enhance their appreciation, it is not unreasonable that we should also teach mathematics for an appreciation of the nature of the subject.

In the words of Shlain (1991, p. 271), *"visual art is an exploration of space; music is the art of the permutation of time."* He also states that the painter and the composer both used their art form to express ideas of the age and culture in which they live. But ... what is mathematics, really and what are its products?

2.4.1 Mathematics

In his book, What is Mathematics, Really?, Reuben Hersh (1997, p. xi) tries to describe mathematics in humanistic terms, as a human activity which is a special part of culture that has historically evolved within a social framework. As such, Felix (1960, p. 15) says that mathematics can only be understood from the perspective of its time and place. Kleiman (1991, p. 48) states that mathematics is a necessary part of the human experience. According to Kleiman, *"without mathematics ... the nature of humanity and human society would have to be fundamentally different."* He iterates several aspects of the necessary use and function of mathematics from a humanistic point of view: exploration, participation in society, design and construction, prediction and options for the future, play, and an instrument of thinking, creation and communication.

Furinghetti (1993, p. 34) states that mathematics possesses a very unique characteristic compared to other disciplines because, whether mathematics is *"loved or hated, understood or misunderstood,"* everyone has some perception of it. According to Hannula, Maijala and Pehkonen (2004, p. 17) that while some might see mathematics as *"a combination of calculation skill and competence in reasoning,"* neither aspect alone fully describes the nature of mathematics. They explain the intertwining of these concepts: the *"what"* of the facts, the *"how"* of the procedures with the *"why"* of mathematical reasoning. Felix (1960, p. 15) states, *"mathematics is a rigorous language with practically no synonyms."* There have been many other interpretations of the meaning of mathematics and its definition *"has changed several times during the course of history"* (Devlin, 1994, p. 1).

Courant and Robbins (1958) and Hersh (1997) despite the titles of their books, respectively, What Is Mathematics? and What Is Mathematics, Really?, never really answer the question. It is no wonder, therefore, that students are so confused.

- *"Mathematics is the subject in which we never know what we are talking about nor whether what we are saying is true."* (Bertrand Russell, quoted in Cajori, 1950, p. 286)
- *"Mathematics as an expression of the human mind reflects the active will, the contemplative reason, and the desire for aesthetic perfection."* (Courant and Robbins, 1958, p. xv)
- *"Mathematics is the science of learning how to count and measure."* (Anderson, 1960, p. 9)
- *"Mathematics is an art because it creates forms and patterns of pure thought ... a method of expressing, explaining and communicating man's total behaviour."* (Fehr in Felix, 1960, p. ix)
- *"Mathematics is a method of inquiry known as postulational thinking."* (Kline, 1976, p. 4)
- *"Mathematics is the science of patterns."* (Devlin, 1994, p. 6)
- *"Most people perceive mathematics as the 'discipline of certainty' and, consequently associate the ideals of objectivity, absolute truth, and rigor with mathematics."* (Borasi, 1996, p. 260)
- *"Mathematics is a universal, utilitarian subject ... also has a more specialized, esoteric, and aesthetic side. It epitomizes the beauty and power of deductive reasoning ... the efforts made over thousands of years by every civilization to comprehend nature and bring order to human affairs."* (National Research Council, 2002, p. 15)

What students judge to be true about the nature of mathematics, whether consciously or not, influences the ways in which they approach mathematical tasks including the effort they make to complete those tasks and amount of time they will spend to do so (Schoenfeld, 1985, p. 45). In their tome, What Is Mathematics?, Courant and Robbins (1958, p. xix), seeking to impart insights into its essence, write that for an individual that question can only be answered through an *"active experience"* in mathematics.

De Morgan (1943, p. 5) asserts that mathematics has a very different nature than the other subjects. In his view, *"our senses are our first mathematics instructors"* in providing us with mathematical understanding, from our sense of 'how many' (one, two, three, etc.) to our geometric sense (point, line, plane). Demonstration of these concepts enables the rest of mathematics to evolve, by *"means of reasoning."* De Morgan (1943, p. 6) notes that for students there are no other subjects that start as simply as geometry and then, none in which difficulties increase quite quickly as we study it. Whitehead (1958, p. 1) asserts that for students the study of mathematics is capable of bringing about great disappointment because of their struggle to learn a mass of technical procedures and skills as a foundation before any of the interesting applications of mathematics without any understanding of why they are learning it.

In his view, Kleiman (1991, p. 48) believes that mathematics is taught in an *"impoverished"* way by stressing the facts and rules without ever letting students understand the nature of mathematics or its value. He explains that teaching mathematics in this way is like teaching students music by just having them practice their scales without ever letting them play a song or teaching art by having students practice drawing lines and shapes without letting them ever draw a picture. Skemp (1987, p. 207) makes the same musical analogy and states that the solitary paper and pencil way that mathematics is taught to students prevents *"the majority"* of them from ever enjoying it. He calls this *"the silent music of mathematics."*

For some, mathematics is considered an art with all of the characteristics that we would ascribe to the arts. In looking for ways to change our students' perceptions of mathematics, we could adopt Henle's view (1996, p. 28):

> *"Mathematics is not divine; it is mortal. Mathematics is not law; it is taste. Mathematics is not calculation, but communication. The best mathematics is not true; it is beautiful."*

2.4.2 The Arts

The first activities of early humans were to draw, dance and drum and these skills enabled them to express themselves and communicate with one another.

> *"The student artist (musician, dancer, visual artist, writer, or actor) learns by doing. In traditional schools students don't often* do *much of anything: they often learn about doing things by watching someone else do them."* (Perrin, 1994, p. 453)

Perrin elaborates further that artists learn from their mistakes because they are willing to take risks. They recognize that *"mastery of an art form is a life-long process."* Students of art gain self-esteem through their work and understand that its quality is directly related to the time spent practicing their art. *"The student artist feels a level of personal investment not always present in the traditional academic classroom."*

According to Howard Gardner (1994, p. 30), aesthetically, art is the *"communication of subjective knowledge."* He states that this communication is an intentional means of artists sharing their perceptions with those who are willing to look at or listen to their work. Gardner (1994, p. 14) presents the view that the arts have evolved because of the development in humans of knowing and feeling and the ability of the mind to create abstractions to represent these in symbolic form.

2.4.2.1 Art

While many would believe that to be a visual artist requires a type of creative genius, Edwards (2001, p. 2) believes that the experience of an artist is possible in all of us: *"ability to draw depends on ability to see as an artist sees, and this kind of seeing can marvellously enrich your life."* She claims that while an explanation of how to draw may be difficult, it is a skill that anyone can attain by just learning how to see differently. In sharing the feelings of artists, Edwards (2001, p. 4 - 5) explains that in this state of seeing, artists go through a somewhat altered state of consciousness in which they are *"at one with the work."* They find these events pleasurable and relaxing in much the same way as meditation, jogging or listening to music. Edwards (2001, p. 8) believes that learning to draw is important to students because, like learning to read and write, it too improves their thinking.

Gardner (1994, p. 8) states that there are many aspects to the visual arts, from one of an aesthetic point of view that the artist is someone who is an *"inexhaustible fount from whom masterpieces flow"* to the view of art as *"a mirror of the universe"* in looking for models of *"beauty and proportion."* Gardner (1994, p. 10) takes the view that, art primarily reflects the properties of nature or the world are totally naïve and useless because *"individuals do not exactly imitate what they perceive."* This notion is even less valid in the domain of music, which Madsen and Madsen (1997, p. 5) describe as a *"live art."*

2.4.2.2 Music

According to Madsen and Madsen (1997, p. 42), *"music is sound and silence expressively organized in time."* They list five attributes of music, it can be: composed, performed, listened to, verbalized, conceptualized and used for extra musical purposes. According to them (1997, p. 44), listening is an enjoyable activity for many and that after a while of listening, people do not concentrate on the music itself, choosing to only *"drift with a mood."* Madsen and Madsen consider listening to be *"one of the most important aspects of music"* (1997, p. 45). Madsen and Madsen (1997, p. 39 - 40) also assert that music has been called a universal language and a means of non-verbal communication. They tell us that music affects people differently with regard to their moods, tastes and abilities. The well known neurologist, Oliver Sacks (in Matthews, 2008, p. 1), believes, through his brain research, in the power of music to heal. *"Music taps into primitive brain structures involved with motivation, reward and emotion."* (Levitin, 2006, p. 187)

In their chapter on the *"Seven Habits of Highly Effective Composers",* Pogue and Speck (1997, pp. 9 - 12) explain that while composers differ from each other in many aspects, they write music which emanates from themselves with structures and patterns that the listener can feel. Composers are creative and imaginative. Within their music, composers can express all forms of human emotion. Music keeps the listener interested and involved. The mere act of listening to a piece of musical composition possesses the capability of changing the life of an individual. Pogue and Speck (1997, p.318), in describing their own experiences state, *"nothing is so liberating and uplifting as participating in the creation of music."*

Daniel Levitin writes of two kinds of effects music may have on an individual. He (2006, p. 233) states that because the rules and form of different types of music are unique, an unfamiliarity with a certain musical structure may be disturbing thereby creating a lack of appreciation for a piece of music. Levitin (2006, p. 236) asserts that music is pleasing in a sensory way to an individual because of previous positive encounters and that the individual finds comfort and safety in the perceived familiarity. *"We let music take us somewhere outside of ourselves."*

2.5 Interdisciplinary Connections

As teachers, our function is the sharing of what we know and understand about our specific subjects or disciplines with our students. In order to do so, communication is necessary and communication requires a common language. I have many times described learning mathematics to my students as similar to learning a foreign language. It is not surprising to find, in the literature, references that compare each of these three seemingly diverse disciplines to 'language.' Part of the commonality which exists between mathematics and the arts is, perhaps, derived from how they are communicated.

- The language of mathematics to Bronowski (1979, p. 42) is one in which *"we discuss those parts of the real world which can be described by numbers or by similar relations of order."* Kline (1976, pp. 7 - 8) explains that the language of mathematics *"expresses quantitative relations and special forms symbolically"* so *"precise that it sometimes can be confusing."*

- The language of art to Smith (1993, p. 27) is a special visual way of communicating knowledge and understanding. Janson and Janson (1987, p. 7) call art a *"visual dialogue"* where man is an *"inventor of symbols to convey complex thoughts in new ways."*

- The language of music to Levitin (2006, p. 187) *"conveys some of the same emotions ... in a nonreferential, and non-specific way."*

When any form of communication is received, we try to make our own personal sense of what has been conveyed to us through reflection and analysis. Bronowski, in The Visionary Eye (1979, p. 8), says that the ability to use *"words or symbols ... to manipulate your own ideas inside your own head"* is what makes us human. Bronowski (1979, p. 10 - 11) calls this translation from a language of symbols into a language which uses ideas, imagination. According to Augustus De Morgan (1866, p. 132) *"the moving power of mathematical invention is not reasoning but imagination."* In what others aspects are there connections between mathematics, art and music?

> *"... artists are teachers, because they persuade their public ... music appeals to the emotions and can strengthen or weaken our control over them; painters do not copy what they see, but select very carefully, and the elements which they choose to select carry a meaning ... What visual artists like painters want to teach is easy to make out but difficult to explain ... When Piet Mondrian spends his life on constructing pictures made purely of straight lines and rectangles he is ... saying that a certain world of pure mathematical shapes is more valuable ... than the world of people, animals, and nature."* (Highet, 1950, p. 246)

It is Oddleifson's (1994, p. 448) opinion that, focussing on intellectual aspects, *"back to the basics"*, has not been successful for the education of our students. He suggests that an *"arts-integrated education or education in and through the arts"* as a possible solution. Beall (2000, p. xi) has noted that the use of interdisciplinary connections is a powerful motivator for students in the study of mathematics by providing experiences for students which are *"interactive and interdependent."* According to Biller (1995, p. 3), the integration of the disciplines enables the student to appreciate the work of an *"artist and his painting"* and *"the mathematician and his problem."* He further states (p. 8) that the combination of mathematics and the arts provides students with a balanced between *"creative imagination and logic."*

Lambdin and Amarasinghe (1998, p. 161) and Phillips and Bickley-Green (1998, p. 49) have found that integrated instruction of mathematics with art can improve students' attitudes toward and motivation for learning mathematics. Sonia Woodbury (1998) writes of many of the issues in integrating the high school mathematics curriculum with other disciplines. She notes (1998, p. 309) that while teachers may expose their students to examples of interdisciplinary connections, *"teachers and students are still doing mathematics in mathematics class."* Perhaps this stems from the need of teachers to have the training, time, patience and flexibility to learn and understand more for themselves about the value and usefulness of mathematics applications in other areas beyond the *"real-world problems posed at the end of each textbook section."*

Neu (1985, p. 21) suggests that museums can and should conduct programs for teachers to broaden their knowledge of the connections between different disciplines. In order to understand the integration of mathematics and art, I have done independent research on my own and attended workshops at the Albright-Knox Art Gallery, in Buffalo, New York, United States.

Despite the benefits of integrating mathematics instruction with the arts, it is important to maintain the integrity of each discipline without weakening the respective curricula. Gardner (1994, p. 11) states that in many instances, mathematics and the arts have not been combined together with equal significance. In his words, *"either the mathematics is only brought in metaphorically, or the arts are only brought in tangentially."*

The link between mathematics and the arts can be traced throughout the course of history and civilization. Felix (1960, p. 15 – 16) explains that the influences of thought have usually begun in mathematics. She claims that by looking at how music and painting have changed, one is better able to understand the changes in mathematics that occurred during that same period. Mathematician and poet, Jacob Bronowski (1979, p. 63), believed that neither science nor the arts could ever have existed in the history of various cultures without the other. In his view, this was the result of *"something deeply embedded in the human mind ... specifically in the imagination."*

Cohen (1995, p. 4) supports making interdisciplinary connections rather than focussing intently on one subject, because our brains need to understand the functionality of learning something in order to grasp its meaning ... *"we use our emotions to determine what is important to learn and remember."*

Does emotion play any role in our perception of mathematics beyond the negative feelings individuals sometimes express about mathematics? Most people accept without question that the arts are expressions of feelings and emotions. Does the emotional aspect create a conflict in truly believing that there exists any real connection between mathematics and the arts? Kline

(1976, pp. 466 - 467) makes a simple suggestion that mathematics does have an emotional facet because of the *"delight"* one might feel by successfully solving a problem or understanding a difficult concept. Kline (1976, p. 467) also recognizes that *"mathematics generally appeals to the emotions less than music, painting or poetry do."* Hickman and Huckstep (2003, p. 5), commenting on Kline's views, assert that while both mathematicians and artists can experience the same types of positive emotional experiences by doing their work, *"the artist can additionally, intentionally, express emotion in his or her work, this is not true of the mathematician."*

Henle (1996, p. 19) states that, while not well understood by the general public, the aesthetics of mathematics are complex and varied depending on the 'beholder.' According to Henle, *"mathematicians are attracted not only to the beautiful, but also to the grand, the picturesque, and the gothic."* Conklin (1970, p. 28) likens the function of intuition in mathematical discovery to that of aesthetic sensitivity in creation in the arts. The 'discovery' could be a new piece of mathematical knowledge or just something already known but newly made by an individual, independently or in the appreciation of the work of another. *"Each discovery is original for the person who makes it."* Conklin (1970, p. 28) further illustrates his point by stating that there exists an aesthetic sense to discovery whether one is composing a symphony or just listening to one, creating a *"beautiful proof"* or just following its arguments.

> *"Whenever people try to figure out what something means, they search for patterns that make sense to them ... 'patterns that*

connect' ... each learner must put it all together personally." (Caine, 2004, p. 12)

The next section will look at the *patterns that connect* the disciplines of mathematics, art and music in order to personally put together an understanding of relationships between the thinking of mathematics and the feelings in the arts. Caine (2004, p. 13) asserts that *"cognition and affect"* are no longer considered to be separate.

2.5.1 Mathematics and Art

Many consider the subjects of art and mathematics to be totally diverse disciplines. Cipra (1992, p. 748) explains this misconception by suggesting that the differences between the rigid, logical world of mathematics with all of its rules and the creative, emotional nature of art result because, *"art is personal, ambiguous, intuitive; mathematics is strictly axiom, number, and deduction."* He quotes the views of several mathematicians-artists, a.k.a. 'marths' (a term coined at a Mathematics and Art Conference that I attended in Albany, New York, United States in 1993), who find the appeal of *"crossing disciplines is strong."* To them, both subjects involve restrictions and choices in their own way. At that Mathematics and Art Conference, organizer Nat Friedman, a mathematician and sculptor, expressed his opinion that the commonality between art and mathematics lies in that they are both about seeing relationships and patterns. To Friedman, creativity is the seeing of things from a new perspective. Cipra (1992, p. 748) and Peterson (2001, p.9) both see mathematics as providing a framework for *"artistic expression"* and art as making it possible to *"awaken mathematical intuition, revealing aspects of mathematics that are otherwise hidden within abstract formulations."* Peterson describes the similarities between mathematicians and artists in the ways they experience delight, passion and extreme fulfilment in the creation of something new whether it be a piece of art work or a theorem.

Howard Levine (1994, p. 87) asserts that mathematics and art possess the same essential core: *"interpreting the fundamental nature of both the universe and our place within it."* Hickman and Huckstep (2003, p. 3) compare mathematics and art in their intent to convey a representation of nature – art imitates its properties and mathematics is applied to create mathematical models. Levine (1994, p. 88 - 89) goes on further to state that both mathematicians and artists can both be considered as problem solvers. He refers to Polya's four step *"heuristic technique"* to make his point about this aspect of the relationship between mathematics and art. Sir Roger Penrose is a mathematician who, by combining his talents in both mathematics and drawing, is well known for his tiling patterns that *"bridge the yawning gap between the two cultures"* (Mullins, 2000, p. 18). Penrose was greatly influenced by the work of M.C. Escher. Penrose visually manipulates his images to understand mathematics within his tiling problems. It is the way, Penrose feels, that he can convey the beauty of mathematics.

The mathematician Helaman Ferguson, whose sculptures are actual theorems carved out of stone, e.g. 'Alexander's Horned Wild Sphere' which is related to topology, continues to write traditional research papers on mathematics. In comparing his written mathematics to his art, he states: *"In terms of communicating all the mathematics of my soul, this is not enough."* Ferguson (NCTM, 1994, p. 7) gets great satisfaction by representing mathematics through his art because of the chance it gives people to *"touch mathematics"* through his sculptures. He credits his ability to integrate these two subjects that he loves to the influence of a teacher that he had in high school: *"... who encouraged me in both directions [art and mathematics] which was unusual."* Ferguson (1999, p. 134) believes that *"mathematics is an art form, which need not remain invisible. Some math evokes art, some art evokes math."* Ferguson (1999, pp. 136 - 137) speaks of mathematics as a *"conceptual language with an aesthetic of its own"* and that conjecture is *"one of the most creative acts in mathematics."* He claims that, in the philosophy of mathematics, it has often been frowned upon to communicate mathematical ideas with pictures. It is no wonder then that many of the mathematicians Ferguson meets in his travels and lectures *"shyly confess their artistic side."*

Many have studied and written about other 'marths' including, Atalay (2004) and Peterson (2001). Peterson (2001, p. v) elaborates on the *"creativity and imagination at the intersection of mathematics and art."* The view that mathematicians and artists are disjoint sets has certainly been changing.

2.5.2 Mathematics and Music

> *"Music is the pleasure the human soul experiences from counting without being aware without being aware it is counting."* (Gottfried Leibniz in Pappas, 1995, p. 41)

The connection between mathematics and music has long fascinated individuals and cultures. Mathematics and music are connected in many ways such as the beat of a rhythm, the explanation for why some notes sound higher than others or why some sounds are more pleasing to hear than others.

Felix (1960, p. 16) asserts that the structures of mathematics and music evolved in parallel ways. People made and wrote about music in the same ways as they did mathematics - from needs that were *"instinctive, intuitive, esthetic, mystic, religious."* Garland and Kahn (1995, p. 4) add that mathematics is used in various ways by composers and is helpful in understanding music as it has evolved over the ages by different cultures. Garland and Kahn (1995, p. 5) claim that both mathematics and music are *"universal"* and *"cross cultural, historical and intellectual boundaries."* They explain that music is *"grounded in mathematics"* and mathematics is *"reflected in music."*

Garland and Kahn (1995, pp. 96 - 109) and Papadopoulos (2002, pp. 65 - 73) expound upon many human connections between mathematics and music from notable mathematicians such as Pythagoras, considered responsible for the first scientific study of musical scales, to Keppler and his 'music of the spheres' and Euler who published a 'New Theory of Music.' Papadopoulos (2002, p. 66) asserts that both musical theory and composition require *"a certain abstract way of thinking and contemplation which are very close to mathematical pure thought."* Garland and Kahn (1995, p. 103) suggest that the existence of more child prodigies in mathematics and music than in any other area may be related to the use and manipulation of symbols which is important in both of these disciplines.

In his informative book, Mathematics: The Science of Patterns, Keith Devlin (1994, p. 4) discusses the uses of symbolic notation as representations of both mathematics and music with their own structural rules. Devlin does however, note that not being a trained musician and able to read notes does not prevent one from experiencing and enjoying music. This is not the case in mathematics, where for the most part, *"the only way to appreciate mathematics is to learn how to 'sight-read' the symbols."*

Beall (2000, p. viii) compares composing music to mathematical problem solving and likens musical systems and structures to mathematical modelling. According to Beall, an analysis of the unifying structure of what is aesthetically pleasing in sound, rhythm and harmony *"leads directly to mathematics."*

Furinghetti (1993, p. 34), comparing the creative aspects of the activities of various disciplines, places mathematics and music in the lead because their outcomes are *"an intrinsic product of the spirit"* and not the discovery of something already in existence or derived from nature which is where Furinghetti places the work of an artist.

2.5.3 Mathematics and Art and Music

In his 1989 Pulitzer Prize winning book, Godel, Escher, Bach, Douglas Hofstadter weaves a story of the relationship between these three intriguing subjects at the heart of my research. Hofstadler's (1989, p. 28) objective is to inform us of the *"central solid essence"* which connects the three seemingly diverse disciplines of mathematics, art and music. The reader is immersed by Hofstadler into the concept of 'Strange Loops,' the movement from one point to others which eventually takes us back to the place from which we started. Hofstadler explains his hypothesis with *"information preserving transformations"* in the recurring themes of the musical canons of Bach (1989, p. 9), the *"most beautiful and powerful realization"* in the art works of Escher (p. 10), and the 'Epimenides paradox' and Godel's *"idea to use mathematical reasoning in exploring mathematical reasoning itself"* (p. 17).

In going more deeply into the associations between mathematics, art and music, v. Maur expounds (1999, p. 83) on the ways in which artists, by recognizing certain limitations in their format made *"a transition to three dimensions, ... scientific theories of corresponding sounds and colours"* through the influence of the work of Newton which had been evolved out of a *"universal music of the spheres"* which in turn had been derived through the proportions an harmonic systems from Pythagoras and Keppler. She goes on to discuss the *"trope diagrams developed from twelve-tone music, both sounds and colour"* and related to and derived from a higher formula. Other examples of the connections which exist among these subjects are referenced by, v. Maur (1999, p. 86) including the building of an early colour keyboard instrument by the Jesuit priest and mathematician, Castel. His invention made a flash of colour appear when a note on a harpsichord was played.

The *"Boogie Woogie,"* paintings by Mondrian are described by v. Maur (1999, p. 97 - 100) as having been influenced by the artist's love of jazz music and the street grids and neon lights of Manhattan, New York. In explanation, she (1999, p. 100) tells the viewer of its union of musical and visual rhythms in a steady duplication of a *"right-angled theme."*

Another commonality between mathematics, art and music, Gardner (1994, pp. 127 - 131) says is the use of symbolism in each of these disciplines which *"have a complicated series of associations and organizations among themselves."* (p. 127) Mathematical symbols are combined in specific ways into formulae and notation in an object-based system; musical symbols are usually organized to form scales and rules of composition with regard to patterns and emotions; in art, to create effects, we find the use of lines, colours and textures with regard to balance, rhythm, mood and tone. While different from each other, mathematics, art and music each contain an organization of their symbols by meaning and association within its domain. Robert Root-Bernstein (1987, p. 20) believes that *"the search for beauty, and the nature of pattern forming and pattern recognition transcend disciplinary boundaries."* Conklin (1970, p. 31) explains the connection between mathematics, art and music by claiming that the mathematical qualities of *"internal consistency, temporal progression, 'logical' inevitability or predictability ... enable us to appreciate a finished product in any of the arts."*

The individual appreciation of each of these disciplines is completely possible but cannot be learned all at once, state Rademacher and Toeplitz (1957, pp. 6 – 7). Awareness begins for a person in small pieces, with a single line of music or the basic motive of a problem, in order to gain a full understanding of *"the essential ideas of each subject."* In their view (1957, p. 7),

> *"He will then get a glimpse of what a few great thinkers have created when they have occasionally left the realm of their comprehensive theoretical production and have built, from simple beginnings, a small self-contained piece of art, a fragment of the prototype of mathematics."*

Conklin (1970, p. 32) asserts that,

> *"Any subject can be made more inspiring and aesthetically enjoyable for students if teachers and curriculum planners organize the subject matter to maximize its logical beauty."*

2.6 Student Attitudes While Learning

The ultimate purpose of educating students should be more than to attain knowledge. Lewis and Shaha (2003, p. 538) and others believe that school experiences should not only provide academic benefits for students, but they should also result in the improvement of students' attitudes towards school, learning, self-confidence and specific subject areas.

Much research has been done on student attitudes towards the learning of mathematics. McLeod (1992, p. 581) refers to attitudes as *"affective responses."* They can be either positive or negative and can develop in two ways: 1) the attitude becomes an automatic emotional response which evolved from the same repeated reaction to a given stimulus each time the same stimulus is presented; 2) an already established attitude from previous experiences is transferred to a different but similar stimulus when it is presented.

The Oxford English Dictionary (1961) contains the following definitions:

- attitude (of mind) - a deliberately adopted, or habitual, mode of regarding the object of thought (vol. I, p. 553)

- confidence - the feeling sure or certain of a fact or issue; assurance, boldness, fearlessness arising from reliance (on oneself) (vol. II, p. 803)

- enjoyment - the action or state of deriving gratification from an object (vol. III, p. 189)

In Aiken's view (1963, p.476), there are two theories about students' attitude toward the learning of mathematics. One has to do with the relationship between the rewards students receive for their efforts in mathematics and the other has to do with students' personality traits. Aiken found (1963, p. 479), given students with similar abilities and experiences, those with a more positive attitude toward mathematics *"tend to be more socially and intellectually mature, more self-controlled, and place more value on the theoretical"* than those with a negative attitude toward mathematics. By using 'Aiken's Revised Math Attitude Scale', Miller (1987, p. 142) found that *"overall"* a group of twelfth grade students bordered *"between neutral and having a tendency to dislike mathematics."* In follow-up interviews with these students, Miller (1987, p. 143) found while some students felt that mathematics was useful, they could not give examples of specific areas with mathematical applications leading to the belief that the utility of mathematics stems more from the views of society as a whole. According to Miller (1987, p. 144) and others, students' attitudes toward learning mathematics seem to *"deteriorate"* as they go further in years in their school experience.

Hensel and Stephens (1997, p. 28) state that *"it is still not totally clear whether achievement influences attitude or attitude influences achievement."* Hoyles (1982, p. 360), when asking students about good and bad experiences in learning mathematics, found that student satisfaction was related to their involvement or success in learning mathematics and their dissatisfaction was more likely due to their experiences with a teacher. With relationship to the self, negative stories were mainly related to a lack of confidence if *"teachers left them behind or put pressure on them."* (Hoyles, 1982, p. 369)

The analysis of the data on student attitudes of Tapia and Marsh (2004, p. 1) yielded four factors: self-confidence; value of mathematics; enjoyment of mathematics; and motivation. Chouinard and Roy (2008, p. 45) found that as high school students grew older their motivation in mathematics and their perception of the value of mathematics both decreased. They suggest that more research is necessary to learn why. As it is imperative for the individual student and society as a whole for students to persevere and continue to take mathematics courses, Williamson (2002, pp. 8 - 9) investigated students' reasons for not doing so. She found that student confidence in their ability to do mathematics and their enjoyment of learning mathematics played very important roles in their decisions.

Lewis and Shaha's results (2003, p. 541) found that by integrating the curricula of different subject areas, students' attitudes toward mathematics and their confidence in learning mathematics improved significantly when compared to the group of students who had not been taught with an integrated approach.

Patricia S. Davidson, mathematics professor and researcher into the ways the human brain processes mathematical information, in an article by Loviglio (1981, p. 11) believes that teachers should find opportunities to enhance their own appreciation of mathematics in order to be able to present it with a balance of computation and other aspects of mathematics. Davidson (in Loviglio, 1981, p. 11) asserts that if students *"had some time to work in an area of math they can enjoy and shine in, they would have a better chance of developing mathematical competence – and confidence."*

According to Fennell (2007, p. 3), even though there is more to a student's self-concept than confidence and enjoyment, unless students are confident in their ability to do mathematics and enjoy doing so, they probably will not continue to take challenging mathematics courses, major in mathematics or a related field and appreciate the subject as educated citizens. A study conducted by Jane Armstrong (1985, p. 80) found that students who both liked mathematics and thought that they were good in it took more high school mathematics courses than those who had negative attitudes. Similarly, Alma Lantz (1985, p. 338) was not surprised that confidence and enjoyment were among the highest risk factors for students not taking additional mathematics courses.

Sansone and Morgan (1992, p. 250) question whether having a positive sense of personal competence is enough to promote a student's enjoyment of and interest in learning. The study of 1000 students (ages 6 – 17) across different academic domains conducted by Denissen, Zarrett and Eccles found a strong relationship between student interest and self-concept of ability (2007, p. 440) and *"thinking one is good at everything across the* *board and liking all academic subjects indiscriminately leads to less coupling between these two beliefs"* (2007, p. 444). At the heart of my research is the desire to understand student attitudes toward mathematics and the arts based upon their levels of *confidence* and *enjoyment* in these subjects, to find out why they feel as they do and uncover what may be done to encourage students to persevere in their mathematics education regardless of their attitudes.

2.6.1 Confidence

Skemp (1987, p. 193) considers confidence as an indicator of competence, the *"ability to achieve one's goals by one's own efforts."* In a study of students' general attitudes towards learning mathematics, Bassarear (1986, p. 15) found that some students whose performance in mathematics was good had a negative attitude toward mathematics and in addition, that some of those students who performed poorly in mathematics had a positive attitude toward mathematics. This would seem to be supported by the work of Heubner and Mc Cullough (2000, p. 334), who state that academic self-confidence did not always predict a student's satisfaction with school. Mc Leod (1987, p. 134) states that confidence is an important factor in the desire of students to further study mathematics.

McLeod (1987, p. 136) compares confidence and anxiety by their effects on the abilities of individuals to have the power to either move toward (in the former) or away from (in the latter) a *"goal state."* According to Burton (2004, p. 357) and Hannula, Maijala and Pehkonen (2004, p. 23), confidence in mathematics and success in this subject are directly related to each other. Burton claims this relationship to be true despite the lack of agreement on the definition of confidence or how to measure this attitude in mathematics. Burton (2004, p. 358) also adds that on some occasions a decrease in students' interest and motivation in mathematics can sometimes be mistakenly attributed to a lack of confidence. She feels (2004, p. 359) that the relationship between confidence and learning in mathematics is a complex issue.

The aim of Burton's study was to understand the views of students and teachers on the meaning of confidence. She found (2004, pp. 367 - 369) that in the opinions of students, confidence was described as a sense of *"can do,"* the ability to understand and do without any help, rather than a result of memory. Other factors were mentioned by students as related to confidence such as, competition, a collaborative classroom working style and success. Students related confidence more to feelings than the teachers surveyed did. According to Burton (2004, p. 369) students thought that in order to promote the confidence in learning mathematics of their students, teachers should encourage *"discussion, teamwork, a light hearted approach, and a relaxed classroom where you are not afraid of making errors ... explain well and not rush the work"* while being sensitive to the needs of those students who are *"struggling to understand."* This would seem to agree with the opinion of Maxwell (1989, p. 225) who believes that mathematics, as a means of communication involves not only *"methods, hypotheses and answers but also feelings."* Maxwell suggests that time should be given, when teaching mathematics, to *"play, relax, discuss, absorb, assimilate and understand"* in order to improve student confidence and she attributes the lack of this approach to teachers' views of mathematics themselves as *"cold, impersonal and subhuman."*

Chouinard and Roy (2008, p. 39) in their study, found that students' competence beliefs were stable across grades 7, 8 and 9 and then decreased for males and slightly increased for females in grades 10 and 11. They also found that competence beliefs for both genders were higher at the beginning of a year rather than at the end in grades 7, 8 and 10.

In the view of the National Council of Teachers of Mathematics (2000, p. 374 - 375), learning mathematics can sometimes be difficult. It is the job of the teacher to assist students to become confident and engaged in their learning. They suggest finding ways for students to utilize activities that illustrate the connections of mathematics to their daily lives and to provide students with multiple ways of demonstrating their learning.

According to Cooley (1964, p. ix), *"mathematics is loved by many, disliked by a few, admired and respected by all."* He goes on to say that the methods of mathematics inspire confidence in those who can understand them. Tulock (1957, p. 572) believes that student confidence in mathematics tied into a sense of accomplishment. Tulock, Cooley (1964, p. ix) and Boerkaerts (1994, p. 2) assert that those who experience repeated failure in mathematics lose their confidence and tend to build up an aversion toward learning it. Cooley (1964, p. ix) believes that students lose confidence because of a lack of adequate teaching.

Boerkaerts (1994, p. 5) found that student confidence affected performance but not effort and that student anxiety affected effort but not performance.

She also found that students' views of the relevance of a task did affect effort but not confidence or anxiety. Tulock (1957, p. 575) lists ways to change a student's *"emotional mathematical block"* in confidence. Among her suggestions for a teacher to use are those that enhance student interest in mathematics by inclusion of recreational mathematics and *"resource materials from the environment that have a mathematical content or implication."*

In general, Kloosterman (1988, p. 345) feels that when mathematical situations arise, those students who are more confident in their abilities are less likely to experience anxiety. He found (1988, p. 349 - 350) that self-confidence can, at least partially, be clarified by students' reasons for students' success or failure in mathematics, i.e. thoughts about ability and confidence in learning mathematics are related. Mann (2006, p. 250) claims that,

> *"mathematics is a powerful tool that can be used at varying levels of complexity in almost every occupation. Yet, many students leave school disliking mathematics and with the belief that they just cannot do math."*

Fennell (2007, p. 3) and many other mathematics educators (myself included) are distressed by the overwhelming number of parents who routinely excuse and accept their children's difficulties in mathematics because of their own problems in learning mathematics. Mathematician David Thomas (1992, p. 135) notes that in society there seems to be a stigma attached to the *"fear of figuring."* This would not compel anyone to excuse their own personal inadequacies as an artist or musician when introduced to someone whose expertise is in the arts!

The Brookings Institution (2006) took a look at the TIMSS (The Third International Mathematics and Science Study) data with the purpose of understanding three aspects of student learning in mathematics, student self-confidence in learning mathematics, the level of student enjoyment in learning mathematics and whether teachers try to make the learning of mathematics relevant to the lives of their students. According to the Brookings Institution report (Loveless, 2006, p. 15) results, students in different nations possess varying degrees of self-confidence in learning mathematics. It was found that,

- Within countries, *"high confidence is positively associated with achievement."* Those students with higher confidence score higher.
- Between nations, *"nations with highly confident students have lower scores than those with less confident students."*

2.6.2 Enjoyment

Csikszentmihalyi, Rathunde and Whalen (1993, p. 218) believe that what has gone wrong in education is the view of many that *"learning cannot be enjoyable."* As a mathematics teacher, I would hope students could find the acquisition of mathematical knowledge a pleasurable experience. When 201 students in the United States, ages 12 – 13 years, were asked by Picker and Berry (2001, p. 205) to respond to the Likert-scale statement: *"I don't enjoy my mathematics class"*, 57.5% disagreed, suggesting that in the Picker and Berry study most of those students surveyed did enjoy their mathematics classes. What about the other 42.5% of the students in their study?

Rademacher and Toeplitz (1957, p. 5) suggest that just as one does not have to be gifted in music to enjoy it, not being gifted in mathematics should not prevent one from understanding and enjoying mathematics. In conversations with mathematicians, artists and musicians, they found it clearly evident that these professionals enjoy their work. What can be done to communicate to our students a sense of what those working in the field of mathematics feel?

Eperson (1988, p. iv), in his teaching experiences, found the use of mathematical patterns in visual form to be a valuable method for enhancing student understanding of mathematical concepts, building vocabulary, stimulating curiosity and making algebra and geometry *"enjoyable activities."* Langer (1997, p. 59) found that the enjoyment of tasks stemmed from the *"process of not knowing to knowing"* and the doing of something that required the determination of a solution from several possibilities rather than just finding one fixed answer.

In a study by Tapia and Marsh (2001, p. 14) it was found that achievement levels affected motivation and enjoyment at all grade levels 7 through 12. Goetz, Frenzel, Hall and Pekrun (2008, p. 26) worked with students in mathematics classes and German language classes in the German school system. They determined there to be *"positive within-domain effects of achievement"* with student enjoyment.

Ma (1997, p. 228) and Schiefele and Csikszentmihalyi (1995, p. 173), however, found that neither achievement nor the level of mathematical ability are much related to enjoyment in the subject for students, indicating that just because a student performs well in mathematics does not mean that he or she enjoys learning it. Ma (1997, p. 228) also claims that enjoyment does not change a student's view of the difficulty in or importance of learning mathematics. Schiefele and Csikszentmihalyi (1995, p. 173) state that the level of quality of an experience was especially related to student interest in the event. They suggest (1995, p. 179) that teachers should foster experiences which enhance and increase the interest of their students by actively engaging them in student-centred activities.

In the study by Jo Boaler (1998, p. 52) comparing the mathematics instruction of two schools, when students were asked to comment on what they liked and disliked about their mathematics lessons, the students from the school with the more open-ended style commented most about the enjoyment of their interest in the lessons and the open-ended approach. The students from the school with the more traditional style commented most about their dislike of lack of understanding and the use of the textbooks.

Dickey and Clawson (1988, pp. 3 - 7), in studying the value and enjoyment of mathematics of college business and education majors, found that these two attitudes may be perceived differently based upon a student's major areas of study. Enjoyment was not significantly different between the students in the two majors. Their value of mathematics was markedly higher than their enjoyment of the subject. Hartley (2006, p.7) relates enjoyment to personalisation and explains that enjoyment stems from emotions or feelings which culturally has been overshadowed by thinking. Hartley (2006, pp. 7 - 12) explains the contradiction between enjoyment and excellence which he relates to creativity and the accumulation of knowledge. Furinghetti (1993, p. 37) wondered how students could enjoy the learning of mathematics if the only approach that they experienced focused on the learning of procedures and techniques and the sharpening of skills. Tapia and Marsh (2004, p.2) mention that *"students may find math to be simply unappealing or socially unacceptable, although they may actually have high aptitude."*

Boaler (1999, P. 130 - 133) found four themes in student responses in interviews with students about what they liked and disliked about mathematics lessons. One of those themes was 'monotony' because each class seemed to comprise of the same routine with textbook exercises. Many of those students interviewed could not give examples of lessons which were enjoyable for them. Mann (2006, p. 245) and Burton (2001, p. 65) found the same result. According to Burton (2001, pp 65 – 67), students enjoyed lessons which involved class discussions, helping one another, learning together in a relaxed atmosphere and not just listening to the teacher. For Boaler (1999, p. 133), the few who did give enjoyable mathematics lessons used those in which no work was done or because the instructor used a different approach to teach the lesson. Another theme was 'lack of meaning' in which students expressed their perception that mathematics involved rules and procedures which required rote memorization without any understanding of meaning attached. Lucock (1987, p. 131) states that when this occurs, students *"become disillusioned with mathematics and give up trying."* Boaler (1999, p. 134) also found in this theme the lack of relevance of the mathematics studied in school to the real world with its inability to be used outside of the school setting as another factor for student lack of enjoyment of the subject. In addition, Boaler suggests that for these students, aside from the enjoyment of learning mathematics, doing well in mathematics makes one seem *"less than human."* Fennell (2007, p. 3), recognizing this misconception, mathematics is not just for *"nerds,"* it is for everyone!

The findings of Boaler, William and Zevenbergen (2000, p. 8) regarding students in schools in the United States seem to contend that students who are high achieving in mathematics don't necessarily enjoy learning it. Reasons for these students' lack of enjoyment appear to be that their views of mathematics as *"abstract, absolute and procedural"* are not in accord with their views of themselves. These results were the same as students in the United Kingdom, but were not held as strongly.

Nardi and Steward (2002) investigated reasons for student engagement in and the enjoyment of their mathematics lessons. They state that there are some students who are not engaged not because of the lessons themselves, but because of their abilities to understand the mathematics (2002, pp.42 – 43). Nardi and Steward (2003, p. 350) assert that there a number of students who engage in classroom mathematical activities not because they enjoy the experience, but more out of a sense of obligation. They call these students the *"quietly disaffected."*

Nardi and Steward categorized the reasons for students not enjoying the learning mathematics as: *"Tedium* (boring, irrelevant)*; Isolation* (no collaboration with peers)*; Rote Learning* (rules with only one way to work and one right answer); *Elitism* (difficult, demanding); *and Depersonalisation* (does not acknowledge the individual needs of students)" (2003, p. 350). Langer (1997, p. 69 – 70) explains that rote memorization and learning results in *"no personal meaning"* for students and contributes to students' boredom. The students that Nardi and Steward (2003, p. 352) worked with felt that activities with relevance, excitement, variety and challenge made mathematics learning fun and enjoyable which helped them learn and remember better. Some students also enjoyed mathematics lessons which were not routine. Steward and Nardi (2002, p. 5) also found that students did not enjoy mathematics because it was difficult and not relevant to their lives. Some of the students that Steward and Nardi (2002, p. 5) worked with felt that the use of interdisciplinary projects, using mathematics and art, for example, connecting mathematics and their interests would be more enjoyable. These students felt proud of the work that they had done in other subjects, liked their freedom to do their own thing without any right or wrong in art and that creativity in mathematics was *"ridiculous."*

In response to a mathematics and art project I assigned, a student wrote: *"I was surprised that I actually enjoyed it ... it's a different approach to math ... with its unique perspective.* "(Harasymowycz, 2007a, p. 13) Other mathematics teachers have also used the arts as a key to involving students in the learning of mathematics and found that it enhances both the mastery of the related mathematics curriculum and students enjoyment of mathematics (Jehlen, 2008, p. 26).

The Brookings Institution report (Loveless, 2006, p.13) states that many reform attempts in mathematics education *"place children's happiness on* *equal footing with their learning"* with the belief that happy students, experiencing a relevant mathematics curriculum, will learn more. Loveless (2006, p. 14) goes on to say that merely making students *"contented"* does not ensure higher achievement. The Brookings Institution report (Loveless, 2006, p. 16) found,

- Within countries, students who enjoy mathematics score higher than students who do not enjoy mathematics

- Between countries, the reverse seems true. The nations whose students are at the top of the scale for the enjoyment of mathematics score lower than those countries whose students are in the low end of the scale for enjoyment of learning mathematics

This seems to imply that *"the more math a nation's children know, the less likely they are to enjoy mathematics."* (Loveless, 2006, p. 17)

2.6.3 Looking For Answers

Scherer (2006/ 2007, p. 7), commenting on the Brookings Institution's report on what has begun to be called 'the happiness factor', asks wouldn't it be better to use research to guide educators by suggesting ways to create a generation of students who are *"both confident and competent, creative and knowledgeable?"* According to Loveless, this report (2006, p. 20) suggested the need for student involvement in their learning and that *"real student engagement is not about keeping students happy, boosting their self esteem, or convincing them that what they are learning is relevant."*

Skip Fennell (2007, p. 3), former President of the National Council of Teachers of Mathematics of the United States, explains his view on the

Brookings' recommendation, *"the mathematical intent of any activity should not be lost in attempts to make it real, relevant or meaningful."* In Fennell's view, *"curiosity encourages imagination and a genuine interest in mathematics learning."* If we can promote an environment in which *"our students enjoy the subject and approach it with curiosity and confidence as well as the perseverance that embraces struggle within mathematics learning ..."* who knows what might happen?

Is it possible, that the encouragement of curiosity and imagination may create experiences for students more like their experiences in the arts? In her book <u>The Sound of Painting</u> (1999), Karin v. Maur takes great effort to point out the many ways in which artists and musicians have always found to become inspired by each other's work. Without directly making a specific reference to the subject of mathematics in describing their works, v. Maur shares with the reader many that clearly have also been inspired by the inclusion of mathematics.

Chapter 3 Methodology

3.1 Introduction

The formal systematic process of inquiry and analysis has many tools and procedures with which the researcher may advance down a road which begins with a problem or question and ends with the discovery of a solution or final conclusion. Cohen, Manion, and Morrison (2000, p. 73) argue that there isn't one right way to plan a design for research. According to them, the choice of methodology is based solely on *"fitness for purpose."* Wiersma (1975, p.23) states similarly, that the techniques used must *"fit the requirements of the research problem."*

The road on which I began this journey led me toward insights into the role that confidence and enjoyment play in students' attitudes about the learning of mathematics, art, and music. If the focus of a study is an understanding of the behaviour of individuals or a group, Levin (2005, p.44) advises, *"investigate it, describe it, and seek to explain it."* This chapter will clarify the decisions about and rationale for the methodology chosen for this research. The next three chapters will describe in greater detail how the data were collected and analyzed with a questionnaire and interviews.

3.2 Qualitative versus Quantitative Methodologies

Wellington (2000, p. 200) differentiates between quantitative versus qualitative approaches to research by the kind of data with which each deals. Simply put, qualitative methods deal with words as opposed to the numeric data of quantitative methods. He asserts that even though the two are often separated because they are considered to be contradictory to each other, the two approaches could be used to *"complement and enrich each other."*

Strauss and Corbin (1990, p. 17), explain that while the findings of qualitative research do not result from any kind of numeric/ statistical formula, the data can be looked at quantitatively. It's the analysis which is qualitative. Both research methods can be successfully combined in a study but most researchers choose either the quantitative or qualitative approach exclusively.

Generally, quantitative research begins with a hypothesis to be tested. The collection of data enables the researcher to ascertain the truth value of this hypothesis. In a qualitative approach to data analysis, the *"explicit goal is description."* (Glaser and Holton, 2004, par. 2) Glaser and Strauss (1967, p. 17) see the only real conflict between these two methods or data as lying primarily with what is most important, *"verification or generation of theory."*

3.2.1 The Rationale For a Qualitative Approach

Glaser and Strauss (1967, pp. 15 - 17) explain that, in the 1930's, due to a use of qualitative data in ways which lacked organization and precision, there was criticism that such research was not theoretical enough or was overly based upon impressions of the researcher.

Borman, LeCompte and Goetz (1986, pp. 43 - 44) have expressed some concerns about qualitative research. In their view, the researcher utilizing qualitative methods is essentially the research tool existing simultaneously as the *"filter and interpreter."* They suggest that qualitative researchers develop a rigorous and disciplined strategy for monitoring their biases in each decision that is made during the research process. They propose collaboration with outside sources for other perspectives on *"clarifying concepts, developing and refining questions, and regaining insight."* Some claim that qualitative research is without value because it does not assess hypotheses or theories. Borman, LeCompte and Goetz (1986, p. 50) state that although it is mainly used to generate theories grounded in data, qualitative methods can also be utilized to verify them and address the criticism that qualitative findings lack accuracy because they are not mathematically analyzed with quantified amounts. Borman, LeCompte and Goetz state that *"qualitative studies can be even more 'empirical' than quantitative studies because the data are closer to the objects or phenomena observed."* (1986, p. 50)

In qualitative research, understanding is subjectively derived from the perspectives of the people in which the researcher is interested. According to Strauss and Corbin (1990, p. 19), qualitative research lends itself to the discovery and understanding of an individual's experiences and can give *"intricate details of phenomena that are difficult to convey with quantitative methods."* Kirk and Miller (1986, p. 9) state that, *"quantity"* refers to the amount of something while *"quality"* refers to the nature of something. It is not surprising then that, Ely et al (1991, p. 2) claim that qualitative research is synonymous with naturalistic inquiry. They, agreeing with Lincoln and Guba (1985, p.8), suggest that the two have no real stated definition. Instead, the meaning of qualitative research and naturalistic inquiry can best be understood by the procedures and activities that take place in the course of conducting the research. Ely et al (1991, p.4) state that there is *"no one general method"* of qualitative research.

Lincoln and Guba (1985, pp. 39 - 43) describe fourteen characteristics of doing research in a naturalistic mode of operation:

- *Natural setting*
- *Human instruments* for data gathering
- *Utilization of tacit knowledge* - that which is unspoken
- *Qualitative methods* - more flexible in dealing with multiple realities
- *Purposive sampling* - increases the scope of the data
- *Inductive data analysis* - ideal for recognizing multiple realities
- *Grounded theory* - emerging from the data
- *Emergent design*
- *Negotiated outcomes* - of meanings and interpretations with the human sources
- *Case study reporting mode*
- *Idiographic interpretation* - interpret data in terms of the particulars of the case
- *Tentative application* - of the findings because realities are multiple and different, dependent on the interaction between the investigator and the respondent
- *Focus-determined boundaries* - multiple realities define the focus
- *Special criteria for trustworthiness*

After careful consideration of the issues, procedures, and characteristics of both methodologies, I decided on a qualitative approach for this project. For me, the method which would best fit the purpose of my study needed to be open-ended, guided by the values of students, humanistic and descriptive. I began this research with no predetermined hypothesis. Instead, I sought to have the data drive the outcomes of this project. Ely et al (1991, p. 4) add two other aspects of qualitative research: the desire for those being studied to be understood through their own voices and a comprehension of events as a unified whole (without separate variables) through the eyes of the participants. Although I knew the analysis of the data for this research would be time consuming, the richness of student responses and the opportunity to understand students in a more personal way far outweighed the time that the analysis would take for me to complete.

One of the fourteen characteristics of a naturalistic mode of operation is grounded theory. Glaser and Holton (2004, par. 12 - 13) explains that grounded theory deals with data *"as it is."* In grounded theory, there are no preconceived or forced notions. It seeks to *"generate a conceptual theory that accounts for a pattern of behaviour which is relevant and problematic for those involved."* The goal of this research was to create a premise about the attitudes and actions of students from their own voices as they were. Grounded theory was well suited to accomplishing this objective.

3.2.2 Grounded Theory

According to Glaser and Holton (2004, par. 41), grounded theory

> *"... is not findings, not accurate facts and not description. It is just straightforward conceptualization integrated into theory – a set of plausible, grounded hypotheses ... readily modifiable as new data come from whatever source ... the constant comparative method weaves new data into the sub-conceptualization."*

Grounded theory, claims Leigh (1998, p. 219), had its origins as a response against the prevailing *"a priori schemes with claims on universality"* or those methods which took a deductive approach in arriving at a related effect based upon a known or assumed cause. Inductively, the qualitative process of grounded theory, is simply defined as the *"discovering of theory from data"* by Glaser and Strauss (1967, p.1). Later Strauss and Corbin (1990, pp. 23-24), explain that conclusions from grounded theory are,

> *"discovered, developed, and provisionally verified through systemic data collection and analysis of data pertaining to the phenomenon."*

Dick and Jankowicz (2001, p. 188) used the method of constant comparisons taken from grounded theory to do a content analysis on the data collected in their study using repertory grids. This was consistent with the analysis which I felt best met the needs of my research goals.

The grounded theorist is excited by the notion of discovery. The researcher begins with a topic for investigation and anticipates that what is germane will begin to appear. Glaser and Strauss (1967, pp. 5 - 6) further clarify the nature of grounded theory as being *"derived from data and then illustrated by characteristic examples of the data."* The hypothesis originates from the data and is methodically developed in the course of the research. Although Strauss and Corbin (1997, p. vii) assert that grounded theory methodology and methods are considered to be prominent as a strategy for generating theory in research, Dey (1999, pp. 14 – 19) reports that criticisms have been raised about the use of grounded theory regarding: *"deviation"* from the method, procedures, coding paradigms, verification, conflicting interpretations, the possibility for analysis to change over time, and confusion with other qualitative methods. He has difficulty with the paradox that *"a methodology based upon interpretation should prove itself so hard* *to interpret"* (1999, p. 23) and which expert views to accept. Dey (1999, p. 24) claims that there are more questions than answers in the use of grounded theory. He insists that it is the responsibility of each qualitative researcher to be thoughtful on *what* and *how* is being done at each step along the way, in order to be faithful to the voices of the participants in the research. Glaser and Holton (2004, par. 4) on the other hand, states that *"the conceptual nature of classic grounded theory renders it abstract of time, place, and people"* and that grounded theory with its systematic procedures does not encounter the dilemmas of accuracy that trouble other qualitative data analysis methods. These opinions helped guide me along each step in the creation and implementation of my research plan.

3.3 Underlying Considerations in Planning the Research Design

According to Denzin (1989, p. 14), *"there is no clear window into the inner life of a person."* The reason, he claims, is the nature of language with its symbolism and various connotations. To optimize our understanding of the meanings and intentions of the language of the research participants without ambiguity, it is imperative that our communication with them takes more than one form. Pope and Denicolo (1993, p. 542) have found it to be of value to utilize interviews in combination with other strategies. They believe that,

> *"Commitment to hearing the message should always take precedence over adherence to a particular medium."*

The blend of various strategies contributes the features of each to benefit the research and *"enhance its rigour."* Levin (2005, p.47) also advises the use of various investigative tools such as surveys complemented with interviews.

In order to hear the message that students wanted understood, I decided to collect and analyse the data from two complementary sources: questionnaires distributed in the United States and the United Kingdom followed up with interviews in the United States.

3.3.1 Questionnaires

The job of a questionnaire as a research tool, according to Oppenheim (2000, p. 100), is measurement. Just like other aspects of the research process, much planning and thought must be given to the design of a questionnaire. This also includes decisions on the exact issue, topic, or variable to be measured and the specific manner in which that will be done.

Oppenheim (2000, pp. 112 - 114) explains that care must be given to the type of question format to best suit the needs of the project and choosing whether the responses will be open or closed. He asserts that it may be beneficial sometimes to ask the same question in both closed and open form. Among the advantages of open-ended questions are the chances they provide to the participants to answer freely and provide rich data that can be collected. As a result, *"we get a free, spontaneous sketch in the respondents' own language and containing their own ideas."* Oppenheim (2000, p. 114) Closed questions are easier and take less time for participants to respond to and for the researcher to quantify. Wellington (2000, p. 104) feels that a questionnaire should begin with the closed questions, leaving the open-ended ones to the end where they can yield *"fascinating qualitative data."*

Initial data for this research was collected with a questionnaire [See Appendix C] designed to identify students in four categories based upon their confidence and enjoyment. There were seven closed questions with Likert-type scale responses related to student confidence and enjoyment in the learning of art, mathematics, and music. The eighth question, which was open-ended, asked participants to specifically categorize themselves as to their confidence and enjoyment in art, mathematics, and music and then explain their response to one of the categorizations. The duplication of questions regarding confidence and enjoyment in both the closed section (questions two and six) and the open-ended question eight, enabled me to identify students with consistent responses and select them for the interview phase of this research. Chapter 4 contains a detailed account of the development and analysis of the questionnaires used in this project.

3.3.2 Interviews

> *"Skilled interviewers are remarkable for the economy of what they say ... it is the interviewee who has the information."* (Gillham, 2000, p. 28)

Knight (2003, p. 61) defines an interview as a *"face-to-face interaction."* Cohen, Manion, and Morrison (2000, p. 273) assert that there are four different types of interviews that are used for research purposes: structured – prepared in advance with little choice for the interviewer to make changes; unstructured – an open conversation, led by the reasons for the research, that while carefully planned gives the interviewer more flexibility, freedom, and leeway; non-directive – very little direction and control by the interviewer with the interviewee having the ability to express themselves as fully as they'd like; and focused – developed in order to let the interviewer retain more control than in the non-directive type as the interviewee responds to a particular situation in which he or she has been involved. Each of them has their own purpose and process.

There is a flexible range of structure underlying any interview (Knight, 2003, p. 61). In order to ensure the validity and reliability of the responses collected from interviewees, I developed a careful, organized plan to fine tune my final interview structure as advised by Gillham (2000, p.3). Knight (2003, p. 66) reminds the researcher that *"good pilot interviews can identify topics that need to be added to the list."* These are described in detail in Chapter 5.

The purpose of this research was to examine the ways in which students construed their learning experiences in mathematics, art and music. In order to gather and understand their underlying preferences, a type of interview tool developed by George Kelly as an outcome of the psychology of personal constructs was used. After reading about repertory grid interviews as a unique form of structured interviewing in Jankowicz (2004b, p. 8) and other sources, I determined that it was an ideal method for understanding how students see all or part of their world in their own terms.

3.3.2.1 Personal Construct Theory and Repertory Grid Interviews

> *"Man looks at his world through transparent patterns or templates which he creates and then attempts to fit over the realities of which the world is composed."* (Kelly, 1991, p. 7)

The patterns which an individual creates and then tries to *'fit over the realities'* are called constructs. Through life experiences an individual builds up and adapts other constructs into a bipolar system or 'repertoire' that is recalled to understand and predict an experience each time a similar situation occurs. The bipolarity of the constructs refers to how the individual perceives things as being similar to or different from each other, e.g., *involves other people* versus *the student is working alone*. The meaning of a construct is not fully known until it is compared to its opposite.

Kelly (1991, p. 35) considers construing as *"placing an interpretation"* by an individual on a set of elements in terms of the traits that are common or not common to them. This example from my data illustrates the process: the elements *working on examples, experiencing feelings and emotions*, and *possessing ability/ talent*, when taken as a triad, elicited this response from an interviewee regarding the learning of mathematics: *working on examples* and *possessing ability/ talent* are alike because *they help understanding* as opposed *to experiencing feelings and emotions* which *may confuse students.*

Rogers and Ryals (2007, p. 597) view the act of construing, of which the individual may or may not be aware, *"not as thinking or feeling but discriminating."* Adams-Webber (1979, p. 31) and Mackay (2004, p. 294) state that the interpretations individuals make in expressing their constructs are more than just verbal labels. Adams-Webber asserts that constructs are the way in which individuals communicate *"underlying cognitive structures used in processing information."* Owens (1987, p. 163) sees individuals as,

> *"... active processors of knowledge – assimilating and organizing experience through an evolving system of bipolar images ... this system is both modified by experience and determines how experiences are perceived by the individual."*

The theory of personal constructs, from its inception in the 1930's by George Kelly, is an attempt to understand the emotional responses of subjects to certain people, things or events. It contends that each person has their own individual theory or way of interpreting experiences that they assess, amend and use to make sense of their world. These theories may vary for each person. Bannister (1985, p. 192) calls this Kelly's *reflexive* theory and suggests that Kelly assimilated a research design in his theory because it represented, as research does, a prescribed form of human investigation. He later elaborates (1985, p. 194) on the significance of reflexivity for the research method. Bannister asserts that the research questions should be personally meaningful and important to the researcher and that the researcher should view their investigations as part of life and society. This research began as a means to improve and inform my role as a mathematics teacher and enhance the learning of my students, how much more meaningful and personal could that be?

The repertory grid is the tool introduced by Kelly in 1955 as a method to learn about the ways in which individuals construe their experiences in the world. Originally, it was used to understand how patients construed their relationships with other people in their lives. Besides its initial applications in psychology, repertory grids have been used in a variety of other fields, business (Honey, 1992; Easterby-Smith et al, 1996; Rogers and Ryals, 2007), police work (Dick and Jankowicz, 2001), travel and tourism (Embacher and Buttle, 1989; Coshall, 1991) and education (Yorke, 1978; Kreber et al, 2003). Repertory grids have also been of value to researchers in mathematics education, Owens (1987), Lengnink and Prediger (2003), Hoskonen (1999) and Lucock (1987).

3.3.2.2 Components of the Repertory Grid

The name repertory grid is derived from the word 'repertoire' which can be defined in this context as a range or collection of constructs that individuals possess through which they make sense of their worlds. Pope and Keen (1981, p. 55) and Pope and Denicolo (1993, p. 530, p. 542) see a repertory grid *"as a catalyst within a conversation."* Pope and Denicolo view the technique of conducting this type of interview as both an art and a science. Even though the participant's full repertoire of constructs may not be elicited in the process, the repertory grid technique according to Beail (1985, p. 2) is

> *"a way of standing in the shoes of others, to see the world from their point of view, to understand their situation, their concerns."*

Jankowicz (2004a, p. 10) states that every repertory grid contains four parts: 1) topic; 2) elements; 3) constructs; 4) ratings. While Jankowicz considers the construct as the most important component, Wright and Lam (2002, p. 113) consider the elements as the *"backbone of all construct elicitation. They ... determine the direction and level of the entire grid interview."* The selection of the elements is also recognized by Lengnink and

Prediger (2003, p. 46) and Yorke (1978, p. 64) as a very critical part of the process. Neimeyer (2002, p.92) advises the researcher to be aware of the important responsibilities and demands in the proper selection and elicitation of elements and constructs and the use of the rating scale. In addition, the researcher should understand, identify and articulate the effects that different variations may have.

My topic for each interview was *how students viewed their learning*. Each interview was focused, however, on only one of the three subjects, mathematics, art or music. Chapter 5 contains a detailed account of the decisions that I made in developing the structure and the components of my repertory grid interviews. To illustrate each concept, six examples of the repertory grid interviews I conducted are contained in the appendices listed below:

- **D**: **Confident/ Enjoy** mathematics
- **E**: **Confident/ Not Enjoy** mathematics
- **F**: **Not Confident/ Enjoy** mathematics
- **G**: **Not Confident/ Not Enjoy** mathematics
- **H**: **Confident/ Not Enjoy** art
- **I**: **Confident/ Not Enjoy** music

Appendix J contains a partial transcript of the exchange between me and the student whose repertory grid interview is referenced in Appendix E to clarify the repertory grid process.

As the word grid implies, a repertory grid is simply a matrix through which an individual compares a set of elements based upon his or her personal constructs using a rating scale (numerical score) to indicate how closely each element is related to either the emergent (most similar) pole or implicit (most different) pole. A new grid is created for each participant. The topic is the issue being investigated. It must be clearly specified in order to elicit constructs that the interviewee has with regard to the issue.

The elements are examples of people, places, things or events that exist/ happen within the topic from the interviewee's environment. Honey (1979a, p.361) advises that six elements are too few and the data is richer with at least nine elements. They may be fixed by the interviewer or elicited from the interviewee (Jankowicz, 2004a, pp. 29 - 33; Beail, 1985, pp. 3 - 4; Pope and Keen, 1981, p. 39; Stewart and Stewart, 1981, pp. 32 - 33; Easterby-Smith, Thorpe and Holman, 1996, p. 9; Yorke, 1978, p. 64; Bjorklund, 2008, p. 14). They must be homogeneous and representative of the topic. It is a customary practice to have each of the elements written on individual index cards for interviewees to manipulate for a random selection of three elements at a time. The elements are then systematically compared to bring out the interviewee's bipolar constructs one triad at a time.

These contrasting construct statements are elicited by the interviewer asking the interviewee, 'how are two of these alike ... (in terms of the topic) ... in such a way that makes them different from the third?' The only constraints on the number of triads compared and therefore, the number of constructs elicited, are the time available and/ or when no new constructs emerge from the interviewee (Easterby-Smith, Thorpe and Holman, 1996,

p. 11). A good construct is suitably detailed, expresses a clear contrast, and is openly related to the topic of the grid. Depending upon the aims of the interviewer and the research, constructs can also be fixed by the interviewer or elicited from the interviewee (Jankowicz, 2004a, p. 56; Beail, 1985, pp. 4 – 6; Pope and Keen, 1981, p. 40; Stewart and Stewart, 1981, p. 44; Adams-Webber, 1979, p. 23; Kreber et al, 2003, p. 436; Slater, 1965, p. 969; Easterby-Smith, Thorpe and Holman, 1996, p. 9; Yorke, 1978, p. 64; Bjorklund, 2008, p. 15). Adams-Webber (1979, p.23) explains that constructs that have come from the interviewees themselves may be more significant to them. Those constructs which are supplied should be expressed in terms which are within the frame of reference of the interviewees and related to the topic. Constructs which have been supplied, however, do offer a level of standardization with which the interviewer may make certain comparisons between the responses of several interviewees. Bjorklund (2008, p. 16) believes that the labels the interviewees place on their constructs are very subjective and we must be careful to understand them in the way the interviewee intended. Jankowicz (2004b, p. 20) adds a reminder to the researcher that since there may be varying interpretations of a construct, it is imperative to *"negotiate meaning with the other person, until you understand <u>their</u> construction, not yours."*

There are various options for the range of scores that can be used effectively in the <u>rating</u> scale. Kelly mainly used a 2 point scale in his work, but most popular are the 5 or 7 point scales. Each element is rated on each construct to uncover a precise understanding of what the interviewee feels about each element in his/ her own terms. The ratings signify the relative position of each element on a continuum between each pair of bipolar constructs. Because a 5 point scale was used in the repertory grids of this research, the continuum went from a score of 1 for those elements exactly construed as the emergent (most similar) pole and a score of 5 for those elements construed exactly as the implicit (most different) pole. Interviewees also gave scores of 2, 3, or 4 for those elements somewhere in-between in relationship to the two poles. Kreber, Castleden, Erfani, Lim and Wright (2003, p. 437) and Kreber (2004, p.35) compare the repertory grid rating scale to a Likert-type scale.

All of the components of a repertory grid are interdependent on each other and as such contribute their own value to allowing the researcher insights into how participants anticipate events in their lives. Jankowicz (2004a, p. 54) summarizes the relationship of the components by explaining that the constructs tell *how* one is thinking about the topic and the ratings tell *what* one is thinking about the elements.

Kreber et al (2003, p. 443) and Neimeyer (2002, p.91) view the flexibility of the repertory grid technique and the multiple ways that it can be adapted to suit the researcher's purpose as one of its much-loved aspects. Pope and Denicolo (1993, p. 531) say it this way, *"there are no absolute rules regarding the repertory grid procedure."* For this research, in order to create a more uniform set of data, elements were fixed and based upon students' responses in the pilot stages and constructs were both fixed and elicited. The fixed constructs were related to confidence and enjoyment, the two key concepts being investigated. A more detailed description follows later Chapter 5.

Neimeyer (2002, p. 90) claims that one of the reasons for the popularity of repertory grids is its ability to *"develop an intersection between objective and subjective methods of assessment."* Rogers and Ryals (2007, p. 601) state that one advantage of using the repertory grid technique for researchers is the opportunity to, *"capture interviewees' perceptions of nebulous concepts, probe below the surface into areas of 'unawareness' ... and understand how they use their past experiences to make judgments."* They also maintain that there are other strengths of this technique (2007, pp. 610 – 611) namely that, the findings of research using repertory grids emerge by being grounded in the experiences of the participants, the researcher can explore the tacit ways that individuals make sense of their worlds, and the reduction of bias that an in-depth interview can create because of the constructs being expressed in the interviewees own words. Cohen, Manion and Morrison (2000, p. 344) find excitement in the opportunities for repertory grids to provide an *"abundance and richness of interpretable material."* Honey (1979a, p. 358, 1992, p. 79) and Dick and Jankowicz (2001, p. 187) agree.

My choice to use the repertory grid technique was made because its unique style bridged the gap between the personal thoughts of the interviewees in the words of their constructs and the detached numeric ratings they applied to each of the elements based upon their constructs. It seemed to me the best of both worlds for an interview structure in order to understand the roles that confidence and enjoyment play for students. Honey's method of analysis of repertory grid data was ideal for doing just that by finding those constructs which students most closely associate with the fixed constructs related to confidence and enjoyment. Honey's method is described in Chapter 6.

Rogers and Ryals (2007, pp. 609 – 610) find four limitations on the utilization of the repertory grid technique: vocabulary - the possibility of the meanings of the bipolar constructs being contested based upon the context in which they were used: *"are terrorists and freedom fighters bipolar opposites or the same thing?"* and expressions used to elicit constructs and the rating of elements; sorting of constructs into themes; grid responses may change for the same individual over a period of time due to the nature of personal construct theory; length of time for completion and too much information for the participants to take in at one time. Others have mentioned the time consuming nature of conducting repertory grid interviews and then analyzing the data collected from them (Pope and Keen, 1981, p. 55; Easterby-Smith, Thorpe and Holman, 1996, pp. 25 - 26). Honey (1992, p. 88) agrees and quite accurately warns that *"the data are a great deal messier to analyse and interpret!"* Honey and Easterby-Smith et al do see, however, substantial benefit from a qualitative analysis of repertory grid data. Honey (1979a, p. 358) and Slater (1976, p.21) also warn that the repertory grid technique is difficult to explain, especially to children. Cohen, Manion and Morrison (2000, p. 345) consider the broadening differences in the way technology has been used to analyze grids compared to the original intensions of personal construct theory, as the most important difficulty with repertory grids. This is also noted by Neimeyer (2002, p. 90) and others like Dick and Jankowicz (2001, p. 196) who suggest that in order to be mindful of the issues that may arise that the researcher operate in a mode of constant critical reflection throughout the investigation. All of these issues were given consideration along each step of my research.

3.4 The Importance of Triangulation

Triangulation is defined by Cohen, Manion and Morrison (2000, p. 112) as *"the use of two or more methods of data collection in the study of some aspect of human behaviour."* The use of multiple means of collecting data is an important way to strengthen the findings of any research project, but especially as one which involves a qualitative approach. To employ only one method can result in limited or biased findings. Lincoln and Guba (1985, p. 283) contend that *"triangulation is crucially important in naturalistic studies."* Wiersma (1975, p. 135) identifies both the questionnaire and the interview as tools for data collection in survey studies.

Triangulation was achieved in this research by using two discrete instruments to obtain responses from students with regard to their confidence and enjoyment in mathematics and the arts. Wiersma (1975, p.137) explains the relationship between the two as follows,

> *"A questionnaire is sometimes referred to as a written, self-administered interview ... an interview is an oral questionnaire."*

Knight (2003, pp. 87 - 89) similarly differentiates questionnaires and interviews. I used questionnaires to see how prevalent the particular attitudes of confidence and enjoyment were in a student population and then the repertory grid interviews to explore those attitudes to a greater extent. Knight (2003, p. 89) sees interviews as *"far better for exploring* *things in depth, learning about the informants' perspectives and about what matters to them."* He does indicate, however, that an open-ended question on a questionnaire can do the same. Sample size for questionnaire can be quite large, while those for interviews are somewhat smaller which is better for a deeper investigation. A benefit of the self-administered questionnaire is the anonymity of the responses. Interviewees may tend to be cautious in their responses in a face-to-face situation.

Care was given to address all of these issues in this research adding to the assurance that both the questionnaire with its open-ended responses and the repertory grid interviews, with their unique nature, provided rich information which complemented each other in the generalizations of my findings.

3.5 Validity and Reliability

Essential aspects of any research are its reliability and validity. Reliability involves stability and consistency over time; that what has been recorded is exactly what has occurred. Validity is concerned with the credibility, accuracy, and trustworthiness of the study. Does it measure what was purported to be measured? In an external sense, that means can the results be generalized. In an internal sense, that means can it be replicated.

As concern must be given to 'trustworthiness' in all research, Lincoln and Guba (1985, pp 300 – 331) suggest a different set of terms for naturalistic inquiry from the traditional ones with activities to assist in meeting these criteria:

- *Credibility* (Internal validity) - triangulation and collaborative discussions with colleagues are ways to ensure that results and interpretations will be convincing and realistic

- *Transferability* (External validity) – The expectation for the conventional researcher is that external validity can be established by, for example, statistical confidence limits. For the naturalist, that is an impossible task. Instead, in order for the naturalist to establish transferability, he or she must provide as thorough and exhaustive portrayal of the data so as to make it possible for another, interested in the findings, to make their own transferable decisions

- *Dependability* (Reliability) – in the traditional sense, if validity exists so does reliability, since research cannot possess the quality of validity if it is not reliable. Therefore the establishment of validity/ credibility should be enough, in the theoretical sense, to demonstrate reliability/ dependability. Several options are suggested for the establishment of reliability on its own merits such as, an inquiry audit to examine the data, process, interpretations, and findings to verify their accuracy

- *Confirmability* (Objectivity) – triangulation and a similar audit as described above are two methods that can establish confirmability

Kirk and Miller (1986, pp. 10 - 11) state that objectivity is a vague idea but to them it can mean that everything has a reason to explain why it happened or in intellectual terms, the willingness to take a risk on something and be wrong. They later (pp. 20 - 23) explain that in the social sciences, the efforts to assert reliability are emphasized more because at least partly, it is impossible to achieve flawless validity.

Ely et al (1991, pp. 94 - 95) remind the researcher that it is important to consider the issues and aspects of trustworthiness throughout the course of qualitative research to ensure that the findings are sincere and authentic. Borman, LeCompte and Goetz (1986, pp.46 - 50) assert that reliability for qualitative research cannot be assessed in the same ways as would be done for quantitative methods. This is based on the reality that qualitative research is impossible to duplicate exactly even when the same researcher and the same project are involved. Because of this lack of internal and external reliability, the concerns of reliability must be handled differently. Their solution is a rigorous scrutiny of why the changes may have occurred and a detailed description of how the study was conducted. This was a very important consideration for me in order for others to have the ability to replicate the thinking and direction of my research.

Borman, LeCompte and Goetz (1986, p. 48) also examine the problems of validity in qualitative research. They believe that to compensate, the researcher must "*have a reasonable, logical, and rational basis for claiming that research sites, populations, or events are comparable and that the populations used and conclusions reached can be translated from one study to another.*" To them the adherence to this mode of operation is as important to the qualitative researcher as external validity is to those using quantitative methods. The case against the existence of internal validity in qualitative methods lies mainly in the subjective nature of its research state Borman, LeCompte and Goetz (1986, p. 48). To counter this claim, the qualitative researcher needs to be diligent in eliciting participant constructs and ensuring that they were not imposed by the researcher.

Gathercole, Bromley and Ashcroft (1970, pp.513 - 516) performed a study to evaluate the reliability of generalizations made from a single repertory grid. They found the conclusions made by the analysis of one grid to be unreliable, advising *"extreme caution"* especially if constructs were elicited. Their study used grids with different elements and the same grid on more than one occasion. One might predict, however from Kelly's Experience Corollary (1963, p. 73) that even the same individual might construe their world differently at different times because *"experience is made up of the successive construing of events."*

Jankowicz (2004a, p. 150) states that, if data from more than one repertory grid is being analyzed, there are a few different forms of reliability to consider: stability, reproducibility, and sheer accuracy. Stability is the level to which the content analysis is constant over time with regard to the theme definitions. Are they strong enough to ensure that if the process were done again, you would end up with the same categories and the same constructs included in each theme. Reproducibility assures the researcher that others would make the same sense of the constructs as he or she did. Sheer accuracy refers to how consistently the theme definitions are applied during the content analysis. Adams-Webber (1979, p. 36) maintains the importance of evaluating reliability of repertory grid data and it is the responsibility of the researcher to have an appropriate guideline to follow. It is understood however, that because people and situations may change over time, so also may the way they construe their worlds.

Questions about the reliability and validity of repertory grids are those which arise mostly out of misunderstanding. Slater states that there are a variety of means with which to compare entries but what would that prove?

> *"Constructs which are relatively independent of one another may reveal more about the informant's attitudes to the elements than constructs that only tend to replicate one another."* (Slater, 1965, p. 973)

Fransella, Bell and Bannister (2004, pp. 132 - 152) have much to say about reliability and validity as related to repertory grids and their analyses. They define stability taking into account Kelly's notion of change and 'man as a form of motion' with the understanding that with time and different experiences, one's personal construing system may or may not change. Fransella et al (2004, p. 133) see reliability as being able to *"do our best to assess predictable stability and predictable change."* They consider reliability as only one piece of validity. It is their view that we shouldn't expect that a grid will give the same result each time, more importantly we should look to what any change in the grid results means. This coincides with the previously mentioned views of Borman, LeCompte and Goetz (1986, pp. 46 - 47) with regard to qualitative research and reliability. Another problem that exists in arguing the issue of reliability of the repertory grid, claim Fransella et al (2004, p. 134), occurs because there is *"no such thing as the grid."* Due to the flexibility of the grid technique, there have been multiple ways that it has been adapted and utilized. As a result, repertory grids have appeared as research tools in a variety of forms, with different content format and strategies for analyzing them. Fransella et al (2004, p. 134) liken discussing the reliability of the grid in the same way as they would the reliability of the questionnaire with its endless possibilities for variation ... *"Clearly nonsense to talk about"*! This is much the same view that Bannister and Mair (1968, pp. 156 - 200) took. After citing previous studies on the reliabilities of various types of grids, Bannister and Mair (1968, p. 175) *"stressed the pointlessness of trying to make any generalized statement about 'the reliability of the grid'."*

Bannister and Mair (1968) and Fransella, Bell and Bannister (2004) use this same assertion, because of the various forms that a repertory grid can take, on a discussion of validity. In addition, for validity in repertory grids, Fransella et al (2004, p. 144) find that issue very different from the validity of a questionnaire. Repertory grids do not measure some aspect of the research as a questionnaire might because their function is to explore relationships between a person's constructs. The validity of a repertory grid can be discussed with regard to its ability to make it possible for the researcher to recognize patterns and relationships which occur in certain types of data. Fransella et al (2004, p. 144) claim that repertory grids possess an intrinsic validity because, more times than would happen randomly, an analysis reveals significant links between the constructs. To them a grid is basically a structure for data and while one might consider the validity of its format, one cannot argue the validity of the grid itself. In the end, Fransella et al (2004, p. 152) disallow the traditional opinion that validity of a repertory grid can only be attested to by means of a correlation with another test or its being able to forecast some minor detail of human conduct. Rather, they and Bjorklund (2008, p. 17) associate, as Kelly did, *"validity with usefulness and to see understanding as the most useful of enterprises."* My view on the validity of repertory grid interviews agrees with those of Kelly, Fransella, Bell, Bannister, and Bjorklund. It is judged by the opportunity it provides to the researcher to understand the significant relationships between the constructs elicited from the interviewees.

Kreber et al (2003, p. 441) chose not to do a reliability analysis on their study using repertory grids because it had *"little relevance"* to their purpose which was much aligned to Kelly's personal construct theory and how their subjects construed their learning. Hoskonen (1999, p. 102) using a single grid in a pilot study asked the subject to *"check the validity of the results."*

Others, like Adams-Webber (1979), have differing opinions on the reliability and validity of repertory grids in different applications and methods of analysis used. Latta and Swigger (1992, p. 118) feel that the studies have been insufficient to make a decision on the validity of the grid technique for signifying a consensus of opinion on specific areas of expertise. Yorke (1985, pp. 383 – 399) specifically presents a different view than Fransella et al (2004) as explained in the previous paragraph. While he may agree somewhat the arguments of Fransella, Bell and Bannister (2004), he feels that it is necessary for researchers to explore more deeply to determine whether the data intended to be collected is actually what was collected. He states that Kelly also viewed validity as being able to tell us what we already know (concurrency) and not just usefulness. In Yorke's (1985, pp. 383 – 399)view, the ideas of validity and concurrency are circularly related to each other and he also chides that given Fransella et al's opinions, validity should also be determined by the repertory grid's ability to inform the researcher of something that he or she did not already know. Yorke does not refute all of the assertions of Kelly, Fransella, Bell, and Bannister as to the factors important to the concept of the validity of a repertory grid. He advises the researcher though, that there is more to assessing validity than their beliefs when it comes to the mechanics of the procedures through which the repertory grid data was collected and analyzed. Yorke (1985, pp. 383 - 399) goes into detail on his issues of concern with grid context, elements, constructs, and ratings as data is collected. He warns the researcher that validity of a repertory grid should not be taken for granted. He advises the researcher to not let that assumption deter the researcher from a rigorous inspection of the method. In terms of analysis, Yorke's commentary focused mainly on two computer driven analysis methods. Turning data over to technology did not fit my purposes and desire to interactively see the theory emerging from the responses that had been collected from the students.

Jankowicz (1987, p. 485) recognizes that the difficulties in repertory grids stem from the fact that each is different in the interviewees constructs and ratings. If both the elements and constructs were fixed, the grids would simply be a ratings scale. That would strip away the unique advantages the repertory grid has in learning about the interviewees in their own terms. Content analysis was my method of choice for examining the data in this research. Tan and Hunter (2002, p. 51) assert that the discovery of emerging themes is a common strategy for handling repertory grid data with a qualitative approach and that a quantitative approach to handling repertory grid data would employ mathematical and/ or statistical analyses. At the end of his critique of repertory grid validity, Yorke acknowledges that any method of human inquiry will possess some amount of imperfection:

> *"The problem is to decide whether, all things considered, the particular grid in question can be regarded as an acceptable instrument of inquiry."* (1985, p.397)

Bannister and Mair (1968, p. 207) emphasize that while the repertory grid technique is not a trouble-free method, the difficulties it presents can simply, from the start, be resolved by *"listening to the subject."*

3.6 Summary of the Research Process

> *"... the overall function of educational research is to improve educational procedures through the refinement and extension of knowledge."* (Wiersma, 1975, p. 28)

There is a major disagreement among qualitative researchers regarding how much analysis should be done on the data. Some of them feel that rather than analyze the data, the researcher should gather the information and present it without interpretation, resulting in a truthful report letting, in the words of Strauss and Corbin (1990, p. 21), *"the informants speak for themselves."* The large amount of data involved makes this alternative not a practical means of representing research findings. Recognizing this issue, some researchers concentrate on correct descriptions, weaving their own interpretations among informants' responses. It is the *"building of theory',* claim Strauss and Corbin (1990, p. 21) however, that motivates still another group of researchers. Strauss and Corbin (1990, p. 22) believe that by interpretation and conceptualization of the data, a theory can be constructed to describe the reality of the informants' experiences while providing a structure for moving forward. Researchers are cautioned by Reason and Rowan (1985, p. 491) about the right and wrong kinds of research, believing that there is no middle ground. The concern of Reason and Ryal (1985, p.491) about doing it 'right' stems from the effects of research, *"The outcome of research is knowledge. Knowledge is power."* They believe that an incorrect kind of research might result in the *"wrong kind of power."* This may stem from the fact that some feel that qualitative research results in *"trivial conclusions."* (Borman, LeCompte and Goetz, 1986, p. 49) In response to that accusation, they state that it does fulfil its purpose when it is done well by making connections between the findings and what is already known to explain what has been discovered through the research. Jankowicz (1987, p.482), seeking to clarify Kelly's theory, explains that the *"construct system may be inaccurate according to some external criterion"* should not be the issue. Instead, because it is the only way the individual has to make sense of their world, we must listen *"at face value ... paying attention to the content for its own sake."*

Each of the forty-two repertory grid interviews I conducted, fourteen each in mathematics, art, and music, by the nature of the process and the diversity of the participants took on a different tone. I eagerly anticipated the theory which would surface from this glimpse into the ways these students viewed their worlds and the significance it would have for the teaching and learning of mathematics. The methodology of this research is clearly linked to the qualitative character of Kelly's (1991, p. 241) first principle of personal construct theory, self-characterization: *"If you do not know what is wrong with someone, ask them – they may tell you."*

Chapter 4 Analysis of the Questionnaires

4.1 Introduction

> *"Mathematics has many important practical applications, but should not be of interest only to the scientist. There is much in mathematical thought which should interest the arts student, and much which is beautiful and should interest everyone. Those who profess mathematics do so because they enjoy it. To understand and share in this enjoyment, the reader is invited to follow some of the arguments."* (Pedoe, 1958, p.5)

It is with these words that Dan Pedoe begins the preface to his 1958 book, The Gentle Art of Mathematics. In this chapter of the thesis, the reader will gain insights into how students compare their confidence versus enjoyment in mathematics, art and music. Afterwards, in the words of Pedoe, the *"reader is invited to follow some of the arguments"* of students, in their own words, as to why they don't enjoy mathematics even when they are confident in their abilities to learn and do mathematics and, by comparison, learn about the attitudes of students regarding their enjoyment of art and music even when they lack confidence in their abilities to learn and do art and music.

4.2 Background

A study by Anderson (1971, p.54) found that,

> *"Some open-minded individuals may be positive in their attitudes toward art and artists, but they may also have an inferiority complex, so to speak, about their own personal art behaviours. Having knowledge about, and actually doing, a particular activity can be two very different kinds of behaviour."*

This statement began a train of thought that led to the questions which prompted my research:

- Is it possible for students to have an inferiority complex about their own personal mathematics behaviour and yet still have a positive attitude toward the study of mathematics itself?
- Is there a difference in students' attitudes toward the learning of mathematics when their confidence is positive versus when their confidence is negative?
- What is the effect of students' negative images of themselves as mathematicians on their attitudes toward mathematics?
- Will student attitudes toward mathematics differ based upon their confidence in their ability to do mathematics?

In the version one pilot questionnaire [See Appendix A], students were asked to explain their thinking in reaction to the statement: **Do you think that it is possible for a student to state: "I can't do mathematics, but I like learning it?"** Responses to this open ended question indicated that students <u>could have</u> a lack of confidence in doing mathematics and still enjoy learning it. In looking to probe more deeply, a second pilot version of the original questionnaire [See Appendix B] was designed to survey students and gather information more personally related to each student's own attitudes and beliefs about their own confidence versus enjoyment. In addition to data on students not confident in their abilities who enjoyed learning each subject, data was also collected for the other three permutation possibilities of confidence versus enjoyment in each of the subjects, mathematics, art, and music. Later, in the third and final version of the questionnaire, I sought to do the same [See Appendix C].

4.3 Design and Purpose

The purpose of my research project was to explore the attributes of confidence and enjoyment that a random sample of students possesses while learning mathematics versus music and/or art. The goal: <u>to discover factors and reasons for students' attitudes toward learning and enjoying mathematics versus the arts based upon their degree of confidence in each specific subject.</u>

In beginning this investigation into the question of how students perceive their learning in mathematics compared to art and music, an instrument was required to classify students with regard to their confidence versus their enjoyment in each of these subjects.

> *"The need for an appropriate research design arises whenever we wish to generalize from our findings, either in terms of the frequency or prevalence of particular design attributes or variable, or about relationships between them."* (Oppenheim, 2000, p.5)

In order to discover any relationship between student confidence and enjoyment, I designed a questionnaire to identify students in each of the following categories in mathematics, art and music:

- **Confident and Enjoy (+/+)**
- **Confident and Not Enjoy (+/-)**
- **Not Confident and Enjoy (-/+)**

- **Not Confident and Not Enjoy (-/-)**

The first part of the questionnaire was a six question Likert-type scale. Students were asked to respond to each statement regarding the degree of their confidence and enjoyment in their school experiences, for each of the subjects: mathematics, art and music. A five point scale was used*: Strongly Disagree, Disagree, Uncertain, Agree and Strongly Agree*. Students were also asked to select the category which best expressed their feelings and/or experiences for those subjects from the following choices:

- I am confident ... and I like learning ...
- I am confident ... but I don't like learning ...
- I am not confident ... but I like learning ...
- I am not confident ... and I don't like learning ...

To better understand the meaning behind their choice(s), students were asked to explain one of their selections as completely as they could.

4.4 Pilot History

> *"Questionnaires do not emerge fully fledged; they have to be created or adapted, fashioned and developed to maturity after many abortive test flights."* (Oppenheim, 2000, p.47)

In the autumn and winter of 2002 – 2003, the first version of a questionnaire for this research was distributed to 234 students of mixed ages in mathematics classes of some schools in New York, United States and in various schools in Devon, England. It contained ten Likert statements and a general open ended response which sought to determine if students thought it was possible for someone who was not confident in their ability to do mathematics to still enjoy learning it. [See Appendix A] A second version of the questionnaire was created later in 2003. All participants were given the same ten Likert scale statements as Appendix A. A variation of the open response question 11 was used. [See Appendix B]

To better understand the students' attitudes and beliefs regarding their own experiences in learning mathematics, art and music, each participant was asked to specifically categorize themselves for one of the subjects and then explain their choice. Each questionnaire contained one of the four variations of question 11: two related to mathematics, one related to art, and one related to music. The subject categorization variations were randomly distributed to respondents. A total of 115 students in 7th and 9th grade from some schools in New York, United States completed this pilot version in their English classes.

Early in this study, in order to pare down the number of questions, fine tune the research questionnaire, and ensure that the wording of statements did elicit the intended concepts of confidence versus enjoyment, the Likert response statements were sent to several teaching colleagues for their feedback. These colleagues were asked to identify the aspects of teaching and learning that they felt that students were being asked to respond to in each. The response comments received validated the initial intent of the questionnaire. The final questionnaire was developed in April 2004. The number of Likert scale responses was reduced from ten to six. Five were exactly the same as the pilot versions' statements and the 6th was a variation of another of the original statements. A Likert attitudinal scale was selected to enable participants to provide a broad range and degree of responses regarding perceptions of their own confidence and enjoyment in the learning of mathematics, art and music. Cohen, Manion and Morrison (2000, p. 255) are *"useful for tapping attitudes, perceptions and opinions of respondents."* They suggest the need for pilot stages in order to *"devise and refine categories, making them exhaustive and discrete"* if the questionnaire method of data collection is used.

In the open ended response part in this final version, each participant was asked to categorize themselves on confidence versus enjoyment in each subject: mathematics, art, and music and then explain their response to one of the categorizations. [See Appendix C] The combination of the rating scale along with an opportunity for participants to describe their thoughts, feelings and opinions was used to give me a clearer picture of the similarities and differences in the thinking, doing and enjoying of mathematics versus the arts.

> *"It is the open ended responses that might contain the 'gems' of information that otherwise might not have been caught in the questionnaire. Further it puts the responsibility for and ownership of the data much more firmly into the respondents' hands ... the open ended question can catch the authenticity, richness, depth of response, honesty and candour..."* (Cohen, Manion, and Morrison, 2000, p. 255)

What better way to truly understand the nature of students' personal sense of mathematics and identify the factors that affect the way they view learning it!

The results of the pilot versions of the questionnaire did more than serve as a tool with which to polish and refine the version that was finally used to collect the data. From the responses received, I used the data collected to identify students in each category to interview at a later date. These were pilot interviews that helped me formalize and create the repertory grid method that was ultimately used to interview students in the second phase of this research.

The final version of this research questionnaire was distributed to 14 -15 year old students during the next academic school year. The first distribution was to forty-four students from a school in New York, United States and thirty students from a school in Devon, England. This group provided interviewees for the pilot repertory grids interviews in preparation for the final phase of this research. The second survey group contained an additional 1152 students from various secondary schools in the United States and England. In was from this latter group that my final repertory grid interviewees were selected from participants in the United States.

4.5 Participants

The research questionnaire was completed by a total of 1226 students, aged 14 – 15 years old from sixteen secondary schools in the United States and England. A total of 975 students were surveyed from nine schools in the United States. These American schools are located in the states of: Delaware (1 school), Georgia (1 school), New Jersey (1 school), New York (5 schools), and South Carolina (1 school). A total of 249 students were surveyed from seven schools in England. These British schools are located in the counties of: Devon (5 schools), Hertfordshire (1 school), and Somerset (1 school). Two respondents did not indicate a school on their questionnaires. By gender, 48.8% of the respondents were male and 51.2% of the respondents were female.

4.6 Data and Analysis

The data collected from the questionnaires was first analyzed by looking at the frequencies of students who had self-selected a place for themselves in each of the categories based on their confidence versus enjoyment in each subject. The responses to the open ended question 8 were used to calculate these frequencies. The data was entered into SPSS. The numbers one through four were designated to represent the categories **+/+**, **+/-**, **-/+**, and **-/-** respectively. Where a student found themselves in between two categories, a numerical code of .5 was noted. For example, a student categorizing themselves as "confident" but <u>somewhere between</u> "enjoy" and "not enjoy" was given a designated as a 1.5. To reiterate, the categories of students in this research are:

- **1: +/+ Confident and Enjoy**
- **2: +/- Confident and Not Enjoy**
- **3: -/+ Not Confident and Enjoy**
- **4: -/- Not Confident and Not Enjoy**

The frequency tables of category results for each subject are given below in Tables 4.1- 4.3 (note that percentages which do not add up to 100% are accounted for by students who responded in an "in-between" category and in some cases* rounding):

Category	Frequency	Percent	Valid Percent	Cumulative %
+/+	502	40.9	42.0	42.0
+/-	509	41.5	42.6	84.6
-/+	81	6.6	6.8	91.4
-/-	97	7.9	8.1	99.5
Not Clearly in Category	5	.4	.5	100
Total	1194	97.4	100	
Missing	32	2.6		
Total	1226	100		

Table 4.1 Category Frequency for Mathematics

Ranked in order, from highest to lowest, the frequency of each category for

mathematics:	**+/-**	**42.6%**
	+/+	**42.0%**
	-/-	**8.1%**
	-/+	**6.8%**

From Table 4.1, it can be seen that 84.6% of the students surveyed are confident in their ability in mathematics and 14.9% are not. With regard to the initial question, while not a very large percentage (6.8%), there are students who are not confident in their abilities to do mathematics but who still enjoy learning it. Of the students surveyed, 48.8 % enjoy learning mathematics and 50.7% do not. A surprising statistic to me, however, is that more than half of those students who are confident in mathematics (509 out of 1011) do not like/enjoy learning it! In fact, the results indicate that the category with the greatest frequency in mathematics is **+/-** (Confident but Not Enjoy)!

Category	Frequency	Percent	Valid Percent	Cumulative %
+/+	384	31.3	33.1	33.1
+/-	94	7.7	8.1	41.2
-/+	411	33.5	35.4	76.6
-/-	270	22	23.3	99.9
Not Clearly in Category	2	.2	.2	100.1*
Total	1161	94.7	100.1*	
Missing	65	5.3		
Total	1226	100		

[* percents do not add up to 100 due to rounding]

Table 4.2 Category Frequency for Art

Ranked in order, from highest to lowest, the frequency of each category for

art:

-/+	**35.4%**
+/+	**33.1%**
-/-	**23.3%**
+/-	**8.1%**

Table 4.2 shows that 41.2% of the students surveyed are confident in their ability in art and 58.7% are not. Of the students surveyed, 68.5% enjoy learning art and 31.4% do not. The results of this questionnaire do support the Anderson (1971) study. A student can feel inferior in his/her ability in art and still feel positively toward art. In fact, the category with the greatest frequency in art is **-/+** (Not Confident but Enjoy)!

Category	Frequency	Percent	Valid Percent	Cumulative %
+/+	432	35.2	37.5	37.5
+/-	169	13.8	14.7	52.2
-/+	241	19.7	20.9	73.1
-/-	307	25.0	26.7	99.8
Not Clearly in Category	2	0.2	0.2	100
Total	1151	93.9	100	
Missing	75	6.1		
Total	1226	100		

Table 4.3 Category Frequency for Music

Ranked in order, from highest to lowest, the frequency of each category for

music:	**+/+**	**37.5%**
	-/-	**26.7%**
	-/+	**20.9%**
	+/-	**14.7%**

Table 4.3 shows that 52.2% of the students surveyed are confident in their ability in music and 47.6% are not. Of the students surveyed, 58.4% enjoy learning music and 41.4% do not. The category with the greatest frequency in music is **+/+** (Confident and Enjoy).

In looking at the subject category results as a whole, some interesting comparisons come to light:

- At 84.6%, the highest level of student confidence is in mathematics
- At 58.7%, the greatest lack of student confidence is in art
- At 68.5%, the highest level of student enjoyment is in art
- At 50.7%, the greatest lack of student enjoyment is in mathematics
- In rank order, the <u>top two categories</u> are sorted by:
    - ➢ Confidence then, not enjoy/ enjoy for mathematics
    - ➢ Enjoyment then, not confident/ confident for art

- Confidence = Enjoyment, that is, **+/+** then **-/-** for music
- In rank order, the category Confident but Not Enjoy is:
  - Highest for mathematics
  - Lowest for art and music
- In rank order, the category Confident and Enjoy is
  - Highest for music
  - Second highest for mathematics and art

As the above data was reviewed, I found the result that <u>42.6% of the students surveyed were confident in mathematics but did not enjoy learning it</u> was quite interesting! Would this percentage for categories in mathematics be similar when participants were compared by gender and by school? After sorting student responses with SPSS, Tables 4.4 and 4.5 were created:

Gender	Number of Students	Percent +/+	Percent +/-	Percent -/+	Percent -/-	% Not Clearly in Category
Male	570	44.7	44.2	4.4	6.3	.5
Female	601	40.3	40.3	9.0	10.1	.3
Total	1171					

[**Note** – 55 student participants did not indicate their gender]

Table 4.4 Category Frequency for Mathematics by Gender

There appear to be no remarkable differences in these frequencies compared to those in the questionnaire population as a whole. This suggests that gender is not a factor in understanding the relationship between student confidence and enjoyment in the learning of mathematics.

School	Number of Students	Percent +/+	Percent +/-	Percent -/+	Percent -/-	% Not Clearly in Category
1	268	48.5	41.8	3.0	6.7	
2	31	41.9	51.6	0.0	6.5	
3	117	33.3	46.2	6.0	14.5	
4	65	35.4	55.4	1.5	7.7	
5	52	46.2	23.1	17.3	13.5	
6	26	46.2	46.2	3.8	3.8	
7	33	42.4	48.5	3.0	6.1	
8	27	11.1	77.8	0.0	3.7	7.4
9	55	47.3	43.6	5.5	3.6	
10	72	48.6	38.9	2.8	9.7	
11	131	45.8	32.1	15.3	6.1	.8
12	88	33.0	29.5	21.6	14.8	1.1
13	48	27.1	56.3	2.1	12.5	2.1
14	23	8.7	87.0	4.3	0.0	
15	127	57.5	37.0	3.1	2.4	
16	29	20.7	55.2	6.9	17.2	
Total	1192					

[**Notes**: Due to rounding, some percentages that do not add up to 100%;
34 student participants did not indicate their school]

Table 4.5 Category Frequency for Mathematics by School

After comparing the percentages in each category for mathematics, the data from five of the schools participating in this research jumped out at me. Schools 5 (23.1%) and 12 (29.5%), one from the United Kingdom and one from the United States, have much lower percentages of students in the "Confident and Not Enjoy" category. School 5's percentage is half as much as the same category when all schools are analyzed together. In both schools, the "Confident and Enjoy" category ranked first. The percentages for "Not Confident and Enjoy" and "Not Confident and Not Enjoy" are much higher than these same categories in the participant population taken as a whole. School 12's percentages are more spread out among all four categories.

For schools 8, 14, and 16 the mathematics categories were ranked almost the same as the total participant population as a whole. The percentages for **+/-** students, 77.8%, 87.0% and 55.2% respectively, however, were dramatically higher than the total population! Two of these schools are in the United Kingdom and one is in the United States. Questions arise:

- When more than 60% of the students in each of these schools (in some schools the rate is over 90%) are confident in their ability to do mathematics, what do students mean by "confidence" in mathematics?
- What are the issues in schools, like 8 and 14, where such a great percentage of confident students do not enjoy the learning of mathematics?

Due to practical constraints on the arranging of the repertory grid interviews, none of the students from these five schools were among those interviewed.

4.7 Student Comments

Strauss and Corbin (1990, p. 28) state that,

> *"... many researchers find the data collection deeply satisfying. Yet, the most gratifying moments of research for analytically minded researchers will be those that bear on their discoveries. They may be matters of quick flashes of 'intuition', or major breakthroughs in understanding the meanings and patterns of events, or the deeper satisfaction of having solved the research's major puzzles ..."*

The exciting and 'gratifying' next phase of this research for me was to be able to try to understand the similarities and differences in the reasons for student confidence and enjoyment and/ or lack thereof in learning mathematics versus the arts and 'discover' the 'meanings and patterns of events' as related to the learning of mathematics. In short, how and why do students construe the experiences that shape their confidence and enjoyment? What can we uncover about student confidence and enjoyment in learning the arts that can have an impact on their confidence and enjoyment in learning mathematics?
In qualitatively looking at the data collected, it is important to let the students own words articulate their thoughts and feelings. This gives, according to Strauss and Corbin (1990, pp. 21 – 23), an opportunity for a truthful report based on the spontaneous responses of students of the words as they were actually expressed. Any biases on the part of a researcher does not interfere with the data and in the end, *"what is relevant ... is allowed to emerge"* (1990, p. 23).

Students' comments were sorted into categories based on their confidence and enjoyment. Investigating themes that arise in the data with an element of

> *"creativity ... enables the researcher to ask pertinent questions of the data and make the kind of comparisons that elicit from the data new insights into phenomenon ..."* (Strauss and Corbin, 1990, p. 31)

I was able to identify students in all four categories for all three subjects and to formulate additional 'pertinent' questions with the data that was collected:

- Why don't students like learning mathematics when they are good at it?
- What makes students who don't perform well in mathematics still like learning it?
- How do these compare with students' attitudes and beliefs in the same categories above, for art and music?
- What can we learn from students' attitudes in art and music that can improve the teaching and learning of mathematics and help foster a positive attitude toward learning mathematics in all students?

Through this approach, this research will result in a grounded theory *"inductively derived from the study of the phenomenon it represents."* (Strauss and Corbin, 1990, p.23) This theory will help describe, by category and subject, the perceptions that students have about learning mathematics, art and music. In the end, what is relevant will emerge.

4.7.1 Comment Characterizations

Understanding the views of students expressed in 1350 individual comments on these questionnaires was an ineffective and daunting task!

> *"One can count "raw" data, but one can't relate or talk about them easily.* ***Therefore, conceptualizing our data becomes the first step in analysis.*** *By breaking down and conceptualizing we mean taking apart an observation, a sentence, a paragraph, and giving each discrete incident, idea, or event, a name, something that stands for or represents a phenomenon. Just how do we do this? We ask questions about each one, like: What is this? What does it represent? We compare incident with incident as we go along so that similar phenomena can be given the same name. Otherwise, we would wind up with too many names and very confused!"* (Strauss and Corbin, 1990, p. 63)

In beginning to analyze student responses to question 8, comments were first sorted by subject and "confidence versus enjoyment" category. By comparing the comments and "asking questions" of the data many "names" to represent similar "phenomena" were decided upon. Strauss and Corbin add:

> *"How do categories get named? ... most names come from YOU! The name you choose is usually the one that seems most logically related to the data it represents, and should be graphic enough to remind you quickly of its referent. But, it must be a more abstract concept than the one it denotes ... The important thing is to name a category, so that you can remember it, think about it, and most of all begin to develop it analytically."* (1990, pp. 67 - 68)

As the word "category" is used in this research to refer to the grouping of students based on their confidence and enjoyment (+/+, +/-, -/+, -/-), the term, "characterization" is used as its replacement. Following the grounded theory approach, these eight comment characterizations were decided upon to represent the data seen in this research:

- Atmosphere/Environment – experiences while learning and doing the subject
- Emotional Appeal – how the subject makes the student feel, how much the student likes/ dislikes the subject
- Impressions of the Subject – nature of the subject, level of difficulty, effort and time required, tedium, intellectual stimulation, understanding
- Motivation – enthusiasm for learning the subject, interest, engagement, makes the students want/ not want to learn
- Nature of the Tasks – learning process and activities involved
- Personalization – expression/ challenge of self, impression of self: ability, achievement

- Relevance – importance for the improvement of skills, level of necessity and usefulness now and in the future, practical use/ application, value
- Teacher – presentation of the lesson, teaching style, teacher

4.7.2 Comments by Category

There were a total of 1,350 responses made by participants explaining the reasons for their category selection. It should be noted that some students did not explain any of their choices and others explained more than one of their choices. The breakdown of the comments by subject and category was:

Category / Subject	+/+	+/-	-/+	-/-	Total
Mathematics	224	248	40	49	561
Art	156	30	129	68	383
Music	211	57	76	62	406
Total	591	335	245	179	1350

Table 4.6 Comments by Subject and Category

In order to understand the reasons behind students' choices for self categorization more deeply, the comments were broken down by category, subject and characterizations. At the beginning of each category discussion, examples of comments representative of each of the characterizations for each category are listed. The headings include the number of comments that were attributed to each characterization by subject. These numbers do not match the total in each category, because some comments overlap into more than one characterization. The numbers in parentheses identify the student participants.

4.7.2.1 Category +/+ Comments

These students are confident in their ability to do the individual subject and they enjoy learning it.

+/+ Mathematics

There were a total of 224 comments made by students in this category.

- Atmosphere/Environment – 16 comments
  - *Math just falls into place for me.* (1082)
- Emotional Appeal – 74 comments
  - *I feel satisfied when I understand certain topics without feeling frustrated.* (998)
  - *It makes me feel good that I can do it.* (220)
  - *Math is super!* (1167)
- Impressions of the Subject – 86 comments
  - *There's always an answer.* (147)
  - *I always like to learn with numbers.* (309)
  - *Mathematics is fun to me because it is kind of like a puzzle.* (439)
  - *I like how structural mathematics is/ I enjoy working with numbers.* (689)
  - *I don't mind figuring out how to solve math problems.* (179)
  - *I enjoy working hard to expand my ability.* (1143)
  - *Math is fascinating, intriguing ... enlightening.../as a very analytical & logical thinker, math is a natural strength of mine.* (1102)
- Motivation – 74 comments

- *I enjoy learning different techniques into solving problems.* (46)
- *I find working with numbers interesting.* (89)
- *My whole family is good at math.* (225)
- *Like learning new things in math.* (269)

- Nature of the Tasks – 60 comments
  - *It's fun to be able to solve problems.* (324)
  - *You don't really have to remember a lot of notes ... once you do some problems & know how to do them.* (35)
  - *... I like working equations and doing sums in my head.* (54)
- Personalization – 111 comments
  - *I can learn new topics & methods quickly & use them efficiently.* (49)
  - *It's a subject that I am really good at.* (260)
  - *I like math and I am Asian/ Asian is good at math.* (944)
- Relevance – 44 comments
  - *Plan to seek a career somewhere in the field of math.* (2)
  - *One of the most important subjects employers look for.* (74)
  - *It gives reasonable explanations for things.* (614)
- Teacher – 16 comments
  - *I enjoy math as long as I have a good teacher who explains well and allows you to figure problems as many ways as possible.* (413)

+/+ Art

There were a total of 156 comments made by students in this category.

- Atmosphere/Environment – 14 comments

- *I really like art because it's relaxing.* (167)
- *Doing art is a release for me.* (566)

- Emotional Appeal – 52 comments
  - *I feel proud to say I drew that.* (138)
  - *Non-frustrating.* (242)
  - *When I finish a piece of work, it makes me feel happy.* (453)
  - *It makes you feel like you can do anything.* (1185)
  - *It's a way to express feelings.* (144)
  - *I love art no matter what.* (237)
- Impressions of the Subject – 27 comments
  - *I like to be creative and make things.* (23)
  - *It's something where you can let your "mind run free."* (139)
  - *Don't have to be thinking ... learning art is simply watching how others interpret ... never right or wrong.* (380)
  - *Imaginative.* (475)
  - *You can do what you want.* (564)
  - *You don't really learn anything ... just draw ...* (432)
- Motivation – 52 comments
  - *It's a fun class because you can talk.* (37)
  - *I like learning how to make different things.* (734)
  - *Learning new things & using new material.* (177)
  - *It interests me a lot and never bores me.* (1198)
  - *Constantly love learning new forms of art.* (268)
  - *Natural ... people in my family do art ... novelties to it.* (28)
  - *Enjoy in my free time ... learning new techniques.* (33)
  - *... different styles, techniques, history absolutely fascinating.* (83)

- *I like learning about past artists.* (31)

- Nature of the Tasks – 87 comments
  - *I love drawing and painting.* (31)
  - *I like drawing and using my hands to make things.* (201)
  - *I love to design things.* (357)

- Personalization – 96 comments
  - *I have talent in art and I enjoy doing it.* (8)
  - *Art for me is expressing myself.* (96)
  - *Art helps express my feelings and ideas ... creativeness.* (80)
  - *Express feelings/ art can trigger memory of a person ...* (144)
  - *Art is definitely a part of my life.* (1133)
  - *I'm good at drawing.* (218)
  - *I'm better in art than the academics.* (252)
  - *I'm in control.* (600)
  - *... make something I never thought I'd be able to create.* (654)
  - *Describes me.* (791)
  - *Everyone has their own good style.* (138)

- Relevance – 22 comments
  - *I also want to be an expert so I can go further.* (830)
  - *Art is my hobby.* (143)
  - *Interesting ... I might also pursue a career in it.* (629)
  - *I enjoy learning ... and ways to improve my work.* (1204)
  - *The better you can do something, the more you like it.* (282)
  - *... I get better and better.* (339)
  - *Apply what I learn to my art.* (382)

- Teacher – 5 comments
  - *Enjoy being taught by the teacher.* (259)
  - *My teacher sets us good work.* (450)

+/+ Music

There were a total of 211 comments made by students in this category.

- Atmosphere/Environment – 15 comments
  - *Because it's relaxing and I enjoy music.* (119)
  - *... calming and soothing ...* (388)
  - *... think that when I'm taking tests, I do better hearing music.* (795)
  - *... I like it because it calms me down when I am angry.* (34)
- Emotional Appeal – 60 comments
  - *... playing ... for others to enjoy makes me feel good inside.* (110)
  - *... make me feel good about my special talent.* (81)
  - *... some songs that I listen to explain exactly how I feel.* (155)
  - *... I love the emotions in it.* (169)
  - *always feel accomplished when I can sing or learn a new song.* (303)
  - *... it makes me feel happy and clears my head.* (568)
  - *... I enjoy learning ... it makes me feel like I have a talent.* (930)
- Impressions of the Subject – 31 comments
  - *I like music, it is art for the soul ...* (34)
  - *Music is very hard but fun ...* (242)
  - *... get to be creative with noise.* (1031)
  - *... challenging/ I enjoy having to work out difficult rhythms.* (200)

- *... don't have to spend much time at a piece to play it well.* (396)
- *Because it is the easiest to learn.* (714)

- Motivation – 64 comments
  - *... everyone in my family played a musical instrument ...* (112)
  - *... find music interesting.* (127)
  - *... I like to learn new pieces and then play them well.* (130)
  - *... it can get boring and some theories are hard ...* (1036)
  - *Music is very interesting ... some music is fun to play.* (200)
  - *... always loved learning new things.* (148)
  - *... interesting how music works.* (613)
  - *I like ... learning how music is created.* (774)
  - *... look forward to band every day to learn something new.* (82)
  - *Music is fun and it's great.* (504)
  - *... I don't like playing the same thing over and over ...* (1130)
  - *I like all types of music ...* (840)
  - *Music is fun to learn and is everywhere ...* (1092)
- Nature of the Tasks – 104 comments
  - *I love to play music and listen to it.* (42)
  - *I play trumpet and sing/ I love listening and making music ...* (130)
  - *I love to sing and learn about music.* (111)
  - *I like to play violin and I do not mind practicing.* (141)
  - *Figure out how to ... play cool songs and techniques.* (1093)
  - *I play piano ... learning new songs and symbols in music.* (238)
  - *I can sing very good but I don't like reading notes ...* (347)

- *I like learning about the history of music and listening to different kinds of music* ... (497)
- ... *it's fun to just make music.* (604)
- ... *a lot of time listening to, learning and composing music* ... (648)
- *Love to perform for other people.* (704)

• Personalization – 80 comments

- *I'm good at playing and singing music and I love to learn it.* (5)
- *Music has always come pretty easily to me* ... (84)
- *I love music/ it is easy for me to learn and I can relate to the subjects studied because I play the piano.* (125)
- ... *beautiful way of expressing* ... *feelings and emotions* ... (930)
- ... *it comes pretty natural.* (365)
- ... *I love making up my own music.* (649)
- ... *big part of my life.* (727)
- ... *born for the love of music.* (884)
- ... *sing and play the piano well* ... *love learning new things in music and expressing it in my own way.* (602)

• Relevance – 31 comments

- ... *I like* ... *I'm good at it* ... *plan on continuing/ it's very interesting to learn how to play more advanced music.* (142)
- ... *field I want to go into when I am older.* (990)
- *Hobby* ... *release from the real world* ... (1008)
- ... *would maybe like a career in music if I can.* (226)
- *I like learning music because it's a nice hobby.* (248)
- ... *I like to learn more* ... *to help improve my musical skills.* (355)

- Teacher – 8 comments
  - ... *my music teacher and overall the class is interesting to me.* (313)
  - ... *I have an excellent teacher and it is really fun.* (327)
  - ... *I didn't like my teacher.* (392)
  - ... *teacher makes it fun.* (466)

Comparative Discussion: "Confident and Enjoy"

The characterizations, ranked by the frequency of the number of comments in each, are listed below. Percentages of the total number of comments that were classified in each characterization to the nearest $\frac{1}{10}$ % are included:

Mathematics **(224)**	***Art*** **(156)**	***Music*** **(211)**
1 Personalization 49.6%	1 Personalization 61.5%	1 Nature of Tasks 49.3%
2 Impressions of Subject 38.4%	2 Nature of Tasks 55.8%	2 Personalization 37.9%
3 Motivation 33.0%	3 Motivation 33.3%	3 Motivation 30.3%
3 Emotional Appeal 33.0%	3 Emotional Appeal 33.3%	4 Emotional Appeal 28.4%
5 Nature of Tasks 26.8%	5 Impressions of Subject 17.3%	5 Relevance 14.7%
6 Relevance 19.6%	6 Relevance 14.1%	5 Impressions of Subject 14.7%
7 Atmosphere/Environment 7.1%	7 Atmosphere/Environment 9.0%	7 Atmosphere/Environment 7.1%
7 Teacher 7.1%	8 Teacher 3.2%	8 Teacher 3.8%

Table 4.7 +/+ Rankings and Ratings of Comment Characterizations

Comments are quoted using only the appropriate text for the content being discussed. In comparing these results for the students who are **Confident and Enjoy** (**+/+**), interesting observations can be made:

- <u>Teacher</u> ranked last for all three subjects. It was about the same percentage for art (3.2%) and music (3.8%). While still low in mathematics, at 7.1%, <u>Teacher</u> was twice as high as for other subjects. No specific comments were repeated by any number of students in any subject.

- <u>Atmosphere/Environment</u>, ranked seventh for mathematics, art, and music. The percentage for mathematics and music was 7.1% while the comments for art had a slightly higher rate of 9.0%. Interestingly, 8 out of the 14 comments for art and 15 out of the 15 comments for music expressed the following sentiments:
    - *Doing art is a release for me ... doing it relaxes me* (566)
    - *... when I play instruments it sometimes helps me relax/ music can be very calming and soothing to me* (388)

  There was no such comment expressed for mathematics and no comment from those made which was repeated by any number of students.

- While the percentages for <u>Relevance</u> were similar, the percentage for mathematics was highest in this characterization. Ranked fifth for music (14.7%) and sixth for both mathematics (19.6%) and art (14.1%), many students expressed their views on the importance of the subjects to their future, improving their skills, and relevance to their world in these ways (43 out of 44 comments for mathematics, 16 out of 31 for music, and 12 out of 22 for art):
    - *I am very interested in math and plan on seeking a career somewhere in the field of math ...*(2)
    - *... Its* (mathematics) *applications are everywhere and without it you'll surely perish*. (140)

- ... *You need it* (mathematics) *for everyday life. Plus you would use it more than music or art in your everyday life! ...* (343)
- *I have liked art since I was a little kid and will probably have a future in it.* (1056)
- *I like art more than math or music/ I don't want to be an artist or anything when I'm older ...*(390)
- *I am good at art and I enjoy improving it.* (547)
- *I have listened to music since I was little and can relate to it more than say art or math.* (931)
- *... music is the field of work that I want to get into ...* (990)
- *I see myself as a musical person before a mathematician and an artist/ I do extra music stuff outside of school.* (1208)

- <u>Personalization</u> ranked first for mathematics (49.6%) and art (61.5%). It ranked second for music, but with a much lower percentage (37.9%). The repeated reference to "expression of self and their feelings" (17 out of 96 comments for art and 10 out of 80 comments for music) conveyed the following sentiments:
  - *Art for me is expressing myself and how I feel ...* (96)
  - *... I love learning new things in music and expressing it in my own way.* (602)
  - *... I also love making up my own music on the piano and for the guitar ...* (649)

  There was no such comment expressed for mathematics.

- Nature of Tasks had a diverse ranking for **+/+** students in mathematics versus the arts. It was ranked first with students in music (49.3%), a second place ranking but a higher percentage with students in art (55.8%), and a fifth place ranking in mathematics (26.8%). The ratings in art and music were about twice the rate in mathematics, which can be attributed to the differences in the learning activities and outcomes that take place during lessons in mathematics compared to lessons in the arts. A common theme in the Nature of Tasks comments was the "doing" of something ... students described actively participating in their learning. Hands-on engagement was an overwhelming happening in the learning of both art and music but not, it seemed, in mathematics: 57 out of 87 for art, 103 out of 104 comments for music versus 1 out of 60 comments for mathematics. Examples of this theme in responses are:
    - *I like maths because we make posters.* (468)
    - *I like drawing and using my hands to make things.* (201)
    - *You don't really learn anything/ you just draw and it's not boring to do like other subjects.* (432)
    - *I play the piano and like playing it and learning new songs and symbols in music.* (238)
    - *... it's fun just to make music.* (604)
    - *I like to play the violin and do not mind practicing it.* (141)

- Impression of Subject ranked fifth for art (17.3%) and music (14.7%). In contrast, mathematics ranked second with more than twice as high a rate (38.4%) as the other two. In examining students' Impression of Subject, there were several popular themes that occurred within this characterization. As opposed to the physical aspect of "doing" in Nature of Tasks commented on by so many students, there appeared to be a more mental view of the behaviours involved in the learning of mathematics and music. In 37 out of 86 comments for mathematics and 16 out of 31 comments for music, the following views were expressed:

- *... I like solving math problems. Learning mathematics allows me to solve more complicated problems ...* (1174)
- *In maths ... you are just following patterns ...* (496)
- *... I like to have to think things out and solve problems out.* (14)
- *Math ...easy for me/ when it's not I enjoy the challenge ...* (172)
- *... I love listening and making music ...* (130)
- *Music ... can be challenging/ I enjoy having to work out difficult rhythms ...* (200)

No comments like these were made for art. "Creativity" and "imagination" were mentioned by 9 out of 27 comments for art and 2 out of 31 comments for music:

- *... art is also enjoyable because it's very imaginative.* (475)
- *I adore being creative/ I know I'm good at my creativity.* (503)
- *... I like doing it* (music) *because you get to be creative with noise.* (1031)

As opposed to being creative or imaginative, this view of mathematics was noted in 1 out of 86 comments:

- *I like how structural mathematics is and I enjoy working with numbers.* (689)

- Emotional Appeal ranked similarly for all three subjects: third for mathematics (33.3%) and for art (33.3%), but fourth for music (28.4%). Several students expressed feeling good, happy and/or proud doing the specific subject commented upon. Examples of the 9 out of 74 comments for mathematics, 9 out of 52 for art and 5 out of 60 for music are:
  - *It* (mathematics) *makes me feel good that I can do it.* (220)
  - *I feel proud to say I drew that ...* (138)
  - *... playing music for others makes me feel good inside.* (110)

  Even with this group, there were some comments that conveyed the possibility of a somewhat negative emotional side to mathematics:
  - *... it can be annoying when we learn things that may not be essential to our curriculum.* (63)
  - *... I'm not afraid to walk in the class thinking there's a quiz every day.* (257)
  - *... I was offered to take accelerated but declined in fear ...* (290)
  - *... but there's always something that confuses you.* (886)
  - *... math is something important and you have to learn it so I just try to be positive about it.* (1082)

  This type of sentiment was not found in art or music.

- Motivation ranked third for each subject with rather similar percentages: 33.0% for mathematics, 33.3% for art, and 30.3% for music. In 11 out of 74 comments, students who were confident and enjoyed mathematics were fascinated and excited by working with numbers:

  - *Maths is fascinating, intriguing ... enlightening ...* (1102)
  - *... I find working with numbers interesting ...* (89)

  There were no such comments made for art or music. Another aspect of Motivation that received repeated comments was "fun versus boring" across the subject areas. With regard to the subject being *fun/never boring*, students reported this to be true for 25 out of 74 comments in mathematics, 15 out of 52 comments in art, and 22 out of 64 comments in music ... about one third of the comments for each subject!

  - *... I never fall asleep in math class.* (1196)
  - *It's fun to be able to solve problems* (in mathematics). (324)
  - *... it's* (art) *a fun class because you can talk.* (37)
  - *I don't consider myself an artist, but art is fun to me ...* (121)
  - *... I find my music lessons in school fun and interesting.* (503)

  Even in this group however, these comments appeared (2 out of 74 comments in mathematics, 2 out of 63 comments in music):

  - *... it's* (mathematics) *quite interesting but it can be very boring.* (1200)
  - *... learning about Beethoven is boring.* (42)

  A key issue students expressed in the Motivation comments was the idea of newness. In 13 out of 74 comments for mathematics, 17 out of 52 comments for art, and 23 comments for 64 for music, these **+/+** students explained it this way:

- *I know how to do math but I like learning new things in math that I did not know before, ex. new formulas or ways to do problems easier.* (385)
- *I love learning art, learning new things and using new material.* (177)
- *... I like to learn new pieces and then play them well.* (130)

+/+ Summary

What are the key issues that unite and separate the views of students who are "Confident and Enjoy" when they are learning the subjects of mathematics, art and music? The characterizations of Personalization, Motivation, Relevance, Atmosphere/ Environment, Emotional Appeal, and Teacher all rank very similarly for the **+/+** students. From them, an understanding of the common factors that promote their enjoyment of the learning these subjects can be derived. In contrast, Nature of Tasks and Impressions of Subject, as the characterizations which rank quite differently, enable us to differentiate their attitudes and beliefs toward mathematics versus the arts.

The high ranking of Personalization in all three subjects indicates a strong connection for these students between themselves and their abilities, achievement, enjoyment and learning. While no students stated that mathematics was a way to *express themselves* as in the arts, their *self* connection is conveyed in other ways: *I like learning math because I know* *how to do it right.* (235) The ranking of Motivation, third for all three subjects with about a third of the comments made in each area, indicates that the **+/+** students' enthusiasm for learning and those factors making them want to learn are equitable. The idea of novelty is definitely a uniting motivational factor for these students. *New ... formulas, ideas, things, techniques, topics, methods, materials, styles, pieces ...* were responses that came up time and time again. Ranking just below Motivation across all subjects was Emotional Appeal. Fifteen students claimed that mathematics was their "favourite" subject. Of all of the responses, this only occurred three times for art and once for music. Listed in the last three positions for all three subjects for **+/+** students are Relevance, Atmosphere/ Environment and Teacher respectively. This would seem to indicate that for these students the necessity, usefulness and value of the subject and their experiences while learning the subject are not as noteworthy for their enjoyment of these subjects as other characterizations. By its last place ranking for each subject, the Teacher has the least significant impact on the enjoyment of these confident students.

In addition to the positive Emotional Appeal comments from these **+/+** students, there were some that expressed some negative aspects to the learning of mathematics: *I like learning math not to the extent of an exclamation point, but ...* (661). Thoughts like these seem to create the disparity in the ranked order of Impressions of Subject for mathematics versus the arts. While not with any great frequency, other themes than ran through the comments:

- The idea that in mathematics there is *one* definite answer as opposed to art where there is no right or wrong.
- In mathematics some students did not like having to work at it whereas in the arts others did not mind having to do so.
- Mathematics is something that you have to or are made to learn whereas in art students have the freedom to do what they want, if they want.

As mentioned earlier in this thesis, the Nature of Tasks characterization comments for the arts related very heavily to the hands-on actively engaging activities that students experienced in those learning environments. Mathematical tasks were more inclined to be those that involve figuring out and solving. The challenges that **+/+** enjoy as a result of working on equations and problems were positive tasks for these students. In the subsequent sections of this thesis, these last two characterizations will be looked at more closely as a possible link to the differences in students' attitudes and beliefs while learning mathematics versus the arts.

4.7.2.2 Category +/- Comments

These students are confident in their ability to do the individual subject but they do not enjoy learning it.

+/- Mathematics

There were a total of 248 comments made by students in this category.

- Atmosphere/Environment – 17 comments
  - *Learning all the formulas is a big pain.* (322)
  - *... concentration is poor because I don't really enjoy the topic.* (70)

- Emotional Appeal – 44 comments
  - *Mathematics cannot let you have a way to express your feelings.* (144)
  - *I'm good at math but learning is boring and stressful.* (237)
- Impressions of the Subject – 117 comments
  - *I don't really enjoy learning it because sometimes it can get frustrating and confusing/ eventually it all "clicks" and I can do it, but I hate when I learn math and I know the concepts, just not how to apply them to every problem.* (340)
  - *... your answer is either right or wrong/ if wrong you correct yourself until it is right ...* (393)
- Motivation – 123 comments
  - *Math is really boring to learn/ it just isn't fun and you get way too much homework.* (341)
  - *It is boring to sit there and take notes from the board to learn it.* (317)
- Nature of the Tasks – 48 comments
  - *I don't like math because it is dull and all you do is write.* (454)
  - *... I can't stand taking notes and doing 10,000,000 problems at one time.* (584)
- Personalization – 140 comments
  - *I get good grades in math but classes start to get boring and easy.* (615)
  - *I think I have the ability to do math, I just don't like cramming it into my head ...* (651)

- Relevance – 33 comments
  - *I do it because I feel a good grade in maths will benefit me a lot in the future/ I don't enjoy it ….* (1221)
  - *… most of it I won't use out of school.* (107)
  - *… I wonder why we need to know some of the things we learn.(12)*
- Teacher – 31 comments
  - *… very boring and repetitive/ we do the same thing every day.* (21)
  - *… the teacher is really annoying because she does not explain things properly.* (461)

+/- Art

There were a total of 30 comments made by students in this category.

- Atmosphere/Environment – 1 comment
  - *… relaxes you when you sit and draw …* (430)
- Emotional Appeal – 9 comments
  - *… not one of my favourites ! It doesn't have a vibe to it.* (452)
  - *… like to express feelings through art …* (514)
- Impressions of the Subject – 7 comments
  - *Art is something that can't be learned/ art is something that expresses your feelings.* (580)
- Motivation – 19 comments
  - *I like to draw but I don't like learning about it/ I just like being creative.* (749)

- *I dislike learning art ... they do not cover the kind of art I like.* (514)

- Nature of the Tasks – 18 comments
  - *I have always been pretty bad at art for drawing but I'm good at designing things.* (567)
- Personalization – 11 comments
  - *I am very artistic ... I just like to draw what I draw.* (737)
  - *I am an artist ...* (944)
- Relevance – 3 comments
  - *... I don't think art can be taught, it has to come out from the inside.* (331)
- Teacher – 9 comments
  - *... when teacher teaches how to use paint brush and paints, it gets boring ...* (430)
  - *I hate listening to the teacher.* (160)
  - *... don't like to be told what to draw.* (947)
  - *Don't need a teacher for artsy-ness.* (1186)

+/- Music

There were a total of 57 comments made by students in this category.

- Atmosphere/Environment – 4 comments
  - *... it's fun playing and having a good time.* (265)
- Emotional Appeal - 33 comments
  - *I love music but I wouldn't want to learn about it.* (76)
  - *I like to play but when the teacher gets mad at us I hate it.* (162)

- Impressions of the Subject – 16 comments
  - *I don't like to learn music but I like to hear it and play music.* (295)
  - *... have a talent for playing it but I hate practicing.* (1091)
- Motivation – 39 comments
  - *... it's so boring to practice ... I do like to perform in recitals.* (1178)
  - *I like music but I don't want to learn the basics.* (901)
- Nature of the Tasks – 44 comments
  - *I love to sing and play guitar, but I hate and have a hard time reading music ...* (31)
  - *... I would prefer to play and compose more in the lessons which doesn't happen.* (64)
- Personalization – 12 comments
  - *... I like doing things in my own way, not what the sheet of music says.* (101)
- Relevance – 3 comments
  - *Music is useless unless you are amazing at it.* (163)
  - *... put it on my resume and help me in my future career.* (310)
- Teacher – 6 comments
  - *... the teacher doesn't make our work enjoyable and doesn't seem sympathetic when someone has a problem ...* (64)
  - *... I would prefer a more practical approach. (50)*
  - *... some songs the teacher assigns are REALLY stupid and boring ...* (650)

Comparative Discussion: "Confident and Not Enjoy"

The characterizations, ranked by the frequency of the number of comments in each, are listed below. Percentages of the total number of comments that were classified in each characterization to the nearest $\frac{1}{10}$ % are included:

Mathematics **(248)**	***Art*** **(30)**	***Music*** **(57)**
1 Personalization 56.5%	1 Motivation 63.3%	1 Nature of Tasks 77.2%
2 Motivation 49.6%	2 Nature of Tasks 60.0%	2 Motivation 68.4%
3 Impressions of Subject 47.2%	3 Personalization 36.7%	3 Emotional Appeal 57.9%
4 Nature of Tasks 19.4%	4 Emotional Appeal 30.0%	4 Impressions of Subject 28.1%
5 Emotional Appeal 17.7%	4 Teacher 30.0%	5 Personalization 21.1%
6 Relevance 13.3%	6 Impressions of Subject 23.3%	6 Teacher 10.5%
7 Teacher 12.5%	7 Relevance 10.0%	7 Atmosphere/Environment 7.0%
8 Atmosphere/Environment 6.9%	8 Atmosphere/Environment 3.3%	8 Relevance 5.3%

Table 4.8 +/- Rankings and Ratings of Comment Characterizations

Comments are quoted using only the appropriate text for the content being discussed. In comparing these results for the students who are **Confident and Not Enjoy** (+/-), interesting observations can be made:

- The Atmosphere/ Environment percents were similar for all three subjects and the same can be said for its rankings: last for mathematics (6.9%) and art (3.3%) and second last for music (7.0%). Although the percentages for mathematics and music were more than twice those of art, no specific comment regarding the experiences of students while learning any of the subjects was repeated with any significance.

- In the case of Relevance, the percentage for mathematics and its ranking (sixth place, 13.3%) were similar to those of art (seventh place, 10.0%) and about twice the 5.3% percentage for music, which was ranked last for that subject. In the **+/-** category, 6 out of 33 comments in mathematics and 2 out of 3 comments for music acknowledged those subjects as valuable and important to the future of students.

- *Mathematics is very boring to learn but it is good to know it.* (902)
- *I can put it* (music) *on my resume and help me in my future career.* (310)

Surprisingly, in this group of confident mathematics students, quite a number of them (27 out of 33) found the study of mathematics "pointless":

- *... I don't see how certain topics we cover are ever going to apply to real life.* (180)
- *... I don't see the point of learning it if I'm not going to find out the area of a triangle in an emergency.* (1018)

Few students made the same remarks for art or music.

- Comments related to the Teacher were similar for mathematics (seventh place, 12.5%) and music (sixth place, 10.5%), but quite different for art with 30.0% of the comments and a fourth place ranking for that subject. While a few students mentioned positive experiences in mathematics like, *meeting with the teacher for help* (164) and the *teacher tries to make it fun* (1017), these examples conveyed some of the negative views (16 out of 31 comments for mathematics, 5 out of 9 for art and 6 out of 6 for music):

- *I do not like the teacher* (mathematics) *at all.* (102)
- ... *it's the same types of things to learn it* (mathematics). (176)
- ... *teachers make it* (mathematics) *uninteresting* ... (424)
- ... *teacher doesn't explain* (mathematics) ... (191)
- *I like art although I dislike being taught it.* (486)
- ... *I hate listening to the art teacher.* (160)
- ... *I don't like to listen to teachers about it (music).* (219)

- Near the top for all subjects, Motivation ranked first for art with 63.3% of the comments and second for mathematics (49.6%) and music (68.4%). While students clearly had a need to express their views in this area for mathematics, two key areas dominated their views: the repetitive way in which mathematics is taught (12 out of 123 comments) and the boring factor (121 out of 123 comments):
  - ... *we always do* (mathematics) *exercises from the same book.* (48)
  - *I am very good at math but it is a boring subject that doesn't interest me.* (270)
  - *I think math is just boring/ sitting in class and learning new ways to use numbers.* (431)
  - ... *I find it (mathematics) quite boring as there is never a fun way of learning it.* (1021)

  Though not in such a great number, student also found that art (3 out of 19 comments) and music (9 out of 39 comments) both possessed the boring factor:

- *... don't like art lessons because they teach you things that are boring ...* (1020)
- *I sing good, but learning notes and history is boring.* (237)

The last four characterizations ranked and rated very differently across the subjects. Personalization and Emotional Appeal ranked and rated diversely for all three subjects. Nature of Tasks and Impressions of Subject, made the divide between mathematics versus the arts quite clear.

- Personalization, students' expression/ impression of themselves ranked first for mathematics (56.5%), third for art (36.7%) and fifth for music (21.1%). For **+/-** students, in mathematics there was a much greater need to assert their confidence and ability in explaining the way they categorized themselves:
- *Maths comes very easily to me but I don't enjoy it very much/ I would prefer doing something I enjoy but aren't very good at.* (67)
- *I am the best at math but ...*(92)

This did not occur as frequently for the same category of student in art or music. With regard to the expression of self, 2 out 11 students noted that art expresses your feelings and 2 out 140 specifically noted that mathematics did not provide students with the opportunity to express their thoughts and feelings.

- In reversed order to Personalization, Emotional Appeal was third for music (57.9%), fourth for art (30.0%) and fifth for mathematics

(17.7%). For **+/-** arts students, there was a greater need to assert their love/ like of the doing the specific activities of drawing, painting, singing, playing, and listening to music in explaining the way they categorized themselves. The most repeated comments in this characterization conveyed the way the subject made the students feel. In only 1 out of 44 comments was it stated that mathematics made the student feel good. In contradiction, in 16 out of 44 comments words like frustrating, tiring, annoying, confusing were used:

- ➢ *I go to the Gifted Math Program, but it is really annoying to try and learn it.* (185)
- ➢ *... I get so frustrated with the problems that it makes me feel like I don't want to do it anymore because of the frustration.* (325)

No such statements were made by students for art or music.

- Nature of Tasks ranked at the top for music (first place, 77.2%) and art (second place, 60.0%). The physical/ active aspect of students' experiences while learning these two subjects accounts for high rates of responses even if they were not completely positive:
  - ➢ *I can play piano pretty well, but I don't enjoy it at all and would much rather be doing something else.* (136)
  - ➢ *I write music/ I'm learning how to play the guitar/ I don't want to learn about it in class.* (403)
  - ➢ *I like making art not learning about art in the past.* (3)

The Nature of Tasks in mathematics (fourth place, 19.4%) is suggested differently by students:

- *Even though I like to do mathematical exercises, I don't like to learn new things because it involves a lot of listening and repeating.* (56)
- *After learning how to complete a mathematical problem, I usually understand it. I just don't like doing math problems since they take so much time.* (156)

Another theme which is repeated in this characterization was the work that is involved in the learning process. Dislike for tasks like homework, practicing and studying were expressed in 17 out of 48 comments for mathematics, 7 out of 44 for music and not once for art:

- *It* (mathematics) *comes easy, I don't like to study*. (249)
- *... there's so many steps in solving a problem ...* (350)
- *Math is easy but too much homework is given.* (367)
- *I enjoy listening to music and playing music/ I hate to sit down and practice difficult songs.* (685)

- Differences between mathematics and the arts were also evident when Impressions of Subject was examined. Ranked third for mathematics with 47.2% of the comments, fourth for music (28.1%) and sixth for art (23.3%), there were two key areas with high repetitions of comments. The first relates to the more mental/ passive experiences that occur during the learning of mathematics.

This type of comment prevailed in 12 out 117 comments for mathematics:

- *I like doing things but I find it boring when we just sit there and learn.* (546)
- *... I don't like sitting down learning it step by step.* (642)
- *I can do math but I don't like learning it because it's boring and you have to think a lot.* (1065)

"Reading notes" is the equivalent activity to this in music*: I don't like reading music and reading notes. (29*) However, a passive experience like "listening" was anything but negative in learning music:

- *I like to listen to it ...* (818)
- *I like listening and learning about what's going on in music ... (304)*

While 18 out of 117 comments reported that mathematics was easy, 55 out 117 used words like <u>hard, tedious</u> and <u>complicated</u>. By comparison, only 1 out of 7 and 9 out of 16 used those words to describe art and music respectively:

- *... it (mathematics) is quite hard too, and it's also hard to learn it all as there is lots to learn.* (61)
- *... too linear and technical/ a technical error can cause the entire* (mathematics) *problem to be incorrect ...* (122)
- *... I guess I'm kind of lazy, but I just hate taking the time to do the* (mathematics) *work.* (738)
- *It's very hard to read and understand music/ it's like learning a different language.* (695)

+/- Summary

While the characterizations of Atmosphere/ Environment, Emotional Appeal, Motivation, Personalization, Relevance, and Teacher are not as consistently similar for mathematics, art and music as they were for the **+/+** category, once again Nature of Tasks and Impression of Subject characterizations give reason to suspect that these are the areas which differentiate the attitudes and beliefs in learning mathematics compared to the arts for the **Confident and Not Enjoy** students.

Atmosphere/ Environment, Motivation, and Relevance rate and rank similarly for mathematics, art and music. Atmosphere/ Environment and Relevance rank toward the bottom indicating that, for the **+/-** student, the necessity, usefulness and value of the subject and their experiences while learning the subject are not as noteworthy to explain their lack of enjoyment of these subjects as other characterizations. Motivation is ranked at the top for mathematics, art and music. This signifies that for the **Confident and Not Enjoy** students their enthusiasm for learning and those factors making them want or not want to learn are highly important to explain their attitudes in all three subjects. Lack of interest, being bored and the repetitiveness of daily lessons in mathematics were dominant themes students expressed as reasons for their lack of enjoyment of learning. In art and music, much of a students' motivation stemmed from the opportunities presented to them to *do* and *create* as expressed in: *Not fun/ I used to take violin/ I didn't like learning/ I just like playing*. (333) and *I like creating art and drawing pictures for people but I don't like learning it.* (909)

As suggested earlier, the rankings and ratings for Emotional Appeal and Personalization are distinct for each of the subjects. The actual feelings that students experience in the learning of mathematics outweigh any moments of possible enjoyment: *I can do math and all the calculations but at times the numbers, there* (are) *so many and they become annoying. (1131)* Student emotions in music and art are tied to an act of "doing": *I'm in band and I hate it ... I love to sing.* (271) and *I am artistic, but I like to keep it to myself/ when I show my art in front of people I feel like I will mess up.* (720)

The last three characterizations, Teacher, Nature of Tasks, and Impressions of Subject, pair two subjects against one in their similarities of ranking and rating. Student comments about Teacher are similar for mathematics and music compared to art. In content, however, the comments for mathematics deal with the teacher personally and the way in which the lessons were presented. Those for art and music related to the students' desires to do their own thing as opposed to being told what to do.

In both Nature of Tasks and Impressions of Subject the arts were united together and separated from mathematics in rankings and ratings. The high percentage of comments and ranking (first place, 77.2% for music and second place, 60.0% for art) devoted to the Nature of Tasks in the arts is quite dramatic when compared to the fourth place, 19.4% of the comments for mathematics! Once again, this difference can be attributed to the hands-on actively engaging tasks that seem commonplace in the arts classroom, but not in the mathematics classroom. Impressions of Subject ranked third for mathematics with 46.8% of the comments as opposed to the similar 28.1% of music comments and 23.3% of art comments. The passive engagement approach to learning paired with students' conceptions of daily tedium in their mathematics lessons contribute greatly to the "do not enjoy" part for this group of students "confident in mathematics". While not with any great frequency, other themes also ran through the Impressions of Subject comments:

- Creativity and imagination in art versus the structure of mathematics.
- Mathematics it is either right or wrong which is not true for art.
- Mathematics is a course which students are forced to take and the arts are optional for students.

Overall, the comments of **+/-** students conveyed here with regard to their learning in mathematics are quite eye opening for the mathematics teacher who has not thought this scenario possible! What can we do to change the attitudes and beliefs of these students and how can we have been so blind? In the words of a student respondent to this questionnaire, identified here as #451: *My teacher thinks I enjoy it.*

4.7.2.3 Category -/+ Comments

These students are not confident in their ability to do the individual subject but they do enjoy learning it.

-/+ Mathematics

There were a total of 40 comments made by students in this category.

- Atmosphere/Environment - 1 comment
  - *When I try to pay attention everything seems confusing.* (978)

- Emotional Appeal – 8 comments
  - *... learning something new boosts my confidence and makes me feel good ...* (1212)
  - *... don't understand math but it doesn't really bother me.* (1064)
- Impressions of the Subject – 13 comments
  - *... people with a logical way of thinking are good at maths ...* (1199)
  - *I do like math but it is hard to learn. (796)*
- Motivation – 8 comments
  - *... now I'm understanding some things I didn't before and it's sort of fun to be able to do things ...* (437)
  - *I don't really understand math a lot but I do like learning it because it's exciting.* (811)
- Nature of the Tasks – 2 comments
  - *I like learning equations, I just can't remember them.* (219)
- Personalization – 32 comments
  - *I do like math but I have a hard time learning it.* (898)
  - *... it doesn't come easy when I apply it.* (31)
  - *I'm not as good in mathematics, but I'm willing to try.* (884)
- Relevance – 3 comments
  - *I like learning maths to help me in my jobs.* (537)
  - *... I acknowledge how important it is.* (543)
- Teacher – 2 comments
  - *... depending on my teacher/ a good teacher is patient, understanding and isn't all serious ...* (337)
  - *... teacher makes it fun ...* (942)

-/+ Art

There were a total of 129 comments made by students in this category.

- Atmosphere/Environment – 4 comments
  - ➢ *Fun and relaxing.* (45)
  - ➢ *... relaxed environment with a lot of freedom to do what you want to, rather than doing very specific tasks (i.e. like in math).* (574)
- Emotional Appeal – 7 comments
  - ➢ *... it's an easy way to help express feelings.* (652)
  - ➢ *I'm not so good at art but I like art class.* (289)
- Impressions of the Subject – 16 comments
  - ➢ *Art doesn't require much skill/ you get to be creative and colourful/ there's no right or wrong.* (394)
  - ➢ *... I have trouble coming up with original ideas at times.* (591)
- Motivation – 35 comments
  - ➢ *I can't draw but it's fun to try and learn how to do it.* (608)
  - ➢ *... I like learning about artists and different things about art/ sometimes I think it's boring and won't participate.* (893)
- Nature of the Tasks – 52 comments
  - ➢ *... but the pictures, painting are nice.* (832)
  - ➢ *... I always enjoy working on art projects ...* (1084)
  - ➢ *I like making things and using art materials to paint and draw but I can't do art.* (1210)
- Personalization – 65 comments
  - ➢ *... I like it but I need some improvement.* (150)
  - ➢ *... I'm not very good but that doesn't mean I don't like it.* (425)

- Relevance – 10 comments
  - *... like to learn how to draw better.* (256)
  - *I can't draw, but strive to be a great artist.* (778)
- Teacher – 0 comments
  - (No student comments with this characterization)

-/+ Music

There were a total of 76 comments made by students in this category.

- Atmosphere/Environment – 3 comments
  - *... one of the greatest things/ relaxing just to sit and listen ...* (399)
- Emotional Appeal – 18 comments
  - *... it makes me feel better when I listen to it or play it.* (939)
  - *... I would be happy if I could play an instrument.* (153)
- Impressions of the Subject - 5 comments
  - *... easy to learn.* (445)
  - *... I like the class as a change in the more studious classes like math ... a lot more fun to learn than math.* (88)
- Motivation – 22 comments
  - *... I think that learning it is pretty fun ...* (322)
  - *I do like learning it and playing on the instruments and trying out new things ...* (515)
  - *I really want to learn music ... because it's fun and I really want to play instruments.* (938)
- Nature of the Tasks – 36 comments
  - *I do like listening to music.* (788)

- *I like to sing music and learn the lyrics ...* (291)
- *... I enjoyed composing and playing musical pieces.* (471)

- Personalization – 52 comments
  - *I am not good at all in music but I am teaching myself the electric guitar.* (1222)
- Relevance – 17 comments
  - *... I dance and am learning to sing/in the future I want to take a course in music theory.* (414)
  - *... someday I would like to be like Usher or at least learn his moves ... so please can you help me to become like ...*(783)
  - *... I do try to learn it as a hobby.* (1109)
- Teacher - 0 comments
  - (No student comments with this characterization)

Comparative Discussion: "Not Confident and Enjoy"

The characterizations, ranked by the frequency of the number of comments in each, are listed below. Percentages of the total number of comments that were classified in each characterization to the nearest $\frac{1}{10}$ % are included:

Mathematics **(40)**	***Art*** **(129)**	***Music*** **(76)**
1 Personalization 80.0%	1 Personalization 50.1%	1 Personalization 68.4%
2 Impressions of Subject 32.5%	2 Nature of Tasks 40.3%	2 Nature of Tasks 47.4%
3 Motivation 20.0%	3 Motivation 27.1%	3 Motivation 28.9%
3 Emotional Appeal 20.0%	4 Impressions of Subject 12.4%	4 Emotional Appeal 23.7%
5 Relevance 7.5%	5 Relevance 7.6%	5 Relevance 22.4%
6 Nature of Tasks 5.0%	6 Emotional Appeal 5.4%	5 Impressions of Subject 6.6%
6 Teacher 5.0%	7 Atmosphere/Environment 3.1%	7 Atmosphere/Environment 3.9%
8 Atmosphere/Environment 2.5%	8 Teacher 0.0%	8 Teacher 0.0%

Table 4.9 -/+ Rankings and Ratings of Comment Characterizations

Comments are quoted using only the appropriate text for the content being discussed. In comparing these results for the students who are **Not Confident and Enjoy** (**-/+**), interesting observations can be made:

- For the "Not Confident and Enjoy" students the least significant of the characterizations were Atmosphere/ Environment and Teacher. Ranked seventh for art and music and eighth for mathematics with very similar percentages for all three subjects was Atmosphere/ Environment. A relaxed and comfortable environment was listed in 4 out of 129 comments for art and 2 out of 76 comments for music. In mathematics students were far from relaxed and comfortable, instead they experienced confusion.

- The Teacher characterization was ranked eighth for art and music. No specific comments were made regarding the Teacher in these subjects which suggested that for the **-/+** arts students there were other factors which influenced their enjoyment. The sixth place, 2 out of 40 comments for this characterization for mathematics offered suggestions for how the Teacher could provide an enjoyable experience for this group despite their lack of confidence in mathematics:
    - *... our teacher makes it fun ...* (942)
    - *... a good teacher is patient, understanding and isn't all serious, no fun ... depending on how it is explained to me ...* (337)

- Ranked in fifth place for all three subjects was Relevance with similar percentages for mathematics (7.5%) and art (7.6%) and about three times that rate (22.4%) for music. Students shared their views on the importance of these subjects and their applications in their futures. More students in music expressed their plan/desire to learn how to play a musical instrument.

- Personalization ranked first for all three subjects. For mathematics, student responses repeated experiences such as difficulties in understanding and remembering (14 out of 40 comments), not being able to get it and needing more time. Despite their lack of confidence, students commented on their willingness to try in 10 out of 40 comments for mathematics, 23 out of 129 for art and 10 out of 76 for music:
    - *Math is difficult but I try real hard in it ...* (342)
    - *... I try my best to learn and understand* (mathematics). (816)
    - *... I enjoy having a go and trying to draw.* (466)
    - *Although I can't do music ... you tried your best, they can't ask for anything more.* (520)

- Motivation was ranked in third place for all subjects with very similar percentages. The most repeated comment for this characterization was interesting for all three subjects with 4 out of 40 comments for mathematics, 10 out of 129 for art and 4 out of 76 for music. Liking to learn new techniques was mentioned in 7 out of 129 comments for art and 4 out of 76 for music. Boring was mentioned only once for art and music. The fun factor was described 20 out of 129 comments for art, 10 out of 76 for music, and 5 out of 40 for mathematics.

- The responses by students of fun in the **-/+** mathematics group was closely followed each time with an emotional factor:
  - ... *now I'm understanding something I didn't before and it's sort of fun to be able to do things.* (437)
  - ... *when I can do something hard, that no one else can do, I feel good.* (510)
  - ... *it* (mathematics) *is hard for me to learn, but when I do understand, I feel accomplished.* (691)

  The Emotional Appeal characterization was ranked and rated similarly for mathematics (third place, 20.0%) and music (fourth place, 23.7%) but very differently for art (sixth place, 5.4%). Other responses that appeared shared different views: expression of thoughts and feelings in art and nervousness and confusion in mathematics.

- For the Nature of Tasks, which ranked second for art and music with similar percentages, students felt that even though they were not good in drawing or playing they still loved to do so (36 out of 129 comments for art, 31 out of 76 for music):
  - ... *it seems interesting how to mix colours and to paint.* (212)
  - ... *I always love to sketch things and see how the outcome will be/ I love to make models ...* (359)

- *... just because you are not good at something it doesn't mean you can't enjoy or do the subject.* (1027)
- *I am not musical but I would like to learn how to do beats.* (821)
- *I'm not very good at music and do not play any instruments ... it is very fun to compose ...* (1211)

In the 2 out of 40 comments in mathematics, for which Nature of Tasks ranked sixth with 5.0% of the comments, students liked learning and solving equations and problems.

- Students' Impressions of Subject in this category ranked second for mathematics (32.5%), fourth for art (12.4%) and fifth for music (6.6%). The views of mathematics as hard (10 out of 40 comments), tedious, and something that you have to work at create a different picture when compared to the arts where students used phrases like creative (9 out of 129 comments), easy and no right or wrong. An important view regarding students' confidence versus enjoyment was repeated for art in 9 out of 129 comments and not at all for mathematics or music:
  - *... you don't have to be good at drawing to be good at art.* (481)
  - *I'm not really good at it but I like stuff having to do with art ...* (879)

-/+ Summary

In looking to determine the key issues which unite and separate the attitudes and beliefs of the "Not Confident and Enjoy" student, it is interesting to note that Atmosphere/ Environment, Emotional Appeal, Motivation, Personalization, and Teacher all rank and rate similarly for mathematics, art and music. *Relevance* ties the views of students for mathematics and art together as opposed to those of the music students. The characterizations that differentiate the learning experiences of **-/+** students in mathematics versus the arts are Nature of Tasks and Impressions of Subject.

What observations can be made regarding the characterizations of similar ranking and rating? Atmosphere/ Environment and Teacher are at the bottom of the lists and therefore provide very little significance in explaining why students with a lack of confidence would enjoy the subject. Emotional Appeal and Motivation ranked third provide somewhat of an insight into the reasons for the enjoyment of learning these subjects for "Not Confident and Enjoy" group. Students seemed to be rather mellow about their experiences: *I don't like maths, but I don't mind learning it.* (908), *I like learning music but I am finding that I don't do as well as others.* (661), and *I am not artistic but I like art because I can express my feelings without words.* (955) Relevance was ranked fifth for all three subjects but, by percentages of comments, almost three times as many were made for music compared to mathematics and art. While not a strong rationale for enjoyment, the value and importance of each subject do play a role for some of this group. Personalization was first for all three subjects with mathematics at 80.0% having the highest rate. In art (50.1%) and music (68.4%), students' impressions and expressions of themselves conveyed their thoughts of being able to improve in the aspect of doing/ performing: *... I can learn it probably better then I can use my skills to do projects in* *art.* (238) and *I am not musical but I want to be and I think learning it is fun.* (646) Many characterizations are expressed in this one student's personalized view of mathematics: ... *I am very under confident with my maths and I don't like being put on the spot/ I take longer to learn things. Maths is a very interesting subject and I enjoy getting things right and understanding them.* (493)

In both Nature of Tasks and Impressions of Subject the arts are united together and separated from mathematics. With more than 40% of the comments for each in art and music, the second place rankings for Nature of Tasks are quite dramatically different from its sixth place (5.0% of the comments) for mathematics. The hands-on actively engaging tasks of the arts have a major impact on student enjoyment. The reverse significance is true for Impressions of the Subject. In second place for mathematics with 32.5% of the comments, it ranks and rates quite differently for art (fourth place, 12.4%) and music (fifth place, 6.6%). Some of the same thoughts expressed in the comments of the **+/+** and **+/-** categories of students were found again: creativity and no right or wrong in art and the required mathematics course. Related to this optional idea of taking an arts course, some art students articulated their difficulties in learning art: *I'm interested in art and when I learned it I did like it/ but I found it really hard to do so I didn't continue learning it and I also didn't think art would benefit me in the future.* (1197) and *Art class is usually a goof off class and I like to draw ... you don't have to work in art class.* (1167) What seemed to be conveyed in the attitudes of **-/+** mathematics students is their willingness to work and do what is necessary to perform better in mathematics despite any difficulties they may experience while learning it: *I think that I am not incapable but capable of learning simple things in maths, but when it gets complicated I tend to stick to it and try but I might not be any good at it.* (501) Might their views be different if mathematics was an optional course?

4.7.2.4 Category -/- Comments

These students are not confident in their ability to do the individual subject and they do not enjoy learning it.

-/- Mathematics

There were a total of 49 comments made by students in this category.

- Atmosphere/Environment – 4 comments
  - ➢ *It's too confusing and I can't do it …* (223)
- Emotional Appeal – 25 comments
  - ➢ *I hate math.* (152)
  - ➢ *… I go to math I get so stressed out and feel stupid …* (354)
- Impressions of the Subject – 25 comments
  - ➢ *I'm not good at math. It's usually hard to understand.* (281)
  - ➢ *… it takes lots of time and effort and understanding/ way too many things to learn …* (242)
- Motivation – 8 comments
  - ➢ *I hate maths and find it unrewarding and boring …* (1022)
  - ➢ *… not interesting … why it's hard to pay attention and learn.* (178)
- Nature of the Tasks – 0 comments
  - ➢ (No student comments with this characterization)

- Personalization – 32 comments
  - *... I don't like learning about something that I don't understand.* (19)
  - *I struggle in math and I hate working with numbers.* (755)
- Relevance – 5 comments
  - *... don't understand why we have to take it.* (13)
  - *... don't know how we're going to use the things we learn ...* (178)
  - *A lot of the stuff we learn does not seem relevant.* (230)
- Teacher – 3 comments
  - *... teacher pays no attention to what anyone says ... when they need help ...* (1022)
  - *Hard ... can only learn when the teacher makes it interesting* ...(201)

-/- Art

There were a total of 68 comments made by students in this category.

- Atmosphere/Environment – 2 comments
  - *... I enjoy looking at other people's work.* (1212)
- Emotional Appeal – 16 comments
  - *I extremely hate drawing art ...* (1060)
  - *Not being able to do something really frustrates me and I feel like I can't do it anymore.* (214)
- Impressions of the Subject – 8 comments
  - *... art just seems too hard.* (305)
  - *... to get a good grade in art you have to have a special gift.* (74)

- Motivation – 26 comments
  - ➢ *... don't want to spend my time trying to do something I can't do.* (10)
  - ➢ *... art is boring/ it just does not and never has interested me.* (137)
- Nature of the Tasks – 23 comments
  - ➢ *I cannot draw and I don't want to draw.* (499)
  - ➢ *I quite like drawing but I'm not good at it.* (55)
- Personalization – 44 comments
  - ➢ *... bad at drawing and worse than my other classmates ...* (308)
  - ➢ *I just can't use my hands to create art...* (22)
- Relevance – 8 comments
  - ➢ *I don't feel confident enough to pursue this area ...* (17)
  - ➢ *... I just take it for the credit.* (213)
  - ➢ *... complete waste of my time ...* (483)
- Teacher – 1 comment

➢ *... we never seemed to learn how to do it ...* (483)

-/- Music

There were a total of 62 comments made by students in this category.

- Atmosphere/Environment – 0 comments
  - ➢ (No student comments with this characterization)
- Emotional Appeal – 18 comments
  - ➢ *I don't take music because I hate it.* (213)
- Impressions of the Subject – 11 comments
  - ➢ *... it is hard to learn and takes a lot of time.* (669)

- ... *learning so many notes and all of that.* (611)

- Motivation – 22 comments
  - ... *I got bored with the lessons.* (765)
  - ... *boring to learn because it's simply not interesting to me.* (393)
- Nature of the Tasks – 24 comments
  - ... *like listening to it ... couldn't perform a piece to save my life.* (215)
  - *I cannot sing very well ... don't enjoy learning music ... just like to sing ...* (601)
- Personalization – 15 comments
  - *I have a good ear for music ... I love music not piano, guitar ...* (984)
  - ... *I am a big fan of music but I can't play music which makes me not want to learn it.* (1118)
- Relevance – 4 comments
  - *I don't like to be a musician.* (944)
  - *Music is a complete waste of time to have in school ...* (362)
- Teacher – 1 comment
  - ... *I really don't like being taught it as I can' t sing or play an instrument.* (544)

Comparative Discussion: "Not Confident and Not Enjoy"

The characterizations, ranked by the frequency of the number of comments in each, are listed below. Percentages of the total number of comments that were classified in each characterization to the nearest $\frac{1}{10}$ % are included:

Mathematics (49)	*Art* (68)	*Music* (62)
1 Personalization 65.3%	1 Personalization 64.7%	1 Nature of Tasks 38.7%
2 Impressions of Subject 51.0%	2 Motivation 38.2%	2 Motivation 35.5%
2 Emotional Appeal 51.0%	3 Nature of Tasks 33.8%	3 Emotional Appeal 29.0%
4 Motivation 16.3%	4 Emotional Appeal 23.5%	4 Personalization 24.2%
5 Relevance 10.2%	5 Impressions of Subject 11.8%	5 Impressions of Subject 17.7%
6 Atmosphere/Environment 8.2%	5 Relevance 11.8%	6 Relevance 6.5%
7 Teacher 6.1%	7 Atmosphere/Environment 2.9%	7 Teacher 1.6%
8 Nature of Tasks 0.0%	8 Teacher 1.5%	8 Atmosphere/Environment 0.0%

Table 4.10 -/- Rankings and Ratings of Comment Characterizations

Comments are quoted using only the appropriate text for the content being discussed. In comparing these results for the students who are **Not Confident and Not Enjoy** (-/-), interesting observations can be made:

- Teacher had little significance for these **-/-** students, ranking second to last for both mathematics (6.1%) and music (1.6%) and last for art (1.5%). The comments all had negative connotations. Those made for mathematics all are directly related to the teacher's behaviour during lessons: *Maths is confusing/ when the teacher doesn't explain the problems' answers, it's kind of hard to learn it.* (113) In the arts the comments were geared toward the presentation of the lesson.

- Atmosphere/ Environment also ranked low for students: mathematics (sixth, 8.2% of the comments), art (seventh place, 2.9%), music (last place, 0.0%). As for Teacher, the mathematics comments in this characterization occurred at a higher rate. Confusion and/ or frustration were mentioned by students in all four mathematics comments.

The remaining characterizations for the "Not Confident and Not Enjoy" were similarly ranked and rated for two subjects versus one. Personalization and Relevance paired up mathematics and art versus music. Interestingly, in relationship to this thesis, Emotional Appeal, Motivation, Nature of Tasks and Impressions of Subject all paired up the arts compared to mathematics.

- Relevance was ranked in fifth place for both mathematics and art with almost twice as high a rate as for music which was in sixth place. There were no positive views of importance, value and necessity expressed for learning any of the subjects. Instead, the students repeated comments which conveyed an opposite view: *I find math very difficult/ I don't understand how I will ever use most of the information in my daily life.* (398) appeared in 5 out of 5 comments; ... *I don't really have any reason to learn art either.* (1108) appeared in 5 out of 8 comments. In addition, some of these **-/-** students referred to learning these subjects as a *waste of time*: ... *don't want to spend my time trying to do* (music) *something I can't do.* (10)

- Personalization ranked first for both mathematics and art both with close to 65% of their respective comments. This is quite dramatic compared to the fourth place music (24.2% of its comments). Students spoke of their own lack of ability as a primary reason for the lack of their enjoyment more emphatically in mathematics and art: *I* *can't stand math/ I'm not good at it and I never have been good.* (216) and ... *when I'm not good at something I usually don't like learning it* (art). (97) These same sentiments were expressed for music, but those students, even if they possessed no musical talent, could appreciate the subject in another way: ... *I like listening to music of my choice at home.* (524)

- Motivation ranked second with similar rates in art and music. For mathematics, it was fourth with less than half the percentage of comments. The separation of mathematics compared to the arts for -/- students in Motivation was more markedly seen by the repeated expression in this category of the terms boring and uninteresting and not just for mathematics:
    - Mathematics – 8 comments: boring 5, uninteresting 2
    - Art – 26 comments: boring 16, uninteresting 11
    - Music – 22 comments: boring 9, uninteresting 9

  These descriptors were not used for the arts with this much frequency in any of the other categories.

- The same sort of issue occurred with a comment involving Emotional Appeal which was rarely seen even for mathematics in other categories. This characterization ranked second for mathematics (51.0%), third for music (29.0%) and fourth for art (23.5%). The percentage of comments is quite different for mathematics versus the arts. In all three subjects, the students emphatically used the word hate when describing how much they liked/ disliked the subjects:

- Mathematics – 8 out of 25 comments
- Art – 10 out of 16 comments
- Music – 9 out of 18 comments

In addition, the mathematics students included other emotional factors in their responses: *Math is really hard for me and when I try to learn I get more frustrated* (956) and *I don't like math and find learning it stressful* (959).

- There were no Nature of Tasks comments made for mathematics, hence its last place ranking. For music and art it was ranked in first and third places respectively with very similar percentages. All of the art comments and nearly all of the music comments mentioned actively engaging hands-on tasks which they didn't like doing because of their lack of ability: *I don't really like drawing because I do not have a steady hand.* (669) and *I don't play an instrument and I tried to learn but it is not something I'm interested in.* (1110)

- Ranked second for mathematics (51.0% of the comments) and fifth for both art (11.8%) and music (17.7%) was Impressions of Subject. The vastly different percentages of comments for mathematics versus the arts indicate that **-/-** students' perceptions of mathematics (more than half of the comments) are much more significant in effect on their enjoyment of maths than the **-/-** students' perceptions of the arts are on their enjoyment of the arts. In addition to complicated with one right answer, 22 out 25 comments expressed how hard and difficult mathematics is. By comparison, 3 out of 8 **-/-** art students and 6 out of 11 **-/-** music students felt that those subjects were <u>hard</u> and <u>difficult.</u>

-/- Summary

In summarizing the responses of the "Not Confident and Not Enjoy" students, the negative aspects of each characterization across mathematics, art and music jump out at the reader. It is the factor that unites these students even when specific characterizations separate them.

According to these results, the characterizations of <u>Atmosphere/ Environment</u> and <u>Teacher</u>, with their low rankings, and even <u>Relevance</u> (in fifth/ sixth place), with similar percentages to the other two, have the least impact on the enjoyment for the **-/-** student in mathematics or the arts.

In examining the top two places for all subjects, there are no totally common characterizations. The first place ranking of <u>Personalization</u> for mathematics and art indicates that for these students there is a strong connection between themselves and their lack of abilities, achievement, lack of enjoyment and learning in mathematics and art: *I'm not very good at maths and I don't enjoy learning it.* (513) and *I'm not artistic, so it's no fun to do art.* (163) <u>Personalization</u> ranked fourth for music, a middle of the road response. For these students, ability and achievement were not as significant as the aspects of its first place <u>Nature of Tasks</u>. Despite a lack of ability and confidence in music, **-/-** students suggested a different factor that influenced their enjoyment: *I don't like learning music, but I like listening to it.* (309) In second place for art and music, <u>Motivation</u> had more than one third of the comments for each. For the "Not Confident and Not Enjoy" students, there was little enthusiasm for learning art and music. In second place for mathematics were Emotional Appeal and Impressions of Subject. Clearly, for this group, aspects of how the subject made the students feel (... *feel really bad when I can't do it as well as most or some kids* ... (323)) , level of difficulty, time required to learn, tedium and one right answer overshadowed their willingness to persevere and enjoy the learning mathematics.

4.7.3 Summary of Student Comments

"Following the arguments" of students in their own words, a picture begins to emerge of the categories under investigation in this research based on student confidence and enjoyment in the subject of mathematics, art and music. What are the issues and factors that affect confidence and enjoyment of students?

The students who are "Confident and Enjoy" take pride and satisfaction in their abilities and in the work that they do whether that involves the challenges of solving problems, the creativity of producing a piece of art/ music expressing their thoughts and feelings, or presenting that work to others. The subject comes easily to them, seldom causing any frustration in the learning or understanding of it. **+/+** students love being exposed to new ideas and techniques. Their motivation comes from the nature of the subject and within themselves. They feel a sense of happiness and achievement when they have completed a job well done. Many of them see themselves as continuing in a career related to the subject and value the subject as a tool for the improvement of their skills. The subject seems to be a part of their being and comes naturally to them in unique ways: a fascination with numbers, puzzles and the sense of one right answer versus the act of making something distinctly their own by hand in which there is no right or wrong. **+/+** mathematics students love to do the mathematics. They are intrigued by numbers, formulas and figuring things out. A good teacher to them is one who explains well. **+/+** arts students adore the creativity and freedom these subjects allow them to experience. They like learning about arts in the past, but their greatest pleasure comes from feelings and emotions rather than thinking.

The students who are "Confident and Not Enjoy" are bored by learning the subject either because of the repetitive way in which it is taught, the easiness of the subject for them or by being told what to do and how to do it. They see little value in the learning of the subject for their lives, present and future. **+/-** students find it difficult to be motivated. They view the subjects as something than cannot be taught or that the learning of it requires more time and energy than they are willing to spend. **+/-** mathematics students find no satisfaction in the *"too much"* ... work, time, steps in process, problems ... that, to them, mathematics entails. **+/-** arts students find dissatisfaction when learning takes away their independence to do things their own way.

The students who are "Not Confident and Enjoy" recognize their lack of ability in the subject and are willing to try their best to improve their skills. Despite negative experiences related to their difficulties in the subject, they strive for success and the feelings of pride and satisfaction when they achieve that goal. **-/+** students value the subject as essential to their future jobs, careers and avocations. They are motivated by learning different ideas and techniques and the desire to do better. They find fun in the learning process either through a teacher's efforts, in performing the tasks or having opportunities to express themselves. **-/+** mathematics students feel nervous and dislike being put on the spot. Even if they don't always understand, they have an optimistic outlook on the outcomes of their efforts. To them, a good teacher is patient. **-/+** arts students feel that anyone can learn these subjects. To them, the arts are relaxing and they find it amazing to create.

The students who are "Not Confident and Not Enjoy" can see little reason to spend any time and effort doing something in which they do not perform well. They "hate" their lessons. Their negative experiences and emotions prevent them from being motivated to enjoy learning. **-/-** students are bored and disinterested. They find no relevance to their lives, feeling that the subjects are a complete waste of time in school. They find it hard to pay attention and become frustrated when they try. **-/-** mathematics and arts students were consistent in their views of learning.

In comparing the rankings of the comment characterizations across all categories within each subject separately, similarities can be found.

CHAR / CAT	Atmosphere/ Environment	Emotional Appeal	Impression of Subject	Motivation	Nature of Tasks	Personalization	Relevance	Teacher
+/+	7	3	2	3	5	1	6	7
+/-	8	5	3	2	4	1	6	7
-/+	8	3	2	3	6	1	5	6
-/-	6	2	2	4	8	1	5	7
AVG Rank	7.25	3.25	2.25	3	5.75	1	5.5	6.75

Table 4.11 Comment Characterization Rankings By Category – Mathematics

The top ranking characterizations for mathematics are Personalization and Impressions of Subject. This indicates that for mathematics, the students' impressions of themselves and the subject of mathematics are most important in understanding the relationship between their confidence and enjoyment.

CHAR / CAT	Atmosphere/ Environment	Emotional Appeal	Impression of Subject	Motivation	Nature of Tasks	Personalization	Relevance	Teacher
+/+	7	3	5	3	2	1	6	8
+/-	8	4	6	1	2	3	7	4
-/+	7	6	4	3	2	1	5	8
-/-	7	4	5	2	3	1	5	8
AVG Rank	7.25	4.25	5	2.25	2.25	1.5	5.75	7

Table 4.12 Comment Characterization Rankings By Category – Art

The top ranking characterizations for art are Personalization and Nature of Tasks. Although Motivation ties in average rank with Nature of Tasks, by actual rankings Nature of Tasks is second for all categories but **-/-**. This indicates that for art, the students' impressions of themselves and the activities involved in the learning of art are most important in understanding the relationship between their confidence and enjoyment.

CHAR / CAT	Atmosphere/ Environment	Emotional Appeal	Impression of Subject	Motivation	Nature of Tasks	Personalization	Relevance	Teacher
+/+	7	4	5	3	1	2	5	8
+/-	7	3	4	2	1	5	8	6
-/+	7	4	5	3	2	1	5	8
-/-	8	3	5	2	1	4	6	7
AVG Rank	7.25	3.5	4.75	2.5	1.25	3	6	7.25

Table 4.13 Comment Characterization Rankings By Category - Music

The top ranking characterizations for music are Nature of Tasks and Motivation. This indicates that for music, the activities involved in the learning of music and the factors that make the students want/ not want to learn are most important in understanding the relationship between their confidence and enjoyment. Personalization averages in a third place ranking.

Even across categories, it is clear that the key factors affecting students' confidence and enjoyment are Nature of Tasks and Impressions of Subject. In the arts, Motivation also provides some important reasons. It is at this point that I can suggest answers to the questions posed earlier in this chapter:

Why don't students like learning math when they are good at it? Students who are confident but do not enjoy mathematics believe that it is boring, tedious, complicated, hard, confusing and frustrating. They see mathematics lessons as repetitive ... endless days of sitting back, taking notes and doing problems/ exercises from the board/ book. **+/-** students place no value and relevance in the study of mathematics. They believe it is a fun-less compulsory course requiring work and time without any rewards: *Math is tedious and half of it I will never use in my life after college so we are being taught what we don't need to know which to me is a waste of my time and my teacher's time ... I think we should be given the option of math based on the career we are going to pursue.* (379)

<u>What makes students who don't perform well in mathematics still like learning it?</u> Students who are not confident in mathematics but enjoy learning it take extreme pleasure in tackling the difficult study of mathematics and overcoming obstacles to succeed. When they can do something that previously they or others found hard or impossible, their sense of accomplishment and happiness is well worth the effort. Learning something boosts their confidence and they feel good. Understanding is fun for these students and they are willing to try.

<u>What can we learn from students' attitudes in art and music that can improve the teaching and learning of mathematics and help foster a positive attitude toward learning mathematics in all students?</u> Comments expressed by **+/-** students conveyed a mainly negative attitude toward the learning of mathematics: *My school is based on math and science. I am very able to do math, but I don't enjoy it. It is useful to life but not very interesting to me. Maths is a subject that few people enjoy, but it is required so we deal with it.* (1188) Students should not be allowed to feel that they just have to

"deal with it"! It is certainly clear from the data collected in this research that not all subjects will be enjoyed by all students but, ... to find that ***42.6%*** of students categorized themselves as ***"Confident and Not Enjoy"*** in maths is unconscionable: *I have always been able to do math without having to study ever and I don't pay good attention in class. Even though I'm good at math, I think it's abominable. It's not fun and it's not supposed to be.* (597) It's hard to read that there is a student with this view of mathematics and not try to do something about improving that view.

Strauss and Corbin (1990, p.59) suggest that the researcher *"should be asking questions all along the course of your research project".* According to what has been discovered thus far, several factors differentiate the learning experiences students have in the arts versus their learning in mathematics. There are also several similarities. Students do not like studying, practicing or working in music or art in much the same way as they don't in mathematics. There are, however, experiences taking place during the learning of those subjects that provide a balance and enhance their enjoyment. Novelty in the daily routine in the mathematics classroom and the acknowledging that mathematics can be creative are two suggestions. Students who enter their mathematics class each day expecting the unexpected will be less inclined to sit back and not pay attention to the lesson. A discovery approach can supply "aha" moments and promote the freedom to think and act independently that arts students describe.

In addition, according to the data from these questionnaires, students in art and music are provided with something to do with the skills they have learned and what sets the enjoyment of mathematics so far afield from that of the arts are the actively engaging hands-on experiences. From the comments of students regarding Nature of Tasks and Impressions of Subject, it is in these characterizations wherein the divide between mathematics and the arts exists. Art and music actively involve students in learning. Students enjoy, for example, making, doing, composing and creating in the arts. When their lack of ability prevents them from enjoying the doing in music, a mental/passive aspect, listening, gives them enjoyment. Mathematics is viewed as a totally mental/ passive subject by students. Providing them with something to do with the mathematics which they have learned make available options, inherent in the arts, to allow for greater student enjoyment of mathematics.

The results of the analysis of the questionnaire have provided much food for thought in the understanding of students' attitudes and beliefs toward the learning of mathematics compared to the arts ... but, have all of the questions been answered and are there more questions to ask? Strauss and Corbin (1990, p. 95) caution the researcher:

> *"Each of us brings to the analysis of data our biases, assumptions, patterns of thinking, and knowledge gained from experience and reading. These can block our seeing what is significant in the data ...* (The researcher must) ... ***Periodically step back and ask****: What is going on here? Does what I think I see fit the reality of the data?"* (Strauss and Corbin, 1990, p. 44)

It was at this point that I needed to "step back and ask" exactly "what is going on here?" and probe more deeply, with a smaller number of students, into these findings. In the following chapters, the results of repertory grid interviews conducted with a select group of students who had participated in the questionnaire are described and analyzed. The combination of both of these methodologies will provide the reader with a more complete picture and richer understanding of the answers to the questions inherent in my research seeking to create a more positive attitude toward the learning of mathematics by examining learning in the arts. In the words of Jacob Bronowski,

> *Neither art nor science is dull; no imaginative activity is dull to those who are willing to reimagine it for themselves.* (1979, p. 23)

Chapter 5 Repertory Grid Early Implementation and Refinements

5.1 Introduction

> *"As researchers and as instructors, we face the daily challenge of diagnosing student ideas by interpreting their written and spoken statements. Individual student interviews are often considered the gold standard for listening accurately to student ideas. However, even in an open-ended, time-unlimited, one-on-one conversation, accurate listening requires careful effort. We can and do ignore student statements when our own research agenda limits our attention. Explicit consideration of possible research agendas can increase our awareness of the richness of interview data."* (Scherr and Wittmann, 2002, p.1)

Having faced the *challenge of diagnosing student ideas* from their written words in the questionnaire phase of this study, it was necessary to make a *careful effort* to listen *accurately* to their *spoken* words in order to gain a deeper understanding. Interviews with these students were required so that *attention* would not be limited and the door to gaining further insights into the issues of confidence and enjoyment in the learning of mathematics, art, and music would be opened. This chapter will provide the reader with an overview of how the repertory grid technique was used to further explore and answer the questions related to this research.

5.2 Background

> *"All questions are responded to within some context ... In an interview, the interviewer can note the most sensitive kinds of contexts which could influence the results ... A face-to-face interview can avoid some of the ignorance about contexts under which those who use questionnaires labour. The face-to-face interviewer can be more certain that the question is being answered within the context*

of the question itself than if he uses a questionnaire or test." (Skager and Weinberg, 1971, p.118)

In addition to explicitly considering other *possible research agendas* when the data from the questionnaires is compared and contrasted to the *richness of interview data* and being assured of the appropriate *context* for the collection of this data, triangulation is another reason for following up the questionnaires in this study with student interviews. Lincoln and Guba state:

> *"Triangulation of data is crucially important in naturalistic studies. As the study unfolds and particular pieces of information come to light, steps should be taken to validate each against one other source ... and/ or a second method ... No single item of information (unless coming from an elite and unimpeachable source) should ever be given serious consideration unless it can be triangulated."* (1985, p. 283)

Much consideration was given to the type of interview that would best suit the needs of understanding how students make their own personal sense of confidence and enjoyment in the learning of mathematics, art, and music. In the end, the repertory grid technique, a form of structured interviewing, was decided upon because it is,

> *"... a very simple method for going 'beyond words' ... to see how one idea has linkages with a number of other ideas, and how one person can be seen as similar to some people and yet different from others. These linkages are such that the person may not easily be able to put them into words ... grids and personal construct theory are about people ..."* (Fransella, Bell and Bannister, 2004, pp. xi - xii)

This research is focused on discovering the *linkages* between confidence and enjoyment compared to mathematics and the arts for a group of *people*, namely high school students. Repertory grids also give the interviewer the benefits of a qualitative face-to-face conversation along with a quantitative way of assessing a student's own personal construct system when words may fail.

> *"You're trying to understand the interviewee in his or her own terms, and not to collect 'right answers' ..."* (Jankowicz, 2004a, p. 23)

5.3 Design and Purpose

> *" ... People can be seen as differing from each other, not only because there may have been differences in the events which they have sought to anticipate, but also because there are different approaches to the anticipation of the same events ... no two people can play precisely the same role in the same event ... each experiences a different person as the central figure (namely, himself) ... does this mean that there can be no sharing of experience? Not at all; for each may construe the likenesses and differences between the events in which he is involved, together with those in which he sees that the other person is involved. Thus, while there are individual differences in the construction of events, persons can find common ground through construing the experiences of their neighbours along with their own. It is not inevitable that they should come upon such common ground ... individuals can be found ... in altogether different subjective worlds."* (Kelly, 1963, pp. 55 - 56)

This purpose of the repertory grid used in this study was to recognize both the *common ground* and *individual differences* of how its participants construe the **topic**: Students views of learning mathematics, art and music. After evaluating the purpose of these interviews and taking into account all of the essential considerations for this research, it was decided that a variation of Kelly's repertory grid would be used. One of the beauties and advantages of a repertory grid is the flexibility of its design for the researcher (Jankowicz, 2004a, p.27).

> *"The reader should not be mesmerized by the particular examples of grid forms that have been included here. The grid is truly a technique, and one which is only limited by the user's imagination ... all forms of grids are sorting tasks which enable the subject to tell us something of the way in which he or she sees and orders the world."* (Fransella, Bell and Bannister, 2004, pp. 80 - 81)

Twelve fixed **elements** related to students' learning experiences were used:

- receiving grades
- watching a demonstration lesson
- discussing ideas with teacher/ classmates
- working on examples
- listening to the teacher
- being influenced by family experiences
- understanding new material
- experiencing feelings and emotions
- possessing ability/ talent
- relating topics to the real world
- handling objects/ materials
- creating something new

Each element was intended to be compared systematically on a **rating scale** of one to five based on seven **constructs**. Five of these constructs were elicited and two were fixed. In the words of Jankowicz,

> *"Each element is rated on each construct to provide an exact picture of what the person wishes to say about each element within the topic."* (2004a, p. 14)

The rep grid interviews were carried out in four phases with each student:

I. 12 fixed elements, 3 random triads resulting in 3 elicited constructs, all elements were rated

II. 12 fixed elements, 2 fixed triads resulting in 2 elicited constructs, all elements were rated

III. 12 fixed elements, 2 fixed constructs, all elements were rated

IV. Time permitting, participants were asked to explain the comments on their questionnaires and add any other constructs that had not already been mentioned

To ensure that the underlying foci of this research were included, in phase III, if constructs related to confidence and enjoyment had not been elicited from interviewees, these fixed constructs were used:

- *Makes learning fun* versus *learning is boring*
- Student feels *'Yes, I can do this'* versus *student feels like a failure*

If constructs of confidence and/ or enjoyment were elicited from students, other fixed constructs involving motivation, understanding, achievement, learning and/ or the student personally were used. Other rationales for this researcher's choices are included later in this chapter.

5.4 Pilot History

Each of the four rounds of "pilot repertory grids" that were conducted contributed to the evolution of the component parts of the "final version repertory grid". These were the issues that needed to be considered:

- **Elements** - What, how many, and would they be fixed or elicited?
- **Constructs** - How many and would they be fixed or elicited?
- **Ratings** - The first decision was to use a 5-point **rating scale.**

> " ... *You should decide the range of your rating scale in advance. All of my examples use a 5-point scale, but there's no reason why you shouldn't use a 4-, 6-, or 7-point scale. To use a wider range is probably spurious, since you'd be asking people to make finer discriminations than they can accurately express in a consistent way across the whole grid. You could always emulate Kelly, who generally used a 2-point scale ... This throws the whole focus of the grid, and how you use the information obtained, onto the constructs and their meaning rather than on the numbers, which was, very broadly speaking, his intention at the time. However, it does limit the information potentially available from an analysis, and current practice is normally to use a 5- or 7-point scale."* (Jankowicz, 2004, pp. 36 - 37)a

The mid-range 5-point rating scale provided interviewees with some latitude for *discrimination* and was less limiting of the *information potentially available*.

- **Number of interviews** - What number of interviews would provide the most thorough investigation possible while satisfying certain practical constraints? How would students be selected?

5.4.1 The Selection of Interviewees

In order to have a reliable representative sample of students to interview from all of the categories in mathematics, art, and music: Confident and Enjoy, Confident and Not Enjoy, Not Confident and Enjoy, and Not Confident and Not Enjoy, it was decided that the combination of their responses to questions two, six and eight on the questionnaire would be the determining factor. The list of students identified by category from the open ended question 8, in which students self-selected a confidence/ enjoyment type for themselves, was cross referenced with their responses in the Likert-type scale questions:

- **Q2**: I am confident in my ability to do mathematics/ art/ music.
- **Q6**: I enjoy learning mathematics/ art/ music.

The students who matched categories within these three questions were then identified as definitive members of the "preferred interviewee" list for their indicated subject and category.

5.4.2 First Pilot Repertory Grid

In order to get a feel for the repertory grid technique and explore ideas for eliciting elements, a preliminary "trial" was carried out in April 2004. This was a single interview which took place in the United Kingdom with a volunteer student from a school in which the questionnaire was not distributed. The student was asked to be a test case for the newly finalized version of the questionnaire and then to participate in a rep grid interview.

By category, the student was identified as a **+/+** (Confident and Enjoy) for mathematics. The goal was to explore and elicit elements related to the learning of mathematics. Care was given so as not to "steer a path" for the student, just "open it up" ... to let the data emerge in the students' own words without the interviewer providing any information. During the interview with this student there was enough time to create two different grids with the following prompts:

- List eight things you do to learn mathematics. These elements were elicited from the student: *solve problems, listen, ask questions, understand, use something you already know, check answers, review mistakes, read the maths book.* Four bipolar constructs were elicited as these elements were compared in random triads. Each element was then rated for each construct on a 5-point scale.

- List nine students in your mathematics class: four who are good mathematics students, four who are poor mathematics students and yourself. The student's responses became the elements. They were compared in random triads and elicited five bipolar constructs. Each element was then rated for each construct on a 5-point scale.

Discussion for Moving Forward to Pilot Two

This first time experience with repertory grids enhanced this researcher's practical familiarity with the process and understanding of the repertory grid components. Constructs expressing confidence and/ or enjoyment were not elicited in either grid. It was noticed that the elements in this student's view of learning mathematics involved mostly mental/ passive contexts, agreeing with the analysis previously noted in the last chapter on student questionnaires. In addition, it became quite clear that depending upon the number of interviews and number of elements, organization and analysis of grids with totally elicited elements could become unwieldy! As the repertory grid was to be ultimately used with students in mathematics, art and music, some careful planning was in order. Any kinds of "things" can serve as elements but,

> *"The best set of elements is one that covers the whole field of the topic evenly."* (Jankowicz, 2004a, p. 29)

In the next pilots, special attention was given to determining the elements.

5.4.3 Second Pilot Repertory Grid

The students selected to be interviewed in the next two pilot groups were chosen from among the first seventy-four students who responded to the research questionnaire. Originally, twelve students from the United States and seven from the United Kingdom were selected to be interviewed. The topic to be explored in this early stage of the study was still related to the learning of mathematics. The plan was to be able to interview at least three students from each category for mathematics. Due to schedules, arrangements and other changes, only eleven students were actually interviewed. Eight took place in the United States and three were in the United Kingdom. Six of the interviewees were female and five of them were male. The numbers of students in each category for the second repertory grid interviews were:

- **+/+** *Confident and Enjoy* – 3 students
- **+/-** *Confident and Not Enjoy* – 4 students

- **-/+** *Not Confident and Enjoy* - 1 student
- **-/-** *Not Confident and Not Enjoy* - 3 students

It was preferable to have more students in the **+/-** category because of the intrigue with the high percentage of mathematics students found to be in this category. The percentage of students in the **-/+** category was small to begin with, so it was not surprising and acceptable to have less of them.

At the beginning of each interview in the second pilot, students were shown photographs taken during lessons in mathematics classrooms. Scenes in these photographs included: students working and talking in groups to solve problems and discover concepts, students working and thinking alone, students at the blackboard, teachers working through problems at the board, teachers lecturing, students getting extra help by working one-on-one with a teacher or a peer. The photographs were used as a catalyst to prompt students to think about learning mathematics prior to eliciting elements with the following instructions: <u>List eight objects, people, events or experiences that would affect a student's confidence and/ or enjoyment in learning mathematics either positively or negatively. The first four should be related to confidence and the next four should relate to enjoyment</u>.

As in the first pilot, each interview followed the steps of Kelly's procedure:

- Elements elicited in the interview were written on cards.

  "Many people find that it helps them to think about and clarify their constructs if they have something physical to move around on the table." (Jankowicz, 2004a, p.33)

- The student shuffled the cards and then drew three cards at random

- The student was asked, with regard to the learning of mathematics, "How are two of them alike in such a way that makes them different from the third?"
- Bipolar constructs were elicited:
    - On the left the aspect of similarity is written – rating of 1
    - On the right the aspect of difference is written – rating of 5
- The student then rated each of their elements with respect to the constructs with a rating score of any number from 1 to 5 according to which end of the pole it is most closely aligned

Students were interviewed during their free times in the school schedule. The interview process continued with each one until their schedule required them to be elsewhere. On the average, each interview lasted 30 – 40 minutes. The elicitation of elements and constructs was not always easy for interviewees.

> *"The way to identify a set of constructs on a given topic is very straightforward. You provide an interviewee with plenty of examples of that topic, and discover the ways in which s/he puts those examples together ... these 'examples of a topic' are known as elements."* (Jankowicz, 2004a, p.13)

Eight elements were elicited from each of the eleven interviewees. Three constructs were elicited from each of nine of the interviewees and four constructs were elicited from each of the remaining two interviewees. By examining the elicited elements and constructs from these interviews several duplications were found. After the doubles were eliminated, the list created contained the following numbers of items:

- Elements for confidence - 24
- Elements for enjoyment - 28
- Elements that overlapped for confidence and enjoyment - 12
- Constructs - 28 bipolar pairs

In order to create a more structured grid and provide interviewees with *plenty of examples* representative of the topic for the final version of the repertory grid used in this study, both the elements and constructs were grouped by themes. In the next paragraphs the process by which the elements and constructs from this pilot data were consolidated and the strategic decisions for moving forward that were made are described.

Elements

Along with the flexibility and freedom for the researcher in designing a repertory grid comes a great deal of decision making to create just the right model for the investigation at hand.

> *"The nature of the elements selected by the researcher is likely to have an important bearing on the constructs elicited, and they should be chosen carefully to be a valid and representative sample of the field under study. Thus, if one wishes to study perceptions of teaching, the elements should be teaching situations rather than teachers themselves."* (Yorke, 1978, p. 64)

For this study focused on the *perceptions* of learning of mathematics, art and music, the elements needed to be learning *situations*. At this point in the study, to make certain that judgements on the authenticity of the elements were well founded, colleagues were asked to collaborate in the process. Even though the elements and constructs were elicited in this pilot with a prompt related to mathematics, they were deemed valid for student learning in the arts also.

It was noticed that three elements were also elicited as constructs:

- "Understanding" was used by three students as an element for confidence and by four students as an element for enjoyment.
- "Motivation" was used by one student as an element for confidence.
- "Interest" was used by one student as an element for enjoyment.

These three were eliminated as elements and considered only as constructs because they seemed more appropriate and relevant in that role.

> *"Anything can be an element ... provided it doesn't include constructs."* (Jankowicz, 2004a, p. 29)

Despite its low ranking in the analysis of questionnaire data, it was interesting that teacher related comments appeared most often as elements in several different contexts. "Teacher" alone was an element for confidence six times and enjoyment one time. "Teacher reaction" was an element for confidence one time. "Teacher support" was an element for confidence twice. "Teacher feedback" was an element for enjoyment twice. "Teacher presentation/style" was an element one time for confidence and three times for enjoyment. After much deliberation, "teacher" was given two different classifications as an element: <u>teacher - feedback, reaction, support</u> and <u>teacher presentation/style of teaching</u>. In addition to these two, the list of elements agreed upon also included the other ten elements found to be common to both confidence and enjoyment: understanding, grades, classmates, homework, tests, parents, curriculum/ kinds of maths/ topics studied, student feeling/ mood, location/ environment, and relevant topic/ examples to life. After this consolidation, the list of elements was reduced to ten items.

The next step was to return to the original confidence and enjoyment lists of elements to determine which of them could be classified within these ten themes. To be sure that all of the ideas important to the interviewees were taken into account, those that did not belong under the umbrella of any of the ten existing elements created a need for two additional elements. This analysis was, for some items, more difficult than for others. After more brainstorming and discussions with colleagues, a total of twelve categories of elements were decided upon from responses elicited from previous interviewees.

Fransella, Bell and Bannister state that,

> *"It is common practice for the elements to be chosen by the grid designer."* (2004, p. 21)

According to Jankowicz,

> *"A good set of elements will evoke a feeling of ownership on the part of the interviewee ... four feasible alternatives in deciding how far to involve the interviewee in the choice of elements ... you choose the elements based on your background and knowledge, you let the interviewee choose the elements, you share the reasons for your investigation ... and identify elements jointly, and you can elicit the elements ... by providing general categories ... that the interviewee responds to specifically."* (2004a, p. 30)

It was decided that a combination of the alternatives suggested by Jankowicz would be used. If the researcher solely chooses the elements, there is a possibility that elements important to the interviewee may be left out. If the interviewee chooses the elements, there is a possibility that key issues may be left out. Using the list of elements elicited from the students in their own words to create a set to be supplied to all future interviewees, maximized the opportunity to have elements that were at once both important to the interviewee and representative of the whole topic completely. The use of supplied elements also presented to the researcher the chance to compare the ways in which the same examples of the topic were construed by different students in different or same categories for each subject. The twelve **fixed elements**, developed after pilot two for this study's repertory grid, follow below with descriptors to best encompass the meaning of each category:

- **1** - Achievement – grades/test results
- **2** - Teaching Methods - lesson presentation, style of teaching, hands-on-activities, working in groups, alternative activities, discovery experiments, relaxing lessons
- **3** - Friends & Classmates
- **4** - Work Ethic - homework, studying, dedication, time spent
- **5** - Student-Teacher Interaction – teacher: feedback, reactions, support
- **6** - Family – parents, siblings
- **7** - Maths Topics – kinds of maths, topics studied, challenge of coursework

- **8** - Student Emotion – student feeling, mood, time of day, student expression
- **9** - Surroundings – location, environment
- **10** - Relevancy – relevant topics and examples to life, connections to other subjects
- **11** - Student ability – being smart, getting an answer
- **12** - Resources – textbooks, calculator, maths games, memory devices

Constructs

Understanding the definition of the repertory grid as a tool for data collection and the purposes of this research, it was conflicting to create a model with totally supplied constructs. Jankowicz (2004a, p. 56 - 57) writes that,

> *"... eliciting the interviewees' own constructs. That's the fundamental definition of a repertory grid."*
>
> *"Think of the issue as follows:*
>
> - *If you want to discover what the interviewee's own constructs are, and how s/he uses them, don't supply any of your own.*
> - *If you want to check a personal belief about the interviewee's own constructs and how s/he uses them, supply a construct related to that belief and see how it compares with the interviewee's own.*
> - *If you want a reflection of the different ways in which a sample of people construes an issue, but you don't need to capture the respondents own personal constructs, then don't elicit any constructs at all. Supply your own for all of them to use."*

Taking all aspects into consideration, the ideal design for this repertory grid component was a combination of both fixed and elicited constructs. Elicited constructs indicated how different interviewees made sense of the same elements in their own terms. The analysis of a grid with fixed constructs related to the important issues of confidence and enjoyment (plus perhaps others) in this study allowed this researcher to also see how all of the interviewees viewed them with the same elements.

After more thought and discussions with colleagues, the constructs elicited during pilot two were categorized into groups by commonality. There appeared to be several: <u>motivation</u> (M), <u>achievement</u> (A), <u>confidence</u> (C), <u>understanding</u> (U), <u>enjoyment</u> (E), <u>self</u> (S), and <u>learning styles</u> (L). The original twenty-eight constructs were reduced down to ten, trying to have each theme be represented by at least one of them. Those ten were consolidated further into the seven key **fixed constructs** that embodied the way pilot two interviewees construed their learning of mathematics.

The following two constructs were to be used as a <u>must</u> if they had not already been elicited from students. They are the key issues in this study:

- <u>Increases student confidence // makes student less confident</u> (C) (It was also elicited as a construct)

- <u>Makes learning more enjoyable // lessens student enjoyment</u> (E)

Two of the following were used if they had not already been elicited from students:

- <u>Makes student want to learn // does not drive student to learn</u>(M) (It was elicited as both an element and construct)

- Builds student understanding // not necessary for understanding(U)

  (It was elicited as both an element for confidence and enjoyment and a construct)

- Makes class more personalized // not personally related to students(S)

  (Elicited as an element, especially in comparison to issues related to music and art)

The following were other choices if they had not already been elicited from students:

- Necessary for achievement // no effect on achievement (A)

- Individual experience // other people are involved (L)

  (These are from key themes not already represented)

Discussion for Moving Forward to Pilot Three

As a result of the analysis of the data collected in this pilot, the following model was designed:

- **Topic**: Student Views of Learning Mathematics

- **Elements**: Twelve fixed

- **Constructs**: Nine - five elicited and four fixed

The model would be carried out with students in four phases:

I. 12 fixed elements, 3 random triads resulting in 3 elicited constructs, all elements were rated

II. 12 fixed elements, 2 fixed triads resulting in 2 elicited constructs, all elements were rated

III. 12 fixed elements, 4 fixed constructs, all elements were rated

IV. Time permitting, participants were asked to add any other constructs that had not already been mentioned

Mathematically, there are 220 ways of choosing 3 elements out of 12. Since in any one interview session there is no guarantee that a certain combination will be selected at random, for phase II of my model, the constructs were also elicited from fixed triads of elements. Choices for particular fixed triads were made because certain combinations of them had potential to be strong indicators of confidence and enjoyment. In addition, since a given triad has three possible combinations of comparing for similarities and differences, it was thought to be interesting to explore the ways individual students would group the elements of these fixed triads. The elements chosen for the fixed triads were:

- #'s **2, 7 and 11**: teaching methods, math topics and student ability
- #'s **3, 4 and 5**: friends and classmates, work ethic and student-teacher interaction
- Alternate fixed triad #'s **1, 8 and 12**: achievement, student emotion and resources. These were to be used if the others were chosen at random.

The time frame planned on for the next pilot was 45 - 60 minutes per interview.

5.4.4 Third Pilot Repertory Grid

A short time later, in order to try out the repertory grid model that was developed, three additional students from the initial questionnaire group were interviewed. They were a convenient sample of three female students in the United Kingdom. As the interviews continued to focus on the learning of mathematics, each began with students being shown the same photographs as in pilot two. Two of the students were **+/+** (Confident and Enjoy) and one was **+/-** (Confident and Not Enjoy). Although it was interesting to note the similarities and differences of these students in construing their views of learning mathematics, the purpose of these interviews was to test out the model and evaluate how well interviewees would handle the process, the fixed elements, the fixed and elicited constructs and the time frame.

Each interview took about 40 minutes. In that time:

- Each student reacted to five (fixed and elicited) constructs
- **+/+** interviews resulted in:
    - **2 random triads** leading to **2 elicited constructs**
    - **1 fixed triad** (#'s **2, 7 and 11**) leading to **1 elicited construct**. The elements were grouped differently for their similarities and differences by the two **+/+** students. Student **+/+** 1 paired <u>Teaching Methods</u> (2) and <u>Math Topics</u> (7) as those that are alike versus different from <u>Student Ability</u> (11).

The bipolar construct elicited was *what student learns* as opposed to *how well you can learn*. Student **+/+**2 paired Teaching Methods (2) and Student Ability (11) as those that are alike versus different from Math Topics (7). The bipolar construct elicited was *more flexible* as opposed to *stays the same, can't be controlled*.

- **2 fixed constructs** related to: *increasing confidence* versus *making students less confident* and *makes learning enjoyable* versus *lessens student enjoyment*. Student **+/+**1 rated Achievement, Work Ethic and Student Ability as most likely to increase confidence and Resources as the element to most likely to lessen confidence in learning mathematics. Student +/+2 thought Resources was most likely to increase confidence and Teaching Methods was most likely to lessen confidence. As for the construct for enjoyment of learning, for student **+/+**1 Achievement, Teaching Methods and Student Ability made learning mathematics most enjoyable and Family most likely to lessen enjoyment. In comparison, student **+/+**2 viewed Resources as most likely to make learning enjoyable and Relevancy most likely to lessen enjoyment in learning mathematics. The views of these students were quite diverse, even though they are both "Confident and Enjoy" for mathematics.

* The **+/-** student's interview resulted in:
  - **2 random triads** leading to **2 elicited constructs.** The second random triad (#'s **1, 8 and 10**) elicited one of the

important constructs for this study but in slightly different words: *increases student confidence* as opposed to *might scare students* by pairing Achievement (1) and Student Emotion (8) as those which are alike while different from Relevancy (10). This **+/-** student viewed Achievement and Resources as most likely to increase confidence in mathematics and Relevancy most likely to scare.

- **1 fixed triad** (#'s **2, 7 and 11**) leading to **1 elicited construct**. This **+/-** student used a different combination that the **+/+** students and paired Math Topics (7) and Student Ability (11) as those that are alike versus different from Teaching Methods (2). The bipolar construct elicited was *affects how well a student does* as opposed to *doesn't affect how well a student does/outside of school*. Work Ethic, Student-Teacher Interaction, Math Topics and Student Ability were most likely to affect how well students do and Relevancy was least likely to do so.
- **2 fixed constructs:** Since the confidence construct was already elicited from this student, the two that were used involved enjoyment and motivation. The elements that this student felt *makes students want to learn* were Student-Teacher Interaction and Relevancy as opposed to Friends and Classmates which *does not drive students to learn*. This **+/-** student felt that Achievement was most likely to make learning mathematics enjoyable and Family as most likely to lessen enjoyment.

Interesting similarities and differences had already begun to emerge in this study along with other issues of the repertory grid process itself.

Discussion for Moving Forward to Pilot Four

In reflection on the process and experiences during these last interviews, certain insights were gained and other issues needed to be re-evaluated:

- **Time** – Twelve fixed elements required more time than anticipated. The use of the photographs was eliminated. Not as many constructs were elicited and/ or reacted to as predicted.

- **Elements** – While the meaning of the elements themselves were understood by students, it was sometimes difficult for interviewees to compare and contrast them. According to Jankowicz (2004a, p. 29 - 30),

  > *"Elements which are nouns are easier to handle than those which are verbs ... when you are using verbs, try to express them as activities, each ending in –ing, since this is easier when you present each triad ... A usable set of elements has an obvious 'neatness' about it representing the topic. The set should be 'all of a kind'. If you can, try not to mix abstract nouns with concrete nouns, activities, and complicated verbal forms. How you construe 'the Olympic ideal', 'mountain bikes', and 'Saturday afternoons spent acting as a sports coach'? It's possible, but rather messy to handle for you and the interviewee."*

  The difficulty interviewees experienced appeared attributable to the mixture of nouns that were concrete with those that were abstract in the pilot three set of elements. To alleviate the 'messiness' of the elements, it was decided that each element would be expressed as an '-ing' to ensure some homogeneity in the set of elements. The list of elements from pilot two was revisited. Care was given to being faithful to the meaning of the words that the students originally expressed and to being sure that the new list also represented the whole topic. The new set of elements is listed below. Expressing all of these elements as 'all of a kind' '-ing' phrases sometimes became a tricky task. The numbers from the first set of elements to which each is connected are given in parentheses:

- **1** - <u>Receiving Grades</u> (1)
- **2** - <u>Watching a Demonstration Lesson</u> (2, 5)
- **3** - <u>Discussing Ideas with Teachers/ Classmates</u> (3)
- **4** - <u>Working on Examples</u> (4)
- **5** - <u>Listening to the Teacher</u> (2, 5)
- **6** - <u>Being Influenced by Family Experiences</u> (6)
- **7** - <u>Understanding New Material</u> (7)

The views of students in pilot two that became <u>Math Topics</u> in pilot three included expressions like: *curriculum/ materials/ kinds of math*, *tone of math*, and *challenge of coursework*. Underlying their comments in the interviews was the ability to 'get it'. *Understanding* itself was mentioned three times as an element for confidence and four times for enjoyment in pilot two. A choice was made at that time not to include it as an element but allow certain expressions of it as a construct. This time around, a conscious decision was made to combine the two to mean a student's 'getting' the mathematics ... and later, art and music ... topics. There exists a subtle difference between Understanding New Material as an element and *builds understanding* as opposed to not necessary for understanding as a construct. The responsibility for differentiating the two for interviewees will lie with the interviewer.

- **8** – Experiencing Feelings and Emotions (8)
- **9** – Possessing Ability/ Talent (11)
- **10** – Relating Topics to the Real World (10)
- **11** – Handling Objects/ Materials (2, 12)
- **12** – Creating Something New

This element was related to one of the original elements for enjoyment, 'discovery lessons', in which students learned something that they had not known before. Since the repertory grid interviews would also be conducted with students regarding the arts and questionnaire comments mentioned 'creating', it was deemed to be an appropriate addition to the list.

Element **9** from the original list, Surroundings, was eliminated due to its low or mediocre ratings on the pilot three repertory grids. To include it would make thirteen elements and twelve was a large enough sample for interviewees to handle. These changes were validated after discussions with colleagues.

Since the elements were changed, the fixed triads were also changed:

- #'s **5, 7 and 8**: listening to the teacher, understanding new material, experiencing feelings and emotions
- #'s **3, 4 and 9**: discussing ideas with teacher/ classmates, working on examples, possessing ability/ talent
- #'s **2, 3 and 7**: watching a demonstration lesson, discussing ideas with teacher/classmates, understanding new material

For variation, in some interviews, other fixed constructs were used.

- **Constructs** – offering constructs was difficult for students, sometimes in getting ideas and other times in finding ways to express them in bipolar terms. Jankowicz (2004) has several thoughts on this issue:

> *"... a good construct is one which expresses your interviewee's meaning fully and precisely, and it is a matter of three things:*
>
> *(a) a clear contrast*
>
> *(b) appropriate detail*
>
> *(c) a clear relationship to the topic in question*(p. 33)

> *"... there is a substantial amount of skill involved in obtaining an accurate description of the other person's constructs and values ... The end result is a description which stays true to the constructs being offered by the other person, rather than your own. This involves you in questioning, checking, and mulling over what exactly the other person means – in other words negotiating your understanding of what the other person means. It's very much a two-way process."* (p.15)

> *"... use the interviewees words as much as possible, but do feel free to discuss what s/he means, and to negotiate a form of words that makes sense to you both."* (p. 24)

Fransella, Bell and Bannister (2004, p. 40) suggest that

> *"... it works best if the process is more like a conversation than a task."*

The repertory grid model for this research was amended based on the above considerations. It was tested a short time later in a fourth and final pilot with four additional interviewees from the United Kingdom.

5.4.5 Fourth Pilot Repertory Grid

This last pilot also focused on the learning of mathematics, being reminded of the real interest in this study but always aware that the model would also be used with students in the arts. The interviewees were selected from a later questionnaire group according to the same process described earlier. This pilot group contained two male and two female students. The mathematical category breakdown was: one "Confident and Not Enjoy", two "Not Confident and Enjoy", one "Not Confident and Not Enjoy" and zero "Confident and Enjoy". Even though others had been asked to participate, these were the only students available given the time commitment each of these repertory grid interviews required along with the need for a common time in the school day for the interviewer and the interviewee to meet.

The model was carried out as before. It did seem to get easier for the interviewees to understand the process after one or two triads had been explained and construed.

- **Constructs** – These continued to make interviewees think a little harder when asked to offer constructs. If that occurred, the meanings of their constructs were 'negotiated' in a 'conversation' with them. It was helpful to use a suggestion of Jankowicz (2004a, p. 35) to help clarify,

  > *"... simply by reminding the interviewee of the topic each time you offer a triad of elements."*

  Sometimes the difficulties occurred because there was repetition in the elements randomly selected in the triads when the cards on which the elements were written were replaced in the deck, for example, #'s **3, 4 and 7**, followed by #'s **1, 3 and 7** and then #'s **4, 7 and 10**. As a result, the same types of construct seemed to be offered. Jankowicz advises,

  > *"The idea is to help the interviewee to arrive at a completely different construct each time, and this is helped by offering a different combination of elements ... might result in an interviewee 'stuck' on the same construct or construct. Use triads whose element combinations don't repeat, as far as possible."* (2004a, p. 42)

  In order to alleviate this concern, once a triad was selected at random, the element cards were not replaced in the deck. This tactic did not deter from the process because: the triads were still random, the possible number of constructs elicited was limited anyway because of the time frame for interviewing in a school setting, and the fixed triads brought back some of the elements for further discussion.

All students in pilot four were given the fixed triad #'s **5, 7 and 8**. Interestingly enough, three out of the four students paired Listening to the Teacher (5) and Understanding New Material (7) as alike to each other while different from Experiencing Feelings and Emotions (8). The two **-/+** students expressed similar views on these elements with subtle differences from the constructs of *required for understanding* as opposed to *depends on level of understanding*. The **-/-** student stated it this way: *helps with learning new topics* as opposed to *may cause you to give up*. The **+/-** student paired Understanding New Material (7) and Experiencing Feelings and Emotions (8) as alike while different from Listening to the Teacher (5) because the two are *happening within the student/ active* as opposed to *passive/ listening/ hearing*.

The number of constructs in an interview depended on the time frame and the interviewee. A student who was thoughtful and/ or talkative took much more time in an interview than a student who was quick to make decisions and/ or was a person of 'few words'. This one aspect of repertory grids cannot be predicted with certainty ahead of time.

- **Ratings** – It needed to be more clear to the interviewees as to how each element is rated once the 'emergent' pole (how two of the elements are alike) and the 'implicit' pole (what makes them different from the third) had been established and expressed. Interviewees were reminded that a rating of 1, 2, 3, 4, or 5 can be given to any element as long as it is appropriate and represents their views. More emphasis had to be made that the two elements which were rated alike should be assigned a rating of 1 and the one element which was different should be assigned a rating of 5. There could be other 1's and 5's, but by virtue of their comparison, the triad of elements had already been rated.

> *"I do find that, for the first few constructs, until the interviewee gets used to the procedure, it helps to record the ratings of the particular triad first, before completing the rest. It seems more logical to the interviewee. And it makes it easier for you to check that the interviewee hasn't reversed the ratings unintentionally.* (Jankowicz, 2004a, p. 47)

This technique was used in the final repertory grid interviews. Both the interviewees and this interviewer felt that Jankowicz's suggestion was both logical and helpful to the process.

- **Technology** – In order to have an audio record to back up this researcher's memory and the actual grid elicitations and ratings rationales when vocalized, attempts were made to tape record each pilot interview session. The purposes for taping were explained to the interviewees. None of them objected. While most interviews experienced no problems, there were issues with two out of the three recorders used in the pilot interviews.

5.5 Moving on to the Final Repertory Grid Interviews

All of the previously described pilots served as stepping stones in enhancing my understanding and comfort level with the carrying out of the repertory grid methodology. Jankowicz explains,

> *"Constructs tell you* ***how*** *a person thinks. The ratings of elements on constructs tell you* ***what*** *the person thinks."* (2004a, p. 19)

In the next chapter of this thesis, the final repertory grid interviews with forty-two students, representative of all categories comparing confidence and enjoyment in mathematics, art and music, are described. An analysis of *how* and *what* the forty-two interviewees were thinking is also included.

> *"Whilst it is doubtful that most educational researchers would go so far, the nature of personal construct theory, and its derivative the repertory grid, suggest a paradigm which is closer to the illuminative than to the classical scientific."* (Yorke, 1978, p.73)

This researcher eagerly looked forward to being illuminated on the views of students when they are learning mathematics as opposed to the arts!

Chapter 6 Analysis of Repertory Grid Interviews

6.1 Introduction

> *"Science provides us some of the most elegant, stimulating puzzles that life has to offer ... occasionally, it improves our lives. I love science, and it pains me to think that so many are terrified of the subject or feel that choosing science means that you cannot choose compassion, or the arts, or be awed by nature. Science is not meant to cure us of mystery, but to reinvent and invigorate it."* (Sapolsky, 1998, pp. viii - ix)

For me, the puzzle of why students, confident in their ability to do mathematics, do not enjoy learning it was truly a stimulating one. What is it about the subject of mathematics that makes so many feel that to choose mathematics means that they *"cannot choose compassion, or the arts ..."*? This chapter will provide an in-depth look at the perspectives of forty-two students, across categories and disciplines, who participated in the repertory grid interviews used in this research. It is hoped that by comparing the learning of mathematics versus the arts a solution to this puzzle will be found, *"... not to cure us of mystery, but to reinvent and invigorate ..."* students' attitudes and beliefs about the learning of mathematics.

6.2 Background

In order to understand the recurrent themes in the experiences of the students interviewed for this research, it was important to find an appropriate tool for analysis. It was expected that each interviewee would have their own unique personal sense of construing the learning of mathematics, art and music. The question, however, was: Would there also be a commonality to the recurrent themes of the individual students, by category and/ or by subject?

Fransella, Bell and Bannister (2004, p. 6) describe the repertory grid technique devised by George Kelly in 1955 as,

> *"... an attempt to stand in others' shoes, to see their world as they see it, and to understand their situation and their concerns."*

Through the analysis of the repertory grid data collected I hoped to *"see the world"* of students, *"understand"* how confidence and enjoyment affect their views of learning, and offer some suggestions for addressing their *"concerns."*

Kelly's premise (1963, pp. 4 – 5) in the psychology of personal constructs is that each person is a *"scientist."* This analogy allows us to realize that as a scientist, each of us is motivated to *"predict and control"* life's experiences. Adams-Webber (1979, p.3) clarifies Kelly's theory this way,

> *"... everyman, in his 'scientist-like-aspect,' tries to make sense out of his experience and anticipate events ... each individual must be understood in terms of his own efforts to anticipate his experience."*

Kelly explains his theory of the psychology of personal constructs in greater detail through eleven corollaries. Two of these corollaries are the *Construction Corollary*, in which Kelly (1963) states that we *"anticipate events by construing replications"* (p. 50) and the *Individual Corollary*, in which he states that persons are different from one another in *"their construction of events"* (p. 55). Adams-Webber (1979, p. 4) explains that according to Kelly, individuals don't face the same things repeatedly, but their previous experiences make it possible for them to recognize certain recurring patterns in these events. Adams-Webber adds that important to understanding the process of creating *"conceptual structures"* is *"interpersonal communication."*

Jankowicz (2004a, p.146) suggests that content analysis is a technique to be considered for the analyzing of repertory grid data. Holsti (1969, p.2) asserts that content analysis, a method with many purposes, is an ideal vehicle for enquiry into any issue in which *"the content of communication serves as the basis of inference."* The communication that *"serves as the basis of inference"* in the repertory grid technique is both verbal and numeric and so I felt that content analysis was ideally suited for the analysis of my data. In the next sections my reasons for the number of interviews that were conducted and the specific way in which the data collected was analyzed are explained.

6.2.1 Rationale for the Number of Interviews

An important issue for me was the number of interviews that would best meet the needs of my research. Evenly distributing the number of interviews by conducting two in each category (**Confident/ Enjoy, Confident/ Not Enjoy, Not Confident/ Enjoy, Not Confident/ Not Enjoy**) for each subject (mathematics, art and music) would give a basic cross section of data to explore. Since it was, however, the attitudes of the **Confident/ Not Enjoy** and **Not Confident/ Enjoy** students that prompted the most interest, a second option was to interview five students in each of these categories and none in the others for each subject. Doing that, however, would not provide a complete picture so I decided that a combination of these two strategies would be used. In order to delve into the issues of the **+/-** and **-/+** categories, five repertory grid interviews in each of these categories for each subject would be conducted. While the **+/+** and **-/-** students were not as key to the questions of my research as the other two categories, I felt that they might provide some unexpected insights and so I decided that repertory grid interviews with two students in each of these categories for each subject would also be included. The resulting forty-two repertory grid interviews provided data from all of my confidence/ enjoyment groups with more emphasis on those students in the **Confident/ Not Enjoy** and **Not Confident/ Enjoy** categories.

6.2.2 Selection of Interviewees

The students chosen for the repertory grid interviews were selected from among the 1226 students who completed the questionnaire in the first phase of this research. A prime criterion for the pool of students considered for interviewing were those who lived within a reasonable driving distance for me. This meant that I had a group of 459 students from which to choose the participants for my repertory grid interviews. These students were all from suburban high schools and they had been randomly surveyed on the initial questionnaires in their social studies classes and not mathematics, art or music classes. This was purposely done so as not to influence any of their responses with regard to their views on those three subjects which were at the focus of my research. Also characteristic of this sample of students was the diverse natures of their school and family populations. The schools from which these students came were, as a whole, academically, economically and ethnically mixed which also helped make this sample from which interviewees were chosen as unbiased and random as possible.

Another prime concern of mine was the ability to have enough students available for interviewing in each category for each subject. Table 6.1 below indicates the frequency breakdown of categories in each subject based on student responses to question eight (self-categorization) of the questionnaire.

Category/ Subject	**+/+**	**+/-**	**-/+**	**-/-**	**Totals**
Mathematics	192	202	16	40	450
Art	168	38	130	110	446
Music	178	85	66	112	441
Totals	538	325	212	262	1337

Table 6.1 Frequencies by Category & Subject of Repertory Grid Pool

Although 459 students were in this group, the differences in the totals column can be accounted for by the following:

- Each student was asked to categorize themselves in each subject by the level of their confidence and enjoyment
- Not all students categorized themselves in each subject
- In music, two students categorized themselves in in-between categories and are not counted in the totals

I also felt confident that this group was a representative sample for the repertory grids because when the category frequencies by subject in Table

6.1 from the interview pool were compared with those found in Tables 4.1 - 4.3 for the total questionnaire population, the category rankings were exactly the same for mathematics and similar for art (first and second place reversed) and music (third and fourth place reversed):

Mathematics	**Art**	**Music**
+/- 44.9%	**+/+ 37.7%**	**+/+ 40.4%**
+/+ 42.7%	**-/+ 29.1%**	**-/- 25.4%**
-/- 8.9%	**-/- 24.5%**	**+/- 19.3%**
-/+ 3.6%	**+/- 8.5%**	**-/+ 15.0%**

As explained in section 5.3.1, the selection of interviewees was made by choosing those students who were consistent in their responses to the open ended question 8 on the questionnaire where students were asked to categorize themselves by category and enjoyment and the Likert-type questions 2 and 6. The low number of students (16) in the pool available for category **Not Confident/ Enjoy** for mathematics needed to be addressed. After discussions with colleagues regarding an alternative approach for students who were not a clear cut match between question 8 and the Likert-type scale responses in questions 2 and 6 on the questionnaire, I decided that the spirit and objective of the selection process would not be undermined by using some *undecided* responses in the process. The responses for the four students who were designated as **-/+** for mathematics in this manner were as follows: two were *undecided for confidence* (Q2)/ *enjoy* and two were *not confident/ undecided for enjoyment* (Q6).

Special care was also given to gender equity in the selection of interviewees by category and subject. See Table 6.2 below:

Category/ Subject	**+/+ Male**	**+/+ Female**	**+/- Male**	**+/- Female**	**-/+ Male**	**-/+ Female**	**-/- Male**	**-/- Female**
Mathematics	1	1	3	2	2	3	1	1
Art	1	1	3	2	2	3	1	1
Music	1	1	2	3	3	2	1	1
Totals	3	3	8	7	7	8	3	3

Table 6.2 Repertory Grid Interviewees by Gender

6.2.3 The Repertory Grid as Used in This Research

After selection, a meeting was held with each interviewee to discuss the purpose and process of this research, distribute permission slips for participation (to be signed by the student and a parent/ guardian) and schedule an appointment time and place for the repertory grid interview. Each student was notified that a tape recorder would be used and why. Each interview took, on the average, forty minutes to complete.

The specifics of the repertory grid design used in this research were outlined in section 5.2. To summarize, there were twelve fixed elements:

1. Receiving Grades
2. Watching a Demonstration Lesson
3. Discussing Ideas with Teachers/ Classmates
4. Working on Examples
5. Listening to the Teacher

6. Being Influenced by Family Experiences

7. Understanding New Material

8. Experiencing Feelings and Emotions

9. Possessing Ability/ Talent

10. Relating Topics to the Real World

11. Handling Objects/ Materials

12. Creating Something New

There were <u>two fixed constructs</u>:

1. Makes learning fun versus learning is boring

2. Student feels 'yes, I can do this' versus student feels like a failure

The amount of data collected in each interview depended on the nature of the responses of each interviewee, interruptions (for the most part, interviews took place within the school day) and, in a few cases, the interviewee being unable to suggest any new constructs after a period of time. The first part of each interview took longer to explain and understand before each interviewee felt comfortable with the process.

There were different fixed triad of elements available to be used with each interviewee based upon the elements that had already been selected for the random triads. The fixed triad, "watching a demonstration lesson (2), discussing ideas with teacher/ classmates (3) and understanding new material (7)" was used most frequently: in 10 out of the 14 mathematics related interviews, 12 out of the 14 art related interviews, and 13 out the 14 music related interviews.

Each interview was comprised of four parts. The first three used all twelve fixed elements and each element was rated on a continuum from 1 to 5 based on the bipolar constructs involved:

I. 3 random triads leading to 3 elicited constructs

II. 1 fixed triad leading to 1 elicited construct

III. 2 fixed constructs to which interviewees responded by rating the elements

IV. Each student was asked to re-categorize themselves, comment on their choices and add any other thoughts

Parts II through IV were used as stated with all interviewees. Any divergence from this plan came up in Part I. Based on the factors mentioned earlier affecting the amount of time allotted to each interview, the number of constructs, elicited and fixed, varied as follows for the forty-two interviews:

- Five constructs – one interview
- Six constructs – fourteen interviews
- Seven constructs – twenty-three interviews
- Eight constructs – four interviews

6.3 Options for Analysis

Fransella, Bell and Bannister (2004, p.xi) tell us that, while repertory grids come in many varieties and can be used in multiple ways to *"ask questions and give answers"*, it is important to use the process well. Just as a repertory grid can be designed to suit and meet the needs, goals and objectives of a researcher, the analysis of the data collected can also be accomplished in a variety of ways. I intended that the repertory grid interviews of this research would ask some questions, give some answers and be "used well"!

Many have utilized and written about various strategies for repertory grid interviews and the analysis of the data collected through them for example, Fransella, Bell and Bannister (2004, p. 82); Neimeyer, Neimeyer, Hagans and Van Brunt (2002, p. 162); Stewart and Stewart (1981, p. 46). A unique feature of this methodology and its analysis is the way in which it can be adapted from a very basic to a more complicated technique. In his introduction to outlining some basic approaches to repertory grid analysis, Jankowicz (2004) reminds the reader that to consider the words of the constructs as qualitative and its ratings as quantitative would be deceptive and that it is naïve to consider the words of the constructs as content and the ratings as structure. *"Meaning"* to Jankowicz (2004a, p. 72) is *"what has been captured in the grid ... both the words and the numbers."* In his view to analyze repertory grid data requires, therefore, a *"blend of both."* Important to me was the ability that I would have *"to come to understandings which are useful"* about the learning of mathematics, which Jankowicz (2004a, pp. 75 – 76) later reminds the reader is the function of all research and analysis. He adds that one's choice of analysis depends on the level to which the method "*summarizes your interviewees' meanings ... allows you to draw inferences and conclusions* ... (and) *communicates this to your own constituency.*"

Beail (1985, pp. 11 – 19) and Neuendorf (2002, p.2), like Jankowicz, assert that the choice of a method of analysis depends on the preferences and resources of the researcher. Beail mentions the use of computers as do Easterby-Smith, Thorpe and Holman. According to Easterby-Smith, Thorpe and Holman (1996, p. 4), "*repertory grid technique has obtained something of a reputation for being dependent on computers.*" Even though Easterby-Smith, Thorpe and Holman (1996, p.26) hold the view that "*...repertory grid technique is a most stimulating task master,*" they comment that it has the most promise with an "*interactive*" approach and that the use of a highly technological method of analysis can allow the researcher to "*get carried away with the statistical approach to grid analysis*" by using a process that seems easier than qualitative methods. Beail (1985, p. 22) adds that despite the fact that repertory grids are not "*handy or convenient,*" they can "*yield valuable data in a variety of contexts.*"

My decision on how the data collected through my repertory grid interviews was analysed was influenced by an assumption of Kelly's theory that,

> "*... each individual relies on his own personal constructs to plan, regulate, and explain his own actions. It follows that specific information about the content and structure of a person's construct system will provide a source of useful cues to anyone ... who attempts to interpret or anticipate his or her behaviour.*" (Adams-Webber, 1979, p. 102)

I wanted a method of analysis capable of capturing the essence of the *"content and structure"* of students' construct systems as expressed by interviewees in both their words and their ratings of the elements. An interactive approach, which did not rely upon computers to manipulate the data, would enable me to have a personal relationship with the meanings articulated by the interviewees. I felt that the source of useful *"cues"* for a deeper comprehension of students' views of the learning of mathematics versus the arts would require a content analysis of their constructs which included an exploration into the ways they used the ratings on the elements based on those constructs and the truest understanding of students' behaviours would come from directly dealing with data myself.

Jankowicz (2004a, p.169) states that the content analysis of several repertory grids is a means of generally describing the gist of what has been expressed in a set of grids as a whole while also deliberating on the meanings of the *"personal and, at times, idiosyncratic knowledge"* conveyed by individuals.

Holsti (1969, pp.3 - 5) asserts that while there are different forms that content analysis can take, it is widely held that each one must possess:

- *Objectivity* where research is performed with specific *"rules and procedures"* so that others using these rules can come to the similar conclusions with the same data

- *System* in which themes or characteristics have been determined with *"consistently applied rules"*

- *Generality* which ensures that the results have *"theoretical relevance"* to the purpose and nature of the research project being undertaken

Holsti (1969, pp. 5 – 10) adds that there is debate with regard to two other characteristics of content analysis: the differences between quantitative versus qualitative methods as applied to content analysis and whether the analysis should not just be restricted to the 'manifest' content, but also include the underlying aspects of the data. Holsti (1969, p. 14) believes that while *"only the manifest attributes of text may be coded,"* conjectures about the hidden meanings are allowed. He also concludes that quantitative and qualitative strategies complement each other, allowing the researcher to acquire more insights into meaning of the data. He adds, however, *"asking the right questions of the data is even more important than the system of enumeration used to present the findings"* (1969, pp. 11 – 12). Guba and Lincoln (1981, p. 242) present the same view. Both Holsti (1969, p. 12) and Lincoln and Guba (1985, p. 338) quote the opinion of the statistician, John W. Tukey (1962, pp. 13 – 14) who more fully writes,

> *"The most important maxim for data analysis to heed, and one which many statisticians seem to have shunned, is this: 'Far better an approximate answer to the 'right' question, which is often vague, than an 'exact' answer to the wrong question, which can always be made precise.' Data analysis must progress by approximate answers, at best, since its knowledge of what the problem really is will at best be approximate. It would be a mistake not to face up to this fact, for by denying it, we would deny ourselves a great body of approximate knowledge, as well as failing to maintain an alertness to the possible importance in each particular instance of particular ways in which our knowledge is incomplete."*

Lincoln and Guba (1985, pp. 338 – 339) assert that there is a commonality between the naturalistic and conventional approaches to content analysis, but that there are also very important differences. They feel that while the conventional style may *"guide"* a naturalistic approach to data analysis, it should not limit the latter especially with regard to *"the timing of rule formulation, need for a priori guiding theory (and deduced categories), utility of generalizable findings, and rejection of constraint to the quantitative arena."*

I decided to utilize the technique of content analysis developed by Honey in 1979 (as outlined in Jankowicz, 2004a, pp. 169 - 177). It provided a strategy to best address the intentions of my research while being faithful to the definitions and characteristics of content analysis as described and outlined by Holsti (1969) and Guba and Lincoln (1981). The next sections are devoted to Honey's method of content analysis and how I used it.

6.4 Content Analysis of Repertory Grid Interviews

Holsti's (1969, p. 14) broad definition of content analysis is, *"Any technique for making inferences by objectively and systematically identifying specified characteristics of messages."* Jankowicz (2004a, p. 146) suggests that the approach taken in the content analysis of repertory grid data depends on two factors: the number of grids needing to be analyzed and the design format of the grids. According to Jankowicz's (2004a, p.292) definition, content analysis is *"the only feasible way of aggregating the information present in a large set of repertory grids, by collecting and categorising the different meanings of constructs present in the set."*

6.4.1 Rationale and Decision Making

To be able to understand and communicate the meanings found in forty-two grids would be complex and not as straightforward as doing the same for just one grid. Since, in this research, each interviewee was provided with exactly the same twelve elements, the focus was to compare, contrast and categorize the constructs used by all of the interviewees. The unit of analysis for this research, which expressed the unit of meaning to be explored, was the interviewees' constructs. An effective strategy, with which to systematically deal with the two hundred bipolar elicited constructs and eighty-two fixed constructs of confidence and enjoyment, was required.

My purpose for using the repertory grid technique was to investigate how students construe their learning in mathematics, art and music and compare and contrast their views when these students were sorted into subsets based on their confidence and enjoyment. The constructs of the twelve category subsets required a sorting to enable me to identify any similarities and differences between how experiences were construed by each group.

A four digit labelling code was created to identify each interviewee's constructs so as to determine which subgroup each belonged during the content analysis: **#.#.#.#**, e.g. 1.2.3.4

- The first digit indicated the subject: 1 - mathematics, 2 - art, 3 - music

- The second digit indicated the category:

  1) +/+, 2) +/-, 3) -/+, 4) -/-

- The third digit was the identification number for the student within the category

- The fourth digit indicated the order in which each construct (whether elicited or fixed) was considered by the interviewee

For example, 1.2.3.4, identifies the fourth construct of the third student in the confident/ not enjoy category for mathematics.

In chapter seven of Jankowicz (2004), a generic content analysis approach is described along with a variant approach, the Honey Method. All interviewees were provided with the same two fixed constructs of confidence and enjoyment. Jankowicz (2004a, p. 148) states that the use of a provided *"overall assessment construct with the full sample of respondents lies at the heart of the procedure",* i.e. Honey's Content Analysis. My repertory grid interviews provided all students with fixed constructs related to confidence and enjoyment with the purpose of matching those constructs which students most closely aligned with those fixed constructs. Jankowicz (2004a, pp. 169 - 170) best expresses my rationale for selecting Honey's Method of analysis as opposed to other methods which only focus on the constructs:

> *"We haven't been able to use the 'ratings' of the elements on constructs available in the original grids. Somewhere in the whole set, we've retained each person's constructs; but we've lost what each person was telling us about the topic by means of those personal constructs ... we've lost a considerable amount of information available on each grid ... there is a technique* (Honey) *... it*

> *aggregates different constructs across a sample and provides a way in which we can make use of some of the individual meanings being conveyed by each person's ratings ... the technique assumes that what we are interested in is each individual's personal understanding of the topic in question, and treats each construct offered by the individual as more closely related, or less closely related, to the overall issue s/he has in mind while thinking about the topic."*

The Honey Method is faithful to Kelly's personal construct theory, in which his *Organization Corollary* (1963, p. 56) states, each individual creates "*ordinal relationships between constructs.*" Meaning that, some constructs have a higher order value than others. In terms of his *Fragmentation Corollary*, Kelly (1963, p. 83) explains individuals may utilize an assortment of construct "*subsystems which are inferentially incompatible with each other.*" Jankowicz (2004, p. 170) adds that the diverse ways in which individuals make of certain circumstances may not necessarily fit with each other and that it is expected that some constructs may be more associated to the subject at hand than others. Others, although pertinent, may only add "*grace notes, as it were, without being 'what the whole thing is about'.*"

My adaptation of Honey's Method by supplying all interviewees with the two fixed constructs of confidence and enjoyment, along with others which were elicited, was a uniquely powerful way to summarize how students construed their learning in mathematics versus the arts and determine the *ordinal relationships* inherent in their *construction systems* based upon their confidence and enjoyment categories.

6.4.2 The Honey Technique: Construct Indices

Before the content analysis on the constructs was begun, each construct was assigned two coded indices:

- A percentage of similarity score

The calculations for this index were done with an Excel spreadsheet. The percentage of similarity scores were calculated with the formula in Jankowicz (2004a, section 6.1.2, p. 115),

$$PS = 100-(\{SD/[(LR-1)*E]\}*200).$$

SD is the sum of differences between the ratings along a particular construct row and those of a fixed construct row for each element, **LR** is the largest possible rating and **E** is the number of elements. The use of this formula allowed me to look at the relationship between individual constructs in comparison to the supplied/ fixed constructs of confidence and enjoyment, each in turn. Given the bipolar nature of the constructs, the same procedure was carried out for each construct with the poles transposed. This was necessary due to the possibility that the similarity between a given construct and the fixed construct(s) would be closer in meaning to a reversal in order from how the construct was originally expressed. Appendix K contains a worked out example of how a percentage of similarity score was calculated for the grid in Appendix E.

This process enabled me to compare the grids and constructs of forty-two interviewees and determine the level to which the constructs were the same, first with the confidence construct and then with the enjoyment construct. A 100% score meant that a particular construct matched exactly, in the interviewee's view, with the fixed construct. These two examples from the data illustrate constructs with a 75% of similarity to enjoyment construct: construct **2.4.2.5** (art **-/-**), *helps learning* versus *takes away from learning* and construct **1.3.3.4** (mathematics **-/+**), *student is doing/ student is active* versus *student is not doing/ sitting back.* These provided me with some food for thought on how enjoyment can be construed in mathematics versus art. A complete analysis of the data is contained in a later section of this thesis.

- A range of similarity index (High-Intermediate-Low)

Allowing for the fact that different people may vary in their distinctive percentage of similarity scores, this index enabled me to compare and recognize each interviewee's range of similarity. Some interviewees used a very narrow range and others a very wide range of scores. For the examples above, construct **2.4.2.5**'s range of values went from 12.5% to 75%. Construct **1.3.3.4**'s range of scores was from 29.17% to 75%. It is important to observe that for both constructs used as examples, 75% was a high score for the students in question. Construct **3.2.5.1** (music, **+/-**), *hands on/ interaction between people* versus *student alone* had a 29.17% of similarity with the confidence construct. This was an intermediate score in similarity scores that ranged from 12.5% to 33.33% for this student.

It is clear that different individuals will have differing ranges of percent of similarity scores and that a score of 85% may be in a high range for one person and a low range for another. Jankowicz (2004a, p. 171) calls this *"different personal metrics"* and notes the importance of recognizing whether each individual's percentage of similarity scores are in a high, intermediate or low range (H-I-L).

For each grid, by looking at the percentage of similarity scores, the constructs were divided into thirds: the third with the highest percentages (H), the third with the intermediate percentages (I) and the third with the lowest percentages (L). Jankowicz (2004a, pp. 174 - 175) suggests that this be done *as best you can* ... an odd number of percentages won't necessarily divide into three equal groups. In addition, it should also be noted that some percentages of similarity were equal or close in value. It would not have been appropriate to assign one of two equal or even close scores to two different ranges of similarity. In order to establish the reliability of all of the designations, two colleagues were asked to independently assign the H-I-L indices to the percentages of similarity scores. Discussion followed until an index could be agreed upon by consensus. The individual percentage similarity scores were then labelled as high (H), intermediate (I) or low (L).

Since the repertory grids of this research contained two supplied/ fixed constructs, the data was analyzed across the sample twice, once where the focus was confidence (C) and second with a focus on enjoyment (E).

The data collected were looked at by using both the percentage of similarity and H-I-L indices. After careful observations of the indices and the characterizations, the views and constructs of the interviewees were explored. Some constructs had a high correspondence with the constructs of confidence and enjoyment while others did not. Honey (1992, pp. 88 – 86) uses the terms *top* (high) and *tail* (low) to describe those constructs, which after the percents of similarity had been calculated, would represent *the ones most closely associated* with the topic and *the ones least closely associated* with the topic respectively.

The next step was the coding of the constructs into thematic categories. Guba and Lincoln (1981) extend Holsti's (1969) thoughts on content analysis by adding two observations about the topic. First, that the researcher is not, as a rule in control of the content and second, that because of this lack of control, the attributes of what is communicated must be allowed to emerge without presuming a pre-conceived theory. This is ideal for Guba and Lincoln because that *"virtually guarantees that the categories will be grounded in the data and, hence, in the context."* (1981, p.240) Jankowicz (2004a, p. 149) claims that each construct expresses a *"single unit of meaning"* and considers the construct to be a unit of both content and context.

Coding to Guba and Lincoln (1981, p. 243) is basically an intuitive process which requires careful reading which must be informed by practice and theory in order to arrive at conclusions which are relevant, true and complete. Honey cautions the researcher of the dangers of the sorting and categorizing process. Because one of the strengths of the repertory grid is the pureness of their raw data, Honey (1979b , p. 457) states,

> *"If we now proceed to contaminate it by forcing items into categories for our convenience this is a serious distortion of data ... I am always prepared to have miscellaneous items that won't fit my main category headings. The items come first and my categories second. I will ... have as many categories as it takes to accommodate the data. I never set any kind of limit on the number of categories and never have preconceived ideas about what the categories will be ... I invite at least two other people to sort the data into categories quite independently ... sorting and categorising the data is not only the single most important stage in the analysis process it is also the most time consuming."*

Although Honey (1992, p. 86) sorts his constructs and identifies categories in a process which deals with the *top* and *tail* constructs separately, I decided for purposes of my research (with two fixed constructs of confidence and enjoyment) to complete the categorising of constructs from one total pool, regardless of their percentage of similarity to either fixed construct. The complete analysis 'painted the picture' of the interrelationships between confidence and enjoyment and students' perceptions of them in mathematics versus the arts.

6.4.3 Aggregation of the Constructs

> *"The aggregated set of constructs for the sample as a whole will, in other words, represent the categorised views of 'all' the individuals in the sample, but will also preserve information about 'each' individual's view in terms of how he or she severally, personally, 'idiosyncratically' if you will, thought about the topic. And it will take into account both the % similarity value and the individual's personal metric! ... I have taken off my hat to Peter Honey ... he gives us exactly what we need when we seek to aggregate large samples of respondents."* (Jankowicz, 2004a, pp. 171 – 173)

Content analysis must be reproducible and make sense to others. Jankowicz (2004a, p. 150) states three key factors of reliability for content analysis,

- *Stability* - the extent to which the categories, with the same constructs under each, would stay the same over time

- *Reproducibility* - the extent to which others would make the same sense of the constructs

- *Sheer accuracy* – the extent of how consistently the category definitions have been applied

In order to establish the reliability of the characterizations of the constructs, I met with colleagues to discuss and negotiate the groupings and meanings in the categories of the constructs elicited in this research using procedure suggestions made in Jankowicz (2004a, pp. 148 – 165). For my purposes, since I used the word category with regard to student confidence and enjoyment groups, the word 'theme' is used for the characterizations of the constructs. Before the analysis took place, I went through my thematic procedure with all of the constructs for a second time. Except for a minimum number of changes, mainly in the theme descriptors, the results were the same.

At the start, there were 282 constructs, including the fixed ones for confidence and enjoyment. Eliminating the fixed constructs but taking into consideration that, in a few cases, they were elicited from some interviewees, the number to be categorised was 200 elicited bipolar constructs. Preliminary grouping arranged these constructs into 46 sets (44 plus ones for confidence and enjoyment). Through discussions in which similarities and differences in individual meanings and intentions were recognized, questions were raised and answered regarding the kinds of categories that were emerging, the sorts of constructs that were contained in each and possible descriptors and definitions for each category. Thought was given to the mutual exclusivity of each category developed. The process continued for several revisions until colleagues were in agreement on the proper allocation of construct subsets to each category. The themes of the constructs were finalized into 12 categories, 10 from the elicited constructs plus one each for the fixed, and sometimes elicited, confidence and enjoyment. No percentages of agreement were calculated because of the nature of the way in which the discussions were held and the meanings of the constructs were negotiated. The principles of reliability were assured because consensus among colleagues was an important part of the decision making process on the aggregation of the constructs.

For the purposes of this thesis, since 'category' refers to the confident/ enjoy classification of the student subjects of this research, the characterizations of the constructs as aggregated were designated as <u>themes</u>. Below is the list of the themes of the repertory grid constructs, with examples and descriptors. The numbers in parentheses indicate the number of constructs in each theme.

1. <u>Applications</u> – utilization of what has been learned (10)

   1.2.5.6 *Can be applied to jobs and future* versus *no need/ extra*

2. <u>Intellectual Stimulation</u> – thinking, creativity, imagination, ideas (14)

   2.4.1.1 *Student gets ideas/ open ended* versus *closed/ fixed/ precise*

3. <u>Social Aspect</u> – involvement with others (17)

   3.3.3.2 *Working with teacher or another* versus *individual activity*

4. Nature of Tasks (24)

1.3.3.4 *Student doing/ active* versus *student not doing/ sitting back*

5. Feelings and Emotions (10)

1.2.3.4 *Feels good/ less stressful* versus *frustrating*

6. Knowledge and Comprehension (30)

1.1.1.4 *Helps student understanding* versus *confuses student*

7. Learning - process, level, cause-effect (37)

3.2.5.4 *Learning is easier* versus *learning is not necessarily easier*

8. Personal Self - ability, expression (35)

3.1.1.2 *Internal to student* versus *teacher's opinion/ view*

9. Motivation (18)

2.3.1.4 *Builds student interest* versus *gets in the way of interest*

10.) Enjoyment (43)

1.2.5.7 *Makes learning fun* versus *lessens student enjoyment*

11. Confidence (42)

2.3.2.4 *Builds confidence* versus *lessens confidence*

12. Miscellaneous (2)

3.2.3.1 *Related to inside school* versus *outside of school* (L - 25%)

3.3.4.4 *Outside factors* versus *inside classroom* (I - 20.83%)

These last two constructs, although similar to each other, did not fit into the other eleven themes without being *forced*. Jankowicz (2004a, p. 149) notes,

"... a small number are usually unclassifiable without creating categories of just one item ... <u>all unclassifiable items are placed in a single category labelled 'miscellaneous</u> ... consider redefining so that, at the end, <u>no more than 5% of the total are categorised as 'miscellaneous'</u>."

A Dick and Jankowicz (2001, p. 189) study which began with 380 constructs, cast off five constructs because they could not be assigned to any theme. The two miscellaneous constructs that did not fit in my research, constituted 2 out of 200 or 1% of the elicited constructs. I felt justified in discarding these constructs, leaving 198 elicited constructs and 82 fixed constructs for a total of 280 constructs that were allocated across my 11 themes.

Guba and Lincoln (1981, pp. 244 - 246) claim that there is no simple norm or explanation of how to code while differentiating between what and how things have been said. They assert (p. 246) that coding is many times a *"trial-and-error process, forcing the investigator to move between the data and either an 'a priori' or a grounded theory."* They describe a strategy similar to Jankowicz (2004a, p. 149) and Glaser and Strauss (1967, pp. 102 – 103). This strategy is essentially one of constant comparisons in which the data are sorted and when a new piece of data does not seem to fit into an already existing theme, a new tentative theme is created. The themes are continuously compared and modified until all data can be sorted. This system allows for a organization generation of theory, which is, according to Glaser and Strauss (1967, pp. 102 – 103), *"a theory that is integrated, consistent, plausible, close to the data."*

Following the guidelines for a systematic and objective approach to the coding of the interviewees' constructs, which included a repeated deliberation on them and a constant comparison of the constructs with each other until themes were identified, as set forth by Holsti, Guba and Lincoln, and Glaser and Strauss, I proceeded on to the completion of the analysis phase of this project, confident in the construct themes and their reliability and validity.

6.5 Analysis of the Forty-Two

According to Glaser and Strauss (1967, p. 115) this qualitative process of content analysis is *"an inductive method of theory development"* which forces the researcher generate often abstract ideas to account for the causes of the *"underlying uniformities and diversities"* especially when there is a great deal of variety in the data.

The power of Honey's content analysis compared to others is that, as stated before, it allows more than just a thematic representation of the constructs. I was also able to consider the meanings expressed by individual students based upon the ratings they utilized. In order *to bring out the underlying uniformities and diversities*, the analysis of this research focused on the *meanings shared* by the students interviewed, first by their subject (mathematics, art, music) and then by their category (**+/+**, **+/-**, **-/+**, **-/-**). The percentages of similarity calculated for each construct were then used as indicators of the definition of relevance to confidence and enjoyment that each student possessed. Insights were gained into each individual's attribution of the private sense of learning each of these subjects that each student held.

In this analysis samples from the data are cited to exemplify the construct similarities of students while learning mathematics versus the arts. As the percentages of similarity are all relative, different personal metrics for students based on their ranges of percentage of similarities and high (H), intermediate (I), and low (L) indices are key to determining which constructs will represent the views of the individuals in each group as being important, mixed importance, or not very important. In his analysis, Honey dismisses those constructs which are in the intermediate range:

> *"The penultimate step is to compare the 'top' and 'tail' data. In particular look for similarities within the two sets of data. Similarities tend to indicate that there isn't a clear consensus on* (the topic) *... Taken by itself, the 'top' data would have shown a strong link ... the 'tail' data warn us that this is by no means the whole story since there is a similar number of items that are 'neutral' about* (the topic) *... Comparing the 'top' and 'tail' doesn't take long once they have all been categorized ... The Repgrid is an excellent way to 'surface' people's perceptions ... the advantage of using the Repgrid routine is that there is no need to design a questionnaire. The disadvantage is that the data are a great deal messier to analyse and interpret!"* (1992, pp. 87 - 88)

I did the same as Honey, concentrating on those constructs which provided a *strong link* (the *tops* - H) to confidence and enjoyment and those that helped to complete a *whole story* (the *tails* – L) in order to understand the perceptions of students when learning mathematics versus the arts. Only those constructs with the highest and lowest percentages of similarity were considered. In Honey's (1992, p. 86) words,

> *"Top data: 33 1/3 percent of data compiled ...; 33 1/3 percent of data <u>untouched</u> – this is the middle wadge; Tail data: 33 1/3 percent of data compiled ..."*

Jankowicz (2004a, p. 176) says that for those items which reveal no particular type of consensus, *"there's no point in preserving ambiguity or ambivalence."*

Since all interviewees were provided with the fixed constructs of confidence and enjoyment, comparisons were first made among the themes in the other 195 constructs. If those 195 constructs were sorted randomly into nine themes, each theme should have contained about 22 constructs or about 11% of the total number of constructs. The themes ranked in order, by frequency in that group, were: Learning - 19%, Personal Self - 18%, Knowledge and Comprehension - 15%, Nature of Task - 12%, Social Aspects and Motivation tied with 9%, Intellectual Stimulation - 7% and Application and Feelings and Emotions tied with 5%. By looking deeper within each theme some interesting observations were made regarding confidence and enjoyment in mathematics versus the arts.

6.5.1 Percentages of Similarity by Category with Confidence

There were a total of 238 constructs that were compared by percentages of similarity to the fixed confidence construct. Included in that number were the 43 enjoyment constructs and 195 constructs from the other themes. Table 6.3 below lists for the reader a breakdown of the constructs with the highest (H) and lowest (L) percentage of similarity to confidence by theme. A designation of 'R' next to a construct number indicates that it is most/ least similar in reverse order. The percentage of the numbers of H-L constructs contained in each theme is calculated to the nearest whole percent.

Theme / Number	Constructs	Percent of Theme	Percentage Similarity	H-I-L Range
1. Application (10)	1.2.5.6	30%	54.17	H
The utilization of what has been	1.4.2.2		58.33	H
learned, relevance to the real world	2.3.3.3		70.83	H
	1.2.3.1	40%	37.50	L
	1.2.4.2R		29.17	L
	2.2.5.2		16.67	L
	2.3.2.1R		25.00	L
2. Intellectual Stimulation (14) Thinking, creativity, imagination, ideas	NONE	0%		H
	2.3.4.1	50%	29.17	L
	2.4.2.2		16.67	L
	3.1.1.3R		8.33	L
	3.3.3.1		50.00	L
	3.3.3.4		50.00	L
	3.3.5.5		33.33	L
	3.4.2.2		29.17	L
3. **Social Aspects (17)** Involvement with others.	3.4.2.3	6%	45.83	H
	1.2.5.4	41%	33.33	L
	1.3.1.4		16.67	L
	1.3.5.2		37.50	L
	2.2.1.4		8.33	L
	2.3.3.4R		20.83	L
	3.3.1.1R		33.33	L
	3.3.4.5		12.50	L
4. **Nature of Task (24)**	2.2.4.1	8%	54.17	H
Engagement, physical/ mental experience	2.4.1.4		45.83	H
	1.2.1.2	42%	16.67	L
	1.2.4.4		20.83	L
	1.2.5.5		33.33	L
	1.4.2.1R		29.17	L
	2.1.2.2R		25.00	L
	2.2.3.3		25.00	L
	2.3.3.1R		12.50	L
	3.1.2.3		16.67	L
	3.2.1.2R		25.00	L
	3.4.1.4R		16.67	L
5. **Feelings and Emotions (10)**	1.1.2.4	20%	54.17	H
Student feelings during lessons/ about the subject	2.2.2.1		66.67	H
	1.3.1.5	40%	12.50	L
	1.3.2.4		12.50	L
	2.4.1.3		8.33	L
	3.3.2.5R		25.00	L
6. **Knowledge & Comprehension**	1.2.4.5	23%	54.17	H
(30)	1.3.2.2		58.33	H
Understanding, facts acquired	1.3.4.4R		54.17	H
	1.4.1.4R		50.00	H
	2.3.3.5		79.17	H
	2.3.5.1R		58.33	H
	3.2.3.3		70.83	H
	1.1.1.4	30%	16.67	L
	1.1.2.1R		12.50	L
	1.3.4.1		4.17	L
	2.2.3.5		29.17	L
	2.2.5.5		25.00	L
	2.3.2.2		25.00	L
	2.3.4.5R		29.17	L
	3.3.4.1R		16.67	L
	3.4.1.3R		8.33	L

7. Learning (37) Process, level, cause-effect	1.2.2.1 1.3.2.1 1.3.2.5 1.3.3.5 1.4.1.2R 1.4.2.3 2.2.1.5 2.2.1.2 2.4.2.5 3.2.1.1 3.2.4.4 3.2.5.4 3.3.4.3	35%	58.33 58.33 58.33 62.50 54.17 62.50 58.33 41.67 54.17 58.33 62.50 33.33 50.00	H H H H H H H H H H H H H
	1.1.1.2 1.2.1.4 1.2.2.2 2.1.1.3 2.1.1.4 2.2.1.3R 2.2.4.4 2.3.1.1 3.2.2.1 3.2.4.3 3.3.1.2R 3.3.2.3R	32%	12.50 16.67 33.33 45.83 41.67 12.50 12.50 50.00 29.17 37.50 20.83 8.33	L L L L L L L L L L L L
8. Personal Self (35) Ability, expression, student's ideas/ opinion	1.3.3.6R 2.1.1.1 2.3.3.2 2.3.5.3	11%	58.33 62.50 70.83 62.50	H H H H
	1.1.1.5 1.2.1.1 1.3.1.3 1.3.3.3 1.3.5.3 1.4.1.5 2.3.4.4 2.3.5.4R 2.4.1.2 3.1.2.2R 3.2.2.2R 3.2.3.4R 3.2.5.5 3.3.5.2	40%	16.67 16.67 20.83 37.50 45.83 16.67 33.33 29.17 4.17 25.00 33.33 29.17 12.50 33.33	L L L L L L L L L L L L L L

9. Motivation (18) Interest, making student involved in learning	1.3.1.6 1.3.5.5 2.2.3.4R 2.4.2.1 3.3.3.3 3.4.2.4	33%	70.83 79.17 54.17 54.17 62.50 45.83	H H H H H H
	1.3.3.1R 1.4.1.3 2.2.2.5 2.3.1.4 3.2.1.4R 3.3.2.2R	33%	33.33 25.00 50.00 50.00 25.00 29.17	L L L L L L
10. **Enjoyment (43)**	1.1.1.6 1.1.2.5 1.2.1.5 1.2.2.5 1.2.3.5 1.2.4.6 1.2.5.7 1.3.1.7 1.3.4.5 1.3.5.6 1.4.2.6R 2.1.1.5 2.1.2.5 2.2.3.6 2.2.4.5 2.2.5.6 2.3.1.6 2.3.2.5 2.3.4.6 2.3.5.7 2.4.1.6 2.4.2.6 3.1.1.5 3.1.2.6 3.2.1.6 3.2.2.5 3.2.3.6 3.2.4.6 3.2.5.6 3.3.1.5 3.3.2.6 3.3.3.6 3.3.5.6 3.4.1.5	79%	58.33 62.50 70.83 58.33 58.33 58.33 58.33 62.50 54.17 75.00 58.33 58.33 70.83 50.00 50.00 58.33 58.33 70.83 87.50 58.33 54.17 62.50 37.50 62.50 58.33 66.67 70.83 66.67 33.33 83.33 91.67 66.67 79.17 41.67	H H H H H H H H H H H H H H H H H H H H H H H H H H H H H H H H H H
	1.2.5.1R 2.2.2.3 3.3.4.6R	7%	29.17 45.83 12.5	L L L

Table 6.3 Confidence H-L Range by Themes

The next section of this thesis focuses on an examination of interviewees' perceptions of confidence as related to each of the themes in mathematics, art and music.

6.5.1.1 Factors Related to Confidence

The following paragraphs contain the summary of the observations that were made by examining the constructs contained in each of the themes as related to confidence in mathematics, art and music. The themes were considered in their reverse rank order. Examples that express the meanings of constructs are included. Where a fuller meaning would be expressed, the complete bipolar construct is given. Numbers in parentheses indicate each construct's four digit identification code.

The two lowest ranking themes within those constructs which were elicited were Application and Feelings and Emotions each with 5% of the constructs. No students considered any constructs in either of these themes as highly related to confidence in music. No Application construct was considered as least associated to confidence in either music or art. In Feelings and Emotions, however, for one music student this construct was least related to confidence: *Determines student's likes or dislikes about the subject* versus *teacher presents what student is told about the subject* (**3.3.2.5**)

The most related to confidence Application constructs were, for mathematics:

- *Can be applied to jobs and future* (**1.2.5.6**)

- *Student sees/ feels mathematics is useful* (**1.4.2.2**)

Least related to confidence in mathematics was,

- *Relevant to real world/ makes you better* (**1.2.3.1**)
- *How to do* versus *where mathematics is applied outside* (**1.2.4.2**)

In art, the construct most related to confidence in Application was,

- *Enables student to apply learning* (**2.3.3.3**)

The Application constructs least related to confidence in art were,

- *Life lessons/ things needed but not learned in class* versus *more practical part of learning*(**2.3.2.1**)
- *Student uses what they are learning* versus *student learning how to do* (**2.2.5.2**)

In the theme of Feeling and Emotions, the only highly associated construct to confidence in mathematics was:

- *No internalized highs and lows* (**1.1.2.4**)

The constructs least associated to confidence in mathematics:

- *Mental knowledge gained* versus *how student feels about it* (the subject) (**1.3.1.5**)
- *Involves student feelings while learning* versus *just right answer/ no feelings involved* (**1.3.2.4**)

The art constructs related to Feelings and Emotions and confidence were:

- *The subject itself* (**2.2.2.1**) as most related
- *Student expression of feeling* (**2.4.1.3**) as least related

Ranked seventh among the themes with 7% of the constructs was Intellectual Stimulation. There were no constructs related to confidence in mathematics at all in this theme. There were no highly associated constructs for either art or music. These constructs were least related to confidence in art and music:

- Art: *'Going with the flow'* versus *thinking and planning are required* (**2.3.4.1**)

  Open ended versus *right or wrong/ cut and dry* (**2.4.2.2**)

- Music: *Creativity/ imaginative side of learning* (**3.3.5.5**)

  'Spark to light the fire' (**3.1.1.3**)

  Related to activity versus using your imagination (**3.3.3.1**)

  More in depth versus overview (**3.3.3.4**)

  Open ended learning/ changeable (**3.4.2.2**)

Ranked fifth among the themes with 9% of the constructs were Social Aspects and Motivation. There were no highly important constructs expressed for confidence in Social Aspects for mathematics or art. The only construct highly related to confidence in music was:

- *Involves others* versus *individual* (**3.4.2.3**)

As least related to confidence in Social Aspects, the original and reversal of this construct were also expressed:

- *Involves others* versus *individual*

  1.2.5.4, 1.3.1.4, 2.2.1.4, 3.3.1.1, 3.3.4.5

- *Involves student alone* versus *someone else involved*

  1.3.5.2, 2.3.3.4

In Motivation, the constructs of *makes students want to learn* and *needed for achievement* were mixed for all three subjects in their H-L indices.

- *Makes students want to learn; helps motivate; builds interest*

  H – Mathematics: **1.3.1.6, 1.3.5.5**; Art: **2.4.2.1**; Music: **3.4.2.4**

  L – Mathematics: **1.4.1.3**, Art: **2.3.1.4**

  The construct, *no need to try/ no motivation* (**1.3.3.1**), was also low for mathematics. Another low for music was: *not necessarily interesting if you know it already* (**3.2.1.4**)

- *Needed for student achievement*

  H – Music: **3.3.3.3**

  L – Art: **2.2.2.5**

The construct, *not necessary for a good grade* (**2.2.3.4**), was also high for art. *Developing student ideas* versus *affecting student achievement* (**3.3.2.2**) was low for music.

The four top ranked themes all contained more constructs than the 11% that would have occurred if placement had been random. In fourth place with 12%, <u>Nature of Task</u> had no highly related to confidence constructs in mathematics or music. The two in art were expressed this way:

- *The process is a physical experience* (**2.2.4.1**)
- *Student is actively engaged* (**2.4.1.4**)

The construct, *physical experience* **2.1.2.2** for art was also in the low range.

Other constructs in the low range of percentage of similarity to confidence in the <u>Nature of Task</u> theme, involved the topic of *active* versus *passive engagement*. Both this statement and its reversal appeared as least related to confidence:

- *Student doing something* versus *student observing*

  1.2.4.4, **1.2.5.5**, **2.2.3.3**, **3.4.1.4**

- *Laid back/ student is passive* versus *student is actively engaged*

  1.2.1.2, **1.4.2.1**, **2.3.3.1**

Music had two other least associated constructs to confidence:

- *Materials are needed* (**3.1.2.3**)
- *Reaction to learning* versus *involving mental process/ thinking* (**3.2.1.2**)

In third place was Knowledge and Comprehension with 15% of the constructs.

These two constructs were highly related to confidence in their subjects:

- Mathematics - *things are clear for student* (**1.3.4.4**)
- Music – *uses/ involves student understanding* (**3.2.3.3**)

The other constructs in this theme had mixed H-L indices and could be broken up into these sub-themes:

- Timing

  Happens after; reflection of understanding

  H – mathematics: **1.2.4.5**, art – **2.3.5.1**

  L - mathematics**:** **1.1.2.1**, art – **2.3.4.5**

  At beginning/ student gets ideas

  L - art:**2.2.3.5**, **2.2.5.5**

- Effect on understanding

  Helps/ increases understanding

H - mathematics: **1.3.2.2**, art – **2.3.3.5**

L - mathematics: **1.1.1.4**, **1.3.4.1**, art – **2.3.2.2**

Takes away/ interferes with understanding

L - music: **3.4.1.3**

No relationship/ little effect on understanding

H - mathematics: **1.4.1.4**

L - music: **3.3.4.1**

Student's Personal Self, with 18% of the constructs, ranked second. There were no highly important constructs related to confidence in music and only a few in mathematics and art:

- Mathematics: *Immediate understanding* versus *getting different views from outside influences* (**1.3.3.6**)
- Art: *Put yourself into your work* (**2.1.1.1**)

  Makes expressing ideas easier (**2.3.3.2**)

  Internal to student/ interest/ passion (**2.3.5.3**)

There were quite a number of least associated to confidence constructs. In mathematics:

- *Guess from inside student/ 'gut feeling'* (**1.1.1.5**)

- *Something student develops* versus *given to student/ no choice* (**1.2.1.1**)
- *Student must work at these* versus *student already possesses this* (**1.3.1.3**, **1.4.1.5**)
- *Student involved in learning* versus *student not involved in learning/ not first hand experience* (**1.3.3.3**)
- *Student makes their own sense of learning* (**1.3.5.3**)

The constructs least related to confidence in art in this theme were:

- *Bring in ideas of others* versus *personally related to students own ideas* (**2.3.4.4**, **2.4.1.2**)
- *Student has to learn these* (**2.3.5.4**)

Those least related to confidence in music:

- *From yourself* (**3.1.2.2**)
- *No need for someone to tell you what to do/ student already knows* (**3.2.3.4**)
- *Student's own ideas expressed* (**3.2.2.2**)
- *Possessing understanding* (**3.2.5.5**)
- *Highest achievement/ student is beyond the 'struggle'* (**3.3.5.2**)

Learning was the highest ranking theme of the elicited constructs with 19% of the constructs. In mathematics, most closely related to confidence were:

- *Helps students learn better* (**1.2.2.1**, **1.3.2.1**)
- *Comes before learning* (**1.3.2.5**)
- *Makes learning easier* (**1.3.3.5**, **1.4.2.3**)
- *The way you learn* (**1.4.1.2**)

Those least associated to confidence in mathematics were:

- *Helps students learn better* (**1.1.1.2**)
- *Important to the process of learning* (**1.2.1.4**)
- *Distracts students as they are learning* (**1.2.2.2**)

Those constructs of Learning most closely related to confidence in art:

- *How students get the information* (**2.2.1.2**)
- *Needed for learning* (**2.2.1.5**)
- *Helps learning* (**2.4.2.5**)

The least related constructs to confidence in art in Learning:

- *Happens in beginning/ before learning* (**2.1.1.4**, **2.2.1.3**)
- *Ideas needed to learn are given* (**2.1.1.3**)
- *Increases student learning* (**2.2.4.4**)

- *Necessary for learning/ essential for success* (**2.3.1.1**)

Learning constructs closely related to confidence in music:

- *Useful in learning* (**3.2.1.1**)
- *Improves ability/ knowledge* (**3.2.4.4**)
- *Learning is easier* (**3.2.5.4**)
- *Occurs before learning* (**3.3.4.3**)

Least related to confidence in music:

- *Happens while student is learning* (**3.2.4.3**, **3.2.2.1**)
- *Comes after learning has taken place* (**3.3.1.2**)
- *Inspires students to further out of school* (**3.3.2.3**)

The last theme to be compared by percentage of similarity to the fixed construct of Confidence was the construct of Enjoyment. During analysis, an unexpected result was obtained! Overwhelmingly, in most cases, regardless of the subject or confidence/ enjoyment category of the student, Enjoyment was very highly associated with Confidence. The breakdown was interesting, given that there were 14 interviewees per subject!:

- Mathematics – 11 **H, 1 L**
- Art – 11 **H**, 1 **L**
- Music – 12 **H**, 1 **L**

Four different cases were examined a little further in order to get a better understanding of what issue or circumstance was responsible for this difference they conveyed. The first case was the construct **1.4.2.6R**. It was in the high range in reverse order. The construct would then have read: *learning is boring* versus *learning is fun*. For this student there was a high relationship between *boring* and confidence. That was not a surprise, given that the student was in the **Not Confident/ Not Enjoy** category for mathematics. For mathematics, the enjoyment construct which was in the low (L) range of similarity to confidence was **1.2.5.1R**. This was an elicited construct that received a percentage of similarity score of 29.17%. The other percentages of similarity of the constructs of this interviewee were: **H**- **1.2.5.7** (58.33%), **1.2.5.6** (54.17%); **I**- **1.2.5.3** (50.00%), **1.2.5.2** (45.83%); **L**- in addition to **1.2.5.1R**, **1.2.5.5** (33.33%), **1.2.5.4** (33.33%). Expressed in the reverse polar order as indicated, construct **1.2.5.1R** becomes: *Different each time* versus *boring because every day is the same thing/ routine*. Construct **1.2.5.7** was the fixed enjoyment construct: *Makes learning fun* versus *learning is boring*. This construct, in the high range, did come out as closely related to confidence for this student. It could have just been the way in which the elicited construct was worded to bring out a different aspect of *boring*. In the third case, the elicited construct **2.2.2.3**: *Makes learning more interesting* versus *learning is boring*, had a low percentage of similarity to confidence of 45.83%, the range of percentages of similarity were quite close: **H** – **2.2.2.1** (66.67%); **I** – **2.2.2.2** and **2.2.2.4** (both with 54.17%); **L** – in addition to **2.2.2.3**, **2.2.2.5** (50.00%). Once again, it may be in the definition of the term, *boring*, that the answer lies. The last case fixed construct, **3.3.4.6R** received a low (L) score of 12.50% in reverse order. The range of scores for this interviewee was: **H** – **3.3.4.3** (50.00%); **I**- **3.3.4.2** and **3.3.4.4** (20.83%); **L** – in addition to **3.3.4.6**, **3.3.4.1** (16.67%), **3.3.4.5** (12.50%). As low in reverse order, it may be that, boring has little to do with confidence or for this student in the category of **Not Confident/ Enjoy** in music, as his/ her category implied, there really was no link between enjoyment and confidence!

In the next section the themes are explored by similarities and differences of student's constructs to confidence within each theme by subject.

6.5.1.2 Discussion of the Subjects, Constructs and Confidence

The question behind this thesis sought to understand how students construe their learning in mathematics versus the arts. The following paragraphs summarize the observations made by examining the themes of the constructs and the way in which they were used by interviewees to express how they construe their experiences in each of the subjects as related to Confidence. Using the advice of Honey (1992, p. 87), duplication of types of constructs in the **H**/ *'top'* and **L**/ *'tail'* data would indicate that there was not as *clear-cut* a consensus on the *strong association* of those constructs. This was taken into consideration in the following paragraphs which provide glimpses into how students construed their learning in mathematics and the arts.

Students expressed that these had strong associations with confidence:

- ***Mathematics*** – It *applies to their jobs and the future* and the student sees it as *useful*, although it should not be relevant just to *make you better* or just for *how to do*. They can *internalize their highs and lows*, but not when the feelings involve *just right answers.* Confidence occurs when understanding is increased and at the time that concepts are made *clear for the student. Immediate understanding* is more valued than *getting different views from outside influences.* Learning experiences were important in several ways: to *help students learn better, make learning easier* and *the way that you learn*. There were mixed views that being motivated to learn is important for confidence.

- ***Art*** – Enables student to apply learning. There were mixed views on whether the *physical experience* the amount of engagement influences confidence. The feeling about the subject itself. It *helps with understanding* and the students being able to *follow through after*. Important in learning is *how the students get the information,* things *needed for learning* and that are *helpful* to learning. Students can put themselves *into their work*, it *makes expressing ideas easier,* and it is *internal to the student/ interest/ passion*. It is not necessarily the *good grade* but other motivators to learn that are valued for confidence.

- ***Music*** – There are mixed views on the importance of being involved with others. It uses and involves *understanding* and several issues related to learning: *useful in learning, occur before learning, improve ability/ knowledge*, and *make learning easier*. Important for confidence, motivationally is, it is *helpful for strengthening achievement* and *makes learning interesting*.

Other observations, related to factors having an impact on confidence in mathematics versus the arts, from this data were made. The involvement of many people in the learning of a subject versus the individual learning alone held mixed H-L association for music and only low association in mathematics and art. Involvement with others during learning did not appear to be important for confidence in any of the subjects. Motivational constructs neutralized each other in regard to an association with confidence with no consensus on the constructs in this theme: *makes students want to learn* had H – L percentages of similarity to confidence for mathematics and art. Student achievement was only a key factor in music. *Applying* what has been learned is related to confidence in mathematics and art as long as it is not based on *how to do*. Intellectual Stimulation had no highly important constructs related to confidence in any of the three subjects. Types of activities did not really differentiate the subjects as related to confidence. None were highly related for mathematics and music and those in art did not have a strong significance for confidence. Active versus passive engagement also did not seem to have an impact. Other similarities related to confidence in mathematics when compared to confidence in the arts involved helping/ clarifying understanding and some issues of learning - *helps students learn better, makes learning easier*, and the process of how one learns.

Some factors differentiating confidence in learning mathematics versus the arts were motivational. While there was no clear consensus on *needed for* *achievement* in art and important in music, it was not mentioned at all for mathematics. In addition, in art confidence is highly related to *putting yourself in your work*, *makes expressing ideas easier*, and being *internal to students/ interests/ passions*. In mathematics, the only highly personally related factor to the students themselves with regard to confidence was *immediate understanding*. The feelings and emotions involved in mathematics had to do with *students' internalized highs and lows* regarding confidence as opposed to the way students feel about the subject of art itself.

The next sections take a look at how the constructs related to the enjoyment of learning mathematics, art and music.

6.5.2 Percentages of Similarity by Category with Enjoyment

There were a total of 237 constructs that were compared by percentage of similarity to the fixed enjoyment construct. Included in that number were the 42 confidence constructs and 195 constructs from the other themes. Table 6.4 below lists for the reader a breakdown of the constructs with the highest (H) and lowest (L) percentage of similarity to enjoyment by theme. A designation of 'R' next to a construct number indicates that it is most/ least similar in reverse order. The percentage of the numbers of H-L constructs contained in each theme is calculated to the nearest whole.

Theme / Number	Constructs	Percent of Theme	Percentage Similarity	H-I-L Range
1. Application (10) The utilization of what has been learned, relevance to real world	1.3.2.3 1.4.2.2 2.3.3.3	30%	58.33 58.33 70.83	H H H
	1.2.3.1 1.2.4.2 2.2.5.2 2.3.2.1	40%	4.17 29.17 33.33 12.50	L L L L

2. Intellectual Stimulation (14)	1.3.2.6	21%	50.00	H
Thinking, creativity, imagination, ideas	2.4.1.5		70.83	H
	3.3.3.1		58.33	H
	2.1.2.4	29%	37.50	L
	2.4.2.2		12.50	L
	3.1.1.3		12.50	L
	3.3.3.4		33.33	L
3. Social Aspects (17)	3.3.4.5	6%	41.67	H
Involvement with others.				
	1.3.1.4	29%	12.50	L
	1.3.5.2		20.33	L
	2.3.3.4R		20.83	L
	3.2.4.1		25.00	L
	3.4.2.3		37.50	L
4. Nature of Task (24)	1.2.5.5	25%	58.33	H
Engagement, physical/ mental	1.3.3.4		75.00	H
experience	2.2.4.1		54.17	H
	2.2.5.3		66.67	H
	2.3.5.5R		62.50	H
	3.4.1.4R		58.33	H
	1.2.4.4	33%	20.83	L
	1.4.2.1		37.50	L
	2.1.2.2R		45.83	L
	2.2.3.3		25.00	L
	2.3.3.1R		29.17	L
	3.1.2.3R		20.83	L
	3.2.1.2R		16.67	L
	3.2.3.5R		37.50	L
	3.2.4.2		33.33	L
5. Feelings and Emotions (10)	1.1.2.4	10%	58.33	H
Student feelings during lessons/ about the subject				
	1.2.5.2	70%	29.17	L
	1.3.1.5R		16.67	L
	1.3.2.4		4.17	L
	2.2.2.1		20.83	L
	2.4.1.3		54.17	L
	2.4.2.3		25.00	L
	3.3.5.3		25.00	L
6. Knowledge & Comprehension (30)	1.2.4.5R	20%	70.83	H
Understanding, facts acquired	1.3.4.4R		50.00	
	1.4.1.4R		50.00	H
	2.3.5.1R		66.67	
	2.3.5.6		66.67	
	3.3.4.1		37.50	H
				H
				H
				H
	1.1.1.4	40%	8.33	L
	1.1.2.1R		33.33	L
	1.2.1.3		16.67	L
	1.3.3.2		29.17	L
	1.3.4.1R		25.00	L
	2.1.2.3		45.83	L
	2.2.3.5R		29.17	L
	2.2.5.5		25.00	L
	2.3.4.5		33.33	L
	3.1.2.1R		25.00	L
	3.2.5.2		8.33	L
	3.4.1.3		0.00	L

7. Learning (37) Process, level, cause-effect	1.3.3.5 2.2.1.3 2.4.2.5 3.2.4.4	11%	58.33 54.17 75.00 62.50	H H H H
	1.1.1.2 1.2.1.4R 1.2.2.1 1.2.2.2R 1.2.3.3 1.2.4.3R 1.3.2.5 1.4.1.2R 1.4.2.3R 2.1.1.3 2.1.1.4 2.2.1.2 2.2.1.5 2.2.4.4R 2.3.1.1 3.1.1.4R 3.2.2.1 3.2.4.3 3.2.5.4 3.3.1.2R 3.3.2.3R 3.3.4.3R 3.4.1.1	62%	4.17 20.83 33.33 33.33 8.33 25.00 25.00 4.17 37.50 20.83 25.00 8.33 16.67 20.83 41.67 4.17 29.17 29.17 16.67 20.83 16.67 12.50 20.83	L L L L L L L L L L L L L L L L L L L L L L L
8. Personal Self (35) Ability, expression, student's ideas/ opinion	2.1.1.1 2.3.1.5 3.1.1.2 3.2.5.5	11%	70.83 62.50 45.83 37.50	H H H H
	1.3.3.6R 1.3.5.3 1.4.1.5 1.4.2.5R 2.3.1.2 2.3.2.3 2.3.3.2 2.3.4.2 2.3.4.4 2.3.5.3 2.3.5.4R 2.4.1.2R 3.1.2.5 3.2.2.2 3.2.3.4R 3.3.5.2	43%	37.50 29.17 0.00 37.50 45.83 29.17 37.50 33.33 37.50 45.83 29.17 50.00 29.17 25.00 41.67 20.83	L L L L L L L L L L L L L L L L
9. Motivation (18) Interest, making students involved in learning	1.2.2.3 1.3.1.6 1.3.5.5 2.2.2.5R 2.2.3.4R 2.3.1.3 3.1.2.4 3.2.1.5 3.3.3.3 3.4.2.4	56%	54.17 75.00 95.83 45.83 62.50 70.83 75.00 70.83 62.50 70.83	H H H H H H H H H H
	1.3.4.3 1.4.1.3 3.2.1.4 3.4.2.1	22%	29.17 8.33 25.00 33.33	L L L L

11. Confidence (42)	1.1.1.7	69%	58.30	H
	1.1.2.6		62.50	H
	1.2.1.6		70.83	H
	1.2.2.6		58.33	H
	1.2.3.6		58.33	H
	1.2.5.8		58.33	H
	1.3.1.1		62.50	H
	1.3.2.8		50.00	H
	1.3.4.6		54.17	H
	1.3.5.7		75.00	H
	1.4.2.7R		58.33	H
	2.1.2.6		70.83	H
	2.2.2.6		45.83	H
	2.2.4.6		50.00	H
	2.2.5.7		58.33	H
	2.3.2.4		70.83	H
	2.3.4.7		87.50	H
	2.4.2.7		62.50	H
	3.1.1.6		37.50	H
	3.1.2.7		62.50	H
	3.2.1.7		58.83	H
	3.2.2.6		66.67	H
	3.2.3.7		70.83	H
	3.2.4.7		66.67	H
	3.2.5.7		33.33	H
	3.3.1.6		83.33	H
	3.3.2.7		91.67	H
	3.3.3.7		66.67	H
	3.3.5.7		79.17	H
	2.4.1.7	5%	54.17	L
	3.3.4.7R		12.50	L

Table 6.4 Enjoyment H-L Range by Themes

The next sections of this thesis focus on an examination of interviewees' perceptions of enjoyment as related to each of the themes.

6.5.2.1 Factors Related to Enjoyment

The following paragraphs contain the summary of the observations that were made by examining the constructs contained in each of the themes as related to enjoyment in mathematics, art and music. The themes were considered in their reverse rank order as was done for the comparison to confidence. To express the meanings of constructs, examples are included. Where a fuller connotation needed to be expressed, the complete bipolar construct is given. Numbers in parentheses indicate each construct's four digit identification code.

The constructs of Application important for enjoyment in mathematics were:

- *Applies to real world* (**1.3.2.3**)
- *Student sees/ feels that mathematics is useful* (**1.4.2.2**)

The Application constructs least related to enjoyment in mathematics were:

- *Relevant to real world/ makes you better* (**1.2.3.1**)
- *Where mathematics is applied outside* versus *how to do ...* (**1.2.4.2**)

In art, this Application construct was highly related to enjoyment,

- *Enables student to apply learning* (**2.3.3.3**)

Art's least related constructs in this theme were:

- *More practical part of learning* versus *life lessons/ things needed but not learned in class* (**2.3.2.1**)
- *Student uses what is learned* versus *learning how to do* (**2.2.5.2**)

There were no constructs with high or low range percentages of similarity to enjoyment for music in the Application theme.

As for Feelings and Emotions, there were no highly associated constructs in art or music. In mathematics, the only highly related construct to enjoyment was:

- *Internalize highs and lows* (**1.1.2.4**)

Each subject contained constructs with a low relevance to enjoyment:

- Mathematics: *Helps students continue working* versus *makes student working frustration* (**1.2.5.2**)

  How student feels about it (mathematics) (**1.3.1.5**)

  Involves student feelings when working versus *just right answers/ no feelings involved* (**1.3.2.4**)

- Art: *The subject itself versus how you are feeling about the subject* (**2.2.2.1**)

  Student expression of feeling (**2.4.1.3**)

  Affect student feelings/ morale about the class (**2.4.2.3**)

- Music: *Support/ inspirational/ encouraging* (**3.3.5.3**)

The only construct in the <u>Intellectual Stimulation</u> theme related to enjoyment in mathematics was the highly associated:

- *Allows for more in depth thinking* (**1.3.2.6**)

In art, *sparks student creativity/ imagination* (**2.4.1.5**) was closely related to enjoyment and these were least closely related to enjoyment in art:

- *What you/ student thinks* versus *closed ideas* (**2.1.2.4**)
- *Open ended/out there* versus *right or wrong/ cut and dry* (**2.4.2.2**)

In Intellectual Stimulation, *related to activity* versus *using your imagination* (**3.3.3.1**) was closely related to enjoyment in music and these were least related to enjoyment:

- *Helps you keep knowledge* (**3.1.1.3**)
- *More in depth learning* (**3.3.3.4**)

There were no constructs highly related to enjoyment for Social Aspect in mathematics or art. The only construct highly related to enjoyment in music was: *involves others* versus *student alone* (**3.3.4.5**). This same construct meaning or its reversal was least related to enjoyment in all subjects.

- *Involves many other people* (**1.3.1.4, 2.3.3.4, 3.2.4.1, 3.4.2.3**)
- *Involves student themselves* versus *someone else involved* (**1.3.5.2**)

Motivation contained more constructs important for enjoyment than not important in each subject. There were no constructs with little importance for enjoyment in art. The constructs covered just two strands in this theme. Some had mixed H- L indices.

- *Makes students want to learn, related to interest in student world, student has reason to learn, gets student involve in learning*

  H: 1.2.2.3, 1.3.1.6, 1.3.5.5, 2.3.1.3, 3.1.2.4, 3.2.1.5, 3.4.2.4

  L: 1.4.1.3, 3.2.1.4, 3.4.2.1

- *Needed for achievement/ grades*

  H: 2.2.2.5, 3.3.3.3

L: **1.3.4.3**

Not necessary for a good grade (**2.2.3.4**) was high for art.

A similar situation arose in all three subjects in the Nature of Task constructs with the topic of active versus passive engagement and its relationship to enjoyment:

- *Involved/ active/ engaged* versus *sit back/ relax/ passive*

  H: **1.2.5.5, 1.3.3.4, 2.2.5.3, 3.4.1.4**

  L: **1.2.4.4, 1.4.2.1, 2.2.3.3, 3.2.3.5, 3.2.4.2**

  In reverse order, *passive* versus *active engagement* (**2.3.3.1**) was low in music.

The other Nature of Task constructs with a close association to enjoyment in the arts:

- *Physical experience* versus *mental* (**2.2.4.1**)
- *Hands-on experiences* versus *talking for understanding* (**2.3.5.5**)

These were also least related to enjoyment in the arts:

- *Physical experience* (**2.1.2.2**)
- *Ideas/ concepts* versus *materials are needed* (**3.1.2.3**)
- *Reaction to learning* versus *involved mental process/ thinking* (**3.2.1.2**)

In the theme of Knowledge and Comprehension, these constructs were closely related to enjoyment in mathematics:

- *Happens while student is trying to understand* (**1.2.4.5**)
- *Things are clear for student* versus *time of confusion* (**1.3.4.4**)
- *Little effect on student understanding* (**1.4.1.4**)

These had little importance for enjoyment in mathematics:

- *Helps student understanding* (**1.1.1.4**)
- *Reflection of understanding* (**1.1.2.1**)
- *Very rarely used for student understanding* (**1.3.4.1**)
- *Builds understanding* versus *measures your understanding* (**1.2.1.3**)
- *Important for student understanding* versus *may be confusing to student* (**1.3.3.2**)

Those constructs in Knowledge and Comprehension with a close relationship to enjoyment in art:

- *Student follow through after* versus *before teacher presents lesson* (**2.3.5.1**)
- *Clarifies understanding* (**2.3.5.6**)

Those constructs least related to enjoyment in art:

- *Happens before/ beginning of understanding* (**2.1.2.3**, **2.2.5.5**, **2.3.4.5**)
- *Result of understanding* (**2.2.3.5**)

In music, *required for understanding* (**3.3.4.1**) was most closely related to enjoyment in Knowledge and Comprehension. These were the constructs least associated to enjoyment in music in this theme:

- *Deeper understanding/ allows students to do work* (**3.1.2.1**)
- *Necessary for understanding* versus *result of understanding* (**3.2.5.2**)
- *Helps understanding* versus *interferes with understanding* (**3.4.1.3**)

In the Personal Self theme there were no constructs highly related to enjoyment in mathematics. These four were least related:

- *Immediate understanding* versus *getting different views/ understanding from outside influences* (**1.3.3.6**)
- *Students can be influenced by others* (**1.3.5.3**)
- *Student works to learn* (**1.4.1.5**)
- *Teacher not aware of what student knows or not* (**1.4.2.5**)

There were fewer constructs highly related to enjoyment in art in Personal Self than there were least related constructs. The highs were:

- *Put yourself into your work* (**2.1.1.1**)
- *Enhances ability* (**2.3.1.5**)

The lows in this theme in art were:

- *Student's own ideas* (**2.4.1.2**)
- *Brings in ideas of others* (**2.3.4.4**)
- *Student doing* versus *you possess/ innate* (**2.3.1.2**)
- *Independent activity* versus *rely on someone else* (**2.3.2.3**)
- *Makes expressing ideas easier* (**2.3.3.2**)
- *Work ethic required to learn artificial process* (**2.3.4.2**)
- *Student has to learn these* (**2.3.5.4**)
- *Internal to student/ interest/ passion* (**2.3.5.3**)

In music, enjoyment has a close relationship with the following constructs in the Personal Self theme:

- *Internal to the student* versus *teacher's opinion/ view* (**3.1.1.2**)
- *Possessing understanding* (**3.2.5.5**)

These have little to do with enjoyment in music:

- *Students work at these* (**3.1.2.5**)
- *Other people's ideas expressed* (**3.2.2.2**)
- *Highest achievement/ student is beyond the struggle* (**3.3.5.2**)
- *No need for someone to tell you/ student already knows* (**3.2.3.4**)

Across all three subjects, there were many more constructs in Learning with little relationship to Enjoyment than there were those considered closely related. In mathematics, the construct most closely related to enjoyment was: *makes learning easier versus student gets distracted* (**1.3.3.5**). Those which were least related to enjoyment in mathematics were:

- *Helps students learn better* (**1.1.1.2, 1.2.2.1, 1.2.3.3**)
- *Result/ Comes after learning* (**1.2.1.4, 1.2.4.3**)
- *Important to process of learning* (**1.2.1.4, 1.2.4.3, 1.4.1.2**)
- *Doesn't keep students from learning* (**1.2.2.2**)
- *Comes before learning* (**1.3.2.5**)
- *Makes learning mathematics easier* versus *not easy/ student may not know how to do it* (**1.4.2.3**)

This was the breakdown of constructs in the Learning theme as related to enjoyment in art:

H: *Comes after student learns* (**2.2.1.3**)

Helps student learning (**2.4.2.5**)

L: *Happens in the beginning of learning* (**2.1.1.4**)

Ideas needed to learn are given (**2.1.1.3**)

How student gets information (**2.2.1.2**)

Needed for learning (**2.2.1.5, 2.3.1.1**)

Not important for student learning (**2.2.4.4**)

In the theme of Learning, *improves ability/ knowledge* (**3.2.4.4**) is the one construct highly related to enjoyment in music. Those least related to enjoyment were:

- *Student should have/ needed for understanding* versus *building blocks of learning* (**3.1.1.4**)
- *Taken in while learning* (**3.2.2.1, 3.2.4.3**)
- *Learning is easier* (**3.2.5.4**)
- *Comes after/ result of learning* (**3.3.1.2, 3.3.4.3**)
- *Inspires students to go further out of school* (**3.3.2.3**)
- *Helps students learn* (**3.4.1.1**)

The final comparisons of percentage of similarity to the enjoyment construct were made by looking at the fixed constructs of confidence. As happened with Confidence, there was a high association between the constructs of confidence when compared to those of Enjoyment. Given that there were 14 interviewees per subject, these results spoke for themselves:

- Mathematics - 11 **H**, 0 **L**
- Art - 7 **H**, 1 **L**
- Music - 11 **H**, 1 **L**

Seven of the art interviewees expressed that confidence was highly related to enjoyment, one felt that confidence had little association with enjoyment, and the other six were in the intermediate range ('middle wadge')! This number breakdown made me recall the Anderson (1971, p. 54) study that became the catalyst for this research as cited in chapter 4 of this thesis:

> *"... some open-minded individuals may be positive in their attitudes towards art and artists, but they may also have an inferiority complex ... about their own personal art behaviours ..."*

There were three cases which needed to be examined a little further because of their H – L indices. They were all *student feels 'yes I can' do the work* versus *student feels like a failure*, the fixed construct. One construct in mathematics was high for confidence, but in the reverse order. For **1.4.2.7R** that *student feels like a failure* as closely associated to enjoyment was not a surprise due to this student's category of **Not Confident/ Not Enjoy**. The art construct, **2.4.1.7** had a low range of 54.17%. The range of values for this interviewee was small with higher values in the range: **H** – **2.2.4.1** (70.83%); **I** – **2.4.1.4** (66.67%), **2.4.1.1** (62.50%); **L** - in addition to **2.4.1.7**, **2.4.1.3** (54.17%), **2.4.1.2** (50.00%). What was highly associated in reverse order for the **-/-** in mathematics, is least associated in original order for this **-/-** in art! The music construct **3.3.4.7R**, had a low percent of similarity to enjoyment of 12.50% in reverse order. The other constructs from this interviewee: **H** – **3.3.4.5** (41.67%), **3.3.4.1** (37.50%); **I** – **3.3.4.2** (33.33%); **L** – in addition to **3.3.4.7**, **3.3.4.3** (12.50%). It appeared that these low percentages of similarity to enjoyment could be attributed to the little relationship that confidence has to enjoying for this **Not Confident/ Not Enjoy** student.

In the next section the themes are explored by similarities and differences of student's constructs to enjoyment within each theme by subject.

6.5.2.2 Discussion of the Subjects, Constructs and Enjoyment

The question behind this thesis sought to understand how students construe their learning in mathematics versus the arts. The following paragraphs summarize the observations made by examining the themes of the constructs and the way(s) in which they were used by interviewees to express the views on their experiences in each of the subjects as related to Enjoyment. Following the advice of Honey (1992, p. 87), duplication of types of constructs in the **H**/ *'top'* and **L**/ *'tail'* data indicate that there is not as *clear-cut* a consensus on the *strong association* of those constructs. The next paragraphs give a quick look at how students construed their learning in mathematics and the arts.

Students expressed that these had strong associations with enjoyment:

- ***Mathematics*** - It applies to the real world and students find it useful, although not in the context of *making you better* or concentrating on the *how to do* parts. It *allows for more in depth thinking*. There are mixed views on whether actively engaging activities have a strong association. They can *internalize their highs and lows*, but not highly related to enjoyment are their feelings about mathematics when *student working* involves *frustration*, the lack of feelings involved in *just right answers*, and the *mental knowledge gained*. Understanding is clarified. *Learning is made* *easier* because the *student is not distracted*. Overall, it *makes students motivated* to learn.

- ***Art*** – It *enables students to apply learning*, although not the *practical* or *how to do* parts. *It sparks creativity* and *imagination*. It *makes students motivated* to learn and it is *needed for achievement/ grades*. There were mixed views on the physical experience and active engagement, *hands-on experiences* versus *talking for understanding* was strongly associated to enjoyment. Understanding is clarified with student follow-through after. Helps learning. Personally for students, enjoyment was closely related to putting themselves *into their work* and when their *ability is enhanced*. Motivating factors are involved but not just because of *how well they do* or to get a *good grade*.

- ***Music*** – Students can *relate to an activity* versus *using their imagination*. There were mixed views on the involvement of others for enjoyment. The same is true for active engagement and required for understanding. It *improves ability/ knowledge*. Personally, student enjoyment in music is *internal to the student* and that the student *already possesses understanding*. Motivating factors include students *having a reason to learn* and it is *helpful for achievement*.

Some thoughts related to the factors related to enjoyment from this data came to the fore. As with confidence, the <u>Social Aspect</u> theme of the involvement of many people versus the individual held mixed H-L association for music and only low association in mathematics and art. Work in art is mainly completed by the individual artist. In most of the experiences of interviewees, the learning of mathematics does not involve anyone but the teacher presenting the lesson and the students *'sitting back'*. For the most part, students do not view mathematics as something that is done with others, merely an individual endeavour. Working with others, therefore, is not perceived as an possible part of enjoyment. Related to the idea of *'sitting back'*, there were some constructs in the Nature of Task theme which provided more food for thought. *Active* versus *passive engagement* had high and low percents of similarity to enjoyment in all three subjects. In art and music, that might indicate that, in Honey's words, there was no *clear-cut strong association* between actively engaging activities versus those of a passive spirit and enjoyment in the learning of those two subjects. In contrast, during the repertory grid interviews, a very common view about the learning of mathematics was expressed by a student, who was a **+/-** (**Confident/ Not Enjoy**), regarding enjoyment in mathematics: *It's boring, the same thing every day – taking notes and listening to the teacher.* That perception of a mathematics class could explain the mixed association in this construct to enjoyment. The highly related percentage of similarity of active versus passive engagement to enjoyment was from **+/- (Confident/ Not Enjoy)** and **-/+** (**Not Confident/ Enjoy**) categories, the two middle categories of students of interest in this research. The **+/-** (**Confident/ Not Enjoy**) and **-/-** (**Not Confident/ Not Enjoy**) categories had low percentages of similarity.

When enjoyment of mathematics is compared to enjoyment of the arts, there are similarities in many of the themes: applying what has been learned in mathematics and art, learning, motivation, and understanding. The differences between enjoyment and learning in mathematics versus the arts occur in the themes involving Intellectual Stimulation, Feelings and Emotions, and Personal Self. In mathematics enjoyment was highly related to *allows for more in depth thinking*, as opposed to the arts where enjoyment was associated with *creativity and imagination*. There were no *internalized highs and lows* in the arts, however, in mathematics that was strongly related to student enjoyment. In the arts, there was a very strong association between things personally related to the students and enjoyment: putting themselves in their work and things internal to student, especially. There was no construct highly related to enjoyment involving students' personal self at all in mathematics!

The above summaries of the previous sections did not make any differentiation in the constructs of interviewees based on their confidence and enjoyment categories. Characterizations of the twelve subgroups of students, based upon their confidence/ enjoyment in each subject and the data collected through the repertory grid interviews, follows in the next section.

6.5.3 Confidence and Enjoyment

A key question in my research: why do so many students, confident in their ability to do mathematics, not enjoy learning the subject? My next focus was to look more deeply at how the similarities and differences by subject are affected by a student's individual confidence and enjoyment.

6.5.3.1 The Connection between Confidence and Enjoyment

There were numerous constructs in each of the nine themes of elicited constructs that were determined to be in the same H-L index for both confidence and enjoyment, i.e. high for confidence and high for enjoyment. There were also others with reverse indices, i.e. high for confidence and low for enjoyment.

- High for both confidence and enjoyment - 17
- Low for both confidence and enjoyment - 46
- Reverse indices - 20

The specifics of the connections between confidence, enjoyment, and the constructs are indicated in the category pen pictures in the next section. According to Embacher and Buttle (1989, p. 5) it is important find a way to analyse the grids individually because each person has their own personal sense of the world and at the same time view them collectively to look for "*patterns, similarities and overlaps.*"

6.5.3.2 'Summary Pen Pictures' of Students by Subject and Category

Honey's (1979b, p.458) final step is to,

> *"Summarise the top data, taking the findings ... to produce two pen pictures. One describing favourable attitudes ... the other about unfavourable attitudes ..."*

Pen pictures are created in this section to summarize both the favourable and unfavourable attitudes conveyed in the views of the students involved in this research according to their subject and category and the constructs that were elicited during the repertory grid interviews. The reader will recall that there were two interviewees for each subject in the categories, **Confident/ Enjoy** and **Not Confident/ Not Enjoy** and five interviewees for each subject in each of the categories, **Confident/ Not Enjoy** and **Not Confident/ Enjoy**. It was expected, by the intention of this research of wanting to know more about the two middle categories, that there would be more to the characterizations of the **+/-** and **-/+** than the **+/+** or **-/-** to answer the questions that began this research.

+/+ Mathematics

The **Confident/ Enjoy** student in mathematics views confidence and enjoyment both as highly associated with each other. For them the feelings that they experience when they are learning mathematics, whether positive or negative, are closely related to both confidence and enjoyment. There is little impact on confidence in and enjoyment of learning mathematics for the **+/+** student from the aspects of lessons which are meant to enhance their learning and understanding. In addition, confidence is not affected by ideas which originate from outside sources as opposed to those which involve the *guesses* and *gut feelings* of the students themselves.

+/- Mathematics

Students who are **Confident** but do **Not Enjoy** mathematics generally felt that confidence and enjoyment were closely related to each other. For only one of this group was the confidence construct in the intermediate range of percent similarity to enjoyment, which indicated that this student had no firm view on its importance to enjoyment. Highly associated with confidence for the **+/-** students were the aspects of lessons which enhance student learning and the results of their achieving an understanding of the concepts. The acknowledgement that mathematics can be *applied to jobs and the future* also had a strong impact on their confidence. Confidence for the **+/-** student had little to do with the process of learning itself, the routine nature of mathematics lessons and activities which involve others. There did not seem to be a firm consensus on confidence in mathematics based upon active as opposed to passive engagement for these students. Relevance to the real world in the context of *how to do* or that it *makes you better* had very little association with confidence or enjoyment for the **Confident/ Not Enjoy** student. Enjoyment of mathematics for the **+/-** student was not associated with those aspects of lessons aimed at enhancing learning, strengthening understanding and the outcomes of learning that enable students to continue learning. There was no consensus on active versus passive engagement for these students as related to enjoyment. The factors that the **+/-** students did find highly associated with enjoyment in learning mathematics were those which connected ideas to something of interest in the students' world and the 'aha' moments which they experience when they understand the concepts. Their lack of enjoyment was very much related to the *boring* routine nature of their daily mathematics lessons.

-/+ Mathematics

In the **Not Confident/ Enjoy** category for mathematics, only three out of five students felt that enjoyment was highly related to confidence and four out of five students felt that confidence was related to enjoyment. Other closely related constructs to confidence in mathematics were those aspects of lessons which enhanced learning and increased student understanding. In the low range of percentage of similarity to confidence was what the student already possesses or knows as opposed to those things at which they must work. Strongly related to both confidence and enjoyment were things that motivate students to learn, *make learning easier* and the moment at which their understanding of the concepts is clear and they are no longer confused. Also highly associated with enjoyment of mathematics for the **-/+** student are the lessons which provide opportunities for students to be actively engaged, experience more in depth thinking and utilize applications to the real world. The factors least associated with confidence and enjoyment in mathematics for these students are lessons which involve others, the ability of the student to make their own sense of what is being learned, the *mental knowledge gained*, aspects that have little to do with understanding and the feelings that students experience while learning as opposed the answers just being right or wrong. Also not much related to enjoyment for the **Not Confident/ Enjoy** student were those aspects of lessons which were important for learning, understanding and achievement.

-/- Mathematics

Only one of the students in this category of **Not Confident/ Not Enjoy** felt that confidence and enjoyment were strongly related to each other but in reverse order. That is, that the key characteristics of confidence and enjoyment in mathematics respectively, are that the *student feels like a failure* and that *learning is boring*. The usefulness of mathematics and aspects of lessons which had little effect on student understanding were both highly related to confidence and enjoyment for the **-/-** student. In addition, things that make learning easier and the process of learning were also highly related to confidence in mathematics. Aspects of lessons which motivate students to learn and the need for students to work at learning were not much related to either confidence or enjoyment. Passive engagement of students during lessons had a low association with their confidence. The easiness of the concepts, student ability to know how to do something, active engagement and a teacher's lack of understanding of what the students know had a low percentage of similarity with enjoyment for the -/- student. There was no consensus on the relationship of the process of learning to student enjoyment.

+/+ Art

One student in the **Confident/ Enjoy** category in art felt that confidence and enjoyment were highly associated with each other. The other felt that *putting yourself into your work* was strongly related to both confidence and enjoyment and that only enjoyment was highly related to confidence. The prerequisites of learning and physical experiences had little association to confidence and enjoyment for these students in art. The students' own ideas had little relationship to enjoyment for the **+/+** art student.

+/- Art

Three out of the five students in this category felt that enjoyment was closely related to confidence and the same was true (although not the same three) for a high association of confidence with enjoyment in art. Highly related to both confidence and enjoyment for these **Confident/ Not Enjoy** students were factors that were *not necessary for good grades* and their opportunities to have physical experiences during lessons. Least related to both confidence and enjoyment were student utilization of what they were learning and the beginning of the process of learning through which students obtain ideas. There were mixed views on whether active student engagement was related to enjoyment in art for these students. Also highly associated with confidence for this group were those aspects necessary for learning, the ways in which students get their information and the subject of art itself. By contrast, least associated with confidence were the aspects of lessons which occur before understanding, involve and enhance student learning and their interest in learning and those that may have an effect on achievement. As for enjoyment, the **Confident/ Not Enjoy** students felt that their abilities to use the concepts that they have learned in art and that what they do has little effect on how well they do are highly related to enjoyment. Least associated with enjoyment in art for the **+/-** student are aspects which are related to learning, the results of understanding, *how students get information* and the subject of art itself.

-/+ Art

In this category, four out the five students felt that enjoyment was highly related to confidence and only two out of five felt that confidence was highly related to enjoyment. The ability of students to apply and follow through with what they have learned was highly associated to confidence and enjoyment in art. For the **Not Confident/ Enjoy** student in art, aspects that enhance their understanding and their ability to express their ideas and those aspects of learning art that come from inside the student were strongly associated to confidence. Motivating factors, aspects of lessons that enhance student ability, clarify their understanding and provide them with hands-on experiences were strongly associated with enjoyment in art for the **-/+** student. Least related to confidence in art were the aspects of learning art that are: required for learning and to gain knowledge, build student understanding and occur after understanding has taken place, lessons that are learned in life, involve the student working alone, passive engagement and incorporating the ideas of others. There were many factors least related to enjoyment for the **-/+** student in art: aspects of lessons that require the student to work at art because that they do not have an innate ability in the subject, the necessary and practical aspects of learning and understanding, things that make expressing ideas easier, working alone, passive engagement, incorporating the ideas of others into their work and things which involve students' own interests and passions.

-/- Art

One of the students in this **Not Confident/ Not Enjoy** category felt that confidence and enjoyment were closely related to each other in art. The other held mixed views on the relationship between confidence and enjoyment. This second student felt that something that sparked *student creativity* during lessons was strongly related to the enjoyment of learning art. Highly related to both confidence and enjoyment was something that helped student learning. Also closely related to confidence for the **-/-** student in art were lessons that both actively engaged and motivated students. The **Not Confident/ Not Enjoy** student found that open ended activities and the expression of a student's own ideas and feelings had little association to confidence or enjoyment. In addition, an aspect of a lesson that affected a student's feelings or morale also had little to do with enjoyment of art for the **-/-** student.

+/+ Music

For the **Confident/ Enjoy** student in music, confidence and enjoyment are both highly related to each other. Ideas which initiate with the student as opposed to the teacher's opinion, student ability to understand concepts and the reasons for learning were also closely related to enjoyment in music for these students. Least associated to confidence and enjoyment for the **+/+** music is the ability to retain knowledge. Confidence was least associated with aspects of learning that came from the student himself as opposed to coming from outside influences. Enjoyment in music was not related to things required for understanding concepts, the need for students to work at learning music and student possession of a deeper understanding of what has been learned. The bipolar construct, *materials needed* versus *ideas/ concepts*, was low for confidence in its original order and low for enjoyment in reverse order. Which would indicate that the use of materials in music had little to do with confidence for the **+/+** student and ideas and concepts had little to do with their enjoyment.

+/- Music

All students in this **Confident/ Not Enjoy** category felt that confidence and enjoyment were closely related. Factors that improve student ability and knowledge and students being able to understand the material were closely related to both confidence and enjoyment for this music student. Closely associated to confidence were those aspects of lessons which supported student learning and understanding. Motivating experiences were highly related to enjoyment for the **+/-** music student. Those aspects with little association for both confidence and enjoyment had to do with learning, whether it was while learning was taking place or afterwards as a reaction to the learning, and the when the teacher explains a concept that the student already knows. For this student, the expression of a student's own ideas and the lack of interest, because a student already knows what is being taught, have very little relationship to student confidence in music. Those aspects with little association to the enjoyment of learning music for the **+/-** student involve lessons which make the learning of music interesting, expose students to the expression of other people's ideas, involve active engagement and the participation of others, and the inclusion of ideas to make learning easier and enhance student understanding.

-/+ Music

One of the students in this **Not Confident/ Enjoy** category felt that confidence and enjoyment were both little related to each other but in reverse order, i.e. *student feels like a failure* and *learning is boring* respectively. The other four students all felt that confidence and enjoyment were both highly related to each other. The use of strategies to help strengthen student achievement was highly related to confidence and enjoyment. In addition, the opportunities available to students as a result of learning were closely associated to confidence for the **-/+** music student. The other factors closely related to enjoyment were the use of specific activities, the involvement of others and things required for student understanding. The aspects least associated to both confidence and enjoyment for the **Not Confident/ Enjoy** student in music were those which involved student experiences after learning had taken place: high achievement, students are able to go more in depth and be inspired to continue to go further in music after they are out of school. Confidence itself was least associated to the participation of others, the involvement of students, specific activities, understanding the development of student ideas, creativity and imagination, and those things which determine whether a student likes or dislikes music. Enjoyment for the **-/+** student in music had little to do with the results of learning music or the support and encouragement they receive while learning.

-/- Music

For the **Not Confident/ Not Enjoy** student in music, lessons that made learning interesting were highly related to both confidence and enjoyment. Confidence was closely related to enjoyment and the involvement of others. Active engagement was closely related to enjoyment for the **-/-** student. Least related to confidence were active participation, deterrents to understanding and lessons which are open ended. Enjoyment for these students had little association to those aspects which enhanced learning and understanding, involved others and engaged students.

In the next section, to summarize the important connections for confidence and enjoyment from the repertory grid interviews, these *pen pictures* were 'developed into snapshots' for the reader.

6.5.3.3 Close Associations with Confidence and Enjoyment

The *pen pictures* of the previous section contain a portrayal of the behavioural indicators of categories of students by subject. *Snapshots* focusing only on those constructs with a high percentage of similarity to confidence and enjoyment made an interpretation of the data less messy.

Mathematics: Confident/ Enjoy

- Confidence – *Internalized highs and lows*
- Enjoyment - *Internalized highs and lows*

Mathematics: Confident/ Not Enjoy

- Confidence – *Helping students learn better*, what *happens after understanding*, and applying what has been learned to jobs and the future
- Enjoyment – *Related to interest in the students' world, happens while student is trying to understand*, and *boring because every day is the same thing/ routine*

Mathematics: Not Confident/ Enjoy

- Confidence – *Useful for learning, came before learning, increased understanding, motivate students to learn, makes learning easier*, and when *things are clear*

- Enjoyment – *Motivates student to learn*, *makes learning easier*, when *things are clear*, it *applies to the real world*, *allows for more in depth thinking*, and *doing something* active

Mathematics: Not Confident/ Not Enjoy

- Confidence – *Student feels sees the usefulness of mathematics*, *little effect on understanding*, *makes learning easier*, and the *way you learn*

- Enjoyment - *Student feels sees the usefulness of mathematics* and *little effect on understanding*

Art: Confident/ Enjoy

- Confidence – *Putting yourself into your work*

- Enjoyment - *Putting yourself into your work*

Art: Confident/ Not Enjoy

- Confidence – *Not necessary for a good grade*, *physical experience*, *needed for learning*, *how the student gets information*, and the *subject itself*

- Enjoyment - *Not necessary for a good grade*, *physical experience*, *comes after learning*, and *no effect on how you do*

Art: Not Confident/ Enjoy

- Confidence – *Enables students to apply learning, student follow through later, helps understanding, makes expressing ideas easier,* and *internal to student*

- Enjoyment – *Enables students to apply learning, student follow through later,* motivating factors, *enhance ability, clarifies understanding,* and *hands-on experiences*

Art: Not Confident/ Not Enjoy

- Confidence – *Helps learning, student actively engaged,* and *motivates student*

- Enjoyment - *Sparks student creativity* and *helps learning*

Music: Confident/ Enjoy

- Confidence – (Only enjoyment)

- Enjoyment – *Internal to the student* and *student has understanding/ reason to learn*

Music: Confident/ Not Enjoy

- Confidence – *Improves ability/ knowledge, possessing understanding, useful in learning, uses/ involves student understanding,* and *makes learning easier*

- Enjoyment - *Improves ability/ knowledge, possessing understanding, and motivates student*

Music: Not Confident/ Enjoy

- Confidence – *Helpful for strengthening achievement* and *the result of learning*
- Enjoyment – *Helpful for strengthening achievement, related to activity, required for understanding*, and *involves others*

Music: Not Confident/ Not Enjoy

- Confidence – *Makes learning interesting* and *involves others*
- Enjoyment - *Makes learning interesting* and student *being active*

The following generalities are proposed to understand the recurrent themes in the experiences of these students in learning mathematics versus the arts.

For the **+/+** student, regardless of the subject, their confidence and enjoyment were strongly connected to something internal and personally related to themselves, whether it be the feelings they had, the work that they did, or the motivation they possessed.

Confidence, for the **+/-** student, hinged on learning, understanding, and utilization of what had been learned for mathematics and music. In art, confidence was more about the physical experience and art itself, not necessarily a good grade. Enjoyment for the **Confident/ Not Enjoy** had some similarities related to interest and understanding. In this category, each subject had a factor that differentiated their enjoyment. In music, it was the improvement of their ability and knowledge. In art, it was the *physical experiences* during the learning and what *came afterwards* in addition to no relationship to achievement. In mathematics, the issue that seemed to determine their lack of enjoyment was *boring because every day is the same thing/ routine*.

The students who, while not confident in the subject still enjoy learning, saw confidence as related to learning and understanding in all three subjects. Each subject had other factors to set apart their confidence. In music, the issue was *strengthening achievement*. Art students also viewed confidence as associated with applying learning with follow through later, and aspects *internal to the student* such as, *expressing ideas easier*. **-/+** students in mathematics primarily related their confidence to learning and understanding but added motivation as a reason. Enjoyment was also related to learning and understanding in all three subjects. Music students also felt that enjoyment was affected by achievement, *activity*, and the *involvement of others*. In art, the aspects of enjoyment also included applying and follow through, motivation, enhancing abilities, and *hands-on experiences*. The **Not Confident/ Enjoy** students felt that their reasons for enjoyment in mathematics included motivation, application to the real world, more *in-depth thinking*, and active engagement.

For the **-/-** student, there was more differentiation in the reasons influencing confidence and enjoyment between mathematics and the arts. Confidence in art and music was related to interest, motivation, and active engagement. In mathematics, the issues were those of *usefulness of mathematics*, *little effect on understanding*, and learning. In the latter, confidence was related to the process of learning, *making it easier* and *how* *you learn*. Enjoyment for the **Not Confident/ Not Enjoy** was associated with interest, being active and *creativity/ imagination* in the arts. In mathematics, enjoyment was related to two of the same issues of confidence for this group, its usefulness and the *little effect on understanding*.

Due to "*the great diversity of viewpoints on education held by particular individuals,*" Pope and Keen (1981, p. 3) claim that to classify these perspectives may do an injustice to them. The analysis of the repertory grid interviews contained in this chapter gives, as Coshall (1991, p. 356) states, "*a snapshot of part of a person's overall construct system and thereby be liable to change.*" According to Coshall, *snapshots* only seek to gauge and describe.

Through my snapshots, I have sought to assess and explain students' attitudes and beliefs about learning mathematics versus the arts based upon their confidence and enjoyment in this analysis. What possible interventions could produce changes in these students' construct systems for a more favourable view toward learning mathematics? The final chapter of this thesis ties together the implications of my research for mathematics education.

Chapter 7 Conclusions and Implications

7.1 Introduction

This chapter of my thesis marks the end of a long journey which was undertaken to enhance the teaching and learning of mathematics in my own classroom and hopefully those of countless other mathematics teachers. In addition to my love of mathematics, I have long had an appreciation of art and music. Early in my research, it amazed me to discover that many students enjoy learning art and music but claim not to be very good at these subjects. This rarely happens in mathematics!

An early and surprising finding of this research is that, students must perceive themselves good at mathematics before they can enjoy the subject and, sadly, many of those who do perceive themselves to be good in mathematics do not enjoy learning it!

In her 1997 book, The Power of Mindful Learning, Ellen Langer (p. 61) expresses her belief that approaching any disagreeable activity with a different attitude can reverse an already established mindset about the activity. It is the frame of mind which creates the difficulty, not the activity. The mindset held by so many students is that learning mathematics is not an enjoyable experience. In order to find ways to approach the teaching of mathematics and create a different outlook towards the subject, I deliberately chose to compare student learning in mathematics with their experiences in the arts which they do find pleasurable. These are the questions which guided my way:

- Why don't students like learning mathematics when they are good at it?
- What makes students who don't perform well in mathematics still like learning it?
- How do these compare with students' attitudes and beliefs in the same categories above for art and music?
- What can we learn from students' attitudes in art and music that can improve the teaching and learning of mathematics and help foster a positive attitude toward learning mathematics in all students?

During my investigation, based upon their responses, four categories of learners were considered: **Confident/ Enjoy (+/+)**, **Confident/ Not Enjoy (+/-)**, **Not Confident/ Enjoy (-/+)** and **Not Confident/ Not Enjoy (-/-)**. A summary of my findings and conclusions regarding the attitudes of these students will be reviewed in the next sections.

7.2 Attitudes of the Four Categories of Learners

The open-ended response on the questionnaire (n=1226) from students from the United States and the United Kingdom, asked participants to categorize themselves in mathematics, art and music by their levels of confidence and enjoyment and then explain the choice they selected for one of the subjects. The rich data which resulted was analyzed in two different ways, first by calculating the frequency of each confidence and enjoyment category by subject and then by classifying the explanations that participants gave for their category choices, which were later summarized into category profiles.

7.2.1 Frequencies of Categories by Subject

Table 7.1 below, lists the percentages of students surveyed who self-selected in each category by subject in question 8 [See Appendix C]. Surprising information emerged in the comparison of the rankings of the categories for each subject.

Category	Mathematics	Art	Music
+/+	42.0%	33.1%	37.5%
+/-	42.6%	8.1%	14.7%
-/+	6.8%	35.4%	20.9%
-/-	8.1%	23.3%	26.7%

Table 7.1 Category Frequencies

The **Confident/ Not Enjoy** category ranked first in a choice of category for mathematics with 42.6% of the students. This data supports the findings of Heubner and McCullough (2000, p. 334) and Boaler, William and Zevenbergen (2000, p. 7) who state that even students who do well in school or mathematics don't always report being satisfied with their experiences. In contrast, in both art and music this category ranked last with 8.1% and 14.7% of those student responses respectively. What is it about the learning mathematics that would cause the percentage of students who are confident but do not enjoy mathematics to be so many more times the percentage of students in the arts in the same category?

Anderson's (1971, p.54) opinion that students lacking confidence in art can still enjoy the subject itself is corroborated by this research in which the highest ranking percentage of frequency in art was the **Not Confident/ Enjoy** with 35.4% of the student responses. It was third highest in music with 20.9%.

Considered as a whole, the highest level (84.6%) of student confidence from the three subjects is in mathematics, which also has the greatest level (50.7%) of lack of enjoyment. By sharp contrast, and paradoxically, the greatest level (58.7%) of lack of confidence is in art, which has the greatest level (68.5%) of student enjoyment.

7.2.2 What Their Comments Tell Us

When the details of Tables 4.11, 4.12 and 4.13 were combined I looked at the top three averaged rankings out of the eight comment characterizations (<u>Atmosphere/ Environment</u>, <u>Emotional Appeal</u>, <u>Impressions of the Subject</u>, <u>Motivation</u>, <u>Nature of the Tasks</u>, <u>Personalization</u>, <u>Relevance</u> and <u>Teacher</u>)

for all categories together by subject, as organized in Table 7.2 below.

Rank	Mathematics	Art	Music
1	Personalization	Personalization	Nature of Tasks
2	Impression of Subject	Nature of Tasks	**Motivation**
3	**Motivation**	**Motivation**	Personalization

Table 7.2 Top Three Comment Characterizations by Subject

Personalization, the expression/ challenge of self and impression of self in ability and achievement, and Motivation, the enthusiasm for learning the subject, interest, engagement and those factors which encourage/ deter from wanting to learn, ranked in the top three for all subjects and in exactly the same positions for mathematics and art. This indicates that challenging or expressing themselves is a personally important factor in determining student confidence and enjoyment, especially in mathematics and art and a little less so in music. Motivation is also a strongly related factor in confidence and enjoyment. The comments contained under the Motivation characterization whether positive (*fun*) or negative (*boring*) clearly have an impact on students' attitudes.

It is not surprising that the two characterizations of Personalization and Motivation are both ranked highly in the comment characterizations for all three subjects when we recall the work of Dewey (1913, p. 16), who says that psychologically, taking interest in anything must be related directly and personally to an individual. Caine (2004, p. 12) in an article relating creativity, imagination and learning, states that in the act of trying to figure something out, it is very important that *"each learner must put it all together personally."* Amabile (1989, pp.54 - 55) says that intrinsic motivation is the result of an individual doing something for their own personal reasons and no one else's.

There exists a clear separation between mathematics and the arts when it comes to the third of the top three comment characterizations. In art and music, students' rationales for their self-categorization fall under the Nature of the Tasks characterization which ranked first for music and second for art. Students in the arts primarily explain their confidence and enjoyment based upon the learning process and the activities involved. Most of their comments were related to doing something: *drawing, painting, listening* and *playing*.

Sinclair (2001, p. 29) states that during her task-based interviews related to the role of aesthetics in learning mathematics, students commented that her tasks were different from their regular mathematics lessons because they actually had *"to do things."* Nature of the Tasks' average ranking in mathematics was sixth. In those comments students mostly referred to doing a different kind of something: solving ... *equations* or *problems*.

For mathematics students, Impression of the Subject was second in average ranking and fifth for both art and music meaning that students seem to use reasons based on the nature of the subject, its level of difficulty, the effort and time required in the subject, tedium, intellectual stimulation and understanding much more for their category choices in mathematics than they did in art and music. Students' specific comments paint two very different pictures about their notions of mathematics (logical, takes time and effort, right or wrong, always an answer) compared with the arts (creative, easy, fun, imaginative, feelings, freedom).

7.3 Confidence and Enjoyment in Mathematics, Art and Music

Honey's Method of analysis was uniquely suited for developing an understanding of the relationship between students' confidence and enjoyment from the data collected in the 42 repertory grid interviews I conducted for this research. It enabled me to calculate and compare percentages of similarities between each elicited construct and those which were fixed, namely Confidence and Enjoyment. In addition, a percentage of similarity was calculated for the fixed constructs with each other. As was stated in Chapter 6, these scores were then sorted into High, Intermediate and Low ranges of similarity. The determination of whether the relationship of the individual constructs to Confidence or Enjoyment was considered strong or weak depended upon whether each had a High or Low range of percentage of similarity score.

The constructs were sorted into nine themes: Applications, Intellectual Stimulation, Social Aspect, Nature of Tasks, Feelings and Emotions, Knowledge and Comprehension, Learning, Personal Self and Motivation, plus the fixed constructs, Confidence and Enjoyment.

Student confidence in mathematics, art and music was strongly related to Learning – its process, level and cause and effect, and Knowledge and Comprehension indicating that student confidence in mathematics, art and music depends on how and how well students have learned and the level of what they perceive to be their understanding in what they have learned.

A few differences were noted in the constructs related to Confidence in mathematics versus the arts in the themes of Motivation, Personal Self and Feelings and Emotions. The Motivation theme was not at all important for Confidence in mathematics, but it was strongly linked to Confidence in art and music. Under the theme of Personal Self, art students felt strongly that *putting yourself into your work, expressing ideas easier* and involved with *interests and passions* were strongly related to Confidence. Under Feelings and Emotions, feelings in art were related to the subject of art and in mathematics *the internalized highs and lows* referred mainly to how students felt depending on their level of understanding.

In mathematics, art and music, Enjoyment is related to Application, the utilization of what has been learned, Knowledge and Comprehension, when understanding is clarified, and Motivation. There are also differences in Enjoyment when mathematics is compared to the arts. In mathematics Intellectual Stimulation, which involves thinking, creativity, imagination and ideas, is closely linked to enjoyment as is also true for the Feelings and Emotions of the *internalized highs and lows*. Personal Self, putting themselves into their work with ability and expression, was strongly linked to Enjoyment in art and music. This can be understood by realizing that as Levitin (2006, p. 225) and Perrin (2004, p. 23) explain it, the teenage years are when students experience a time of self-discovery which creates in them a need to find and communicate each with their own unique voice. Brown (2007, p. 759) asserts that "*Mathematics, like art, can teach us* *about ourselves."* It does not appear that many students believe in the ability of mathematics to do that except to say how right or wrong they are.

Freedom for the individual to work on their own, under the Social Aspect, was strongly related to Enjoyment in art as well. Working with others under the Social Aspect, was weakly linked to Enjoyment in mathematics. An explanation for this may be the impression many students have that in most classrooms the only 'other' person they work with in the classroom is the teacher, a situation which involves the *teacher presenting the lesson* and the individual student *sitting back and listening* to the teacher!

7.4 Returning to the Research Questions

My project began with a group of research questions for which I was seeking answers. The numbers indicated in parentheses are the identification codes representing the individual students.

Why don't students like learning mathematics when they are good at it?

The data gathered during the course of this study show that students who are confident in their ability to do mathematics but don't like learning the subject believe that mathematics is boring, tedious, complicated, hard, confusing and frustrating. From their own findings, Ruffell, Mason and Allen (1998, p. 12) suggest perhaps that it is *"cool to be or profess to be bored in school."* **Confident/ Not Enjoy** students in my study find no value to the study of mathematics, *"I don't see the point of learning it"* (1018). This is made worse by the fact that they have no freedom to choose whether to enrol in it or not, as it is a mandatory course in school. *"I don't like maths* *because it's boring, but I do it because I have to."* (578)They find no pleasure in the time and effort required to learn mathematics outside of class and the daily recurring routine of lessons presented by the teacher as they listen, take notes and do exercises. As one student put it, *"Math is very boring and repetitive; we do the same thing everyday"* (21).

While there was no clear consensus, the idea of *sitting back* passively for some students of this group is a primary issue as active engagement is strongly associated with enjoyment for them, *"I can do math but I don't like to sit and learn about it"* (1058).The process of learning and building understanding had little connection to enjoyment for the **Confident/ Not Enjoy** group, although the point at which understanding takes place did. Caine (2004, p. 11) states that *"Getting it at any age is exciting, joyous, fun, satisfying."* The enjoyment for these **+/-** students is not sustained beyond that point of getting it! Once they do get it, students want to move on. Repetition and review for them is unnecessary and boring.

What makes students who don't perform well in mathematics still like learning it?

The key aspect contributing to enjoyment in learning mathematics for those students, in the **Not Confident / Enjoy group**, is the feeling of accomplishment they experience when they have faced the challenges of learning mathematics and have overcome the previous obstacles to their success. Understanding brings them happiness and pleasure and that, they feel, makes it well worth the effort. Perhaps it is their lack of confidence to begin with that makes their enjoyment so strong. These students also appear to believe that applications to the real world, time for more thinking and active engagement are strongly related to enjoyment.

How do these compare with students' attitudes and beliefs in the same categories above for art and music?

The more prevalent comments for the **Confident/ Not Enjoy** students in the arts were those related to the learning process and the activities in which students were engaged. They sometimes did not enjoy their actual lessons and following the directions of the teacher, but art and music students in this category were most dissatisfied when they did not have the freedom to work on their own: *"... I like doing things in my own way, not what the sheet music says"* (101); *"I like to draw but I don't like learning about it; I just like being creative"* (749).

Comments in art and music for the **Not Confident/ Enjoy** group of students were more related to a personal part of themselves. Students in music found listening to music enjoyable and even if they were not good at it they thought that it would be interesting to learn and do. *"Although I am not really good at playing music, I love it and always try playing it"* (1169). Students in the arts commented on improving their skills and expressing themselves. *"It's an easy way to help explain feelings"*(652).

What can we learn from students' attitudes in art and music that can improve the teaching and learning of mathematics and help foster a positive attitude toward learning mathematics in all students?

One issue that students brought up during this research is that for them the study of mathematics is compulsory while the choice to study art and music is solely at the discretion of the student. Ruffell, Mason and Allen (1998, p. 6) found that students' attitudes sometimes depended on the type of mathematics being studied. Within art classes students have choices, e.g., photography or sculpture and the same is true for music, e.g., instrumental, vocal or a course such as 'music in our lives.' While I am not advocating non-mandatory mathematics courses, perhaps being given some choice of shorter courses built into one school year from different strands of mathematics combined to meet school and government requirements could make a difference for this particular group of students.

G.H. Hardy spoke of mathematics as an art in A Mathematician's Apology (1967) and explained that the job of a mathematician is to do something. It is certainly understood by students in art and music that their purpose in those classes is to do, to make, to compose or to play something. They don't see that role for themselves in mathematics classes where they expect that they will only be called upon to think or listen, something they appear to believe is less active. Students find that the long periods of passive engagement during lessons cause them to daydream and get off task. Students believe from the experiences we offer to them that mathematics is purely cerebral while art and music are more physical. We need to change, even in some small way, our approach to teaching mathematics. Providing students with opportunities to do something with the mathematics that they have learned is imperative. Interdisciplinary projects involving mathematics and the arts are ideal vehicles with which to enhance student understanding, enjoyment and motivation. Nascimento and Barco (2007, p. 67) agree.

7.5 Conclusions of this Study

My purpose for embarking on this project was to be able to gain new insights into the teaching and learning of mathematics that would benefit the community as a whole. This research provides evidence that:

- Students in art and music classes have issues with listening to the teacher, studying, practicing and working in much the same way as students in mathematics do. The time spent involved with these **repetitive and mundane activities**, however, does not appear to monopolize their lives and lessen their enjoyment during classes in the arts as students feel that it does in mathematics. Students in mathematics classes spend most of their time listening to the teacher, copying notes and/ or working on exercises. In their arts classes, students spend most of their time engaged with the very personal creation and/ or demonstration of what they can do with what they have learned.

- Enjoyment in art and music exists in a greater degree because of other more **active experiences** which provide a balance to those tasks that are often perceived by students as drudgery. Students don't view learning mathematics as a dynamic event because of the long periods of passive engagement that they regularly endure during their lessons.

- Students in the arts are used to encountering new and different experiences each time they walk into the classroom and many of their comments point to their liking the opportunities afforded to them in their art classes to utilize their **creativity and imagination**. A mathematics class that is not so predictably routine, which provides students with the chance to know, to understand and to experience that beauty and creativity do exist in mathematics as well as in the arts is important to counterbalance the impression that there is no inventiveness in the 'thinking' that they are called upon to do in mathematics.

- Students are able to **put themselves and their emotions into their work** in art and music, something which Perrin (2004, p. 28) sees as an important factor during the adolescent years. Students indicated in this study that they have little opportunity for positive emotions in mathematics classes. The emotional aspect of mathematics classes comes from what mathematics makes the students feel internally: fear, anxiety or satisfaction at getting the one right answer. There is no right or wrong 'answer' in the arts. By contrast, the emotional aspect of the arts shared by students was the outlet these classes provide for something that they truly value, the expression of how and what they are feeling.

- Students view mathematics as a subject with **countless rules, formulas and exercises** that they feel have been foisted upon them. Barzun (2003, p. 34) states that *"rules in art are guidelines"* and not like those in mathematics which are compulsory and cannot be broken! Comments of students regarding mathematics consistently referred to their impressions of the subject: difficult, tedious and time consuming. In the arts they commented mainly on the nature of the tasks in which they were engaged. The nature of the tasks for students in mathematics involved just solving problems. Art and music students also solved problems but the solutions to those problems were instrumental in accomplishing a task directly related to the creation or performance of a piece of work.

- This research project has also uncovered a most interesting paradox. **Students who do well in mathematics often don't enjoy it**, yet students may very well enjoy their classes in the arts even if they don't feel confident. Among the comments these students made was their view that learning mathematics was pointless. The **Confident/ Not Enjoy** students find little to sustain their enjoyment of learning mathematics beyond the moment where the understanding of the concepts developed during lessons has been achieved. Their enjoyment of learning mathematics is undermined when their desire to 'move on' past the process of repeating what they have already learned is stifled. In art and music, the **+/-** students also want to 'move on' but for them that means past the time of teacher direction to the point in the lessons when they can begin to do things in their own way. This indicates a similarity in the cause for a lack of enjoyment experienced by these confident students both in mathematics and the arts. A rationale for the high frequency of this category in mathematics as opposed to its low frequency in the arts, however, appears to stem from the fact that the brief moments of intellectual enjoyment for students during lessons are never followed by occasions for them to do their own 'mathematical things.'

7.6 Implications for Pedagogy

> *"If the artist does not perfect a new vision in his process of doing, he acts mechanically and repeats some old model fixed like a blueprint in his mind ... The real work of an artist is to build up an experience that is coherent in perception while moving in constant change in its development."* (Dewey, 1980, pp. 50 - 51)

As mathematics teachers, we face daily challenges in meeting the needs of all of the students who enter our classrooms whether they are artists, musicians or mathematicians while attempting to inspire in them all the joy of mathematics. Williams (1983, pp 189 - 190) reminds us that *"children come to school as integrated people with thoughts and feelings, words and pictures, ideas and fantasies."* If we fail to recognize these attributes of our students and continue to present our mathematics lessons using the same predictable routines, we will continue to lose so many of our students.

How can we begin to develop a *new blueprint* for the teaching and learning to encourage a positive attitude in students toward mathematics?

Repetitive and Routine Experiences

While we, as mathematics educators, have found it easy to walk into our classrooms each day armed with our notes and homework answers to share and explain during our allotted class time frame, we must be aware that our students really resent the repetitive nature of our lesson plans even if they appear to be engaged with us. If we want to foster the belief that mathematics is an adventure and full of surprises, we must start with the way we teach it to them. In the arts, students enter their classes without being able to predict how lesson's outcome. They know that after a certain amount of teacher direction, they will have the opportunity to explore on their own. In a lifetime of school experiences where most mathematics lessons entail 'sit down, listen, take notes and try these,' students admittedly shut down and lose focus, only believing that mathematics is something they must endure until the bell rings. Without occasions to create or recreate mathematics to explore on their own, students appear to have no personal stake in believing that mathematics can provide any meaning, value or enjoyment for them. Art and music seem recreational to students but that is not the case in mathematics which student perceive as work. Recreational mathematics or decision/ discrete mathematics may provide students with those kinds of experiences. Wouldn't taking students outside with coloured chalk, a straight edge and string one day to demonstrate circle theorems on the pavement surprise and delight them?

Active Engagement

During mathematics class students feel that much more time is spent passively engaged in a one-sided discourse with the teacher. In art and music classes, students feel they spend most of their class time actively engaged with their work. The daily reality for students in learning mathematics overwhelmingly involves concentrating on the attainment of basic skills (Fuhringhetti, 1993, p. 37) while rarely getting a sense of what mathematics really is and does. Yet in art and music classes, students experience practice in the attainment of knowledge they can recognize. They know what music sounds like. They know what a work of art may look like. Mathematics, however, is harder for them to say they really know as a subject and a field. It would be like a team of football players coming in every day to practice, but never playing in a game. In many ways in relation to mathematics, students never get to experience a game. Yet in art and music, once students have acquired some basic knowledge, the teacher allows them 'to play' on their own, giving reinforcement and support as they encounter problems and difficulties. The process becomes enjoyable to students because they know it leads, in the end, to a real experience of art or music. And in mathematics not only do students have no idea about what it may be, but they are often given no time to discover it on their own. They must instead faithfully reproduce what they are given by their teacher. Whole class discussions, experiments, the use of manipulatives, 'Maths Trails' and student presentations are a few suggestions for actively engaging students in learning mathematics.

The Creative Side of Mathematics

Another implication for pedagogy is that teachers must spend time pulling back the curtain, so to speak, on mathematics so that students can understand and therefore appreciate the why and how of mathematics, rather than just finding solutions to exercises and problems. What is lacking in our classrooms is the promotion of mathematics as a creative medium. Students' experiences in learning the arts teach them that gradations of sound in music and colour in art can creatively come together to produce a result which is new to them and each step along the way on the road to creation delights their sense of imagination and enjoyment. Students experience very few 'aha' moments in mathematics while in art and music flashes of unexpected thrill and discovery are regular occurrences. From my data, intellectual stimulation, thinking, creativity and imagination do spark enjoyment in learning mathematics. Anecdotes of the young Carl Gauss amazing his teacher with his rule for the sum of an arithmetic sequence or the young Andrew Wiles being intrigued with the Pythagorean Theorem which resulted in his proof of Fermat's Last Theorem many years later can provide students with a view of mathematics as a creative endeavour.

The Personalization of Mathematics

Students want to do things their own way. The freedom for them to work on their own toward a goal is supported in art and music. In the arts, student enjoyment is directly related to being able to put themselves and their feelings and emotions into their work. In mathematics, students find no such outlet and even many of our good students see no relevance to their lives and therefore are not motivated to continue to study mathematics. In order for students to take an interest in an activity, they must perceive it as personally significant to their interests and their lives. Projects integrating mathematics with other disciplines are ideal for allowing students to understand that mathematics is related to various fields in which they may be interested. It would enhance their enjoyment of mathematics to recognize the ways in which mathematics could be an important part of their future, for example as an artist (ratio and proportion in the work of Mondrian and polar coordinates in anamorphic art) or a musician (transformations in the work of Bach) with their strong mathematical connections. From an emotional aspect, we must avoid focusing our lessons to be the need for right answers. Not an issue in art and music, the emphasis of right versus wrong reduces enjoyment for students in mathematics. Making a mistake in mathematics is embarrassing. In art making a mistake is a way of improving. Students must understand that there is not always a correct answer and sometimes there is more than one correct answer. We must let mathematics lead students through their mistakes down a road to a discovery they never knew existed before they started like Mandelbrot and his work with fractal images.

Creating a New Frame of Mind

As Langer (1997, p. 61) states, it is the frame of mind and not the activity that creates difficulties for individuals. It is clear that most students and much of society have no idea of the real nature of mathematics and how much it is the foundation for almost everything that we do in life. We don't see, hear or feel the mathematics because someone else has used it to improve our lives, whether that exists in the construction of the cars we drive with navigation systems that can be programmed or the construction of a new building. Even when students state that there are applications of mathematics in the real world, many cannot give specific applications. One of the reasons for this is that, according to Stewart (2006, p. 2), mathematics is left *"behind the scenes."* Too often, we focus on how to manipulate variables through operations and formulas without really having students utilize mathematics to actually make or do something with what they have learned. Unfortunately, because the development of mathematical ideas and methods are too often invisible to them, our students have built up no sense or appreciation of its beauty. As opposed to imposing rules, let students value discovery to see clearer that mathematics does not 'drop out of the sky.' Instead of merely writing out the quadratic formula and working out how to use it, a student can be fascinated to understand why it works. One student (616) expressed it this way, *"I can do math, but it can get boring; I like to calculate things but it's the process getting me there is what I don't like."* Our over emphasis on skill attainment as opposed to invention and innovation in the derivations of the formulas that we use and in the applying of what has been learned is definitely a reason for our students lack of enjoyment of mathematics. Art and music lead to 'products' which students create and discover for themselves. Unfortunately, in mathematics the only 'products' students are aware of are the answers to problems. Application was very much an aspect of enjoyment of learning mathematics, art and music!

"My Teacher <u>Thinks</u> I'm Enjoying" It

This quote on one of my questionnaires about mathematics from a **+/-** student (#451) has the most dramatic implication for us as mathematics educators. The realization that 42.6% of students are confident in their ability to learn mathematics but do not enjoy learning it is mind-boggling! This is especially amazing when considered with the percentages of student in art and music in this same category, that is, 8.1% and 14.7% respectively. To stand and look out at a class of mathematics students, who sit smiling and nodding at us on the outside so as not to alert us to the strong dislike they harbour for learning it on the inside and to think that out of 25 of them, about 11 of them are putting on a good show. As teachers of mathematics, **we <u>cannot</u> conclude that those who are good at mathematics really do enjoy it**!

7.7 Ideas for Future Research

Each step along this research project provided food for thought and even further questions which suggest additional areas for future research:

- The results presented in sections 4.6 and 7.2 of this thesis indicate that 84.6% of the students claimed that they were confident in their abilities to do mathematics. Burton (2004, p. 357) recognized the lack of accord on the definition of confidence and its measurement. As an experienced mathematics teacher, this large percentage of confident students in my findings makes me feel the need to probe more deeply into what it means to students when they claim to be confident in mathematics? By what criteria do students ascertain their confidence? More research is necessary in this area.

- Using the breakdown of category frequency by school in Table 4.5, the mean percentage of students **Confident/ Not Enjoy** is calculated to be 48.1375%. Schools **5** (23.1%) and **12** (29.5%) have a much lower frequency in this category and schools **8** (77.7%) and **14** (87.0%) have a much higher frequency in this category. Further investigation into the policies and practices of these specific institutions may provide additional information regarding student attitudes of confidence and enjoyment and why they are so different from the other schools.

- While my research has provided reasons for the attitudes of students in each of my four confidence and enjoyment categories, more research is needed to investigate more deeply into the individual characteristics of students in each group, especially the **Confident/ Not Enjoy**. How do they think? What are their strategies for learning mathematics? In the terminology of Lowman (2004, p. 60) are they the *"creators"* or *"re-creators?"* Is there a relationship of the **+/-** students and learning styles, multiple intelligences or ...?

- An exploration into the attitudes and beliefs of talented students and those professionals in the fields of mathematics, art and music would seem indicated. What are the enjoyable aspects of these subjects? What are the common characteristics in thinking, doing and enjoying these subjects?

7.8 The End of this Journey is Really a New Beginning

The conclusion of this project marks the end of an arduous and fascinating period in my life, but it really is a new beginning. There are other roads which lie ahead and new questions to answer. After completing this research, my approach to the teaching of mathematics is forever transformed and enriched.

I know that I will never greet a new class of students without wondering behind which of these smiling faces lies the soul of a quietly disaffected student who, while I might think or hope otherwise, doesn't enjoy mathematics. It will be my responsibility to try to improve their mathematical experiences. In seeking to reach out to all of my students, I will continue to engage them in projects integrating mathematics and art, and look for new ways and creative ideas to make connections between these subjects. Despite constraints of time and curricula, I will find more ways to use a discovery approach to my lessons, enabling my students to get a glimpse of the beauty of mathematical thought.

This process has taught me much and I sincerely hope that readers take away from this thesis new ideas to stretch their minds to a new dimension about the teaching and learning of mathematics.

Appendices

Appendix—A The Preliminary Student Questionnaire

Student Questionnaire: Mathematics & Other Subjects

AGE: ______________ GRADE: ________________ GENDER: Male____ Female ____

The purpose of this questionnaire is to find out how students feel about mathematics and other subjects. Base your answer to each item i the questionnaire upon your own experiences.

In answering items # 1 - 10, read each statement *carefully* and circle:

SD -- if you STRONGLY DISAGREE with the statement

D - if you DISAGREE with the statement

U - if you are UNDECIDED as to whether you agree or disagree with the statement

A - if you AGREE with the statement

SA - if you STRONGLY AGREE with the statement

Select only one response for each statement. Be as accurate as you can.

1. I am *confident in my ability* to do:

Art	SD	D	U	A	SA
Mathematics	SD	D	U	A	SA
Music	SD	D	U	A	SA

2. I am sure that *I can learn*:

Art	SD	D	U	A	SA
Mathematics	SD	D	U	A	SA
Music	SD	D	U	A	SA

3. This subject is *interesting* to me:

Art	SD	D	U	A	SA
Mathematics	SD	D	U	A	SA
Music	SD	D	U	A	SA

4. Only students with a very *special talent* can learn:

Art	SD	D	U	A	SA
Mathematics	SD	D	U	A	SA
Music	SD	D	U	A	SA

5. I *enjoy learning*:

Art	SD	D	U	A	SA
Mathematics	SD	D	U	A	SA
Music	SD	D	U	A	SA

6. I have a *good feeling* about this subject:

Art	SD	D	U	A	SA
Mathematics	SD	D	U	A	SA
Music	SD	D	U	A	SA

7. *Anyone* can learn to do:

Art	SD	D	U	A	SA
Mathematics	SD	D	U	A	SA
Music	SD	D	U	A	SA

8. I *feel secure* when I attempt to do:

Art	SD	D	U	A	SA
Mathematics	SD	D	U	A	SA
Music	SD	D	U	A	SA

9. I'm *not the type* to do well in:

Art	SD	D	U	A	SA
Mathematics	SD	D	U	A	SA
Music	SD	D	U	A	SA

10. I am *good* in:

Art	SD	D	U	A	SA
Mathematics	SD	D	U	A	SA
Music	SD	D	U	A	SA

(Page 2)

11. Do you think that it is possible for a student to state:
"I can't do mathematics, but I like learning it!"

IF YES –

Why do you think that is true?

IF NO –

Why not?

Appendix—B Second Pilot Questionnaire

Questionnaire: *School Subjects* **AGE:** _____ **GRADE:** _____ **GENDER:** Male____ Female ____

The purpose of this questionnaire is to find out how students feel about certain subjects. Base your answer to each item in the questionnaire upon your own experiences.

In answering items # 1 – 10, read each statement *carefully* and <u>circle</u>:

SD -- if you STRONGLY DISAGREE with the statement

D – if you DISAGREE with the statement

U – if you are UNDECIDED as to whether you agree or disagree with the statement

A – if you AGREE with the statement

SA – if you STRONGLY AGREE with the statement

Select only one response for each statement. Be as accurate as you can.

1. I am *confident in my ability* to do:

Art	SD	D	U	A	SA
Mathematics	SD	D	U	A	SA
Music	SD	D	U	A	SA

2. I am sure that *I can learn*:

Art	SD	D	U	A	SA
Mathematics	SD	D	U	A	SA
Music	SD	D	U	A	SA

3. This subject is *interesting* to me:

Art	SD	D	U	A	SA
Mathematics	SD	D	U	A	SA
Music	SD	D	U	A	SA

4. Only students with a very *special talent* can learn:

Art	SD	D	U	A	SA
Mathematics	SD	D	U	A	SA
Music	SD	D	U	A	SA

5. I *enjoy learning*:

Art	SD	D	U	A	SA
Mathematics	SD	D	U	A	SA
Music	SD	D	U	A	SA

6. I have a *good feeling* about this subject:

Art	SD	D	U	A	SA
Mathematics	SD	D	U	A	SA
Music	SD	D	U	A	SA

7. *Anyone* can learn to do:

Art	SD	D	U	A	SA
Mathematics	SD	D	U	A	SA
Music	SD	D	U	A	SA

8. I *feel secure* when I attempt to do:

Art	SD	D	U	A	SA
Mathematics	SD	D	U	A	SA
Music	SD	D	U	A	SA

9. I'm *not the type* to do well in:

Art	SD	D	U	A	SA
Mathematics	SD	D	U	A	SA
Music	SD	D	U	A	SA

10. I am *good* in:

Art	SD	D	U	A	SA
Mathematics	SD	D	U	A	SA
Music	SD	D	U	A	SA

11. Which of the following statements would best express your feelings and/or experiences:

- *"I am artistic, and I like learning art!"*
- *"I am artistic, but I don't like learning art!"*
- *"I am not artistic, but I like learning art!"*
- *"I am not artistic, and I don't like learning art!"*

Explain your answer as completely as you can. Be sure to include *why* you chose the statement

you did.

11. Which of the following statements would best express your feelings and/or experiences:

- *"I am musical, and I like learning music!"*
- *"I am musical, but I don't like learning music!"*
- *"I am not musical, but I like learning music!"*
- *"I am not musical, and I don't like learning music!"*

Explain your answer as completely as you can. Be sure to include *why* you chose the statement

you did.

11. Which of the following statements would best express your feelings and/or experiences:

- *"I am mathematical, and I like learning mathematics!"*
- *"I am mathematical, but I don't like learning mathematics!"*
- *"I am not mathematical, but I like learning mathematics!"*
- *"I am not mathematical, and I don't like learning mathematics!"*

Explain your answer as completely as you can. Be sure to include *why* you chose the statement

you did.

11. Which of the following statements would best express your feelings and/or experiences:

- *"I can do mathematics, and I like learning it!"*
- *"I can do mathematics, but I don't like learning it!"*
- *"I can't do mathematics, but I like learning it!"*
- *"I can't do mathematics, and I don't like learning it!"*

Explain your answer as completely as you can. Be sure to include *why* you chose the statement

you did.

Appendix—C The Final Questionnaire

Questionnaire: *School Subjects*

NAME: ______________________________ **AGE:** ____ **GENDER:** Male____ Female ____

SCHOOL: ______________________________

1. Which of the following subjects you are *currently studying* in school (*circle **all*** that apply):

Art Mathematics Music

The purpose of this questionnaire is to find out how students feel about certain subjects. Base your answer to each item in the questionnaire upon your own experiences.

In answering items #2 - 7, read each statement *carefully* and circle:

SD -- if you *STRONGLY DISAGREE* with the statement

D - if you *DISAGREE* with the statement

U - if you are *UNDECIDED* as to whether you agree or disagree with the statement

A - if you *AGREE* with the statement

SA - if you *STRONGLY AGREE* with the statement

Select only one response for each statement. Be as accurate as you can.

2. I am *confident in my ability* to do:

Art	SD	D	U	A	SA
Mathematics	SD	D	U	A	SA
Music	SD	D	U	A	SA

3. I am sure that *I can learn*:

Art	SD	D	U	A	SA
Mathematics	SD	D	U	A	SA
Music	SD	D	U	A	SA

4. This subject is *interesting* to me:

Art	SD	D	U	A	SA
Mathematics	SD	D	U	A	SA
Music	SD	D	U	A	SA

5. Only students with a very *special talent* can learn:

Art	SD	D	U	A	SA
Mathematics	SD	D	U	A	SA
Music	SD	D	U	A	SA

6. I *enjoy learning*:

Art	SD	D	U	A	SA
Mathematics	SD	D	U	A	SA
Music	SD	D	U	A	SA

7. Doing this subject *makes me feel good*:

Art	SD	D	U	A	SA
Mathematics	SD	D	U	A	SA
Music	SD	D	U	A	SA

Question 8.

Complete each of the sections below (A - C).

Then for *one* section write an explanation of why you answered that particular section as you did. Use the bottom of the page for your response.

A. *Circle* the one of the following statements that best expresses your feelings and/or experiences:

- *"I can do mathematics, and I like learning it!"*
- *"I can do mathematics, but I don't like learning it!"*
- *"I can't do mathematics, but I like learning it!"*
- *"I can't do mathematics, and I don't like learning it!"*

B. *Circle* the one of the following statements that best expresses your feelings and/or experiences:

- *"I am artistic, and I like learning art!"*
- *"I am artistic, but I don't like learning art!"*
- *"I am not artistic, but I like learning art!"*
- *"I am not artistic, and I don't like learning art!"*

C. *Circle* the one of the following statements that best expresses your feelings and/or experiences:

- *"I am musical, and I like learning music!"*
- *"I am musical, but I don't like learning music!"*
- *"I am not musical, but I like learning music!"*
- *"I am not musical, and I don't like learning music!"*

Explain your answer to *one* of the sections A, B, or C as completely as you can. Be sure to include *why* you chose the section *and* statement you did.

Appendix—D Repertory Grid Maths +/+

FINAL INTERVIEWS: STUDENT # 147 1.1.1 SUBJECT: MA A-MU CATEGORY: +/+

12 – 10 – 1
8 – 4 – 9
11 – 7 – 6
2 – 3 – 7
5 – 7 – 8

1	1 RECEIVING GRADES	2 WATCHING A DEMONSTRATION LESSON	3 DISCUSSING IDEAS WITH TEACHERS & CLASSMATES	4 WORKING ON EXAMPLES	5 LISTENING TO THE TEACHER	6 BEING INFLUENCED BY FAMILY EXPERIENCES	7 UNDERSTANDING NEW MATERIAL	8 EXPERIENCING FEELINGS & EMOTIONS	9 POSSESSING ABILITY / TALENT	10 RELATING TOPICS TO THE REAL WORLD	11 HANDLING OBJECTS OR MATERIALS	12 CREATING SOMETHING NEW	5
ALREADY THERE ~~THESE ARE ESTABLISHED~~	1	1	1	1	3	6	1	5	1	1	5	5	SOMETHING TO BE DISCOVERED
HELPS ST LEARN BETTER	1	6	1	1	1	5	1	5	1	3	4	5	NO RELATIONSHIP TO LEARNING
MAKES PROB SOLVING EASIER	2	4	1	1	2	5	1	5	2	3	1	5	MIGHT TAKE AWAY FROM PROB SOLV.
HELPS UNDERSTANDING	5	5	1	1	2	5	1	4	3	1	3	5	CONFUSE ST.
OUTSIDE OF ST. COMES FROM TCHR.	1	1	3	4	1	5		5	5	1	4	1	GUESS / FROM INSIDE ST. GUT FEELING
MAKES LEARNING FUN	3	1	1	3	3	5	5	5	1	1	1	1	LEARNING IS BORING
"YES I CAN" DO THE WORK	3	3	3	3	3	3	3	5	1	1	1	3	ST. FEELS LIKE FAILURE

Appendix—E Repertory Grid Maths +/-

FINAL INTERVIEWS: STUDENT # 393
1.2.1

SUBJECT: MA - A - MU
CATEGORY: +/-

6 - 11 - 12
9 - 4 - 2
1 - 7 - 10
2 - 3 - 7

1	1 Receiving grades	2 Watching a demonstration lesson	3 Discussing ideas with teachers & classmates	4 Working on examples	5 Listening to the teacher	6 Being influenced by family experiences	7 Understanding new material	8 Experiencing feelings & emotions	9 Possessing ability/talent	10 Relating topics to the real world	11 Handling objects or materials	12 Creating something new	5
~~[illegible]~~ Something St. develops	1	5	4	4	5	5	1	5	3	1	1	1	Given to St/no choice
Laid back / St. passive	3	1	4	5	2	1	5	1	1	4	3	4	St. is actively engaged
Build St. understanding	5	[illegible]	5	2	2	1	1	4	5	1	2	4	Measures your understanding
Important to process of learning	5	1	1	5	4	[illegible]	5	5	3	4	2	4	Result of learning
Makes learning fun	3	4	1	2	2	3	1	3	1	1	1	1	Learning is boring
"Yes I can" do the work	3	1	1	3	3	1	1	3	1	1	1	1	St. feels like failure

Appendix—F Repertory Grid Maths -/+

FINAL INTERVIEWS: STUDENT # 320 SUBJECT: MA - A - MU
1.3.3 CATEGORY: -/+

1 - 8 - 9
2 - 6 - 12
4 - 11 - 5
2 - 3 - 7

1	RECEIVING GRADES (1)	WATCHING A DEMONSTRATION LESSON (2)	DISCUSSING IDEAS WITH TEACHERS & CLASSMATES (3)	WORKING ON EXAMPLES (4)	LISTENING TO THE TEACHER (5)	BEING INFLUENCED BY FAMILY EXPERIENCES (6)	UNDERSTANDING NEW MATERIAL (7)	EXPERIENCING FEELINGS & EMOTIONS (8)	POSSESSING ABILITY/TALENT (9)	RELATING TOPICS TO THE REAL WORLD (10)	HANDLING OBJECTS OR MATERIALS (11)	CREATING SOMETHING NEW (12)	5
MOTIVATE ST. TO LEARN	1	2	2	2	4	4	4	1	5	5	1	3	NO NEED TO TRY / NO MOTIVATION
IMPORTANT FOR ST UNDERST.	3	1	2	1	2	5	1	3	3	4	2	1	MAY BE CONFUSING TO ST.
ST. INVOLVED IN LEARNING	3	1	1	1	3	5	5	3	3	4	1	1	NOT FIRST HAND EXPERIENCE / ST. NOT INVOLVED
ST. DOING / ST. ACTIVE	3	5	2	1	5	4	3	3	3	5	1	2	ST. NOT DOING / SITTING BACK
MAKES LEARNING EASIER	3	5	3	1	5	4	1	4	1	3	1	2	ST. GETS DISTRACTED / MUST PULL THINGS BACK
GETTING DIFFERENT VIEW/UNDERST. FROM OUTSIDE INF	3	1	1	4	2	2	5	3	5	3	3	2	IDEAS ARE UNDERSTANDING NOT VERY DIVERSE
MAKES LEARNING FUN	3	5	3	2	5	3	4	2	3	4	1	2	LEARNING IS BORING
ST. FEELS "YES I CAN" DOING	3	3	3	3	3	3	1	3	1	3	2	2	ST. FEELS LIKE FAILURE

BOTH FROM 2 - 6 - 12

BOTH FROM 4 - 11 - 5

FINAL INTERVIEWS: STUDENT # 398 1.4.2 SUBJECT: MA - A - MU CATEGORY: -/-

1 - 2 - 4
5 - 7 - 10
11 - 6 - 9
2 - 3 - 7 *

1	1 RECEIVING GRADES	2 WATCHING A DEMONSTRATION LESSON	3 DISCUSSING IDEAS WITH TEACHERS & CLASSMATES	4 WORKING ON EXAMPLES	5 LISTENING TO THE TEACHER	6 BEING INFLUENCED BY FAMILY EXPERIENCES	7 UNDERSTANDING NEW MATERIAL	8 EXPERIENCING FEELINGS & EMOTIONS	9 POSSESSING ABILITY / TALENT	10 RELATING TOPICS TO THE REAL WORLD	11 HANDLING OBJECTS OR MATERIALS	12 CREATING SOMETHING NEW	5
~~TRYING~~ ST. ACTIVELY WORKS	1	5	1	1	5	3	5	3	4	3	1	4	ST. IS PASSIVE
~~MAKES POSSIBLE FOR ST TO APPRECIATE~~													~~RULES OF MATH~~
ST. FEEL/SEE MATH USEFUL	4	4	2	3	5	2	1	3	4	1	2	2	RULES/STEPS/INSTRUCTIONS
MAKES LEARNING MATH EASIER	3	2	2	2	4	1	3	2	1	1	5	2	NOT EASY - ST. DOES NOT KNOW HOW TO IT
~~HELPS UNDERSTANDING~~	~~2~~	~~4~~ 5	~~2~~ 1	~~2~~	~~4~~	~~3~~	~~1~~	~~3~~	~~1~~	~~1~~	~~4~~	~~4~~	~~TAKES AWAY FROM UNDERST.~~ POSSIBLE TO BE OFF
TCHR. KNOWS ABILITY UNDERSTANDING	1	5	1	3	4	3	1	3	2	1	2	2	TCHR. NOT AWARE OF WHAT ST. KNOWS OR NOT *
MAKE LEARNING FUN.	4	4	2	3	5	4	2	3	4	4	4	4	LEARNING IS BORING
"YES I CAN" DO THE WORK	3	2	2	3	3	2	1	2	1	2	2	2	ST. FEELS LIKE FAILURE

* BOTH OF THESE WERE ELICITED FROM 2-3-7

Appendix—H Repertory Grid Art +/-

FINAL INTERVIEWS: STUDENT # 264
2.2.1
SUBJECT: MA (A) MU
CATEGORY: +/-

9 - (8) - (3)
1 - (2) - (5)
7 - (10) - (11)
(2) - 3 - (7)
3 - (4) - (9)

1	1 RECEIVING GRADES	2 WATCHING A DEMONSTRATION LESSON	3 DISCUSSING IDEAS WITH TEACHERS & CLASSMATES	4 WORKING ON EXAMPLES	5 LISTENING TO THE TEACHER	6 BEING INFLUENCED BY FAMILY EXPERIENCES	7 UNDERSTANDING NEW MATERIAL	8 EXPERIENCING FEELINGS & EMOTIONS	9 POSSESSING ABILITY/TALENT	10 RELATING TOPICS TO THE REAL WORLD	11 HANDLING ARTIFACTS OR MATERIALS	12 CREATING SOMETHING NEW	5
~~SOMETHING~~ SHARE W/ OTHERS	4	2	1	4	2	5	3	1	5	1	3	2	JUST FOR SELF / ST. HAS THIS
HOW ST. GETS INFORMATION	5	1	2	1	1	3	2	3	5	1	3	3	MEASURE OF STUD. LEARNED
COME AFTER ST. LEARNS BRAINSTORMING WHAT THEY KNOW	3	5	1	5	4	2	5	1	3	1	1	1	NECESSARY BEFORE ST LEARNS
HELPS ST. LEARN TOGETHER	5	1	5	2	1	3	1	5	5	3	5	4	INDIVIDUAL ST. LEARNING
NEEDED FOR LEARNING	3	1	5	1	1	2	2	1	1	2	2	1	NOT NECESSARY
MAKES LEARNING FUN	1	3	1	4	3	4	1	1	3	1	1	1	LEARNING IS BORING
"YES I CAN" DO THE WORK	3	1	3	1	1	5	1	3	1	1	1	1	ST. FEELS LIKE FAILURE

Appendix—I Repertory Grid Music +/-

FINAL INTERVIEWS: STUDENT # 150 3.2.4 SUBJECT: MA - A - MU

CATEGORY: +/-

4 – 12 – 2
1 – 5 7
11 – 3 – 6
8 – 9 – 10
2 – 3 – 7

1	1 RECEIVING GRADES	2 WATCHING A DEMONSTRATION LESSON	3 DISCUSSING IDEAS WITH TEACHERS & CLASSMATES	4 WORKING ON EXAMPLES	5 LISTENING TO THE TEACHER	6 BEING INFLUENCED BY FAMILY EXPERIENCES	7 UNDERSTANDING NEW MATERIAL	8 EXPERIENCING FEELINGS & EMOTIONS	9 POSSESSING ABILITY/TALENT	10 RELATING TOPICS TO THE REAL WORLD	11 HANDLING OBJECTS OR MATERIALS	12 CREATING SOMETHING NEW	5
DONE W/ OTHER PEOPLE	4	1	1	1	5	5	1	3	5	2	2	5	ST. DOES (ALONE) BY THEMSELVES
ST. INVOLVED/ENGAGED	5	1	1	2	1	3	1	3	3	1	5	5	ST. NOT INVOLVED/ ST. HAVE NO PART IN
HAPPENS WHILE ST. LEARNING	5	1	1	1	1	5	2	4	4	5	1	2	NOT NECESSARY FOR LEARNING
IMPROVES ABILITY/KNOWLEDGE	4	1	1	1	1	4	2	5	1	1	1	2	MAY BE DISTRACTING TO ST
DOESN'T MEAN ST. UNDERSTANDS	1	1	1	1	3	1	5	1	2	3	2	2	ST "GETS IT"/UNDERSTANDS
MAKES LEARNING FUN	3	1	1	2	3	3	3	2	1	1	1	2	LEARNING IS BORING
ST. FEELS "YES I CAN" DO THE WORK	3	1	1	1	3	3	1	4	1	2	3	2	ST. FEELS LIKE A FAILURE

Appendix—J Example From a Repertory Grid Interview

Student # 393: Mathematics, +/- Confident and Not Enjoy

MH - Researcher/ Interviewer; **ST** - Student Interviewee

MH: The first thing that I want you to do is pick any three cards. We can shuffle them up. Pick any three cards from here. Turn them over to tell what they are. They've got numbers on them. You've got [cards] 6, 11, and 12.

ST: Yes

MH: "Being Influenced By Family Experiences" [6], "Handling Objects or Materials"[11] and "Creating Something New"[12]. The first time through is the hardest because I need to explain to you what I want you to do.

ST: O.K.

MH: And make sure you understand. After that it will be easier. In terms of learning maths, because you and I are talking about maths, how are two of these alike to each other that make them different from the third one? Pair two of them together. You say these [two] are alike because ...

ST: Um-hm.

MH: ... and in the reason why they are alike what makes them different from the other thing. What does the other thing not have? Does that make sense?

ST: Like these two are alike because they __ [student indicates the pair she has selected]

MH: So you say "Handling Objects or Materials" and "Creating Something New" in terms of learning maths how are they alike to each other?

ST: Because they're both used in a way of learning.

MH: O.K. So these are things that are important to learning. Is that what you are saying?

ST: Yes.

MH: These are important [while writing on the sheet] for learning and how is this not important? ["Being Influenced by Family Experiences"]

ST: Because it's given to you by what you do in life.

MH: So this is given to you. Now you see at this part we are going to go back and forth.

ST: O.K.

MH: [Indicating the card 6 on the table] So you say this is given to you in life- this is just given to you, you have no choice in this. Is that about right?

ST: Yes.

MH: [While writing on the sheet] Given to student/ no choice.

ST: Yes.

MH: And then this is not given but maybe the student has to do? Is that what you mean?

ST: Yes. It's something that's ... [student is trying to find the words]

MH: Now this is something that's given to you – "Being Influenced by Family Experiences."

ST: And that's something you develop.

MH: O.K. So this [6] is something given to you and these [11, 12] are something you develop [while writing on the sheet]. Each of these twelve items [referring to the elements] is listed at the top. What you need to do is give each of them a score from one to five. It's not good or bad, right or wrong.

ST: O.K.

MH: Just how much it [the element] applies to each end. One means it's something that the student has to develop and five means it's something that is given to the student. The student has no choice in it.

ST: Yes

MH: Does that make sense?

ST: Yes.

MH: So two of your ones are these two. 11 and 12 are ones for you. 6 would be a five for you. You can do that. [Student enters ratings for the triad of elements compared in this round, i.e. 6, 11, 12] Now every other item, give it a score from one to five. You can give more ones and fives, but you can also give two, three or four based on how close they are to this side [one] or that side [five]. Does that make sense?

ST: Yes.

MH: And if you want to talk while you are doing it [rating] you can. Some students feel very comfortable. If I have a question I might interrupt you to ask you to tell me more.

ST: O.K. [student begins rating the elements on each pair of constructs].

MH: Would it help if I read these [the elements one at a time]?

ST: Yes.

MH: "Watching a Demonstration Lesson."

ST: Um.

MH: Is that something the student develops or something that's given to you?

ST: It's given.

MH: Again you can give two, three or four [as a rating].

ST: Um-hm.

MH: "Discussing Ideas with Teacher and Classmates."

ST: That's, you develop it somewhat so I'll give it a four.

MH: "Working on Examples."

ST: I say more of a four..

MH: "Listening to the Teacher."

ST: Given too.

MH: "Understanding New Material."

ST: Something you develop.

MH: "Experiencing Feelings and Emotions."

ST: It's given.

MH: "Possessing Ability and Talent."

ST: Something that's given. [Without any comment the student changed her initial five for this element with a rating of three].

MH: "Relating Topics to the Real World."

ST: Something that develops.

MH: Pick another 3 cards ...

[The process begins for the second pair of random triads and elicited constructs].

Appendix—K Percentage of Similarity Calculation

This is an example of how the percentages of similarity were calculated. The data in the table below is from the first row of the repertory grid interview regarding the learning of mathematics with student #393 (+/-, Confident/ Not Enjoy) [See Appendix E]. The construct used in this example is 1.2.1.1, the first construct elicited from this student.

ID # 393E MA 1.2.1	E1	E2	E3	E4	E5	E6	E7	E8	E9	E10	E11	E12
	1	2	3	4	5	6	7	8	9	10	11	12
Construct 1.2.1.1 Ratings	1	5	4	4	5	5	1	5	3	1	1	1
Enjoyment Construct Ratings	3	4	1	2	2	3	1	3	1	1	1	1
Enjoyment Ratings in Reverse	3	2	5	4	4	3	5	3	5	5	5	5
Differences between construct and:												
Original Ratings	2	1	3	2	3	2	0	2	2	0	0	0
Ratings in Reversed Order	2	3	1	0	1	2	4	2	2	4	4	4

The formula for calculating the percentage of similarity (Jankowicz, 2004a, p. 115) between any given construct and a fixed construct is:

PS = 100-({SD/[(LR-1)* E]}*200).

SD is the sum of differences between the ratings along a particular construct row and those of a fixed construct row for each element, **LR** is the largest possible rating and **E** is the number of elements. The largest possible rating (**LR**) for my repertory grid interviews was 5 and there were 12 elements (**E**).

The calculation of the percentage of similarity in the original order was begun by finding the differences between the respective ratings of each element as compared with the specific construct (1.2.1.1) and the enjoyment construct. For example, E1 (element 1) had a difference of 2, E2 had a difference of 1, E3 had a difference of 3, etc. The row of differences was then added: 2+1+3+2+3+2+0+2+2+0+0+0 = 17 to determine the sum of the differences for this construct (**SD**).

The values were then substituted into the formula:

PS = 100-({17/ [(5-1)*12]}*200) which evaluated the percentage of similarity for this construct in original order with enjoyment as 29.17% to the nearest hundredth percent.

Given the bipolar nature of the constructs, the same procedure was carried out for the construct with the poles of the enjoyment construct transposed. This was necessary due to the possibility that the similarity between the given construct 1.2.1.1 and the enjoyment construct might be closer in meaning to a reversal in order from how the construct was originally expressed.

To reverse the poles and consider the ratings in reverse order, a rating of 1 for an element became a 5 and, in the same way, a rating of 2 became a 4, a rating of 3 remained a 3, a rating of 4 became a 2 and a rating of 5 became a 1. The calculation of the percentage of similarity in reverse order was begun by finding the differences between the respective ratings of each element as compared with the specific construct (1.2.1.1) and the reversed enjoyment construct ratings. For example, E1 (element 1) had a difference of 2, E2 had a difference of 3, E3 had a difference of 1, etc. The row of differences was then added: 2+3+1+0+1+2+4+2+2+4+4+4 = 29 to determine the sum of the differences for this construct (**SD**).

The values were then substituted into the formula:

PS = 100-({29/ [(5-1)*12]}*200) which evaluated the percentage of similarity for this construct with enjoyment in reversed order as -20.83% to the nearest hundredth percent.

By comparison, construct 1.2.1.1 had a lower sum of differences and therefore a higher percentage of similarity to enjoyment in its original order than when the poles were reversed. According to Honey's method of analysis this indicates that, one characteristic of enjoyment related to mathematics for this student is when it involves *something the student develops* for himself and a lack of enjoyment as related to something which is *given to the student* and over which he has *no choice*.

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for secondary and college teachers of mathematics

Guest Editorial

> "Mathematics has many important practical applications, but should not be of interest only to the scientist. There is much in mathematical thought which should interest the arts student, and much which is beautiful and should interest everyone. Those who profess mathematics do so because they enjoy it. To understand and share in this enjoyment, the reader is invited to follow some of the arguments." (Pedoe, 1958, p.5)

It is with these words that Dan Pedoe begins the preface to his 1958 book, *The Gentle Art of Mathematics*. As a mathematics teacher, it was my long-held conviction that if students were confident in their ability to do mathematics it would certainly guarantee that these very same students enjoyed learning mathematics. Through my current research investigating how students compare their confidence versus enjoyment in mathematics, art and music, my beliefs have been confronted with reality! In surveying and interviewing students, I have heard, in the words of Pedoe, "some of the arguments" of an overwhelming number of students, confident in their abilities to learn and do mathematics, as to why they don't enjoy learning mathematics. The attitudes of students regarding their enjoyment of learning art and music, however, even when they lack confidence in their abilities to learn and do art and music, are quite another story.

Questions arise from the data that I have collected: Why don't students like learning mathematics even when they are good at it? What can we learn from students' attitudes and beliefs regarding art and music to feed back into the teaching and learning of mathematics to create a positive attitude toward learning mathematics in all students?

The theme of this issue is the making of connections between mathematics and art. In a study by Sonia Forseth (1980), it was found that: "The use of art activities that are designed to reinforce mathematical concepts seems to affect children's attitudes toward mathematics. These attributes tend to be more favourable when art activities are used in mathematics class as part of or as a supplement to the regular lesson.

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Front cover: **Kaleidoscopic photo design from *Maths and art activities for students* by Mary Ann Harasymowycz**
Credit: **K.DESIGN, Winscombe, Somerset**

Editorial Committee

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This does not mean that the use of art activities will alleviate learning difficulties a child may have in mathematics, but it does suggest that art may improve the child's attitude toward learning and in turn this may affect achievement. In other words, the use of art activities seems to help create a more favourable predisposition toward learning mathematics without any impairment to achievement in math."

Like many other teachers of mathematics, I have concerns about providing my students with the mathematical background and understanding necessary for them to move forward in the study of mathematics. I agree with Skip Fennell, President of the USA's National Council of Teachers of Mathematics, when he says: " ... the mathematical intent of any activity should not be lost in attempts to make it real, relevant, or meaningful ... must not take away from the mathematics to be learned" (2007). I fear, however, that we are losing our best and our brightest students to other disciplines because they don't understand that mathematics is an art with all of its aesthetic values. Like Fennell (2007), I "... prefer to see students who enjoy mathematics, who are confident in their ability to do mathematics, and teachers who regularly strive to make mathematics come to life for their students."

When I asked students, confident in their ability to do mathematics, why they did not enjoy learning it these were some of the reasons:

- I usually can understand maths, but sometimes I find it hard to see the point of learning it.
- Maths comes easily to me ... I would prefer doing something I enjoy but aren't very good at.
- Maths is very boring and repetitive ... we do the same thing every day.

What a challenge we face in overcoming the school lifetime of experiences that our students bring to us! How do we find ways to make mathematics relevant to the lives of our students, share with them our joy of maths and break out of any predictable routine to our lessons?

> "If the artist does not perfect a new vision in his process of doing, he acts mechanically and repeats some old model fixed like a blueprint in his mind ... The real work of an artist is to build up an experience that is coherent in perception while moving in constant change in its development." (Dewey, 1980, pp. 50–51)

As you read this issue, devoted to the integration of mathematics and art, ask yourself how you can "build up an experience" using the ideas contained within for your students in learning mathematics that is "coherent in perception while moving in constant change in its development."

My accelerated pre-calculus students just completed an anamorphic art project as an extension of our study of the polar coordinate system. These are some of their responses reflecting on how "the project changed your view of mathematics":

- It was something different ... It showed me there is more to math than just numbers. Everything isn't just formulas and ridiculous problem solving. It gave me a chance to see how math could be used with art.
- I was surprised that I actually enjoyed it ... it's a different approach to math ... with its unique perspective.
- I thought that math was rather ... not "useless" *per se*, but just not incredibly necessary to those whose jobs would not involve math. But here's where I was wrong: math, and especially in this project, can get you to think in a way you never have before much like art.
- It showed me that beautiful things can result from carefully following mathematical concepts.

Wow! These students have certainly acquired a "new vision" of mathematics and all through the integration of a little art!

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Mary Ann Harasymowycz
Williamsville, New York

Maths & Art Activities for Students

by Mary Ann Harasymowycz

I have been a teacher of mathematics for many years and a lover of art for much longer! More recently I have had the opportunity to put together my interests in art, music and mathematics by working through a PhD study with colleagues at the Centre for Teaching Mathematics at the University of Plymouth. It has been interesting to discover the number of students who enjoy art and music but claim to be not very good at the subjects. This rarely happens in mathematics! It seems to be that students have to perceive themselves good at mathematics before they can enjoy the subject and, surprisingly, many of those students who do perceive themselves to be good in mathematics do not enjoy it!

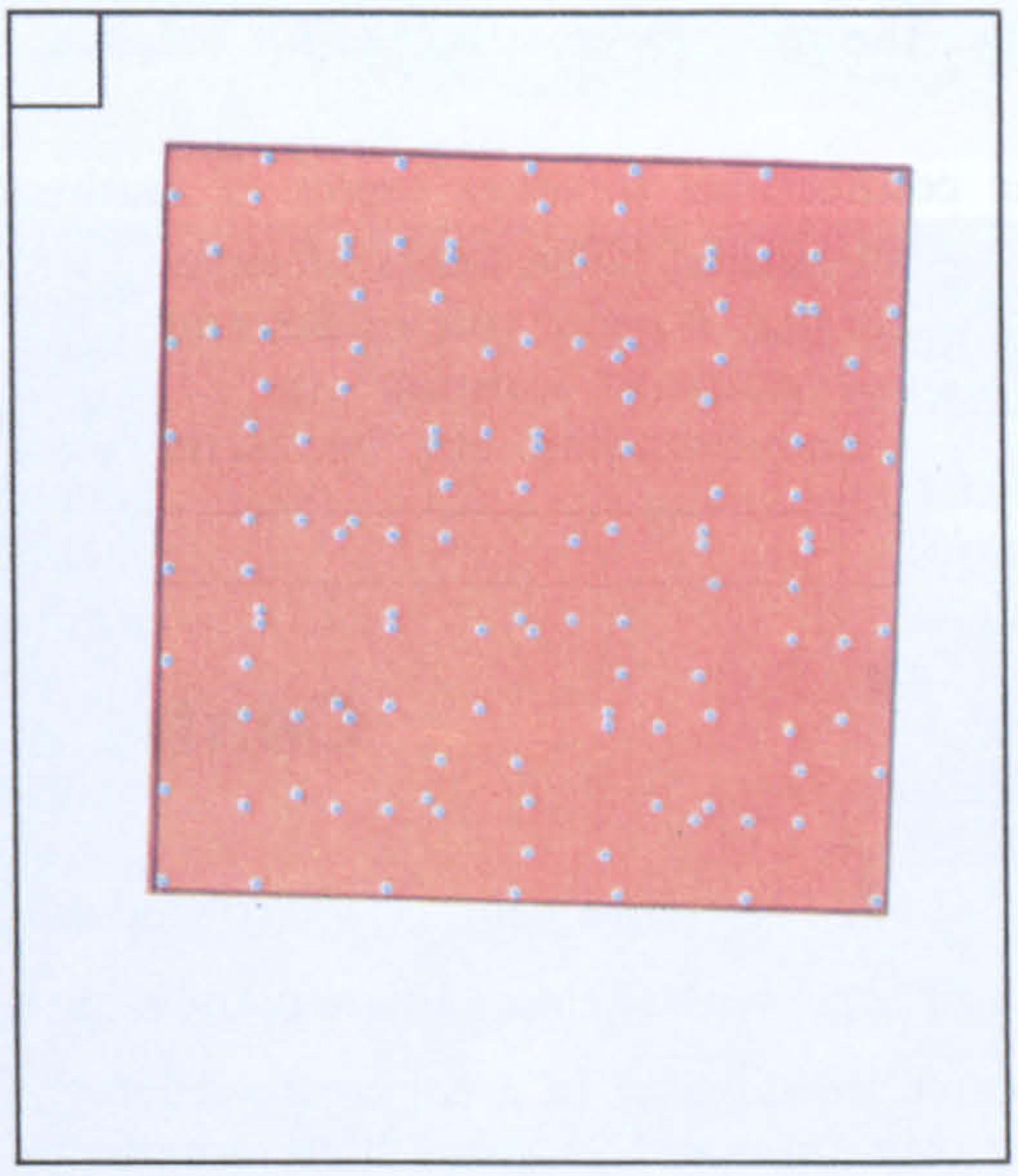
During a recent discussion with my accelerated pre-calculus class a student asked,

> "Mrs. H, so what do mathematicians *do* with mathematics?"

G.H. Hardy noted, "... the function of a mathematician is to do something ..." Our students must believe that each of them is, to some extent, a mathematician. We can help by providing them with something to do with the mathematics they've learned! By expanding student awareness of the 'connections' between mathematics and other disciplines, they learn that mathematicians do more than just maths! Interdisciplinary projects involving mathematics and art are ideal vehicles with which to achieve that goal. The deeper we probe, the more we find mathematicians whose function it is to *do* art ... Dürer, Escher, Seurat and Mondrian to name but a few. "Let no one read me who is not a mathematician" was penned by none other than Leonardo DaVinci. In one of our pre-calculus units, I introduce my students to anamorphic art. The feedback from my students in the past has always been very positive. After they create their own pieces of mathematical art, my students have at least one answer to the question about what you can *do* with mathematics!

In this article I will describe two other activities that I have used to provide a link between art and mathematics. Each activity is rich in opportunities to review and to introduce ideas in geometry. Students with varying abilities in maths get a chance for success and enjoyment of the artistic kind and then realize *they are doing* maths!

Activity 1 *Orange Crush* Op Art



***Orange Crush* by Larry Poons**

Orange Crush was painted in 1963 by Larry Poons. It is part of the collection of the Albright-Knox Art Gallery in Buffalo, New York, USA. Measuring 80″ × 80″, *Orange Crush* is one of Poons' earliest optical paintings. He named it for a popular soda of the same name, with its bright orange colour and effervescent bubbles. Larry Poons, born in 1937, first studied music at the New England Conservatory in Boston, Massachusetts, USA. In 1957, he switched from music to art partly after reading *Lust for Life*, the fictionalized biography of Vincent Van Gogh by Irving Stone. The kinetic 'motion' of the dots with their flashing after-images in *Orange Crush* is created by the optical illusion produced with the complementary colours of orange and blue on his canvas. It is suggested that through the optical 'motion' of the dots, Poons used his musical background to symbolize musical notes moving along their staffs in a staccato rhythm. On close inspection of *Orange Crush*, it is noticed that not all of his dots were randomly placed. Poons carefully and mathematically planned his pieces. Perhaps in seeking a balance and a harmony of his own, in the final design he revised the placement of some of his dots.

I created the op art activity outlined here as part of an interdisciplinary project connecting mathematics and art. The students involved were grade 7 (USA). Working with an art teacher, the end product was a 4′ × 4′ painted wooden panel which was later hung in one of the school's hallways. It could also be expanded to include science, the eye and optical illusions. As described here for the reader, this activity has also been used as a purely mathematics-related activity. The mathematical content is an introduction or review of some of the basic concepts of Coordinate Geometry and would be suitable for students ages 11–14.

The objectives of the activity are to:

- *Review* concepts of coordinate geometry and graphing linear functions.
- *Understand* the relationship between complementary colours and optical illusions.
- *Learn* the relationship between mathematics and art in the work of op artist, Larry Poons.
- *Use* knowledge of plotting points in the coordinate plane to create a random op art work in the style of Larry Poons' *Orange Crush*.
- *Analyse* the patterns created, e.g. for balance, symmetry, etc.
- *Make* connections to other topics of mathematics, e.g. rational numbers, probability.
- *Investigate* and *interpret* the differences between expectation and reality in the placement of 'random dots'.
- *Evaluate* and *judge*, as a class: which group's 'op art' creates the most effective optical illusion, which random placement of dots is most aesthetically pleasing, and which *Crush* name is the favourite.

The materials required are:

- Envelope of 121 *ordered pairs*, (x, y), printed on cardstock.
- *Coloured paper* with the coordinate plane printed upon it.
- Adhesive *circular dots* in a colour complementary to the paper. Measurement of space between the grid lines is based on the diameter of the circular dots available to the teacher. For this activity, the dots were 0.75″ in diameter and the grid lines were 1.5″ apart.
- Worksheet with questions.

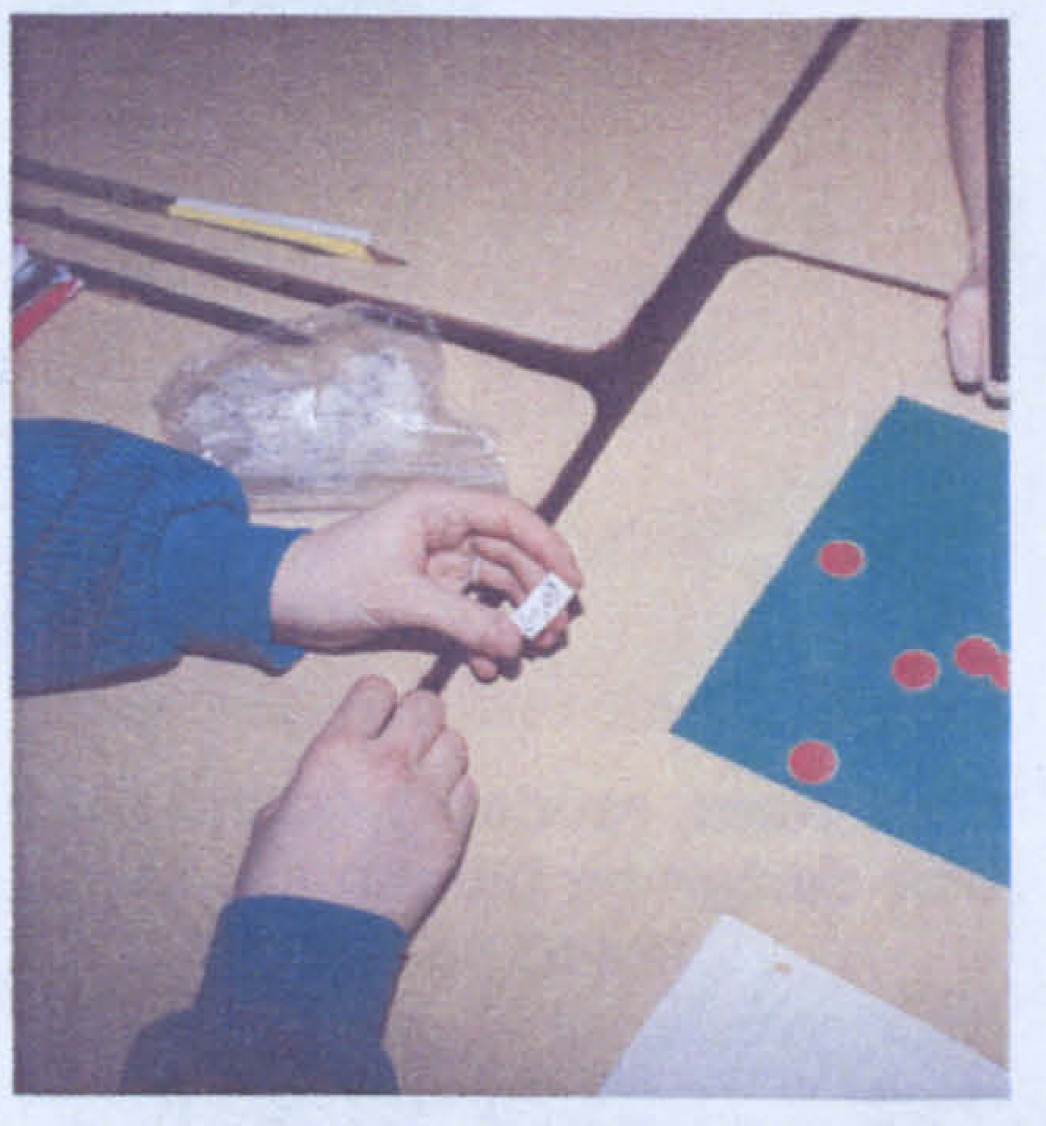
Having gathered together the materials, I encourage the following strategy:

Working in cooperative learning groups of 2–3 students, students will:

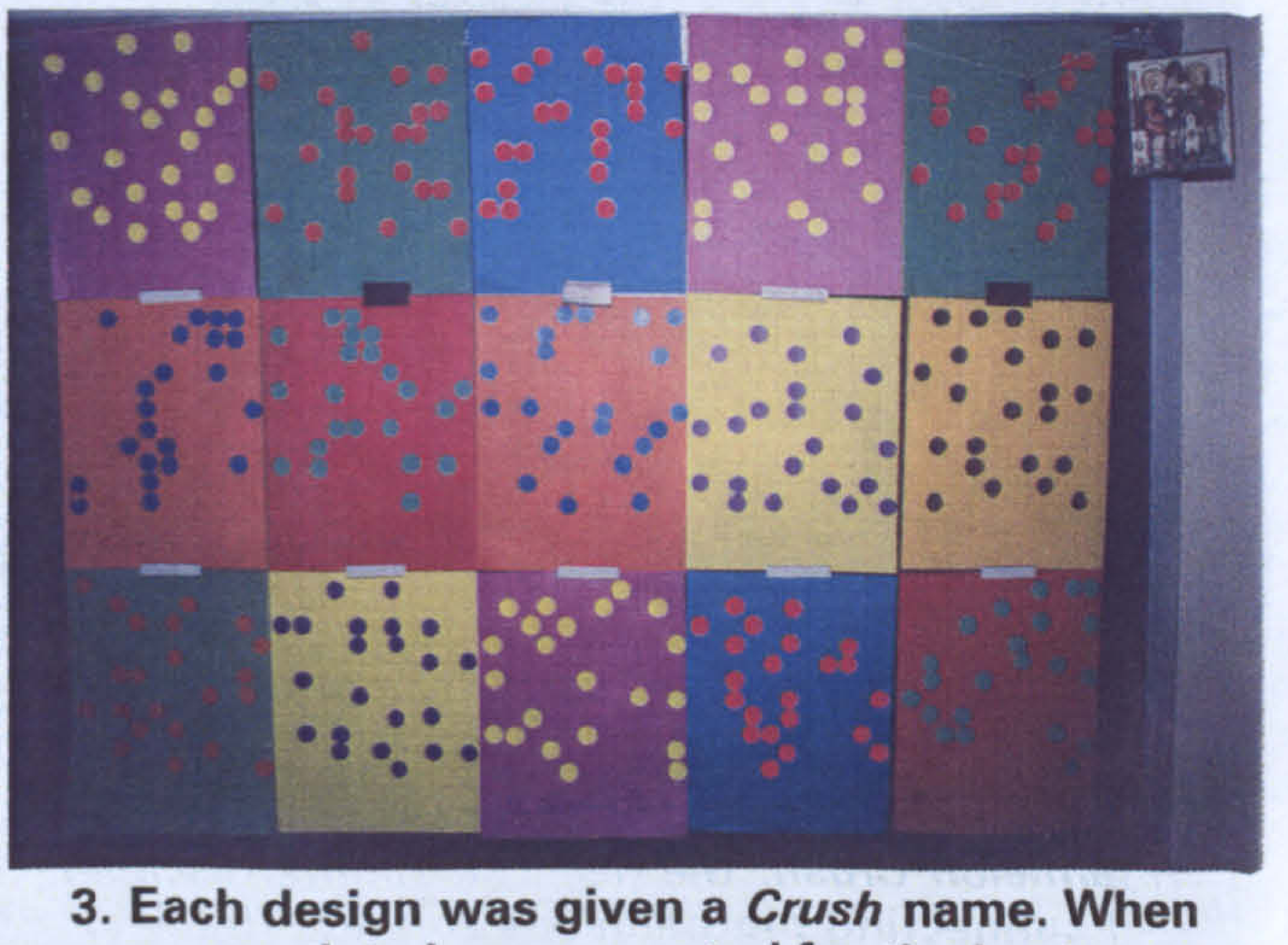
- Select, at random, 22 coordinate pairs from the envelope.
- Take on alternating cooperative learning roles of:
  1. *Plotter* – plots point selected with an adhesive dot in its position in the coordinate plane on the coloured paper.
  2. *Checker* – checks the correctness of the other's point plot.
- Complete the worksheet's chart and questions.
- Discuss, within the group, the random pattern created, questions, concepts and ideas.
- Share, as a class, each group's analysis of this activity and op art *Crush* creations.
- Select a class winner for the most effective optical illusion.
- Submit the group question/reflection sheet.

The following photographs and comments show a group of students working through the activity.

1. Students work in cooperative groups to create random placement of coloured dots while reviewing the plotting of points in the coordinate plane



2. Dots are placed on the coordinate grid as per those 22 ordered pairs selected at random from the set of 121 they have been provided



3. Each design was given a *Crush* name. When completed, groups voted for the best: (a) *colour* contrast of dots and paper creating best optical illusion (green on red), (b) random *design* created (top left), (c) *name* (*Watermelon Crush*)

4. In art class, students painted the base coat and background colour on the 4′ by 4′ wood panel

5. Back in art class, students complete their own Op Art *Crush* design by painting in the dots on the wood panel

6. *Watermelon Crush,* the result of an interdisciplinary project connecting mathematics and art. The finished mural was hung in the school hallway for all to see and enjoy. These middle school students were proud of the optical illusion they had created and they learned to view mathematics in a whole new way!

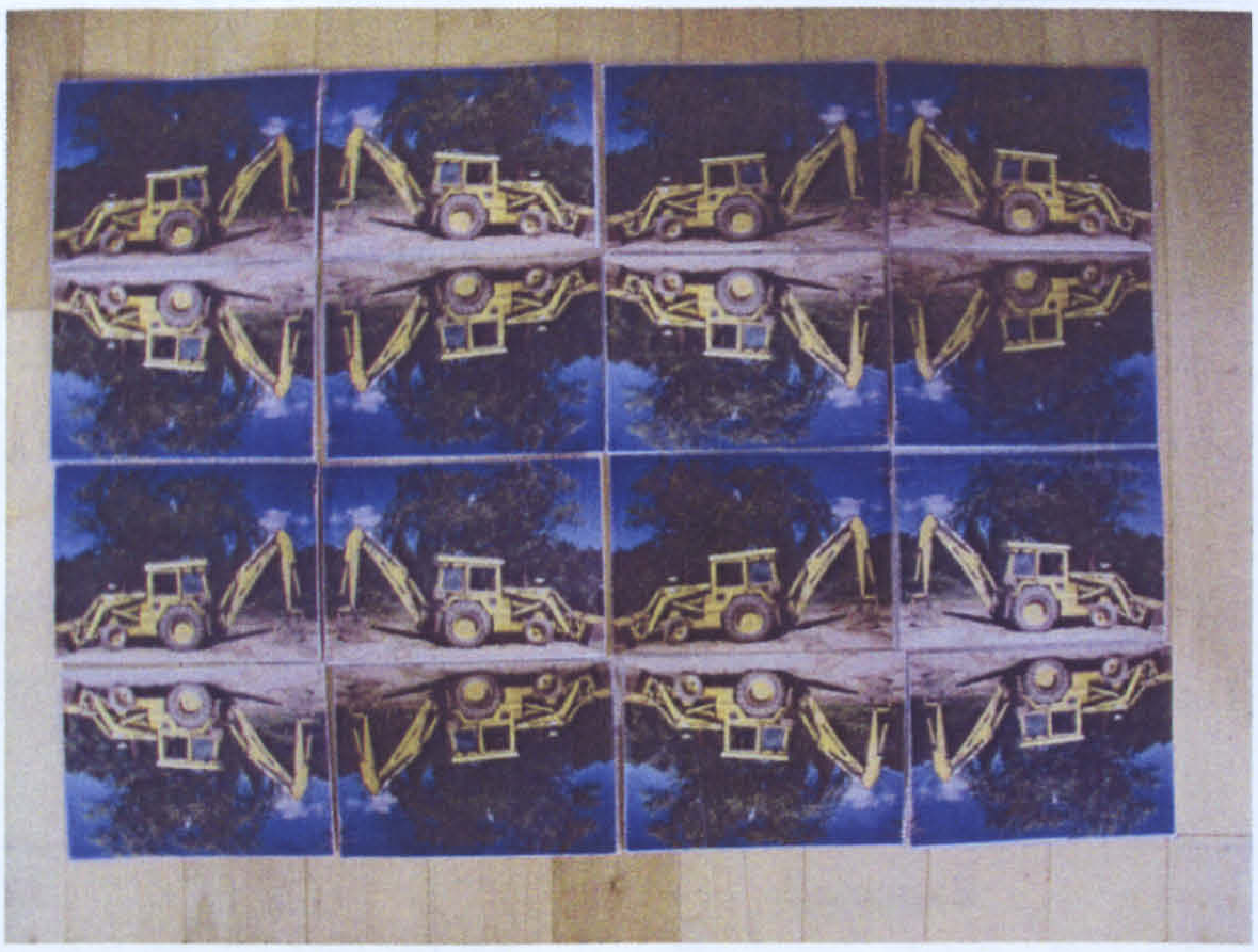
Activity 2 – Kaleidoscopic Photo Designs

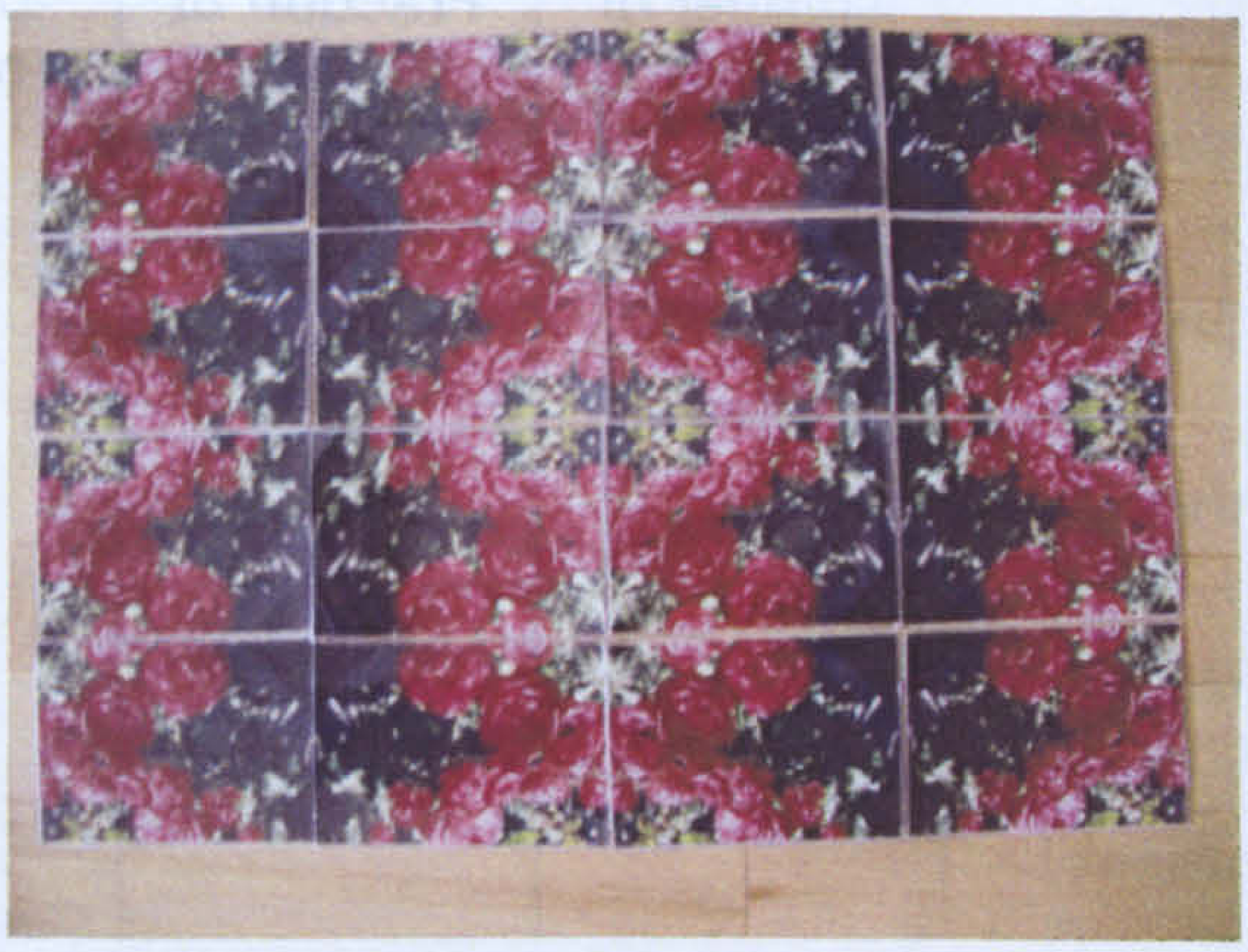
This activity allows students to explore a range of transformations in geometry as well as be creative artistically! It is suitable for students of any level from primary years upwards. The basic idea is to make a collage from sixteen photographs, eight of which are printed normally and the other eight are reversed. Scenes/objects with bright contrasting colours in broad areas work best. I usually have 8–10 *different photo* packets available for students in the class. The figure shows the photograph of a backhoe and its reverse image.

The procedure for this activity is as follows:

- Distribute packets of photographs to students (individuals or cooperative groups).
- Ask the students to create their own designs by organizing the photos in the packets they have been given into a 4×4 rectangular grid.
- Ask students to discuss their designs and the patterns they see.
- Introduce or review vocabulary such as translation, reflection, rotation.
- Have individual students/groups share and discuss their designs with the whole class making sure that they use the appropriate terminology to describe how the images relate to each other.

The following figure shows photographs of three different displays produced by some of my students: the backhoe collage and two others from photographs of a hot air balloon and roses.





For those of us who understand the beauty of mathematics, its value and usefulness in relationship to our world is unquestionable. Jerry King writes:

> " ... the aesthetic experience of mathematics both excites and soothes mathematicians ... the excitement brings them to the subject ... the soothing becomes addictive and draws them back again and again ... the aesthetic pleasures are unavailable to the humanists ... because the right view of mathematics has been hidden from them."

At the end of each activity, I encourage my students to write a reflection on the mathematics and art connection they have experienced in the activity and whether or not the activity has changed their beliefs and attitudes toward mathematics. Of course, one activity is unlikely to change a school lifetime's experience of mathematics. Several students, however, recall the activities described in this article, and others, as high points in their mathematics lessons in school. Some wrote:

> "... math is more than numbers and equations ... it's art too! ... I enjoyed the project more than I thought that I would ... I like math and art, it put both of my interests together ... I was amazed to see that maths ties in with almost everything people do in life ... it showed me a whole new way to look at maths!"

For me, the teacher, the last comment validates making a maths and art connection for students the most. In addition to having provided my students with the understanding of mathematical concepts as per the curriculum, their eyes have been opened to uncover part of the hidden '*right view of mathematics*.' I enjoy seeing my students fully engaged in being creative artists and mathematicians simultaneously. Experiences like these generate a high degree of excitement and enthusiasm for learning. At the end of last year, one of my accelerated pre-calculus students wrote me a note:

> " ... Some of the maths we learned this year was difficult and tedious. Sometimes I would get very frustrated with the homework and what we learned in class ... I enjoyed the extra things we did this year like ... drawing pictures in grids (*anamorphic art*) ... stuff like that really helped to make math more fun ... thank you for making math more interesting this year!"

I hope that you might try out these ideas. Even if it makes a positive impact on just one student's view of mathematics, it is well worth the effort!

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Author

Mary Ann Harasymowycz, Williamsville North High School, Williamsville, New York 14221, USA.

e-mail: **m.harasymowycz@plymouth.ac.uk**

Orange Crush Op Art Worksheet

Name ______________________________

After you have completed your 'Optical Art Work' based upon the work of Larry Poons, complete the table and questions below:

1. Given: "Domain"= $\{(x, y): -5 \leq x \leq 5, -5 \leq y \leq 5; x, y \in \text{integers}\}$

Total number of points = 121; number of points in sample = 22

Number of Outcomes Data / Event investigated	Expected			Actual		
	Number of points	Fraction of domain	Decimal nearest 1/100	Number of points	Fraction of sample	Decimal nearest 1/100
A) Origin						
B) x-axis						
C) y-axis						
D) Quadrant I						
E) Quadrant II						
F) Quadrant III						
G) Quadrant IV						
H) $y = -5$						
I) $x > 3$						
J) $x = y$						
K) $-4 \leq y < 2$						
L) *						

(*Make up one of your own.)

2. Why is there a difference between what was expected before you selected points and what actually happened?

3. (a) Why do you think the number of points in the sample was 22?
   (b) If the sample number of points was 44, how many 'dots' would you have expected on the *x*-axis?

4. (a) (1) Describe your optical art work by studying the random placement of your dots. Does it exhibit balance, symmetry, etc.?

   (2) Name your Optical Art Work: "________________________ -Crush"

   (b) (1) How effective is the 'after image' of your 'optical art'? Explain.

   (2) Compare your work with the others in the class. What colour combinations make the most effective after-image? Why?

5. If your domain was $\{(x, y): -8 \le x \le 8, -8 \le y \le 8; x, y \in \text{integers}\}$:
   (a) What is the total number of points in the domain?
   (b) What is the likelihood of a dot being placed on either the x-axis or the y-axis?
   (c) How many points should the sample of points selected for the op art have if this domain were used? Explain.

(0,0)	(1,0)	(2,0)	(3,0)	(4,0)	(5,0)	(–1,0)	(–2,0)	(–3,0)	(–4,0)	
(0,1)	(1,1)	(2,1)	(3,1)	(4,1)	(5,1)	(–1,1)	(–2,1)	(–3,1)	(–4,1)	
(0,2)	(1,2)	(2,2)	(3,2)	(4,2)	(5,2)	(–1,2)	(–2,2)	(–3,2)	(–4,2)	
(0,3)	(1,3)	(2,3)	(3,3)	(4,3)	(5,3)	(–1,3)	(–2,3)	(–3,3)	(–4,3)	
(0,4)	(1,4)	(2,4)	(3,4)	(4,4)	(5,4)	(–1,4)	(–2,4)	(–3,4)	(–4,4)	
(0,5)	(1,5)	(2,5)	(3,5)	(4,5)	(5,5)	(–1,5)	(–2,5)	(–3,5)	(–4,5)	
(0,–1)	(1,–1)	(2,–1)	(3,–1)	(4,–1)	(5,–1)	(–1,–1)	(–2,–1)	(–3,–1)	(–4,–1)	
(0,–2)	(1,–2)	(2,–2)	(3,–2)	(4,–2)	(5,–2)	(–1,–2)	(–2,–2)	(–3,–2)	(–4,–2)	
(0,–3)	(1,–3)	(2,–3)	(3,–3)	(4,–3)	(5,–3)	(–1,–3)	(–2,–3)	(–3,–3)	(–4,–3)	
(0,–4)	(1,–4)	(2,–4)	(3,–4)	(4,–4)	(5,–4)	(–1,–4)	(–2,–4)	(–3,–4)	(–4,–4)	
(0,–5)	(1,–5)	(2,–5)	(3,–5)	(4,–5)	(5,–5)	(–1,–5)	(–2,–5)	(–3,–5)	(–4,–5)	
(–5,0)	(–5,1)	(–5,2)	(–5,3)	(–5,4)	(–5,5)	(–5,–1)	(–5,–2)	(–5,–3)	(–5,–4)	(–5,–5)

Points in the coordinate plane $\{(x,y): x,y \in \text{integers}, -5 \le x \le 5, -5 \le y \le 5\}$

These are printed out on card, cut up and put in an envelope for selecting 'at random'.

Mathematics and Art: TESSELLATIONS

by Mary Ann Harasymowycz

Have you ever noticed the doodles and drawings of your students in their notebooks? Over my years as a mathematics teacher, I continue to be amazed at the artistry of some of my students. Early in my career, I often wondered how I could tap into these abilities to enhance my students' learning of mathematics. Interdisciplinary projects linking the seemingly diverse subjects of maths and art are the ideal vehicles with which to do so.

Many years ago, into one of my lower level upper secondary classes walked a young man who, try as he might, performed poorly in maths. During that year, hanging on the bulletin board of an art colleague was an outstanding sketch by this very same student. My experience with him became the catalyst for my investigation into finding ways to make connections between maths and art for students ... to strengthen their understanding of maths and improve their attitudes and beliefs about mathematics by using it as a tool to guide students in the creation of their own works of art.

Grunbaum and Shephard (1989, p. vii) find it:

> "... curious that almost all aspects of geometry relevant to the 'man on the street' are ignored by our educational systems ... what remains is rarely of any use to people who wish to apply geometric ideas in their work ... engineers, scientists, architects, artists ... the essence of the subject – its visual appeal – has been completely submerged into technicalities and abstractions ..."

Yet the integration of art into the maths curriculum fits so aptly in the geometric strand. It brings the *visual appeal* to the forefront and provides students with an alternative approach to the *technicalities and abstractions* with which they are often perplexed.

A student reflection on an anamorphic art project my classes completed last spring included this thought:

> "I thought that math was rather ... not 'useless' per se, but just not incredibly necessary to those whose jobs would not involve math. But here's where I was wrong: math, and especially in this project, can get you to think in a way you never have before—much like art."

We can better understand the experiences of many of our students in these words of the artist M.C. Escher from 1965:

> "When one speaks about 'lucid' and 'logical,' one thinks involuntarily of mathematics. In school in Arnheim I was a particularly poor student in arithmetic and algebra because I had, and still have, great trouble with the abstractions of numbers and letters. Things went a little better in geometry when I was called upon to use my imagination, but I never excelled in this subject either while I was in school."
>
> (Escher on Escher, 1989, pp. 21–22)

Through the influence of a graphic arts teacher, Escher started down a "path through life (that took) strange turns". Further along this path he begin to experience a new and different view of mathematics:

> "Although I am even now still a layman in the area of mathematics, and although I lack theoretical knowledge, the mathematicians ... have had considerable influence on my work of the last twenty years ... above all I take pleasure in the contacts and friendships with mathematicians ... they have often provided me with new ideas, and sometimes an interaction between them and myself often develops. How playful they can be, those learned ladies and gentlemen!"

I have worked with so many students who have trouble in maths like the young Escher. Do students lacking confidence in their ability to do mathematics ever get the chance to understand the usefulness of mathematics like Escher did? Students interviewed during my doctoral project undertaken at the Centre for Teaching Mathematics at the University of Plymouth very rarely used words like *imagination*, *new ideas* or *playful* when discussing their views of mathematics. We must ask ourselves what experiences we can provide our students that will inspire them mathematically.

Despite his early school experiences, many of Escher's works have mathematical connections. For example, the five platonic solids are represented in *Reptiles*, and another of his works, *Bond of Union*, visualizes his relationship with his wife in a Moebius band.

Escher is perhaps best known for his tessellations. He became captivated with tessellations after a trip to the Alhambra, in Granada, Spain in 1936. In writing about his work after that visit he stated:

> "... the ideas that are basic to them often bear witness to my amazement and wonder at the laws of nature which operate in the world around us ... by keenly confronting the enigmas that surround us, and by considering and analysing the observations that I had made, I ended up in the domain of mathematics ... I often seem to have more in common with mathematicians than with my fellow-artists."
>
> (Bool, Kist, Locher and Wierda, 1992, p. 55)

In 1956 Escher met and corresponded with a mathematics teacher, Bruno Ernst:

"A remarkable person, who wrote to me out of the blue that my prints fascinated him (and the boys he taught) ... he demonstrated a method to me of easily 'inverting' all sorts of objects ... it is so completely astonishing ..."
(Bool, Kist, Locher and Wierda, 1992, pp. 86–87)

Bruno Ernst was greatly interested in the mathematical characteristics of Escher's work and he created a system to mathematically analyse and describe it. Bruno Ernst was only one of the mathematics teachers that no doubt influenced Escher's *path*. Let us consider that each of us could be a Bruno Ernst having such a positive impact on the paths on any number of Eschers in our classes and look for our own ways to do so!

The definition of *tessellation* listed by Millington and Millington in the *Dictionary of Mathematics* (1966, p. 235):

"Originally a term used to describe the pattern formed by covering a plane surface with congruent *squares*, from 'tessera' ... The term is often applied to patterns using congruent equilateral *triangles* or congruent regular *hexagons* ... all possible patterns formed by covering a plane with shapes in some ordered sequence ... plane tessellations can be classified into three types: *regular* – one kind of regular polygon is used. Only three patterns are possible (triangles, squares or hexagons); *semi-regular* – regular polygons of any kind are used, but all common vertices must be congruent; *non-homogeneous* –these are infinite in variety and include patterns using only one irregular shape ..."

Other sources also define a *demi-regular* tessellation. A demi-regular tessellation is also formed with regular polygons. Their vertices, however, have differing arrangements of polygons.

Mathematics can be defined in a variety of ways. Keith Devlin (1994, pp. 1–3) tells us that the answer to the question *What is mathematics?*

"Has changed several times over the course of history ... within the last twenty years ... a definition of mathematics has now emerged on which most mathematicians now agree: mathematics is the *science of patterns*."

Devlin classifies tessellations and tiling as part of the patterns of *symmetry and regularity*. In a discussion among participants at a conference on Math and Art in 1993, the connection between the two subjects was explained in this way:

"The creativity of mathematics is not that different from art ... we're creating ... beautiful patterns of thought ... some of them can be visualized in form."

This issue of *Mathematics in School* is focused on discrete mathematics. *Repetitive patterns and processes* is a major theme inherent in discrete maths. The exploration of tessellations fits quite appropriately within the study of mathematics as a whole and discrete maths as a part of that whole. It allows students the opportunity to discover the ways that the plane can be covered and analyse the repeated patterns that are created. Decorative tilings abound in the environment of our students, from the floors and walls of their homes to the stonework on the buildings and streets in their communities. The practical problems posed in discrete maths contain applications that are real and relevant to the world of our students and the study of tessellations is no exception!

Over the years, I have used tessellations with grade 7 (Year 8) students in a variety of ways. My first ideas for making this maths and art connection with students was undertaken on a small scale. Students were asked to:

- *Understand* the meaning of a tessellation from photos of tiled walls and brickwork I had taken in the neighbourhood around the school. Students were assigned to bring in to class photos/pictures of other places where there are tessellations in the real world.
- Work in cooperative learning groups with sets of regular polygons cut out of cardboard, *discover* which of them tessellate alone and/or with others (Ask: why do they cover/not cover the plane?)
- *Analyse* and *classify* tessellation patterns as regular, semi-regular or demi-regular by the arrangements of polygons at each vertex. (Ask: why do these work?)
- *Investigate* the total number of different semi-regular/demi-regular tessellations that are possible and sketch out your findings. (Have a class discussion on these student findings – which arrangements were missed or are invalid?)

In subsequent years, by collaborating with my art teacher colleague, students were also asked to:

- *Use* markers and pencils to colour some ready-made design worksheets in maths class (see references for a list of some resources).
- *Paint* their own 'tessellated' cubes in art class using ideas from the worksheets and/or discovery work done in class.

As this project evolved, in addition to the above and in conjunction with a unit on geometric transformations, students:

- *Made templates* using translation, rotation, or reflections on a polygon. (Cardstock parallelograms and regular hexagons were given to students. After cutting out a piece from one side of the polygon, they used a transformation and taped the cut out section to another side of the polygon.)
- *Traced* their templates in at least six positions and *brainstormed ideas* within their groups as to what their completed templates 'look like'. (This enabled the students not to be locked into one idea. Talking it out with others also helps to inspire. You may wish not to allow 'faces' as designs)
- *Traced* around their templates in black ink on a white 7" × 7" square with their own Escher-type tessellation and then detailed their designs in black ink.
- Took their templates to art class and *painted* in water colour a 9" × 12" version of their Escher-type tessellation designs.

Student reflections on their Escher-type tessellations were overwhelmingly positive! The energetic discussions as they shared their completed designs with one another ... the oohs, aahs, WOWs, and AHAs ... were *loud* and *clear*! Maths class became more than numbers and equations. My students' reactions and their understanding of the maths content after this project reinforced my belief that teaching mathematics with a little art could provide an opportunity to change students' views of maths and carve out new turns for some students in their mathematical 'path through life!'

A few weeks after our tessellations unit, we were working on finding the area of various polygons. As I demonstrated with a cardboard figure the development of the area of a parallelogram, one young man experienced a true Aha! moment: "Oh, I get it! You just used a translation to transform a parallelogram into a rectangle!"

Tessellations can also be extended to include symmetries, star polygons, Penrose tilings, space tiling, and hyperbolic Escher patterns.

Included with this article are the names of some resources for more activities, ideas and information on tessellations. Even upper secondary students enjoy having a story read to them: *A Cloak for a Dreamer* is a children's book which tells the story of a tailor and his three sons who have been asked to make three cloaks for an Archduke. The story is sweet, the illustrations are lovely and there is an 'About the Mathematics' section discussing the geometry of tessellations in the back.

Kaleidocycles is a book which contains cardboard polyhedral models with Escher's designs to punch out and make. In the opening paragraph of the booklet that is included, Doris Schattschneider (1987, p.5) writes:

> "Everyone loves surprises. There are two types of surprises – the one is a happy accident or coincidence; the other is meticulously planned, perhaps cunningly disguised to appear natural, and brings double pleasure. It is often hard to say who has the greater delight – the person who is surprised or the one who devised the magic. The Dutch artist M.C. Escher was an ingenious planner of surprises of the second ... cunningly planned visual surprises."

I have used tessellations with students of all ability levels from accelerated to lower level/basic maths classes. One year, as an extension of the tessellation project, I encouraged five of my students to enter Dale Seymour Publication's Second *'Escher-like' Original Student Art Contest* with their designs. Four of the students who entered were from my accelerated class. 'Amy' was one of my lower level students. On a day much like any other, but clearly very different, Amy ran down the hall to my classroom with a beaming smile on her face and a letter in her hand. Her tessellation design, ***Fruit Basket***, had won an ***Honorable Mention*** in the contest!

What a wonderful surprise for both of us! This was not a happy accident ... but a double pleasure! As it had been for Escher, the key to Amy's success was mathematics. She struggled with the mathematics of numbers and equations but surprised herself with her ability to visually express her understanding of the important mathematical concepts needed in the creation of her design. I was happy for her and confident that her mathematical path had taken a delightful turn. Amy signed my copy of the book in which her design was published, very simply: "Dear Mrs. H., Thank you!" Nothing was more surprisingly special to me!

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Profiles in Math and Art URL: **mathartfun.com/shopsite_sc/store/html/index.html**

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Author
Mary Ann Harasymowycz, Williamsville North High School, Williamsville, New York 14221, USA.
e-mail: **mharasymowycz@plymouth.ac.uk**