

Graphical User Interface (GUI) In MatLab for Solving the Pulsatile Flow in Blood Vessel

Badreddin Giuma^{1C}, Vijayajothi Paramasivam³, Kahar Osman¹, Mohamed Rafiq Abdul Kadir^{1,2} and Kanesan Muthusamy³

¹*Faculty of Mechanical Engineering, Universiti Teknologi Malaysia (UTM), Malaysia*

²*Medical Implant Technology Group (MediTeg), Faculty of Biomedical Engineering and Health Science, Universiti Teknologi Malaysia (UTM), Malaysia*

³*Faculty of Science & Technology, Open University Malaysia (OUM), Malaysia*

Received: 16/07/2009 – Revised 28/07/2009 – Accepted 30/07/2009

Abstract

Blood flow analysis is a study of measuring the blood pressure and finding its equivalent flow rate, velocity profile and wall shear stress. In this article, the relationship between blood pressure gradient, velocity profile, centreline velocity, volumetric flow rate and wall shear stress is determined analytically through a Graphical User Interface (GUI) developed using MatLab. If one of these time-dependent blood flow properties is known, i.e. pressure gradient, velocity profile, volumetric flow rate or wall shear stress, then the remaining properties can be calculated. A code is developed to solve these blood flow properties. Any time-dependent blood properties can be used as input data. These data are then digitized and saved in this code. Subsequently, these data are curve-fitted using the Fourier series. The corresponding coefficients of Fourier series are then used to calculate the blood property. Once this is obtained, the remaining three other flow properties can be subsequently calculated. This GUI serves as a learning tool for students who wish to pursue his/her knowledge in understanding the relationship of various blood flow properties of pulsatile blood flow as well as the mathematics governing pulsatile flows.

Keywords: Exact Solution; Pulsatile Flow; Blood Flow Properties; Fourier Series; GUI MatLab.

1. Introduction

Pulsatile flow theory has been the topic of intense research over several decades. Stokes's second problem can be considered as the first mathematical treatment of a simple oscillatory motion. Following this, various studies, both theoretical and experimental have taken place and their summaries can be found in the survey carried out by Rott and White [1]. Studies from Womersley [2] have also been universally cited. Hale, McDonald, and Womersley [3] studied the velocity distribution across the femoral artery of a dog. The velocity profiles were drawn of the harmonic components of this flow based upon experimental data derived from the femoral artery. The corresponding maximum viscous drag had been calculated. Womersley J. R. (1955) [2] derived

^C Corresponding Author: Badreddin Giuma

Email: shaligt@iitm.ac.in Telephone: +91 04422574729

© 2009-2012 All rights reserved. ISSR Journals

the exact solution for a homogeneous, incompressible, Newtonian fluid flow in a rigid, cylindrical tube and introduced a dimensionless variable, called the Womersley Number, α . The solution proposed for the motion of a liquid in a circular tube was in the form of a pressure gradient which was a periodic function of time. The corresponding calculation of flow rate from an observed pressure gradient was described. It was also shown that there exist a phase-lag between the flow rate of the liquid and the pressure gradient which served as a driving function. The calculation of flow rate from an observed pressure gradient was described.

With the rise in availability of computer programs and softwares, the Womersley calculation can now be performed easily without involving extensive manual mathematics. The Womersley analysis is suitable for an arbitrary periodic waveform. Therefore, the calculation of the solution for a range of physiological blood properties can be determined if one of the flow properties is known. This in return, can provide important physical insight of fluid mechanics of pulsatile flows. X. He, David. N. Ku and James E. Moore Jr. [4] developed a computer program code using Mathematica that can be used to perform the Womersley calculation. It also calculates the pulsatile centreline velocity and wall shear stress from an input driving function of pressure gradient.

This article outlines the exact solution method for calculating the blood flow properties if one of the driving functions are given. It also provides an explanation of the GUI MatLab program that is developed to perform the calculation. The waveform shape of pulsatile hemodynamics in a rigid straight cylindrical tube can be explored by manipulating the program code.

2. Methods

2.1 Mathematical formulation

In order to design better strategies for diagnosis and treatment, a better understanding of both physiological and pathological aspects of blood flow is needed. Since most conduits for the flow of fluids within the human body are cylindrical or nearly so, such as arteries and veins and because of the sensitivity of vessel wall cells to the applied shear stress, it is important to have full solutions for these blood flows.

The flow of fluid has been considered to be incompressible ($\nabla \cdot \mathbf{v} = 0$), homogeneous, Newtonian ($\mu = \text{constant}$), laminar, axis-symmetric ($\frac{\partial v}{\partial \theta} = 0$), and fully developed ($\frac{\partial v}{\partial z} = 0$). Consideration was only given to axial flow ($v_r = v_\theta = 0$) and body forces along a sufficiently long, straight cylindrical tube with rigid wall were neglected ($g=0$)[5]. By imposing these assumptions and further algebraically manipulating the continuity and momentum equations in cylindrical coordinates, we arrive at;

$$-\frac{\partial p}{\partial r} = 0 \quad (1)$$

$$-\frac{\partial p}{\partial \theta} = 0 \quad (2)$$

$$-\frac{\partial p}{\partial z} + \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_z}{\partial r} \right) \right] = \rho \frac{\partial v_z}{\partial t} \quad (3)$$

The first two equations (1) and (2) above show that the pressure must be a function of z and time: $p = p(z, t)$. Equation (3) is linear in both the pressure $p(z, t)$ and velocity $v_z(r, t)$. The steady and unsteady parts of the flow will be dealt independently. Subscripts s and u is used to denote

steady flow and unsteady flow, respectively. The unknown pressure and velocity fields are written respectively, as;

$$p(z, t) = p_s(z) + p_{un}(z, t) \quad (4)$$

$$v(r, t) = v_s(r) + v_{un}(r, t) \quad (5)$$

Substituting equation (4) and (5) into (3), yields;

$$\underbrace{\left\{ \frac{\partial p_s(z)}{\partial z} - \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(\frac{\partial v_s(r)}{\partial r} \right) \right] \right\}}_{steady} + \underbrace{\left\{ \frac{\partial p_{un}(z, t)}{\partial z} + \rho \frac{\partial v_{un}(r, t)}{\partial t} - \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v_{un}(r, t)}{\partial r} \right) \right] \right\}}_{unsteady} = 0 \quad (6)$$

The terms that depend on time is separated from those that do not. Each group must equal zero separately because of the differences between the two groups of terms. The derivation of the steady part equation with respect to velocity distribution, volumetric flow rate and wall shear stress can be found in any fluid mechanics textbook. Hence, the final equations are given for brevity i.e. equation (7)-(9).

$$v = -\frac{R}{4\mu} \frac{\partial p_s}{\partial z} \left(1 - \frac{r^2}{R^2} \right) \quad (7)$$

$$Q = -\frac{\pi R^4}{8\mu} \frac{\partial p}{\partial z} \quad (8)$$

$$\tau_w = \left| \frac{\partial p R}{\partial z} \right| \quad (9)$$

In order to derive the governing equation for pulsatile flow, the unsteady part of equation (6) is focused upon next. For simplicity, the subscript u is omitted and the unsteady part of the equation is;

$$\left[\frac{\partial p(z, t)}{\partial z} + \rho \frac{\partial v(r, t)}{\partial t} - \mu \frac{1}{r} \frac{\partial}{\partial r} \frac{\partial v(r, t)}{\partial r} \right] = 0 \quad (10)$$

The cardiac cycle is a periodic phenomenon. Any flow property which is periodic in time can be expressed as the sum of a Fourier series. Therefore, if any one of the blood flow properties is known i.e. pressure gradient, velocity profile, flow rate or wall shear stress, its data can then be extracted and subsequently curve-fitted using the Fourier series expansion, that takes the form of;

$$\xi(t) = \underbrace{\xi(t)}_{steady} + \underbrace{\sum_{n=1}^N \left[\Phi_n \cos(n\omega t) + \Psi_n \sin(n\omega t) \right]}_{unsteady} \quad (11)$$

where, ξ represents any one of the blood flow property, $\xi(t)$ represents the steady state part of the blood flow property, ω is the fundamental frequency and ϕ_n and ψ_n are the Fourier coefficients for n-harmonics of the unsteady part of the blood flow. If the pressure gradient as an input blood property is known, then;

$$\frac{\partial p}{\partial z} = \underbrace{\frac{\partial p}{\partial z}}_{\text{steady}} + \underbrace{\sum_{n=1}^N [\Phi_n \cos(n\omega t) + \Psi_n \sin(n\omega t)]}_{\text{unsteady}} \quad (12)$$

and for the unsteady part, is then written as;

$$\frac{\partial p}{\partial z} = A_n e^{in\omega t} \quad (13)$$

where $A_n = \phi_n - i\psi_n$. With the above expression, the corresponding velocity profile, centreline velocity, volumetric flow rate and wall shear stress in the z-direction can be developed as mentioned by J.R. Womersley [2] as follows;

$$v(r, t) = \underbrace{v(r)}_{\text{steady}} + Re \left[\sum_{n=1}^N \frac{A_n}{\mu \lambda_n^2} \left(1 - \frac{J_0(\lambda_n r)}{J_0(\lambda_n R)} \right) e^{in\omega t} \right] \quad (14)$$

$$Q(t) = \underbrace{Q(t)}_{\text{steady}} + Re \left[\frac{\pi R^4 A_n}{\mu (\lambda_n R^2)} \left(1 - \frac{2J_1(\lambda_n R)}{\lambda_n R J_0(\lambda_n R)} \right) e^{in\omega t} \right] \quad (15)$$

$$\tau_w = \underbrace{\tau_w}_{\text{steady}} + Re \left[\frac{|A_n|}{\lambda_n} \left(\frac{J_1(\lambda_n R)}{J_0(\lambda_n R)} \right) e^{in\omega t} \right] \quad (16)$$

where, the respective steady terms are defined by equations (7), (8) and (9). For all the above equations, from the pressure gradient, the other three properties can be found.

Another example would be, if the volumetric flow rate as the input blood property is known, Q, then;

$$Q = \underbrace{Q}_{\text{steady}} + \underbrace{Q_n e^{in\omega t}}_{\text{unsteady}} \quad (17)$$

From $\underbrace{Q}_{\text{steady}}$ steady state part of the pressure gradient can be calculated using equation (9) and from

$Q_n e^{in\omega t}$ i.e. unsteady part, A_n can be calculated using equation (15). Similarly, if any one of the four blood flow properties is known, the rest can be found.

2.2 GUI-MatLab program explanation

The MatLab software and its Graphical User Interface (GUI) functions have been used to perform the calculation. The explanations of the program steps are as follows and as seen in Figure 1. The raw input data of any blood flow properties i.e. ξ and t is imported into the MatLab workspace. This data is then curve-fitted to find the Fourier coefficients by pressing the “Curve Fitting Tool” button. If, a set of Fourier coefficients corresponding to any of the blood flow properties is readily available, press directly the “Fourier Coefficient” button to insert these coefficients into the code. Next, the corresponding flow property of these input data is selected from the pop-up menu (1). If the input data is velocity, then the program will ask at what radius this velocity represents, as shown in Figure 1. Furthermore, the dimensions of the artery can be defined i.e. radius of artery, density and the viscosity of the blood. Once these input steps are completed, click the “Compute” button and the program will calculate all the unknown blood flow properties. By selecting the pop-up menu (2) in the “Output” column, program will show us (i) any flow property profiles, (ii) the relationship between any two of these properties and (iii) 3D velocity profiles.

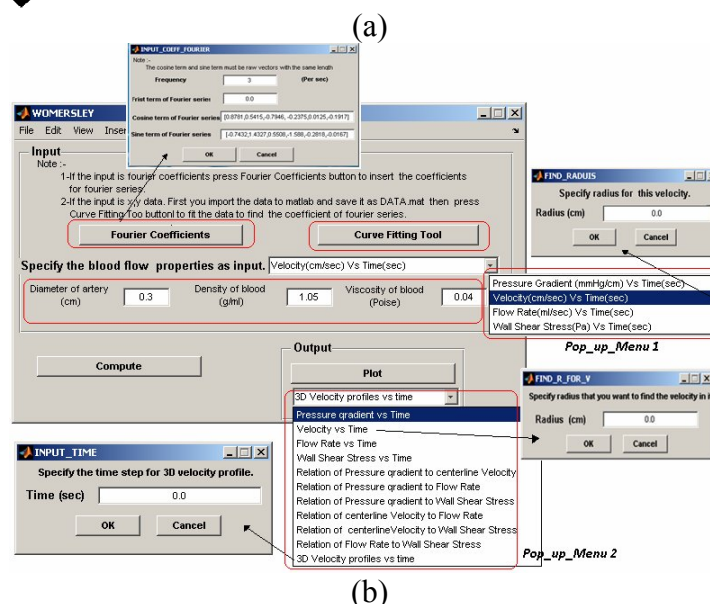
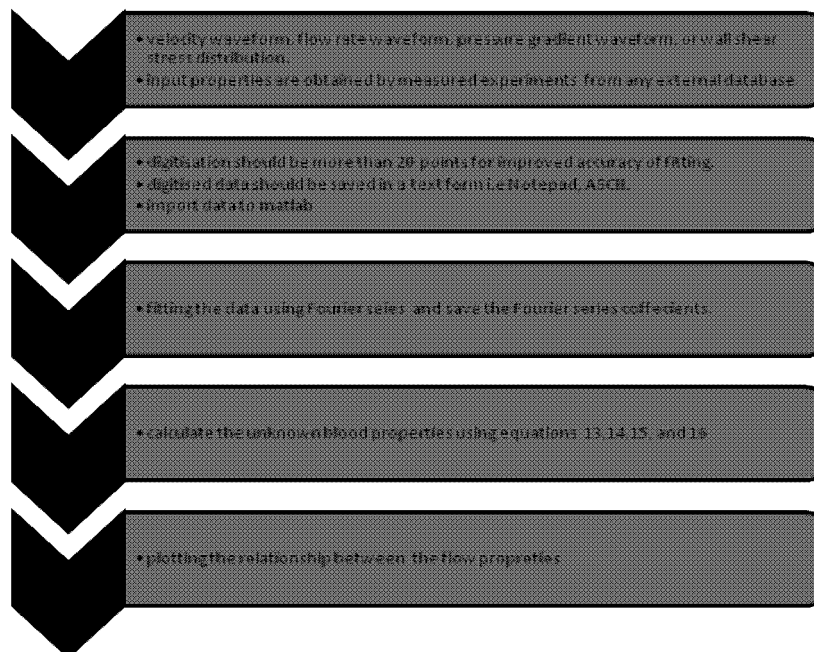


Figure 2. (a) Flowchart of program steps (b) Pulsatile blood flow GUI created with MatLab

3. Results and discussion

As a test to validate the developed program code, the classic Womersley example was used to present our findings. The input data is taken from McDonald (1955) [6] that is, the pressure gradient, being measured in the femoral artery of a dog. This input data is then digitized and its series of data is saved in a text file. The number of digitized data should be as many as possible in order to increase its accuracy. Next, in MatLab, this digitized text data is imported into the workspace of MatLab and this data is renamed as DATA. The pressure gradient waveform was the input i.e. Figure 2 and the corresponding rate of flow was computed and its results are in good agreement with the figures shown in the J. R. Womersley [2] and McDonald [6]. As an extension to this, the corresponding velocity profile distribution, centreline velocity and wall shear stress with regards to the pressure gradient was computed. It is observed that there is a phase lag between zero pressure and zero flow as shown in Figure 3. The positive peak pressure reaches at 35° and subsequently drops back to zero at 78° . During this time, the flow rate increases and reaches its maximum peak at 65° . Next, when the pressure drops almost to its negative peak, the flow rate drops to zero at 112° . The negative pressure gradient continues up to 200° and during this time, the flow decelerates further. Once the pressure gradient increases further in the positive direction from 200° onwards, the flow reverses back to a forward flow. The flow reaches zero at 280° . Besides the flow rate, the axial velocity profile with respect to various radii was computed. Figures 4-6 show the axial velocity distributions at different radius ranging at $r=0.0$ (at centre of the tube), $r=0.1\text{cm}$, and $r=0.13\text{cm}$, respectively. The velocity profile for all these figures have similar shape but with different magnitude, whereby the maximum magnitude is observed at the centreline of the tube and decreases to zero at the wall of the tube.

It is useful to derive the shear stress from the unsteady flow because the quantity is difficult to measure experimentally. Figure 7 shows the relationship between the pressure gradient and wall shear stress. There is a phase lag between these two properties. The development of wall shear stress along the wall of the tube is faster than the pressure gradient.

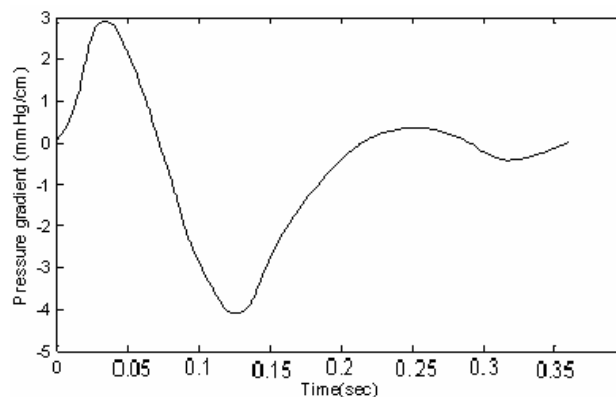


Figure 2. Pressure gradient waveform taken from the femoral artery of a dog [6].

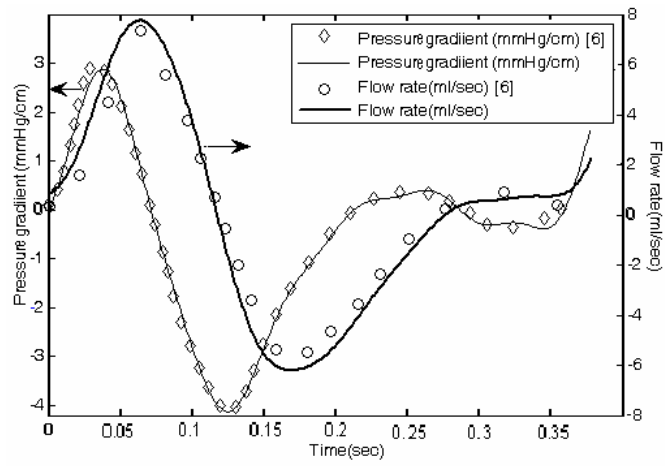


Figure 3. Comparison between experimental results shown in [6] with regards to our calculated results

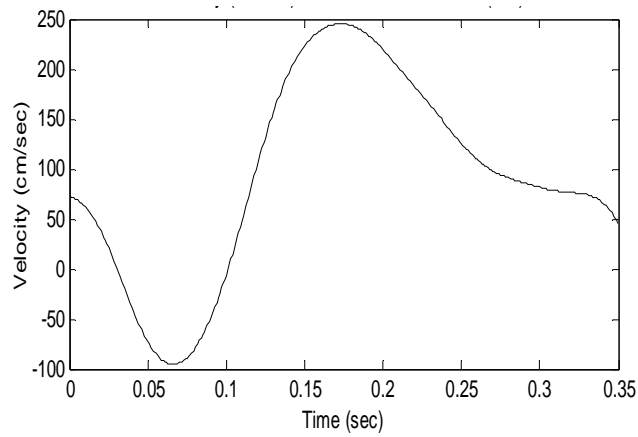


Figure 4. Velocity profile distribution at radius $r=0.0\text{cm}$

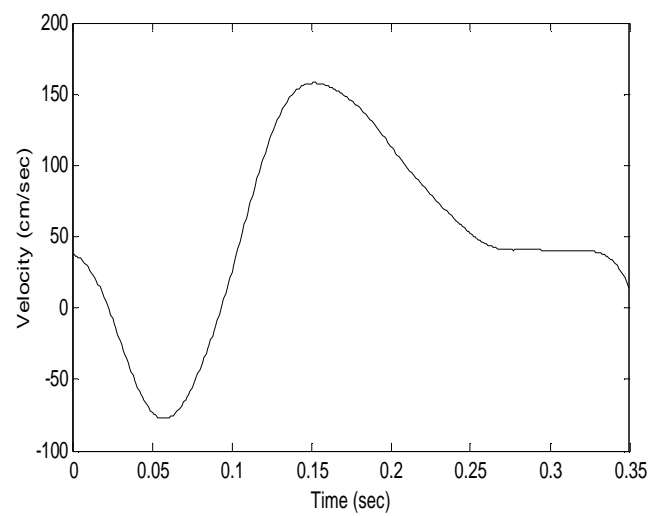


Figure 5. Velocity profile distribution at radius $r=0.1\text{cm}$

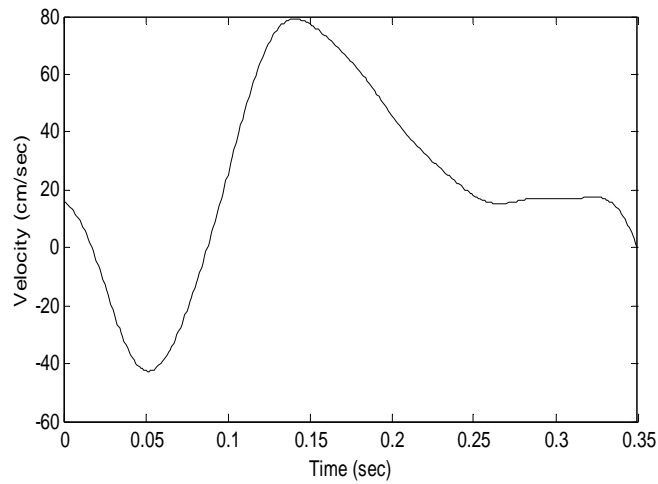


Figure 6. Velocity profile distribution at radius $r=0.13\text{cm}$

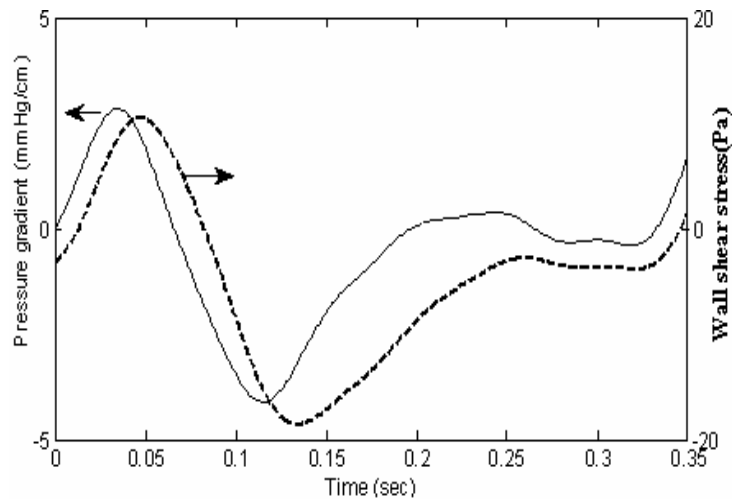


Figure 7. Relationship between wall shear stress and pressure gradient

Conclusion

The exact solution of pulsatile flow was derived. A graphical user interface (GUI) using MatLab was developed to solve these blood flow properties. Any time-dependent blood properties can be used as input data. If one of these time-dependent blood flow properties is known, i.e. pressure gradient, velocity profile, volumetric flow rate or wall shear stress, then the remaining properties can be calculated. This GUI serves as a learning tool for students who wish to pursue his/her knowledge in understanding the relationship of various blood flow properties with pulsatile characteristics as well as the mathematics governing the pulsatile flows.

References

- [1] J. Majdalani, "Exact Navier-Stokes solution for pulsatory viscous channel flow with arbitrary pressure gradient," *Journal of Propulsion and Power*, vol. 24, no. 6, pp. 1412-1423, 2008.
- [2] J. R. Womersley, "Method for the calculation of velocity, rate of flow and viscous drag in arteries when the pressure gradient is known," *J. Physiol*, vol. 127, pp. 553-563, 1955.

- [3] J. F. Hale, D. A. McDonald and J. R. Womersley, "Velocity profiles of oscillating arterial flow with some calculations of viscous drag and the Reynolds Number," *J. Physiol*, vol. 128, pp. 629-640, 1955.
- [4] Xiaoyi He, David N. Ku, and James E. Moore Jr., "Simple calculation of the velocity profiles for pulsatile flow in a blood vessel using Mathematica," *Annals of Biomedical Engineering*, vol. 21, pp. 45-49, 1993.
- [5] Lee Waite and Jerry Fine, "Applied Biofluid Mechanics," McGraw Hill, 2007.
- [6] D. A. McDonald, "The relation of pulsatile pressure to flow in arteries," *J. Physiol*, vol. 127, pp. 533-552, 2005.
- [7] Christopher Depcik, Dennis N. Assanis, "Graphical user interfaces in an engineering educational environment," *Wiley Periodical Inc. Comput. Appl. Eng. Educ.*, 13, pp. 48-59, 2005.