Proposed Simple Multicapillary Flow Model to Generate Constant Pressure: Electrofiltration Equation

S. P. MOULIK

Department of Chemistry, Jadavpur University, Calcutta 700032

With the help of a proposed simple parallely arranged multicapillary system model for the filter and filter-cake equations for constant pressure filtration and electrofiltration have been generated on the basis of the fundamental flow equation of Poiseuille. These are identical in forms with d'Arcy and Sperry equations for normal filtration and the equation of Moulik for electrofiltration, suggesting that the fundamental basis of the proposed model is real. The picture of the permeability (reciprocal of resistance) of the filter as well as the cake has been made physically meaningful.

L IQUID flow through a filter-bed of constant thickness is known to be governed by d'Arcy law[1]. In the case of a bed that continuously grows in thickness, as in filtration, modification of d'Arcy equation made by Sperry[2] and Carman[3] has been considered to be adequate. The underlying principle recognizes two separate resistances, one offered by the filter and the other by the growing filter-cake. The final equation has been derived for liquid transport under a gradient of constant hydrostatic pressure. When the filtration is effected under an applied electric field[4], electrokinetic flows must appear in addition to the normal flow[5], and modification of the equation of Sperry has been made accordingly and has been tested taking measurements in a laboratory-scale apparatus[5-7]. In a very recent paper[8], a complementary analysis of the basic phenomena occurring in electrofiltration has been given applying nonequilibrium thermodynamics.

It is the purpose of the present paper to show that proposition of a simple parallely arranged model multicapillary system can generate d'Arcy and Sperry equations as well as the constant pressure electrofiltration equation put forward earlier by Moulik[5-7].

The Proposed Model

The cross-section of the flow model is shown in Fig. 1. It is considered for simplicity that the assembly is a bunch of parallely arranged capillaries of equal dimensions. In this arrangement in total area, A, formed by the placement of n capillaries, some more pores should develop due to the circular shape of the individual parent capillaries. These are the extra voids. A part of the whole area would of course be the non-void regions which are formed by the participation of the walls of the capillaries. If these voids are blocked or made dead, the bunch of the capillary arrangement will represent a matrix of uniform void and non-void regions. Permeability of such a matrix would depend on the proportions
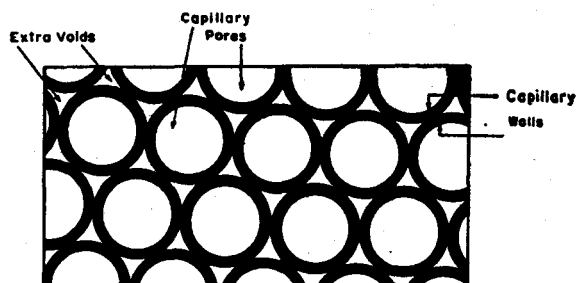

Fig. 1 — Cross-section of a simple parallely arranged multicapillary flow model

of the voids and the non-voids present in the total area. If the extra voids remain as such or the matrix is consisted of capillaries of varying dimensions, the cross-section of the matrix is non-uniform in respect of the void and non-void regions. A membrane filter can have close resemblance to such a model.

Recognizing that the capillary lengths are equal but the diameters different, the volume rate of flow, V_1/T, of a fluid of viscosity, η, at a constant pressure differential, P, through the bunch would be given by Eq. (1) proposed by Poiseuille.

$$\frac{V_1}{T} = \frac{P\pi}{8\eta L}[n_1 r_1^4 + n_2 r_2^4 + n_3 r_3^4 + ...] = \frac{P\pi}{8\eta L}\sum n_i r_i^4 \quad ...(1)$$

where n_i refers to the number of capillaries of type i having radius r_i and length (L).

Now in a total area, A, of the cross-section of a multicapillary bunch if there are n total capillaries of types, 1, 3, 2, ..., the void area per unit face area is

$$\beta = \frac{\pi\sum n_i r_i^2}{A} \quad ...(2)$$

β may be defined as the permeability or the transporting factor of the capillary assembly, and $n = n_1 + n_2 + n_3 + ... = \sum n_i$.

Taking Eq. (2) into consideration, Eq. (1) can be written as

$$\frac{V_1}{T} = \frac{PA\beta}{8\eta L}\left(\frac{\Sigma n_i r_i^4}{\Sigma n_i r_i^2}\right) \qquad ...(3)$$

or

$$\frac{V_1}{T} = \frac{PA}{\eta R_1'} = \frac{P}{R_1} \qquad ...(4)$$

where the total system resistance,

$$R_1 = \frac{8\eta L}{\beta A}\left(\frac{\Sigma n_i r_i^2}{\Sigma n_i r_i^4}\right) \qquad ...(5)$$

and its conventional (partial) resistance,

$$R_1' = \frac{8L}{\beta}\left(\frac{\Sigma n_i r_i^2}{\Sigma n_i r_i^4}\right) \qquad ...(6)$$

(When the dimensions of the capillaries are equal, $R_1 = 8\eta L/\beta A r^2$, and $R_1' = 8L/\beta r^2$).

Eq. (4) is the d'Arcy flow equation[1] for the flow of a clean liquid through a filter-bed. The permeability constant is then the reciprocal of R_1'.

Let us now consider two individual bunches of multicapillary systems, type-1 and type-2, through which a liquid flows at a constant pressure, P. The volume rates of flow will depend on the pressure as well as the dimensions of the capillary systems and should follow equation (4). If these two bunches are now joined in series, and the overall pressure is maintained again at P, then the overall volume rate of flow will be

$$\frac{V_{12}}{T} = \frac{P}{R_1+R_2} = \frac{P}{\frac{TP}{V_1} + \frac{TP}{V_2}} \qquad ...(7)$$

Such a situation prevails in practical electrofiltration experiments[4-6]. The physical significance of Eq. (7) is that the resistance terms R_1 and R_2 are dimensional quantities, and for the same bunches, their magnitudes are independent of the applied pressure and are given by the ratios between the pressure and the volume rates of flow.

The inverse of Eq. (7) is

$$\frac{T}{V_{12}} = \frac{T}{V_1} + \frac{T}{V_2} \qquad ...(8)$$

This equation means that working at a constant pressure helps to eliminate it in the equation and the inverse of the total flow is just the linear combination of the inverse of the individual flows.

The detailed form of Eq. (8) is

$$\frac{T}{V_{12}} = \frac{8\eta L_1}{PA\beta_i}\left(\frac{\Sigma n_i r_i^2}{\Sigma n_i r_i^4}\right) + \frac{8\eta L_2}{PA\beta_2}\left(\frac{\Sigma n_{ij} r_j^2}{\Sigma n_j r_j^4}\right) \qquad ...(9)$$

In Eq. (9), i and j terms are used to differentiate between the two types of the multicapillary systems.

With reference to Eq. (4) we can also write

$$\frac{T}{V_{12}} = \frac{\eta R_1'}{PA} + \frac{\eta R_2'}{PA} \qquad ...(10)$$

and

$$\frac{T}{V_{12}} = \frac{\eta R_1'}{PA} + \frac{\eta L_2 \alpha}{2PA} \qquad ...(11)$$

where

$$\alpha = \frac{16}{\beta_2}\left(\frac{\Sigma n_j r_j^2}{\Sigma n_j r_j^4}\right) = \frac{16A}{\pi \Sigma n_j r_j^4} \qquad ...(12)$$

(For capillaries of uniform dimensions, $\alpha = 16A/\pi n r^4$).

Eq. (11) is identical in form with the equation of Sperry[2] put forward for filtration of a suspension that can form a filter-cake. The resistances of the filter and the cake can be then modelled with the resistances of the multicapillary systems considered above.

Theory of Filtration

Normal filtration — Let us consider that a membrane filter is equivalent to a bunch of tiny capillaries arranged in parallel. In a filtration experiment, suspended particles would be deposited on this filter forming a cake. With the progress of filtration, this growing cake can be imagined to be equivalent to continuous coupling in series of a second bunch of capillaries with the parent matrix. If the deposition is made slow and smooth, then an uniform cake-matrix is possible to be formed, which may impart a constant specific cake resistance[3], α. For such a system Eq. (11) is applicable. Taking ν as the volume of the cake formed per unit volume of the filtrate, L_2 of Eq. (11) can be eliminated[2,3] by $L_2 = \nu V_{12}/A$.

$$\frac{T}{V_{12}} = \frac{\eta R_1'}{PA} + \frac{\eta \alpha \nu V_{12}}{2PA^2} \qquad ...(13)$$

This is the exact filtration equation proposed by Sperry[2] and Carman[3].

Electrofiltration — The increased flow of an aqueous electrolyte solution at constant pressure through a filter under an applied electric field was considered earlier[5] to be a phenomenon of decreased filter resistance caused by the contribution of filter-pore electroosmosis[9]. Under conditions of no surface conductance[9-12], the new resistance, R_1'', was equated with the original resistance, R_1', by a relation

$$R_1'' = \frac{R_1'}{\left(1 + \frac{K\zeta_1 E}{V_1}\right)} \qquad ...(14)$$

where K is a constant, ζ_1 is the electrokinetic potential of the double layer existing at the solid-liquid interface inside the filter-pores, E is the electric field and V_1 is the volume flowed through the filter (the filter has been considered to be the bunch of the multicapillary system, type-1) at zero voltage. The volume rate of flow is then given by

$$\frac{V_1'}{T} = \frac{PA}{\eta R_1''} = \frac{PA(V_1 + K\zeta_1 E)}{\eta R_1' V_1} \qquad ...(15)$$

A similar equation holds also separately for the filter-cake.

$$\frac{V_2'}{T} = \frac{PA}{\eta R_2''} = \frac{PA(V_2 + K\zeta_2 E)}{\eta R_2' V_2} \qquad ...(16)$$

In this equation all terms have the same significance as those appearing in Eq. (15).

The rate of flow for the combined system is then

$$\frac{T}{V_{12}} = \frac{T}{V_1'} + \frac{T}{V_2'} = \frac{\eta R_1' V_1}{PA(V_1 + K\zeta_1 E)}$$
$$+ \left[\frac{\eta \alpha \nu}{2PA^2} \left(\frac{V_2}{V_2 + K\zeta_2 E} \right) \right] V_{12}' \quad \ldots(17)$$

It has been shown earlier[5-7] that at the critical voltage, E_{cr}, the particle flow due to electrophoresis can be made equal to the fluid flow making the net transfer of the suspended particles zero. Then at any voltage, $E < E_{cr}$, the resultant flow towards the filter is

$$V_f = \frac{V_{12}'}{T} - \mu E A \quad \ldots(18)$$

where μ is the electrophoretic mobility of the suspended particles. Eq. (18) will guide the formation of the cake, and at $E = E_{cr}$, $V_f = 0$, yielding

$$V_f = \mu E_{cr} A - \mu E A \quad \ldots(19)$$

The appropriate thickness of the cake, L_2', in electrofiltration[6] is then

$$L_2' = L_2 \left(\frac{V_f}{V_{12}'/T} \right) = L_2 \left(\frac{E_{cr} - E}{E_{cr}} \right) \quad \ldots(20)$$

Introduction of this correction modifies Eq. (17) to

$$\frac{T}{V_{12}'} = \frac{\eta R_1' V_1}{PA(V_1 + K\zeta_1 E)}$$
$$+ \left[\left(\frac{\eta \alpha \nu}{2PA^2} \right) \left(\frac{E_{cr} - E}{E_{cr}} \right) \left(\frac{V_2}{V_2 + K\zeta_2 E} \right) \right] V_{12}' \quad \ldots(21)$$

At the early stages of filtration the factor $(V_2/V_2 + K\zeta_1 E)$, may be nearly equal to unity[5-7]. This may also be the case for systems where $V_2 \gg K\zeta_2 E$. Eq. (21) is then

$$\frac{T}{V_{12}'} = \frac{\eta R_1' V_1}{PA(V_1 + K\zeta_1 E)} + \left[\left(\frac{\eta \alpha \nu}{2PA^2} \right) \left(\frac{E_{cr} - E}{E_{cr}} \right) \right] V_{12}'$$
$$\ldots(22)$$

Eq. (22) is exactly the electrofiltration equation deduced earlier by Moulik[6].

References

1. D'ARCY, H. P. G., Les fontaines publique de la ville de dijon (Victor Dalmont, Paris), 1856, 113.
2. SPERRY, D. R., Chem. metall. Engng, 15 (1916), 198; 17 (1917), 161.
3. CARMAN, P. C., Trans. Inst. chem. Engrs, London, 15 (1937), 150.
4. MOULIK, S. P. & BIER, M., Proceedings of the third American water resources conference, Ser. No. 3, 1967, 524.
5. MOULIK, S. P., COOPER, F. C. & BIER, M., J. Colloid Interface Sci., 24 (1967), 427.
6. MOULIK, S. P., Environmental Sci. & Tech., 5 (1971), 771.
7. MOULIK, S. P., Kolloid Zeit und Zeit Polymere, 254, (1976), 39.
8. MOULIK, S. P., J. Colloid Interface Sci., 53 (1975) 447.
9. ADAMSON, A. W., Physical chemistry of surfaces (Interscience, New York), 1960, 193.
10. KRUYT, H. R., Colloid science, Vol. I (Elsevier, Amsterdam), 1952, 76.
11. RUTGERS, J., Trans. Faraday Soc., 36 (1940), 69.
12. GHOSH, B. N., BANDOPADHYAY, S. & MOULIK, S. P., Proc. Natn Inst. Sci., India, 27 (1961), 193.
13. GHOSH, B. N., MOULIK, S. P. & SENGUPTA, S. K., J. electroanal. Chem., 9 (1965), 372.