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Inventory Level Policy for an Integrated Supply Chain with Time Variable Demand

M. Adel El-Baz [1,2]* and Boushra Taha [2]

[1] Deanship of Scientific Research, King Abdulaziz University, Jeddah, Saudi Arabia

[2] Department of Industrial and Systems Engineering, Zagazig University, Zagazig, Egypt

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Find ways to decrease the total costs related to supply chain activities have given more attention to a more globalized trade. This paper proposes a way to react to time variable demand in an integrated inventory model that enables the manufacturer to reduce lead times and the corresponding variable demand effect. The objective of the model is to control the inventory level in such a way that the total cost of the supply chain is minimized. A solution procedure is suggested and the behavior of the model is analyzed in a numerical example.

Keywords: Time-varying demand, Production-inventory management, Supply chain management

Introduction

Providing customer with high service level at a reduced cost is one of the strategic objectives of supply chain management[1]. Supply chain face challenge and time-varying demand that reflects the practical situation and increasingly compete on delivery lead times to support the responsiveness of the supply chain[2]. To overcome this circumstance, integrated participants of supply chain to deal with time-varying demand resulted in minimizing the system total cost[3,4]. Researches dealt with this problem by controlling the production rate[5,6,7]. Other researcher[8]studied two-stage supply chain under delivery lead time and uncertain demand and made reduction in delivery lead time by crashing setup and transportation times. Many inventory researchers as [9], who studied how varying the production rate influences the inventory level and minimize the system cost of two-stage production facility with multiple parallel machines. Other researches concentrated their research on how to manage the production-inventory system under time-varying demand throw controlling the production rate [10]. Studying the environment of time-varying demand with the controlled inventory level and production lead time has not been addressed enough in the literature.

Problem descriptions and notations

This study presents an integrated production-inventory model, which includes raw material (RM) replenishment, production, transportation and distribution. The manufacturer purchases RM from its suppliers, then converts them into finished goods (FG) through its production facility with a constant production rate, and finally delivers the FG to the subsequent stage. Fig. 1 illustrates how the system under study configuration build up. Due to variation in the market, the customer demand variation is unknown. The required demand is transported instantaneously to customer, if the FG inventory level adequate that, if not the operation time is computed to meet the demand with a processed policy to maintain a variable inventory stock level and is consistent with the gradual

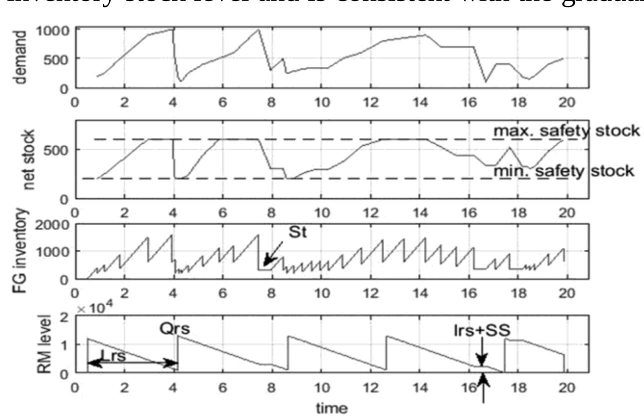

Fig. 1 — production-inventory level under controllable net stock for supply chain system

*Author for Correspondence
E-mail: melmansy@kau.edu.sa

change in demand both in the increase or decrease direction in such a way to achieve reduction in the cost of supply chain. The manufacturer places an order of RM when the level of RM inventory reaches the minimum level of safety stock. In addition, variation in demand leads to variation in RM replenishment times, as shown in Fig. 1 RM level part. Assumptions used in this paper are: A single item production-inventory is considered over the planning horizon, demand is unknown and time-varying, Production rate is constant, the FG transported to customer in un-equal batch sizes with variable times, and the system permits the existence of remaining inventory of RM and FG. Notations are represented in Table 1.

Model formulation

In this section, a mathematical model is developed, which consists of the objective function subject to some constraints. The objective function represents the system total costs of retailer, transportation, manufacturer and RM replenishment, as shown in eq. (1). Retailer includes ordering and holding cost. Transportation consists of fixed cost and variable cost. Manufacturer includes setup cost and holding cost. RM consists of ordering cost and holding cost. The model is restricted by: 1- No initial inventory of FG and RM at the beginning of time horizon; 2- RM remaining inventory at the end of s^{th} replenishment equal SS_h in case of increasing demand and SS_l in case of decreasing demand; 3-FG remaining inventory of each patch is restricted by maximum and minimum FG safety stock level; 4- each RM replenishment quantity is a multiple of production rate. 5- time required to deplete each delivery at customer stage equal production time of next production patch adding to setup time if setup occurs; 6- number of production stop during s^{th} replenishment equal zero or one or more than one; 7- total stopping length during s^{th} replenishment equal setup time multiply by number of production stop during this replenishment.

$$\min z = m * Ar + \frac{1}{2} hr \sum_{j=1}^{m} L_j Q_j + m * At + ht * Lt \sum_{j=1}^{m} Q_j + N * As$$
$$+ h \left\{ \frac{1}{2} * tp_1^2 * P + \sum_{j=2}^{m} \left[\frac{1}{2} * tp_j^2 * P + tp_j \sum_{g=1}^{j-1} \left(tp_g * P - Q_g \right) \right] + St \sum_{i=1}^{N} Irem_{i-1} \right\} + k * Arm$$
$$+ hrm \left\{ \sum_{s=1}^{k} \left[\frac{1}{2} Qr_s (Lr_s - X_s) + SS_{s-1} * Lr_s + St \sum_{t=1}^{b_s} Ir_{s_t} \right] + \frac{1}{2} * IRf * (Lr_k - X_k) \right\}$$

Table 1 — Notations

j	Time-varying demand, $j=1:m$	P	Production rate
n	number used to determine RM replenishment quantity	K	Number of RM replenishments during time horizon (D.V)
Ar	Ordering cost at retailer	m	Number of shipments to customer during time horizon
Arm	Raw material ordering cost per order	IRf	Final inventory level of RM at the end of time horizon
At	Transportation fixed cost	N	Number of production cycle during time horizon
St	Setup time	As	Setup cost
r	Number of RM required to produce a single unit of FG	If_i	The remaining FG quantity at the end of i^{th} production cycle and equal to the initial inventory for $i+1^{th}$ production cycle (D.V)
Lt	Transportation time	$Irem_i$	Net stock after delivering any demand
hr	Retailer holding cost per unit per unit time	Qr_s	RM quantity at s^{th} replenishment, $s=1:k$
h	Holding cost per unit of FG per unit time	Tp_j	Controllable production time required to produce Q_j (D.V)
ht	Transportation variable cost per unit per unit time	Lr_s	Time required to deplete Qr_s at s^{th} replenishments (D.V)
hrm	RMholding cost per unit per unit time	$IRafter_i$	RM remaining inventory after production of i^{th} sub-batch
LR_s	Time s^{th} replenishment from the begging of time horizon	$IRrem_s$	Raw material remaining inventory at the end of s^{th} replenishment
SSh	RM safety stock in case of increasing demand	SSl	RM safety stock in case of decreasing demand
$Nstop_s$	Number of production stop during s^{th} replenishment	SSs	Dynamic safety stock of RM at the end of s^{th} replenishment
$Irs + SSs\text{-}1$	RM inventory level at the end of a production cycle during s^{th} replenishment	$Xstop_s$	Time at which production stops from the begging of time horizon at s^{th} replenishment
X_s	Total stopping time during s^{th} replenishment	Il	minimum safety stock of FG
Ih	maximum safety stock of FG		

Solution methodologies

This section indicates the proposed algorithm and the adaptation policy between demand and controllable inventory level.

Proposed algorithm

The following procedure can be proposed to find a solution of *N*, *K*, If_i, CT_i, tp_j, $Irem_j$ and Lr_s from the input data, such as follows:

Main-algorithm

Step 1:

a) Initialize *Ar, AS, Arm, At, hr, h, hrm, ht, Lt, St, P, r, SSl, SSh, Il, Ih, n.*
b) Set the upper bound value of *m* for *j*;
c) Set the initial inventory level ($Irem_o = 0$) and $j = 0$

Step 2: if$j >= m$, then go to step 10; otherwise, set $j = j + 1$ and go to next step

Step 3: reading the market demand at this decision point (Q_j)

Step 4: if $Q_j + Il > Irem_{j-1}$, then go to the next step; else go to step 9.

Step 5: calculate instantaneous net inventory level at this decision point by eq. (2)

Step 6: calculate the required production quantity and its production time by eq. (3) and (4)

Step 7: calculate retailer cost, transportation cost, and FG holding cost for this sub-batch

Step 8: go to sub-algorithm 1 for RM schedule

Step 9: no production; $tp_j = 0$, $Irem_j = Irem_{j-1} - Q_j$ and repeat step 7, after that go to step 2

Step 10: calculate total cost of retailer, transportation, manufacturer, and RM required

Step 11: end

Sub-algorithm 1

i) If *j=1*, *S=1* and go to 2; else go to 3
ii) Calculate $Qr_s = Ir_s = nrP$, $LR_s(time\ of\ replenshment\ from\ the\ begging\ time\ horizon) = St, and$ $IRafter_j$ (RM remaining inventory after production of i^{th} sub_batch) $= Qr_s - rI_j$, after that go to step 2 in main algorithm
iii) If $IRafter_{j-1} >= r * I_j + SS$, go to 4; else go to 5
iv) $IRafter_j = IRafter_{j-1} - rI_j$, go to step 2 in main-algorithm
v) If $IRafter_{j-1} == SS$, go to 6; else go to 10
vi) $IRrem_{s-1} = SS, and\ s = s + 1$
vii) $Qr_s = n * r * P$, $Ir_s = Qr_s + IRrem_{s-1}$
viii) $IRafter_j = Ir_s - r * I_j$
ix) Time of replenishment is: $LR_s = sum(tp(1{:}\,j - 1 + sum(X1{:}s-1)$, go to sub-algorith 2 to calculate X_s
x) Calculate the reminder (R) of RM required to complete $I_j\left(R = rI_j - IRafter_{j-1} + SS\right)$
xi) Repeat step 6 and 7
xii) $IRafter_j = Ir_s - R$
xiii) $LR_s = sum(tp(1{:}\,j - 1)) + (IRafter_{j-1} - SS)/rP + sum(X(1{:}\,s - 1))$, go to sub-algorith 2 to calculate X_s

Sub-algorithm 2

1) Define a counter in number of production cycle, *count=1*
2) For *i=1:j-1*
3) If $tp_j == 0$, a new production cycle start at *i+1* and go to 4; else go to 2
4) $X\ stop$ (Time at which production stops from the begging of time horizon at s^{th} replenishment) $= sum(tp(1{:}\,i - 1)) + count * St$
5) *Count =count+1*
6) End for, save $count_s$
7) $Nstop_s$(Number of production stop during sth replenishment) $= count_s - count_{s-1}$
8) $X_s = St * Nstop_s$
9) Counter on number of stopping during s^{th} replenishment, *t=0*
10) For *i=1:length(X stop)*
11) If $LR_s < Nstop_s < LR_{s+1}$, *t=t+1*; else go to 10
12) $Ir_{s_t} = Qr_s - r * P(Xstop_{s_t} - LR_s - St(t - 1))$, go to 10
13) End for
14) Go to step 2 in main-algorithm

Demand and controllable inventory level adaptation policy

Knowing that products demand has a continuous variation. An adaptation policy to react with the variation of the demand is developed such that determine the corresponding net stock at each given new demand. This policy is formulated using curve fitting of polynomial type aiming to minimize the mean absolute deviation. Curve interpolation is used to generate the net stock which is scaled by the tangent of incremental demand function, as represented in eq. (2).

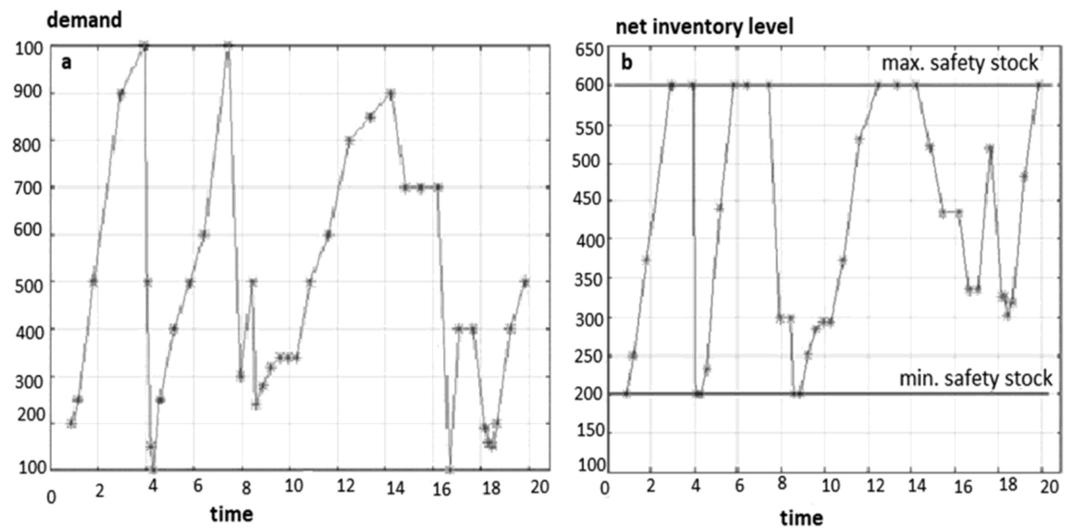

Fig. 2 — controllable net inventory level according to demand variation

$$Irem_j = Irem_{j-1} + \Delta t * \frac{\partial Q(t)}{\partial t} \quad \dots (2)$$

Continuous change in the net stock might depend on the nature of production line; hence the updated net stock is determined based on the available lower/upper boundary of finished goods safety stock related to specific production line which changes from business to another. The instantaneous controllable inventory level and production time can be calculated by eq. (3) and (4) respectively.

$$I_j = Irem_j - Irem_{j-1} + Q_j \quad \dots (3)$$

$$tp_j = {}^{I_j}/_{P} \quad \dots (4)$$

Numerical example and sensitivity analysis

To illustrate the behaviour of the proposed model, a numerical example is derived to device the optimal production-inventory management. The time-varying demand is illustrated in Fig. 2a and the following parameters are used to explain the behaviour of the proposed model**:** *As=$100/setup, At=$80/trip, Arm=$40/ order, Ar=$60/ order, St=0.5 unit time, hr=$0.1/unit, ht=$0.01/unit, h=$0.08/unit, hrm=$0.02/unit, Lt=0.05 unit time, P=1000 units/unit time, r=3, n=4,SSl=50 units, SSh=1000 units, Ih=600 units, Il=200 units.*For this particular problem, the proposed algorithm is employed besides curve fitting of three points of demand is used each time, if a new demand is obtained the last one in another side is eliminated to get the fresh three points of demand. The minimum total operational cost of the integrated supply chain is $ 10712. The computed values of $Irem_j$ according to demand variation are represented in Fig. 2b. The system output has five RM replenishments, as shown in Fig. 1; each one has a quantity equal 12000 units. The time of replenishments' are 0.5, 4.167, 8.67, 12.67,and 17.48 with final inventory equal 1000, 1000, 1000, 50,and 6420units respectively. In addition, the level of RM inventory at the time production stop soccurs during second, fourth, and fifth replenishment sequal 3150, 2412.2, and 11467.7 units respectively. From our knowledge, no researchers studied the same details of the problem exactly as we could compare the results. A comparative calculation is made by fixing net stock during the time horizon. The optimal solution under this policy has a total cost equal $11183.Comparing varying net stock verses fixed net stock; it is observed that applying the adaption between net stock and demand variation is more economical than implementing fixed net stock, which has a reduction equal 4.21%

Conclusion

In this research, an integrated production-inventory model under the consideration of unknown demand fluctuation is studied. A coordination policy is formulated and updated the instantaneous net inventory and production time that reacts with updated continuous variation of product demand in such a way that the total cost is minimized. The results show that, in the environment of demand variation implementations of variable net inventory and production time that interact with demand change can lead to good advantages in reducing the total cost. As a future work, the model can be expanded by integrating multi players in the system instead of serial supply chain. Another option to extend the work is to study the model with multiple product setting in addition to a logistic change is more realistic.

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