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An Integrated Production-Inventory Model with Controllable Production Rates and Dynamic Price

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This paper presents an integrated production- inventory model that involves procurement, production, and delivery, in which product price is an unknown fluctuating pattern with time. A coordination policy is formulated to adapt the instantaneous production rate that reacts with continuous variation of product prices in such a way that the total cost is minimized. A solution procedure is coded to manage the optimal production rates, the economic raw material procurement quantity, the economic finished goods shipments to the customer, the number of shipments to the customer, and the number of raw material replenishment. The behaviour of the model is analysed in different numerical examples. The result of the model has been compared with a particular case of decreasing price over time in one of the previous studies. The results show that the proposed model supports and confirms to the success of reducing total cost in real time of the supply chain in such an environment.

Keywords: supply chain management, integrated inventory control, variable production rate, time varying price, batch size

Introduction

Supply chain is a continuous process from raw material supplier to finished goods consumption stage[1]. Focusing on cost reduction in one zone of the supply chain may lead to a higher cost in another zone. Cooperation between members of the supply chain is essential and earning a competitive advantage with a lower total operational cost of the overall supply chain[2]. Integrating various processes of the supply chain activities is essential to produce products with the right quantity, to the right customer, at the right time through which the system benefits[3]. Most of the studied in this area considered fixed cost during the time horizon that not reflect the practical situation in which product price has a continuous variation. The unpredicted trend of price variation causes a circumstance of fluctuating in the industrial environment. To overcome this circumstance, the supply chain manager should manage their inventory and production schedule efficiently to minimize the system total cost. Many studies overcame the variation in demand by controlling the production rate [4, 5,6,7]. The production rate was taken as an inverse function of system efficiency[8,9]. In addition[8] controlled price variation by production time. Works associated with price change that adapted the purchaser side of supply chain known as EOQ (economic order quantity)[10]. [11]studied the raw material cost fluctuation in a Markovian fashion on the inventory control system. Another studying of price variation on controlling the service inventory system[12]. Studying the environment of dynamic price with the controlled production rate has not been addressed enough in the literature.

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Problem description and notation

This study presents an integrated production-inventory system, which includes raw material replenishment, manufacturer, and customer. The manufacturer purchases raw material form its suppliers, then converts them into finished goods through its production facility, and finally delivers the finished goods to the subsequent stage. Figure 1 illustrates how the system under study configuration build up, in which partial shipments are delivered to the consumption stage before the entire production lot is finished. Therefore, the production schedule looks like a saw tooth with zero beginning and ending inventory level of raw material and zero starting inventory level at the manufacturer. It is shown that the first batch is directly shipped to the customer after its completion time. Due to production rate is greater than the demand rate, the second batch is completely manufacturing before the first batch is completely used up. Therefore, it has to be kept in the stock for a time. Due to manufacturing with constant production rate, the finished goods accumulate in the stock for a time which leads to an increase in inventory cost that leads to increasing the total cost of the system. In this paper, an integrated production-inventory model is developed to minimize the total operational cost and react with a continuous variation of product price through which manager can determine the optimal production rates, the economic raw material procurement quantity and the economic finished goods shipments to the customer. To achieve this objective, a coordination policy is formulated to adapt the instantaneous production rate that reacts with continuous variation of product prices in such a way that the total cost is minimized. Moreover, the proposed model addressed shipment constraints in which the manufacturer delivered the finished goods to the customer of equal or unequal sized batch shipments. Assumptions used in this paper are: There is no initial inventory, Shortage are not allowed, Transportation time is neglected, Holding cost is directly related to the purchasing price, Demand is known and uniform. Notations are represented in Table 1.

Model formulation

In this section, a mathematical model is developed for an integrated production-inventory system. The formulated model is variable in shipping quantity to the customer, time between two successive shipping quantity, production rate, product price, and raw material cost. The decision variables are production rates, the economic raw material procurement quantity and the economic finished goods shipments to the customer, the number of shipments to the customer, and the number of raw material replenishment. Costs for each part of the system are discussed step by step.

Raw material total cost

It composites of raw material ordering cost, raw material procurement cost and raw material inventory holding cost. The ordering cost is calculated by multiplying the number of raw material replenishment by the replenishment cost per order. Procurement cost is equal to the sum of the raw material replenishment quantity multiplied by its purchasing cost. The raw material holding cost is calculated by the sum of time-weighted inventory multiplying by the unit holding cost. Time-weighted inventory can be calculated by multiplying the average quantity by the time in which this quantity is depleted. The time between two raw material replenishments can be calculated by dividing the production uptime of a production cycle by the number of raw material replenishment. The unit

Table 1 — Notations

Symbol	Definition	Symbol	Definition
D	Required demand (units)	d	Demand rate (units/year)
P	Required production rate	T_P	Production uptime
L_j	Interval time required for depleting Q_j ($L_j = Q_j / d$), $j=1,\ldots,m$	L_{rm}	Interval time between two successive raw material replenishment
k	The available increase/decrease in production rate	f	conversion factor of raw material to finished good, $f \leq 1$ [15]
C_m	Fixed cost for manufacturing one unit	C_p	Firm profit per unit
A_m	Setup cost	A_{rm}	Raw material ordering cost per order
A_r	Customer ordering cost per order	N_{req}	Available number of full-truckload
C(t)	Dynamic raw material unit cost	Pr_j	Dynamic finished goods price per unit, $j=1{:}m$
S	Number of raw material replenishment during a production cycle	m	Number of finished goods shipment to the customer
US	Upper bound of raw material replenishment number	Um	Upper bound of finished goods shipments number
P_{max}	Maximum production rate (units/year)	$P1$	Initial Production rate (units/year)
QM	Total production quantity in a production run (units)	Q_j	Quantity delivered to the customer (units/order)
Qrm	Raw material ordering lot size (units/order)	a	The number of shipments at which production stop
i	Interest rate per year	Q_{cont}	Contract shipping quantity (units/order)
TWI	Total time-weighted inventory at the manufacturer	$Irem_f$	Finished good final inventory
TC	System total cost	tc	*System total cost per unit time*

holding cost equal to the interest rate multiplied by the unit procurement cost of raw material.

Manufacturer's total cost

It composites of a manufacturer's production setup cost and manufacturer's finished goods inventory holding cost. The manufacturer inventory holding cost is directly related to the raw material purchasing cost. Inventory holding cost can be calculated by time-weighted inventory times the unit holding cost. Time-weighted inventory is calculated by the area (area 1, area 2 and area 3) under Figure 1, as represented in equation (1). The unit holding cost can be calculated by multiplying the interest rate by the average unit procurement cost of raw materials based on the conversion factor in addition to the unit manufacturing cost.

$$TWI = \frac{Q_1^2}{2P_1} + \sum_{j=2}^{a}\left[\frac{1}{2}\left(\frac{Q_{j-1}}{d}\right)^2 P_j + \frac{Q_{j-1}}{d}\sum_{i=1}^{j-2}\left(\frac{Q_i}{d}P_{i+1} - Q_{i+1}\right)\right] + \sum_{j=a+1}^{m}\frac{Q_{j-1}}{d}\left[\sum_{i=2}^{a}\left(\frac{Q_{i-1}}{d}P_i - Q_i\right) - \sum_{i=a+1}^{j-1} Q_i\right] \quad \text{... (1)}$$

Buyer's total cost

It aggregates the buyer's ordering cost, inventory holding cost and purchasing cost. Buyer's ordering cost is calculated by multiplying the number of order by the cost per order. The finished goods purchasing cost is equal to the sum of the delivered quantity multiplied by its purchasing price. Buyer's holding cost is calculated by the sum of time-weighted inventory multiplying by the unit holding cost. Time-weighted inventory is calculated by multiplying the average quantity by the time in which this quantity is depleted. The unit holding cost can be calculated by multiplying the interest rate by the unit purchasing price of the finished goods. The unit purchasing price at any time is calculated by adding the unit manufacturer cost and the unit profit to raw material unit cost at the same time considering the conversion factor, as shown in equation (2).

$$\Pr(t) = 1/f * C(t) + Cm + Cp \quad \text{... (2)}$$

Finally, the total cost can be calculated by combining the costs elements of retailer, manufacturer and raw material, as shown in equation (3).

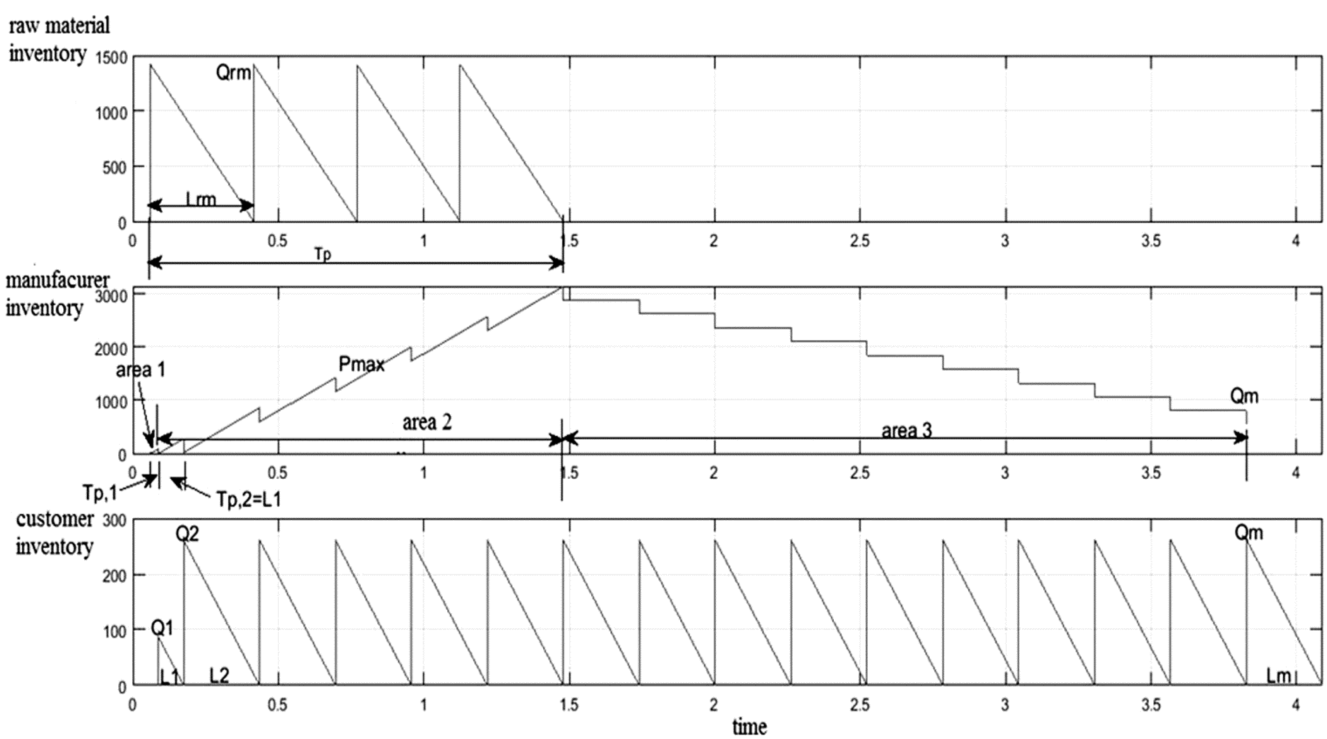

Figure 1 — Inventory profile in a production-inventory system under fixed production rate

$$TC = m\,A_r + \sum_{j=1}^{m} Pr_j Q_j + \frac{i}{2d}\sum_{j=1}^{m} Q_j^2 Pr_j + A_m + \left\{\frac{Q_1^2}{2P_1} + \sum_{j=2}^{a}\left[\frac{1}{2}\left(\frac{Q_{j-1}}{d}\right)^2 P_j + \frac{Q_{j-1}}{d}\sum_{i=1}^{j-2}\left(\frac{Q_i}{d}P_{i+1} - Q_{i+1}\right)\right] + \sum_{j=a+1}^{m}\frac{Q_{j-1}}{d}\left[\sum_{i=2}^{a}\left(\frac{Q_{i-1}}{d}P_i - Q_i\right) - \sum_{i=a+1}^{j-1} Q_i\right]\right\} * \left\{i\left[\left(\frac{1}{f\,S}\sum_{k=1}^{S} C_k\right) + C_m\right]\right\} + SA_{rm} + \frac{\sum_{j=1}^{m} Q_j + Irem_f}{f\,S}\sum_{k=1}^{S} C_k + \frac{i}{2fS^2}\left(\sum_{j=1}^{m} Q_j + Irem_f\right)\left(\frac{Q_1}{P_1} + \sum_{j=2}^{a}\frac{Q_{j-1}}{d}\right)\left(\sum_{k=1}^{S} C_k\right) \quad \dots (3)$$

Solution methodology

This section indicates the proposed algorithm and the adaptation policy between pricing and controllable production rate.

Proposed algorithm

The following procedure can be proposed to find a solution of *S*, *m*, *Q*, *Qrm*,*P* and *tc* from the input data, such as follows:

Step 1:

a) Initialize A_r, A_m, A_{rm}, C, C_m, C_p, d, D, i, f, $P1$, P_{max}, k, N_{req};
b) Set the upper bound values of *Um* and *US* for *m* and *S* respectively ;
c) Set the initial contract value (Q_{cont}), the initial inventory level (*Irem=0*) and *m=0*, *S=0*, *tc* (S*) = inf*.

Step 2: if *m>=Um*, then go to step 9; otherwise, set *m=m+1* and go to next step

Step 3: if $D_m<=0$, then go to step 9; otherwise, get product price (Pr_m) and go to next step

Step 4: if $D_m >= Irem_m$, then go to the next step; else go to step 8.

Step 5: calculate instantaneous production rate by equation (5)

Step 6: go to case 1, after that go to step 2

Step 7: calculate the production uptime

Step 8: go to case 2; after that, go to step 2

Step 9: calculate the total cost for all the shipping quantity at the customer and then go to step 10

Step 10: if (*S >=US*), go to step 12; otherwise set *S=S+1* and go to next step

Step 11: get the unit cost of raw material and calculate the following partial steps

1. Calculate the total cost at the manufacturer
2. Calculate total cost at the raw material stage
3. Compute the total cost per unit time for the integrated system$\left(tc(s) = \frac{d}{\sum_{i=1}^{m} Q_i}(TC)\right)$
4. If *tc(S)<=tc* (S*)*, set *S*=S*, *tc*=tc(S)*; after that, go to step 10

Step 12: the minimum total cost is obtained.

Case 1: production uptime schedule

1. Calculate inventory level $\left(I_m = \frac{Q_{m-1}}{d}P_m + Irem_{m-1}\right)$
2. determine the shipping quantity according to the shipment policy (Q_m)
3. Calculate the remaining demand ($D_m = D_{m-1} - Q_m$)
4. Calculate the remaining inventory after shipment ($Irem_m = I_m - Q_m$)
5. Calculate the time that this shipping quantity depleted at retailer stage $L_m = \frac{Q_m}{d}$
6. Finally, calculate the cost of this shipment ($TCr_m = Ar + Pr_m Q_m + i(Q_m^2 Pr_m/(2d))$)

Case 2: production downtime schedule

a. $I_m = Irem_{m-1}$
b. determine the shipping quantity according to the shipment policy (Q_m)
c. $Irem_m = I_m - Q_m$
d. Calculate $D_m = D_{m-1} - Q_m$and Calculate $L_m = \frac{Q_m}{d}$
e. Finally, calculate the cost of this shipment ($TCr_m = Ar + Pr_m Q_m + i(Q_m^2 Pr_m/(2d))$)

Pricing and controllable production rate adaptation policy

The Knowing that product price has a continuous variation. An adaptation policy to react with the variation of the price is developed such that determine the corresponding production rate at each given new price. Figure 2 represents four decision points and their estimated production rate.

This policy is formulated using curve fitting of polynomial type aiming to minimize the mean absolute deviation, as depicted in Figure 3b. Curve

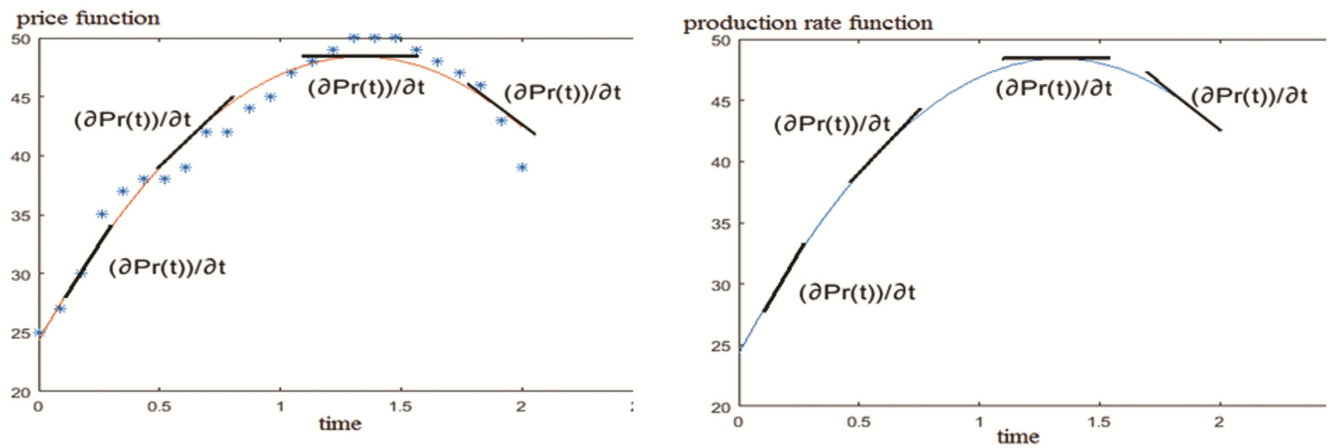

Figure 2 — Updated curve interpolation at each given new price

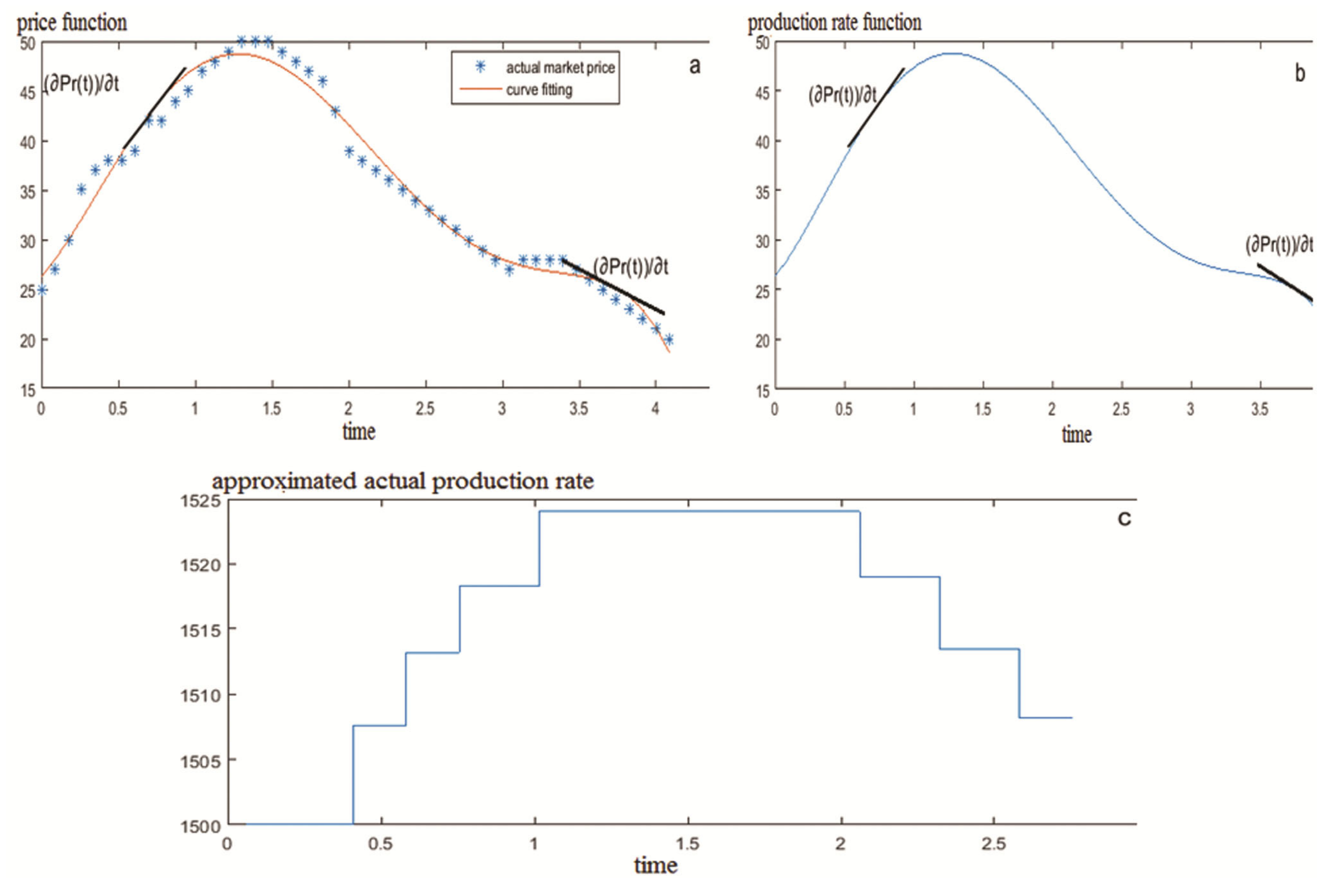

Figure 3 — Updated curve interpolation of updated price curve fitting.

interpolation is used to generate the production rate which is scaled by the tangent of price function, as represented in equation (4):

$$\frac{\partial Pr(t)}{\partial t} = \frac{P_{m+1} - P_m}{\Delta t} \quad \text{... (3)}$$

Substituting by equation (2) in equation (4) then, the instantaneous production rate can be calculated by equation (5):

$$P_{m+1} = P_m + \frac{\Delta t}{f} * \frac{\partial C(t)}{\partial t} \quad \text{... (4)}$$

Continuous change in the production rate might depend on the nature of production line, hence when and for how long to use the updated production rate are determined based on the available increase/decrease in production rate. It can be approximated to the actual production rates, as depicted in Figure 3

Numerical examples and sensitivity analysis

To illustrate the behaviour of the proposed model, using these parameters: D=4000 Units; A_m=$400/setup; A_{rm}=$2500/manufacturer order; A_r=$25/customer order; C_m=$7.5/unit; P_{max}=3200units/year; $P1$=1500 units/year; C_p=$5/unit; d=1000units/year; k=5units; Q_{cont}=87 units; f=0.8; i=2% annually; N_{req}=3 track.

Three natures of product price fluctuation are used; 1- unknown price trend, 2- price with increasing trend, and 3- price with decreasing trend. Each type of product price is analysed under the three policies of shipments. The first policy is to deliver an equal-sized batch shipment to the customer. The second one is to deliver unequal-sized batch shipment that increased by a fixed factor[13]. The last one is to deliver multiple numbers of the contract quantity which is restricted by the available full-truckload. Sensitivity analysis is made for the different scenarios are shown as follows:

Results of scenario 1: unknown price trend

It represents a continuous variation of product price, as depicted in Figure 3a. In which an analysis of three Different shipments policies is made. The result obtained that the optimal solution for this particular example takes place in policy 1, as depicted in Table 2. The optimal solution takes place at minimum total cost when the order of raw material needed for a production run is procured in single shipments with quantity equal to 5057 units. The number finished goods shipments to the customer should be 46 with lot-size equal 78 units and the total quantity produced equal 4046 units. The optimal output points out that delivering of finished goods should be frequent in small lots to take advantage of the price variation, which is continually decreasing in down-stream corresponding to the time horizon of the numerical study.

Results of scenario 2: price with increasing trend

The optimal output of the proposed system occurs at delivering unequal-sized shipments policy to the customer, as depicted in Table 2. This policy allows transporting the available manufactured finished good to the customer with manufacturer zero remaining inventories after every sub-batch. Transporting the available manufactured finished goods to customer leads to that the customer buys the finished product with lower price as possible which minimize the total cost of the system. Besides, the optimal number of raw material procurement equal one which leads to minimizing the integrated total cost of supply chain.

Results of scenario 3: price with decreasing trend

It is concluded that frequent delivery of finished goods to the customer and frequent raw material procurements in small lots is more economical that simulates logic, as represented Table 2. A sensitivity analysis is made by fixing the production rate during

Table 2 — System outputs for all scenarios

Scenario	Policy	TC	S	M	Q	QM	Qrm	Lrm
1	1	95098	1	46	87	4046	5057	2.668
	2	100556	1	8	87,130,196,297,452,687, 1037,1113	4442	5552	2.945
	3	95755	1	17	87,87,174,261,261,261,261,261,261,2 61,261,261,261, 261,261,261,259	4173	5217	2.755
2	1	122669	1	46	87	4073	5092	2.668
	2	120307	1	8	87,130,197,298452,690, 1060,1085	4568	5710	2.973
	3	121979	1	17	87,87,174,261,261,261,261,261, 261,261,261,261,261, 261,261,261,259	4212	5265	2.755
3	1	99998	8	46	87	4075	637	0.344
	2	104301	8	8	87,130,196,292,434,642, 944,1275	4105	641	0.348
	3	101080	8	17	87,87,174,261,261,261,261,261, 261,261,261,261,261, 261,261,261,261	4079	637	0.344

the production run, through which analyses of three Different shipments policies for all scenarios are constructed. Under the consideration of fixed production rate, the calculation of un-equal sized shipment quantity can be determined by $Q_m = Q_{cont} (P / d)^{(m-1)}$. The system outputs are in the case of scenario 1 and scenario 3, adapting instantaneous production rates lead to lower system total cost than fixed production rate for all policies. But, in the case of scenario 2, the developed algorithm gives that it is more economical to produce with manufacturer full capacity under policy 2. This is because of purchasing the raw material and selling the finished goods earlier as possible, leads to minimize the total system cost, which matches with the logic.

Comparison with previous work

A typical example used by[14] is used to verify the proposed model. The most significant difference between the paper at hand and their paper is that they assumed only a continuous decrease in price for the product in the system. Besides, they considered neither variation in production rates nor different shipping quantities. Thus, considering the adaptation between production rate and continuous decrease in price are applied. It is observed that 9.95% reduction is obtained by the integrated production-inventory system currently under attention when the proposed algorithm is employed.

Conclusion and future works

In this research, an integrated production-inventory model under the consideration of unknown price fluctuation is studied. The model developed to determine the optimal production rates, the economic raw material procurement quantity and the economic finished goods shipments to the customer, the number of shipments to the customer, and the number of raw material replenishment. A coordination policy is formulated and updated to adapt the instantaneous production rate that reacts with updated continuous variation of product prices in such a way that the total cost is minimized. The result shows that, in the environment of price fluctuations, implementations of variable production rates that interact with price change can lead to good advantages in reducing the total cost. As a future work, the model can be expanded by integrating quality and cost issues from variable production rate, which may be taken in to account when coordinating the production policies of the supplier and the manufacturer. Another option to extend the work is to study the model in case the demand is unknown and variable time.

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