Indian Journal of Geo Marine Sciences
Vol. 48 (06), June 2019, pp. 886-898

Introduction of loose ribbons in geographic information system

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Received 18 November 2017; revised 02 May 2018

In a geographic information system, we use principally many models, such as points, polylines and regions to represent spatial objects. But, usually, lines represent linear objects that have a width, whereas from a mathematical point of view, lines have no width. To solve this paradox, in previous papers, the notion of rectilinear lines was replaced by rectangular ribbons. The rectangular ribbon was used to represent longish objects such as streets, roads and rivers. However, the problems come from their mathematical modeling because in reality, rivers and roads can have irregular widths and measurement errors must be taken into account. So, not all longish objects have rectangular shapes, but they can have loose ones. To solve this problem, the concept of a loose ribbon need be developed. In this paper, we address the eventual mutation of the topological relations between loose ribbons into other topological relations, according to certain criteria, when downscaling.

[**Keywords**: Generalization; Loose ribbon; Loose topological relationships; Visual acuity; Downscaling]

Introduction

In mostly geographic configurations represented in a geographic information system, such as the running of a road along a sea, the running of rivers such as the Rhone in Lyon…etc, the rectangular ribbons proposed in Lejdel et al. (2015)[1] cannot represent well the reality because rivers or roads, possibly with an irregular width, must be taken into account. To represent more exactly the reality, another representation of these objects must be given to get a more robust model. In this new configuration, we have to consider that these objects have shapes that can be considered as loose rectangles for two main reasons: (1) Width of so-called linear objects can slightly vary and (2) Small measurement errors must

In Figure 1, we show the difference of representation of longish objects such as rivers, by two different models: Rectangular ribbons and loose ribbons. In this figure, the representation that uses the loose ribbons shows all the details of the object such as the bends of the river (Fig. 1a), whereas the second representation that uses the rectangular ribbons illustrates only the general shape of the object (Fig. 1b). So, we choose the loose ribbon representation because it represents more exactly the reality.

Thus, the concept of loose ribbons was developed in this work. Depending on the scale, or more exactly on visual acuity and granularity of interest, a loose ribbon will be mutated into a rectangular ribbon, when a line will disappear. In other words, loose ribbons can be seen as an extension of rectangular ribbons. Moreover, to not get stuck to cartography, the concept of granularity of interest was introduced.

In different cases, we need to change the scale for certain detailed representations when the demanded representation does not exist in the geographic database. When applying this process, various changes have been held in the representation contents of database; such as geometry, topology, etc. For example, in Figure 2, the road is running along part of

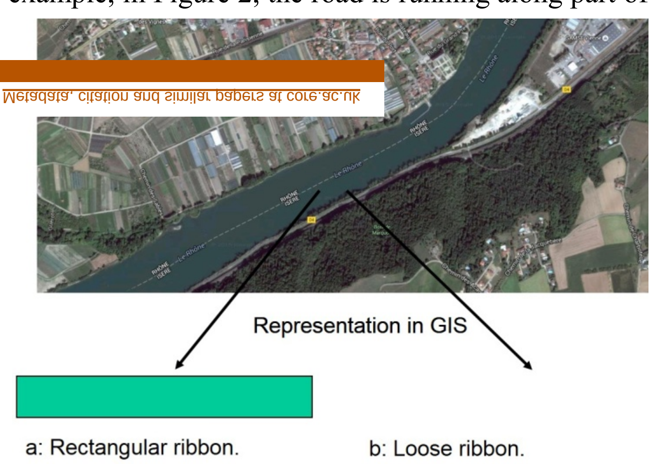

Fig. 1 — Two representations of the same object (river of Rhone).

the Rhone; at some scales, they both appear disjoint because there are some trees between the road and the Rhone (Fig. 2a), whereas at some smaller scales, they meet (Fig. 2b).

Also, when somebody says « this road runs along the sea », what are exactly the spatial or geographic relations concerned? Sometimes, either the road touches the sea or a small beach is located between the road and the sea. From a mathematical point of view, mostly there is a disjoint relation between the road and the sea, whereas for people the relation is different. So, the topological relations mutate according to scale[1]. These issues was be treated in this work. Taking these considerations into account, any reasoning system will generate difficulties because the spatial relations hold differently: any conceptual framework dealing with spatial relationships must be robust against the scales. Thus, a mathematical model composed of some mathematical assertions can be proposed to formalize the topological relations that can be held between the loose objects and their mutations into other topological relations, when downscaling. Thus, a mathematical theory based on metric relations, as area, distance and certain thresholds…etc, was developed to preserve a topological consistency between loose objects, when downscaling. In this work, we used principally two objects: Loose ribbons and regions.

Generalization and Topological Relations

The process of generalization, not only involves the generalization of geographic objects, but it can also generalize topological relationships which can be held between these objects. Thus, there is a need to describe relationships between all objects in space, the points, lines and areas for all possible kinds of deformation. It was started initially by the Douglas and Peucker algorithm published in 1973 and used to generalize polylines[2]. Recently, complete processes use different modern methods as multi-agent-systems (MAS)[3,8]. These use MAS to automate the generalization process.

State of the art for generalization of rectangular ribbons

From historical point of view, different topological models have been proposed. First, Laurini proposed a model for organizing pieces of a linear model as rectangular ribbons and introduced this concept to represent longish objects in GIS[9]. Then, in our previous paper, we used this new concept to define different topological relations between them. Also, we treated different eventual mutation of the topological relations into others relations, when downscaling[1]. For regions, Egenhofer and Herring (1990) proposed the first topological model for two-dimensional objects[10-11] and then a second model family named RCC was proposed by Randell et al. (1992)[12]. Also, Winter (2000) presents a statistical model for quantitative assessment of uncertain topological relations between two imprecise regions[13]. This model was based on a morphological distance function to determine the type of topological relations. Those relations must be reconsidered for ribbons.

Rectangular ribbons: In a recent paper[9], ribbon relations were proposed to describe streets, roads and rivers. Four relations can be defined with ribbons as exemplified in Figure 3, side-by-side, end-to-end,

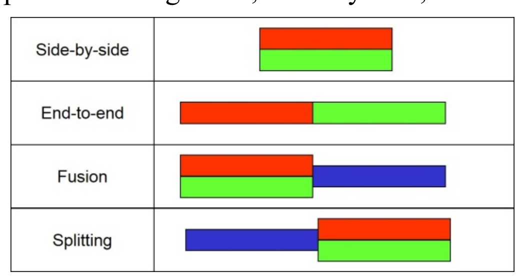

Fig. 3 — Basic ribbon relations

a : Trees between the Rhone and the road b: The Rhone meet the road.

Fig. 2 — According to scale, the Road meets the Rhone.

fusion (or merging), and splitting[9]. For describing a section of a highway, we based the approach essentially on the Allen relation[14].

A real world feature (exp. a road or a river) can be modeled by a single composite ribbon, that is a set of ribbons linked by side-by-side and/or end-by-end relations. As the scale diminishes, ribbons will be reduced to lines, for instance to their axes *(axis(R))* or can disappear.

In the sequel of this paper, sometimes some of those relations will be used. To avoid ambiguities with other relations, the name "Allen" will be used as a prefix when necessary, to designate Allen relations.

In some cases, the user does not know whether two segments are aligned or not. If they can be considered as aligned, Allen relations can hold. For that, let us consider two segments A and B supported respectively by equations $y=m_a x+p_a$ and $y = m_b x+p_b$. Also, two thresholds must be defined (ε_1, ε_2). The segments A and B can be considered as aligned if the following condition holds:

$$\forall A, B \in segment \wedge \left|m_a - m_b\right| < \varepsilon_1 \wedge \left|p_a - p_b\right| < \varepsilon_2$$
$$\Rightarrow AllenAligned(A, B).$$

Rectangular ribbons generalization model: Lejdel et al. (2015), defined a mathematical model for the generalization of topological relationships between two rectangular ribbons or between ribbons and regions[1]. Thus, two rectangular ribbons can be disjoint or intersect. The disjunction is defined by a distance separating the two ribbons. The intersection between the two ribbons can be Point (0D), Line (1D) or area (2D) according to certain criteria. In this work, we formally obtained the mathematical description for each topological relationship between objects when we use thresholds and metric measurements, such as area, distance, etc. These topological relationships can be: Disjoint, meet, merge and crossing. When downscaling, these topological relations can be mutated into other topological relations according to certain criteria. In Figure 4, we present an example of a Meet relation described in the topological model[1]. This relation can be described by the following assertion:

$$\forall R^1, R^2 \in \text{Ribbon}, (\forall \sigma \in \text{Scale}) \wedge (R^1_\sigma = 2Dmap\ (R^1, \sigma))$$
$$\wedge (R^2_\sigma = 2Dmap\ (R^2, \sigma)) \wedge (Inters\ (R^1, R^2) = \{P \vee L\} / P(x, y),$$
$$L(y = ax + b)) \Rightarrow Meet\ (R^1_\sigma, R^2_\sigma).$$

When downscaling, the topological relations between the ribbons can be mutated into other relations according to certain criteria. Some samples of this mutation are presented here:

Mutation of Disjoint to Merge :The disjoint relation mutates into Meet relation, when downscaling (Fig. 5). This process can be modeled as follows:

$$\forall R^1, R^2 \in \text{Ribbon}, (\forall \sigma \in \text{Scale}) \wedge (R^1_\sigma = 2Dmap\ (R^1, \sigma)) \wedge$$
$$(R^2_\sigma = 2Dmap\ (R^2, \sigma)) \wedge Disj\ (R^1, R^2) \wedge (Dist\ (R^1, R^2) < \varepsilon_{Dj})$$
$$\Rightarrow Merge\ (R^1_\sigma, R^2_\sigma).$$

When a ribbon becomes very narrow, we apply this assertion:

$$\forall R \in \text{Ribbon}, (\forall \sigma \in \text{Scale}) \wedge (R_\sigma = 2\text{Dmap}(R, \sigma)) \wedge (\text{Width}(R_\sigma) < \varepsilon_{lp})$$
$$\Rightarrow R_\sigma = \varphi.$$

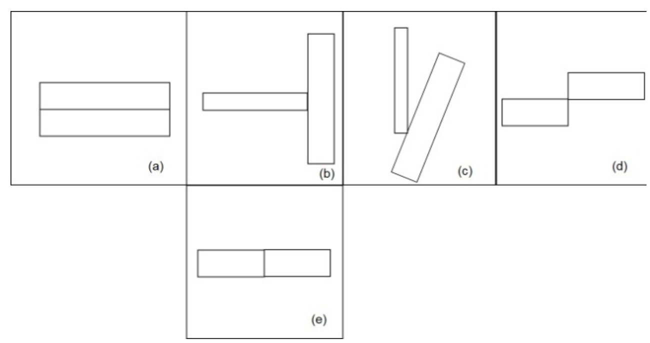

Fig. 4 — Example of meet relation between rectangular ribbons[1]

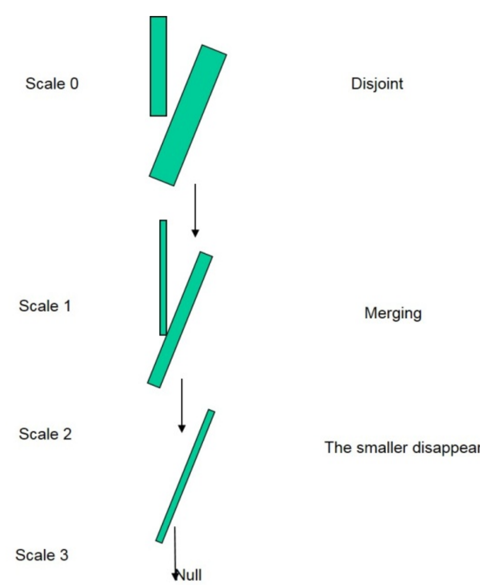

Fig. 5 — Disjoint to meet. In scale 0: the two ribbons are disjoint; In scale 1: Because of the downscaling, the relation between the ribbons became Meet; In scale 2: The most smaller between the two ribbons can be disappear, and in scale 3: All ribbons can disappear.

Mutation of Meet to Merge

The transformation of Meet relation to Merge relation is expressed by the following assertion (Fig. 6):

$$\forall R^1, R^2 \in \text{Ribbon}, (\forall \sigma \in \text{Scale}) \wedge (R^1_\sigma = 2Dmap(R^1,\sigma)) \wedge (R^2_\sigma = 2Dmap(R^2,\sigma)) \wedge (meet(R^1,R^2)) \wedge (Area(R^1 \cap R^2)) > \varepsilon^2_{Mr}) \Rightarrow merge(R^1_\sigma, R^2_\sigma).$$

When a Ribbon becomes very narrow, we apply this assertion:

$$\forall R \in \text{Ribbon}, (\forall \sigma \in \text{Scale}) \wedge (R_\sigma = 2Dmap(R,\sigma)) \wedge (Width(R_\sigma) < \varepsilon_{lp}) \Rightarrow R_\sigma = \phi.$$

State of the art for generalization of regions

Region topological relations: To define a model of topological relationships between simple regions, Egenhofer and Herring (1990) proposed a spatial data model based on topological algebra[10]. The topological algebra model is based on geometric primitives called cells that are defined for different spatial dimensions 0-D, 1-D, and 2-D. A variety of topological properties between two cells can be expressed in terms of the 9-intersection model[15]. The 9-intersection model between two cells *A* and *B* is based on the combination of six topological primitives that are interiors, boundaries, and exteriors of *A* $(A^\circ, \partial A, A^-)$ and $B\,(B^\circ, \partial B, B^-)$.

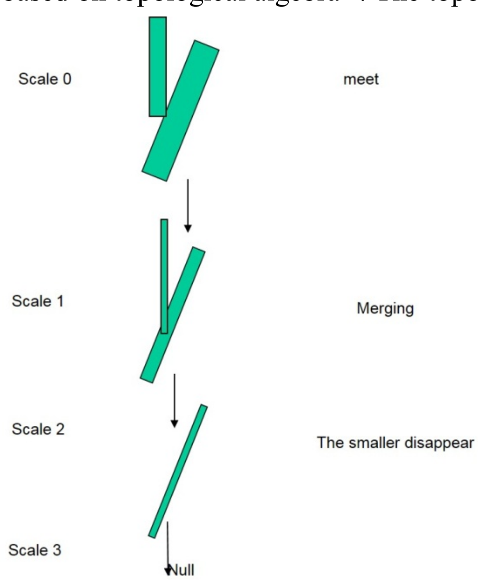

Fig. 6 — Mutation of a ribbon relation Meet to Merge. In scale 0: the two ribbons are Meet; In scale 1: Because of the downscaling, the relation between the ribbons became Merge; In scale 2: The most smaller between two ribbons can disappear; and in scale 3 : All ribbons can be disappear.

These six topological primitives can be combined to form nine possible combinations representing the topological relationships between these two cells. These 9-intersections are represented as a 3×3 matrix[16]:

$$R(A,B) = \begin{pmatrix} A^\circ \cap B^\circ & A^\circ \cap \partial B & A^\circ \cap B^- \\ \partial A \cap B^\circ & \partial A \cap \partial B & \partial A \cap B^- \\ A^- \cap B^\circ & A^- \cap \partial B & A^- \cap B^- \end{pmatrix}$$

The values represented in the matrix will be replaced by a symbol indicating whether the intersection is null (ϕ) or not null ($\neg\phi$) (Fig. 7). For example, the 9-intersection based on null/non-null intersections for a configuration in which region *A* covers region *B* is:

$$Cover(A,B) = \begin{pmatrix} A^\circ \cap B^\circ = \neg\phi & A^\circ \cap \partial B = \phi & A^\circ \cap B^- = \phi \\ \partial A \cap B^\circ = \neg\phi & \partial A \cap \partial B = \neg\phi & \partial A \cap B^- = \phi \\ A^- \cap B^\circ = \neg\phi & A^- \cap \partial B = \neg\phi & A^- \cap B^- = \neg\phi \end{pmatrix}$$

When the value of the intersection is not important, it is represented by (-). If *A* is disjoint from *B*, the intersection between these two regions must be null, for example, if A's boundary is disjoint from B's interior, then the 9-intersection between *A and B* must match the following model:

$$Disjo\,\text{int}(A,B) = \begin{pmatrix} - & - & - \\ \phi & - & - \\ - & - & - \end{pmatrix}$$

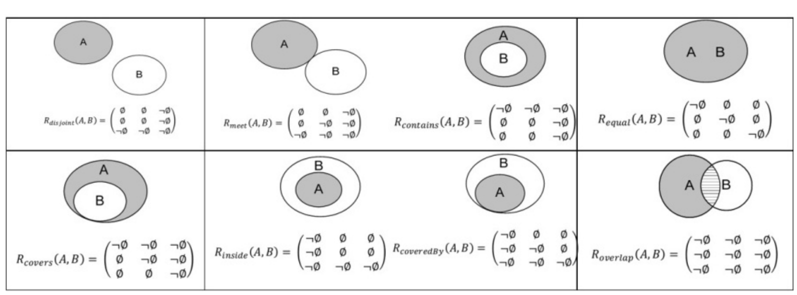

Fig. 7 — The eight topological relations between two regions A and B[15].

Based on these nine possible intersections, one can construct 512 theoretical relations. However, they do not all exist. Therefore, the result implies eight possible topological relations between two regions in $\Re^2$. These eight relations are explicitly represented in Figure 7 (note that sometimes, the MEET relation is called TOUCHES in some papers):

Mutation of topological Region relations: Egenhofer's relations are mainly treated[15]. After generalization, the object geometries are adapted to the perceptual limits imposed by the new (smaller) scale[8]. Laurini treats only the mutation of Disjoint relation to Meet relation[9]. The relation Disjoint mutates to relation Meet (Fig. 8), according to the following assertion:

$$\forall \mathrm{O}^1, \mathrm{O}^2 \in \mathrm{GeObject}, \forall \sigma \in \mathrm{Scale} \wedge O^1_\sigma = 2Dmap(O^1,\sigma) \wedge O^2_\sigma = 2Dmap(O^2,\sigma) \wedge Disjoint(O^1, \mathrm{O}^2) \wedge Dist(\mathrm{O}^1, \mathrm{O}^2) < \varepsilon_1 \Rightarrow Meet(O^1_\sigma, O^2_\sigma).$$

It is noted that 2Dmap is a function transforming a geographic object to some scale possibly with generalization, but a smaller object can disappear or be eliminated if its area is too small to be well visible. So in this case, the initial relation does not hold anymore.

But a smaller object can disappear or be eliminated if its area is too small to be well visible. So in this case, the initial relation does not hold anymore.

$$\forall O \in \mathrm{GeObject}, \forall \sigma \in \mathrm{Scale} \wedge (O_\sigma = 2Dmap(O,\sigma)) \wedge (Area(O_\sigma) < (\varepsilon_{lp})^2) \Rightarrow O_\sigma = \phi.$$

Loose Ribbons and Topological Relationships

In the work of Lejdel et al.[1], the streets, roads and rivers can be modeled as rectangular ribbons and can be mutated to lines, when downscaling. However, in most other geographic configurations, the rectangular ribbons cannot represent well the longish objects, for example rivers do not always have a rectangular shape. Thus, we need to model these objects as loose ribbons with specific properties so that they can be reduced to a rectangular ribbon, lines or disappear when needed. To take these characteristics into account, the concept of loose ribbon is detailed here with some mathematical backgrounds.

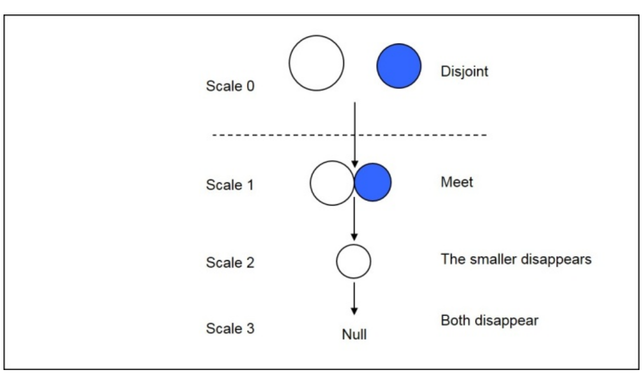

Fig. 8 — The mutation of Disjoint to Meet

Loose ribbons

Starting from any loose ribbon *LR*, we want to qualify it as a rectangular ribbon. For this purpose, let us define the equivalent rectangular ribbon.

The first step is to consider all vertices of *LR* (Fig. 9a), and by the least squares method, to compute the regression line $y=mx+q$ (Fig. 9b). Let us define θ the angle so that $tg(\theta)=m$. Then we make a rotation of $-\theta$ so that the regression line is parallel to the x-axis (Fig. 9c). Then, we sort all vertices according to the ascending values of x and y coordinates. We can determine the minimum and maximum according to those orders. Along x, the mid values of the two first and the two last will determine respectively both ends; and along y, the mid values of the two first and the two last will determine both sides (Fig. 9d); those values will determine the equivalent rectangular ribbon of *LR* noted *ERR(LR)*.

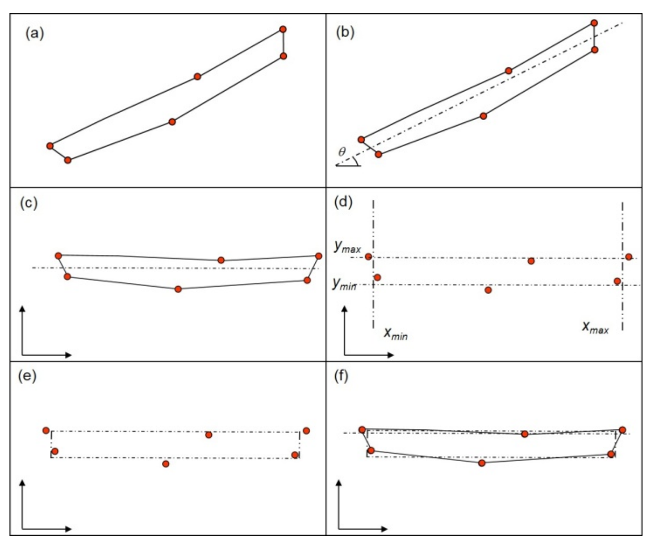

Fig. 9 — Loose ribbons (a) Example of a loose ribbon, (b) By using least squares method, determination of the regression line, (c) Rotation, (d) Determining the elements of the equivalent rectangle, (e) Equivalent rectangle, and (f) Comparing the loose ribbon and its equivalent rectangle.

Now, let us compare the areas of *LR* and *ERR(LR)*. Generally speaking, there is a small discrepancy between those values. A solution is to slightly modify l and w to reach the exact value. Let us note $A_1 = Area(LR)$ and $A_2 = Area(ERR(LR))$. Generally speaking, they are not equal. To force them equal, let us compute the area of $A'2$ in which l and w are modified:

$$A'2 = (w + \Delta w) \times (l + \Delta l) = w \times l \times (1 + \frac{\Delta w}{w}) \times (1 + \frac{\Delta l}{l})$$

$$= A1 \times (1 + \frac{\Delta w}{w}) \times (1 + \frac{\Delta l}{l})$$

Let us suppose that we want to modify those in the same proportion. So, we impose $\frac{\Delta w}{w} = \frac{\Delta l}{l} = t$ the modification ratio, so giving $A'2 = A1 \times (1 + t)^2$. Its value can be given by $t = \sqrt{\frac{A'2}{A1}} - 1$, or $t = \frac{\sqrt{A'2} - \sqrt{A1}}{\sqrt{A1}}$. We reiterate this modification until $|A'2 - A1| < \varepsilon_A$, both areas will be more or less equal.

If the longishness ratio r_l of the equivalent rectangular ribbon is greater than the threshold value r_L, then the loose ribbon *LR* is considered as an extended rectangular ribbon *R=ERR(LR)*. To conclude this step, we need to back rotate the extended rectangular ribbon. So, we can define a loose ribbon from its equivalent rectangular ribbon.

Relations for loose ribbons: Parallel of axis

Let's define loose ribbon relations when the axes of these ribbons are parallel (Fig. 10).

Loose End-to-End relations: Let there be two loose ribbons R_a, and R_b. They will be linked by a loose End-to-End relation $LETE(R_a, R_b)$ provided that:

- The extremities of the skeletons are approximately equal (threshold ε_1),
- The widths are approximately equal ($w_a = w_b$), we must define thresholds ε_2, and
- The skeletons (slopes m_a and m_b) are approximately aligned.

We can model this relation by the assertion:

$$\forall R_a, R_b \in \text{LooseRibbo ns} \wedge (dist(R_a, R_b) < \varepsilon_1) \wedge (|w_a - w_b| < \varepsilon_2) \wedge \mathit{AlignedAll\ en}(Skel(R_a), Skel(R_b)) \Rightarrow LETE(R_a, R_b)$$

Loose Side-by-Side relations: Let there be two loose ribbons R_a, and R_b. They will be linked by a loose Side-by-Side relation $LSBS(R_a, R_b)$ provided that:

- The ends of R_a are respectively approximately aligned with the ends of R_b, and
- A side S_a of R_a is quasi identical to a side S_b of R_b (threshold ε_3).

$$\forall R_a, R \in_b \text{LooseRibbons} \wedge (AlignedAllen(skel(R_a), skel(R_b)) \wedge (dist(S_a, S_b) < \varepsilon_3) \Rightarrow LSBS(R_a, R_b).$$

Other relations of loose ribbons

We can also add other topological relations between loose ribbons when their axes are parallel. Generally, there are three relations (Figure 11).

Loose Contain relation :

Let there be two loose ribbons R_a, and R_b. They will be linked by a loose Contain relation $LContain(R_a, R_b)$ provided that:

- The skeletons of R_a *and* R_b are approximately aligned,
- The loose ribbon R_b is quasi contained in the loose ribbon R_a, and
- The area of relative complement of intersection between loose ribbons R_a *and* R_b is very small.

$$\forall R_a, R_b \in \text{LooseRibbons} \wedge (AlignedAllen(skel(R_a), skel(R_b))) \wedge (R_a \subseteq R_b) \wedge (Area(\neg(R_a \cap R_b)_{R_a}) < \varepsilon^2) \Rightarrow LContain(R_a, R_b).$$

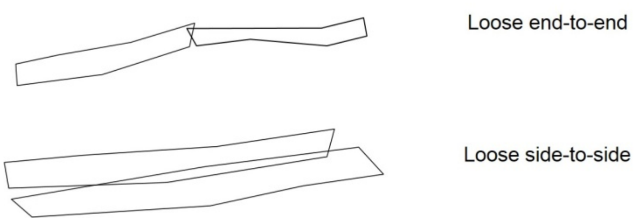

Fig. 10 — Examples of loose End-to-End and loose Side-by-Side relations.

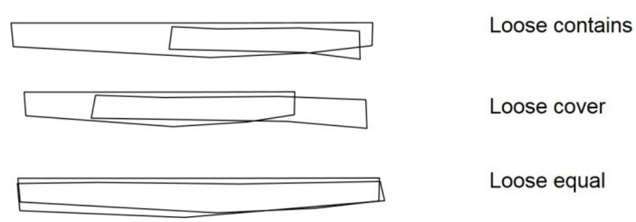

Fig. 11 — Loose ribbon relations when the axis of loose ribbon are parallel.

Loose Cover relation :

Let there be two loose ribbons R_a, and R_b. They will be linked by a loose Cover relation *LCover* (R_a, R_b) provided that:

- The skeletons of R_a *and* R_b are approximately aligned,
- The loose ribbon R_b is quasi contained in the loose ribbon R_a, and
- The area of relative complement of intersection between loose ribbons R_a *and* R_b is great than ε.

$$\forall R_a, R_b \in \text{LooseRibbons} \wedge (AlignedAllen(skel(R_a), skel(R_b)) \wedge (R_a \subseteq R_b) \wedge (Area(\neg(R_a \cap R_b)_{R_a}) > \varepsilon^2) \Rightarrow LCover(R_a, R_b).$$

Loose Equal relation :

Let there be two loose ribbons R_a, and R_b. They will be linked by a loose Equal relation *LEqual* (R_a, R_b) provided that.

- The skeletons of R_a *and* R_b are approximately aligned.
- *The loose ribbon* R_b *is quasi* Equal to the loose ribbon R_a
- The area of intersection between loose ribbons R_a *and* R_b is quasi equal to the area of R_a.

$$\forall R_a, R_b \in \text{LooseRibbons} \wedge (AlignedAllen(skel(R_a), skel(R_b)) \wedge (R_a \subseteq R_b) \wedge (\left|Area((R_a \cap R_b)\;) - Area(R_a)\right| < \varepsilon^2) \Rightarrow LEqual(R_a, R_b).$$

Loose ribbon relations: Intersection of axes

Generally, we can model the relations between the loose ribbons as in Figure 12, where the axes of these loose ribbons can be intersecting. These relations may be seen as an extension of the relations between rectangular ribbons.

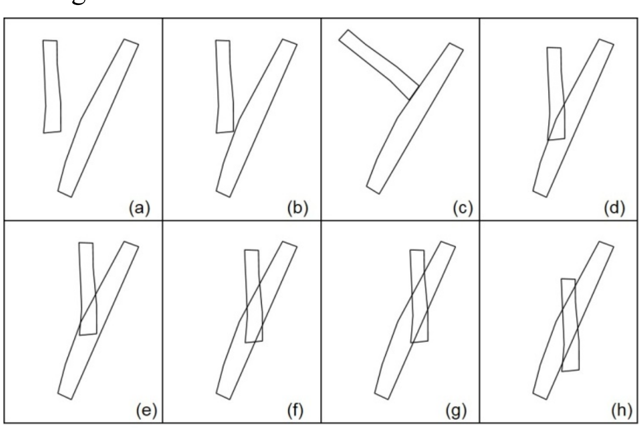

Fig. 12 — (a) Corresponding to a disjoint, (b and c) Corresponding to a meet relation, (d and e) Corresponding to a merge relation, and (f to h) Corresponding to a cross relation.

Loose Disjoint relation: Let there be two loose ribbons R_a, and R_b. They will be linked by a loose Disjoint relation *LDisjoint*(R_a, R_b) provided that:

- The distance between R_a *and* R_b is great then ε, $\text{Dist}(R_a, R_b)$ and
- The skeleton of R_a *and* R_b are not aligned.

This relation is defined by the assertion:

$$\forall R_a, R_b \in \text{LooseRibbons} \wedge \text{Dist}(R_a, R_b) > \varepsilon \wedge \neg AlignedAllen(skel(R_a), skel(R_b)) \Rightarrow LDisjo\text{int}(R_a, R_b).$$

Loose Meet relation: Let there be two loose ribbons R_a, and R_b. They will be linked by a loose Disjoint relation *LDisjoint*(R_a, R_b) provided that:

- The skeleton of R_a *and* R_b are not aligned and
- The intersection between the end of R_a *and* R_b *is a point P(x,y) or Line*

(y=xa+b), $\text{Inters}(End1(R_a) \wedge R_b)$.

The following assertion can model the loose Meet relation:

$$\forall R_a, R_b \in \text{LooseRibbons} \wedge \neg AlignedAllen(skel(R_a), skel(R_b)) \wedge (\text{Inters}(End1(R_a) \wedge R_b) = \{P(x, y) \vee L(y = xa + b)\})) \Rightarrow LMeet(R_a, R_b).$$

Loose Merge relation: Let there be two loose ribbons R_a, and R_b. They will be linked by a loose Disjoint relation *LMerge*(R_a, R_b) provided that:

- The intersection between R_a *and* R_b is great than ε and
- The skeleton of R_a *and* R_b are not aligned.

This relation can be modelled by assertion:

$$\forall R_a, R_b \in \text{LooseRibbons} \wedge (\,Inters(R_a \cap R_b) > \varepsilon^2 \wedge \neg AlignedAllen(skel(R_a), skel(R_b)) \Rightarrow LMerge(R_a, R_b).$$

Loose Cross relation: Let there be two loose ribbons R_a, and R_b. They will be linked by a loose Disjoint relation *LCross*(R_a, R_b) provided that:

- The intersection between R_a *and* R_b is great than ε,
- The skeleton of R_a *and* R_b are not aligned and
- The area of relative complement of intersection between loose ribbons R_a *and* R_b is great than ε.

This relation can be defined by the assertion:

$$\forall R_a, R_b \in \text{LooseRibbons} \wedge \neg AlignedAllen(skel(R_a), skel(R_b)) \wedge (Area(\neg(R_a \cap R_b)_{R_a}) > \varepsilon^2) \wedge \Rightarrow LCross(R_a, R_b).$$

Scaling Mutation of Topological Relations

As previously stated, according to scales, geographic objects can mutate according to two rules (see examples in Fig. 13). As scale diminishes,

- Area will mutate to a point and then will disappear, and
- A loose ribbon will mutate to a rectangular ribbon, and a line will disappear.

General process

The generalization process is very complex[17]. In this context, to simplify the generalization of geographic objects, we adapted the complete process described in Laurini (2014)[9] as follows:

- Step 0: Original geographic features only modeled as areas and/or loose ribbons,
- Step 1: As scale diminishes, small areas and ribbons will be generalized and possibly can coalesce,
- Step 2: As scale continues to diminish, areas mutate to points and ribbons into lines, and
- Step 3: As scale continues to diminish, points and lines can disappear.

This process is called "generalization-reduction-disappearance" (GRD) process.

Visual acuity applied to geographic objects

It is well known that cartographic representation is linked to visual acuity[9]. To define visual acuity, thresholds must be defined. In classical cartography, the limit ranges from 1 mm to 0.1 mm. If one takes a road and a certain scale and if the transformation of this road gives a width more that 1 mm, this road is an area, between 1 mm and 0.1mm, then a line and if less that 0.1 mm the road disappears. The same reasoning is valid for cities or small countries such as Andorra, Liechtenstein, Monaco, etc. In these cases, for example the "holes" in Italy or in France disappear cartographically. With the thresholds previously defined, such as $\varepsilon_{lp} = 0.1\,mm$ and $\varepsilon_i = 1\,mm$, we can formally get:

a/ Disappearance of a geographic object (O) at scale σ:

$$\forall O \in \text{GeObject}, \forall \sigma \in \text{Scale} \wedge O_\sigma = 2Dmap(O,\sigma) \wedge Area(O_\sigma) < (\varepsilon_{lp})^2 \Rightarrow O_\sigma = \phi.$$

Such as *2Dmap* is a function transforming a geographic object to some scale possibly with generalization, in the 2-dimension domain.

b/Mutation of an area into a point (for instance the centroid of the concerned object, for instance taken as the centre of the minimum bounding rectangle):

$$\forall O \in \text{GeObject}, \forall \sigma \in \text{Scale} \wedge O_\sigma = 2Dmap(O,\sigma) \wedge (\varepsilon_i{}^2 > Area(O_\sigma) > \varepsilon_{lp}{}^2) \Rightarrow O_\sigma = Centroid(O).$$

c/Mutation of a ribbon *R* into a line (for instance its skeleton):

$$\forall R \in \text{Ribbon}, \forall \sigma \in \text{Scale} \wedge R_\sigma = 2Dmap(R,\sigma) \wedge \varepsilon_i > Width(R_\sigma) > \varepsilon_{lp} \Rightarrow R = Skel(R).$$

Therefore, one can say that any spatial relation mutates according to scale. As previously stated, one says that a road runs along the Rhone; but in reality, in some places, the road does not run really along the Rhone due to trees, beaches or buildings, etc. At one scale, the road MEETs the Rhone (Fig. 2a), but at another scale at some places, this is a DISJOINT relation (Fig. 2b). For instance, let's consider two geographic objects O^1 and O^2 and $O_\sigma{}^1$ and $O_\sigma{}^2$ their cartographic representations, the following assertion holds:

$$\forall\, \text{O}^1, \text{O}^2 \in \text{GeObject}, \forall \sigma \in \text{Scale} \wedge O^1_\sigma = 2Dmap\,(O^1,\sigma) \wedge O^2_\sigma = 2Dmap\,(O^2,\sigma) \wedge Disjo\,\text{int}(O^1,\text{O}^2) \wedge Dist\,(\text{O}^1,\text{O}^2) < \varepsilon_1 \Rightarrow Meet\,(O^1_\sigma, O^2_\sigma).$$

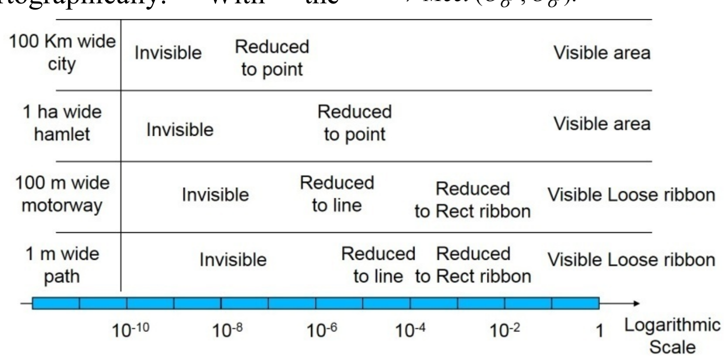

Fig. 13 — Mutation of geographic objects

Similar assertions could be written for Contains, Overlap relations between geographic objects, using Meet relation. In addition, two objects in the real world with a Meet relation can coalesce into a single one.

As a consequence, in reasoning what is true at one scale, can be wrong at another scale. So, any automatic system must be robust enough to deal with this issue.

Granularity of interest

The previous remarks are not only valid for cartography, but also there are other concepts which correspond to cartography. Beyond thresholds of visual acuity, which is a fundamental concept in cartography, let us define granularity of interest: this is the minimum level of interest for a geographic user. For instance, a nationwide politician will be interested at state level, whereas an urban planer will be concerned only at the level of the city for which he works. If we take into account the concept of granularity of interest, the model of the topological relations will be more complex. So, in the sequel of this paper, to simplify the presentation, we will continue to use the thresholds for visual acuity instead of granularity of interest.

Generalization of Topological Relations

The generalization of spatial data implied the generalization of the loose topological relations according to certain accurate rules. We considered here the GRD process to formulate a list of these rules, first between regions. Then, we treated the relations between loose ribbons.

Topological relations of region

Mutation of loose topological relations of region: Often, due to measurement errors and independent processing or generalization, geographic objects do not coincide exactly. Eghenhofer and Dube (2009) investigated the possible connections between topological relations and metrics[18]. When one wants to evaluate the topological relations between them, it is necessary to take this aspect into account. Within the context of granularity of interest, when downscaling, this characteristic will disappear. Let us define loose topological relations when dealing in such cases (Fig. 14). By considering the conventional topological relations, let us immediately say that the Disjoint relation is not concerned, except when the regions are very close.

Loose meet

The criterion to define a loose meet is based on the area of intersection of two regions, A and B. For instance, given a threshold ε_{LM}:

$$\frac{Area(A \cap B)}{Area(A \cup B)} < \varepsilon_{LM} \Rightarrow Lmeet(A,B)$$

When downscaling from σ_1 to σ_2, this mutation Lmeet-to-meet can be defined:

$$Lmeet(A\sigma_1, B\sigma_1) \wedge (\sigma_2 < \sigma_1) \wedge (A\sigma_1 = 2Dmap(A, \sigma_1)) \wedge (A\sigma_2 = 2Dmap(A, \sigma_2)) \wedge \left(\frac{Area(A_\sigma \cap B_\sigma)}{Area(A_\sigma \cup B_\sigma)} < {\varepsilon_2}^2\right) \Rightarrow Meet(A\sigma_2, B\sigma_2)$$

Loose cover

Here one has to evaluate the area of the sliver polygons. This area is composed of two parts, $A \cap B^-$ and $A^- \cap B$. In other terms, this is a symmetric difference defined as follows: $A \oplus B = (A \cap B^-) \cup (A^- \cap B)$. Therefore by defining another threshold, the corresponding criterion can be:

$$\frac{Area(A \oplus B)}{Area(A \cup B)} < \varepsilon_{LC} \Rightarrow \mathrm{Lcover}(A,B)$$

So, the mutation Lcover to cover when downscaling:

$$Lcvr(A\sigma_1, B\sigma_1) \wedge (\sigma_2 < \sigma_1) \wedge (A\sigma_1 = 2Dmap(A, \sigma_1)) \wedge (A\sigma_2 = 2Dmap(A, \sigma_2)) \wedge \left(\frac{Area((A \cap B^-) \cup (A^- \cap B))}{Area(A \cup B)} < (\varepsilon_4)^2\right) \Rightarrow \mathrm{Cover}((A\sigma_2, B\sigma_2)).$$

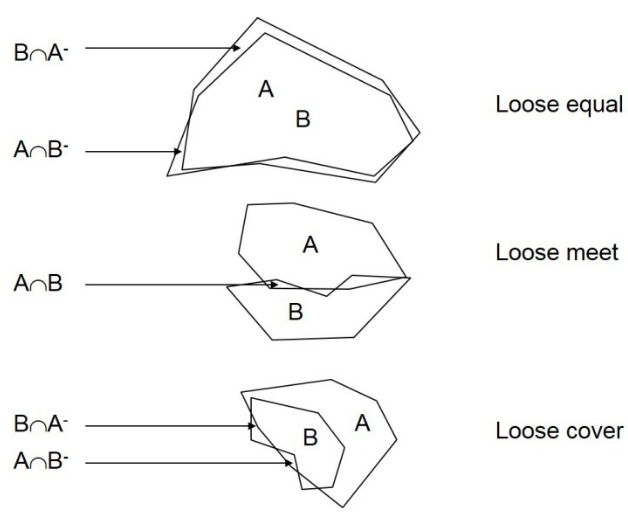

Fig. 14 — Loose topological relations.

Loose equal

The Loose-Equal relation can be defined from the Loose-Cover relation, but the area in the intersection must not be far from the union. So two criteria must be used with another threshold:

$$\frac{Area(A \oplus B)}{Area(A \cup B)} < \varepsilon_{LC} \wedge \frac{Area(A \cap B)}{Area(A \cup B)} < 1 - \varepsilon_{LE} \Rightarrow Lequal(A,B)$$

Similarly, this relation can mutate to an Equal relation when downscaling:

$$Lequal(A\sigma_1, B\sigma_1) \wedge (\sigma_2 < \sigma_1) \wedge (A\sigma_1 = 2Dmap(A,\sigma_1)) \wedge (A\sigma_2 = 2Dmap(A,\sigma_2)) \wedge \left(\frac{Area((A \cap B^-) \cup (A^- \cap B))}{Area(A \cup B)} < (\varepsilon_5)^2\right) \Rightarrow Equal((A\sigma_2, B\sigma_2))$$

Mutation of Eghenhofer relation: The loose relations described in the previous section were mutated into good-standing region relations, when downscaling. We present here the mutations of topological relations which were not treated in the work of Laurini[7], thus, completing the model defined in the work of Laurini (2014)[9].

Mutation of Overlap to Meet

The relation overlap can mutate to Meet relation according to the following condition (Fig. 15):

$$\forall O^1, O^2 \in GeObject, (\forall \sigma \in Scale) \wedge (O^1_\sigma = 2Dmap(O^1,\sigma)) \wedge (O^2_\sigma = 2Dmap(O^2,\sigma)) \wedge (Overlap(O^1, O^2)) \wedge (Area(O^1 \cap O^2) < Area(\neg(O^1 \cap O^2)_{O^2})) \Rightarrow Meet(O^1_\sigma, O^2_\sigma).$$

In addition, similarly, the smaller object can disappear.

Mutation of Overlap to Cover

Also, the relation Overlap may be mutated into relation Cover; to formulate this mutation, one can use the assertion (Fig. 16):

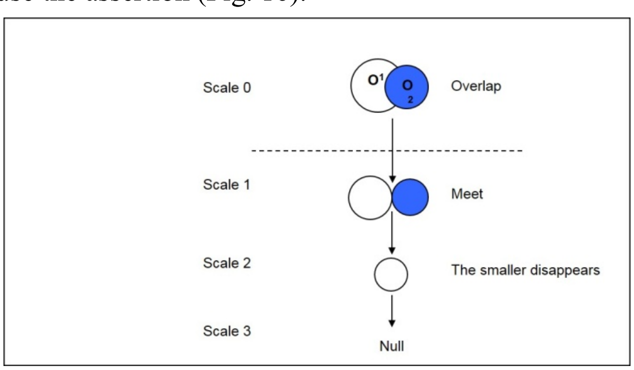

Fig. 15 — The mutation of Overlap to Meet

$$\forall O^1, O^2 \in GeObject, (\forall \sigma \in Scale) \wedge (O^1_\sigma = 2Dmap(O^1,\sigma)) \wedge (O^2_\sigma = 2Dmap(O^2,\sigma)) \wedge (Overlap(O^1, O^2)) \wedge (Area(O^1 \cap O^2) > Area(\neg(O^1 \cap O^2)_{O^2})) \Rightarrow Cover(O^1_\sigma, O^2_\sigma).$$

Mutation of Contains to Cover

The mutation of relation Contains to Cover is expressed by the assertion (Fig. 17):

$$\forall O^1, O^2 \in GeObject, (\forall \sigma \in Scale) \wedge (O^1_\sigma = 2Dmap(O^1,\sigma)) \wedge (O^2_\sigma = 2Dmap(O^2,\sigma)) \wedge (Contains(O^1, O^2)) \wedge (Dist(O^1, O^2) < \varepsilon_1) \Rightarrow Cover(O^1_\sigma, O^2_\sigma).$$

Mutation of Contains to Meet

The mutation of relation Contains to Cover is expressed by the assertion (Fig. 18):

$$\forall O^1, O^2 \in GeObject, (\forall \sigma \in Scale) \wedge (O^1_\sigma = 2Dmap(O^1,\sigma)) \wedge (O^2_\sigma = 2Dmap(O^2,\sigma)) \wedge (Contains(O^1, O^2)) \wedge (Dist(O^1, O^2) < \varepsilon_1) \Rightarrow Cover(O^1_\sigma, O^2_\sigma).$$

Mutation of loose ribbon relations

We consider that loose ribbons as a specific region, the same mutations of topological relations between regions described above can be made for loose ribbons. Thus, the topological relations of loose

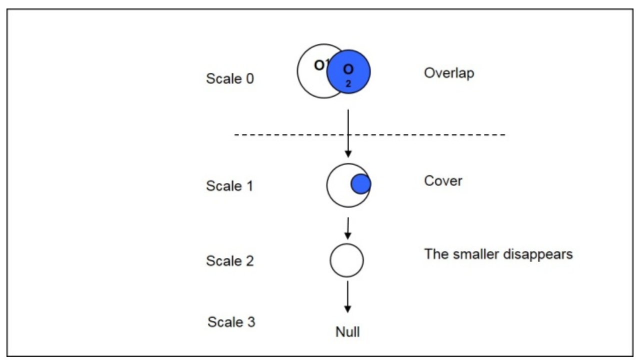

Fig. 16 — The mutation of Overlap to Cover

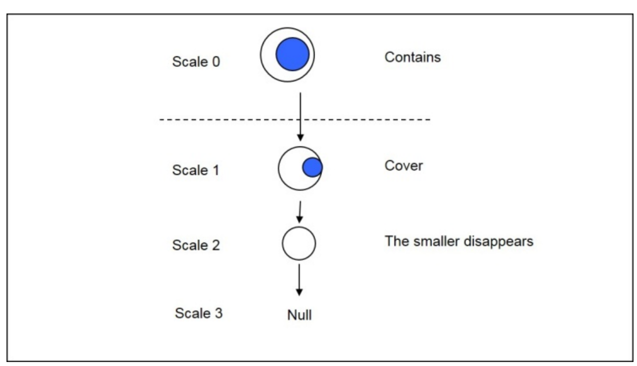

Fig. 17 — The mutation of Contains to Cover

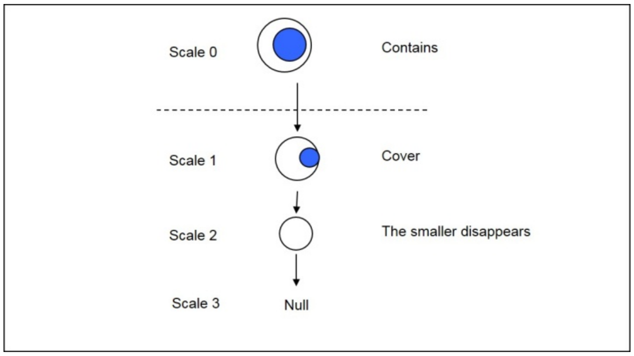

Fig. 18 — The mutation of Contains to Cover

ribbons can be mutating into good-standing ribbon relations. Suppose that one has a road file, a river file and a railway file for the same narrow valley. Since the files are of different origin, during the overlay, sliver polygons can be found and must be removed; these relations cannot be considered as topological relations of loose ribbons. Figures 11 and 12 give different configurations for the Loose Equal, Loose Side-to-Side and Loose End-to-End ribbon relations…etc, which are considered as topological relations of loose ribbons. The same methodology could be applied here as ribbons are specific regions. Thus, the same algorithms can be used to mutate those loose relations into their equivalent good-standing ribbon relations. When downscaling, each of those loose relations is mutated into other topological relations according to the rules mentioned above; also the loose ribbon will be mutating to rectangular ribbons or to polylines or will disappear.

Implementation and Some Results

The application of the generalization process requires changing the geometry of objects and the mutation of their topological relation with other objects. To study the mutation of loose topological relations between objects, we developed the mathematical assertions cited above. Thus, we developed all the necessary functions used to mutate the loose topological relations.

In the implementation, we used the loose ribbons to represent the linear objects which have loose rectangles because the rectangular ribbon cannot represent the reality very well. The majority of the rivers or the roads have irregular width which were taken into account. To validate the concepts proposed in this paper as loose ribbons and loose topological relations, we developed a prototype which can apply the generalization of the topological relations. We present here an example to show the mutations of loose topological relationships when downscaling. The topological consistencies of the map are required when downscaling. However, traditional methods for maintaining the consistency of topological relationships are ineffective, as they do not associate the shape simplification with the mutation of loose topological relationships. Thus, they cannot analyze the mutations of loose topological relationships; this makes them ineffective and weak to preserve topological consistencies in the map.

To implement the GRD process, we defined two thresholds, ε_i for invisibility of objects and ε_{pl} for the reduction of objects (regions or loose ribbons) to points, rectangular ribbons or lines. In our implementation, we took $\varepsilon_i = 0.1\,mm$ and $\varepsilon_{\text{lp}} = 1\,mm$. When downscaling, the rivers and the buildings are generalized and the loose topological relationships are mutated into other relationships. First, the developed prototype can automatically detect the loose topological relationships between objects. Then, it can mutate the loose relations into other relations according to the mathematical assertions cited above. Finally, the prototype can display the result on the screen.

In this paper, we proposed some assertions, which mutate the topological relationships into other ones to maintain the consistency of topological relationships, thus keeping the high quality of the map when downscaling because this is the main objective of the generalization process. Here, we present an example, which is corresponding to different loose topological relationships between spatial objects (loose ribbons or regions) as Loose Disjoint, Loose Meet, Loose Cross, Loose Merge and Loose Covers. They have been successfully tested and indicate the correctness of our concepts and the ability of our mathematical assertions to mutate the loose topological relationships from any given map. In Figure 19, we present a real example. In this example, we applied our model on real spatial data. Before generalization, we have two buildings (building01, building02) and a pathway. We also have a road, river and bridge. When downscaling, we obtain only one building and the Loose Disjoint between the buildings and the pathway mutated into Meet relation. Also, the Cross relation between the river and the road mutated into the Merge relation, and the bridge which is represented by a region, is transformed into a point after generalization.

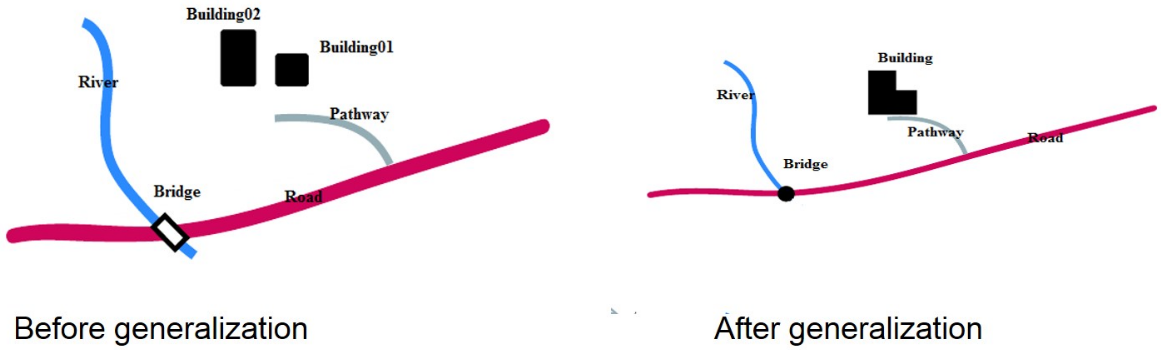

Fig. 19 — Mutation of topological relationships, when downscaling.

In our implementation, we use the loose ribbons to represent the linear objects to verify the correctness of the concepts of the proposed framework. In this study, we present some examples to show the transformations of topological relationships when downscaling. The topological consistencies of the map are required when downscaling. However, traditional methods for maintaining consistencies of topological relationships are ineffective as they do not associate the shape simplification with the transformation of topological relationships. Thus, they cannot analyze the transformations of topological relationships; this makes them ineffective and weak to preserve topological consistencies in the map.

The framework presented in this paper consists to transform the loose topological relationships into other ones to maintain the consistencies of topological relationships, thus, keep the high quality of the map when downscaling. Our collection of cases we tested in the three previous examples is corresponding to different topological relationships between spatial objects (ribbons or regions) as Disjoint, Meet, Cross, Merge and Covers. They have been successfully tested and the test result indicate the correctness of our concepts and the ability of our mathematical assertions to transform the loose topological relationships from any given map.

This study focused only on the transformation of loose topological relationships when downscaling. The mathematical assertions of this framework can be integrated on any simplification algorithm provided by GIS, but this is beyond the scope of this study. This work will be addressed in the future.

Conclusion and Future Works

In a past work, we proposed a topological model to generalize rectangular ribbons which represent linear objects (Lejdel et al., 2015). In this work, we defined loose ribbons which can be seen as an extension of rectangular ribbons with specific characteristics. We develop a topological model of loose ribbons. This model principally is viewed as the extension of the model, which was presented in Lejdel et al. (2015). We considered two objects: Loose ribbons and regions. The application of the generalization's operators may cause topological conflicts. To avoid these conflicts, topological conditions were used to generate the relationships in terms of meeting, overlapping, disjunction, and containment between map objects into other relationships. In this paper, we used these topological conditions to formulate some mathematical frameworks which comprised a set of assertions for treating the variety of loose topological relation according to the scale. When downscaling, a spatial object represented by area can be transformed into a point, or disappear; also, a loose ribbon can be transformed into a ribbon, line, or disappeared. If these geographic objects have loose topological relationships between them, each one will be also generalized using the assertions given in a mathematical framework for each situation.

This work can open various future perspectives, such as:

- Integration of this topological model in the automatic generalization process or on-the-fly web map generation.
- Use of these basic topological relations to model the other relations between complex objects.

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