Indian Journal of Fibre & Textile Research
Vol. 42, September 2017, pp. 359-363

New theoretical modeling for predicting yarn angle on OE yarn influenced by fibre movement on torus coordinate based on classical mechanics approach

Valentinus Galih Vidia Putra[1,a], M Farchani Rosyid [2] & Guntur Maruto[2]

[1]Department of Textile Engineering, Politeknik STTT, Bandung, Indonesia
[2]Physics Department, Universitas Gadjah Mada, Yogyakarta, Indonesia

Received 18 March 2015; revised received 6 October 2015; accepted 8 February 2016

In this study, fibre movement inside yarn has been analyzed using geodesic equation on torus coordinate. By understanding the trace of fibre movement inside the open-end (OE) yarn, the relation of twist and angle of twist can be established well. In this new model, the equation of fibre movement has been found and the relationship of angle of twist to diameter of yarn is well formulated.

Keywords: Angle of twist, Classical mechanics, Fibre migration, Open-end spinning, Torus coordinate, Yarn movement,

Spinning can be defined as a process to produce fibre or filament from natural or synthetic polymer to form a yarn used in textile industry. The study of fibre movement inside yarn and its influence on properties of yarn have been studied by many researchers both theoretically and experimentally [1-9]. According to many reseArachers[1-3,7,9], twist is defined as the ratio of rotor angular velocity to delivery yarn velocity or it may be defined as the amount of turned yarn per length of yarn. Fibre migration is the change in the distance of a fibre (along its length) from the axis of a yarn, which occurs during production of open-end spinning yarn. According to Lawrence[3], the characteristic of spun yarn can be determined by the fibre movement and yarn structure (Fig.1).

According to Backer *et al*[1], the strength of yarn per tex (for filament yarns) is influenced by the rate of twist and the relation is explained as: the lower the twist, the higher is the strength of yarn per tex and vice versa. Rohlena[7] found that breakage rate is influenced by the twist. The higher the twist, the higher is the breakage rate. According to Backer *et al.*[1], Lawrence[3] and Trommer[9], the relation of twist and rotor angular speed can be formulated using the following equation:

$$T = \frac{tan\alpha}{\pi d_{yarn}} = \frac{n_{rotor}}{v_d} \quad \text{... (1)}$$

where α is the angle of twist; d_{yarn},the diameter of yarn; T, the twist in unit (1/m), n_{rotor}, the rotor angular speed in unit (rpm); and v_d, the yarn delivery speed in unit (m/min). According to Lawrence[3], the probability (P) that fibre is laid inside yarn can be written as

$$P = \frac{\sum \Delta l_i}{L_f} \quad \text{... (2)}$$

According to Lawrence[3] and Rohlena[7], fibre migration can influence the type of spinning. Backer *et al.*[1] and Rohlena[7] developed mathematical relationship of fibre migration inside of yarn as shown in Eq. (2).

Thus, if the probability P=1, then the full length of fibre will be spun in and if the probability P =0, then fibre is laid on the surface, and is called hair. If part of the fibre length is spun in and the rest protrudes from the yarn, then $\sum \Delta l_i < P < L_f$. The trace of fibre inside the yarn is shown in Fig.2**.**

Yarn properties can be analyzed and determined from the fibre movement which is shown by the ratio of yarn length to fibre length (K_f), as shown below:

$$K_f = \frac{L_i}{L} = \frac{\sum l_i cos\alpha}{L} \quad \text{... (3)}$$

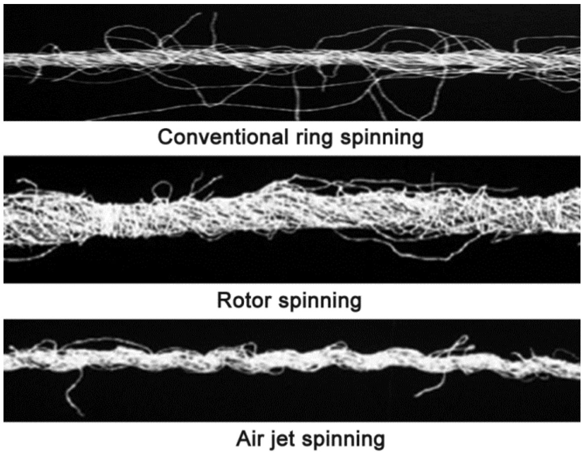


Fig. 1 — Structure of different yarn[3]

Corresponding author
E-mail: valentinus@kemenperin@go.id

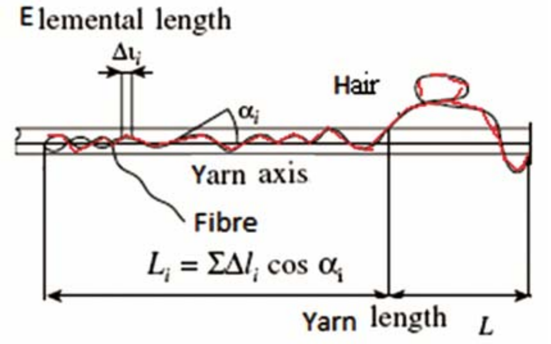


Fig.2 — Fibre movement inside OE yarn

Fig.3 — Yarn movement on rotor spinning

where L_i is the sum of fibre length which is projected to the yarn length L; and α, the angle of yarn length against fibre trace. According to Lawrence[3] and Rohlena[7], the ratio of yarn length to fibre length (K_f) will influence the strength. The higher the value of K_f, the more is the strength of yarn. Rohlena[7] observed that breakage rate was influenced by the twist. The higher the twist, the higher is the breakage rate. Furter[2] found the same results as by Backer *et al.*[1] that the lower the tenacity, the higher is the twist**.** According to Musa and Ayse[4], Penava[5] and Prendzova[6], the relationship of yarn strength is proportional to the diameter of yarn. The wider the diameter of the yarn the higher is the strength of the yarn. The relationship of yarn strength with yarn diameter is related by the movement of fibre which is inside the yarn. The influence of fibre movement on the properties of yarn, such as diameter of yarn, angle twist, strength, hairiness, yarn delivery, and also yarn twist has been discussed and derived in this study considering the fibre-yarn movement on torus coordinate.

Experimental

Fibre Moving on Torus Coordinate

Fibre moving on torus coordinate can be derived using a mathematical method. By knowing the coordinate system, the fibre equation inside yarn can be analyzed. In particular, a fibre with length (dl) moves inside a yarn, whose length is du during a time (dt). A yarn is assumed to be formed as torus coordinate, whose radius (a) and the length of gap (b) are shown Fig.3. A coordinate system, usually to be encountered in addition to the Cartesian, is of the curvilinear type, such as torus. Consider the three quantities of C_m (m=1,2,3) which relate to the rectangular coordinates and the three transformed quantities (torus coordinate) of $\widetilde{C_\mu}$ (μ =1,2,3) related to C_m as shown below:

$$\tilde{C}_\mu = \frac{dx^m}{d\tilde{x}^\mu} C_m \qquad \text{... (4)}$$

Also consider a transformation from rectangular coordinates to torus coordinates. as shown

$$S = (x, y, z) = [(b + r\cos v)\cos u, (b + r\cos v)\sin u, r\sin v] \qquad \text{... (5)}$$

Equation (5) can predict the yarn movement using geodesic equation.

The square of the line element on torus coordinate is given below:

$$ds^2 = (dx^2 + dy^2 + dz^2) = [(b + r\cos v)^2 du^2 + r^2 dv^2 + dr^2] \qquad \text{... (6)}$$

The metric element can be determined, as shown below

$$g_{mn} = \begin{pmatrix} g_{11} & 0 & 0 \\ 0 & g_{22} & 0 \\ 0 & 0 & g_{33} \end{pmatrix} = \begin{pmatrix} (b + r\cos v)^2 & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad \text{... (7)}$$

The yarn is assumed to be circular in cross-section and the radius of yarn (r) is kept constant (a). The fibre is rotated with angular velocity $(\dot{v})$ and moves parallel to the yarn length by speed $(\dot{u})$. The square of the line element on torus coordinate is given below:

$$dS^2 = (dx^2 + dy^2 + dz^2) = ((b + a\cos v)^2 du^2 + a^2 dv^2) \qquad \text{... (8)}$$

The metric element can be determined as shown below:

$$g_{mn} = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} = \begin{pmatrix} (b + a\cos v)^2 & 0 \\ 0 & a^2 \end{pmatrix} \qquad \text{... (9)}$$

The geodesic equation of torus for u and v can be written as (after some calculations):

$$\ddot{u} - \frac{2a\sin v}{b + a\cos v}\dot{u}\dot{v} = 0 \qquad \text{... (10)}$$

$$\ddot{v} + \frac{1}{a}\sin v(b + a\cos v)\dot{u}^2 = 0 \qquad \text{... (11)}$$

Defining that

$$\beta = b + a\cos v \qquad \text{... (12)}$$

$$\dot{\beta} = -a\dot{v}\sin v \qquad \text{... (13)}$$

Substitute Eq (12) and Eq (13) to Eq (10), hence

$$\ddot{u} + \frac{\dot{\beta}}{\beta}2\dot{u} = 0 \qquad \text{... (14)}$$

$$\dot{u} = Q\beta^{-2} = \frac{Q}{(b+a\cos v)^2} \qquad \text{... (15)}$$

The dimension of Q is $[L]^3[T]^{-1}$ and $\dot{v}$ can be expanded as

$$\ddot{v} + \frac{1}{a}\sin v\beta\dot{u}^2 = 0 \qquad \text{... (16)}$$

$$\ddot{v} + \frac{1}{a}\left(-\frac{\dot{\beta}}{a\dot{v}}\right)\beta\dot{u}^2 = 0 \qquad \text{... (17)}$$

Hence, after some calculations, one finds from Eq. (17) that $\dot{v}$ can be written as shown below:

$$\dot{v} = \pm\sqrt{-\frac{Q^2}{a^2(b+a\cos v)^2} + Konst} \qquad \text{... (18)}$$

After some calculations, it can be shown as below:

$$\frac{\dot{u}}{\dot{v}} = \frac{du}{dv} = \frac{\frac{Q}{(b+a\cos v)^2}}{\sqrt{-\frac{Q^2}{a^2(b+a\cos v)^2}+Konst}} \qquad \text{... (20)}$$

Hence

$$\frac{dv}{du} = \frac{n_r}{n_d} = \left(\frac{b}{a} + \cos v\right)\sqrt{\left(\frac{aV}{Q}\right)^2 (b + a\cos v)^2 - 1} \qquad \text{...(21)}$$

For V has a dimension of $[T]^{-1}$ and for $\tan\alpha = \frac{n_r}{n_d}$, hence

$$\frac{\tan\alpha}{2\pi a} \cong \frac{n_r}{v_d} = \left(\frac{b}{2\pi a^2} + \frac{\cos v}{2\pi a}\right)\sqrt{\left(\frac{aV}{Q}\right)^2 (b + a\cos v)^2 - 1} \qquad \text{... (22)}$$

$$T = \frac{\tan\alpha}{\pi d_{yarn}} = \left(\frac{b}{2\pi a^2} + \frac{1}{2\pi a}\right)\sqrt{\left(\frac{aV}{Q}\right)^2 (b + a)^2 - 1} \qquad \text{... (23)}$$

According to Trommer [9], $d_{yarn} = 0.04\sqrt{tex}$ and Lawrence [3], $gap = \frac{v_d}{n_r}$, hence

$$\alpha \approx \tan^{-1}\left\{\left(\frac{\left(\frac{\frac{v_d}{n_r} + 0.02\sqrt{tex}}{0.02\sqrt{tex}}\right)}{\sqrt{\left(0.2\sqrt{tex}\right)^2 M_S \left(\frac{v_d}{n_r} + 0.02\sqrt{tex}\right)^2 - 1}}\right)\right\} \qquad \text{... (24)}$$

$$T = \frac{\tan\alpha}{\pi d_{yarn}} = \frac{\left(\frac{\frac{v_d}{n_r} + 002\sqrt{tex}}{\frac{a^2}{2}\pi tex}\right)}{\sqrt{\left(0.02\sqrt{tex}\right)^2 M_S \left(\frac{v_d}{n_r} + 0.02\sqrt{tex}\right)^2 - 1}} \qquad \text{...(25)}$$

M_S is determined as constant value which depends on A/Q. According to experimental result of Textile Research Center for open end spinning, a yarn of radius $r_{yarn} = 0.02\sqrt{Tex}$ =0.1mm (Tex=25), and gap (b) = $\frac{v_d}{n_{rotor}} = \frac{64.27\text{m/min}}{72000\text{rpm}} = 0.89$mm (ref.3). Suppose a fibre moves along trajectory with angle $\alpha = 35^o$, hence for radius of yarn a = 0,1 mm we obtain the constant (M_S) which is $102 = \frac{1}{(0{,}1)^2} \approx \frac{1}{(r_{yarn})^2}$. Table 1 shows that by knowing the constant (M_S), the angle of twist can be measured considering the gap, the rotor velocity, yarn delivery velocity and yarn count number.

It can be observed from Table 1 that the radius of yarn is influenced by the angle of twist. The higher the angle twist of yarn, the wider is the radius of yarn for a certain condition (the gap b and twist T are constant). The angle of yarn twist can be measured using the following equation:

$$\alpha \approx tan^{-1}$$

$$\left\{\left(\left(\frac{\left(\frac{v_d}{n_{rotor}}\right) + 0.02\sqrt{tex}}{0.02\sqrt{tex}}\right)\sqrt{[102, \left(0.02\sqrt{tex}\right)^2 \left(\frac{v_d}{n_{rotor}} + 0.02\sqrt{tex}\right)^2] - 1}\right)\right\} \qquad \text{...(26)}$$

Table 1 — Relationship of radius of yarn to angle twist [M_S=102]

$Yarn\ count$ (Tt), tex	Radius of yarn (a), mm	Gap (b) mm	Angle twist of yarn (α), deg
25	0.100	0.89	35
30	0.109	0.89	52
40	0.126	0.89	61,6

Simulation of Fibre Movement Inside a Yarn

The prediction of fibre movement has been investigated and is shown in Table 2. The simulation of fibre movement inside the yarn was carried out using Eq (16), Eq (17) and the computer program.

According to Lawrence[3], Rohlena[7] and Trommer[9], twist (number of turned of yarn per length of yarn) is influenced by the angular speed of yarn and velocity of delivery yarn. The higher the angular speed of yarn, the higher is the twist. Using Eq. (26) the angle of twist can be measured. The simulation result also shows that the angular speed of yarn is proportional to the twist.

Results and Discussion

The prediction of fibre movement inside the yarn has been derived using torus coordinate occurred on rotor open end spinning machine. In this new theory, the relationship of yarn diameter , the gap of yarn-rotor and the angle of twist has been established. The model in this prediction has shown the relationship of the angle of twist to diameter of yarn. The higher the yarn diameter, the higher is the angle of twist, twist T and gap b are kept constant.

Based on the theoretical consideration, it can be found that the higher the twist, the higher is the yarn count number. The higher the yarn diameter, the lower is the twist, for the same yarn count number. In this research, the relation of twist and yarn diameter is established. According to Musa and Ayse[4], Penava[5] and Prendzova [6], the relationship of yarn strength is proportional to the diameter of yarn. The wider the diameter of the yarn the higher is the strength of the yarn, The relationship of yarn strength with yarn diameter is related to the movement of fibre inside the yarn. According to Backer *et al.*[1], the strength of yarn per tex is influenced by the rate of twist and the relation is shown as: the lower the twist, the higher is the strength of yarn per tex and vice versa. Rohlena[7] observed that breakage rate is influenced by the twist. The lower the twist, the higher is the breakage rate. Based on the theoretical consideration and also the data from literature, it can be assumed that the relation of twist to yarn properties can be formulated by using following equation:

$$T = \frac{\tan\alpha}{\pi d_{yarn}} = \left(\frac{b}{2\pi a^2} + \frac{1}{2\pi a}\right)\sqrt{\left(\frac{aV}{Q}\right)^2 (b+a)^2 - 1} \qquad \text{... (27)}$$

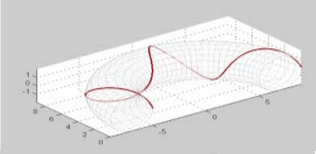
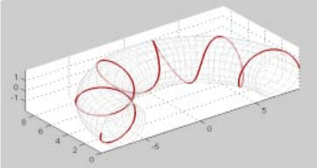
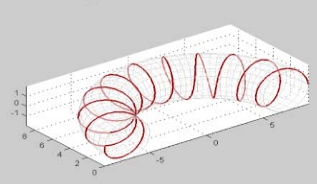
From Eq. (27) it can be assumed that the wider the diameter of yarn, the higher is the angle of twist and yarn strength. Hence adjusting the angle of twist is a need to get a good yarn having a high strength.

In this study it has been found that the angle twist is influenced by the gap of yarn-rotor (***b***) and yarn diameter $d_{yarn} = 0.04\sqrt{Tex}$ in torus coordinate, as shown below:

$$\alpha \approx tan^{-1}$$

$$\left\{\left(\left(\frac{\frac{v_d}{n_r} + 0.02\sqrt{tex}}{0.02\sqrt{tex}}\right)\sqrt{\left(0.02\sqrt{tex}\right)^2 \delta \left(\frac{v_d}{n_r} + 0.02\sqrt{tex}\right)^2 - 1}\right)\right\}$$

Table 2 — Simulation of fibre movement inside yarn.

Influence of angular velocity on fibre inside a yarn $n_{yarn} = \dot{v} = 10$ and delivery yarn $Vd = \dot{u} = 2$. $T_{simulation} = 3$ whereas $T_{count} = \frac{n_{yarn}}{v_d} \approx \frac{n_{rotor}}{v_d} = 5$	
Influence of angular velocity on fibre inside a yarn $n_{yarn} = \dot{v} = 20$ and delivery yarn $Vd = \dot{u} = 2$. $T_{simulation} = 5$ whereas $T_{count} = \frac{n_{yarn}}{v_d} \approx \frac{n_{rotor}}{v_d} = 10$	
Influence of angular velocity on fibre inside a yarn $n_{yarn} = \dot{v} = 40$ and delivery yarn $Vd = \dot{u} = 2$. $T_{simulation} = 11$ whereas $T_{count} = \frac{n_{yarn}}{v_d} \approx \frac{n_{rotor}}{v_d} = 20$	

References

1 Backers S, Hearle J W S & Grosberg P, *Structural Mechanics of Fibres, Yarns and Fabrics* (Wiley-Interscience, USA), 1969.

2 Furter R & Meier S, *Measurement and Significance of Yarn Twist* (Uster Technology AG, Switzerland), 2009.

3 Lawrence CA, *Fundamentals of Spun Yarn Technology* (CRC Press, New York), 2003.

4 Musa K & Ayse O, *Fibres Text J*, 14 (5) (2006) 84.

5 Penava J Z & Oreskovic V, *J Text Inst* , 88 (1997) 21.

6 Prendž*ova* M, *Int Polym Materials, 47* (2000), 701.

7 Rohlena Vaclav *Open-End Spinning* (Elseiver Scientific Publishing Company, New York), 1975,

8 Spinning Fine Open-end Yarn, Text Topics, No.6, Volume XIV(6), 1986.

9 Trommer Gunter, *Rotor Spinning* (Deutscher Fachverlag, Frankfurt), 1995.