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Reduced-order optimal controller design for an underwater glider

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Present study aims to design optimal controllers based on full and reduced-order models of hybrid-driven, buoyancy-propelled underwater Glider. After establishing the glider's mathematical model and linearizing it about the steady glide path, it is reduced to a lower order by balanced realization method. Then, optimal controllers are designed using Linear Quadratic Regulator (LQR) scheme based on original and reduced-order models to control the glider motion in vertical plane for 35° downward glide path. A satisfactory controllability and tracking is observed from the approximated model controller which confirms the advantageous characteristic of such approximation.

[Keywords: optimal controller, underwater glider, steady glide path, Linear Quadratic Regulator, controllability, reduced-order model controller**]**

Introduction

Underwater Gliders has been considered as one of the most substantial platforms in oceanographic research projects. Low-power consumption, the ability to discover large areas and great operational flexibility made them an advantageous choice among other underwater vehicles. Their main tasks are ranged from gathering data from the oceans to monitoring water currents and temperature. These vehicles use buoyancy as their major means of propulsion and they change their net buoyancy to induce motion in vertical direction. Moreover, underwater gliders use their fixed wings to provide lift at non-zero angles of attack and therefore, induce a motion in the horizontal direction[1]. The vehicle moves through the water in a saw-tooth like pattern. On the other hand, Navigation is accomplished using a combination of GPS fixes on the surface and internal sensors that monitor the vehicle heading, depth and attitude during dives. External sensors are continuously used to take samples from the ocean and to collect environmental parameters.

Three of the most typical operational glider models are including the electric 'SLOCUM'glider[2], the 'Spray' glider[3]and the 'Seaglider'[4]. All of these models use an electromechanical actuator pump or piston to change their weight. Due to their maneuverability and successful career in ocean sampling, control of these vehicles is considered to be a crucial task. Thus, various control strategies have been taken into account to control the motion and attitude of underwater gliders. One of the foremost investigations was the feedback control of a laboratory-scaled underwater glider called ROGUE (Remotely Operated Gliding Underwater Experiment) by N. E. Leonard and J. G. Graver from the Princeton University in 2001[5]. Among feedback control schemes, PID is considered as the most popular method because of its simple architecture design and tuning parameters[6-8]. Another widely-used strategy for designing controllers in this specific system is Linear Quadratic Regulator (LQR)[5,9-10]. In addition, a decentralized supervisory control (DSC) system based on the RW (Ramadge & Wonham) supervisory control theory of discrete event dynamic system (DEDS) was implemented for an underwater glider[11]. Due to changeable conditions in the ocean environment and water currents, robustness against parameter variations and disturbances is imperative.

Thus, a robust controller for this vehicle in the vertical plane was designed based on the sliding mode control theory[12].

All of the proposed controllers in these studies are based on the original model of the vehicle while some strategies like order reduction may play a key role in reducing the implementation costs in controller design procedure as well as the computational time to process the variables.

This paper concerns the nonlinear model of laboratory-scaled ROGUE underwater glider which has already been modelled[5].After linearization and considering the energies of states represented in Hankel Singular Values graph, the original model order of this vehicle is reduced using balanced realization method. Subsequently, controllers based on original and reduced-order models are designed using LQR control scheme. Finally, the simulation results are assessed and comparisons between responses will be made.

Material and Methods

To start with, the glider's mathematical model and equations of motion should be derived. All of the equations throughout this paper are based on [5]. In this modeling procedure, glider considered as a rigid body with fixed wings and tail. The hull is assumed to be in ellipsoidal shape with a body frame coordinate (e_1, e_2, e_3) attached to the vehicle body while its origin is placed at the center of the ellipsoid or so called center of buoyancy (CB). The orientation of the glider is given by the rotation matrix R. This matrix maps vectors expressed with respect to the body frame into inertial frame coordinates (i, j, k). Thus, the position of the glider $b = (x, y, z)^T$ is the vector from the origin of the inertial frame to the origin of body frame. The vehicle moves through the fluid with translational velocity $v = (v1, v2, v3)$ and angular velocity $\Omega = (\Omega_1,\ \Omega_2,\ \Omega_3)$ with respect to the body frame. A schematic of frames, Glider position and orientation variables are given by figure 1.

The total stationary mass or body mass, m_s is the sum of m_h or fixed mass which is uniformly distributed throughout the ellipsoid, m_w or a fixed point mass that may be offset from the CB and m_b or the variable ballast point mass which is fixed in location at the CB. Vectors from the CB to the point mass and center of the stationary mass are denoted by r_w and r_s, respectively. Another vector called $r_p(t)$ describes the position of movable mass or $\bar{m}$ with respect to the CB at time t. Therefore, the total mass of the vehicle body is written as:

$$m_v = m_h + m_w + m_b + \bar{m} = m_s + \bar{m} \quad \dots (1)$$

Adding to this, the glider equations of motion for the gliding path restricted to the vertical plane are obtained as:

$$\dot{x} = v_1 cos\theta + v_3 sin\theta \quad \dots (2)\ \dot{z} = -v_1 sin\theta + v_3 cos\theta \quad \dots (3)\ \dot{\theta} = \Omega_2 \quad \dots (4)\ \dot{\Omega}_2 = \frac{1}{J_2}\left((m_3 - m_1)v_1 v_3 - \bar{m}g\left(r_{p1} cos\theta + r_{p3} sin\theta\right) + mDL - rp3u1 + rp1u3 \quad \dots (5)$$

$$\dot{v_1} = \frac{1}{m_2}\left(-m_3 v_3 \Omega_2 - P_{p3}\Omega_2 - m_0 g sin\theta + Lsin\alpha - Dcos\alpha - u1 \right. \quad \dots (6)$$

$$\dot{v_3} = \frac{1}{m_3}\left(m_1 v_1 \Omega_2 + P_{p1}\Omega_2 + m_0 g cos\theta - Lcos\alpha - Dsin\alpha - u3 \dots \right.$$

$$(7)\dot{r}_{p1} = \frac{1}{\bar{m}} P_{p1} - v_1 - r_{p3}\Omega_2 \dots (8)$$

$$\dot{r}_{p3} = \frac{1}{\bar{m}} P_{p3} - v_3 + r_{p1}\Omega_2 \quad \dots (9)$$

$$\dot{P}_{p1} = u_1 \quad \dots (10)\ \dot{P}_{p3} = u_3 \quad \dots (11)\ \dot{m}_b = u_4 \quad \dots (12)$$

In this set of equations, θ is the pitching angle, α is the angle of attack, Ω is the angular velocity, r_p is the position of movable mass, p_p is linear momentum, M_{DL} is the viscous moment, D is drag and L is the lift force. The last three parameters are modelled based on the airfoil theory and potential flow calculations as given by:

$$D = (K_{D0} + K_D \alpha^2)(v_1^2 + v_3^2)) \dots (13)$$

$$L = (K_{L0} + K_L \alpha)(v_1^2 + v_3^2) \dots (14)$$

$$M_{DL} = (K_{M0} + K_M \alpha)(v_1^2 + v_3^2) \dots (15)$$

where the K's are constant coefficients. Also the difference between angle of attack and pitch angle, namely the glide path angle reads:

$$\xi = \theta - \alpha \quad \dots (16)$$

It is also worth noting that the desired glide path angle and the desired speed are denoted by ξ_d and V_d, respectively. Thus, according to the glider position in water, the new inertial coordinates (x', z') is defined, where x' specifies the desired path and z' is the vehicle's perpendicular distance to desired path. The dynamics of the z' states are:

$$\dot{z}' = sin\xi_d(v_1 cos\theta + v_3 sin\theta)$$
$$+cos\xi_d(-v_1 sin\theta + v_3 cos\theta) \quad \dots (17)$$

Following the objectives in [5], the gliding along the prescribed line which include z' (excluding x') in our analysis should be controlled. In other words, the objective is to make z' decay to zero.

Linearization

To perform the linearization, let the variable $x = (z', \theta, \Omega_2, v_1, v_3, r_{p1}, r_{p3}, P_{p1}, P_{p3}, m_b)^T$ be the state vector and $u = (u_1, u_3, u_4)^T$ be theinput vector while u_1 and u_3 are linear momentum rate of $\bar{m}$ and u_4 is the controlled variable mass rate. By defining

$$\delta x = x - x_d \quad \dots (18)$$
$$\delta u = u - u_d \quad \dots (19)$$

The linearization for the planar glider about the steady glide path gives:

$$\delta\dot{x} = A\delta x + B\delta u \quad \dots (20)$$

where for $35°$ downward glide paths, A and B matrices are presented in (21) and (22)

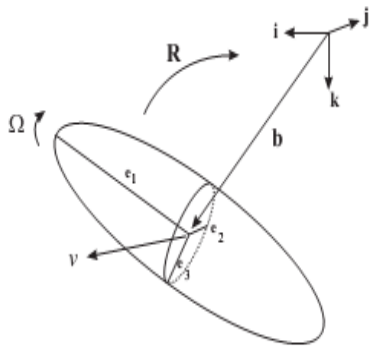

Fig. 1__Glider position and orientation variables

$$A = \begin{bmatrix} 0 & -0.3 & 0 & -0.1 & 0.99 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -7.5 & 0 & 148.67 & -5.71 & -179.65 & 78.86 & 0 & 0 & 0 \\ 0 & -0.32 & -0.06 & 68.66 & -16.33 & 0 & 0 & 0 & 0 & 39.43 \\ 0 & -0.07 & 0.174 & -12.71 & -0.53 & 0 & 0 & 0 & 0 & 0.44 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \dots (21)$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -0.4 & 0.41 & 0 \\ -0.086 & 0 & 0 \\ 0 & -0.04 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \dots (22)$$

Controller Design

In this section, design procedure of the LQR controller is described. This control scheme generates a stabilizing control law that minimizes the cost function, J. This quadratic function is a weighted sum of the squares of the states and input variables as follows

$$J = \int_0^\infty \delta x^T Q \delta x + \delta u^T R \delta u \, dt \quad \dots (23)$$

where Q and R variables are state and control weighting matrices which are positive semi-definite and positive definite, respectively. There is a trade-off between these weights and Depends on our requirements, the regulation of these matrices should be performed.

The weighting matrices that are used for controller based on the original model are set to $Q = diag(50, 40, 100, 100, 120, 40, 50, 100, 150, 20)$ and $R = diag(1, 1, 1)$. After the implementation of controller with these tuning parameters, the gain matrix is obtained as follows:

$$K = \begin{bmatrix} -5.16 & 8.73 & -11.37 & -130.21 & 12.31 \\ 4.82 & -7.34 & 1.25 & 14.71 & -1.75 \\ -0.18 & 1.50 & 15.34 & 428.37 & -76.50 \end{bmatrix}$$
$$\begin{bmatrix} 115.21 & -45.41 & 10.02 & -0.85 & -39.47 \\ -14.34 & 11.78 & -0.07 & 12.61 & 4.41 \\ -147.81 & 63.89 & 3.66 & 0.53 & 179.32 \end{bmatrix} \dots (24)$$

Order Reduction

In a great number of design procedures, it is possible to describe system dynamics by several linear differential equations. But in most of the real processes due to their large-scale systems, analysis of their original form has become a rather expensive and time-consuming task[13]. Thus, approximating these systems with simpler models containing smaller number of values which has substantial role over the eliminated ones is considered highly-required. In other words, by eliminating those state variables with the lowest controllability and

observability properties, a reduced-order model with less complexity would be achieved.

The major issue in order reduction methods is choosing a proper order out of the original model, i.e. the states which may be omitted[14]. One efficient method in literature for determining this order is balanced realization method. By defining the n^{th} order stable linear system with state space realization given by

$$\dot{x} = Ax + Bu \quad \ldots (25)$$
$$y = Cx + Du \quad \ldots (26)$$

The controllability and observability gramian matrices for the system are defined as

$$P = \int_0^\infty e^{At} B B^T e^{A^T t} dt \quad \ldots (27)$$
$$Q = \int_0^\infty e^{A^T t} C^T C e^{At} dt \quad \ldots (28)$$

which meet the following Lyapanov equations

$$AP + PA^T + BB^T = 0 \quad \ldots (29)$$

$$A^T Q + QA + C^T C = 0 \quad \ldots (30)$$

In essence, a realization is balanced if its controllability and observability gramians are diagonal and equal which means

$$P = Q = \Sigma = diag\{\sigma_1 \sigma_2 \cdots \quad \sigma_n\} \ldots (31)$$

where for i=1,2,…,n-1, $\sigma_i \geq \sigma_{i+1}$ and $\Sigma = \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix}$.Thus, in Σ if we have $\sigma_k \geq \sigma_{k+1}$, then k is an appropriate value for reducing the order of the model. Then by taking Hankel singular values into consideration, the energy of system modes are obtained as indicated in figure 2.

As can be seen, the first 8 modes have higher energies compared to the last two modes. Though the 9th mode have lower energy than previous modes, but excluding it could have negative effects on our analysis. Thus, the first 8 modes are considered to have impact on the system and we reduce the model order from 10 to 8. By eliminating P_{p3} and m_b from our analysis. It should also be stated that weighting matrices for the controller based on this reduced-order model are selected similar to the weights mentioned in the previous section.

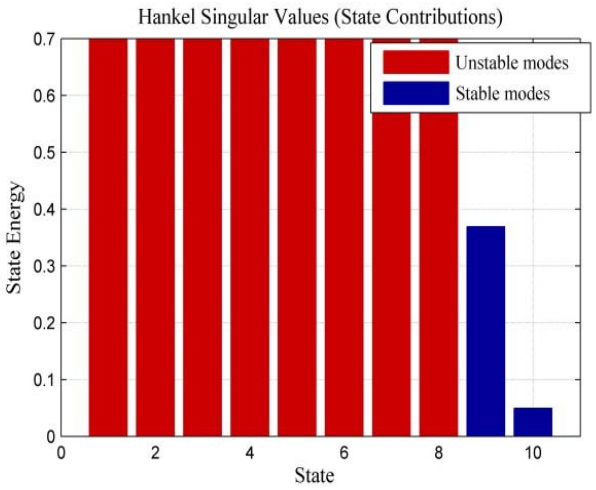

Figure. 2__Hankel Singular Values in terms of state energies

Results and Discussion

This section represents the MATLAB simulated results of the ROGUE underwater glider motion for 35° downward gliding path and demonstrates closed-loop responses of system key variables based on the full and reduced-order models. These responses are obtained by tuning the LQR controller with the mentioned weight matrices. The results of control input and position variable responses as a function of time position variables are depicted in figures 3 and 4, respectively. From the figure 3, clearly the control input results based on both original and reduced-order models have satisfactory behavior and all the responses converge to the desired value of zero with a very low steady-state error. It is also observed that the results based on reduced-order model have more deviations than those of the full-order model.

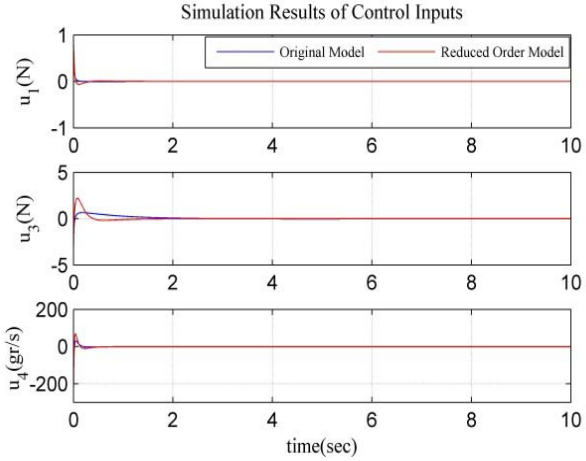

Figure. 3__Simulation results of control inputs

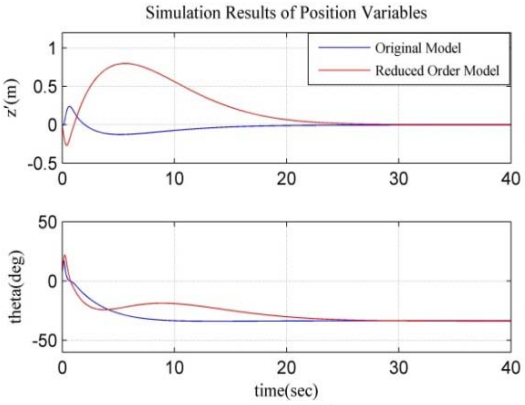

Figure. 4__Simulation results of position variables

In case of position variable results demonstrated in figure 4, responses pertaining to the first state, namely $z'$for both models have met the desired value. Hence, the control objective to control gliding along the prescribed line is accomplished. Likewise, from the graph related to pitching angle responses, good controllability is observed and both controllers obtain the same value of $35°$ downward glide path. It is clear that the graphs based on reduced-order model have significantly larger overshoot peak values and also higher settling time compared to Full-order model responses. But in controlling the pith angle, higher accordance among responses of the two controllers may be observed.

The simulation results of angular and translational velocities as well as the movable mass positions are all presented in figures 5 and 6, respectively. Based on the figure 5, the desired values for linear velocity components (0.3 m/s for v_1 and 0 forv_3) are tracked by both controllers with very low error. Moreover, the value of zero is obtained by the proposed controllers for the angular velocityΩ_2.

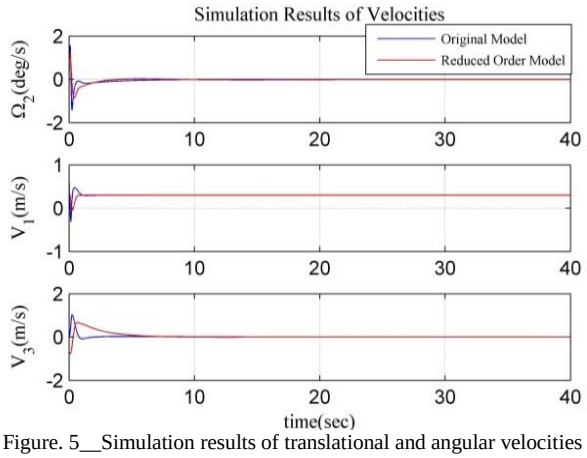

Figure. 5__Simulation results of translational and angular velocities

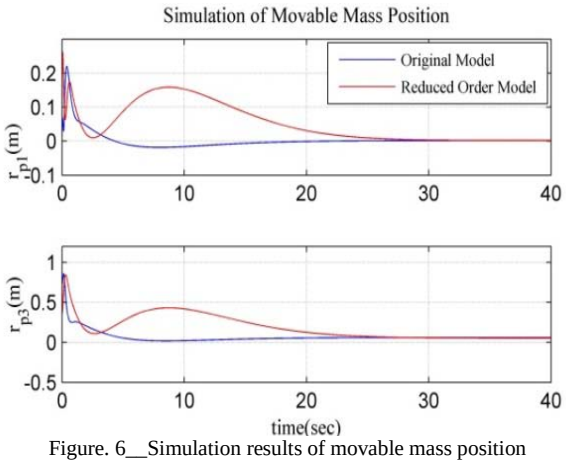

Figure. 6__Simulation results of movable mass position

Also it is interesting to note that the controller based on reduced-order model have slower responses while these have less deviations in comparison with the results based on full-order model. For instance in v_3 responses, the settling time for original controller is 2.6 sec. while it is 5.3 sec. for the controller based on reduced-order model.

The same attitude may be detected from the movable mass position results in figure 6. In both plots in regards with r_{p1} andr_{p3}, blue graphs related to the controller based on original model have faster responses with less overshoot and settling time values. Also both controllers have satisfactory performance from the tracking point of view.

Conclusion

This paper is concerned with the design procedure of LQR controllers based on original and reduced order models of the laboratory-scaled, buoyancy-driven underwater glider ROGUE. After obtaining the mathematical model of the system and linearizing it about steady glide path, state energy contributions with Hankel singular values graph is considered and the original model order is reduced from 10 to 8 by balanced realization method. Afterwards, LQR controllers based on the original and reduced-order models with appropriate values for the weighting matrices are designed and subsequently, the MATLAB simulated results are achieved. By obtaining the responses in accordance with the control inputs, position variables, velocities and movable mass positions, it is perceived that controllers based on the full-order model show faster responses with lower settling

time compared to the results of the controller based on reduced-order model. Additionally, the overshoot peak values for the reduced-order models are higher than those of the original model. It should also be noted that all the approximated model responses show very low steady state error in tracking the desired values compared to those of the original model.

However, it can be observed that the controller based on reduced-order model in this model of the underwater glider captures acceptable behavior. Thus, by taking the advantages of approximated model such as reducing the computational complexity and controller implementation costs into consideration, model order reduction strategies and in this case balanced realization method could play a supportive role in controlling large scale systems.

Regarding the future path of this work, other order reduction methods such as singular perturbations method could be used. Moreover, due to the uncertainties and ocean current disturbances, another controller design strategy namely the Robust Model Predictive Control (RMPC) may possess a superior characteristic to take the simulation process into the more realistic events.

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