**Indian Journal of Radio & Space Physics**
**Vol. 1, June 1972, pp. 136-137**

# Variation of Apparent Ionospheric Height with Frequency & Observed Multiple Reflections*

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*Manuscript received* 16 *March* 1972

**A theoretical analysis of the effect of the apparent height gradient *dh'/df* in the ionosphere on the shape of the reflected pulse has been made. It has been shown that when the gradient *dh'/df* is high, the pulse would normally be split into two. It has been further shown that the observed four split echoes may be the result of such a gradient in the electron density in the ionosphere, when the working frequency is near the critical frequency of the region.**

## Introduction

PROPAGATION of a radio wave in a dispersive medium like the ionosphere has been discussed by many workers. Budden[1], Ginzburg[2] and Knop[3] have investigated the shape of the signal envelope in an unbounded plasma and have given the solutions in the form of Bessel's functions. Recently, Terina[4] has investigated the transient response of the electromagnetic fields when a Gaussian envelope carrier wave is incident normally on an anisotropic inhomogeneous ionized medium. Kozaki and Mushiake[5] have theoretically investigated as to how the shape of a pulse modulated radio wave is deformed by reflection from the ionosphere. They have examined the variation in phase and amplitude by computing the parameters of the reflected pulse obtained after oblique incidence of a rectangular electromagnetic pulse at the ionosphere in which the electron density varies linearly in one direction. They have also investigted[6] the modification of the shape and phase of a sinusoidal carrier, with a Gaussian pulse envelope, which was incident at an arbitrary angle on the plane interface between the plasma and free space. Numerical calculations for the representative cases have also been given by Kozaki and Mushiake[6]. In some experimental observations one of the present authors observed that whenever multiple reflected echoes are observed at low latitude stations, the *dh'/df* at the time of observation is found to be very high. Many more authors have reported similar type of observations from low and equatorial regions. The multiple reflections have been explained as due to the presence of irregularities in the higher regions of the ionsphere. In their extensive study of the nature of the multiple echoes the present authors have found that the phenomenon was observed near the critical frequency only. Above the critical frequency the multiple reflections were absent. It seems desirable to examine the modification of shape of the reflected echoes by the ionosphere at times when *dh'/df* is high rather than associating them with the irregularities only.

In the present study the effect of the high *dh'/df* on the observed multiple reflections near critical frequency has been explained. The authors are of the view that the splitting along with considerable spread of the reflected echoes may be due to irregularities but splitting without much spread is due to high value of *dh'/df* only. Here we shall examine the cases of vertical incidence only.

## Theory and Analysis

Let us assume that the electric field in the transmitted pulse of predominant frequency is given by

$$E^{i}(t) = m(t) \cos 2\pi f_1 t \qquad \ldots(1)$$

and lasts for a duration $T$. The factor $m(t)$ is appreciable only in range $0<t<T$ and it gives the shape of the modulation and describes the envelope of the incident wave. A well-known theorem in Fourier analysis shows that

$$E^{i}(t) = \mathrm{Re}\int_{-\infty}^{\infty} M(f-f_1) \exp 2\pi i f t \, df \qquad \ldots(2)$$

where $M(f)$ is Fourier transform of $m(t)$. The reflected pulse at the ground is given by

$$E^{r}(t) = \mathrm{Re}\int_{-\infty}^{\infty} M(f-f_1) \exp \left[2\pi i f\left\{t-\frac{2}{c}h(f)\right\}\right] df \qquad \ldots(3)$$

where $h(f)$ is the phase height of reflection, and $M(f-f_1)$ is appreciable when $(f-f_1)$ is small. From (3) we get by expanding $fh(f)$ by Taylor's series about $f=f_1$,

$$\begin{aligned} E^{r}(t) = \mathrm{Re} \exp &\left[2\pi i f_1\left\{\tau-\frac{2}{c}\Big(h(f_1)-h'(f_1)\Big)\right\}\right] \\ &\times \int_{-\infty}^{\infty} M(f-f_1) \exp \left[2\pi i\left\{\tau(f-f_1) \right.\right. \\ &\left.\left. -\frac{1}{c}(f-f_1)^2 h_1' - \frac{1}{3c}(f-f_1)^3 h_2' - \ldots\right\}\right] . df \qquad \ldots(4) \end{aligned}$$

here $\tau = t-\frac{2}{c}h'(f_1)$ and $h_1'$, $h_2'$ are first and second derivatives of the apparent height $h'(f)$.

The integral in Eq. (4) represents the shape of the reflected echo. When the frequency $f_1$ is chosen so that $h'(f)$ curve has a positive slope and

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*Paper presented to the Physics Section of the 59th Session of the Indian Science Congress, held at Calcutta in February 1972.

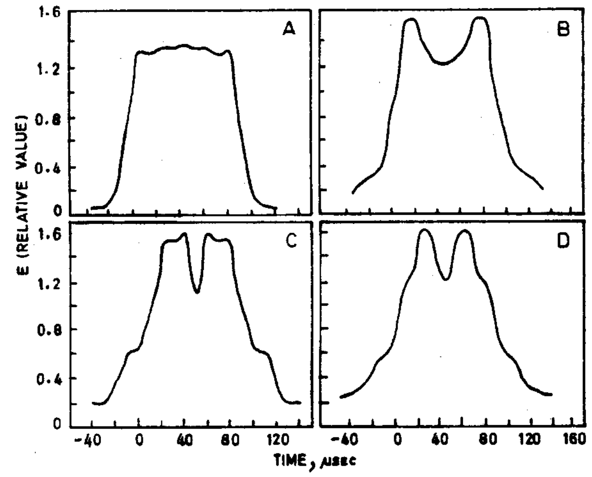

Fig. 1 — Shape of reflected pulse wave for different *dh'/df* when pulse duration $T=100$ μsec [Value of *dh'/df* in km/(Mc/s): A, 10; B, 100; C, 200; and D, 250]

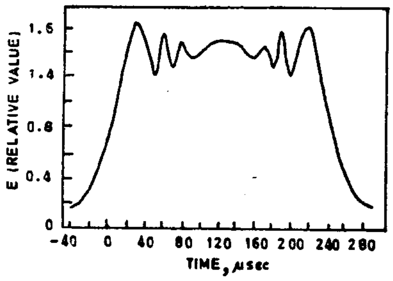

Fig. 2 — Shape of reflected pulse wave for pulse duration $T=250$ μsec and $dh'/df=200$ km/(Mc/s)

only small curvature then $h_1'$ has a positive value and $h_2'$ is small. In this case the pulse shape is given by

$$\int_{-\infty}^{\infty} m(\tau-t) \exp\left[\frac{\pi i c t^2}{2h_1'}\right] dt \qquad \ldots(5)$$

Let us now suppose that the original pulse is rectangular as used in ionosonde, e.g. $m(t)=1$ for $0<t<T$ and $m(t)=0$ for other values of $t$, then expression (5) becomes (apart from constant factor)

$$\int_{\tau-T}^{T} \exp\left[\frac{\pi i c \nu^2}{2h_1'}\right] d\nu \qquad \ldots(6)$$

This integral has been evaluated using Fresnel integrals

$$C)v) = \int_0^v \cos\left(\frac{\pi v^2}{2}\right) dv \text{ and } S(v) = \int_0^v \sin\left(\frac{\pi v^2}{2}\right) dv \qquad \ldots(7)$$

whose values are taken from standard tables $\left(\text{here } v^2 = \frac{c}{h_1'}\nu^2\right)$. For some higher values of $v$ Fresnel's integrals have been evaluated by using their series expansion given by

$$C(v) = C(\infty) - \frac{1}{\pi v}\left[P(v)\cos\left(\frac{\pi v^2}{2}\right) - Q(v)\sin\left(\frac{\pi v^2}{2}\right)\right]$$

$$S(v) = S(\infty) - \frac{1}{\pi v}\left[P(v)\sin\left(\frac{\pi v^2}{2}\right) + Q(v)\cos\left(\frac{\pi v^2}{2}\right)\right]$$

where

$$P(v) = \frac{1}{\pi v^2} - \frac{1.3.5}{(\pi v^2)^3} + \frac{1.3.5.7.9}{(\pi v^2)^5} - \ldots$$

$$Q(v) = 1 - \frac{1.3}{(\pi v^2)^2} + \frac{1.3.5.7}{(\pi v^2)^4} - \ldots$$

Numerical calculations were made to investigate the shape of the reflected echoes for various values of *dh'/df* and some of these results are shown in Figs. 1 and 2.

## Conclusion

On examination, the curves in Figs. 1 and 2 show that:

(i) The original pulse is split into two main peaks and there is a dip at $T/2$.

(ii) There is some spread of pulse duration $T$ but it is very small and is expected to be lost in the noise of the receiving system.

(iii) Splitting is prominent only when *dh'/df* is high.

(iv) When the working frequency is near the critical frequency of the layer, the effect would be prominent in both the ordinary and extraordinary components with the result that four echoes may appear on the oscilloscope instead of the two splits, viz. the ordinary and the extraordinary echoes. The sense of polarization of one pair would correspond to O-component and the other pair would correspond to X-component. A detailed analysis of the observed four split echoes in the light of the above theory would be published elsewhere.

## Acknowledgement

One of the authors (S.K.V.) is thankful to the University Grants Commission, New Delhi, for the financial assistance for conducting the work. They are also thankful to Prof. J. Varma for his keen interest in the work.

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