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Magnetic Field on the Nightside of the Magnetosphere

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The Explorer 18 data have been analyzed to evaluate the coefficients of the spherical harmonic expansion for the magnetic field on the nightside of the magnetosphere between 18 and 31 earth radii. Numerical computation of the first 24 harmonic coefficients has been made using an IBM 360 computer. From an analysis of these computed values it is observed that at large distances exceeding 18 earth radii, the secondary magnetic field is much stronger than the dipole field.

1. Introduction

SIGNIFICANT progress has been made[-3] during the last decade in unravelling the structure of the magnetosphere, the waves, fields and particles inside it through measurements made with Explorer 18[1] and 28, and several other satellites. The results telemetered by some of the earlier satellites led to the conclusion that the field lines both on the dayside as well as on the nightside are compressed, as a result of the interaction of the solar wind with the geomagnetic field. Mead[4] had earlier evaluated the secondary magnetic field generated by the currents flowing on the surface of the geomagnetic cavity in the form of a spherical harmonic expansion and has published tables for the coefficients of the various harmonics for values of (n,m) up to 6. The expression given by Mead can be regarded as representing the geomagnetic field well for distances up to seven or eight earth radii, but beyond this distance its validity is uncertain as it does not take into account the neutral sheet on the nightside.

Through the kind courtesy of the World Data Center, NASA, we received the data regarding the magnetic field in the cavity and interplanetary space as measured by the satellite Explorer 18 or IMP 1. This satellite was a spin stabilized spacecraft instrumented for interplanetary studies of cosmic rays, magnetic fields and plasmas and was launched on 27 November 1963 in a highly elliptical orbit, with an apogee of 32 earth radii. The magnetic field data used by us covered the period from 28 November 1953 to 30 May 1964 and during the course of this period the satellite made a total number of 47 orbits.

We analyzed the data giving the magnetic field for the nightside of the magnetosphere between 18 and 31 earth radii to evolve a model of the magnetic field on the nightside. Assuming that the components of the magnetic field can be expanded in a spherical harmonic expression, we had evaluated the numerical values of the coefficients of the spherical harmonic series up to the terms (4,4). Even this involves inversion of a matrix of the order of 24. The results of these investigations are reported in the present paper.

2. Analysis of the Data

Let the potential of the magnetic field be written as

$$V = \frac{\mathbf{M.r}}{r^3} + a\sum\left(\frac{r}{a}\right)^n (g_n^m \cos m\phi + h_n^m \sin m\phi) P_n^m(\cos\theta) \quad ...(1)$$

where

$$P_n^m(x) = \left[\frac{2\,(n-m)!}{(n+m)\,!}\right]^{\frac{1}{2}} \frac{(1-x^2)^{m/2}}{2^n\,n\,!} \frac{d^{n+m}}{dx^{n+m}} (x^2-1)^n \quad ...(2)$$

The first term gives the potential of the dipole at a point r in the magnetosphere. This term gives the internal contribution to the magnetic field. The remaining terms describe the cavity field and arise from the solar wind interaction with the geomagnetic field. They constitute the contribution to the magnetic field arising from external sources.

A knowledge of the coefficients g_n^m and h_n^m will enable one to fix the potential and consequently the components of the magnetic field. In polar coordinates, the components of the magnetic field are given by

$$B_r = -\frac{\partial V}{\partial r};\ B_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta};\ B_\phi = -\frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi} \quad ...(3a)$$

At this stage, we shall make a few remarks about the assumption that the magnetic field is of potential origin. Generally, the magnetic field at any point is given by the Maxwell equation

$$\text{curl}\,\mathbf{B} = \frac{4\pi}{c}\,\mathbf{j} + \frac{1}{c}\frac{\partial\,\mathbf{D}}{\partial t} \quad ...(3b)$$

with the usual notation. If the right hand side in Eq. (3b) vanishes, curl **B** = O, so that **B** can be expressed as the gradient of a potential, viz. **B** = —grad V. The assumption that the magnetic field is derivable from a potential is thus true only when the currents and the time variation of the electric fields in the magnetosphere are negligibly small.

Our present knowledge of the electric fields and current systems in the magnetosphere is, however, very meagre. Detailed reviews of the electric fields in the magnetosphere have been given by Obayashi and Nishida[5] and Falthammer[6], and an extensive bibliography on this subject can be had from these reviews. One may conclude[6] that the average electric field in the vicinity of the ecliptic plane is normal to it and had a strength of the order of a millivolt per metre in a system fixed in space, but interplanetary space outside the ecliptic plane has not yet been explored. Very little data are available on the time variation of the electric fields in the magnetosphere. As pointed out by Falthammer[6] the actual structure of the current system in outer space is still not clear and should be decided by future satellites and space probes. In view of the uncertainties regarding knowledge about the electric fields and currents in the magnetosphere, it is not possible to make any reasonable evaluation of the contribution of these to the magnetic fields in the magnetosphere, especially in the region covered by Explorer 18. We, therefore, assume that contributions to the magnetic field from the currents and time variation of the electric field are negligibly small and the field itself can be derived from a potential.

The coordinate axes used in the IMP 1 were related to the spin axis of the satellite and the sun-earth line. A geocentric solar ecliptic coordinate system was defined, and this reflects the interplanetary nature of the data and also takes into account the motion of the earth around the sun. In this coordinate system, the origin is at the centre of the earth, the x-axis points to the sun, the z-axis is normal to the ecliptic plane and the y-axis is chosen to complete a right-handed coordinate system. The components (X,Y,Z) of the magnetic field are then related to B_r, B_θ, B_ϕ by a simple orthogonal transformation and are given by

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & -\sin\theta\sin\phi & \cos\theta \\ \sin\phi & \cos\phi & 0 \\ \cos\theta\cos\phi & -\cos\theta\sin\phi & -\sin\theta \end{pmatrix} \begin{pmatrix} B_r \\ B_\theta \\ B_\phi \end{pmatrix} \quad \ldots(4)$$

The magnetic axis of the earth does not exactly coincide with the z-axis and will make a small angle of the order of 20 to 30° with it. If (ψ,ξ) and (θ,ϕ) denote respectively the colatitude and longitude of the magnetic axis and the fixed point r respectively, we have

$$\frac{\mathbf{M.r}}{r^3} = \frac{\mathbf{M}}{r^3}\Big[(\sin\theta \sin\psi \cos(\phi-\xi) + \cos\theta\cos\psi \Big]$$

As the work by Ness[1] does not specify the precise value of ψ and ξ we have chosen a set of typical values $\psi=20°$ and $\xi=0°$ for our calculations, but as can be seen from Table 1 the contribution of the dipole term or the internal sources to the geomagnetic field beyond 18 earth radii is very small and is at least 10^{-2} times smaller than the field due to the interaction of the solar plasma with the terrestrial **field.** The coefficients are, therefore, not sensitive to changes in ψ and ξ. The IMP 1 data supplied by the World Data Center give the values of the following parameters for a period of about 70 days: year, day, hour, orbit, radial distance, the cordinates $(X,Y.Z)$ of the field point, the field magnitude, the field latitude angle, field longitude angle, the field components (X, Y, Z) and the standard deviation. We had chosen for our calculations a set of 259 points from these, giving the values (X_i, Y_i, Z_i) of the magnetic field. It was ensured that all these field points lie on the night-side of the magnetosphere and cover all latitudes ranging from 0 to 85° and all longitudes ranging from 90 to 270°.

The components of magnetic field X, Y, Z as given by Eq. (3a) are functions of the constants g_n^m and h_n^m. Let X_i, Y_i, Z_i denote the observed values of the components of the magnetic field as measured by the satellite during several of its orbits. We then determine g_n^m h_n^m by the condition that

$$S = \sum_{i=1}^{N} (X-X_i)^2 + (Y-Y_i)^2 + (Z-Z_i)^2 \quad \ldots(5)$$

is a minimum. In the above, the sum over i runs over all the N (=259) values of the magnetic field. The condition for a minimum leads to the following two sets of equations:

$$\frac{\partial S}{\partial g_n^m} = 0 \; ; \quad \frac{\partial S}{\partial h_n^m} = 0 \quad \ldots(6)$$

and these could alternatively be written as,

$$\sum_i \sum_n \sum_m \Big[A^g{}_{n_1 m_1 nm}\, g_n^m + B^g{}_{n_1 m_1 nm}\, h_n^m \Big] = \sum_i \Lambda^g{}_{n_1 m^1} \quad (7a)$$

$$\sum_i \sum_n \sum_m \left[A^h_{n_1 m_1 n m}\, g^m_n + B^h_{n1m1nm} h^m_n \right] = \sum_i \Lambda^h_{n1m1} \qquad \ldots(7b)$$

where

$$A^g_{n1m1nm} = -\left[n \left(\frac{r_i}{a}\right)^{n-1} \cos m\,\phi_i\, P^m_n (\cos\theta_i)\, \lambda^g_1 + \left(\frac{r_i}{a}\right)^{n-1} \frac{d}{d\theta} (p^m_n \cos\theta_i) \cos m\phi\, \lambda^g_2 + \frac{m \sin m\phi_i}{\sin\theta_i} \left(\frac{r_i}{a}\right)^{n-1} P^m_n (\cos\theta_i)\, \lambda^g_3 \right] \qquad \ldots(8a)$$

$$A^h_{n1m1nm} = -\left[-n \left(\frac{r_i}{a}\right)^{n-1} \cos m\phi_i\, P^m_n (\cos\theta_i)\, \Lambda^h_1 + \lambda^h_2 \cos m\phi_i \left(\frac{r_i}{a}\right)^{n-1} \frac{d}{d\theta_i} P^m_n (\cos\theta_i) + \frac{m \sin m\phi_i}{\sin\theta_i} \left(\frac{r_i}{a}\right)^{n-1} p^m_n (\cos\theta_i)\, \lambda^h_3 \right] \qquad (\ldots 8b)$$

$$B^g_{n1m1nm} = -\left[+n \left(\frac{r_i}{a}\right)^{n-1} P^m_n (\cos\theta_i) \sin m\,\phi_i\, \lambda^g_1 + \lambda^g_2 \sin m\,\phi_i \times \left(\frac{r_i}{a}\right)^{n-1} \frac{d}{d\theta} P^m_n (\cos\theta_i) - \frac{m \cos m\phi_i}{\sin\theta_i} \left(\frac{r_i}{a}\right)^{n-1} p^m{}_n (\cos\theta_i)\, \lambda^g_3 \right] \qquad \ldots(9a)$$

$$B^h_{n1m1nm} = -\left[n \left(\frac{r_i}{a}\right)^{n-1} \lambda^h{}_1 \sin m\,\phi_i P_{nm} (\cos\theta_i) + \lambda^h_2 \left(\frac{r_i}{a}\right)^{n-1} \times \sin m\,\phi_i \times \frac{d}{d\theta_i} [P^m_n (\cos\theta_i)] - \frac{m \cos m\phi_i}{\sin\theta_i} P^m_n (\cos\theta_i)\, \lambda^h{}_3 \right] \qquad \ldots(9b)$$

$$\Lambda^g{}_{n1m1} = \frac{2M\lambda^g{}_1}{r_i^3} [\cos\theta_i \cos\psi + \sin\theta_i \sin\psi \cos(\phi_i - \xi)] + \frac{M\lambda^g_2}{r_i^3} [\sin\theta_i \cos\psi - \cos\theta_i \sin\psi \cos(\phi_i - \xi)] + \lambda^g_1 (X_i \sin\theta_i \cos\phi_i + Y_i \sin\phi_i + Z_i \cos\theta_i \cos\phi_i) + \lambda^g_2 (X_i \sin\theta_i \sin\phi_i - Y_i \cos\phi_i + Z_i \cos\theta_i \sin\phi_i) - \lambda^g_3 (-X_i \cos\theta_i + Z_i \sin\theta_i) \qquad \ldots(10a)$$

$$\Lambda^h{}_{n1m1} = \frac{2M\lambda^h_1}{r_i^3} [\cos\theta_i \cos\psi + \sin\theta_i \sin\psi \cos(\phi_i - \xi)] + \frac{\lambda M^h_2}{r_i^3} [\sin\theta_i \cos\psi - \cos\theta_i \sin\psi \times \cos(\phi_i - \xi)] + \lambda^h_1 [X_i \sin\theta_i \cos\phi_i + Y_i \sin\phi_i + Z_i \cos\theta_i \cos\phi_i] + \lambda_2{}^h [X_i \sin\theta_i \sin\phi_i - Y_i \cos\phi_i + Z_i \cos\theta_i \sin\phi_i] - \lambda_3{}^h [-X_i \cos\theta_i + Z_i \sin\theta_i] \qquad \ldots(10b)$$

$$\left.\begin{aligned} \lambda_1{}^g &= -n_1 \left(\frac{r_i}{a}\right)^{n_1-1} \cos m_1\phi_i P^{m_1}_{n_1} (\cos\theta_i) \\ \lambda_2{}^g &= -\left(\frac{r_i}{a}\right)^{n_1-1} \cos m_1\phi_i \frac{d}{d\theta_i} P^{m_1}_{n_1} (\cos\theta_i) \\ \lambda_3{}^g &= -m_1 \frac{\sin m_1\phi_i}{\sin\theta_i} \left(\frac{r_i}{a}\right)^{n_1-1} P^{m_1}_{n_1} (\cos\theta_i) \end{aligned}\right\} \qquad \ldots(11a)$$

$$\left.\begin{aligned} \lambda_1{}^h &= -n_1 \left(\frac{r_i}{a}\right)^{n_1-1} \sin m_1\,\psi_i P^{m_1}_{n_1} (\cos\theta_i) \\ \lambda_2{}^h &= -\left(\frac{r_i}{a}\right)^{n_1-1} \sin m_1\,\phi_i \frac{d}{d\theta_i} P^{m_1}_{n_1} (\cos\theta_i) \\ \lambda_3{}^h &= \frac{m_1 \cos m_1\,\phi_i}{\sin\theta_i} \left(\frac{r_i}{a}\right)^{n_1-1} P^{m_1}_{n_1} (\cos\theta_i) \end{aligned}\right\} \qquad \ldots(11b)$$

These are a set of simultaneous equations in g^m_n and h^m_n. To make the numerical solution of the problem tractable, the series in Eq. (1) was truncated with the term $n=4$. Eqs. (7) then involve 24 variables. A programme in Fortran was written for the coefficients in the set of Eqs. (8) as well as for the inversion of the

Table 1— Numerical Values of g^m_n and h^m_n

n, m	g^m_n	h^m_n
1, 0	$0{\cdot}41538635 \times 10^{2}$	0
2, 0	$0{\cdot}2292031$	0
3, 0	$0{\cdot}57000667 \times 10^{-2}$	0
4, 0	$0{\cdot}98402053 \times 10^{-3}$	0
1, 1	0.47303516×10^{3}	$-0{\cdot}99872879 \times 10^{2}$
2, 1	$-0{\cdot}18228699 \times 10^{2}$	$0{\cdot}15337763 \times 10^{1}$
3, 1	$0{\cdot}49940675$	$-0{\cdot}40683351 \times 10^{-1}$
4, 1	$-0{\cdot}78130215 \times 10^{-2}$	$0{\cdot}32834779 \times 10^{-3}$
2, 2	$-0{\cdot}36901093$	$0{\cdot}59953588$
3, 2	$-0{\cdot}17934673 \times 10^{-1}$	$0{\cdot}26388625 \times 10^{-2}$
4, 2	$0{\cdot}78416429 \times 10^{-4}$	$0{\cdot}15959516 \times 10^{-3}$
3, 3	$-0{\cdot}17195869 \times 10^{-1}$	$0{\cdot}10727976 \times 10^{-1}$
4, 3	$0{\cdot}84059095 \times 10^{-4}$	$-0{\cdot}3579834 \times 10^{-3}$
4, 4	$0{\cdot}12963088 \times 10^{-3}$	$-0{\cdot}19347058 \times 10^{-3}$

matrix and the equation was solved using an IBM 360 computer. The output was designed to get the coefficients of the (24×24) matrix as well as the solution for the variables. The numerical values of g_n^m as well at h_n^m are given in Table 1.

It can be seen from Table 1 that the coefficients g_1^0, g_1^1, h_1^1, and h_2^1 have the largest numerical values, but as the expression for the magnetic potential has a factor $(r/a)^n$ which is of the order $(20)^n$, the higher order terms also contribute significantly to the magnetic field. Still it has been noted that the contributions from the terms corresponding to $n=4$ are definitely smaller than those of the other terms and this gives some hope about the fast convergence of the series for this region of the magnetosphere. As mentioned earlier, the contribution of the dipole term is much smaller than those of the others. Further, for any fixed n, the value of the coefficients becomes smaller with increasing values of m.

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