Indian Journal of Radio & Space Physics
Vol. 3, March 1974, pp. 5-8

A Comparative Study of Generalized & Classical Absorption Coefficients in D-Region at Udaipur

S. K. VIJAYVERGIA & R. K. RAI
Department of Physics, University of Udaipur, Udaipur

Received 1 *January* 1973; *accepted* 18 *May* 1973

A study of the ratio γ of the absorption coefficients (K) given by the Sen-Wyller (S-W) generalized theory and the Appleton-Hartree (A-H) classical theory has been made for the D-region for various working frequencies. An empirical relation for γ has been proposed as a function of height and working frequency, which would help one to compute the generalized absorption coefficients, simply multiplying the values of K calculated from the A-H theory by γ. It has been found that γ varies nearly from 1 to 2.5 in the height range of 70-85 km and remains 2·5 for heights above 85 km. The total non-deviative absorptions at various working frequencies have also been computed graphically.

1. Introduction

IN THE derivation of the Appleton-Hartree (A-H) magnetoionic dispersion equation for the study of radio wave propagation in the ionosphere, the collision of electrons with the heavy particles was introduced through a frictional constant term ν independent of the velocity of the electrons. Sen and Wyller generalized the A-H magnetoionic formula taking into consideration the collision frequency of the electron with heavy particles as a function of electron velocities, viz. $\nu=\nu_m f(v)$. It was concluded in their work that for the extreme limits $\nu \ll (\omega + \omega_H)$ and $\nu \gg (\omega + \omega_H)$ the A-H formula can be retained provided the effective collision frequency term is taken as $\frac{5}{2}\nu_m$ and $\frac{3}{2}\nu_m$ respectively. None of these extreme limits is valid in the D-region of the ionosphere. So, for precise work in this region the exact Sen-Wyller (S-W) generalized formula for the absorption coefficient should be used. In the present paper, a study of the multiplying factor (γ) which would enable one to obtain generalized absorption coefficient from the classical A-H absorption coefficient, has been made. A study of the variation of the ratio (γ) of the absorption coefficients calculated from the two theories as a function of true height and working frequency has been made and thereby an attempt has been made to obtain an empirical relation for this ratio or the multiplying factor as it may be called.

The height distribution of the electron density in the D-region of the ionosphere has been taken as Chapman distribution. This is shown in Fig. 1. The collision frequency profile used is also shown in Fig. 1. All the calculations have been made for vertically incident ordinary wave component, for overhead sun position ($\chi = 0$) and the geomagnetic conditions appropriate to Udaipur (gyrofrequency $f_H = 1{\cdot}17$ MHz and dip angle 33° 20′). Longitudinal propagation in the D-region has been assumed. Graphical method has been used for the computation of total absorption in the D-region.

2. Theoretical Considerations

The absorption coefficient K_{AH} is derived from the well-known dispersion expression of Appleton[1] for the ordinary wave:

$$\left(\mu - i\,\frac{K_{AH}c}{\omega}\right)^2 = 1 - \frac{X}{1 - iZ - \dfrac{Y_T^2}{2(1-X-iZ)} + \sqrt{\dfrac{Y_T^4}{4(1-X-iZ)^2} + Y_L^2}}$$

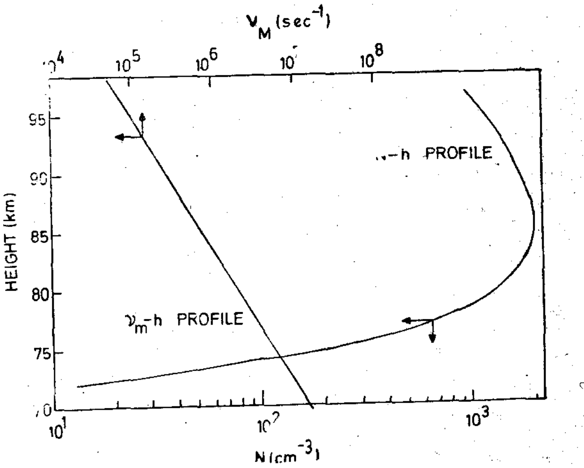

Fig. 1—Model Chapman D-Layer (N-h profile); and collision frequency profile (ν_m vs h)

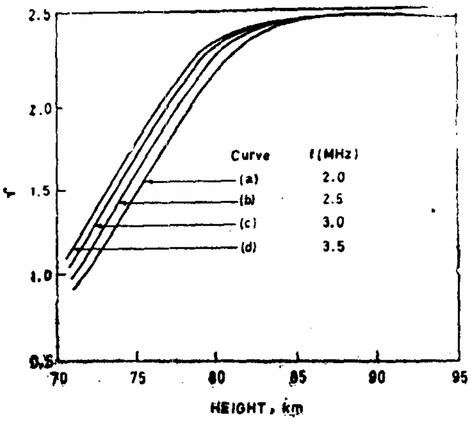

Fig. 2 — Variation of multiplying factor with height

where the symbols used are those as recommended by URSI. Assuming non-deviative absorption, coefficient K_{AH} for longitudinal propagation may be given by

$$K_{AH} = \frac{e^2}{2cm\epsilon_0} \cdot \frac{N\nu}{(\omega + \omega_H)^2 + \nu^2} \left(\frac{\text{Nepers}}{m}\right) \qquad \ldots(1)$$

which reduces to

$$K_{AH} = 4{\cdot}605 \times 10^4 . \frac{N\nu}{(\omega + \omega_H)^2 + \nu^2} \left(\frac{\text{dB}}{km}\right) \qquad \ldots(2)$$

where in Eqs. (1) and (2) N is the electron number density per cm^3.

Sen and Wyller[2] obtained a generalized expression for the complex refractive index. For the ordinary wave component and for longitudinal propagation it is given as follows :

$$\left(\mu - i\,\frac{K_{SW}c}{\omega}\right)^2 = \left\{1 - \frac{\omega_N^2(\omega + \omega_H)}{\omega\,\nu_m^2}\, C_{3/2}\left(\frac{\omega + \omega_H}{\nu_m}\right)\right\} - i\left\{\frac{5\omega_N^2}{2\omega\nu_m}\, C_{5/2}\left(\frac{\omega + \omega_H}{\nu_m}\right)\right\}$$

i.e. $$\frac{2\mu K_{SW}c}{\omega} = \frac{5\omega_N^2}{2\omega\nu_m}\, C_{5/2}\left(\frac{\omega + \omega_H}{\nu_m}\right)$$

where ω_N is the angular plasma frequency.

The non-deviative absorption coefficient in this case is given by

$$K_{SW} = 1{\cdot}152 \times 10^5 . \frac{N}{\nu_m} . C_{5/2}\left(\frac{\omega + \omega_H}{\nu_m}\right) \left(\frac{dB}{km}\right) \qquad \ldots(3)$$

where N is the electron number density per cm^3, ν_m is the collision frequency of mono-energetic electrons and $C_{5/2}\left(\frac{\omega + \omega_H}{\nu_m}\right)$ is the C-script in integral of the argument $\left(\frac{\omega + \omega_H}{\nu_m}\right)$. The values of C-script integral for different arguments are tabulated by Dingle *et al.*[3].

The multiplying factor γ may be defined as

$$\gamma = \frac{K_{SW}}{K_{AH}}$$

or, $$K_{SW} = \gamma\, K_{AH} \qquad \ldots(4)$$

The value of γ in Eq. (4) has been calculated using Eqs. (2) and (3) for different frequencies, viz. 2, 2·5, 3 and 3·5 MHz in the height range 70-95 km. The frictional term ν of the A-H equation has been taken equal to ν_m. The variation of γ with height for different frequencies is shown in Fig. 2. It may be seen that the value of γ, for all the frequencies, increases nearly from 1 to 2·5 and it becomes constant at about 85 km and above. The variation of γ thus nearly resembles the charging phenomenon of a condenser in which the value of the charge on the plates first increases and then becomes constant after a certain time. Hence γ should have the form somewhat similar to the relation for the charging of a condenser. The authors made an attempt to obtain an empirical relation for γ, whereby one would be able to calculate K_{SW} directly from K_{AW} values. The calculation of K_{SW} by Eq. (3) requires the value of C-script integral for various arguments and is a laborious one. The final form of the empirical relation for γ as obtained by the authors, which fitted the curve (Fig. 2), may be represented by

$$\frac{\gamma}{a} = 2{\cdot}5\left[0{\cdot}97 - \exp\left\{-0{\cdot}11\,(h - 65)\right\}\right] - 0{\cdot}05\,(76{\cdot}5 - h) \qquad \ldots(5)$$

where $a = 0{\cdot}11\,(f - 2) + 1$

h = true height in km

and f = working frequency in MHz.

The proposed empirical relation [Eq. (5)] has been tested for a number of working frequencies between 2 and 3·5 MHz and has been found satisfactory for the height range $70 < h < 85$ km in which most of the D-region absorption takes place. Fig. (2) further suggests that the maximum value of γ would be 2·5. The values of K_{SW} as calculated from the empirical relation for γ [Eq. (5)] and the value of K_{AH} [Eq. (2)], and that obtained by expression [Eq. (3)], do not differ much as may be seen from Fig. 3 (A, B, C and D).

3. Discussion

Values of K_{SW} are calculated using Eq. (2) for K_{AH} and empirical formula in Eq. (5) for γ for different frequencies. These are shown in Figs. 3(A)-(D). The values of K_{SW} as obtained from the actual generalized formula [Eq. (3)] are also shown for com-

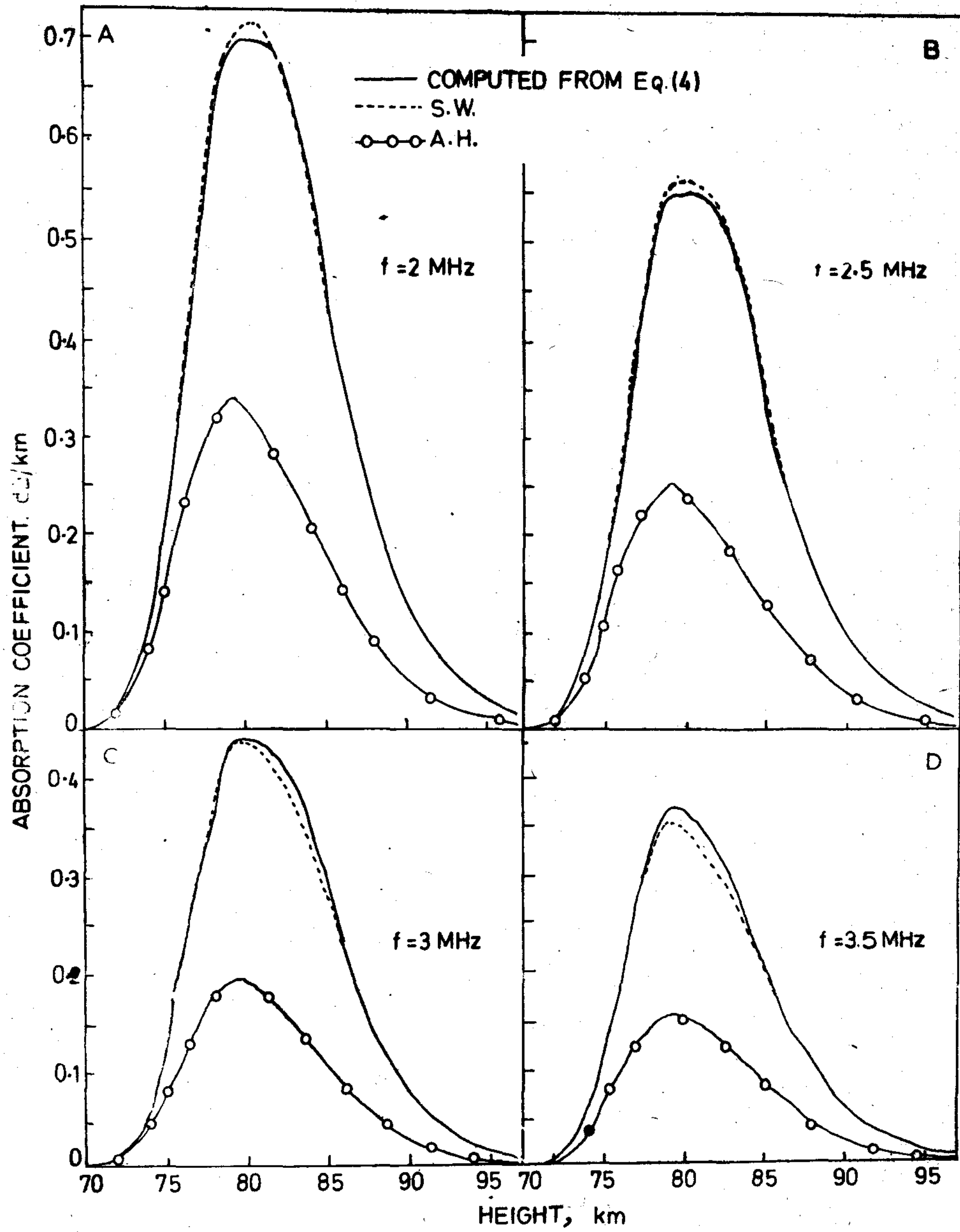

Fig. 3 — Variation of absorption coefficient with height for various frequencies

parison. The two values of K_{SW} are in good agreement.

The total absorptions for the D-region $[2\int_0^{95} K_{SW} dh]$ for the *N-h* and ν_m-*h* profiles, shown in Fig. 1 have been calculated graphically for the two sets of values of K_{SW}, one given by actual formula in Eq. (3) and other using the empirical relation in Eq. (5) and K_{AH} values [Eq. (2)]. The values of total absorption at different frequencies are shown in Table 1. It may be seen from cols. 3 and 4 of Table 1 that there is no appreciable difference between the total absorption value (i) computed from Sen-Wyller theory and (ii) that calculated using empirical relation for γ and K_{AH} values.

In order to study the nature of the frequency dependence of the total non-deviative D-region absorption, as obtained by A-H and S-W theories, a graph has been plotted between the total non-deviative D-region absorption and $(f + f_H)$ on a log-log scale as shown in Fig. 4. The slope of the line in the case of S-W is 1·77 whereas it is 2 in the case of A-H, suggesting inverse square relationship as predicted by most of the theoretical workers. It may be mentioned here that the experimental value of the exponent *m* in the relationship, $L \propto \frac{1}{(f+f_L)^m}$ at low latitudes has been reported[4,5] to lie between 1·5 and 1·8 which agrees well with that of Sen-Wyller's case. Also, the experimental value of *m* as obtained at Udaipur is 1·7

Table 1 — D-Region Total Absorption

Working frequency MHz	Non-deviative total absorption dB From A-H Theory	From S-W Theory	Computed using empirical relation for γ and K_{AH} values
1	2	3	4
2	7·28	14·94	14·86
2·5	5·34	12·12	12·074
3	4·15	9·36	9·53
3·5	3·32	7·50	7·70

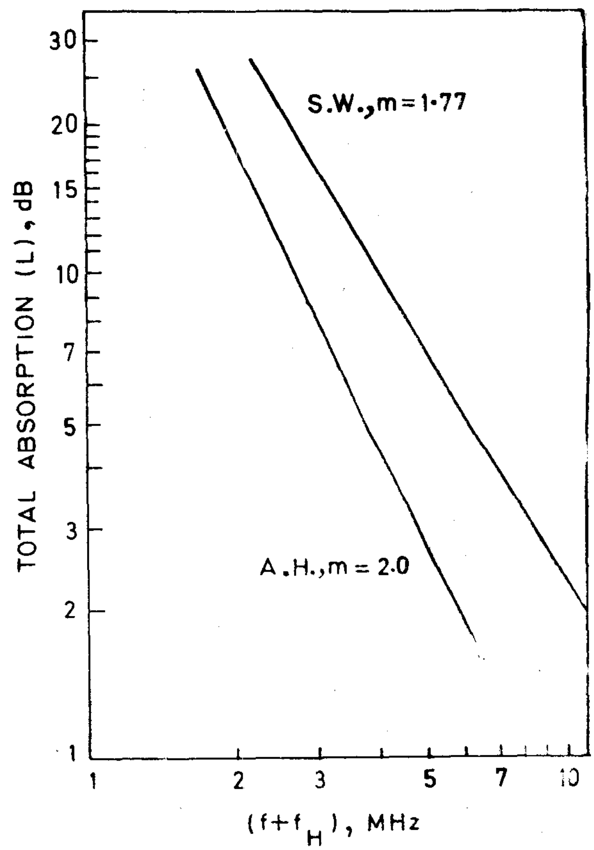

Fig. 4 — Frequency variation of total absorption in D-layer (log-log scale)

(Jain, S. L., unpublished work, personal communication).

4. Conclusions

The following conclusions, as established above, are worthy of mentioning.

(i) The empirical relation in Eq. (5) is suitable as the multiplying factor which would enable one to calculate the generalized absorption coefficient from the classical A-H one.

(ii) The multiplying factor varies from nearly 1 to 2·5 in the height range $70<h<85$ km and afterwards it remains very nearly 2·5.

(iii) The frequency index m comes out to be 1·77 instead of 2 when generalized theory is used, which value agree well with the experimental results.

Acknowledgement

One of the authors (S.K.V.) is thankful to the University Grants Commission, New Delhi, for the financial assistance for conducting the work. The authors are thankful to Prof. J. Varma for his keen interest in the work.

References

1. APPLETON, E.V., *J. Instn elect. Engrs*, **71** (1932), 642.
2. SEN, H. K. & WYLLER, A. A., *J. geophys. Res.*, **65** (1960), 3931.
3. DINGLE, R. B., ARNDT, D. & ROY, S. K., *Appl. scient. Res.*, **6** (**B**) (1957), 155.
4. RAMANA MURTHY, Y. V. & RAO, B. RAMACHANDRA, *J. atmos. terr. Phys.*, **24** (1962), 245.
5. RAO, M. K., MAZUMDAR, S. C. & MITRA, S. N., *J. atmos. terr. Phys.*, **24** (1962), 245.