Indian Journal of Radio & Space Physics
Vol. 5, June 1976, pp. 183-187

Studies on Ionospheric D & E Region Models in Conjunction with Al Absorption Measurements at Waltair*

D N MADHUSUDHANA RAO & K V V RAMANA
Department of Physics, Andhra University, Waltair 530 003

Received 30 *April* 1975; *revised received* 15 *October* 1975

Ionospheric absorption has been studied in model ionospheric layers at Waltair using Sen-Wyller's fully generalized magneto-ionic theory. On the basis of numerical calculations a suitable model is selected that gives values of absorption, virtual height and the cos χ index *n* in reasonable agreement with the experimentally observed values from Al absorption data. It is found that the assumption of a single Chapman layer model along with an exponential collision frequency profile leads to a cos χ index less than 1·5. In particular, a three-tier model of electron density profile consisting of (i) a Chapman type $E(N_{e0} = 2\times10^5$ cm^{-3} and h_0 = 110 km) and a Chapman type $D(N_{e0} = 10^3$ cm^{-3} and h_0 = 75 km); (ii) a constant electron density lower *D*-region ($N_e = 200$ cm^{-3} in the height range 50-70 km) with a uniform scale height of 6 km, and (iii) an exponential collision frequency profile assuming the value of $\nu_0 = 2\times10^4$ sec^{-1} at 110 km, is proposed.

1. Introduction

Ionospheric hf wave absorption measurements are important in computing electron density or collision frequency profiles in the absorbing zone of the region. Although considerable amount of absorption data has been acquired at various places, attempts[1,2] at using this data in deriving electron density and collision frequency profiles are few and far between.

In this paper, data of daily Al absorption and group height measurements on 2·4 MHz are used to derive a model electron density profile for the absorbing zone under interest assuming an exponential collision frequency model. First the absorption is interpreted in terms of an absorbing Chapman layer and calculations are made using the Chapman type distribution of electron density profile for combined E and D regions. Subsequently, the model is refined on the basis of experimental data. A computer method developed by us for calculating the absorption coefficient from Sen-Wyller formula[3] for the complex refractive index and a brief account of experimental results of Al absorption on 2·4 MHz at Waltair, are given.

2. Theory and Method

The Sen-Wyller formula for the complex refractive index *n* as given by Budden,[3] is

$$n^2 = (\mu - i\chi)^2 = \frac{\epsilon_1\epsilon_2 \sin^2\theta + 1/2\, \epsilon_3 (\epsilon_1+\epsilon_2)(1+\cos^2\theta) \pm M^{1/2}}{(\epsilon_1+\epsilon_2)\sin^2\theta + 2\epsilon_3\cos^2\theta} \quad \ldots(1)$$

*Paper presented at the Symposium on Earth's Near Space Environment, 18-21 February 1975 held at the National Physical Labaratory, New Delhi 110 012.

where

$$M = \sin^4\theta\, \{\epsilon_1\epsilon_2 - \tfrac{1}{2}\, \epsilon_3 (\epsilon_1+\epsilon_2)\}^2 + \epsilon_3^2 \cos^2\theta\, (\epsilon_1-\epsilon_2)^2$$

μ and χ are the real and imaginary parts of the complex refractive index and

$$\theta = (\pi/2 - \text{dip})$$

$$\epsilon_1 = 1 - X\,[(1+Y)\, W^2\, \mathscr{C}_{3/2}\{W(1+Y)\} + 5/2\, iW\mathscr{C}_{5/2}\{W(1+Y)\}] = u_1 - i\nu_1 \quad \ldots(2)$$

$$\epsilon_2 = 1 - X\,[(1-Y)\, W^2\, \mathscr{C}_{3/2}\{W(1-Y)\} + 5/2\, iW\, \mathscr{C}_{5/2}\{W(1-Y)\}] = u_2 - i\nu_2 \quad \ldots(3)$$

$$\epsilon_3 = 1 - X\,[W^2\mathscr{C}_{3/2}(W) + 5/2\, iW\mathscr{C}_{5/2}(W)] = u_3 - i\nu_3 \quad \ldots(4)$$

Further $X = \omega_p^2/\omega^2$ where ω_p and ω are respectively the angular plasma and operating frequencies; $Y = \omega_H/\omega$, where ω_H is the angular gyro frequency; and $W = \omega/\nu_m$, where ν_m is the collision frequency of the monoenergetic electrons.

In Eqs. (2)-(4), $\mathscr{C}_{3/2}(x)$ and $\mathscr{C}_{5/2}(x)$ are the Dingle[4] integrals given by

$$\mathscr{C}_{3/2}(x) = \frac{1}{(3/2)!}\int_0^\infty \frac{\epsilon^{3/2}}{\epsilon^2+x^2}\exp(-\epsilon)\, d\epsilon, \text{ and}$$

$$\mathscr{C}_{5/2}(x) = \frac{1}{(5/2)!}\int_0^\infty \frac{\epsilon^{5/2}}{\epsilon^2+x^2}\exp(-\epsilon)\, d\epsilon$$

The evaluation of these integrals is given by Hara.[5] From Eqs. (2)-(4), on separating the real and imaginary parts we get,

$$u_1 = 1 - X\,[xW\mathscr{C}_{3/2}(x)] \qquad \nu_1 = 5/2\; WX\mathscr{C}_{5/2}(x)$$

$$u_2 = 1 - X\,[yW\mathscr{C}_{3/2}(y)] \qquad \nu_2 = 5/2\; WX\mathscr{C}_{5/2}(y)$$

$$u_3 = 1 - XW^2\mathscr{C}_{3/2}(W) \qquad \nu_3 = 5/2\; WX\mathscr{C}_{5/2}(W)$$

where $x = W(1+Y)$ and $y = W(1-Y)$

Eq. (1) may be written, after simplification, in the following form

$$n^2 = \frac{\epsilon_1\epsilon_2 \sin^2\theta + \frac{1}{2}\,\epsilon_3(\epsilon_1+\epsilon_2)\,(1+\cos^2\theta) \pm S}{\epsilon_1+\epsilon_2 \sin^2\theta + 2\,\epsilon_3 \cos^2\theta} \quad ..(5)$$

where

$$S = \cos\theta\,\epsilon_3\,(\epsilon_1-\epsilon_2)\,(1+\lambda^2)^{1/2}$$

with

$$\lambda = \xi + i\eta = \frac{[\sin^2\theta\,\{\epsilon_1\epsilon_2 - \epsilon_3\,(\epsilon_1+\epsilon_2)\}]}{\cos\theta\,\epsilon_3\,(\epsilon_1-\epsilon_2)} \quad ...(6)$$

in which

$$\xi = \frac{P \sin^2\theta}{\cos\theta\,\{(u_1-u_2)^2 + (v_1-v_2)^2\}\,(u_3^2+v_3^2)} \quad ..(7)$$

where $P = (u_1^2+v_1^2)\,\{u_2u_3+v_2v_3-\frac{1}{2}\,(u_3^2+v_3^2)\} - (u_2^2+v_2^2)\,\{u_1u_3+v_1v_3-\frac{1}{2}\,(u_3^2+v_3^2)\}$

and

$$\eta = \frac{Q \sin^2\theta}{\cos\theta\,[(u_1-u_2)^2 + (v_1-v_2)^2]\,(u_3^2+v_3^2)} \quad ...(8)$$

where

$$Q = (u_1^2+v_1^2)\,(u_2v_3-v_2u_3)+(u_2^2+v_2^2)\,(u_3v_1-v_3u_1) + (u_3^2+v_3^2)\,(u_1v_2-v_1u_2)$$

Writing $\{1+\lambda^2\}^{1/2} = (\phi+i\psi)$ we obtain

$$\phi = (1/2)^{1/2}\,[\{(1+\xi^2+\eta^2)^2\}-4\eta^2\}^{1/2}+(1+\xi^2-\eta^2)]^{1/2} \quad ...(9)$$

and

$$\psi = (1/2)^{1/2}\,[\{(1+\xi^2+\eta^2)^2-4\eta^2\}^{1/2}-(1+\xi^2-\eta^2)]^{1/2} \quad ...(10)$$

Replacing $\{1+\lambda^2\}^{1/2}$ by $(\phi+i\psi)$ in Eq. (5) and separating the real and imaginary terms, we get $(\mu^2+\chi^2)$ and $(-2\mu\chi)$ from which the values of μ and χ can be obtained.

The real part of the complex refractive index

$$\mu=\{1/2(\gamma^2+\delta^2)\}^{1/2}\,[\{(\alpha^2+\beta^2)\,(\gamma^2+\delta^2)\}^{1/2}+(\alpha\gamma+\beta\delta)]^{1/2} \quad ...(11)$$

and the imaginary part

$$\chi=\{1/2(\gamma^2+\delta^2)\}^{1/2}\,[\{(\alpha^2+\beta^2)\,(\gamma^2+\delta^2)\}^{1/2}-(\alpha\gamma+\beta\delta)]^{1/2} \quad ..(12)$$

in which α, β, γ and δ are given in terms of u_1, u_2, u_3, v_1, v_2 and v_3 as follows:

$$\alpha = \alpha' \pm a \quad ...(13)$$

$$\beta = \beta' \pm b \quad ...(14)$$

with

$$\alpha' = (u_1u_2-v_1v_2)\sin^2\theta + 1/2\,(1+\cos^2\theta)\,[u_3(u_1+u_2)-v_3(v_1+v_2)]$$

$$\beta' = -(u_1v_2+u_2v_1)\sin^2\theta - 1/2\,(1+\cos^2\theta)\,[u_3(v_1+v_2)+v_3(u_1+u_2)]$$

$$a = \pm\,(m\phi + p\psi)\cos\theta$$

$$b = \pm\,(m\psi - p\phi)\cos\theta$$

$$m = u_3(u_1-u_2) - v_3(v_1-v_2)$$

$$p = u_3(v_1-v_2) + v_3(u_1-u_2)$$

$$\gamma = (u_1+u_2)\sin^2\theta + 2u_3\cos^2\theta$$

$$\delta = -[(v_1+v_2)\sin^2\theta + 2v_3\cos^2\theta]$$

From Eq. (12), the linear absorption coefficient k is obtained as

$$k = (\omega/c)\,\chi, \quad ...(15)$$

where c is the speed of light.

In computing α and β for the ordinary and extraordinary rays from Eqs. (13) and (14), it is to be remembered that if the product ab is positive, then the positive roots of a and b are used for the O-ray and the negative roots for E-ray. If the product ab is negative, the positive root of a and the negative root of b are used for the O-ray and for the E-ray the negative root of a and positive root of b are used.

The derivation of appropriate equation for the group height from the Sen-Wyller formula is found to be laborious. It has been noticed that the Appleton-Hartree and Sen-Wyller formulae give identical results for μ and χ when ν_{eff} (effective collision frequency) and ν_m (monoenergetic collision frequency) are used respectively for ν in the formulae. It may be mentioned here that $\nu_{eff} = 5/2\,\nu_m$ over most part of the absorbing zone. Hence, a numerical method of deducing h' with the help of the Appleton-Hartree formula and the well known relation between group and phase complex refractive indices, has been used to obtain the group height.

3. Results

Some relevant results of studies of diurnal variation of ionospheric absorption based on A1 measurements on 2·4 MHz carried out at Waltair during 1971-73 and which formed the basis for the following model calculations are briefly presented here. The absorption obeys the diurnal variation law, $L \propto \cos^n\chi$, χ being the solar zenith angle. The mean value of n is found to exhibit seasonal variation with maximum in summer (0·89) and minimum in winter (0·63). The D-region absorption separated from the total value using Jaeger's method is found to be about 60% of the total absorption, a result which is in reasonable agreement with earlier calculations.[6]

The hourly values of D-region absorption, obtained after separating the E-region contribution, revealed that part of the D-region contribution varies with zenith angle and the rest remains independent of the zenith angle. It may, thus be concluded that the absorption arises in three layers, E, upper D and lower D regions. Table 1 summarizes the experimental results of absorption (at 2·4 MHz) measurements at Waltair (θ = 70°; magnetic intensity = 0·38 gauss) during 1971-73. The absorption coefficient, using the computer method given earlier, is calculated (for both single and double Chapman models) at 0·1 km intervals from the reflection height down to 60 km. The relevant absorption intergral is evaluated by numerical integration.

3.1 Single Chapman Model

The single Chapman layer model is used here with the E-region datum height at 110 km and an exponential collision frequency profile given by

$$\nu = \nu_0 \ \exp(-z) \qquad \ldots(16)$$

where z is the reduced height. The key parameters of the model are assumed to be: $h_0 = 110$ km; scale height $H = 10$ km and electron density profile $N_e = N_{eo} \exp\{1/2\,(1 - z - e^{-z} \sec \chi)\}$ with $N_{eo} = 2 \times 10^5$ cm^{-3} (corresponding to a f_0E of 4 MHz).

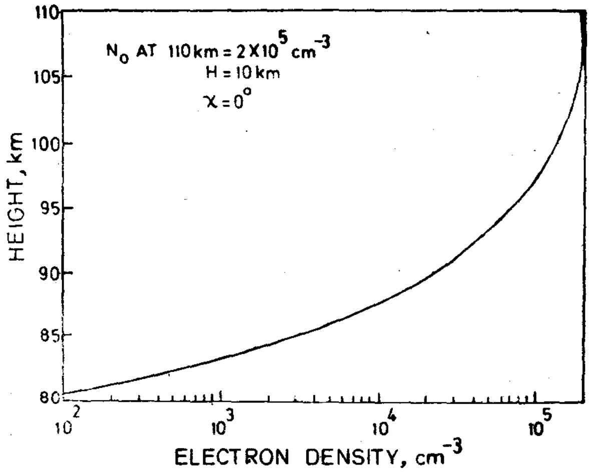
The effective collision frequency ν_0 in Eq. (16) is chosen by trial and error to give an absorption that agrees with the experimental average noon-time value. The total absorption is computed for different cos χ values over a day and the cos χ index is then obtained graphically. Table 2 gives the absorption at $\chi = 0$ and the cos χ index for different values of ν_0 assumed for the single Chapman layer model as shown in Fig. 1. Although the absorption calculated using $\nu_0 = 4 \times 10^4$ sec^{-1} agrees with the experimental value, the D-region contribution is found to be only 13% of the total (owing to the fact that the assumed single Chapman model gives insignificant values for electron density in the D-region). Also, the adoption of full magneto-ionic formula in computing absorption in this model below 110 km has demonstrated that the cos χ index departs significantly from the value deduced using quasi-longitudinal approximation for the linear absorption coefficient.

Table 1—Values of Absorption on 2·4 MHz at Waltair during 1971-73

Season	Mean L at noon dB	cos χ index n
Summer	44	0·89
Winter	40	0·63
Equinox	50	0·70
Yearly	45	0·71

Table 2—Absorption and cos χ Index for Different Collision Frequency Profiles

ν_0 10^4 sec^{-1}	L dB	cos χ index
2	23	0·91
3	35	0·87
4	46	0·92



Fig, 1—A Single Chapman Layer Model

However, the value of cos χ index obtained (0·9) does not agree with the value (1·5) obtained by Appleton.[7] But various investigators[6,8] obtained n (cos χ index) values in the range 0·7 ± 0·2 from experimental measurements of absorption of E region echoes. Hence the adoption of full Sen-Wyller formula in conjunction with a single Chapman layer and an exponential collision frequency model is sufficient to bring the cos χ index value close to the experimental value. Better agreement between the computed value with the experimental value of n may be achieved by modifying the single Chapman layer model. As a first step in this direction, a double Chapman model for D and E regions is considered.

3.2 Double Chapman Model

Chapman type electron density profiles are computed separately for E and D regions and they are superposed to give a double Chapman profile (Fig. 2). The model is computed using the following parameters:

Parameter	D region	E region
h_0	75 km	110 km
N_{e0}	1×10^3 cm^{-3}	2×10^5 cm^{-3}

The scale height (6 km) taken is an average value for E- and D-region values obtained from CIRA (1965). An exponential collision frequency profile with $\nu_0 = 2 \times 10^4$ sec^{-1} at 110 km is used. This value of ν_0 is arrived at, on the basis of the concentrations of various atmospheric constituents.[9]

The total absorption (at $\chi = 0$) computed using the above profiles comes out as 43 dB with a D-region contribution of 29 dB, which is about 66% of the total absorption. Table 3 presents the absorption and group heights computed for different zenith angles. The group height (at $\chi = 0$) also agrees well with the observed noon virtual height (about 100 km) of reflection of (at 2·4 MHz) radio

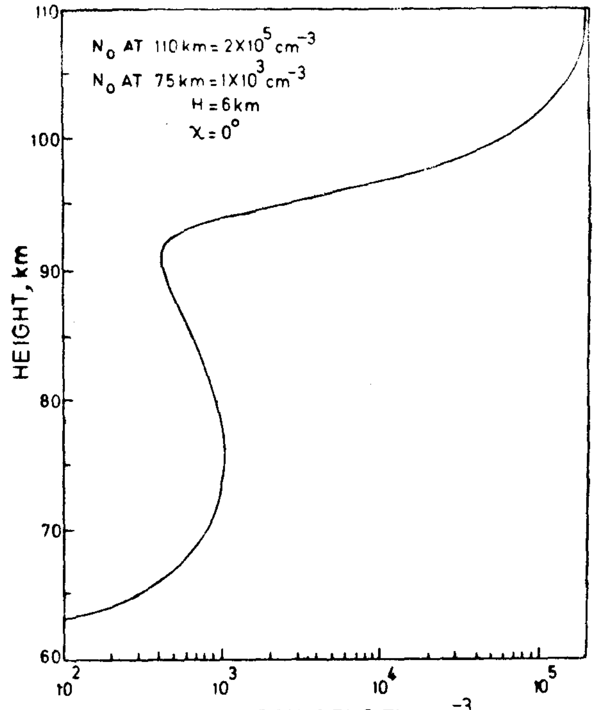


Fig. 2—A Double Chapman Layer Model

Table 3—Zenith Angle (χ) Variation of Absorption and Virtual Height (h') for Double Chapman Model (n = 0·96)

cos χ	L dB	h' km
0·4	18	110·4
0·5	22	107·8
0·6	26	105·5
0·7	32	103·5
0·8	35	103·0
0·9	39	101·0
1·0	43	100·9

waves. From the linear plot of (1+log L) versus (1+log cos χ), the cos χ index is obtained as 0·96, which is higher than the average experimental value of 0·71 or the average summer value of 0·89. Again from Table 2 it is seen that ν_0 has little influence on n value. Therefore, the parameter that influences n is the electron density, especially in the D region. It was mentioned earlier that the absorption can be interpreted as occurring in Chapman type E and D regions, the lower part of the latter being independent of zenith angle. This suggestion is now taken into consideration in the calculation of the cos χ index value.

3.3 Modified Double Chapman Model

For this modification of the model the peak of the D layer is shifted to 80 km and a double Chapman profile is built, adding a constant electron density of 200 electrons cm^{-3} from 70 km down to 50 km. The profile thus obtained is shown in Fig. 3. The absorption, at χ = 0, calculated using this profile comes out as 38 dB. The cos χ index for diurnal variation obtained is 0·86 which is less than that (0·96) obtained for simple double Chapman layer model. This value, although closely agreeing with the average summer value of 0·89, experimentally obtained, is higher than the average cos χ index value of 0·71. It is to be noted that the absorption value is less than the average value of 45 dB. A closer agreement may yet be achieved if the peak of the D-layer is brought back to 75 km as was done earlier. The D-layer maximum height is shifted to 80 km in this calculation because of uncertainty in defining the peak. Usually the peak of the layer is given between heights 70 and 85 km.[10-12] In order to see whether the situation improves if the peak of the D-layer is shifted back to 75 km, calculations are redone for the double Chapman model taking 75 km as the D-layer maximum height (Fig. 4). The total moon-time absorption (at χ = 0) obtained now is 46 dB, in excellent agreement with the experimental value. After calculating absorption at different zenith angles, the cos χ index is obtained as 0·8. The increased absorption from 38 dB to 46 dB is a natural consequence of the shifting of the D layer

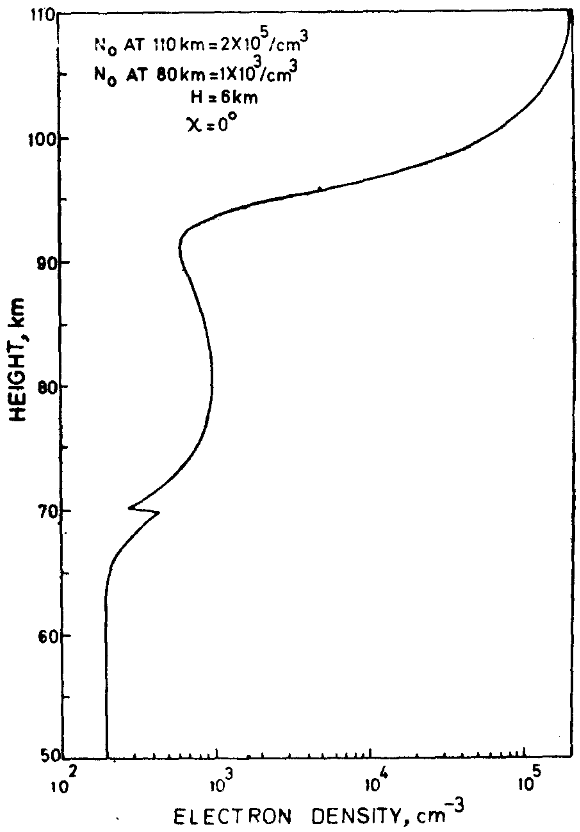


Fig. 3—Modified Double Chapman Layer Model

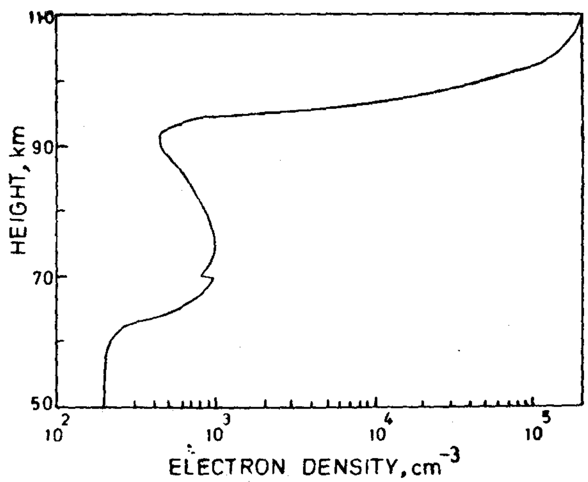


Fig. 4 — Modified Double Chapman Layer Model

peak to 75 km. Also the calculated group height agreed well with the experimentally observed value. Hence, this model may be taken as representative of average ionospheric conditions over Waltair at least up to 110 km. It is worth stressing that the collision frequency model adopted here gives the kind of ratio of E and D region absorption contributions consistent with generally accepted values. If the number density of electrons in height range 50-70 km is increased from 200 cm^{-3}, calculations show that the cos χ index value drops further, apart from the drop in the ratio of E to D-region absorption contributions.

It was observed experimentally that the winter average of the cos χ index is less than the corresponding summer value. This is due to the fact that if the winter collision frequencies are higher at Waltair (summer anomaly),[13-14] there is a tendency for the lower D-region absorption to increase in magnitude with respect to that of E and upper D regions, resulting in a decline in the value of cos χ index.

Acknowledgement

The authors wish to thank Prof. B. Ramachandra Rao for his interest in the work. One of the authors (D.N.M.) expresses his gratitude to the CSIR, New Delhi, for the award of a fellowship.

References

1. Piggott W R & Thrane E V, *J. atmos. terr. Phys.*, **28** (1966), 467.
2. Beynon W J G & Rangaswamy S, *J. atmos. terr. Phys.*, **31** (1969), 891.
3. Budden K G, *Radio Sci.*, **69D** (1965), 191.
4. Dingle R B, Arndt D & Roy S K, *Appl. scient. Res.*, **6B** (1956-57), 155.
5. Hara E H, *J. geophys. Res.*, **68** (1963), 4388.
6. Gnanalingam S, Proceedings of Third International Symposium on Equatorial Aeronomy, (Physical Research Labaratory, Ahmedabad), 1969.
7. Appleton E V, *Proc. R. Soc.*, **A162** (1937), 451.
8. Appleton E V & Piggott W R, *J. atmos. terr. Phys.*, **5** (1954), 141.
9. Banks P N & Kockarts G, *Aeronomy* (Academic Press Inc., New York), 1973.
10. Nicolet N & Aikin A C, *J. geophys. Res.*, **65** (1960), 1469.
11. Jain A R & Setty C S G K, *J. atmos. terr. Phys.*, **30** (1968), 1353.
12. Gnanalingam S, Equatorial Ionospheric Absorption during Half a Solar Cycle, *GFSC Pep.* No: X-625-72-726, 1972.
13. Rao A S M & Rao B R, *J. Instn. Telecommun. Engrs, New Delhi.* **18** (1972), 106.
14. Rao D N M, *Studies on hf absorption at Waltair*, Ph.D. thesis, Andhra University, Waltair, 1974.