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Dependence of Frequency Power Law Exponent of Ionospheric Radiowave Absorption on Solar Zenith Angle

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A study of the effect of the solar zenth angle on the frequency power law exponent derived from simultaneous vertical incidence multifrequency ionospheric absorption measurements available from a number of low and midlatitude stations has shown that the exponent, in fact, increases when deviative absorption becomes relatively more significant compared to non-deviative absorption. A numerical simulation using a double α-Chapman type absorbing zone yielded results identical to the experimental observation. It is suggested that in the formula $L = A/(f+f_L)^m$ (solar zenith angle constant), widely used in such studies, neither A nor m remains independent of the solar zenith angle.

1 Introduction

The absorption measurements used in deriving the frequency power law exponent, in the past, were carried out at different solar zenith angles (SZAs) and over different frequency ranges. The picture that emerges from a careful examination of most of the available data so far indicates that the exponent m in the relation[1]

$$L \propto (f+f_L)^{-m} \quad \text{... (1)}$$

is generally higher at middle latitudes and at higher frequencies compared to that at low latitudes if one considers echoes from the E-region. Since the deviative absorption depends on the SZA at the time of observation, the authors thought it would be desirable to enquire into the dependence of the exponent on SZA. For this purpose, a series of simultaneous multifrequency ionospheric absorption measurements made at different times of a day at Colombo, Waltair, Kokubunji and Alma Ata are used. Results of a numerical evaluation of the frequency power law exponent, as a function of SZA, using the generalized magnetoionic ordinary linear absorption coefficient k_0 for an assumed α-Chapman type ionization profile in the absorption zone to determine the absorption at a set of frequencies, are compared with experimental results.

2 Theory

According to the classical Appleton-Hartree magnetoionic theory, non-deviative ordinary wave absorption coefficient k_0 is inversely proportional to the square of the sum of the operating frequency f and the longitudinal gyrofrequency component f_L in the quasi-longitudinal approximation. In the quasi-transverse approximation, k_0 is inversely proportional to the square of f.

In the non-deviative zone, the ordinary wave linear absorption coefficient

$$k_0 = \frac{Ne^2 \nu_{eff}}{8\pi^2 \varepsilon_0 mc(f+f_L)^2}, \qquad \omega+\omega_L \gg \nu_{eff} \quad \text{... (2)}$$

in the quasi-longitudinal approximation and in the quasi-transverse approximation,

$$k_0 = \frac{Ne^2 \nu_{eff}}{8\pi^2 \varepsilon_0 mcf^2}, \qquad \omega \gg \nu_{eff} \quad \text{... (3)}$$

In Eqs. (2) and (3), N is the electron density; ε_0, the free space permittivity; e, the electronic charge; m, the electron mass and c the velocity of light in vacuum.

The total absorption suffered by a wave from the bottom of the ionosphere to the point of reflection is $2\int k_0 dh$, which implies that irrespective of the ionization and collision frequency profiles employed in relation to Eqs. (2) and (3), the absorption varies inversely as $(f+f_L)^2$ or f^2 in the non-deviative approximation. But, such a situation is not obtained in the ionosphere and the real part of the complex refractive index, a function of N and f among others, appears in the denominators of Eqs. (2) and (3) and complicates the relationship between absorption and frequency.

3 Data, Analysis and Results

In Table 1 a list of the stations and their locations from where simultaneous multifrequency vertical

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Table 1—A1 Absorption Data Used in the Study of Variation of Frequency Power Law Exponent *m* with Solar Zenith Angle

Station	Geographic & geomagnetic coordinates	Period for which data are available	Frequencies of observation
Colombo	Lat. 06°45′N Long. 79°52′E Dip 5°S	July '64-Nov '68 July '57-Dec '58	2.0 and 2.2 MHz 2.55 and 2.85 MHz
Waltair	Lat. 17°43′N Long. 83°18′E Dip 20°N	Aug '78-Jan '79	2.4 and 5.6 MHz
Kokubunji	Lat. 35°42′N Long. 139°30′E Dip 49°N	Jan '58-Dec '59	2.4, 2.8, 3.0 and 3.2 MHz
Alma Ata	Lat. 43°15′N Long. 76°55′E Dip 62°N	Jan '58-Dec '61	2.2 and 3.0 MHz

incidence ionospheric absorption data are available is given indicating also other relevant details.

These absorption data are processed and analyzed in the following manner. Values of absorption at each hour of the day at the available frequencies are employed to determine the power law exponent *m* in Eq. (1). The average values of *m* for each season, i.e. summer (May, June and July), autumn (August, September and October), winter (November, December and January) and spring (February, March and April) at each hour of the day are evaluated with the year as a unit for Colombo, Kokubunji and Alma Ata. For Waltair, since only six months' data are available, the averaging is done over a month. The particular grouping of the months for the seasons is chosen since the SZA is not expected to vary significantly for any local mean time in that particular season.

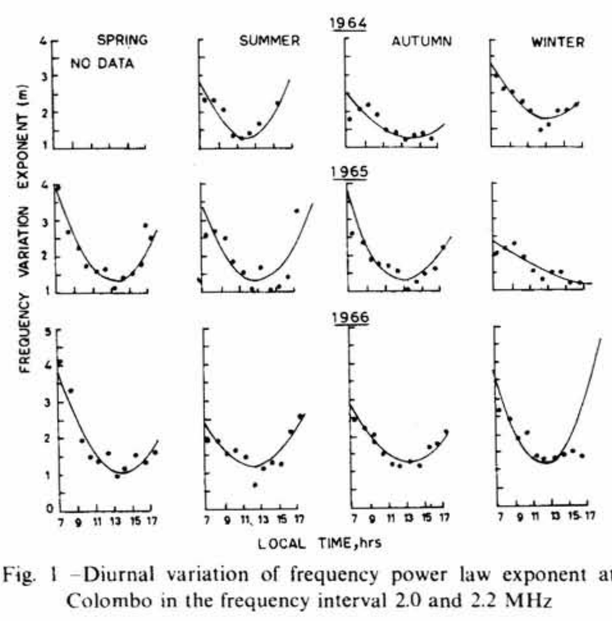

Fig. 1 —Diurnal variation of frequency power law exponent at Colombo in the frequency interval 2.0 and 2.2 MHz

Figs. 1-4 show the dependence of *m* on the local time (or SZA) at Colombo, Waltair, Kokubunji and Alma Ata during the periods indicated. The rest of the plots for Colombo for the period Jan 1967-Nov. 1968, not shown in Fig. 1, were also found to be similar in shape. The pattern that clearly emerges from the plots demonstrates that *m* increases non-linearly with SZA. Ganguly[2] and Vijayavergia and Rai[3] noted a similar pattern in the dependence of *m* on SZA. The interesting aspect of this study is that *m* increases when deviative absorption effects become more dominant consequent on the operating frequencies moving closer to the critical frequency of the E-layer with increasing SZA. The picture is contrary to the assumption made by some earlier investigators that the power law exponent

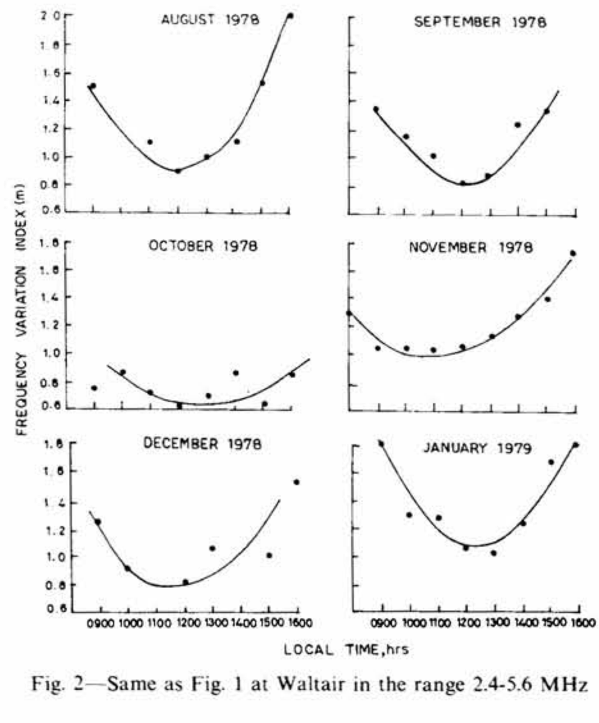

Fig. 2—Same as Fig. 1 at Waltair in the range 2.4-5.6 MHz

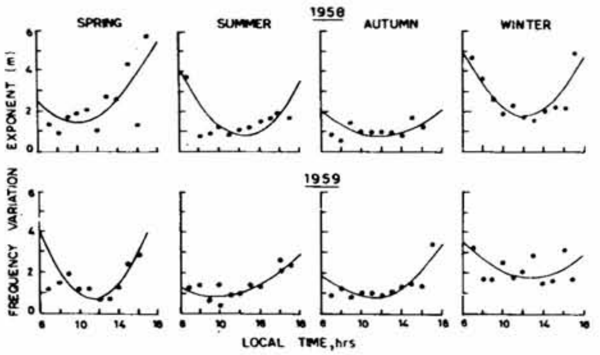

Fig. 3 Same as Fig. 1 at Kokubunji in the range 2.4-3.2 MHz

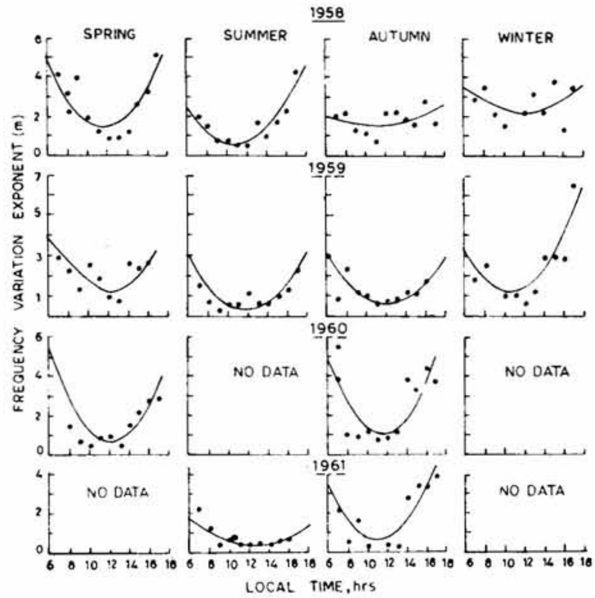

Fig. 4 Same as Fig. 1 at Alma Ata in the range 2.2-3.0 MHz

decreases in magnitude because of deviative absorption. However, previous investigations on the exponent which gave higher values of m at middle latitudes in comparison with those at low latitudes are in conformity with the result of our study since the noon-time SZAs are normally higher on the average at middle latitudes in relation to those at low latitudes for most part of a year. Table 2 contains the results of earlier measurements of the frequency power law exponent at several low and middle latitude stations at local noon.

4 Discussion

The extensive measurements of multi-frequency ionospheric absorption over the last three to four decades are found to satisfy the power law [Eq. (1)] if the SZA is kept constant, irrespective of the magnitude of m. Accepting this proposition, the implication of the variation of m with SZA noted may be discussed. If A is presumed to be the same through the day, an increase in m with SZA means that the absorption falls off more sharply with frequency at higher values of SZA. But, the fractional contribution to the total absorption from the deviative zone should sharply increase as SZA gets higher since the operating frequencies move closer to the critical frequency of the layer[4]. As a consequence, the assumption that A remains independent of the SZA is untenable. In other words, we suggest that A strongly

Table 2—Frequency Variation Exponent from the Experimental Results at Various Stations

Sation	Geographic latitude	Period of observation	Frequency range MHz	cos χ (approximate values)	m
Lwiro	02°15'N	1959	2.0-5.7 and 8.2	1.0	0.78
Colombo	06°54'N	1966-68	2.0, 2.2, 2.6 and 5.6	1.0	1.14
Ibadan	07°22'N	1953-54	2.0, 2.2 and 2.6	1.0	1.00
		and IGY	2.0-5.8		1.00
Waltair	17°43'N	1961-62	2.0-9.0	0.95	1.40
		1963-65	2.0, 2.5 and 3.0		1.00
		1972-74	1.6-3.6		0.82
		1978-79	2.4 and 5.6		0.94
Ahmedabad	23°02'N	1972-77	1.8, 2.2, 2.5	1.0	1.11
		(computed for $R_z = 0$)		0.6	0.71
Udaipur	24°35'N	1974-77	2.3-4.1	0.9	1.79
Delhi	28°35'N	1964-66	5.0-9.0	0.9	1.80
Kokubunji	35°42'N	Nov '57-Dec '58	1.7-4.8		1.00
			2.4-4.5	0.8	0.98
Freiburg	48°03'N	Jul '57-Dec '58	1.7-5.8	0.7	1.27
De Bilt	52°06'N	Jul '57-Dec '58	1.9-3.2	0.7	1.32

depends on the SZA, increasing with its magnitude and the sharp rise in absorption with SZA is partly compensated for by an increase in m. The expected increase in A with increase in SZA has been noted in the data analyzed for this study. However, at Ahmedabad[5], for which measurements at two values of SZA are available, both A and m decrease with increasing SZA for $R_z = 0$ condition.

To provide credence to this argument, we have taken recourse to a numerical simulation of the dependence of m on SZA. The ionization profile of the ionosphere below 105 km is assumed to be basically double α-Chapman. The absorption of vertically incident radiowaves at sounding frequencies 1.4 to 3.6 MHz at intervals of 0.4 MHz and at SZAs 0° to 60° at intervals of 10° is computed by numerically integrating the Sen-Wyller[6] linear absorption coefficient of the 'O' ray, from the bottom of the ionosphere to the reflection point. Absorption values at sounding frequencies greater than f_0E at any value of χ are avoided in determining m. The collision frequency profile employed in this numerical evaluation is the same as that used by Victor *et al*[7]. Table 3 lists the parameters assumed for the ionization profile. The dip angle value employed is 20° N.

Fig. 5 shows a plot of m as a function of SZA. The noon SZA is assumed to be zero degree. The m values in the post-noon period are identical to those in the pre-noon period at each SZA. It is, at once, obvious that the trend in Fig. 5 is almost identical to the trend seen in Figs. 1-4 . It would have been more appropriate to consider in situ measurements of ionization at a number of SZAs at the same station. Unfortunately, such measurements are not available. Although the parameters chosen in numerical simulation are not unique, it is believed that the dependence of m on SZA will not be much different from the calculated trend. In fact, a set of values slightly different from those given in

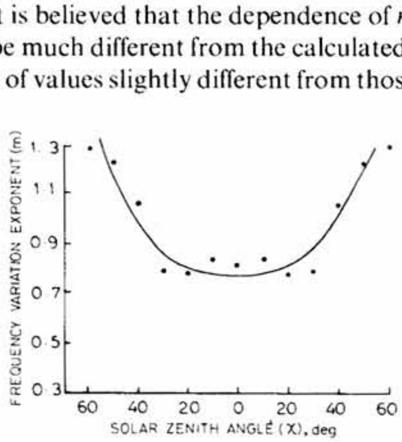

Fig. 5 Numerically derived variation in the frequency power law exponent at frequencies 1.4 to 3.6 MHz with SZA in a double α-Chapman ionization profile

Table 3 Model Electron Density Profile Assumed in Numerical Simulation

Name of the layer	Parameters adopted	Remarks
E	$N_0 = 2 \times 10^5$ cm^{-3} $h_0 = 105$ km $H = 6$ km	h_0 is height of unit optical depth at equator at equinox at $\chi = 0$. H is scale height.
D	$N_0 = 10^3$ cm^{-3} $h_0 = 75$ km $H = 6$ km	
C	$N_h = 10(h-50)$cm^{-3} h = altitude in km	These values are applicable for $50 < h < 70$ km.

Table 3 reproduced a similar trend in the variation of m with SZA.

This study on the dependence of the frequency power law exponent on the SZA shows that multifrequency ionospheric absorption measurements used to derive m should be carried out at the same SZA. The non-deviative approximation of the linear absorption coefficient is inadequate in the sense that even if the operating frequency range is remote from the critical frequency of the layer, the frequency power law exponent is nowhere near 2 as may be noted from Fig. 5. Further, it is not correct to attribute values of m smaller than 2 to either deviative absorption effects or due to significant non-deviative absorption occurring in the absorption zone satisfying $(\omega + \omega_L) \ll \nu_{eff}$. The difference in the magnitudes of m observed at low latitudes in relation to that at midlatitudes is basically due to measurements carried out at different SZAs.

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