Indian Journal of Radio & Space Physics
Vol. 14, October 1985, pp. 126-130

Effect of Corona on Ground Electric Field Beneath a Thundercloud

PRATAP SINGH, T S VERMA & N C VARSHNEYA
Department of Physics, University of Roorkee, Roorkee 247 667

Received 6 *March* 1985

The effect of corona space charge on growth rate of ground electric field beneath a thundercloud has been studied. The maximum electric fields have been calculated at the ground surface and aloft. It has been found that the corona limits the magnitude of the electrostatic field and also influences the time behaviour of the electric field at the ground surface. The computed results are found to be in good agreement with the available experimental observations.

1 Introduction

It is widely accepted that thunderclouds accumulate a net positive charge at their top (P-region) and net negative charge at their bottom (N-region); and thus are known to produce strong electric fields at the ground. Previous investigators have reported the existence of a charge layer near the ground surface when the electric field is intense as it often is under a thundercloud[1-3]. Standler and Winn[2] estimated the threshold value of the electric field required to initiate corona to be about 5 kVm^{-1} and also stated that the rate of production of ions by corona is a rapidly increasing function of the ambient electric field. Rust and Moore[4] found that space charge densities at the ground can be strongly perturbed by the presence of even weakly electrified clouds overhead. Moreover, Standler and Winn[2] and Winn *et al.*[5] found that beneath a thundercloud the magnitude of the electric field at few hundred metres above the ground is several times larger than that at the ground. The experimental evidences elucidate that space charge layer produced by corona from the ground irregularities (elevated objects) is responsible for reducing the magnitude of the ground electric field.

Earlier, Whipple and Scarse[6] and Wormell[7] pointed out that the electric field at the ground level is the result of the superposition of two fields: one from the charges in the thundercloud and other from space charges near the ground surface produced by corona. Mathpal and Varshneya[8] suggested that for a more accurate informative calculations of the ground electric field the effect of corona should be taken into account. In the present work, we present computations for the ground electric field incorporating the effect of corona space charge at the ground surface. The calculations show that corona space charge considerably reduces the electric field at ground surface.

2 Earlier Observations

Livingston and Krider[9] recorded ground electric field at Kennedy Space Center (KSC) on the eastern coast of central Florida using surface electric field sensors. The details of the field sensors have been described by Jacobson and Krider[10]. The electric field records showing complete development of six small air mass thunderstorms were made at sites selected to be near the centre of electrical activity. The ground electric fields produced by these thunderstorms were found to be of the order of $\approx -8\,kVm^{-1}$. Furthermore, Standler and Winn[2] reported simultaneous measurements of the electric field at the ground surface and aloft. These experiments were conducted during July and August 1976 at Langmuir Laboratory in Central New Mexico. The observations were made by balloon-borne meters as well as station field mills. They have shown the magnitude of the observed electric field at ground surface to be about 10 kVm^{-1}. Vonnegut[11] and Uman *et al.*[12] also reported that the absolute value of electric field is usually less than 10 kVm^{-1} at the ground surface. However, Winn *et al.*[5] have reported the magnitude of the ground electric field to be about 15 kVm^{-1}.

3 Theoretical Approach

Following earlier investigators[8,13], a thundercloud of cylindrical shape of radius D and having vertical charge distribution within it, has been assumed. Fig. 1 shows the dimensions of a thundercloud, wherein $2L$ is the length of the charging zone within thundercloud and H is the height for P- and N-regions. We also assume predominance of the positive charge in the upper part and negative charge in the lower part of thundercloud as usually observed. In the intervening space the charge varies linearly with height. P is the point at the ground surface. Calculations for electric

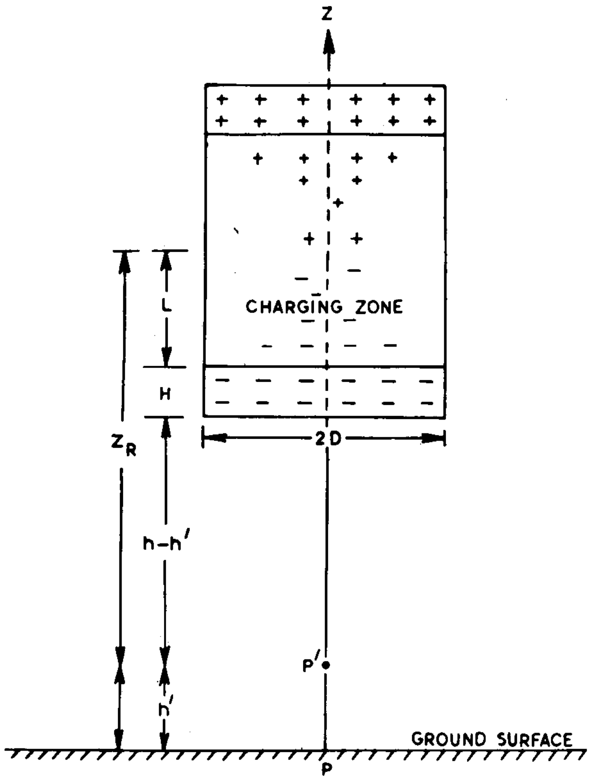

Fig. 1—Charge distribution and thundercloud size

field growth rates at P are performed.

Modifying the equation given by Mathpal and Varshneya[8] to the point P just beneath the thundercloud, the electric field growth rate at this point P at the ground surface may be expressed as

$$\frac{dE_{out}}{dt} = -\frac{D^2[(L+H)^2 - L^2/3] \, . \, J_{in}(t)}{\varepsilon_0 Z^3 (H+L/2)} \quad \ldots (1)$$

where ε_0 is the permittivity of the free space and Z is the vertical height of the cloud centre from the ground surface. The parameter J_{in} represents the current density inside the thundercloud and may be given by

$$J_{in}(t) = F(t)[\alpha - F(t)E_{in}(t)\beta] + A_1 E_{in}(t)[W - A_2 E_{in}^2(t)] \quad \ldots (2)$$

Here W is the downdraft velocity. For the details of Eq. (2) the reader is referred to Mathpal et al[14].

As pointed out previously the electric field at the ground surface is the superposition of fields due to thundercloud and space charge layer produced by corona near the ground surface. Including the corona space charge effect, the electric field, E_{out}, at point P on the ground surface will be given by

$$\frac{dE_{out}}{dt} = -\frac{D^2[(L+H)^2 - L^2/3][J_{in}(t) + J_{cr}(t)]}{\varepsilon_0 Z^3 . (H+L/2)} \quad \ldots (3)$$

where, $J_{cr}(t)$ is the corona current density at the ground surface at the instant t.

Standler and Winn[2], Chauzy and Raizonville[3] and Winn et al.[5] reported space charge densities of 1, 2-4 and 1.7 nCm^{-3}, respectively, near the ground surface. Standler and Winn[2] established a generalized relation between corona current density, J_{cr}, and ambient electric field E_{out}, at the ground surface which is given by

$$J_{cr}(t) = \begin{cases} 0 & \text{for } |E_{out}(t)| \leq E_0 \\ C\,E_{out}(t)[|E_{out}(t)| - E_0]^2 & \text{for } |E_{out}(t)| > E_0 \end{cases} \quad \ldots (4)$$

where E_0 is the minimum magnitude of the electric field to set on corona at the ground surface and C is the function of the number of discharging objects per unit horizontal area. For a generalized case, the estimated values of E_0 and C are 5 kVm^{-1} and 2.0 $\times 10^{-20}$ AmV^{-3}, respectively[2].

Including the effect of corona at ground surface the Eq. (3) for the growth rate of the electric field at point P becomes

$$\frac{dE_{out}}{dt} = -\frac{D^2[(L+H)^2 - L^2/3]}{\varepsilon_0 Z^3 (H+L/2)} \times \{[F(t)[\alpha - F(t)E_{in}(t)\beta] + A_1 E_{in}(t)[W - A_2 E_{in}^2(t)]] + C\,E_{out}(t)[|E_{out}(t)| - E_0]^2\} \quad \ldots (5)$$

where, $E_{in}(t)$ is the electric field inside the thundercloud at instant t and can be represented by[11]

$$\frac{dE_{in}}{dt} = \frac{(1-C_2)}{\varepsilon_0}\{F(t)[\alpha - F(t)E_{in}(t)\beta] + A_1 E_{in}(t)[W - A_2 E_{in}^2(t)]\} \quad \ldots (6)$$

$$F(t) = KF'(t)/[\exp(\langle p \rangle . t/\tau)] \quad \ldots (7)$$

$$F'(t) = \int_0^t E_{in}(t)\exp(\langle p \rangle . t/\tau)dt \quad \ldots (8)$$

$$\frac{dF'(t)}{dt} = E_{in}(t)\exp(\langle p \rangle . t/\tau) \quad \ldots (9)$$

and

$$C_2 = \left\{[(L+H)^2 + D^2]^{1/2} - \frac{(L^2+D^2)^{1/2}}{2} - \frac{D^2}{4L}\ln\left[\frac{(L^2+D^2)^{1/2}+L}{(L^2+D^2)^{1/2}-L}\right]\right\} \Big/ [H+L/2] \quad \ldots (10)$$

Here, $\langle p \rangle$ is the average rebound probability of cloud drops and τ is relaxation time.

Standler and Winn[2] observed that the electric field at a few hundred metres above the ground surface is several times larger than that at the ground surface. Therefore, to evaluate it theoretically we consider a case in which we compute electric field growth rates and maximum electric fields at a point P′ which is at vertical height h' from the ground surface. It has been assumed that corona space charge affects the electric field only at the ground surface. The notable feature of this aspect is that in such cases the charge of real cloud affects more the electric field than the charge of image cloud produced by the conducting earth. It happens due to proximity of the observational point to the real cloud (Fig. 1). Therefore, rate of electric field at a point P′ at vertical height h' from the ground surface may be given by

$$\frac{dE_{out}}{dt} = -\left[\frac{D^2[(L+H)^2 - L^2/3][Z_R^3 + Z_I^3]}{2\varepsilon_0 Z_R^3 . Z_I^3 (H+L/2)}\right] \times \{F(t)[\alpha - F(t)E_{in}(t)\beta] + A_1 E_{in}(t)[W - A_2 E_{in}^2(t)]\} \quad \ldots (11)$$

where

$Z_R = L+H+h-h'$

$Z_I = L+H+h+h'$

It would be worthy of mentioning here that the electric field at the ground surface is produced by real and imaginary cloud because earth is assumed as conducting.

4 Computation and Discussion

A computer programme was run to solve Eqs. (5), (6) and (9) simultaneously using Runge-Kutta method[15]. To solve these equation numerically, we assume that initially at $t=0$, $E_{in} = E_{out} = 100$ Vm^{-1} (fair weather electric field). Generally, in evaluations, we take $L = 1.5$ km, $H = 0.5$ km, $D = 1.25$ km, $p_0 = 10$ mmhr^{-1}, $\langle p \rangle = 1.0$ and $\tau = 100$ s [Mathpal and Varshneya[8]]. The text shows that the growth rate of electric field at the ground surface is associated with the strength of developing electric field inside the thundercloud and the height of cloud base from the ground surface. In the present paper, both cases have been studied including the effect of corona space charge. The downward directed electric field is termed as positive electric field throughout the discussion.

There are many parameters (downdraft velocity W, cloud width D, precipitation intensity p_0, etc.) which affect the development of electric field inside the thundercloud. Electric field inside thundercloud and electric field at the ground surface are interdependent. Therefore, these parameters also influence the build-up of ground electric field. To assess the significance of corona space charge on the growth rates of ground electric field, calculations have been made for several values of W. Fig. 2 shows that the growth rates of ground electric fields are reduced tremendously when electric field exceeds its threshold value required to initiate corona. Moreover, this reduced growth rate of electric field also limits the magnitude of maximum electric field, $(E_{out})_{max}$ reaching at the ground surface.

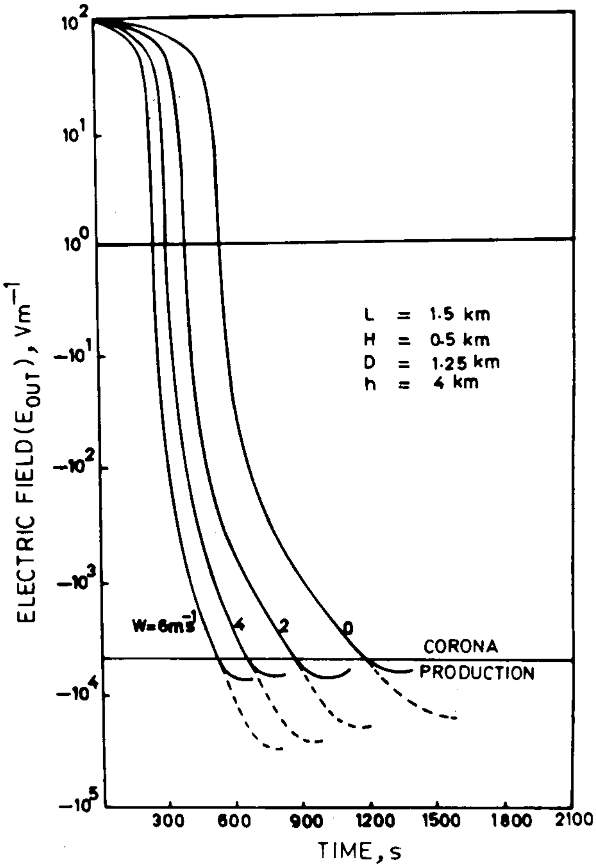

Fig. 2—Rates of growth of the electric field (with and without corona effect) at a point on the ground surface for various values of downdraft velocity

For example, assuming $W = 2$ ms^{-1}, values for $(E_{out})_{max} \approx -6.7 \times 10^3$ Vm^{-1} and -2.1×10^4 Vm^{-1} are obtained with and without considering corona effect, respectively. It is also noted that after production of corona space charge, a higher field growth does not contribute much to the maximum intensity of ground electric field. Of couse, a higher field growth takes less time to reach threshold value of electric field to initiate corona.

The effect of cloud base height on ground electric field growth rates and maximum electric fields, has also been carried out. Again both the cases (with and without corona effect) have been studied simultaneously. The values of electric field, E_{out} and the corresponding times to obtain them are shown in Fig. 3. Here we obtain $(E_{out})_{max} \approx -3.7 \times 10^4$ Vm^{-1} without corona influence, and $(E_{out})_{max} \approx -7.1 \times 10^3$ Vm^{-1} when corona effect is included at the ground surface ($h = 3$ km). Fig. 3 shows that lower base clouds produce higher maximum electric field at the ground surface. It is evident from Fig. 3 that in such clouds the magnitude of electric field is several times reduced due to corona space charge. The calculated

Table 1—Calculated Values of Electric Field at the Ground Level due to a Thundercloud for Various Values of W, p_0 and h
[taking $L=1.5$ km, $H=0.5$ km, $D=1.25$ km]

W ms^{-1}	p_0 mm hr^{-1}	h km	Maximum ground electric field $(E_{out})_{max}$			
			Without corona space charge		With corona space charge	
			$(E_{out})_{max.}$ 10^3 Vm^{-1}	Time s	$(E_{out})_{max.}$ 10^3 Vm^{-1}	Time s
0	10	4	−6.3	1360	−17.0	1550
2	10	4	−6.7	1010	−21.3	1130
4	10	4	−7.1	790	−26.3	900
6	10	4	−7.5	660	−31.8	730
4	10	2	−7.8	990	−72.1	1130
4	10	3	−7.1	1000	−37.0	1130
4	10	4	−6.7	1010	−21.3	1130
4	10	5	−6.4	1030	−13.4	1130
4	5	4	−6.3	1550	−18.2	1750
4	10	4	−6.8	1010	−21.3	1130
4	20	4	−7.2	680	−25.9	750
4	30	4	−7.6	540	−29.5	600

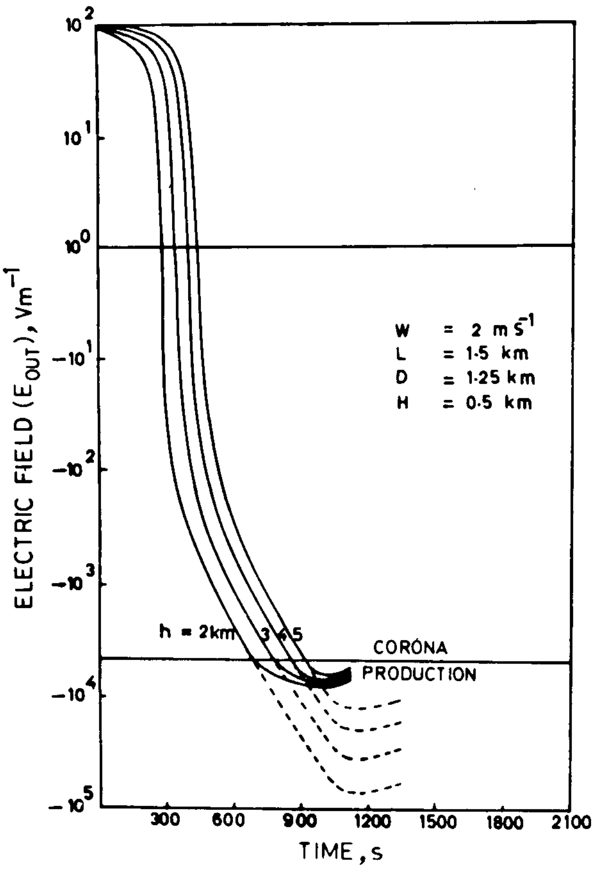

Fig. 3—Rates of growth of the electric field (with and without corona effect) at a point on the ground surface for various values of cloud base height

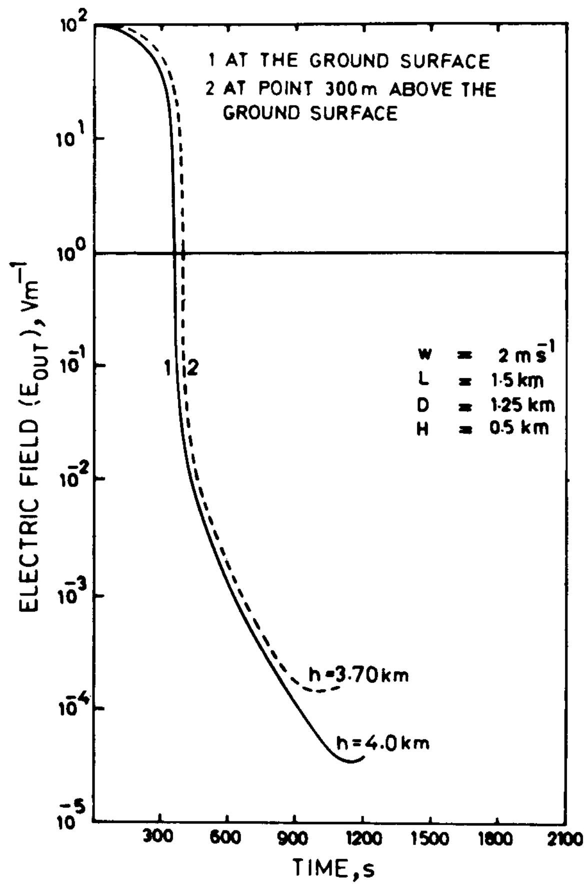

Fig. 4—Rates of growth of electric field on ground and 300 m above the ground surface just beneath the thundercloud

Table 2—Observed Values of Electric Field at Ground Level by Various Investigators

Investigator	Observed magnitude of ground electric field k Vm^{-1}
Standler and Winn[2]	≈10
Winn *et al.*[5]	≈15
Livingston and Krider[9]	≈8
Vonnegut[11]	⩽10
Uman *et al.*[12]	≈10
Reynold and Brook[16]	⩽10

values $(E_{out})_{max}$ for various sets of W, h and p_0 have also been tabulated in Table 1.

Standler and Winn[2] and Winn *et al.*[5] observed that the electric field at 300 m above the ground was about 3-6 times more than electric field at the ground surface. An attempt is now made to evaluate such differences theoretically. We consider a point P′ at 300 m above the ground just beneath the thundercloud. Eqs. (5) and (11) are solved simultaneously to obtain electric field growth rates and maximum electric fields at point P and P′, respectively. The computed values are shown in Fig. 4. Present calculations provide $(E_{out})_{max} \approx -2.8 \times 10^4$ kVm^{-1} within 1130 s at 300 m above the ground surface and $(E_{out})_{max} \approx -6.1 \times 10^3$ Vm^{-1} within 1010 s at the ground surface. These results show that electric field at 300 m above is more than 4 times higher than that at the ground surface. In this way, this theoretical treatment also finds good support from the observation made by Standler and Winn[2] and Winn *et al.*[5]

5 Conclusion

The present study reveals that corona space charge severely reduces the electric field at the ground surface. The magnitudes of electric fields which are obtained including corona effect at the ground surface are less than 10 kVm^{-1}. Such electric fields (Table 2) have been observed by several investigators[2,5,9,11,12,16]. Tables 1 and 2 show a good consistency between theoretically calculated and experimentally observed value of ground electric field under a thundercloud. This numerical simulation assesses the significance of corona on ground electric field and aloft and provides more accurate values of ground electric field.

Acknowledgement

One of the authors (PS) wishes to express thanks to Dr K C Mathpal for his valuable suggestions.

References

1 Toland R B & Vonnegut B, *J Geophys Res (USA)*, **82** (1977) 438.
2 Standler R B & Winn W P, *Q J Meteorol Soc (GB)*, **105** (1979) 285.
3 Chauzy S & Raizonville P, *Abstracts of the sixth international conference on atmospheric electricity, UMIST, Manchester, England*, 28 July-1 Aug. 1980.
4 Rust W D & Moore C B, *Q J R Meteorol Soc (GB)*, **100** (1974) 450.
5 Winn W P, Moore C B & Holmes C R, *J Geophys Res (USA)*, **86** (1981) 1187.
6 Whipple F J W & Scarse F J, *Geophys Memoirs (GB)*, **68** (1936) 272.
7 Wormell T W, *Q J R Meteorol Soc (GB)*, **79** (1953) 3.
8 Mathpal K C & Varshneya N C, *J Met Soc Jpn (Japan)*, **61** (1983) 909.
9 Livingston J M & Krider E P, *J Geophys Res (USA)*, **83** (1978) 385.
10 Jacobson E A & Krider E P, *J Geophys Res (USA)*, **33** (1976) 103.
11 Vonnegut B, *Meteorol Monographs (USA)*, **5** (1963) 224.
12 Uman M A, Beasly W H, Tiller J A, Yung-Tao Lin, Krider E P, Weidmann C D, Krehbiel P R, Brook M, Few A A, Bohannon (Jr) J L, Lennon C L, Poehler H A, Jafferis W, Bulick J R & Nicholson J R, *Science (USA)*, **201** (1978) 9.
13 Illingworth A J & Latham J, *Q J R Meteorol Soc (GB)*, **103** (1977) 281.
14 Mathpal K C, Varshneya N C & Rai J, *Ann Geophys (France)*, **38** (1982) 367.
15 Scarborough J B, *Numerical Mathematical Analysis* (Oxford Book Company, New Delhi), (1962) 356.
16 Jacobson E A & Krider E P, *J Geophys Res (USA)*, **33** (1976) 103.