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On the Determination of Initial Polarization of Signals Transmitted by Geostationary Satellites

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Expressions for determining the initial polarization and ellipticity, as seen from the receiver end, of signals transmitted by geostationary satellite for different signal polarizations in the satellite-based coordinate system have been presented and discussed.

In order to determine the absolute value of total electron content (TEC) of the ionosphere by monitoring the Faraday polarization twist of the signal transmitted by geostationary satellites, a knowledge of the initial orientation of the electric field vector of the transmitted signal relative to the receiving antenna is required. This orientation called initial polarization is defined as the angle made by the projection of the electric field vector of the signal transmitted by the satellite antenna, on a plane perpendicular to the receiver-satellite ray path, with some reference direction at the receiving site, say the local horizontal, in the absence of Faraday rotation. The initial polarization is not normally measured before the launch of a satellite.

For geostationary satellites having a linearly polarized antenna, initial polarization has been determined by various workers[1-4]. But no such effort has been made for the case of those geostationary satellites which transmit either elliptical or circularly polarized signals. In this communication an attempt has been made to fill this gap.

The geometry of the coordinate systems for the satellite and the receiver is shown in Fig. 1. In order to determine initial polarization, we have to know how the electric field pattern in the satellite based coordinate system (x, y, z) is projected on to the plane $(X\ Y)$ perpendicular to the receiver-satellite ray path (Z-axis). The Y-axis is horizontal. The transformation matrix between satellite-based coordinate system, (x, y, z) and the receiver based coordinate system (X, Y, Z) is found to be as given in matrix equation (1), where $\theta = \theta_s - \theta_r$ and $\varphi = \varphi_r$ (latitude of the receiver).

Here θ_s, θ_r stand for longitude of the satellite and receiver, respectively, and γ_s and E_s are, respectively, the right ascension and elevation of the satellite as seen

$$\begin{vmatrix} X \\ Y \\ Z \end{vmatrix} = \begin{vmatrix} \sin E_s & 0 & -\cos E_s \\ 0 & 1 & 0 \\ \cos E_s & 0 & \sin E_s \end{vmatrix} \begin{vmatrix} \cos \gamma_s & \sin \gamma_s & 0 \\ -\sin \gamma_s & \cos \gamma_s & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\times \begin{vmatrix} \cos \varphi & 0 & \sin \varphi \\ 0 & 1 & 0 \\ -\sin \varphi & 0 & \cos \varphi \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & \sin \theta \\ 0 & -\sin \theta & \cos \theta \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} \quad \ldots (1)$$

from receiver end [Fig. 1(b)]. After substituting the values of γ_s and E_s which are given by

$$\gamma_s = \tan^{-1}(\tan \theta / \sin \varphi)$$

$$E_s = \tan^{-1}\left(\frac{\cos \theta \cos \varphi - K_2/6.6}{K}\right)$$

where

$$K_2 = 1/K_1 + H/R$$

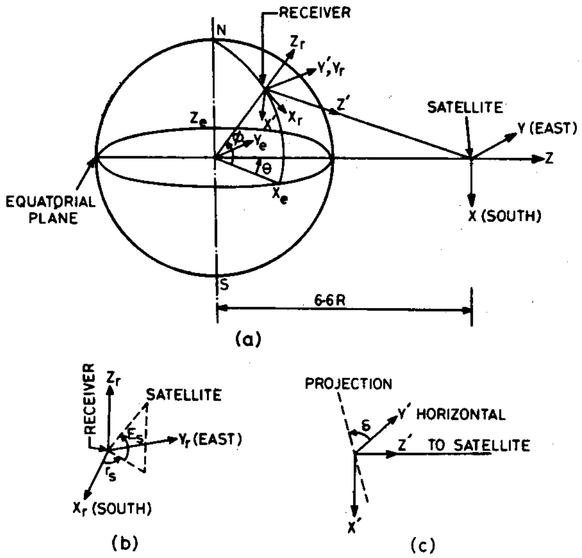

Fig. 1—Geometry of the coordinate systems for satellite and receiver for determining the initial polarization

$K_1 = [1-(2f-f^2)\sin^2\theta]^{-1}$

$K = (\cos^2\theta \sin^2\varphi + \sin^2\theta)^{1/2}$

R Equatorial radius of the earth
H Height above reference ellipsoid
f Flattening of the earth $=1/298.3$

Eq. (1) becomes

$$\begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} A_x & B_x & C_x \\ A_y & B_y & C_y \\ A_z & B_z & C_z \end{vmatrix} \begin{vmatrix} X \\ Y \\ Z \end{vmatrix}$$

where

$$A_x = \frac{R}{K\rho}(\sin\varphi)(6.6 - K_2\cos\theta\cos\varphi)$$

$$A_y = \frac{R}{K\rho}(\sin\theta\cos\varphi)(6.6 - K_2\cos\theta\cos\varphi)$$

$$A_z = -R.K.K_2/\rho$$

$$B_x = \frac{(\sin\theta\cos\varphi)}{K}$$

$$B_y = (\sin\varphi)/K$$

$$B_z = 0$$

ρ Slant range of the satellite

The parameters C_x, C_y, C_z need not be determined.

If the projection of electric field vector (which may be a straight line or major axis of an ellipse) makes an angle δ with Y-axis in the counterclockwise direction, then by rotating the coordinate system about X-axis through the angle δ in counterclockwise direction such that Y-axis becomes parallel to the projected electric field vector, the projection on the $X'Y'$ plane in the new coordinate system (X', Y', Z') can be obtained as

$$\begin{vmatrix} X' \\ Y' \\ 0 \end{vmatrix} = \begin{vmatrix} P_x & P_y & P_z \\ Q_x & Q_y & Q_z \\ C_x & C_y & C_z \end{vmatrix} \begin{vmatrix} x \\ y \\ z \end{vmatrix} \quad \ldots (2)$$

where

$$P_x = A_x\cos\delta + B_x\sin\delta$$

$$P_y = A_y\cos\delta + B_y\sin\delta$$

$$P_z = A_z\cos\delta + B_z\sin\delta$$

$$Q_x = B_x\cos\delta - A_x\sin\delta$$

$$Q_y = B_y\cos\delta - A_y\sin\delta$$

$$Q_z = B_z\cos\delta - A_z\sin\delta$$

Now for signal linearly polarized (passing through x, y, z) in satellite-based coordinate system since the projection is along Y'-axis, $X'=0$ and

$$\tan\delta = -\frac{(A_x.x + A_y.y + A_z.z)}{(B_x.x + B_y.y + B_z.z)} \quad \ldots (3)$$

For signal elliptically polarized in the satellite-based coordinate system, the general equation of the ellipsoid is given by

$$x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$$

where a, b, c have their usual meanings.

Its projection on the $X'Y'$ plane, using Eq. (2), is given by

$$(P_x^2/a^2 + P_y^2/b^2 + P_z^2/c^2)X'^2$$
$$+(Q_x^2/a^2 + Q_y^2/b^2 + Q_z^2/c^2)Y'^2$$
$$+(P_xQ_x/a^2 + P_yQ_y/b^2 + P_zQ_z/c^2)X'Y' = 0$$

which is the equation for an ellipse. Since ellipse has major axis along Y'-axis, we have

$$P_xQ_x/a^2 + P_yQ_y/b^2 + P_zQ_z/c^2 = 0$$

Substituting the values of P_x, Q_x etc., we get

$$\tan 2\delta = \frac{2C}{B-A} \quad \ldots (4)$$

where

$$A = A_x^2/a^2 + A_y^2/b^2 + A_z^2/c^2$$

$$B = B_x^2/a^2 + B_y^2/b^2 + B_z^2/c^2$$

$$C = A_xB_x/a^2 + A_yB_y/b^2 + A_zB_z/c^2$$

Ellipticity ε (ratio of major axis to minor axis) is given by

Table 1—Values of Different Parameters of Different Stations

Station	Lat. °N	Long. °E	Azimuth A_s deg	Elevation E_s deg	Initial polarization deg	Ellipticity
Bangalore	13.00	78.50	99.79	28.75	16.09	26.99
Hyderabad	17.30	78.50	103.31	28.84	21.53	20.38
Waltair	17.72	83.30	106.00	33.53	23.53	19.70
Nagpur	21.10	79.05	106.28	28.35	26.22	16.85
Calcutta	22.90	88.60	113.82	36.73	32.38	15.30
Delhi	28.63	77.22	110.00	24.12	34.17	12.79

$$\varepsilon=\left(\frac{A\cos^2\delta+B\sin^2\delta+C\sin 2\delta}{B\cos^2\delta+A\sin^2\delta-C\sin 2\delta}\right)^{1/2} \quad \ldots (5)$$

In actual practice, the signal will be either elliptically or circularly polarized in a plane. In that case the contribution from remaining axis becomes zero, e.g. when transmitted signal is elliptically polarized in xy-plane ($z=0$), then we put $A_z=B_z=0$ in Eq. (4) and (5). Further if the signal is circularly polarized, we put $a=b=c=1$ in Eqs (4) and (5).

In the case of ATS-6 satellite, the linearly polarized antenna was in the east-west direction, i.e. $x=0$, $y=1$, $z=0$. For Delhi (lat., 28.63°N; long., 77.22°E) and ATS-6 (lat., 0°N; long., 35°E), we get the value of δ from Eq. (3) as 50.7°. The values derived by Klobuchar[3], Sengupta[4] and Garg *et al.*[5] are respectively, 51.1°, 50.7° and 50.5°.

The ETS-II satellite is situated at (lat., 0°N; long., 130°E). It is transmitting signals which are circularly polarized in the yz-plane. The values of δ and ellipticity for different stations are calculated by putting $a=b=c=1$ and $A_x=B_x=0$ in Eqs (4) and (5).

Values of different parameters obtained for a number of stations are given in Table 1.

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