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Excitation of Electrostatic Cyclotron Harmonic Waves in Magnetosphere

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Wave-particle interaction in the magnetosphere has been studied for wave frequencies at the cyclotron harmonics of electrons and ions. A generalized dispersion relation for electrostatic wave, propagating nearly perpendicular to the magnetic field in non-Maxwellian plasma has been considered. Expressions of damping and growth rate for electrostatic electron and ion cyclotron harmonic (ICH) waves have been derived and computed results have been demonstrated for the magnetospheric plasma parameters. The growth of electron cyclotron harmonic waves may be interpreted in terms of the observed electron cyclotron harmonic emissions due to energetic electrons and ICH waves by energetic ions in the magnetospheric plasma. The damping of the waves may explain the heating of the magnetospheric plasma.

1 Introduction

In almost all regions of the magnetosphere, the collisionless plasma distribution functions are significantly non-Maxwellian and they are continuously unstable with respect to the generation of the electrostatic waves. These waves have been first observed by the plasma wave experiments aboard OGO 5 satellite[1-3] and later on the data from several other satellites revealed the occurrence of these waves[4-9]. Earlier theories for the generation of these waves[2,10-12] stated that the thermal component supports the waves with frequencies between the electron cyclotron harmonics, and the energetic component supplies the free energy from pitch angle anisotropy to generate emissions at these frequencies in the presence of the cold plasma. Curtis and Wu[13] have shown that the electrostatic cyclotron harmonic emissions due to energetic electrons and ions are also possible without the cold component of plasma. Excitation of ion cyclotron harmonic waves has been studied by them[14]. These waves have also been observed by GEOS-I[15] and are excited by the energetic ions present in the magnetosphere.

In the present paper, the excitation of the electrostatic electron and ion cyclotron harmonic waves has been dealt with. A generalized dispersion relation has been considered for the electrostatic waves propagating at large angles with respect to the magnetic field in a non-Maxwellian anisotropic magnetoplasma. Expressions of damping and growth rate for electrostatic cyclotron harmonic waves have been derived and computed for the magnetospheric plasma parameters. It has been shown that as the perpendicular temperature increases, the growth rate also increases[16]. For lower perpendicular temperature the excitation is possible at $(n+1)\,\Omega_e$, Ω_e being angular cyclotron frequency of the electron while $(n+1/2)\,\Omega_e$ excitations are occurring at higher perpendicular temperature. The range of frequency for the generation of electron cyclotron harmonics (ECH) is specially indicated from $(n+1/2)\Omega_e$ to $(n+1)\Omega_e$ depending on the anisotropy factor. These excitations are interpreted in terms of the observed ECH emissions in the magnetosphere[9,11]. Since these waves are slow and short lived, they are absorbed by the particles and the plasma gets heated[17] (under energy transfer condition, i.e. the thermal velocity of the particles is less than the phase velocity of the wave). These emissions, therefore, demonstrate the heating of the magnetospheric plasma[18].

A similar study has also been carried out for the damping and the growth rate of the electrostatic ion cyclotron harmonic (ICH) waves. The damping of these waves at the ion cyclotron harmonic frequencies indicates the heating of the ions[19]. The growth rate of the waves has been suggested as one of the generation mechanism of ICH waves in the magnetosphere as observed by GEOS-I[15].

2 Theoretical Analysis

The dispersion relation for electrostatic waves propagating nearly perpendicular to the magnetic field for an anisotropic non-Maxwellian velocity distribution has been given by Stix[20] as follows.

$$1+\sum_{s}\sum_{n=-\infty}^{\infty}\frac{2\omega_{ps}^2}{k^2V_{T_{\perp s}}^2}I_n(\lambda_{\perp s})\exp(-\lambda_{\perp s})$$

$$\times\left[\left(\frac{T_\perp}{T_\parallel}\right)_s+i\left\{\frac{\omega+n\Omega_s}{k_\parallel V_{T_{\parallel s}}}\left(\frac{T_\perp}{T_\parallel}\right)_s\right.\right.$$

$$\left.\left.-\frac{n\Omega_s}{k_\parallel V_{T_{\parallel s}}}\right\}F_0(\alpha_{ns})\right]=0 \qquad \ldots (1)$$

where

$$F_0(\alpha_{ns})=\sqrt{\pi}\frac{k_\parallel}{|k_\parallel|}\exp(-\alpha_{ns}^2)+2i(\alpha_{ns})$$

$$S(\alpha_{ns})=\exp(-\alpha_{ns}^2)\int_0^{\alpha_{ns}}\exp(t^2)\mathrm{d}t$$

and

$k^2=k_\perp^2+k_\parallel^2$ Propagation vector
$\omega_{ps}=(N_e^2/\varepsilon_0 m_s)^{1/2}$ Plasma frequency
$\Omega_s=(e\beta_0/m_s)$ Angular cyclotron frequency
$V_{T_{\perp s}}=(2T_{\perp s}/m_s)^{1/2}$ Thermal velocity across the magnetic field
$V_{T_{\parallel s}}=(2T_{\parallel s}/m_s)^{1/2}$ Thermal velocity along the magnetic field
V_{ds} Drift velocity
$T_{\perp s}$ Thermal energy across the magnetic field
$T_{\parallel s}$ Thermal energy along the magnetic field

$$\lambda_{\perp s}=(k_\perp V_{T_{\perp s}}/\sqrt{2}\Omega_s)^2=\frac{1}{2}k_\perp^2\rho_{\perp s}^2$$

$$\lambda_{\parallel s}=(k_\parallel V_{T_{\parallel s}}/\sqrt{2}\Omega_s)^2=\frac{1}{2}k_\parallel^2\rho_{\parallel s}^2$$

$$\alpha_{ns}=\frac{\omega-n\Omega_s-k_\parallel V_{ds}}{k_\parallel V_{T_{\parallel s}}}$$

S Species of the particles
$\perp$ and $\parallel$ Directions perpendicular and parallel to the background magnetic field
I_n Modified Bessels function of nth order

The Eq. (1) for two component plasma, i.e. $s(e, i)$ can be rewritten as

$$1+\sum_{n=-\infty}^{\infty}\frac{2\omega_{pe}^2}{k^2V_{T_{\perp e}}^2}I_n(\lambda_{\perp e})\exp(-\lambda_{\perp e})A_e$$

$$+\sum_{m=-\infty}^{\infty}\frac{2\omega_{pi}^2}{k^2V_{T_{\perp i}}^2}I_m(\lambda_{\perp i})\exp(-\lambda_{\perp i})A_i \qquad \ldots (2)$$

where

$$A_e=\left[\left(\frac{T_\perp}{T_\parallel}\right)_e-2S(\alpha_{ne})\left\{\alpha_{ne}\left(\frac{T_\perp}{T_\parallel}\right)_e\right.\right.$$

$$\left.-\frac{n\Omega_e}{k_\parallel V_{T_{\parallel e}}}\right\}+i\sqrt{\pi}\frac{k_\parallel}{|k_\parallel|}\{\exp(-\alpha_{ne}^2)\}$$

$$\left.\times\left\{\alpha_{ne}\left(\frac{T_\perp}{T_\parallel}\right)_e-\frac{n\Omega_e}{k_\parallel V_{T_{\parallel e}}}\right\}\right]$$

$$A_i=\left[\left(\frac{T_\perp}{T_\parallel}\right)_i-2S(\alpha_{mi})\left\{\alpha_{mi}\left(\frac{T_\perp}{T_\parallel}\right)_i\right.\right.$$

$$\left.-\frac{m\Omega_i}{k_\parallel V_{T_{\parallel i}}}\right\}+i\sqrt{\pi}\frac{k_\parallel}{|k_\parallel|}\{\exp(-\alpha_{mi}^2)\}$$

$$\left.\times\left\{\alpha_{mi}\left(\frac{T_\perp}{T_\parallel}\right)_i-\frac{m\Omega_i}{k_\parallel V_{T_{\parallel i}}}\right\}\right]$$

and, n, m are the harmonic numbers of electron and ion.

In the above equation, $S(\alpha_{n,m})$ is related with the plasma dispersion function[21]. The expansions of $S(\alpha_{n,m})$ for large and small argument limits are expressed by asymptotic and power series as

$$S(\alpha_{n,m})=\frac{1}{2\alpha_{n,m}}\left(1+\frac{1}{2\alpha_{n,m}^2}+\frac{3}{4\alpha_{n,m}^4}+\ldots\right)$$

$$\text{for } \alpha_{n,m}\gg 1 \qquad \ldots (3)$$

$$S(\alpha_{n,m})=\left(\alpha_{n,m}-\frac{2}{3}\alpha_{n,m}^3+\ldots\right)$$

$$\text{for } \alpha_{n,m}\ll 1 \qquad \ldots (4)$$

The dispersion relation [Eq. (2)] for the asymptotic expansion of $S(\alpha_{n,m})$, where $\alpha_{ne}\gg 1$ and $\alpha_{mi}>1$, neglecting the higher power terms is written as

$$1+\sum_{n=-\infty}^{\infty}\frac{2\omega_{pe}^2}{k^2V_{T_{\perp e}}^2}I_n(\lambda_{\perp e})\{\exp(-\lambda_{\perp e})\}B_e$$

$$+\sum_{m=-\infty}^{\infty}\frac{2\omega_{pi}^2}{k^2V_{T_{\perp i}}^2}I_m(\lambda_{\perp i})\{\exp(-\lambda_{\perp i})\}B_i \qquad \ldots (5)$$

where

$$B_e = \left[\left\{\frac{n}{(x_e - n - \lambda_{de})} - \frac{1}{2}\left(\frac{T_\perp}{T_\parallel}\right)_e \frac{\lambda_{\parallel e}}{(x_e - n - \lambda_{de})^2}\right\}\right.$$

$$+ i\sqrt{\pi}\{\exp(-\alpha_{ne}^2)\}\frac{\Omega_e}{k_\parallel V_{T_{\parallel e}}}$$

$$\left.\times\left\{(x_e - n - \lambda_{de})\left(\frac{T_\perp}{T_\parallel}\right)_e - n\right\}\right]$$

$$B_i = \left[\left\{\frac{m}{(x_i - m - \lambda_{di})} - \frac{1}{2}\left(\frac{T_\perp}{T_\parallel}\right)_i \frac{\lambda_{\parallel i}}{(x_i - m - \lambda_{di})^2}\right\}\right.$$

$$+ i\sqrt{\pi}\{\exp(-\alpha_{mi}^2)\}\frac{\Omega_i}{k_\parallel V_{T_{\parallel i}}}$$

$$\left.\times\left\{(x_i - m - \lambda_{di})\left(\frac{T_\perp}{T_\parallel}\right)_i - m\right\}\right]$$

and

$x_e = \omega/\Omega_e;\ x_i = \omega/\Omega_i$

$\lambda_{de} = k_\parallel V_{de}/\Omega_e = k_\parallel \rho_{de}$

$\lambda_{di} = k_\parallel V_{di}/\Omega_i = k_\parallel \rho_{di}$

Following the procedure adopted by Gwal and Misra[17], the imaginary part of the dispersion Eq. (5) may be given as

$$\left.\frac{\chi_s}{\Omega_s}\right|_{s=e,i} = \left(\frac{\pi}{2}\right)^{1/2}\frac{(A_1 + B_1)}{C_1} \qquad \ldots (6)$$

where

$$A_1 = \frac{1}{\sqrt{\lambda_{\parallel e}}}\{\exp(-\alpha_{ne}^2)\}\, I_n(\lambda_{\perp e})\{\exp(-\lambda_{\perp e})\}$$

$$\times\left[(x_e - n - \lambda_{de})\left(\frac{T_\perp}{T_\parallel}\right)_e - n\right]$$

$$B_1 = \frac{1}{\sqrt{\lambda_{\parallel i}}}\{\exp(-\alpha_{mi}^2)\}\, I_m(\lambda_{\perp i})\{\exp(-\lambda_{\perp i})\}$$

$$\times\left[(x_i - m - \lambda_{di})\left(\frac{T_\perp}{T_\parallel}\right)_i - m\right]$$

$$C_1 = I_n(\lambda_{\perp e})\{\exp(-\lambda_{\perp e})\}\chi_1 + M_1\gamma_1 I_m(\lambda_{\perp i})\exp(-\lambda_{\perp i})$$

$$M_1 = \frac{\omega_{pi}^2}{\omega_{pe}^2}\left(\frac{V_{T_{\perp e}}^2}{V_{T_{\perp i}}^2}\right)\frac{W_e}{\Omega_i}$$

$$\chi_1 = \frac{n}{(x_e - n - \lambda_{de})^2} + 2\left(\frac{T_\perp}{T_\parallel}\right)_e \frac{\lambda_{\parallel e}}{(x_e - n - \lambda_{de})^3}$$

$$\gamma_1 = \frac{m}{(x_i - m - \lambda_{di})^2} + 2\left(\frac{T_\perp}{T_\parallel}\right)_i \frac{\lambda_{\parallel i}}{(x_i - m - \lambda_{di})^3}$$

The Eq. (6) gives the damping of the electrostatic cyclotron harmonic waves. Under energy transfer conditions, when the thermal velocity of particles is greater than the phase velocity of the waves, these waves get growing. In order to obtain the growth of the waves, the power series expansion of Eq. (4), neglecting the higher power terms, is considered for $S(\alpha_{n,m})$ in the dispersion Eq. (2). Following the earlier procedure[17], the expression for the growth rate may be written as

$$\left.\frac{\gamma_s}{\Omega_s}\right|_{s=e,i} = -\left(\frac{\pi}{2}\right)^{1/2}\frac{(A_1 + B_1)}{D_1} \qquad \ldots (7)$$

where

$$D_1 = I_n(\lambda_{\perp e})\{\exp(-\lambda_{\perp e})\}\chi_2 + M_1\gamma_2 I_m(\lambda_{\perp i})\exp(-\lambda_{\perp i})$$

$$\chi_2 = \frac{1}{\lambda_{\parallel e}}\left[n - 2(x_e - n - \lambda_{de})\left(\frac{T_\perp}{T_\parallel}\right)_e - \frac{n(x_e - n - \lambda_{de})^2}{\lambda_{\parallel e}}\right]$$

$$\gamma_2 = \frac{1}{\lambda_{\parallel i}}\left[m - 2(x_i - m - \lambda_{di})\left(\frac{T_\perp}{T_\parallel}\right)_i - \frac{m(x_i - m - \lambda_{di})^2}{\lambda_{\parallel i}}\right]$$

Eqs (6) and (7) are the generalized expressions for the damping and growth of the electrostatic electron and ion cyclotron harmonic waves. Computations have been carried out for the damping and growth of these waves for the magnetospheric plasma parameters.

3 Electrostatic Electron Cyclotron Harmonic Waves

The mode of propagation for the case $k_\parallel = 0$ has been suggested for the observations of electrostatic cyclotron emissions in the earth's magnetosphere[4]. It is reported that the electron cyclotron waves have been generated at $(n + 1/2)$ harmonics. The inclusion of non-zero $k_\parallel$ is to extend the range in frequency over which the growth rate is possible[22]. For the case of non-zero $k_{\parallel}$, the damping and the growth rate of electrostatic ECH has been computed assuming $\omega > \Omega_e$, $\omega \gg \Omega_i$ and $V_{de} \neq 0$ in Eqs (6) and (7). The damping of ECH waves has been shown in Fig. 1 for different values of $(T_{\perp}/T_{\parallel})_e$. For larger temperature anisotropy of electrons, the damping is increased while it is decreasing for higher harmonics as shown in Fig. 2. The maximum damping occurs at $(n+1/2)\Omega_e$ for $k_{\parallel}\rho_{de}=0$ [from Eq. (6)], but for drift velocity distribution of electrons, i.e. $k_{\parallel}\rho_{de} \neq 0$, the maximum damping occurs at higher half harmonics of electron cyclotron frequencies as shown in Fig. 2. As a result of the damping of the ECH waves, the plasma gets heated up[18]. The heating process is controlled by the perpendicular electron energy. Since an adiabatic increase of the magnetic field strength could produce high $T_{\perp}$ electrons, therefore, such magnetospheric electrons resonating with the propagating ECH wave having larger phase velocity as compared with the thermal velocity of the electrons may get energy from ECH wave and the wave is absorbed. Under the conditions, where the thermal velocity of electrons is larger than the phase velocity of the ECH wave, the waves start growing. The growth rate of the ECH waves has been computed for first and second harmonics and plotted in Fig. 3 for different values of temperature anisotropy of electrons. The range of $k_{\parallel}\rho_{de}$ decides the occurrence of the damping and the growth rate of the ECH rate. The frequency range of the occurrence of the resonant growth rate is ranging from $(n+1)\Omega_e$ for lower temperature anisotropy to $(n+1/2)\Omega_e$ for higher temperature anisotropy in electrons. The growth rate is increasing with the increase of the temperature anisotropy [Fig. (3)]. This result shows that the ECH waves are generated at $(n+1/2)\Omega_e$ and $(n+1)\Omega_e$ and have dependence on the temperature anisotropy, which shows that the excess electron kinetic energy perpendicular to the magnetic field is responsible for the generation of the ECH emissions in the magnetosphere[16].

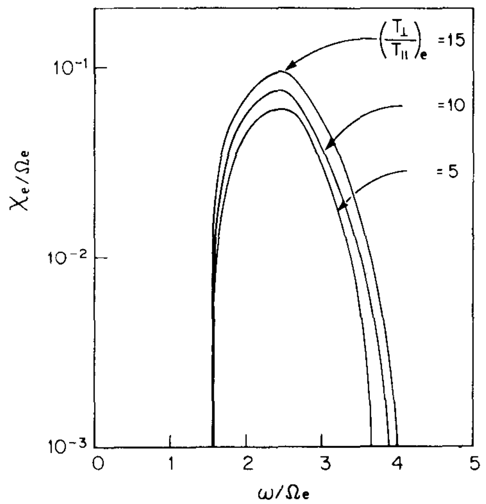

Fig. 1—Damping of ECH waves for different values of $(T_{\perp}/T_{\parallel})_e$ where $k_{\perp}\rho_{\perp e}=10$, $k_{\parallel}\rho_{\parallel e}=1$, $n=1$ and $k_{\parallel}\rho_{de}=0.5$

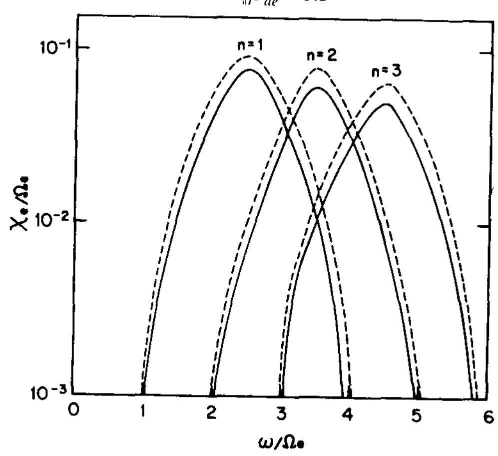

Fig. 2—Damping of ECH waves at different harmonics, where $k_{\perp}\rho_{\perp e}=10$, $k_{\parallel}\rho_{\parallel e}=1$ and $k_{\parallel}\rho_{de}=0.5$ [Solid curve is for $(T_{\perp}/T_{\parallel})_e=10$ and dashed curve is for $(T_{\perp}/T_{\parallel})_e=50$.]

4 Electrostatic Ion Cyclotron Harmonic Waves

The wide band frequency measurements discussed by Gurnett[23] reveal a complicated cyclotron harmonic spectrum of lines whose spacing suggests that cyclotron harmonics of several different ion species are superposed in the spectrum and represent emissions generated by energetic ion species existing in the plasmasphere. Later on Curtis and Wu[14] investigated the types of waves that are excited by the observed energetic ions and supported by the background plasmaspheric plasma. In the present

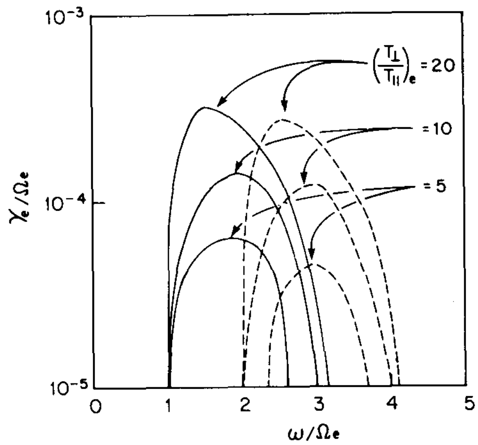

Fig. 3—Growth rate of ECH waves for different values of $(T_{\perp}/T_{\parallel})_e$ where $k_{\perp}\rho_{\perp e}=10$, $k_{\parallel}\rho_{\parallel e}=1$ and $k_{\parallel}\rho_{de}=0.5$ (Solid curve is for $n=1$ and dashed curve is for $n=2$.)

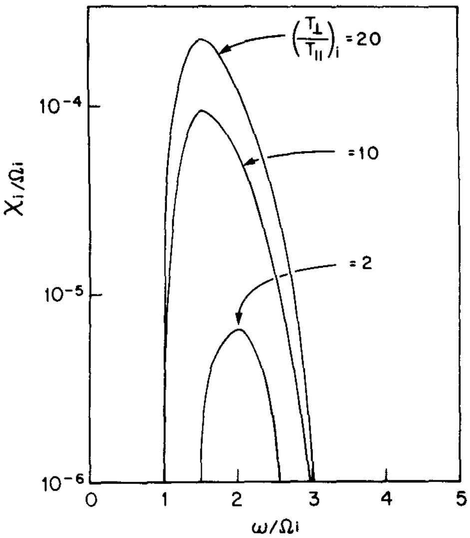

Fig. 4—Damping of ICH waves for different values of $(T_\perp/T_\parallel)_i$ where $k_\perp \rho_{\perp i} = 7$ and $k_\parallel \rho_{\parallel i} = 0.7$

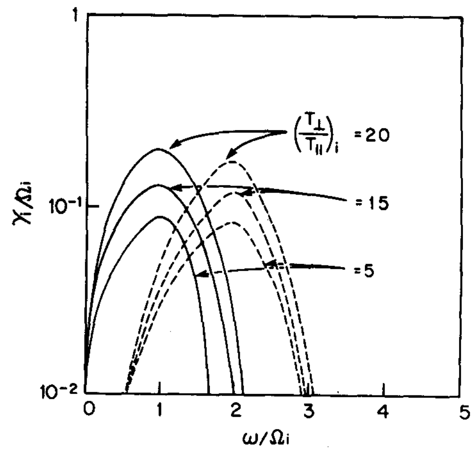

Fig. 5—Growth rate of ICH waves for different values of $(T_\perp/T_\parallel)_i$ where $k_\perp \rho_{\perp i} = 10$ and $k_\parallel \rho_{\parallel i} = 1$ (Solid curve is for $m = 1$ and dashed curve is for $m = 2$.)

analysis, the damping and the growth of the ion cyclotron harmonic (ICH) waves have been considered. Assuming the frequency range from $\omega > \Omega_i$ to $\omega \ll \Omega_e$ in Eqs (6) and (7), the damping of ICH waves has been computed for $k_\perp \rho_{\perp i} = 7$ and $k_\parallel \rho_{\parallel i} = 0.7$. Fig. 4 demonstrates the variation of the damping of the ICH waves with ω/Ω_i for different values of the temperature anisotropy due to ions. The resonant damping occurs at $(n+1)\Omega_i$ for lower value of the temperature anisotropy, while for higher temperature anisotropy the frequency range decreases to $(n+1/2)\Omega_i$. The damping also increases with the increase of the temperature anisotropy due to ions, i.e. as we increase the transverse energy of ions the damping increases. This result may be interpreted in terms of the heating of the ions to superthermal energy by the electrostatic ion cyclotron waves[19].

Under the energy transfer condition[17], the ICH waves, instead of being absorbed, start growing. The growth rate of ICH waves has been shown in Fig. 5 for different values of temperature anisotropy due to ions. The growth of the waves at the harmonics of the ion cyclotron frequency may be interpreted in terms of the observed ICH emission by GEOS-I[15].

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