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Effect of Magnetic Field-aligned Currents on VLF Emissions in the Magnetosphere

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The dispersion relation for the electromagnetic electron cyclotron waves in the presence of magnetic field-aligned currents has been obtained. The kinetic distribution of electrons for the main body of plasma with a temperature anisotropy and a loss cone distribution have been considered. In general, it has been seen that the current moving along the direction of resonant electrons reduce the growth rate. This effect has been analysed in the case of magnetospheric plasma to suggest possible correlations between the Birkeland currents and the emissions of very low frequency (VLF) electromagnetic waves.

1 Introduction

Among the global problems in the magnetospheric plasma physics, the study of magnetic field-aligned currents and magnetospheric and ionospheric interactions are of particular interest[1]. Magnetospheric field-aligned currents (Birkeland currents) link the interplanetary medium with the Earth's upper atmosphere[2]. Observed by rockets and satellites, they flow from the magnetopause and magnetotail into the auroral ionosphere, where large currents flow in the electrojet especially during magnetic storms. Birkeland currents also show large scale features and are related to both visual auroral arcs[3,4] and radar aurora[5]. Energetic particle bursts have been observed aboard IMP 8; these bursts seem to be correlated with Birkeland currents in the geomagnetic tails[6].

There are simultaneous observations of the Birkeland currents in the northern and southern auroral regions. It is seen that the currents flowing upward in the northern cusp have a counter part flowing downward in the southern cusp and vice versa[7]. The resonant geomagnetic field oscillations observed by the Viking satellite in the polar orbit and the AMPTE/CCE satellite in the equatorial orbit suggest that the Birkeland currents flow along the geomagnetic field lines up to a distance of 8.8 R_e, where R_e is the Earth's radius (Ref. 8). Based on these observations, we have assumed that the magnetic field-aligned currents flow between northern and southern hemispheres along the closed magnetic field lines, since the auroral zones also come within this region.

The resonant interaction between energetic particles and whistler mode waves is responsible for the VLF emissions. This has been studied to explain the features of VLF emissions[9]. The non-linear effects have also been included to explain frequency time structures[10,11]. The emission phenomena have also been studied in different situations and conditions occurring in the Earth's magnetosphere[12-14]. However, phenomena associated with the field-aligned currents have not been analysed completely.

In this paper, the effect of field-aligned currents on the VLF emissions has been analysed with the help of dispersion relation. The growth rate of these waves, due to the temperature anisotropy and due to the loss cone distribution of energetic particles, has been calculated. It has been shown that field-aligned currents flowing in the direction opposite to the propagation of whistler mode waves reduce the growth rate. As an example, we have calculated the current needed to suppress completely the growth at a given frequency.

2 Dispersion relation

We assume that the background plasma is uniform and homogeneous with density n_{om} (the subscript m denotes the main plasma and o denotes the equilibrium value) in the presence of uniform magnetic field B_0 along Z-axis. The wave is propagating along the magnetic field and its frequency range is $\omega_{pi}, \Omega_i < \omega < \Omega_e, \omega_{pe}$ (ω is the wave frequency; Ω and ω_p are gyro and plasma frequency; i and e denote ion and electron parameters, respectively). The ion, being massive, provides a charge neutralizing background. Using the perturbation theory and considering electrons only, the dispersion relation is given by[15]

$$c^2k^2/\omega^2 = 1 - \omega_p^2/\omega^2 \times \int_{-\infty}^{\infty} \mathrm{d}V_\parallel \int_{-\infty}^{\infty} \frac{V_\perp^2 \ \mathrm{d}V_\perp \ [A+B]}{(kV_\parallel - \omega + \Omega)} \qquad \ldots(1)$$

where

$$A = (1 - kV_\parallel/\omega)\partial f_0/\partial V_\perp$$

and

$$B = (kV_\perp/\omega)(\partial f_0/\partial V_\parallel)$$

The equilibrium distribution function f_0 is given by, $f_0 = f_{om} + f_{ob}$, where f_{om} is the distribution supporting the wave propagation and f_{ob} is that of the current carrying electrons. Both f_{om} and f_{ob} are normalized to unity. V is the velocity; $\parallel$ and $\perp$ denote components parallel and perpendicular to the magnetic field; and k, ω and c have standard meanings.

The distribution function f_{om} is chosen to be bi-Maxwellian in one case and is given by

$$f_{om} = (m/2\pi T_\parallel)^{1/2}(m/2T_\perp) \times \exp(-mV_\parallel^2/2T_\parallel - mV_\perp^2/2T_\perp) \qquad \ldots(2a)$$

and it is a loss cone distribution in second case and is given by

$$f_{om} = (\pi^{1/2}A_\parallel)^{-3}\exp\left(-\frac{V_\parallel^2}{A_\parallel^2}\right) \times \left\{\frac{\Delta}{\alpha}\exp\left(-\frac{V_\perp^2}{\alpha A_\parallel^2}\right) + \frac{(1-\Delta)}{(\alpha-\beta)}\left[\exp\left(-\frac{V_\perp^2}{\alpha A_\parallel^2}\right) - \exp\left(-\frac{V_\perp^2}{\beta A_\parallel^2}\right)\right]\right\} \qquad \ldots(2b)$$

In Eq. (2a), T is the temperature in energy units, with the components as defined earlier. In Eq. (2b), $A_\parallel$ is the parallel thermal velocity, α is the thermal anisotropy, and Δ and β determine respectively the depth and size of loss cone.

For the beam of electrons of density n_{ob} moving along B_0 with velocity V_b, we take

$$f_{ob} = \{m/(2\pi T_b)\}^{3/2} \times \exp[-m(V_\parallel - V_b^2)/2T_b - mV_\perp^2/2T_b] \ldots(3)$$

This distribution is such that $n_{ob}/n_{om} < 1$ where subscript b denotes the current parameters and T_b is the temperature of current carrying electrons.

We follow the Landau prescription for the $V_\parallel$ integration, thereby accounting for the singularity occurring at $V_\parallel = (\omega - \Omega)/k$. This indicates Doppler shifted resonance between whistler mode waves and electrons moving in opposite direction along the Z-axis, i.e., k is negative for positive $V_\parallel$. We write

$$\int_{-\infty}^{\infty} \cdots \mathrm{d}V_\parallel = P\int_{-\infty}^{\infty} \cdots \mathrm{d}V_\parallel + \pi i R \qquad \ldots(4)$$

where $i = \sqrt{-1}$, P is the principal part and R the residue. The principal part is obtained by expanding

$$(kV_\parallel - \omega + \Omega)^{-1} = (\Omega - \omega)^{-1}\left[1 - \frac{kV_\parallel}{(\Omega-\omega)} + \frac{k^2V_\parallel^2}{(\Omega-\omega)^2} - \ldots\right]$$

which is valid when $V_\parallel < |(\Omega - \omega)/k|$. On evaluation of the integral of Eq. (4), the term containing $V_\parallel$ and its powers change the real part of the dispersion relation by adding thermal corrections.

Neglecting the thermal corrections, i.e. away from $\omega = \Omega$, the dispersion relation becomes

$$c^2k^2/\omega^2 = 1 - \frac{\omega_{pm}^2}{\omega(\omega-\Omega)} + \frac{\omega_{pb}^2(\omega - kV_b)}{\omega^2(kV_b - \omega + \Omega)} + \pi i(R_m + R_b) \qquad \ldots(5)$$

where ω_{pm} and ω_{pb} are electron plasma frequencies corresponding to n_{om} and n_{ob}, respectively.

The term V_b appears in Eq. (5) because the current carrying electrons add an additional Doppler shift in frequency. However, the current introduced by such electrons in deriving the dispersion relation should be corrected for calculations in the laboratory frame, where Eq. (5) is derived.

The residues R_m and R_b can be obtained in a straightforward manner which for Eq. (2a) is

$$R_m = \omega_{pm}^2(\omega k)^{-1}[1 - (\Omega - \omega)(T_\parallel - T_\perp)/(\omega T_\parallel)]f_0(V_{Rm}) \qquad \ldots(6a)$$

and for Eq. (2b)

$$R_m = \omega_{pm}^2(\omega k)^{-1}[1 - (\Omega - \omega)\{(1-\alpha) - (1-\Delta)\beta\}\omega^{-1}]f_0(V_{Rm}) \qquad \ldots(6b)$$

R_b for the distribution (3) is given by

$$R_b = \omega_{pb}^2(\omega - kV_b)\omega^{-2}k^{-1}f_0(V_{Rb}) \qquad \ldots(7)$$

Here $V_{Rm} = |(\omega - \Omega)/k|$ in Eqs (6a) and (6b) and $V_{Rb} = |(\omega - \Omega - kV_b)/k|$ in Eq. (7). In Eq. (7) we see that $V_{Rb} = V_{Rm} - V_b$ because the poles are also Doppler shifted. $f_0(V_{Rm})$ and $f_0(V_{Rb})$ represent the parts of the distribution function with which the whistler mode waves are resonant.

Eq. (5) can be written as $D(\omega, k) = 0$. Taking ω to be complex, i.e. $\omega = \omega_r + i\gamma$, $D(\omega, k)$ is expanded in a Taylor series around the real part of the frequency, $\omega = \omega_r$, for $\gamma/\omega_r < 1$. Thus

$$D(\omega, k) = D(\omega_r, k) + (\omega - \omega_r)\partial D/\partial\omega|_{\omega=\omega_r} + \dots \quad \dots(8)$$

Equating the real parts of Eqs (5) and (8) and putting $\omega = \omega_r$, we get

$$D(\omega, k) = c^2 k^2 \omega^{-2} - 1 + \omega_{pm}^2 \omega^{-1} (\omega - \Omega)^{-1} - \omega_{pb}^2 (\omega - k V_b)\omega^{-2}(k V_b - \omega + \Omega)^{-1} \quad \dots(9)$$

Similarly, the growth rate γ is obtained by equating the imaginary parts

$$\gamma = \pi(R_m + R_b)/|\partial D/\partial\omega|_{\omega=\omega_r} \quad \dots(10)$$

The value of γ for the distributions (2a) and (3) is

$$\gamma = -\pi\omega(\omega - \Omega)^2 k^{-1}\Omega^{-1} \times [\{(1 - (\Omega - \omega)(T_\parallel - T_\perp)\omega^{-1} T_\parallel^{-1}\} f_0(V_{Rm}) + n_{ob} n_{om}^{-1} (1 - k V_b \omega^{-1}) f_{ob}(V_{Rb})] \quad \dots(11)$$

Similarly, for Eqs (2b) and (3), γ is found to be

$$\gamma = -\pi\omega(\omega - \Omega)^2 k^{-1}\Omega^{-1}\{[1 - (\Omega - \omega)\omega^{-1}\{(1 - \alpha) - (1 - \Delta)\beta\}] f_0(V_{Rm}) + n_{ob} n_{om}^{-1} \times (1 - k V_b \omega^{-1}) f_0(V_{Rb})\} \quad \dots(12)$$

The relation (11) reduces to the relation (16) of Ref. 16 when $V_b < \omega/k$. This is true in a beam plasma system, when the velocity V_b is much smaller than the phase velocity of propagating waves; the beam plasma behaves as a part of the main background plasma. Eq. (11) reduces to the growth rate used by Kennel and Petschek[9] when the beam current is absent.

3 Discussion

The sign of γ depends on the sign of the terms inside the curly bracket in Eqs (11) and (12). In the case of Eq. (11) it depends on the sign of $(T_\parallel - T_\perp)(\Omega - \omega)T_\parallel^{-1}\omega^{-1}$. In the absence of beam current, the frequencies satisfying the condition $(T_\parallel - T_\perp)(\Omega - \omega)T_\parallel^{-1}\omega^{-1} < 1$ have positive growth rate. The term containing the beam current is such that it always leads to the damping.

Therefore, with the effect of beam current it is possible to have a reduced growth rate. This will be true in the presence of field-aligned currents. As an example, we can calculate the current density needed to suppress the emissions completely.

Energetic particles in the magnetosphere form an extended tail with either a biMaxwellian distribution or a loss cone distribution, which is favourable for triggering the VLF emissions. Since the magnetic field and the plasma density are assumed to be homogeneous and uniform, the linear theory presented here is applicable to the equatorial region which is the prime region of emissions of these waves.

The electron current density which is required for the zero net growth at a given frequency using Eq. (11) is

$$n_{ob} f_0(V_{Rb}) = n_{bc} = [(\Omega - \omega)\omega^{-1} (T_\perp - T_\parallel)T_\parallel^{-1} - 1] \times \omega(\omega - k V_b)^{-1} n_{om} f_0(V_{Rm}) \quad \dots(13)$$

and using Eq. (12) is

$$n_{ob} f_0(V_{Rb}) = n_{bc} = [(\Omega - \omega)\omega^{-1}\{1 - \alpha - (1 - \Delta)\beta\} - 1] \times \omega(\omega - k V_b)^{-1} n_{om} f_0(V_{Rm}) \quad \dots(14)$$

Here we can put $n_{ob} f_0(V_{Rb})$ as n_h, the fraction of the hot electrons forming the tail of the distribution in the main plasma.

In the magnetosphere, at a distance of 4 R_e and for a typical density $n_{om} \approx 10\,\mathrm{cm}^{-3}$, the resonant velocity is in keV energy range. This is the typical energy of resonant electrons. Knowing V_{Rb} and V_{Rm}, the calculated current density needed to suppress the emissions completely at $\omega = \Omega/2$ will be of the order

$$J = 10^{-2}\,\mu\mathrm{A\,m}^{-2} \quad \dots(15)$$

This current density is a fraction of the Birkeland current which is about 1 mA m^{-2}. The thermal energy of these electrons is around 100 eV (Refs 17 and 18).

From the resonant condition, we also see that current flowing in the direction opposite to that of the VLF waves is responsible for the phenomena discussed in this paper. In the regions where the flow of current is in the direction as well as opposite to the waves, the electrons moving along the wave will have positive Doppler shift of resonant frequency; such electrons do not undergo cyclotron resonance and growth rate is not affected.

The theory presented here is not applicable to the auroral hiss emissions. The main reasons are: (a) the hiss is observed at frequencies far below $\Omega/2$. Apart from increase in the resonant energy, this energy is far above the average energy of current carrying electrons, and (b) in the auroral regions, the electrons have frequent collisions, which reduce the temperature anisotropy. The whistler instability is weak. The electromagnetic emissions are explained as the Cerenkov phenomena. Direct emissions are again weak and, therefore, the Cerenkov instability is used[19].

There is also an increase in the growth rate due to increase in density by plasma movement[12] and also due to increase in the energetic particle density[13]. There is possibility of reduction in growth rate due to plasma drift[14]. All these processes along with the effect of field-aligned currents need to be analysed.

From the known values of Birkeland currents, we find that the number density is comparable to the background density but with higher average energy. Therefore, we expect that the Birkeland currents will affect the VLF emissions.

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