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Propagation of Transionospheric Radio Beam Waves through a Turbulent Layer

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The problem of transionospheric radio waves propagating through a turbulent layer has been studied and analytical expressions for beam-wave wander and spread have been derived with the power-law spectrum of ionospheric irregularities by using phase screen model. The beam-wave spread has been found to be dependent, among other parameters, on the spectral index of the power-law spectrum. Some numerical computations have been carried out to estimate the possible beam-wave spread for SROSS.

1 Introduction

Over the years, radio physicists have been concerned with the effects of random media on the propagation of beam waves in view of their application in light communication and remote sensing[1-3]. This is confined to the problem of light waves propagating in an extended turbulent atmosphere. Because of scattering of light beam by random fluctuations of refractive index, there will be a spreading of the light beam with a corresponding decrease in beam intensity. In addition, there will be scintillations of the beam intensity leading to a decrease in the spatial and temporal coherence. These effects on radio waves can seriously degrade the performance of radar system.

Recently, the transionospheric radio beam waves have found increasing applications in satellite communication and transionospheric radars in regard to antenna pointing and its resolving power. Furthermore, a plan of the Stretched Rohini Orbiting Satellite Series (SROSS) has been proposed by ISRO and is to be launched in 1987-1988 time frame. The mission is jointly being participated by PRL, Ahmedabad, and NPL, New Delhi, for carrying out experiments on remote sensing, aeronomy, and energetics of the low latitude ionosphere[4]. Although the operating frequencies for these applications are in the microwave range (the suggested frequency for SROSS is 2.245 GHz), the effects of ionospheric irregularities may still be appreciable. The existing GHz scintillation data are limited to frequency bands between 1.6 GHz and 6 GHz. The scintillation is much stronger in the equatorial region. It has been reported that in the equatorial regions, the L-band transionospheric signal may undergo more than 20 dB peak-to-peak[5] fluctuations, and 7 dB peak-to-peak fluctuations at 4 GHz have been observed in mid-latitude observations[6]. These observations necessitate the study of the effects of the ionosphere on the propagation of radio waves in GHz band. There are several methods being employed to analyse the problem of beam-wave broadening, including the extended Huygens-Fresnel principle[7], the transport method[8], and the method of Markovian process approximation[9].

In this paper, we have attempted to investigate the transionospheric propagation of a narrow beam of radio waves with a particular reference to SROSS. We have applied stochastic approach analogous to the problems of pulsed wave propagation where temporal moments are introduced[10-12]. We have used the thin phase screen model. For the ionospheric and interplanetary radio wave propagation, the phase screen method[13-17] has proved to be quite successful in interpreting observational data such as scintillation index, intensity correlation, and power spectrum of intensity fluctuations.

Based on the spatial moments and the phase screen, we have derived the expressions for beam-wave wandering and beam-wave spreading and presented the estimates of beam-wave spread, choosing the typical ionospheric input values valid for SROSS.

2 Beam Wandering and Spreading

The electromagnetic wave propagation in a random medium is governed by the Helmholtz wave equation

$$\nabla^2 E + k^2(1+\beta\xi)E = 0 \qquad \ldots(1)$$

where $\beta\xi$ represents the random part of the relative permittivity, and k is the wave number; the depolarization effect has been neglected. In a turbulent plasma such as perturbed ionosphere with fluctuating electron density ΔN, we have

$$\beta = -\frac{f_p^2}{f^2 - f_p^2}, \quad \xi(\mathbf{r}) = \frac{\Delta N(\mathbf{r})}{N_0} \qquad \ldots(2)$$

where f_p is the plasma frequency of the background ionosphere and N_0 its density. Under the parabolic equation approximation, the complex amplitude $u(z, \boldsymbol{\rho})$ of a wave $E = u \exp(-jkz)$ propagating along z-axis satisfies the parabolic equation[18]

$$-2jk\frac{\partial u}{\partial z} + \nabla^2 u + k^2 \beta \xi(z, \boldsymbol{\rho}) u = 0 \qquad \ldots(3)$$

where

$$\boldsymbol{\rho} = (x, y) \quad \text{and} \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

We consider a beam of radio waves propagating along the z-direction with its initial peak intensity at $\boldsymbol{\rho} = 0$. Due to random scattering and diffraction by the irregularities, the beam tends to be broadened. In general, it has been found that those structures which are large compared to the diameter of the beam tend to deflect the beam, whereas small irregularities usually cause the beam to spread but do not deflect it significantly[19]. To describe these phenomena quantitatively, Fante[19] defined the mean square beam wander (or deflection) $\langle \rho_c^2(z) \rangle$, mean square long-term beam spread $\langle \rho_L^2(z) \rangle$, and short-term beam spread $\langle \rho_s^2(z) \rangle$ as follows:

$$\langle \rho_c^2(z) \rangle = \left[\iiiint_{-\infty}^{\infty} \boldsymbol{\rho}_1 \boldsymbol{\rho}_2 \langle I(z, \boldsymbol{\rho}_1) I(z, \boldsymbol{\rho}_2) \rangle d^2\rho_1 d^2\rho_2 \right] \div \left[\iint_{-\infty}^{\infty} \langle I(z, \boldsymbol{\rho}) \rangle d^2\rho \right]^2 \qquad \ldots(4)$$

$$\langle \rho_L^2(z) \rangle = \left[\iint_{-\infty}^{\infty} \rho^2 \langle I(z, \boldsymbol{\rho}) \rangle d^2\rho \right] \div \left[\iint_{-\infty}^{\infty} \langle I(z, \boldsymbol{\rho}) \rangle d^2\rho \right] \qquad \ldots(5)$$

and

$$\langle \rho_s^2(z) \rangle = \langle \rho_L^2(z) \rangle - \langle \rho_c^2(z) \rangle \qquad \ldots(6)$$

where $I(z, \boldsymbol{\rho}) = [u(z, \boldsymbol{\rho}) u^*(z, \boldsymbol{\rho})]$ is the intensity of the beam wave and the asterisk indicates complex conjugate. To compute these quantities, the second-order and fourth-order moments (or mutual coherence functions) of the field

$$\Gamma_2(z, \boldsymbol{\rho}'_1, \boldsymbol{\rho}'_2) = \langle u(z, \boldsymbol{\rho}'_1) u^*(z, \boldsymbol{\rho}'_2) \rangle \qquad \ldots(7)$$

and

$$\Gamma_4(z, \boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \boldsymbol{\rho}_3, \boldsymbol{\rho}_4) = \langle u(z, \boldsymbol{\rho}_1) u(z, \boldsymbol{\rho}_2) u^*(z, \boldsymbol{\rho}_3) u^*(z, \boldsymbol{\rho}_4) \rangle \qquad \ldots(8)$$

are needed.

To date, however, no general solution has been obtained for the fourth moments except for approximate solutions. So we adopt the thin phase screen model to estimate the broadening of a beam propagating through a turbulent ionosphere.

In the phase screen model, it is assumed that the random medium is concentrated within a 'thin screen' and this thin screen causes only 'phase' modulation of the wave propagating through it, and the effect is represented by thin phase changing screen located at $z = 0$ as shown in Fig. 1. For the region $z > 0$, the moments Γ_2 and Γ_4 defined in Eqs (7) and (8) when coupled with Eq. (3) satisfy the following free-space equations respectively[18].

$$\frac{\partial \Gamma_2}{\partial z} + \frac{j}{2k}(\nabla_1'^2 - \nabla_2'^2)\Gamma_2 = 0 \qquad \ldots(9)$$

$$\frac{\partial \Gamma_4}{\partial z} + \frac{j}{2k}(\nabla_1^2 + \nabla_2^2 - \nabla_3^2 - \nabla_4^2)\Gamma_4 = 0 \qquad \ldots(10)$$

where

$$\nabla_i^2 = \frac{\partial^2}{\partial x_i^2} + \frac{\partial^2}{\partial y_i^2}$$

Defining the change of variables for the coordinates[18]

$$\boldsymbol{\eta} = \frac{1}{2}(\boldsymbol{\rho}'_1 + \boldsymbol{\rho}'_2)$$

$$\boldsymbol{\delta} = \boldsymbol{\rho}'_1 - \boldsymbol{\rho}'_2$$

$$\mathbf{R} = \frac{1}{4}(\boldsymbol{\rho}_1 + \boldsymbol{\rho}_2 + \boldsymbol{\rho}_3 + \boldsymbol{\rho}_4)$$

$$\mathbf{r}_1 = \frac{1}{2}(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2 + \boldsymbol{\rho}_3 - \boldsymbol{\rho}_4)$$

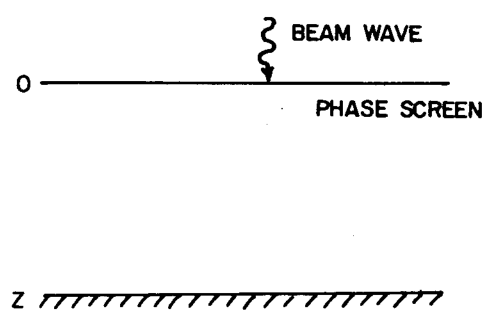

Fig. 1—Geometry of the problem when a beam wave is propagating through a thin phase screen

$$\mathbf{r}_2=\frac{1}{2}(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2-\boldsymbol{\rho}_3+\boldsymbol{\rho}_4)$$

$$\boldsymbol{\rho}=\boldsymbol{\rho}_1+\boldsymbol{\rho}_2-\boldsymbol{\rho}_3-\boldsymbol{\rho}_4$$

Eqs (9) and (10) can be put into the forms

$$\frac{\partial\Gamma_2(z,\boldsymbol{\eta},\boldsymbol{\delta})}{\partial z}+\frac{j}{k}\nabla_{\eta}\nabla_{\delta}\Gamma_2(z,\boldsymbol{\eta},\boldsymbol{\delta})=0 \quad \ldots(11)$$

$$\frac{\partial\Gamma_4(z,\mathbf{R},\mathbf{r}_1,\mathbf{r}_2,\boldsymbol{\rho})}{\partial z}+\frac{j}{k}(\nabla_{\mathbf{R}}\nabla_{\rho}+\nabla_{\mathbf{r}_1}\nabla_{\mathbf{r}_2})\Gamma_4(z,\mathbf{R},\mathbf{r}_1,\mathbf{r}_2,\boldsymbol{\rho})=0 \quad \ldots(12)$$

where

$$\nabla_{\eta}=\hat{x}\frac{\partial}{\partial\eta_x}+\hat{y}\frac{\partial}{\partial\eta_y}$$

similarly for ∇_{δ}, $\nabla_{\mathbf{R}}$, $\nabla_{\mathbf{r}_1}$, $\nabla_{\mathbf{r}_2}$ and ∇_{ρ}. To solve these equations, we employ the Fourier Transform method

$$\hat{\Gamma}_2(z,\boldsymbol{\alpha},\boldsymbol{\delta})=(2\pi)^{-2}\iint_{-\infty}^{\infty}\Gamma_2(z,\boldsymbol{\eta},\boldsymbol{\delta})e^{-j\boldsymbol{\eta}.\boldsymbol{\alpha}}d^2\eta \quad \ldots(13)$$

$$\hat{\Gamma}_4(z,\mathbf{q},\boldsymbol{\sigma},\mathbf{r}_2,\boldsymbol{\rho})=(2\pi)^{-4}\int\iiint_{-\infty}^{\infty}\Gamma_4(z,\mathbf{R},\mathbf{r}_1,\mathbf{r}_2,\boldsymbol{\rho})e^{-j(\mathbf{q}.\mathbf{R}+\boldsymbol{\sigma}.\mathbf{r}_1)}d^2R\,d^2r_1 \quad \ldots(14)$$

Solving Eqs (11) and (12) with the Fourier transforms (13) and (14) and substituting them in Eqs (4) and (5), Kiang and Liu[20] obtained following expressions for beam-wave broadening and deflection:

$$\langle\rho_L^2(z)\rangle=\left[-\iint_{-\infty}^{\infty}\left\{\nabla_{\alpha}^2\left[\Gamma_2\left(0,\boldsymbol{\eta},\frac{\boldsymbol{\alpha}}{k}z\right)e^{-j\boldsymbol{\alpha}.\boldsymbol{\eta}}\right]\right\}_{\alpha=0}d^2\eta\right] \div\left[\iint_{-\infty}^{\infty}\Gamma_2(0,\boldsymbol{\eta},0)d^2\eta\right] \quad \ldots(15)$$

$$\langle\rho_c^2(z)\rangle=\left[\int\iiint_{-\infty}^{\infty}\left\{(-\nabla_{\mathbf{q}}^2+\tfrac{1}{4}\nabla_{\sigma}^2)\times\left[\Gamma_4\left(0,\mathbf{R},\mathbf{r}_1,\frac{\boldsymbol{\sigma}}{k}z,\frac{\mathbf{q}}{k}z\right)e^{-j(\mathbf{q}.\mathbf{R}+\boldsymbol{\sigma}.\mathbf{r}_1)}\right]\right\}_{\mathbf{q}=\boldsymbol{\sigma}=0}\times d^2R\,d^2r_1\right]\div\left[\iint_{-\infty}^{\infty}\Gamma_2(0,\boldsymbol{\eta},0)d^2\eta\right]^2 \quad \ldots(16)$$

Further, we assume that the wave incident on the irregularity layer of thickness L is a Gaussian beam. On emerging from the phase screen which introduces a phase change $e^{-\frac{1}{2}kL\beta\xi}$, the field becomes

$$u(0,\boldsymbol{\rho})=\exp\left[-\frac{\rho^2}{2a^2}-j\frac{k\rho^2}{2F}-j\frac{kL\beta\xi(\rho)}{2}\right] \quad \ldots(17)$$

where a is the effective diameter of the beam and F is the radius of curvature of the beam-wave front. For further simplifications, we assume that the random functions $[\xi(\boldsymbol{\rho}_1)-\xi(\boldsymbol{\rho}_2)]$ and $[\xi(\boldsymbol{\rho}_1)+\xi(\boldsymbol{\rho}_2)-\xi(\boldsymbol{\rho}_3)-\xi(\boldsymbol{\rho}_4)]$ are both normal and zero mean for arbitrary $\boldsymbol{\rho}_1$, $\boldsymbol{\rho}_2$, $\boldsymbol{\rho}_3$ and $\boldsymbol{\rho}_4$.

The initial moments at $z=0$ then become

$$\Gamma_2(0,\boldsymbol{\eta},\boldsymbol{\delta})=\exp\left\{-\frac{\eta^2+\frac{1}{4}\delta^2}{a^2}-jk\frac{\boldsymbol{\eta}.\boldsymbol{\delta}}{F}-\frac{1}{4}k^2L^2\beta^2[\sigma_{\xi}^2-B_{\xi}(\boldsymbol{\delta})]\right\} \quad \ldots(18)$$

$$\begin{aligned}\Gamma_4(0,\mathbf{R},\mathbf{r}_1,\mathbf{r}_2,\boldsymbol{\rho}) =\exp\Big\{&-\frac{4R^2+\frac{1}{4}\rho^2+r_1^2+r_2^2}{2a^2}-jk\frac{\mathbf{R}.\boldsymbol{\rho}+\mathbf{r}_1.\mathbf{r}_2}{F}\\ &-\frac{1}{2}k^2L^2\beta^2\sigma_{\xi}^2-\frac{1}{4}k^2L^2\beta^2[B_{\xi}(\mathbf{r}_1+\mathbf{r}_2)\\ &-B_{\xi}(\mathbf{r}_2+\tfrac{1}{2}\boldsymbol{\rho})-B_{\xi}(\mathbf{r}_1+\tfrac{1}{2}\boldsymbol{\rho})-B_{\xi}(\tfrac{1}{2}\boldsymbol{\rho}-\mathbf{r}_1)\\ &-B_{\xi}(\tfrac{1}{2}\boldsymbol{\rho}-\mathbf{r}_2)+B_{\xi}(\mathbf{r}_1-\mathbf{r}_2)]\Big\}\end{aligned} \quad \ldots(19)$$

where σ_{ξ}^2 is the variance of ξ and $B_{\xi}(\boldsymbol{\rho})$ is its correlation function.

The solutions of integrals (15) and (16) depend on the choice of the power spectrum $\phi_{\xi}(K)$ of the irregularities. In situ measurements indicate that a realistic power spectrum is given by the power law

$$\phi_{\xi}(K)=\sigma_{\xi}^2\frac{\Gamma(p/2)}{\pi^{3/2}\Gamma[(p-3)/2]}\frac{L_0^3}{(1+L_0^2K^2)^{p/2}} \quad \ldots(20)$$

with L_0 as the outer scale of irregularities, K the spatial wave number, Γ the Gamma function, and p the spectral power index. In the ionosphere, usually $2<p<4$, where $p=(11/3)$ corresponds to the Kolmogorov spectrum following the power law $\phi_{\xi}(K)\propto K^{-p}$ for $1/L_0 \ll K \ll 1/l_0$ (l_0 being the inner scale of the irregularities). We modify the corresponding correlation function $B_{\xi}(\mathbf{r})$ of $\phi_{\xi}(K)$ to account for l_0 such that

$$B_{\xi}(\mathbf{r})=\sigma_{\xi}^2\frac{\left(\frac{\mathbf{r}}{L_0}+\frac{l_0}{L_0}\right)^{(p-3)/2}K_{(p-3)/2}\left(\frac{\mathbf{r}}{L_0}+\frac{l_0}{L_0}\right)}{\left(\frac{l_0}{L_0}\right)^{(p-3)/2}K_{(p-3)/2}\left(\frac{l_0}{L_0}\right)} \quad \ldots(21)$$

satisfying the conditions $l_0 \ll L_0$ and $B_\xi(0) = \sigma_\xi^2$. K_ν is the modified Bessel function of second kind with imaginary argument.

Using Eq. (21) and inserting Eqs (18) and (19) into Eqs (15) and (16), we obtain the expressions for the beam-wave broadening and wander after integrations as

$$\langle \rho_L^2(z)\rangle = \frac{z^2}{2k^2a^2} + a^2\left(\frac{z}{F}+1\right)^2 + \left(\frac{1}{4}\right)\sigma_\xi^2\beta^2L^2\left(\frac{z^2}{L_0^2}\right) \times \left[\frac{L_0}{l_0}\frac{K_{(p-5)/2}\left(\frac{l_0}{L_0}\right)}{K_{(p-3)/2}\left(\frac{l_0}{L_0}\right)} - \frac{K_{(p-7)/2}\left(\frac{l_0}{L_0}\right)}{K_{(p-3)/2}\left(\frac{l_0}{L_0}\right)}\right] \quad \ldots(22)$$

$$\langle \rho_c^2(z)\rangle = \frac{L^2\beta^2\sigma_\xi^2 z^2\Gamma[(p-3)/2]a^{(p-7)/2}e^{a^2/4L_0^2}}{2^{(p+5)/4}L_0^{(p-3)/2}} \times \left[2\,W_{(3p-4)/4,(p-5)/4}\left(\frac{a^2}{2L_0^2}\right) - \left(\frac{a^2}{2L_0^2}\right)^{1/2}W_{(1-p)/4,(p-7)/4}\left(\frac{a^2}{2L_0^2}\right)\right] \quad \ldots(23)$$

where $W_{\lambda,\mu}$ is the Whittaker's function[21] defined as

$$W_{\lambda,\mu}(z) = \frac{z^{\mu+1/2}e^{-z/2}}{\Gamma(\mu-\lambda+\frac{1}{2})} \times \int_0^\infty e^{-zt}.t^{\mu-\lambda-\frac{1}{2}}.(1+t)^{\mu+\lambda-\frac{1}{2}}\,dt \quad \ldots(24)$$

$$\left[R_e(\mu-\lambda) > -\frac{1}{2}, |\arg z| < \frac{\pi}{2}\right]$$

Throughout the process, the quantities $(\nabla\Gamma_2)^2$ and $(\nabla\Gamma_4)^2$ are neglected because we have not taken the depolarization effects into account. In obtaining Eq. (23), we have kept $l_0 \to 0$ and $p > 3$ to ensure convergence of integrations.

We note here that the first two terms on the right hand side of Eq. (22) represent the beam-wave spread due to the free-space propagation while the third term gives additional spread caused by the turbulent ionospheric irregularities.

For a limiting case when $l_0 \to 0$ and $p=4$, Eq. (22) reduces to

$$\langle \rho_L^2(z)\rangle = \frac{z^2}{2k^2a^2} + a^2\left(\frac{z}{F}+1\right)^2 + (1/4)\sigma_\xi^2\beta^2L^2\left(\frac{z^2}{L_0^2}\right) \quad \ldots(25)$$

which is fairly identical with the one obtained with $p=4$ power-law spectrum whose correlation function is $B_\xi(\mathbf{r}) = \sigma_\xi^2\, e^{-|\mathbf{r}|/L_0}$.

The factor $(\sigma_\xi^2\beta^2L^2)$ determines the scattering strength of the irregularity layer. Since $l_0 \ll L_0$ for all cases, the Bessel functions in Eq. (22) can be approximated to yield

$$\langle \rho_L^2(z)\rangle - \bar{a}^2 \cong (1/4)\sigma_\xi^2\beta^2L^2z^2 \times \begin{cases} \frac{4}{l_0^2}\left[\frac{1}{2}\frac{\Gamma(|(p-5)/2|)}{\Gamma(|(p-3)/2|)} + \frac{\Gamma(|(p-7)/2|)}{\Gamma(|(p-3)/2|)}\right], & p<3 \\ \frac{4}{l_0^2}\left(\frac{l_0}{L_0}\right)^{(p-3)}\left[\frac{1}{2}\frac{\Gamma(|(p-5)/2|)}{\Gamma(|(p-3)/2|)} + \frac{\Gamma(|(p-7)/2|)}{\Gamma(|(p-3)/2|)}\right], & 3<p<5 \\ \frac{1}{2L_0^2}\frac{\Gamma(|(p-5)/2|)}{\Gamma(|(p-3)/2|)} + \frac{l_0^2}{4L_0^4}\frac{\Gamma(|(p-7)/2|)}{\Gamma(|(p-3)/2|)}, & 5<p \end{cases} \quad \ldots(26)$$

where we have taken

$$\bar{a} = \left[\frac{z^2}{2k^2a^2} + a^2\left(\frac{z}{F}+1\right)^2\right]^{1/2} \quad \ldots(27)$$

as the free-space beam-wave broadening.

Thus, it is clear from Eq. (26) that the beam-wave spread due to ionospheric random part shows its strong dependence on the spectral power index p. In the range $p<3$, the spread is solely caused by the smaller irregularities while in the range $3<p<5$, larger irregularities counteract the effects of smaller irregularities. Further, in the range $5<p$, the spread diminishes sharply.

We now examine the limiting case for the beam-wave deflection (wandering) $\langle \rho_c^2(z)\rangle$. Since the beam-wave dimension is much greater than the outer scale of irregularities, we have

$$\langle \rho_c^2(z)\rangle \cong \frac{1}{2}L^2\beta^2\sigma_\xi^2z^2\Gamma[(p-3)/2]\frac{L_0^2}{a^4} \quad \text{for } \frac{a}{L_0} \gg 1, p>3 \quad \ldots(28)$$

This is very small because $(L_0/a) \ll 1$. The result is also supported by the fact that on choosing the Gaussian power spectrum whose correlation function $B_\xi(\mathbf{r}) = \sigma_\xi^2\, e^{-r^2/r_0^2}$ (r_0 being the irregularity size), we obtain

$$\langle \rho_c^2(z)\rangle = \sigma_\xi^2 \beta^2 L^2 z^2 (r_0^2 - 2a^2)/2(r_0^2 + 2a^2)^2 \quad \ldots(29)$$

This means that the irregularities smaller than the effective diameter of the beam wave will not cause the beam wander.

3 Numerical Results

In this section, we have presented some numerical results for the beam-wave spreads using the power-law spectrum. The values of parameters used for the numerical computations are listed below (typical values for SROSS applications):

$\bar{a} = 1.02 \times 10^4$ m

$L = 2 \times 10^5$ m

$\beta^2 = 4 \times 10^{-10}$,

$f_p = 10$ MHz

$f = 2.245$ GHz

$z = 3 \times 10^5$ m

First, we compute the value of the beam-wave wander $\langle \rho_c^2 \rangle$. For $p = 4$, $l_0 = 10$m, $L_0 = 10^4$m and $\sigma_\xi^2 = 10^{-4}$ (1 per cent fluctuation of ξ), we find that $\langle \rho_c^2 \rangle = 0.36$ m^2, which is much smaller by several orders of magnitude than the random part of $\langle \rho_L^2 \rangle$. We, therefore, neglect the beam-wave wander for the case of SROSS.

In Fig. 2, $(\langle \rho_L^2 \rangle)^{1/2} - \bar{a}$, the excess beam-wave spread due to the turbulence, is plotted against p; we have chosen $\sigma_\xi^2 = 10^{-2}$ (10 per cent fluctuation of ξ), $l_0 = 10$ m and $L_0 = 10^4$ m. It is well known that, in the ionosphere, the range of p varies approximately between 3 and 4 (Ref. 17). From Fig. 2, it is evident that the excess spread decreases gradually in the range $3.1 \leq p \leq 3.3$, but the decrease is very sharp in the range $3.3 < p$. This is due to the fact that the smaller p amounts to more energy distribution in the smaller eddies. It is found that the excess spread of the beam wave for $p = 3.2$ is about 2.5 per cent of $\bar{a}$ which may, indeed, cause an appreciable effect on SROSS application.

The dependence of $(\langle \rho_L^2 \rangle)^{1/2} - \bar{a}$ on l_0, the inner scale of irregularities, is shown in Fig. 3, where $L_0 = 10^4$ m,

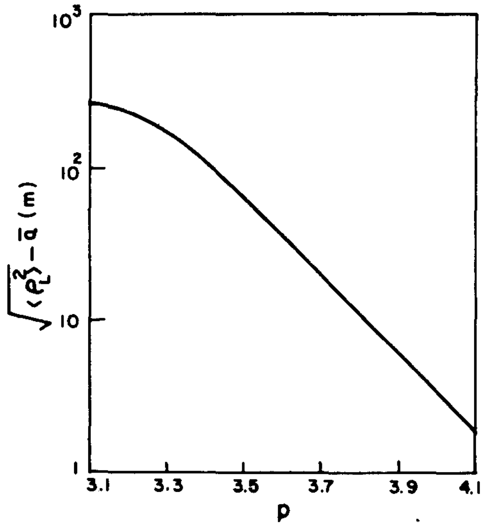

Fig. 2—Excess beam-wave spread as a function of power-law spectral index p for $l_0 = 10$ m, $L_0 = 10^4$ m, and $\sigma_\xi^2 = 10^{-2}$

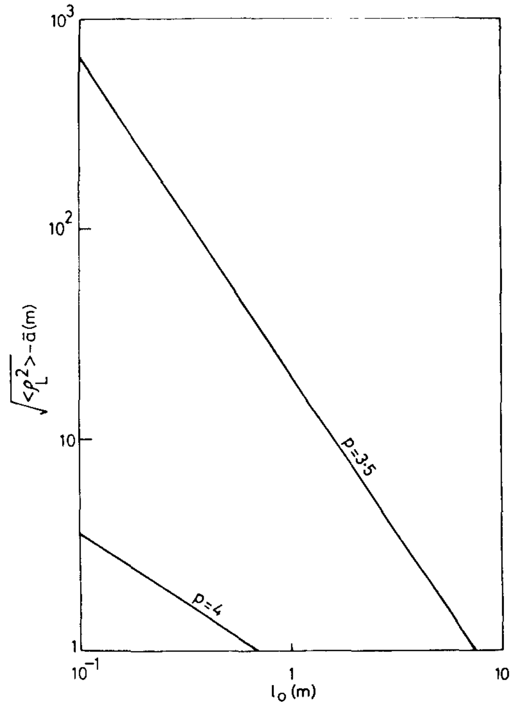

Fig. 3—Excess beam-wave spread as a function of inner scale of irregularities for $L_0 = 10^4$ m and $\sigma_\xi^2 = 10^{-4}$

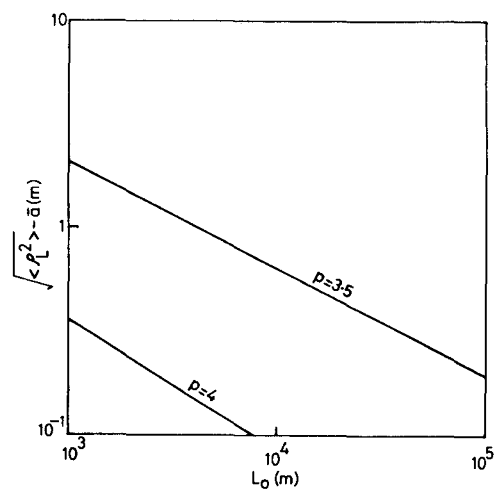

Fig. 4—Excess beam-wave spread as a function of outer scale of irregularities for $l_0 = 10$ m and $\sigma_\xi^2 = 10^{-4}$

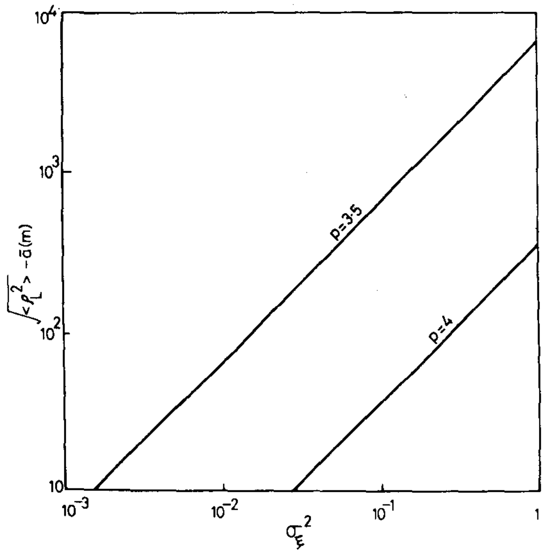

Fig. 5—Excess beam-wave spread as a function of turbulence strength for $l_0 = 10$ m and $L_0 = 10^4$ m

$p = 3.5$ and 4 and $\sigma_\xi^2 = 10^{-4}$ (1 per cent fluctuation of ξ). From Fig. 3, we note that the dependence of $(\langle\rho_L^2\rangle)^{1/2} - \bar{a}$ on l_0 is very strong for small p. The curve for $p = 3.5$ is very steep, indicating the fact that the small eddies with more energy distribution scatter the beam wave more severely, thereby contributing more to the beam spread. Fig. 4 shows the dependence of $(\langle\rho_L^2\rangle)^{1/2} - \bar{a}$ on L_0, the outer scale of irregularities for $l_0 = 10$ m, $\sigma_\xi^2 = 10^{-4}$ and $p = 3.5$ or 4. Throughout the field of turbulence, the effect of larger irregularities is to decrease the excess spread of the beam wave while the effect of L_0 is more pronounced when p increases from 3.4 to 4. In Fig. 5, we have shown the dependence of $(\langle\rho_L^2\rangle)^{1/2} - \bar{a}$ on σ_ξ^2, the turbulence strength. For $p = 3.5$, a 38 per cent fluctuation of ξ can cause a severe excess spread of about 10^3 m (about 10 per cent of $\bar{a}$). In the case of 100 per cent fluctuation of ξ when the turbulence is strong enough, a 66 per cent excess spread of the beam wave can be approached for $p = 3.5$.

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