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# Communications

## Slow Ion Acoustic Double Layers with Non-isothermal Electron Distributions

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The existence of slow ion acoustic double layer in the presence of reflected ions and non-isothermal electrons has been pointed out. The conditions for the existence of this type of double layer have been discussed. The results may be applicable to the recent observations in the auroral plasma.

Studies of ion solitary holes[1,2] and the associated slow ion acoustic double layers[3,4] have been made in recent years. Schamel and Bujarbarua[1] studied ion hole solutions and Kim[3] and Schamel[4] studied the associated slow ion acoustic double layers and showed that such nonlinear solitary structures move with a velocity near the ion thermal speed and need, for their existence, ion trapping (reflection) effects. Recently, Bujarbarua[5] introduced non-isothermal electrons in the propagation of small amplitude ion holes. Since the effects of non-isothermal electrons are manifested in the coefficient of $\phi^2$ (where $\phi$ is the electric potential), Bujarbarua[5] expanded the electron and ion densities up to $\phi^2$ term and looked for soliton solution (note that for usual ion hole solutions with isothermal electrons terms up to $\phi^{3/2}$ are retained in the expansion of ion and electron densities).

In this communication, it is shown that slow ion acoustic double layer (DL) solutions may be obtained in the analysis of Bujarbarua[5] when the appropriate DL conditions are imposed. The motivation for the present study is to explain a recent observation of small amplitude double layers along the auroral field lines[6].

For this purpose, a homogeneous, infinite plasma is considered and the stationary solutions of the Vlasov-Poisson system are studied. We follow closely the analysis of Bujarbarua[5] and assume that the potential $\phi(x)$ is negative with amplitude $\psi$, and $\phi(-\infty)=0$ and $\phi(\infty)=-\psi$. We consider both free and reflected ions. The ion and the electron distribution functions and hence the ion and the electron densities for such a system are similar to those given by Bujarbarua[5]. The 'energy law', as given in Ref. 5 is as follows.

$$\frac{1}{2}\left(\frac{\partial\phi}{\partial x}\right)^2 + V(\phi)=0 \qquad \ldots(1)$$

where the classical potential $V(\phi)$ is given by

$$-V(\phi)=\theta_f\left[\frac{1}{2}A_1\phi^2-\frac{2}{5}A_2(-\phi)^{5/2}+\frac{1}{3}A_3\phi^3+\ldots\right] \qquad \ldots(2)$$

where

$$A_1=[\theta_f^{-1}+1-2\sqrt{u_0^2/2}\; W(\sqrt{u_0^2/2})] \qquad \ldots(3)$$

$$A_2=\frac{4}{3\sqrt{\pi}}[1-\alpha-u_0^2]e^{-u_0^2/2} \qquad \ldots(4)$$

$$A_3=\frac{1}{2}\left[\Delta-e^{-u_0^2/2}-\frac{3}{u_0^4}\right] \qquad \ldots(5)$$

$$\theta_f=T_{ef}/T_i=(\theta_h\theta_c)/(n_{eh}\theta_c+n_{ec}\theta_h) \qquad \ldots(6)$$

$T_{ef}$ Effective electron temperature
$T_i$ Ion temperature

$$\Delta=(n_{eh}\theta_c^2+n_{ec}\theta_h^2)/\theta_h^2\theta_c^2 \qquad \ldots(7)$$

$n_{eh}$ Hot electron density
$n_{ec}$ Cold electron density
$\theta_h=T_{eh}/T_i$
$T_{eh}$ Hot electron temperature
$\theta_c=T_{ec}/T_i$
$T_{ec}$ Cold electron temperature
$\alpha=T_{if}/T_{ir}$

$T_{if}$ Free ion temperature
$T_{ir}$ Reflected ion temperature
$u_0$ Ion drift velocity

Here the space coordinate $x$ is normalized to the effective electron Debye length $\lambda_{De}=(T_{ef}/4\pi e^2 n_0)^{1/2}$, the electric potential $\phi$ is normalized to $T_f/e$, the ion drift velcoity $u_0$ is normalized to the ion thermal speed and $W(x)$ is the Dawson's integral given by

$$W(x)=e^{-x^2}\int_0^x e^{t^2}\,dt \qquad \ldots (8)$$

We shall now impose the DL conditions. The first condition is the nonlinear dispersion relation, $V(-\psi)=0$. Using this condition in Eq. (2), we get

$$A_1=\frac{4}{5}A_2\psi^{1/2}+\frac{2}{3}A_3\psi \qquad \ldots (9)$$

The second DL condition is the charge neutrality condition, viz. $\partial V(\phi)/\partial\phi=0$ at $\phi=-\psi$, and using this condition in Eq. (2), we have

$$A_1=A_2\psi^{1/2}+A_3\psi \qquad \ldots (10)$$

From Eqs (9) and (10), we get

$$A_2=-\frac{5}{3}A_3\psi^{1/2} \qquad \ldots (11)$$

and

$$A_1=-\frac{2}{3}A_3\psi \qquad \ldots (12)$$

Inserting the value of $A_1$ and $A_2$ from Eqs (11) and (12) into Eq. (2) we get

$$-V(\phi)=\frac{1}{3}\theta_f(-A_3)\phi^2\psi[1-\sqrt{-\phi(\psi)}]^2 \qquad \ldots (13)$$

Inserting Eq. (13) in Eq. (1) and integrating we get the DL potential[7] as

$$\phi(x)=-\frac{\psi}{4}[1-\tanh kx]^2 \qquad \ldots (14)$$

where

$$k=[(\theta_f/24)(-A_3)\psi]^{1/2} \qquad \ldots (15)$$

Eq. (14) represents a rarefactive DL, provided

$$A_3=\frac{1}{2}\left[\Delta-e^{-u_0^2/2}-\frac{3}{u_0^4}\right]<0 \qquad \ldots (16)$$

The DL thickness is given by

$$d=[-24/\theta_f A_3\psi]^{1/2} \qquad \ldots (17)$$

The velocity of the DL can be calculated from Eqs (11) and (12) and is given by

$$u_0=\simeq 1.306\left[1+\theta_f^{-1}-0.26(|\alpha|-0.71)\left[\frac{\psi}{4}\right]^{1/2}\right] \qquad \ldots (18)$$

with $\theta_f \geqslant 3.5$ and $\alpha<0.71$

Substituting the linear value of $u_0$ in Eq. (16) we find that DL solutions exist when $\Delta<1.46$. It can be shown that the condition $\Delta<1.46$ can be satisfied for a large number of values of the parameters $n_{eh}$, $n_{ec}$, $\theta_h$, $\theta_c$ etc. For isothermal electrons $\theta_h=\theta_c=\theta_f=\theta$ and $\Delta=\theta^{-2}$. Since for DL solution $\theta>3.5$, $\theta^{-2}$ is always less than 1.46, it may be concluded that for isothermal electrons slow ion acoustic DL exists when the ratio of the electron to ion temperature is greater than 3.5.

We now apply the results of the present theory to observations in the auroral region of the space[6] and take $n_{ec}/n_{eh}=5$ and $T_{ec}/T_{eh}=10^{-2}$. In this case, the effective electron temperature $T_{ef}=T_{ec}=10e\phi$ (For small amplitude DLs, we take $e\phi/T_{ef}=10^{-1}$ so that $T_{ef}=T_{ec}=10e\phi$). According to the observation by Temerin *et al.*[6], the potential of an individual DL is found to be of the order of 1V. Here $T_{ef}=T_{ec}=10e\phi=10$eV and $T_{eh}=100$ $T_{ef}=1$ keV. For $T_i=2$eV, $\theta_f=5$, $\theta_h=T_{eh}/T_i=500$, $\theta_c=T_{ec}/T_i=5$ and $\theta_h/\theta_c=100$. In this case $\Delta=0.03$, $A_3=-0.377$ and the thickness of the DL is found to be $10\lambda_{De}$. This calculated value is of the order of the observed value of Temerin *et al.*[6] which is $40\lambda_{De}$. Therefore, the slow ion acoustic DLs as discussed in this communication may be one of the candidates for the observation of Temerin *et al.*[6] It should be noted that Temerin *et al.*[6] observed both the compressive DL moving downwards from the magnetosphere to the ionosphere and rerefactive DL moving upwards from the ionosphere to the magnetosphere. The slow ion acoustic DL discussed in this communication resembles the rarefactive type of DL observed by Temerin *et al.*[6]

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