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Scintillation Studies of Transionospheric Signal

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Single scattering theory is applied to the problem of ionospheric scintillations. The characteristics of scintillation indices computed with two power spectra, namely, Gaussian and Kolmogorov, are studied in detail. Theoretically computed scintillation index is compared with that observed at 327 MHz at Ooty. It is found that the scintillation index computed with the Kolmogorov spectrum is in closer agreement with that observed at Ooty.

1 Introduction

When radiowave propagates through the ionosphere, the random irregularities present therein can cause fluctuations of the parameters of the wave. This is known as the ionospheric scintillation phenomenon. F-region irregularities are the cause of intense scintillations (irregular phase and amplitude fluctuations) of signals transmitted through the ionosphere over the VHF to 1 GHz frequency range at high latitudes and VHF to *S*-band at equatorial latitudes. While the causative mechanisms of these irregularities remain unresolved and continue to be the subject of multitechnique experiments, their effects cause serious concern to communication engineers. This is because scintillations can degrade the performance of high data rate satellite communication links.

Scintillation of transionospheric signals has been observed by many investigators[1-7]. The scintillation phenomenon is confined to the region ±20° geomagnetic latitudes and high latitudes. The scintillation is much stronger in the equatorial region. Signal fluctuations as high as 20 dB (peak-to-peak) are observed[8] in the *L*-band (1.55 GHz) and 14 dB (peak-to-peak) at 4 GHz (Ref. 9) during solar cycle maximum period. The layer of strong irregularities is often 100-500 km thick ranging from 200 to 700 km in altitude. The relative electron density fluctuations are often of the order of 10% and may be as high as 70%.

When the scintillation is weak, the existing theory based on single-scatter assumptions seems to be quite satisfactory in interpreting the observational data[10] as due to the fact that the irregularities of the size of the Fresnel zone contribute most to the amplitude scintillation observed on the ground. In the single-scatter theory approximation, the field incident on the scattering random medium is assumed to be equal to that of the incident wave in free-space.

Wernik and Liu[11] have studied the scintillation problem based on single-scatter theory summarizing the general properties of the scintillation producing irregularities.

In the present paper, we have studied general characteristics of scintillation index over equatorial region using the Gaussian and power-law spectra. For an ionospheric slab model of 200 km thickness, at an altitude of 350 km and electron density fluctuation of 10%, we have computed S_4 (scintillation index) for nighttime equatorial scintillation with the Gaussian and power-law spectra. The computed values of S_4 are compared with the observed values obtained from the study of ionospheric scintillation at 327 MHz at Ootacamund by Pasricha[12].

2 Scintillation Theory

The single scattering is valid when $<\varepsilon_1^2> k^2 r_0 \ll 1$ and the forward scattering is valid only when $<\varepsilon_1^2> kL \ll kr_0$, where $<\varepsilon_1^2>$ is the variance of the medium fluctuations, k the wave number, r_0 the scale-size of irregularity and L the distance the wave has traversed in the irregularity medium.

Under these conditions, Tatarski[13] has obtained the expressions for the amplitude and phase fluctuations of the wave as

$$\psi_1 = \ln\frac{A}{A_0} + i(S - S_0) = \chi + iS_1 \qquad \ldots (1)$$

where

$$\psi = \psi_0 + \psi_1$$

$$\psi_0 = \ln A_0 + iS_0$$

$$\psi = \ln A + iS$$

and, therefore, χ is the logarithmic amplitude and S_1 is the phase departure of scattered wave. Parameters A_0 and S_0 are the amplitude and the phase of plane wave incident on the irregularity layer.

Fig. 1 shows the geometry of a plane wave incident on the layer of irregularities with the thickness L at a height z above the receiver. From the knowledge of amplitude correlation function[13] we can derive an integral for mean square amplitude fluctuations as follows.

$$\langle\chi^2\rangle = 2\pi^2 kL \int_0^\infty F(K)\phi_N(K)K\,dK \qquad \dots(2)$$

where

$$F(K) = 1 - \frac{2k}{K^2L}\sin\left(\frac{K^2L}{2k}\right)\cos\left[\frac{K^2}{k}\left(\frac{L}{2}+z\right)\right]$$

is called the filtering function and $\phi_N(K)$ is the power spectrum which is assumed to be isotropic.

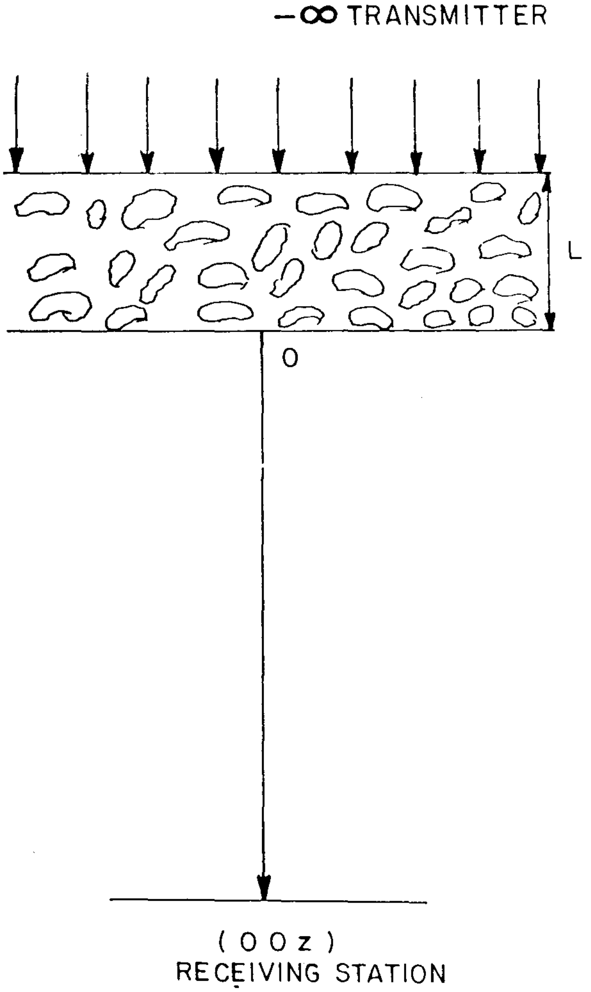

Fig. 1—Geometry ot the scintillation problem

The scintillation index S_4 is generally defined by

$$S_4 = \left[\frac{\langle A^4\rangle}{\langle A^2\rangle^2} - 1\right]^{1/2} \qquad \dots(3)$$

where the angular bracket denotes ensemble average.

For small fluctuations of the field, Rino and Fremouw[14] have shown that

$$S_4^2 = 4\langle\chi^2\rangle \qquad \dots(4)$$

In the following, we choose two power spectra.

(*i*) *Gaussian Spectrum*:

$$\phi_N(K) \doteq \frac{\langle\varepsilon_1^2\rangle}{\pi^{3/2}} a^3 \exp(-K^2a^2) \qquad \dots(5)$$

(*ii*) *Kolmogorov Spectrum*:

$$\phi_N(K) = 0.033\, C_n^2 L_0^{11/3} (1 + K^2L_0^2)^{-11/6} \times \exp(-K^2/K_m^2) \qquad \dots(6)$$

with

$$C_n^2 = 1.9\langle\varepsilon_1^2\rangle L_0^{-2/3}$$

and

$$K_m = \frac{5.29}{l_0}$$

where a is the size of irregularity, L_0 and l_0 are the outer scale and inner scale of the irregularities.

Spectral analyses of observed phase[15] and amplitude[16] fluctuations have shown that the electron density irregularities have a power spectrum that may be characterized by a power-law-shape. This result is not consistent with the description of Gaussian correlation function used to interpret much of the data. These scintillation observations have been further supported by insitu measurements made by satellite-borne sensors that also show a power-law dependence for the power spectrum of electron density fluctuations[17]. We note that for

$$\frac{1}{L_0} \ll K \ll \frac{1}{l_0}$$

the Kolmogorov spectrum reduces to the power-law spectrum; i.e. $\phi_N(K) \propto K^{-p}$ with p lying between 2 and 4.

The Gaussian spectrum in Eq. (5) is characterized by a single scale of the order of the scale size at observing wavelength. The power law in Eq. (6) is more realistic irregularity choice[18,19]. The basic difference between power-law and Gaussian models is that the Gaussian spectrum characterizes electron density spectrum by one scale and power-law demands at least two scales, say, l_0 and L_0. Whether the observed spectrum is Gaussian or power-law can be judged from the high frequency limit in the sense that the Gaussian spectrum yields parabolic distribution with increasing negative slopes with increasing frequency on log-log scale while power-law spectrum yields a straight line distribution.

The filtering function in Eq. (2) is seen to increase until it reaches a maximum at

$$K = K_F = \sqrt{2\pi K/(2z+L)}.$$

It is obvious that the major contribution to the integral in Eq. (2) comes from the neighbourhood of $K = K_F$. This means that for a given frequency, most of the scattered power comes from those which are scattered by irregularities of the dimension $\sim 2\pi/K_F$.

3 Results and Discussion

We have computed scintillation index S_4 from Eqs (2) and (4) using the power spectra Eqs (5) and (6), respectively, for $f/f_p = 100$ (f_p being the

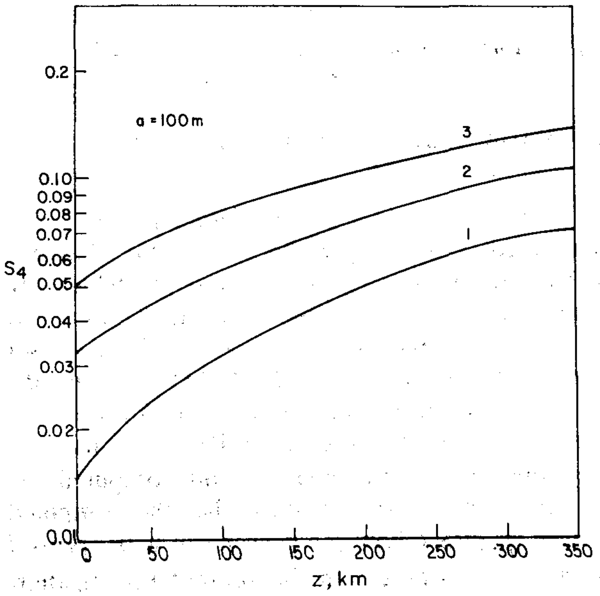

Fig. 2—Scintillation index S_4 versus z (km) for $a = 100$ m when L is varied (Gaussian spectrum)

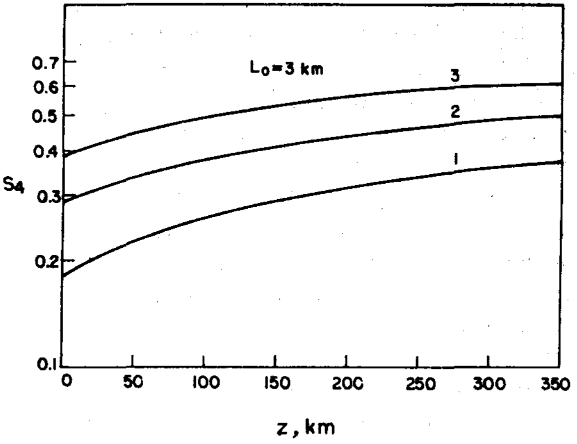

Fig. 3—Scintillation index S_4 versus z (km) for $L_0 = 3$ km when L is varied (Kolmogorov spectrum)

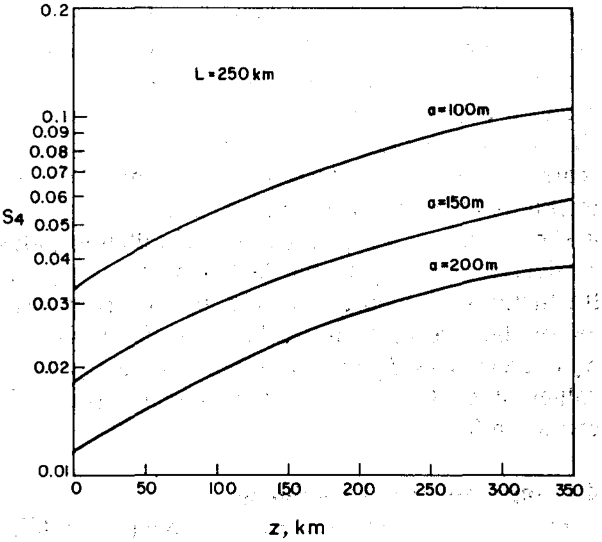

Fig. 4—Scintillation index S_4 versus z (km) for $L = 250$ km when a is varied (Gaussian spectrum)

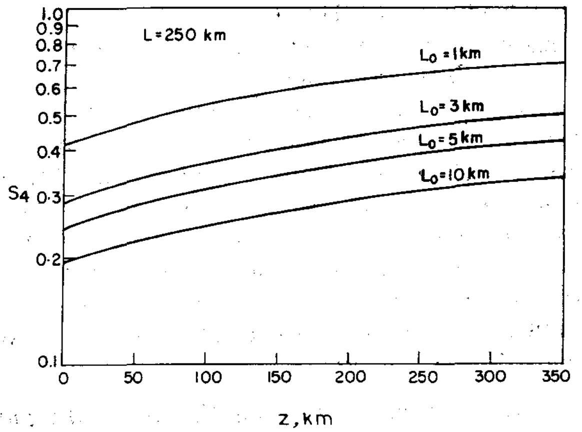

Fig. 5—Scintillation index S_4 versus z (km) for $L = 250$ km when L_0 is varied (Kolmogorov spectrum)

plasma frequency) and $\langle(\Delta N/N)^2\rangle^{1/2}=0.1$. Figs 2-5 depict the behaviour of scintillation index S_4 as a function of z (from slab bottom to the receiver) for the Gaussian and Kolmogorov spectra, respectively. In Figs 2 and 3, the curves 1, 2 and 3 correspond to irregularity slab thicknesses $L=150$, 250 and 350 km, respectively. Figs 2 and 3 show that S_4 increases monotonically with z and finally reaches almost a constant value for both the spectra. The results agree very well with phase screen theory. As L increases, corresponding to a thick random slab, the scintillation index S_4 likewise increases. From a comparison of the results corresponding to the Gaussian and Kolmogorov spectra (Figs 2 and 3) we note that at the bottom of the slab the scintillation index S_4 associated with the Kolmogorov spectrum is larger than that for the Gaussian spectrum. It is due to the fact that in the former case, $\langle\chi^2\rangle$ is proportional to $k^{7/6}$ and $L^{11/6}$ as $z\rightarrow 0$. These powers (7/6 and 11/6) are directly related to the shape of the power-law $K^{-11/3}$. Figs 4 and 5 depict the effects on S_4 of the irregularity size a for the Gaussian spectrum and the outer scale L_0 for the Kolmogorov spectrum when the thickness of irregularity layer, L, is kept fixed. Fig. 4 shows that as a increases, the scintillation index S_4 decreases owing to the fact that the amplitude fluctuations are caused, mostly, by the irregularities of the size $\sqrt{\lambda L}$ or smaller. Fig. 5 shows that S_4 decreases as L_0 increases but sufficiently large values of L_0 do not bring about much appreciable change in S_4.

4 Comparison with Experiment

Pasricha[12] carried out the ionospheric scintillation studies with Ooty radio telescope located near Ootacamund (geogr. lat., 11.4°N; long, 76.7°E) at an altitude of about 2 km in South India. The operating frequency was 326.5 MHz with a bandwidth of 4 MHz. The scintillation observations were made at zenith angle ranging from ~ 0° to 80°. The ionospheric scintillations reported in that paper[12] span over the period 1975-77. No observations were conducted during the June solstice (rainy) period of any of those years. Fig. 6 depicts the diurnal pattern observed at Ooty through a mass plot of the entire S_4 data, irrespective of season, sunspot number and propagation geometry condition. The occurrence of observed scintillations reveals a dominant nocturnal maximum (35%) at ~ 2230 hrs.

In the following, we mention the values of input parameters as chosen by Pasricha[12]

$L=200$ km, $f=327$ MHz, $f_p=9$ MHz

$a=300$ m, $\langle(\Delta N/N)^2\rangle^{1/2}=0.1$, $L_0=10^4$m

$z=350$ km

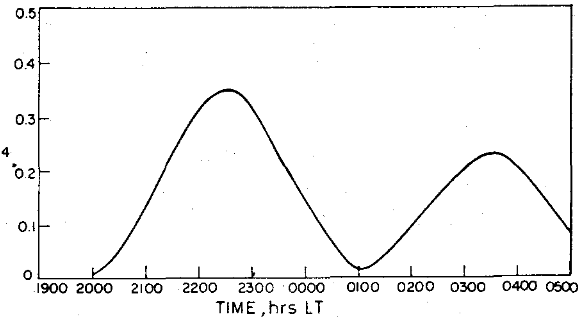

Fig. 6—The diurnal pattern of S_4 at Ooty

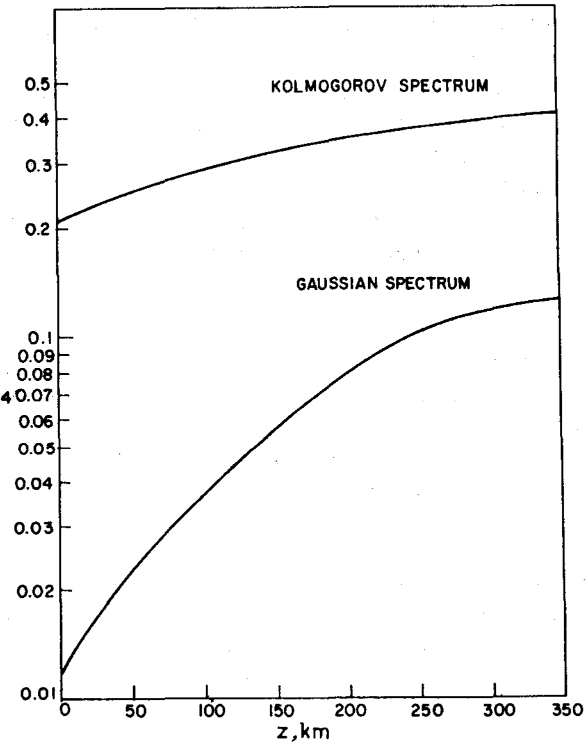

Fig. 7—Scintillation index S_4 versus z (km) for the Kolmogorov and Gaussian power spectra

Using these values of the parameters we have computed S_4 for the Kolmogorov and Gaussian spectra from Eqs (2), (4), (5) and (6), which are shown in Fig. 7. We note that the Kolmogorov spectrum reports 41% of S_4 which is in closer agreement with the observed S_4 (35%), while the Gaussian spectrum reports 20% of S_4, which is lower than the observed value. Pasricha[12] has carried out the spectral analysis and computed the power spectra, and concluded that the computed power spectra follow the power-law with spectral slope ~ −4, whereas in the present investigation we have used the Kolmogorov spectrum as a power-law with the spectral slope −11/3.

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