Indian Journal of Radio & Space Physics
Vol. 20, February 1991, pp. 22-25

Vertical polarization dependence of transient signals above a dielectric layer

Samira T Bishay
Department of Mathematics, Faculty of Science, Ain Shams University, Abbassia, Cairo, Egypt

Received 23 August 1989

The duct model given by Kahan and Eckart [*Ann Phys* (*France*), 5 (1950) 641] has been used for the vertical polarization treatment of the signals. A vertical electric dipole, above the surface layer, is taken as the source of electromagnetic field. The polarization of the primary source, whose moment varies arbitrarily in time, is chosen in such a way that it allows the exact determination of the electric field strength at some fixed point above the duct layer. Two integral transforms, a Laplace transform in time and a two-dimensional Fourier transform in the horizontal coordinates in space are applied to the wave equation for the Hertz vector. This leads to an integral representation of the solution of the wave equation in transform space.

1 Introduction

Modification of propagated electromagnetic waves and their distortion greatly affects radio navigation, identification of targets by means of radar and generally other telecommunication systems. The evaporation duct existing over the sea distorts pulsed signals (of nanosecond durations) most of the time[1].

A number of experimental and theoretical models of the duct refractive index profiles were introduced lately[2,3]. These profiles give a satisfactory description of the mechanism of wave-guiding.

In this paper, the simple duct model by Kahan and Eckart[4] has been applied. The model assumes a discontinuous drop of the usually constant relative permittivity at the upper duct boundary. The earth is assumed to be quite plane and ideal conductor, and thus corresponds to the sea in the range of microwaves.

In a recent paper[5], a similar problem had been treated. A theoretical treatment of the electromagnetic pulse propagation, using Kahan and Eckart model[4] of the evaporation duct, was carried out for a vertical magnetic dipole source. Hence, the results for horizontal polarization were already available. The present work extends the above mentioned theoretical treatment to the case of a vertical electrical dipole, i.e. vertical polarization. The results for vertical polarization are compared with those for horizontal polarization.

2 Formulation of the problem

The model used in this study (Kahan and Eckart model[4]) is shown in Fig. 1. A dielectric layer of relative permittivity, ε_2, is assumed to be overlaid on the infinitely conducting earth, the borderline being the plane $z=0$ in the Cartesian coordinate system (x, y, z). At a height, h, the permittivity acquires a lower value ε_1. The relative permeability, μ, is assumed to be the same for all successive media. We refer the layer (up to h) as medium 2 and the half space $z>h$ as medium 1. The fields which belong to the two media are marked by the corresponding indices.

A vertical electric dipole, as a source of the field, is placed in medium 1 at the point $x=y=0$, $z=d>h$. Its moment is given by $\mathbf{P}=\{0, 0, F(t)\ \delta\ (x, y, z-d)\}$, where t is the time variable, δ the three dimensional Dirac-delta function and $F(t)$ the excitation function. Regarding $F(t)$, we assume that $F(t)=0$ for $t\leqslant 0$ and $dF(t)/dt=0$ for $t=0$. The first assumption guarantees a unique solution and the second describes the time varia-

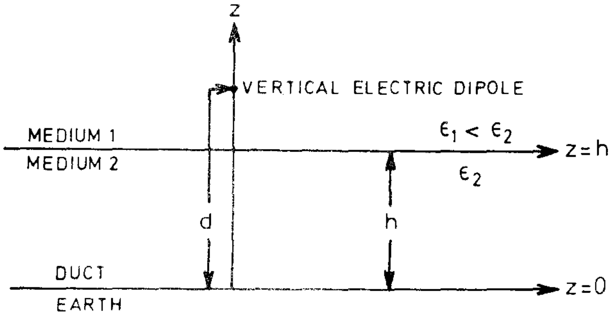

Fig. 1 — Geometry of the model

tion of the current (which flows in an equivalent short linear antenna) whose transient field is not very different from the one radiated by an ideal electric dipole[6]. Zimmer *et al.*[7] have determined the electric field strength at some fixed point within the duct layer, and have chosen certain polarization of the primary source whose moment is allowed to vary arbitrarily with time. Bishay[5] has chosen a horizontal polarization of the primary source which enables the determination of the transient behaviour of the electric field strength at any distance in the ionosphere. This could be applied successfully in the case of radio stars and satellites, as their locations are quite far from the earth.

3 Method of solution and its integral representation

The method is explained with the flow-chart of Fig. 2. We start with the following wave equation for the electric field strength

$$\left(\Delta^2 - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{E} = \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2} - \frac{1}{\varepsilon_0 \varepsilon}\,\text{grad div}\,\mathbf{P} \qquad \ldots (1)$$

Where, $\mathbf{P}$ is the electric moment of the dipole and v the phase velocity. The wave equations for media 1 and 2 are thus given by:

$$\left(\Delta^2 - \frac{1}{v_i^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{E}^i(x,y,z,t)$$

$$= \begin{cases} \mu_0 \dfrac{d^2 F(t)}{dt^2}\delta(x,y,z\text{-}d)\mathbf{e}_z - \dfrac{F(t)}{\varepsilon_0\varepsilon_1}\,\text{grad}\,\dfrac{\partial}{\partial z}\delta(x,y,z\text{-}d), & \text{for } i=1 \\ 0, \ \text{for } i=2 & \end{cases} \qquad \ldots (2)$$

where, $i=1$, 2 and $\mathbf{e}_z$ is a unit vector in z-direction.

Applying Laplace transform with respect to time a three-dimensional Helmholtz equation, in the spatial coordinates for the Laplace-transformed field strength $\mathbf{E}^i(x,y,z;s)$, is obtained. This step uses the initial conditions that the electric field strength and its time derivative are zero for $t=0$. The Laplace transform of the electric field strength and its time derivative are zero for $t=0$. The Laplace transform of the electric field can then be obtained as follows:

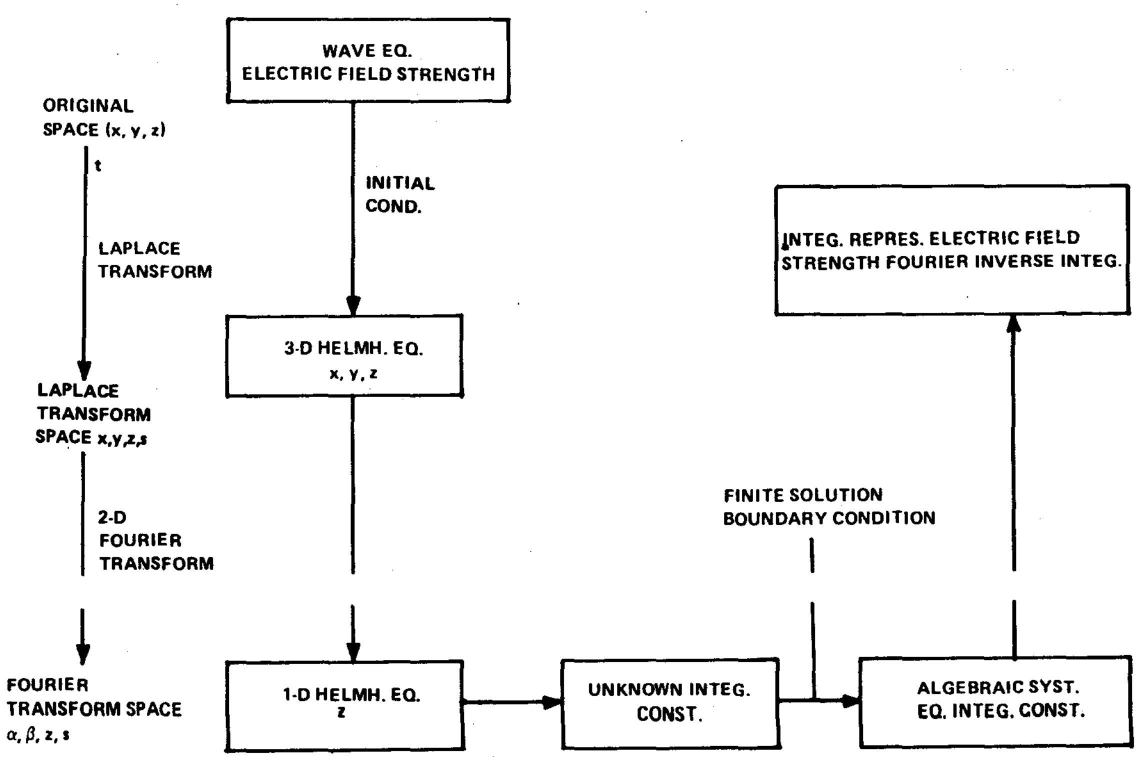

Fig. 2 – Flow-chart for the method of solution

$$\left(\Delta^2 - \frac{s^2}{v_i^2}\right) \mathbf{E}^i(x,y,z;s)$$

$$= \begin{cases} \mu_0 s^2 \mathrm{f}(s)\delta(x,y,z-d)\mathbf{e}_z - \dfrac{\mathrm{f}(s)}{\varepsilon_0\varepsilon_1}\,\mathrm{grad}\,\dfrac{\partial}{\partial z}\delta(x,y,z-d), \\ \qquad\qquad\qquad\qquad\qquad\qquad \text{for } i=1 \\ 0, \quad \text{for } i=2 \end{cases} \quad \ldots (3)$$

The function $\mathrm{f}(s)$ denotes the Laplace transformed excitation function. Representation of this Laplace-transformed field strength, $\mathbf{E}^i(x,y,z;s)$, on a two-dimensional inverse Fourier integral in the following form

$$\mathbf{E}^i(x,y,z;s) = \frac{s^2}{4\pi^2}\iint_{-\infty}^{\infty} \mathbf{E}^i(\alpha,\beta;z;s)\exp\{js(\alpha x+\beta y)\}d\alpha d\beta \quad \ldots (4)$$

leads to a one-dimensional Helmholtz equation in the altitude coordinate z for the Fourier-transformed field, and is given by

$$\left(\frac{\partial^2}{\partial z^2} - \gamma_i^2 s^2\right)\mathbf{E}^i(\alpha,\beta;z;s)$$

$$= \begin{cases} \mathrm{f}(s)\{\dfrac{-js}{\varepsilon_0\varepsilon}(\alpha\mathbf{e}_x + \beta\mathbf{e}_y)\dfrac{\partial}{\partial z}\delta(z-d) + [\mu_0 s^2\delta(z-d) - \\ \qquad\qquad - \dfrac{1}{\varepsilon_0\varepsilon}\dfrac{\partial^2}{\partial z^2}\delta(z-d)]\mathbf{e}_z\}, \text{ for } i=1 \\ 0, \quad \text{for } i=2 \end{cases} \quad \ldots (5)$$

where, $j=\sqrt{-1}$, $\mathbf{e}_x$ and $\mathbf{e}_y$ are unit vectors in the x and y directions, respectively, and

$$\gamma_i^2 = \alpha^2 + \beta^2 + v_i^{-2} \quad \ldots (6)$$

with real part of $\gamma_i \geqslant 0$.

The solution of Eq. (5) could be written in terms of unknown integration constants. These constants are then determined as the solutions of algebraic equations[5] that result from the bounday conditions for the electromagnetic field on the earth's surface and the ionosphere. Further the condition of finiteness of the field at infinity is used. This corresponds to the Laplace-transformed initial conditions and is also used for the determination of the above mentioned constants.

Thus, an integral representation of Laplace transform of the electric field strength in terms of two-dimentional inverse Fourier integral, i.e., for the transformed z component is given by

$$E_z^1(x,y,z;s) = \frac{s^3\mathrm{f}(s)}{8\pi^2\varepsilon_0\varepsilon_1}\iint_{-\infty}^{\infty}(\alpha^2+\beta^2)$$

$$\times\left[\frac{\exp\{-s\gamma_1|z-d|\}}{\gamma_1} + \frac{M}{N}\right]\exp\{js(\alpha x+\beta y)\}d\alpha d\beta \quad \ldots (7)$$

where,

$$M = [\exp\{-s\gamma_1(z+d)\}][1 + r_{12}\exp\{-2s\gamma_2(h-d)\}]$$

$N = \gamma_1[1 - r_{12}\exp\{-2s\gamma_1 h\}]$; and r_{12}, the reflection coefficient at the upper duct boundary, is given by:

$$r_{12} = \frac{v_1^2\gamma_1 - v_2^2\gamma_2}{v_1^2\gamma_1 + v_2^2\gamma_2} \quad \ldots (8)$$

The transformed x and y components of the vertical electric dipole field are not given here, as they are of a similar structure as that of E_z^1 and we concentrate on the vertical polarized component. The same applies for all field components inside the duct layer.

In an earlier paper[5], the components E_x and E_y of the electric field strength were determined in case of a magnetic dipole, i.e., horizontal polarization. In the polar coordinate system E_ϕ can be written as:

$$E_\phi^1(r,\phi,z;s) = -E_x^1\sin\phi + E_y^1\cos\phi \quad \ldots (9)$$

The field representation given by Eq. (7) thus differs from the above horizontal polarization given by Eq. (9). This is due to the differential structure of the pertinent reflection coefficients, C_{12}, for horizontal polarization or, r_{12} for vertical polarization, where

$$C_{12} = \frac{\gamma_1 - \gamma_2}{\gamma_1 + \gamma_2} \quad \ldots (10)$$

Thus it is concluded that the exact solution of the transient behaviour of a dielecric layer, e.g. an atmospheric duct layer over sea, generally depends on polarization.

References

1 Rotheram S, *Marconi Rev* (*GB*), 37 (1974) 18.

2 Kerr D E, *Propagation of Short Radiowaves* (Dover Publications, New York), 1951.

3 Brocks K, Fengler G & Jeske H, *Berichte des Instituts für Radiometereologie und Maritime Metereologie der Universitat*, Hamburg, Germany, 1963, 7.

4 Kahan T & Eckart G, *Ann Phys* (*France*), 5 (1950) 641.

5 Bishay S T, *Can J Phys* (*Canada*), 65 (1987) 376.

6 Langenberg K J & Rech K D, *Proceedings of 6th European Microwave Conference*, Kent, UK, 1976, 86.

7 Zimmer H, Langenberg K J & Becker K D, *Radio Sci* (*USA*), 13 (1978) 261.