Evaluation of the Capacitance and Charge Distribution of Metallic Objects by Electrostatic Analysis

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This paper presents a numerical analysis for computation of free space capacitance of different arbitrarily shaped conductors like square plate, circular disk and T-shaped plate. Accurate evaluations of electrostatic analysis are essential for spacecraft design techniques to control electrostatic discharge (ESD). Capacitance computation is an important step in the prediction of electrostatic discharge which causes electromagnetic interference (EMI). We specifically illustrated capacitance computation of three electrostatic models. Numerical results on capacitance are presented. The results are compared with other available results in the literature. We used the COMSOL multiphysics software for the simulation. The models are designed in three-dimensional form using electrostatic environment and can be applied to any practical design. The findings of this study show that the finite element method (FEM) is a more accurate method for the computation of electrical capacitance.

Keywords: Capacitance, Electrostatic Analysis, Electrostatic Discharge, Finite Element Method, Simulation, Spacecraft

Introduction

The calculation of free space capacitance of different arbitrary shapes like square plate, circular disk and T-shaped plate, which can be considered as significant objects for spacecraft design, is a major unsolved problem in electrostatic theory. The ability of the electrical engineering design depends on how efficiently a physical structure has been modeled. In addition, the studies involving electromagnetic field are usually complex and require a very good working knowledge¹. The evaluation of capacitance of different arbitrarv shapes is important in computational electromagnetics. It deals with the modeling of the interaction between the electromagnetic fields and the physical objects². Compared to the finite difference methods (FDM) and boundary element methods (BEM), the finite element methods (FEM) provide additional elasticity for local mesh refinement, additional rigorous convergence analysis, additional selections of effective iterative solvers for the secondary linear systems and more elasticity for handling the nonlinear equations. The FEM is a standard tool for solving the differential equations in electromagnetics³. It is also one of the most preferred methods in engineering owing to its

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significant ability to deal with complex geometrics. The analyses of three-dimensional square plate. circular disk and T-shaped plate are examined using FEM. This paper investigates the computation of the free space capacitance of the arbitrarily shaped planar conducting objects⁴. The capacitance of arbitrarily shaped conducting bodies had been estimated using of moments with the method rectangular subdomains⁵. The drawback of using rectangular subdomains is that it will not accurately fit into the arbitrarily shaped geometry. Hence, finer anv subdivisions are required to accurately model the surface. In order to avoid the disadvantage, triangular patch modeling⁶ had been employed to exactly modeling the arbitrarily shaped surfaces⁷ encountered in practical situations. In this paper, the capacitance of the different geometrical assemblies was achieved by subdividing the structure into triangular subsections and by computing the charge distribution on each triangular subsection⁸. The capacitances of different conducting structures used in this study are compared with the existing data in the literature. Finding the capacitance of a metallic assembly has been a difficulty with historical significance⁹.

Materials and methods

The expression of capacitance can be introduced by applying the finite element method, i.e., charges and

potentials in any system of conductors that create an electric field, Depending on the nature of the system of conductors measured, the capacitance of a solitary conductor, the capacitance between two conductors, and the capacitance in a system of many conductors can be distinguished. The capacitance of the surface can be computed from

$$\mathbf{c} = \frac{\mathbf{Q}}{\mathbf{V}} \qquad \dots (1)$$

where, Q and V are charge and potential of the conductor. Depending on the type of the system of conductors considered, the capacitance of a solitary conductor, the capacitance between two conductors and the capacitance in a system of many conductors are distinguished. The potential V_e is calculated by

$$Ve(\mathbf{x}, \mathbf{y}) = \mathbf{a} + \mathbf{b}\mathbf{x} + \mathbf{c}\mathbf{y}$$
 (2)

For a triangular element, a, b, and c are constants. The final Jacobian is given by

$$J(\boldsymbol{\xi}_{s}, \boldsymbol{\eta}_{s}) = J(\boldsymbol{\xi}, \boldsymbol{\eta}) \left(\frac{1}{3}\right) (1 - \boldsymbol{\xi}_{s})$$
 (1 (3)

Using the above expressions¹⁰, free space capacitance can be calculated. It is easily fit for the simulation software.

Simulation

Electrical capacitance of square plate

In this section demonstrate the modeling of the square plate by determining free space capacitance using finite element method. The capacitance values are calculated for some cases with respect to the difference in the number of domain elements and boundary elements in square plate. A flat conducting square plate having a dimension of 1×1 m² is

considered for the capacitance calculation. The model is calculated in three - dimensional environments to compare our results with some of the other available results⁶. The topmost of the plate has a specified voltage of 1 V DC, and the bottommost of the plate is specified as ground with a voltage of 0 V. It is assumed that the plates are finished of extremely conductive material. When the unit voltage is applied to the body of the plate, the charge densities nearby the edge of the bodies are much advanced than those away from the edges. The simulation produces the finite element mesh with triangular subsections and 120 boundary elements as shown in figure 1a. It shows the three-dimensional view of the metallic plates with triangular subsections. Figure 1b shows the charge distributions of inhomogeneous media of metallic plates. The charge distribution is constant between the plates, but variances can be seen at the side of the plate. The charge distribution shows that the topmost edge of the plate has more current flow compared to the bottommost of the plate.

The outcomes of the finite element method for the capacitance per unit length of metallic plates compared with the earlier works¹¹. The results are in good agreement with the earlier results. The capacitance value 40.17 pF is obtained. In this section, the square plate simulations with different subsections are analyzed. The same methodology is adopted for all the remaining shapes.

Electrical capacitance of circular disk

In this section, the FEM is used to calculate the electrical capacitance of a circular disk. Figure 2a shows a circular disk with a radius of 1m. The exact formula for circular disk is

í (4)



 $C = 8 \varepsilon_0 a$

Fig. 1ô Square plate (a) Triangular subsection, (b) Charge distribution

where,

⁰ is the permittivity of free space a is the radius of the circular disk.

Calculating the free space capacitance of a simple object like a circular disk is important in spacecraft design. The model is designed based on threedimensional modeling using electrostatic environment. We produced number of subsections from the circular disk model like 2058 domain element of the finite element mesh. From the model, we produced the finite element mesh with triangular subsections and 504 boundary elements⁵. Figure 2b shows the surface charge distribution of the circular disk. The potential distributions of inhomogeneous media of metallic surface are simulated. The capacitance values are calculated for some cases such as with respect to variation in a number of domain elements and boundary elements. The capacitance value obtained, 70.48 pF, is equated with the value obtained from the earlier results. The results tend to converge and the deviation in analytical and numerical results is reduced.

In this section, the circular disk simulations with different subsections are analyzed. The results obtained in this study correspond with the available results¹².

Electrical Capacitance of T-shape plate

In this section, the FEM is used to calculate the electrical capacitance of T-shaped plate. Figure 3a shows a T-shaped plate with a dimension of 3.0 m, 1.0 m, and 3.0 m. The model is designed based on three-dimensional modeling using electrostatic environment. We produced number of subsections from the T-shaped plate model like 180 domain element of the finite element mesh. The charge



Fig. 2ô Circular disk (a) Triangular subsection, (b) Charge distribution



Fig. 3ô T - shaped plate (a) Triangular subsection, (b) Charge distribution

distribution simulations help to better understand the potential distribution of the metallic object. From the model, we produced the finite element mesh with triangular subsections¹⁴ and 504 boundary elements. Figure 3b shows the three-dimensional surface charge distribution of the metallic T-shaped plate with triangular subsections¹⁵. The capacitance values are calculated for some cases such as with respect to variation in a number of domain elements and boundary elements. The capacitance value obtained, 32.45 pF, is equated with the value obtained from the earlier results. The results tend to converge and the deviation in analytical and numerical results is reduced. In this section, the T-shaped plate simulations with different subsections are analyzed. The results obtained in this study correspond with the available results¹⁶.

Results and Discussions

This present investigation demonstrates that the convergence can be obtained for the metallic object for a finite number of elements. A computer program based on the FEM was simulated to determine the capacitance and charge distribution of the square plate, circular disk and T-shaped plate. The results of three geometries are summarized in Table 1 and compared with the earlier results⁶. Using the finite element method, more accurate value is achieved. The process outlining the usage of triangular subsection produces more accurate value. All simulations were performed on a PC with core-i3 processor with 3.1 GHz CPU and 8 GB of RAM. However, this study has certain limitations that have been acknowledged. The simulation was confined to other methods like random walk method and moment method. Nevertheless, in spite of these limitations, this study provided new insights into the capacitance computation of different conducting bodies. Using the finite element method, more accurate value is achieved¹⁴. In the present study, the COMSOL program package has been used and numerical computations have been performed. The capacitance of the circular disk was found to be 70.48 pF, which is similar to capacitance obtained in other studies¹³ based on the method of moments, i.e., 70.73 PF. Moreover, in another study, the capacitance of the square plate was found to be 40.17 pF, which is similar to the capacitance obtained in other studies⁶ based on the method of moments, i.e., 40.27 pF. In another study, the capacitance of the T-shape plate was found to be 32.45 pF, which is similar to the

Table 1ô Comparison of capacitance		
Geometry	Proposed Method	Method of Moment
	Capacitance	Capacitance
Square plate	40.17 pF	40.27 pF
Circular disk	70.48 pF	70.73 pF
T-shaped plate	32.45 pF	32.11 pF

capacitance obtained in other studies⁵ based on the method of moments, i.e., 32.11 pF. The applying of numerical techniques contains the usage of computers and appropriate program packages. The capacitance of the square plate, circular disk and T-shaped plate are evaluated based on FEM. The outcome of the present study shows that the FEM is more effective and accurate than the other methods.

Conclusion

FEM has been found to be the most accurate method for evaluating electrical capacitances. In the present study, different arbitrary shapes are analysed for circuit modeling. Some of the simulations obtained on the study show the usage of FEM with COMSOL Multiphysics software. The results derived from using the software correspond with the results of previous studies. Thus, the method is more suitable for various shapes involved in electric circuit modeling. This approach was simple and can be applied to any practical shapes of the metallic objects. The results were confined to previous results.

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