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# Fractal dimensions of convective clouds around Delhi

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Clouds exhibit fractal structure over wide ranges of scale. Following the method suggested by Lovejoy [*Science* (*USA*), 216 (1982) 185], fractal analysis of convective clouds, observed within 100 km around Delhi during the monsoon season (July-September) of 1977, has been carried out. The study is based on the radar observations collected with a high power 3-cm weather radar. A total number of 2568 radar echoes of convective clouds having areas between 4 and 7420 $km^2$ were analysed for studying the relationship between their perimeters ($P$) and areas ($A$). The analysis showed that the relationship between the above two echo parameters can well be represented by a formula $P \sim A^{D/2}$, where $D$ is described as the fractal dimension of the cloud echo perimeter. However, it was found that the two fractal dimensions of the perimeters are required to describe the above perimeter-area relationship applicable for the entire range of cloud echo area. That is, $D$ =1.34 for cloud echoes having areas up to 600 $km^2$ and $D$ =1.79 for echoes having areas greater than 600 $km^2$.

## 1 Introduction

Clouds are self-similar fractals because they show a consistency in the relation between their areas ($A$) and perimeters ($P$) over a wide range of length scale[1,2]. The concept of fractals provides a systematic approach toward the quantitative characterization of the fragmented structures like clouds. It has appreciably contributed to the understanding of the turbulent diffusion in the atmosphere and the physical mechanisms of the formation of clouds[3,4]. A dimensional analysis of such fractal areas using $A$-$P$ relation helps in understanding the irregularity or degree of contortion of the perimeters enclosing the areas. The term "cloud" in the present context refers to the radar-determined cloud, i.e. the cloud echo as seen on PPI scope of the radar.

According to Lovejoy[1], the $A$-$P$ relation may be expressed as $P \sim A^{D/2}$, where $D$ is the fractal dimension of perimeter of clouds which characterizes the degree of contortion of the perimeter enclosing the horizontal area of the clouds. If the shape of the cloud area is smooth, the length of the perimeter to enclose the area will be minimum and the value of $D$ will be 1, the dimension of a line. The $A$-$P$ relation then takes the form $P \sim A^{1/2}$. As the cloud area becomes more and more irregular the perimeter required to enclose the area will be more and more contorted, increasing thereby the length of the perimeter. This contortion accounts for the fractional value of the dimension of the perimeter. That is, the fractal dimension is a function of irregularity of the surfaces. The upper limit of the contortion of perimeters will tend to double back on itself, filling the area in the plane. This leads the above relation to look as $P \sim A$ and $D$ approaches the value 2.

In our earlier work[5] on the fractal dimension of convective clouds around Delhi, we found that the pre-monsoon convective cloud echoes show consistency in the relation between area and perimeter with area ranging from 4 to 984 $km^2$ and fractal dimension being 1.42. Also, it may be added that out of 397 cloud echoes sampled during the period of that study, most of them related to smaller size ranges.

Table 1—Radar characteristics

| | |
|---|---|
| Wavelength | 3.2 cm |
| Peak power transmitted | 250 kW |
| Pulse length | 1 μs |
| Minimum detectable signal | −90 dBm |
| Pulse repetition frequency | 300 Hz |
| Horizontal and vertical beamwidth | 1.2° |

The present study which is a continuation of the above work deals with the size distribution and the fractal dimension of convective cloud echoes observed within 100 km around Delhi during monsoon season (July-September) of 1977. This study was taken up also to investigate whether there exists any marked difference between the fractal nature of convective clouds in summer season and that in monsoon season.

## 2 Data used and method of analysis

The study is based on hourly radar observations of precipitation echoes from convective clouds within 100 km around Delhi, made during the monsoon (July-September ) season of 1977. An X-band Japanese radar of type NMD-451A has been used for this purpose. The characteristics of the radar set are given in Table 1. The observations were mainly taken from 1000 to 1700 IST (0430 UTC to 1130 UTC). Hourly photographs of PPI observations obtained at an antenna elevation angle of 2.5° were sorted out. From these photographs, precipitation echoes from convective clouds were selected and their areas and perimeters were measured. In all 2568 echo cases were sampled during the period of study. The perimeters of the individual cloud echoes were measured with a resolution of 2 km and the areas were measured with a resolution of 4 km$^2$ by counting the number of 2 km × 2 km grids covered by the individual cloud echoes. The range of radar observations for the present study has been confined up to 100 km only. This is due to the fact that for larger ranges errors may be introduced in the observations of the cloud dimensions due to the effect of radar antenna beamwidth and earth's curvature, and hence the computation of fractal dimension may lead to some erroneous results.

## 3 Results and discussion

### 3.1 Size distribution of convective echoes

Before we proceed to discuss the fractal statistics of the radar echoes from convective clouds sampled for the present study, it is important to discuss first their size distribution. For this purpose, echoes have been classified into different groups according to their areas as shown in Table 2. It may be seen that out of 2568 cloud cases, areas of about 60 per cent of them were less than or equal to 16 km$^2$, areas of as much as 87 per cent were less than or equal to 60 km$^2$, and areas of only 2.9 per cent exceeded 600 km$^2$. Area of the smallest cloud echo was 4 km$^2$ and that of the largest was 7420 km$^2$.

Table 2—Distribution of echo areas

| Size distribution km$^2$ | No. of cases | Percentage of cases |
|---|---|---|
| ≤ 16 | 1530 | 59.6 |
| 20-60 | 711 | 27.7 |
| 64-100 | 121 | 4.7 |
| 104-200 | 62 | 2.4 |
| 204-400 | 43 | 1.7 |
| 404-600 | 27 | 1.0 |
| 604-1000 | 19 | 0.7 |
| 1004-1400 | 18 | 0.7 |
| 1404-2200 | 14 | 0.6 |
| 2204-3000 | 10 | 0.4 |
| 3004-4200 | 7 | 0.3 |
| ≥ 4200 | 6 | 0.2 |
| Total | 2568 | 100 |

### 3.2 Area-perimeter relationship and fractal dimension of convective clouds

Figure 1 shows a scatter plot of logarithm of perimeter (*P*) versus logarithm of area (*A*) for the 2568 convective echoes. It may be mentioned that a large number of echoes have been found to have the same perimeter for a particular area of the echoes mainly in the small echo range. As such, the number of points plotted in the scatter diagram is less than the actual number of cloud echoes sampled. The trend analysis suggests that the points in the scatter diagram can be fitted by two straight lines. As such, the points were fitted to two straight lines by the method of least squares. Line 1 is for cloud echoes with areas less than or equal to 600 km$^2$ (covering 97 per cent of total echo population) and line 2 is for the echoes with areas larger than 600 km$^2$ (covering 3 per cent of the total population).

Equations for the two regression lines, 1 and 2, are given by

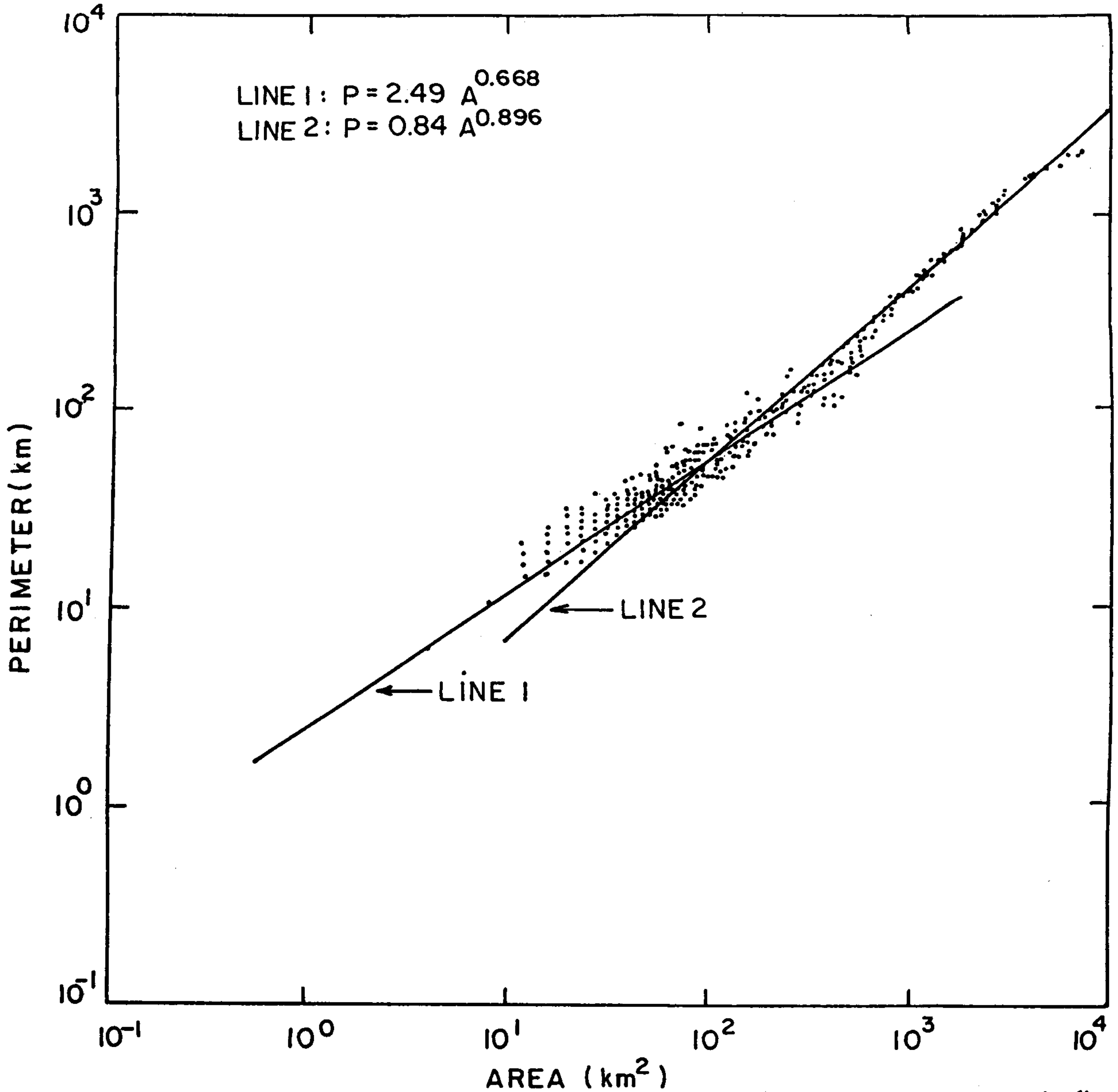


Fig. 1—Log-log plot of the perimeters of radar echoes of convective clouds as a function of their areas. Two regression lines have been fitted to the points representing area ($A$) - perimeter ($P$) relationships of the echoes (Line 1 is for echoes of areas up to 600 km$^2$, Line 2 is for echoes of areas greater than 600 km$^2$; corresponding regression formulae are also indicated in the figure).

$\log P = 0.668 \log A + 0.397$ ...(1)

$\log P = 0.896 \log A - 0.077$ ...(2)

From Eqs (1) and (2) and the relation $P = P_0 A^{D/2}$, where $P_0$ is the constant of proportionality, least square estimates of the fractal dimension ($D$) of perimeters of convective echoes have been deduced for the two groups as

(a) For clouds with echo areas $\leq 600$ km$^2$

$D = 2 \times$ slope of the line 1 $= 2 \times 0.668 = 1.34$ ...(3)

$\log P_0 = 0.397$ or $P_0 = 2.94$ ...(4)

(b) For clouds with echo areas $> 600$ km$^2$

$D = 2 \times$ slope of the line 2 $= 2 \times 0.896 = 1.79$ ...(5)

$\log P_0 = 0.077$ or $P_0 = 0.84$ ...(6)

The present results indicate that the smaller cloud echoes (area $\leq 600$ km$^2$) which occur in majority are less irregular in shape than larger cloud echoes. It is assumed that the clouds are self-similar fractals, i.e. with no characteristic length scale. In other words, they are statistically invariant under certain transformations of scale. The break in the slope, as obtained in our study of *A-P* relation of convective cloud echoes, reveals that the assumption of scale invariance over the whole gamut of cloud size can be discarded. Cahalan and Joseph[4] have also fitted two lines through the points of *A-P* relation, but the cloud echo area at which the break observed by them (3.14 $km^2$) is quite different from that noticed in the present study (600 $km^2$). Rather, scale invariance appears to be an excellent assumption for the smaller and larger cloud groups separately. Although, this property has been found to hold good for larger clouds too, it is difficult to emphasize the result for the reason that the data points are less in number.

In the present study the grouping of smaller and larger clouds has been done on the basis of the trend of scatter plot in Fig. 1. There is a need to search some fundamental criteria for such groupings which must serve in evaluating the break point in the slope of *A-P* relation. Also, there is a need to further investigate the upper limit of larger clouds predicted by line 2.

In our earlier study of fractal dimension[5], the data points representing convective echo areas greater than 600 $km^2$ were meagre (only 0.6 per cent of the total population of 397) which aligned to the same line trend. As such, the property of scale invariance appeared to exist over the whole range of the observed cloud area. This is consistent with the findings of Jain[2] who suggested that the assumption of scale invariance holds good for convective cloud echo areas even up to 5350 $km^2$. The fractal dimensions obtained in our previous and present studies depict that, in general, the wiggliness in perimeters of summer cloud echoes lies in between the wiggliness in perimeters of smaller and larger monsoon cloud echoes. The fractal dimension obtained in the present study for smaller clouds (1.34) is in good agreement with that observed by Lovejoy[1] (1.35) and Jain[2] (1.30).

Visual examination of the radar echoes indicates that most of the small convective echoes which have happened to occur in isolation in the cloud field look less irregular in shape and have uniform brightness throughout. However, many other cloud echoes with areas limited to 600 $km^2$ have been found to show several regions of pronounced and diffused brightness. This suggests cloud merging as a formation process for such clouds. Because of the merging of several cloud echoes, the contact points exhibiting diffused brightness there give rise to a more contorted perimeter enclosing the whole area of the so formed clouds. Such clouds have also been found to occur in the same scaling regime as represented by line 1 of Fig.1. This implies that the process of cloud merging to form larger clouds does not alter the shape of smaller clouds but only magnifies it. The larger cloud echoes with areas greater than 600 $km^2$ are probably formed when convective cells are accompanied by stratiform echoes (as inferred from the presence of melting band). Chatterjee *et al.*[6], in their study of convective clouds within 100 km around Delhi during monsoon season of 1967-72, have shown that the clouds having areas up to 600 $km^2$ are formed either by the process of random entrainment of environmental air or by merging of smaller convective elements and the larger clouds are the result of stable mesoscale extension of the stratiform echoes accompanying the convective cells. This feature explains the higher fractal dimension for the larger cloud echoes.

Cahalan and Joseph[4] have suggested that the smaller clouds develop by the boundary layer convection while the larger clouds by cloud merging. They have also suggested that, perhaps, the cut-off scale occurring in the *A-P* relation can be determined by the thickness of the boundary layer, and change in fractal dimension can be related to a change in three-dimensional turbulence in the boundary layer to two-dimensional turbulence involving cloud mergers in the free atmosphere. But according to their conception smaller clouds are those clouds which have diameters less than 2 km (corresponding to an area of 3.14 $km^2$). In our work, smaller clouds range from an area of 4 $km^2$ to 600 $km^2$. Thus, the study suggests an additional regime which may range from an area in the vicinity of 4 $km^2$ down to the limit of the resolution. There is clearly a need for more research on the factors determining the size of the convective cells, and more generally on the relation of different scaling regimes to different physical processes in the atmosphere.

### 3.3 Fractal dimension of individual convective echoes

Using the relation $P = P_0 A^{D/2}$, where $P_0 = 2.94$ for convective echoes with areas $\leq 600$ $km^2$ and $P_0$

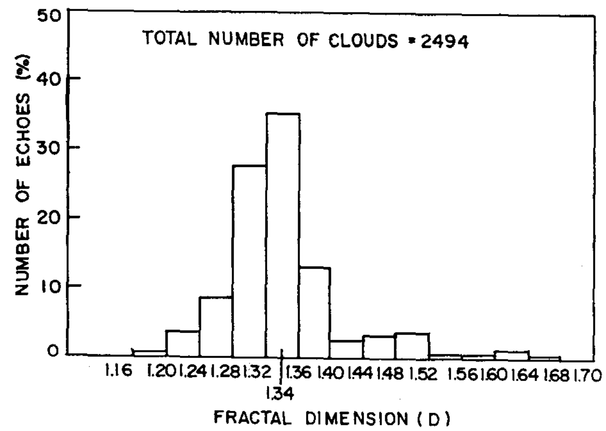


Fig. 2—Percentage frequency distribution of the fractal dimension (*D*) of perimeters of the radar echoes from convective clouds of areas up to 600 km$^2$

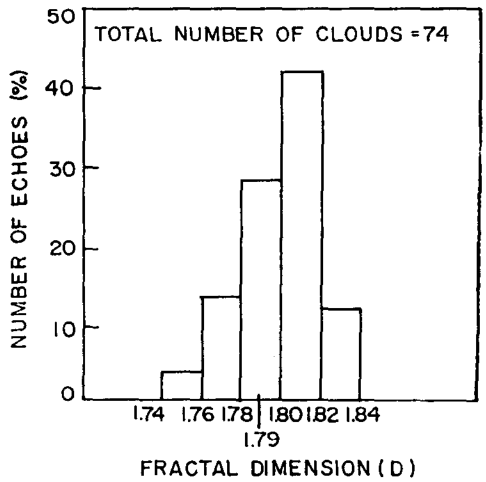


Fig. 3—Same as Fig. 2 but for areas greater than 600 km$^2$

= 0.84 for convective echoes with areas > 600 km$^2$, fractal dimensions of individual convective echoes have been determined in order to study their spread about the least square estimate of fractal dimension $D$ = 1.34 and 1.79 respectively for the two groups of echoes. Frequency distribution of the values of $D$ in different class intervals for both the groups is shown in the forms of histograms in Figs 2 and 3 respectively. It may be seen that for the group of echoes with areas limited up to 600 km$^2$ (Fig. 2) the range of $D$ values lies between 1.16 and 1.68 with standard deviation of 0.065. Their mean, median and modal values are 1.34, 1.33 and 1.34 respectively. The spread of $D$ values from the normal distribution is very small and the deviation shows positive skewness. But when the result is compared with the standard Gaussian and log-normal distributions, it came out to be highly significant (at 0.1 per cent level) in both the cases, i.e. the result is highly deviated from these distributions. Out of the 2494 convective echoes in this group, as many as 2170 echoes (i.e. 87 per cent) exhibited $D$ = 1.34 ± 0.10. For the group of cloud echoes with areas greater than 600 km$^2$, range of $D$ values varied between 1.74 and 1.84 with standard deviation of 0.020. This implies less dispersion of $D$ values around the mean. Their mean, median and modal values are 1.79, 1.80 and 1.80 respectively. This shows a deviation in negative direction from normality. Again the Gaussian and log-normal fittings through this spread showed the results to be highly significant (at 0.1 per cent level). Out of the 74 convective echoes in this group, 62 echoes (i.e. 84 per cent) exhibited $D$ = 1.79 ± 0.03 and 100 per cent of the echoes exhibited $D$ = 1.79 ± 0.05.

## 4 Summary and conclusions

The area-perimeter relationship and fractal dimensions of the perimeters of convective clouds around Delhi have been studied using the radar data of precipitating clouds collected during the monsoon (July-September) season of 1977. A total number of 2568 cloud cases with areas varying between 4 and 7420 km$^2$ have been analysed. A scatter plot of logarithm of perimeter ($P$) versus logarithm of area ($A$), wherein the points are fitted with two straight lines by employing the method of least squares, suggests that two fractal dimensions are needed to describe the $A$–$P$ relationship for convective clouds occurring in Delhi region during monsoon season, i.e. $D$ = 1.34 for convective echoes with areas ≤ 600 km$^2$ (constituting about 97 per cent of the total echoes) and $D$=1.79 for the larger areas (> 600 km$^2$) of more complex in structure. Visual inspection of radar-determined clouds along with the analysis of observations indicates that the process of formation of clouds having areas up to 600 km$^2$ involves merging of smaller cloud cells. In this process the size of the clouds appears as magnified version of the smaller cloud cells and their structure in the horizontal plane remains almost the same.

The break in the slope of the scatter plot (Fig. 1) is a clear indicator of the existence of some different mechanism for the clouds grown to be larger than 600 km$^2$ in area. Formation of such larger clouds could be due to the convective cells accompanied by the stratiform echoes. This feature is considered for the large value of the fractal dimension for the observed larger clouds. The present study points out that one more scaling regime exists in the fractal structure of clouds ranging from an area in the proximity of 4 km$^2$ down to the limit of resolution. It may be mentioned here that in our previous study of the structure of convective clouds of summer season, only one fractal dimension ($D$=1.42) was needed to describe the log (area)-log (perimeter) relationship. This was due to the reason that out of the total convective cloud echoes sampled in the summer season percentage of large echoes was very small. Areas of only 0.6 per cent of the total echo population of 397 exceeded 600 km$^2$ in the summer season as compared to 2.9 per cent in the monsoon season. Maximum echo area observed in the summer season was 984 km$^2$ whereas in the monsoon season it has been found to be 7420 km$^2$.

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