Determination of energy gap directly from phase shift for nuclear systems with large neutron excess

M Murila[a]*, Y K Ayodo[a], K M Khanna[b], E S Miller[a] & J Indumuli[a]

[a]Department of Physics, Masinde Muliro University, Box 190 Kakamega, Kenya

[b]Department of Physics, University of Eldoret, Box 1125 Eldoret, Kenya

Considering $^1S_0$ pairing in infinite neutron matter and nuclear matter, and knowing the fact that in the lowest order approximation, the pairing interaction has been taken to be the bare nucleon-nucleon interaction in the $^1S_0$ channel, the energy gap has been determined directly from the $^1S_0$ phase shifts. The values of energy gaps have been found to increase rapidly for low values of phase shifts, up to around 16°, and it is roughly constant with a value approximately equal to 1.08 MeV for phase shifts[1] greater than 16°.

1 Introduction

The energy gap is determined to a remarkable extent by the available $^1S_0$ phase shifts. Thus, the quantitative features of $^1S_0$ pairing in nuclear matter and neutron matter can be obtained directly from the $^1S_0$ phase shifts. This happens because the nucleon-nucleon interaction is very nearly rank-one separable in this channel due to the presence of a bound state at zero energy, even for densities as high as $k_F = 1.4$ fm^{-1}. This explains why all bare nucleon-nucleon interactions give nearly identical results for the $^1S_0$ energy gap in lowest-order Bardeen Cooper and Schrieffer calculations.

The $^1S_0$ neutron matter super fluid is relevant for phenomena that can occur in the inner crust of neutron stars, like the formation of glitches, which may be related to vortex pinning of the superfluid phase in the solid crust.

The results of different groups are in close agreement on the $^1S_0$ pairing gap values and on its density dependence, which shows a peak value of about 3 MeV at a Fermi momentum[2] close to $k_F \approx 0.8$ fm^{-1}. All these calculations adopt the bare nucleon-nucleon interaction as the pairing force, and it has been pointed out that the screening by the medium of the interaction could strongly reduce the pairing strength in this channel[3].

The calculation of the $^1S_0$ gap in symmetric nuclear matter is closely related to the one for neutron matter.

*Corresponding author (E-mail: murilangu@gmail.com)

Even with modern charge-dependent interactions, the resulting pairing gaps for this partial wave are fairly similar[2].

The size of the neutron-proton (np) $^3S_1$-$^3D_1$ energy gap in symmetric or asymmetric nuclear matter has, however, been a much debated issue since the first calculations of this quantity appeared. While solutions of the Bardeen-Cooper and Schrieffer equations with bare nucleon-nucleon forces give a large energy gap of several MeVs at the saturation density[4,5,6] $k_F = 1.36$ fm^{-1} and there is little empirical evidence from finite nuclei for such strong np pairing correlations, except possibly for isospin $T = 0$ and $N = Z$.

One possible resolution of this problem lies in the fact that all these calculations have negligible contributions from the induced interaction. Fluctuations in the isospin and the spin-isospin channel will probably make the pairing interaction more repulsive, leading to a substantially lower energy gap. One often-neglected aspect is that all non-relativistic calculations of the nuclear matter equation of state (EoS) with two-body nucleon-nucleon forces fitted to scattering data fail to reproduce the empirical saturation point, seemingly regardless of the sophistication of the many-body scheme employed. For example, a Brueckner-Hartree-Fock (BHF) calculation of the EoS with recent parameterizations of the nucleon-nucleon interaction would typically give saturation at $k_F = 1.6$-1.8 fm^{-1}.

In fact, by the advent of Effective Field Theory (EFT) and applying it to the low-energy Quantum Chromodynamics (QCD), we are somehow coming back to the meson-exchange theories with the aid of Chiral Perturbation Theory (CHPT). Describing the atomic-nuclei properties in terms of the interactions between the nucleon pairs is indeed the main goal of nuclear physics.

Apart from relatively weak electric forces, the nuclear interactions between two protons are very similar to those between two neutrons. This yields the idea of charge symmetry of the nuclear forces. Furthermore, the proton-neutron interaction is also very similar.

2 Experimental Results for Phase Shifts and Energy Gap

In determining phase shifts and energy gap experimentally using the CD-Bonn potential[7], the Nijmegen I and Nijmegen II potentials[8], the results are virtually identical, with the maximum value of the gap varying from 2.98 MeV for the Nijmegen I potential to 3.05 MeV for the Nijmegen II potential. The agreement between the direct calculation from the phase shifts and the CD-Bonn and Nijmegen calculation of Δ_F is satisfying, even at densities as high as $k_F = 1.4$ fm^{-1}. It shows that the values of energy gap increase with increase in Fermi momentum then reach saturation where further increase in Fermi momentum the energy gap becomes constant.

Maximal pairing correlations at the Fermi surface for the relativistic versions A, B, and C of the Bonn potential and for the Gogny force $D1$ are Bonn A $\Delta(k_f) = 2.80$ MeV for 0.76 fm^{-1}, Bonn B $\Delta(k_f) = 2.84$ MeV for 0.76 fm^{-1}, Bonn C $\Delta(k_f) = 2.83$ MeV for 0.76 fm^{-1} and Gogny $D1$ $\Delta(k_f) = 2.78$ MeV for 0.80 fm^{-1}.

It is found that the neutron pairing gap Δ_{F_n} is strongly dependent on the Fermi momentum, or equivalently, the nuclear matter density. Energy gap, Δ_{F_n}, increases as the Fermi momentum (or density) goes down, reaches a maximum at $k_{F_n} \approx 0.8$ fm^{-1}in symmetric nuclear matter or $k_{F_n} \approx 0.9$ fm^{-1}in pure neutron matter, and then rapidly drops to zero[9]. A systematical enhancement of about 0.3 MeV for Δ_{F_n} around $k_{F_n} \approx 0.8$ fm^{-1} is revealed in pure neutron matter compared with those in symmetric nuclear matter for all of the adopted pairing interactions[10].

In comparison with the Bonn-B potential, it is seen that the Gogny interactions especially $D1$ have larger

pairing gap when approaching to the saturation density. At $k_{F_n} = 0.8$ fm^{-1}, among the adopted pairing interactions, $D1$ gives the maximum values of Δ_{F_n}, namely, 3.13 MeV in symmetric nuclear matter and 3.40 MeV in pure neutron matter.

As the density decreases, the values of Δ_{F_n} from D_1, D_{1S} and D_{1N} become first consistent with and then smaller than the one from Bonn-B potential gradually. As illustrated in a recent work, the pairing strength required for appearance of a dineutron Bose Einstein Condensation (BEC) state in the low-density limit must be stronger than 1.1 times of Bonn-B potential, and the corresponding pairing gap Δ_{F_n} is 4.12 MeV around $k_{F_n} = 0.8$ fm^{-1}. Therefore, it is expected here again with the Gogny pairing interactions that a true dineutron BEC state cannot occur at any density in nuclear matter.

For the $^1S_0$ channel in nuclear physics, depending upon the number of neutrons, when there is a nucleon-nucleon interaction such that their spins are aligned opposite, the angular momentum is zero. The channel for such neutron excess will be $^1S_0$. There are two weak coupling limits. One is when the potential is weak and attractive for large inter-particle spacing and second when the potential becomes repulsive at $r \approx 0.6$ fm. The potential has a value of some few MeV. In the strong coupling limit, the nucleon-nucleon potential is large and attractive; its value reaches a maximum of around 100 MeV at $r \approx 1$ fm.

3 Theoretical Formulations

At low energies, the effective interaction between two particles is determined by the S-wave scattering length, a_0. For proton-neutron in a singlet spin state, the scattering length is -23.7 fm implying a very strong attraction between two nucleons in the spin singlet state, which is large compared with the range of nuclear interactions of ≈ 1 fm. The two-body Hamiltonian equation for an assembly of nucleon particles in the ground state will be used as a basis for the study as given:

$$\hat{H} = \hat{H}_1 + \hat{H}_2 = \sum_\alpha \varepsilon_\alpha a_\alpha^+ a_\alpha + \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta} a_\alpha^+ a_\beta^+ a_\delta a_\gamma \quad ...(1)$$

where, $\hat{H}_1$ represents the kinetic energy of the system,

$\hat{H}_2$ represents the potential energy of the system, a^+ is the fermion creation operator, a is the fermion annihilation operator, $V_{\alpha\beta\gamma\delta}$ represents the coupled

matrix elements of the two-body interaction $V(r)$. The sums run over all possible single-particle quantum numbers. The pairing gap is the attractive potential for $k \leq 1.74$ fm^{-1}, or for inter-particle distances $r \geq 0.6$ fm.

The second term in Eq. (1) shows that in the nucleus there is interaction of the paired nucleons which are created and annilated which brings about the phase shift and energy gap in the system at a particular potential at the Fermi surface.

The exponential potential is used in this research in derivation of phase shift and energy gap, since it has not been much explored by other researchers fully like the Gaussian well potential and Yukawa well potential[11]:

$$V(r) = -V_0 e^{-2r/\beta} \qquad ...(2)$$

where, V_0 is the potential well depth, $V(r)$ is the interacting potential, β is the range of the nucleon-nucleon force and r is the inter-particle distance. This potential is the basis for this research since it is substituted inthe Born-approximation phase shifts, $\delta_\ell(k_f)$, for scattering from a spherical potential, $V(r)$, in 3-D to find the new values of phase shifts.

The pairing gap[12,13,14] for small values of $k_f|a_0|$ is:

$$\Delta(k_f) = \frac{8}{e^2} \lambda \exp\left(-\pi/2k_f|a_0|\right) \qquad ...(3)$$

where a_0 is the scattering length in the $^1S_0$ channel ($a_0 = -23.7$ fm), λ is a constant $\lambda \approx 1$ and $e = 2.718$. Here a_0 is related to the interaction potential between a pair of nucleons. However, at saturation density $\rho_0 = 0.17$ fm^{-3} and $k_f = 1.36$ fm^{-1}.

For low energy scattering, especially in nuclear physics, the phase shift, $\delta_\ell(k_f)$, due to scattering is given by the relation[2,15]:

$$k_f \cot \delta_\ell(k_f) = -\frac{1}{a_0} + \frac{1}{2} r_0 k_f^2 \qquad ...(4)$$

here r_0 is the effective range of the nuclear force which roughly corresponds to the size of the potential and $\delta_\ell(k_f)$ is the S-wave scattering phase-shift.

Phase shift for nuclear with large neutron excess is given by:

$$\delta_0(k_f) = 1.6899 k_f \qquad ...(5)$$

For scattering length, $a_0 = -23.7$ fm which is the scattering length in the $^1S_0$ channel, epsilon, $e = 2.718$

and $\lambda \approx 1$ substituting in Eq. (5) for energy gap $\Delta(k_f)$ it becomes:

$$\Delta(k_f) = 1.0827 . \exp\left(\frac{-0.0663}{k_f}\right) \qquad ...(6)$$

To determine a formula correlating phase shift $\delta_\ell(k_f)$ and energy gap $\Delta(k_f)$ with the nucleon-nucleon interaction, we make k_f the subject of the formula in Eq. (5) hence the equation becomes:

$$k_f = \frac{\delta_0(k_f)}{1.6899} \qquad ...(7)$$

Substituting Eq. (7) in Eq. (6) the new equation of energy gap $\Delta(k_f)$ in correlation to phase shift $\delta_\ell(k_f)$ becomes:

$$\Delta(k_f) = 1.0827 \exp\left(-0.1120/\delta_0(k_f)\right) \qquad ..(8)$$

This equation is useful in calculation of values of energy gap $\Delta(k_f)$ for specific values of phase shift $\delta_\ell(k_f)$ in large nuclear systems. We consider $^1S_0$ pairing in infinite neutron matter and nuclear matter and show that in the lowest order approximation, where the pairing interaction is taken to be the bare nucleon-nucleon interaction in the $^1S_0$ channel, the pairing interaction and the energy gap can be determined directly from the $^1S_0$ phase shifts. This is due to the almost separable character of the nucleon-nucleon interaction in this partial wave[1].

This means, in turn, that we can, through an inspection of experimental scattering data, understand which partial waves may yield a positive pairing gap and eventually lead to a superfluid phase transition in an infinite fermionic system.

4 Results and Discussions

Equation (8) is useful in calculation of energy gap $\Delta(k_f)$ for specific values of phase shift $\delta_\ell(k_f)$ in large nuclear systems. The variation of energy gap $\Delta(k_f)$ against phase shift $\delta_\ell(k_f)$ is studied using mathCAD 2000 professional software and the data tabulated. A graph depicting the variation of energy gap versus phase shift is drawn as shown in Fig. 1.

The energy gap, $\Delta(k_f)$, increases steadily and faster for low phase shift , $\delta_\ell(k_f)$, up to around 16°
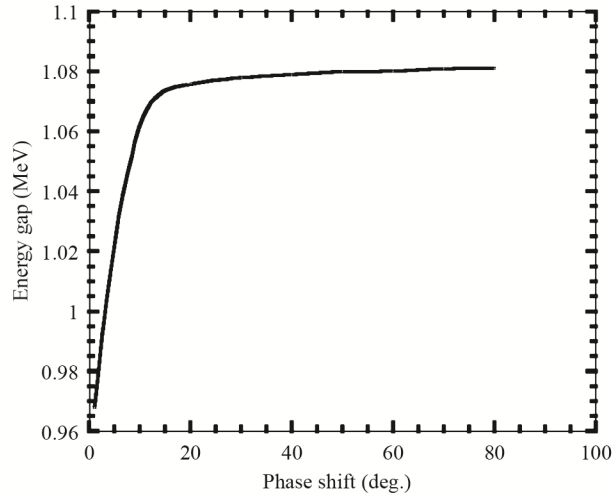
Fig. 1 – A graph of energy gap against phase shift

and it is roughly constant with a value $\cong 1.08$ MeV for phase shifts, $\delta_\ell(k_f) > 16$ degrees. Considering $^1S_0$ pairing in infinite neutron matter and nuclear matter and showing that in the lowest order approximation, where the pairing interaction is taken to be the bare nucleon-nucleon interaction in the $^1S_0$ channel, the pairing interaction and the energy gap, $\Delta(k_f)$, can be determined directly from the $^1S_0$ phase shifts[1], $\delta_\ell(k_f)$.

5 Conclusions

The bare nucleon-nucleon interaction in the $^1S_0$ channel, the pairing interaction and the energy gap, $\Delta(k_f)$, can be determined directly from the $^1S_0$ phase shifts, $\delta_\ell(k_f)$.

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