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LEARNING DIFFERENTIAL EQUATIONS: A METASYNTHESIS OF QUALITATIVE RESEARCH

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ABSTRACT

Teaching methodology is continually developing over time. There is a need to revise the methods that have been planned and examined. This research explores the development of Differential Equations teaching methods from the year 2000 onwards. The methodology used in this paper is metasynthesis research. In the last chapter, the researchers proposed using online learning as a new method of teaching.

***Keywords**: metasynthesis, Differential Equations, teaching methodology, online learning, technology.*

There are many frameworks presented for engaging students in learning mathematics. It is very important to change the teaching techniques or blend the traditional and modern techniques of learning. The rapid development of technology has influenced teaching techniques among teachers and lecturers. It has also affected the teaching and learning of differential equations for undergraduate students.

Kashefi, Zaleha and Yudariah (2010) have found that undergraduate students have difficulties with non routine problems. This issue is caused by the students' weak understanding of the basic concepts and symbols used. In addition, the students also had difficulties in working on graphical representation for 3 dimensional graph.

Albeit, research done at a university in Singapore, by Soon, Lioe and McInnes (2011), found that students did not experience difficulties in drawing the diagrams. However, students still had problems in connecting the real-life contexts with the mathematical representations. Thus, this research looks at the development of differential equations teaching methods in education. There are no previous research on metasynthesis in reviewing the learning differential equations.

As discussed before, students have some difficulties in understanding differential equations. This research is critical to the development of teaching differential equations. Furthermore, technological development over time have resulted in teachers and lecturers utilizing technology tools as teaching aids. Most importantly, the techniques can help engage the students with the subjects and increase their understanding and reasoning. Separating technology from teaching can be costly for the education field and affect the students' abilities. Many frameworks in integrating computer or technology in classroom teaching have been proposed in several studies (Dubinsky &Tall 1991; Kashefi, H, Zaleha Ismail, Yudariah Mohammad Yusof and Roselainy abdul Rahman, 2012; Zaleha Ismail, Zeynivandnezhad and Yudariah, 2012).

Differential Equations is one of the core subjects for undergraduate students majoring in mathematics, science and engineering. For undergraduate students, this subject is demanding since the answers to the questions are not numbers. Secondary

schools, especially in Malaysia, usually teach students to answer the questions using numbers. Thus, it is difficult for students to adapt with this subject essentially.

In traditional teaching methodology, teachers and lecturers give notes in class, demonstrate procedures and give explanation to the students. The students are to copy the notes, listen to the explanation and do exercises in the textbook to sharpen their skills in procedural learning.

Inquiry oriented differential equation has been introduced to help the students understand differential equations by reasoning. The students are exposed to realistic tasks, small group discussions to investigate the problem and also group discussions with their classmates to discuss what they have learnt. This is done so that the students can upgrade not only procedural knowledge, but also conceptual knowledge. (Kwon, 2005)

There are some researchers and the mathematics lecturers who integrate differential equations learning by reasoning using computers or other technologies. Dubinsky and Tall (1991) believe that by fully utilizing computers' abilities in various ways, they can become tools to help students' mathematical thinking. The purpose of this paper is to overview the development of differential equations learning for undergraduate students.

Why learning differential equation is important?

Differential Equations (DE) is one of the most important knowledge that students of science need to acquire. Most colleges and universities place Differential Equations in their mathematics syllabus (Bajpai, calus & Simpsona, 1970). This is also true for universities in Malaysia, where students majoring in Mathematics and Engineering are required to take this subject, as in International Islamic University Malaysia (IIUM) and Universiti Teknologi Malaysia (UTM). Even though both of these universities offer DE in their mathematics syllabus, UTM has put DE in one subject. UTM also include DE as a perquisite subject before they take Calculus. Students should then be more skillful in this subject.

Differential Equations is not a new subject for undergraduate students in Malaysia, especially for science stream students, as they are taught DE since secondary school. However there is a question of what Differential Equation actually is. The most frequent answer is that DE is about function and its derivative (Matd44h3F, 2011; Penick 1997), and Dawkins (2013) added that DE is an equation that contains either ordinary or partial derivatives. Research Objectives are **t**o explore the development of differential equation teaching methods from the year 2000 onwards; and to study students' difficulties in this new era and the effects of technology in their understanding of Differential Equations.

METHOD

Qualitative meta-synthesis approach

Meta synthesis is used in academic writing to bring previous research findings into the new era. This is very important to avoid any repetition of research by the new researchers. Since meta synthesis' focus is on tentative manners, the researchers can follow the development of rising issues. The gathered information from different resources is also based on knowledge. (Hallberg, 2011)

There is a variety of reasons in writing meta synthesis papers. Some studies analyze the connection structure (Chun-Li L & Lin-na H, 2012), acquire a wide understanding about an issue (Ponggenpoel M, 2009: Halai N, 2012) and explore issues that arise (Liu Y, Niu g & Gu, 2007; Douglas et al 2008). Moreover, there is also meta synthesis that is intended to observe the regular view of problems for the next exploration (Tang 2009).

The papers in this research are categorized by academic papers and peer reviews. The papers are retrieved from various online resources such as Science Direct, Emerald, Springer, Sage, e-journals of UTM library, Google and also Google scholars. The keywords used to find the related papers are learning differential equations, differential equations and inquiry oriented differential equations. Several papers from authors such as Rasmussen and Habre are also selected for reviewing.

The number of publications is 50. The inclusion criteria for this research are: 1) all the publications must be qualitative research in design; 2) the data of the research is primary; 3) qualitative data collection including interview, observation and students worksheet; 4) the researchers has defined the sample used in the papers; 5) there is innovative strategy of teaching methodology. After selection, only 35 publications are included. The next step is applying exclusion criteria which are: 1) papers those entirely conceptual, theoretical background and research design; 2) an examples of questions based. The remaining publications for this research are only 8.

RESULTS AND DISCUSSION

For this research there are 8 publications that have been reviewed. Table 1 classifies the studies in terms of the writers, the methodologies used in the each paper, and the respondents. All the papers have Differential Equations as the subjects and the responses from sample as the indicator. Papers written by Rasmussen and Habre are frequently reviewed in this research. The objectives of the study are: 1) to introduced new methodology of teaching to educators; 2) to explore the issues in learning differential equations by the students.

Table 1. Description of studies reviewed

Article (no)	Objective	Methodology	Sample
[1] Wagner F J, Speer M N and Rossa B (2007)	Examine knowledge other than content knowledge needed by a mathematician in his first use of an inquiry-oriented curriculum for teaching an undergraduate course in differential equations	Classroom teaching, videotape, notes in the class, interview, self report	19 undergraduate students, major or minor mathematics and biology, chemistry, Physics Teachers' view
[2] Stephan M &	to present an analysis of the classroom	15 week classroom teaching	Mid-sized university in the US, Engineers

Rasmussen C (2002)	mathematical practices that were established over the first half of the semester including instruction on first order differential equations	experiment Video recording of class and interview, copies of written works, instructor journal, research field notes, audio recordings and debriefing sessions	
[3] Miller RH & Upton S B (2008)	we present the rationale for such computer innovations, the philosophy behind their design, as well as a discussion of their careful development and implementation	Students provided with computer, and the computer screen was videotaped and the verbal exchange was recorded as participants worked	8 students from an introductory ODE course, 4 men, 4 women Different level of mathematical ability
[4] Habre S (2000)	Examines whether students consider slope (or direction) fields as a means for solving first-order ODEs and studies students' success in reading information from these fields. investigates students' abilities converting symbolic information into graphical ones and vice-versa	Classroom observation, observation from lab session, copies of students' exams, copies of IDE assignments, questionnaires, transcripts of semi structured interview- 9 students	26 undergraduate students, major in Biology, Economics, Biometry, Statistics, Chemistry and math
[5] Rasmussen C & Blumenfeld H (2007	Analyze the case of students reasoning with analytic expressions as they reinvent solutions to systems of two differential equations	Classroom teaching experiment Video tape Students' homework, board postings, examinations Interview	37 students for class 21 students for interview Majors in math and science
[6] Rasmussen C (2001)	to offer a framework for interpreting students' understandings of and difficulties with mathematical ideas central to new directions in	Individual interview, interview with instructor, copies of students' quizzes, exams and computer assignments and questionnaire (use of mathematica)	6 students – scientist and engineers

	differential equations		
[7] Kendall M & Stacey K (2001)	his study examines how two teachers taught differentiation using a hand held computer algebra system, which made numerical, graphical and symbolic representations of the derivative readily available.	Questionnaire, assignments, two written test 2 groups with different teaching methods	33 year 11 (17 years old) All female students
[8] Habre S (2003)	This paper investigates students' acceptance of solving a differential equation geometrically.	Students copies of IDE assignments, exam papers, last question on the first exam, photocopies of all students answer, questionnaire Interview with semi-structured question – 6 students	36 students with engineering, math edu and chemistry

Most of the researchers used classroom teaching experiment in their research. Copies of students' works, assignments, quizzes and final examination answers were kept by the researchers for further analysis. In finding the students' reasoning in mathematics, student discussions are recorded either by video tape or audio tape. The majority of the researchers used computer as aided tool to assist students' understanding.

Respondents for the papers were from different universities and undergraduate students only. Only research papers [1] and [7] used teachers as the subjects. Many of the undergraduate students were taking Mathematics or science subjects such as Biology, Chemistry and Physics as minor or major courses. Engineering students are also among the samples analyzed by the researcher.

Table 2 lists the difficulties faced by the respondents in learning and understanding Differential Equations. Not only do students have problems in learning DE but teachers also have problems teaching it. Most of the report on the papers showed the students views of learning DE are still traditional, which is solving by analytical. Another way to solve Differential Equations is by drawing the graph of equation or visualizing the solutions.

Table 2. Respondents' responses in learning differential equation

Article (no)	Students' responses
[1]	1) The teacher beliefs the teaching is not only procedural and analytical, there are should some

Wagner F J, Speer M N and Rossa B (2007)	discussions between the students, to broaden students' experience.
[3] Stephan M & Rasmussen C (2002)	1) Students can solve the situational problem of differential equations 2) Students could answer the questions by analytical and graphical
[5] Miller RH & Upton S B (2008)	1) time framing to do the task is too long, students' did not like it 2) the explanation of the graphs shown were insufficient for students to relate the analytic and geometrical solutions
[6] Habre S (2000)	1) most of the students are reluctant to solve the differential equation by graph 2) they preferred to give solutions in analytic or symbolic because of exactness and pertinence
[7] Rasmussen C & Blumenfeld H (2007)	1) student's assumed, slope is not proportionality constant 2) students can give the solutions by sketched the graphs of differential equations
[10] Rasmussen C (2001)	1) Students cannot made justification to reject or accept the solutions 2) Students' did not sense that graphical expression as the solutions other than analytical expression 3) Equilibrium solution is not the solutions for differential equation 4) Students read the graph line literally
[12] Kendall M & Stacey K (2001)	1) CAS was used by the teacher A for symbolic representation 2) Teacher B used CAS as pedagogy aids to help student interpret the differential equations
[17] Habre S (2003)	1) 67% students define the differential equations in analytic definition 2) 17% students think graph as a solution 3) 25% choose geometrical approach to solve differential equation

Lastly, in the Table 3 shown the technology was used in the changed of teaching approach. The researchers using assorted tools to help the students understand DE. As Mason and Scrivini (2004) believe if the teachers can change the negative belief of mathematics to positive, the students can obtain higher achievement. Almost the tools use gave the answer by visual or graph view and just need the students to interpret it.

Article (#)	Technology	Purpose of technology	Approaching problems	Students responses
[3]	Interactive Differential Equation Calculator	Used graph to look graphical image for further investigation and deep understandin g	Situational problem	Students used the graph to visualize the changing parameters

[6]	+ODE	Visual approach to find the solution		Students had positive view
[17]	With background used mathematica		Conceptual understanding	towards IDE in reading slope and classified the solutions
[5]	Mathlets: JavaTM Applets for Math Explorations	To draw graph – visual image of analytical question		Positive responses toward Mathlets
[10]	Mathematica			
[17]		Solution in visual approach	Conceptual understanding	Positive response Prefer analytical then graphical solution
[12]	Computer Algebra System	1) Symbolic differentiation 2) Pedagogical purpose	Arithmetic	1) Just translating the formula 2) Interpretation
[1]	N/A	1)Maple has been used before by the educators, but have negative feedbacks from students		The lack of seeing the big picture – too much syntax to learn
[7]			Situational problem and analytic problem	Sketching the graph at the board

Interactive Differential Equation

Interactive Differential Equations or IDE is a collection of more than 90 lab activities for mathematical subjects. The characteristics of this tool are dynamics, simple with no struggle for key in the syntax and give illustrations of concepts. Hence, the students and teachers can focus on understanding the concepts by using visualizations. There are many topics covered in this tool, such as first order differential equations, second order differential equations, linear and non linear, and so on. All of the researches applied IDE to visualize graphs and also to understand the connection between graphs and equations.

In paper [17], the researcher conducted a pre test for students about geometrical or visual solutions of differential equations. Most of them preferred to answer differential equations analytically rather than solving them using graphical approach. The accuracy of the answer by graphs is questioned by the students. After the researcher introduced IDE, all the students gave a positive response in using graphs for solving differential equations.

Mathlets

The second tool used in the respective research is Mathlets. Mathlets is a Java applet that covers many topics in mathematics such as calculus, differential equations and linear algebra. Similar to Interactive Differential Equations, students do not need to remember all of the syntaxes to look for the output. Mathlets has covered many topics and can interpret equations and graph the equations for understanding visually.

Computer Algebra System

There are many types of computer algebra system available in this new era. There are those that are accessible freely and those that require purchase. Examples of computer algebra systems are Maxima, Maple and Math lab. Maxima is a free software that can be found on internet. Compared to Maple and Math Lab, the users need to purchase it. The similarity of these three softwares is that the users have to learn the syntax before mastering the software.

CONCLUSION

We have explored the development of technology used by the teachers or educators nowadays. However, there is one method of teaching that has not been tried by researchers, which is learning DE by online learning, such as discussion and e-learning. Online learning has a positive effect on the students grade (Dowell and Small, 2011; Fernandez, Llinares and Valls, 2012). Karadag (2011) stated that in order to reduce students' cognitive load, they can use online cognitive tools. Changes of teaching methods are not unusual in the field of education. The purpose of several teaching styles is to help the students engage in learning from time to time.

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