Economic Geography meets Hotelling: a home-sweet-home effect

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Economic geography meets Hotelling: the *home-sweet-home* effect

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Abstract. We propose a 2-region core-periphery model where all agents are inter-regionally mobile and have Hotelling-type heterogeneous preferences for location. The utility penalty from residing in a location that is not the preferred one generates the only dispersive force of the model: the home-sweet-home effect. Different distributions of preferences for location induce different spatial distributions in the long-run depending on the short-run general equilibrium economic geography model that is considered. We study the effect of two of those: the linear and the logit home-sweet-home effects.

Keywords: Economic Geography; Heterogeneous location preferences; Migration.

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1 Introduction

We propose a general framework for 2-region core-periphery models where agents are heterogeneous regarding their preferences towards living in one region or the other. Our aim is to investigate how spatial distributions evolve as countries become more integrated and whether heterogeneity in preferences for location has qualitative impact on this relationship.

Agent heterogeneity has been progressively incorporated in New Economic Geography (NEG) to explain the observed uneven spatial distribution of economic activities beyond causes based exclusively on pecuniary factors such as real wage differentials (Gaspar, 2018). Potential migrants are heterogeneous, not only in their tastes for manufactured goods or their skill levels, but also in their idiosyncratic preferences for residential location.

Each region (city, province, country) has its own tangible and intangible amenities. Some may be more attractive for most people because they have better provision of facilities such as communal areas, parks, lower crime rates, or a more pleasant landscape. Other regional characteristics are perceived differently by different individuals. This is the case for cultural and historic amenities, such as language, which may be attractive to some individuals but repulsive to others (Rodríguez-Pose and Ketterer, 2012). In this sense, some locations may have clear advantages over others for everyone, while other may only be attractive to certain people (Storper and Manville, 2006; Albouy et al., 2015).

The idea that "there is no place like home" suggests that people may be reluctant to leave their region of origin. Idiosyncratic preferences towards places of residence constitute a local dispersive force,[1] and should be considered in NEG models (Fujita and Mori, 2005; Combes et al., 2008). Steps in this direction are the works of Murata (2003), Tabuchi and Thisse (2002), Akamatsu et al. (2012), Ahlfeldt et al. (2015) and Redding (2016), who considered heterogeneous preferences borrowing from the literature of discrete choice theory (see McFadden, 1974).[2]

In discrete choice theory models of random utility, agents draw utility from a deterministic observable component (e.g., consumption from manufactured goods) and from a random unobservable component which represents their idiosyncratic tastes (e.g., preferences for

[1]See Akamatsu et al. (2017) for a clear distinction between local and global dispersion forces.

[2]Another important contribution is that of Mossay (2003), where geography is treated in a continuous circular space, and individual preferences for residential location follow a random walk process.

residential location that stem from intangible amenities). The unobservable component is assumed to follow some probability distribution, typically Gumbel (Tabuchi and Thisse, 2002; Murata, 2003; Akamatsu et al., 2012), or Fréchet (Ahlfeldt et al., 2015; Redding, 2016). Under the former, the probabilities for each alternative are given by the logit model.[3] Since it has a very simple closed-form, it is widely used for qualitative behavioural choice (Train, 2009). In these models, inter-regional migration responds to the realization of a random variable (Tabuchi and Thisse, 2002; Anderson et al., 1992).

We adopt the framework proposed by Hotelling (1929), which can be adapted to deal with preference heterogeneity across a wide array of domains. Each agent's preferences are described by a parameter x uniformly distributed along the unit interval $[0, 1]$. Regions are points on opposite extremes of the unit interval, with the position of each agent on the line, $x \in [0, 1]$, indicating their relative preference for residing in one region or the other. We thus attempt to reconcile determinants of spatial agglomeration grounded on market factors, with a Hotelling framework describing heterogeneity concerning non-market factors.

While heterogeneity in agent preferences has been tackled in NEG, conclusions have been drawn under specific functional forms concerning the utility agents draw from consumption goods (market factors) and specific functional forms for agent heterogeneity (non-market factors). Instead, we consider a general framework which encompasses several set-ups as particular cases and allows the study of the consequences of agent heterogeneity in general. The logit specification is a particular case of our agent heterogeneity, which we model as a utility penalty *à la* Hotelling,[4] which depends on the agent's personal attachment towards a given region and generates a dispersive force: the *home-sweet-home* effect.

We assume that agents who live in a region draw the same utility from consumption goods. However, each agent bears a utility penalty which is higher the less they enjoy living in the region. Therefore, those who have a lower preference for a region will get a lower overall utility if they reside in that region. In the long-run, each agent chooses the region that offers the highest overall utility. The long-run spatial distribution of agents is the result of two counteracting forces. On the one hand, gains from agglomeration due to increasing

[3]However, both the Gumbel and Fréchet distributions yield qualitative properties that can be generated by the conditional logit model (Behrens and Murata, 2018).

[4]In this sense, the Hotelling model can be reinterpreted as a discrete choice random utility model (Anderson et al., 1992).

returns generate a higher utility from consumption in the more populated region. This promotes the well-known concentration of economic activities driven by pecuniary factors. On the other hand, the less an agent wishes to live in a region the stronger is the dispersive home-sweet-home effect. The utility differential between regions can be seen as the gain from consumption associated with residing in one region instead of the other, minus the home-sweet-home effect, i.e., the Hotelling utility penalty that the borderline agent suffers from residing in the same region instead of the other. In the long-run, the latter effect is analogous to congestion costs that are increasing in the population size of a region.

We characterize the possible long-run spatial distribution of agents contingent on how the overall utility differential depends on the fraction of agents $h \in [0, 1]$ that live in a region. If the utility differential is convex for $h \in \left[\frac{1}{2}, 1\right]$, then symmetric dispersion ($h = 1/2$), agglomeration ($h = 1$), or both are possible long-run equilibria. In the latter case, selection between the two possible spatial outcomes depends on the initial spatial distribution, that is, "history matters" (Krugman, 1991; Matsuyama, 1991; Redding et al., 2011). If the utility differential is concave, then there exists a unique long-run equilibrium which may correspond to symmetric dispersion, agglomeration, or asymmetric dispersion $h \in \left(\frac{1}{2}, 1\right)$.

The convexity of the agent's utility function depends on the short-run equilibrium monopolistic competitive setup which determines regional real wages (market factors) and on the functional form for the utility penalty that stems from agent heterogeneity in preferences for residential location. Our second aim is then to illustrate which spatial distributions arise under a class of different NEG models well established in the literature. Some of these models also account for agent heterogeneity (Tabuchi and Thisse, 2002; Murata, 2003) under different monopolistic competitive settings but assuming the same type of agent heterogeneity, specifically, probabilistic migration according to the logit model.

The logit model corresponds to a situation where the home-sweet-home effect is exponentially increasing in the distribution of agent preferences for residential location. For the agent who has the highest preference toward a region, their utility penalty from residing in the other region tends to infinity. Full agglomeration is precluded, because there is always a fraction of agents (no matter how small) who will want to live in the less industrialized region due to their "infinite" personal attachment towards it. The logit model thus implies extreme convexity of the Hotelling utility penalty, which tends to infinity as h approaches 1. However, if the home-sweet-home effect is, e.g, (non-strictly) concave, agglomeration may be stable, or both the symmetric equilibrium and agglomeration may be simultaneously stable. In the latter case, if initial asymmetries are initially very high, the relative utility in the more industrialized region is so high that all agents will decide to agglomerate in one single region. Intuitively, even the agents with stronger preferences will want to avoid exceedingly low standards of living. Conversely, if regional disparities are initially low, agents will disperse evenly among the countries. The regional utility differential is not enough to offset even the agents who make almost no distinction between the countries.

The goal of this paper is to dwell further into the role of agent heterogeneity, showing that not only does it matter, but it has the potential to produce very different predictions about the spatial distribution of economic activities. While the structure of spatial distributions remains invariant under changes in particular types of heterogeneity (like the logit model), it changes when we introduce preferences that are qualitatively different.

We conclude that different configurations of the home-sweet-home effect may contribute to explain why agent heterogeneity, across different geographical scales in the real world, produce different predictions concerning the spatial distribution of economic activities.

This article is organised as follows: Sections 2 and 3 are devoted to the description of the model and to establishing conditions for existence and stability of long-run equilibria. Section 4 presents three well-known models that provide good illustrations of the effect of the utility differential on the stability of admissible configurations. The same three models are used in Sections 5 and 6 where the outcome of two types of *home-sweet-home* effect is studied. Section 7 concludes. The more technical proofs are presented in an appendix.

2 Model

There are two regions, L and R, located at opposite extremes of the interval $[0,1]$, and a unit mass of agents who are heterogeneous *à la* Hotelling (1929) in their preferences for place of residence. Agents are uniformly distributed along $[0,1]$, with agent of type $x \in [0,1]$ suffering a utility penalty equal to $t(x)$ if they reside in region L and a utility penalty equal to $t(1-x)$ if he resides in region R, where $t : [0,1] \to \mathbb{R}_+$ is continuous and increasing.[5] Note that an agent of type $x = 0$ (resp. $x = 1$) has the strongest preference for residing in

[5]The assumption that agents are uniformly distributed is mild because $t(\cdot)$ can be non-linear.

region L (resp. R), and an agent of type $x = \frac{1}{2}$ is indifferent between the two regions.

In core-periphery models, the utility of residing in a region depends on local population, because the mass of local residents influences wages, prices of consumption goods, and housing costs. Let $h \in [0, 1]$ denote the mass of agents residing in region L, and let $U : [0, 1] \to \mathbb{R}_+$ be a continuous function describing the impact of local population on utility.

The overall utilities for an agent of type $x \in [0, 1]$ of residing in regions L and R are, respectively, given by:

$$\begin{cases} V_L(x, h) = U(h) - t(x) \\ V_R(x, h) = U(1 - h) - t(1 - x). \end{cases} \tag{1}$$

In a short-run equilibrium, the spatial distribution of agents across regions, and thus $h \in [0, 1]$, is fixed. Specific functional forms for $U(\cdot)$ are derived from the short-run general equilibrium of well known core-periphery models in Sections 3 and 4.

In a long-run equilibrium, each agent resides in the region that provides higher utility. Hence, the equilibrium distribution of agents must be such that agents with $x \in [0, h)$ are located in L while agents with $x \in (h, 1]$ are located in R.[6] With this property, spatial distributions are completely described by $h \in [0, 1]$, and the utility levels in regions L and R for a borderline agent of type $x = h$ can be written, respectively, as $V(h)$ and $V(1 - h)$, where $V(z) \equiv U(z) - t(z)$.

For this borderline agent, the overall utility differential of residing in region L instead of region R is given by:

$$\Delta V(h) = \Delta U(h) - \Delta t(h), \tag{2}$$

where $\Delta U(h) \equiv U(h) - U(1 - h)$ is the region-size effect, and $\Delta t(h) \equiv t(h) - t(1 - h)$ is the idiosyncratic preference effect, which we call the *home-sweet-home* effect. Note that $\Delta t(h)$ is positive if $h < \frac{1}{2}$ (the borderline agent prefers to reside in region L) and negative if $h > \frac{1}{2}$, and thus constitutes a dispersive force.

Agglomeration of all agents in a single region, $h^* \in \{0, 1\}$, is a long-run equilibrium if and only if $\Delta V(1) \geq 0$, or, equivalently, $\Delta V(0) \leq 0$. This condition means that if all agents are located in the same region, not even the agent with strongest preference for the other

[6]Given any resident in L, all agents with stronger preference for L also reside in L.

region would gain from migrating. The utility differential associated to residing in the core, $\Delta U(1)$, more than compensates the home-sweet-home effect, $\Delta t(1)$.

Dispersion of agents between the two regions, $h^* \in (0,1)$, is a long-run equilibrium if and only if $\Delta V(h^*) = 0$. This condition means that the borderline agent (of type $x = h^*$) is indifferent between the two regions. Therefore, agents with $h < h^*$ do not gain from migrating to region R and agents with $h > h^*$ do not gain from migrating to region L. *Symmetric dispersion* ($h^* = \frac{1}{2}$) is always a long-run equilibrium because, since $\Delta V(h) = -\Delta V(1-h)$, $\forall h \in [0,1]$, we have $\Delta V(\frac{1}{2}) = 0$.

A long-run equilibrium is *stable* if any small perturbation of the spatial distribution generates a utility differential which induces agents to return to their original location. A sufficient condition for stability of agglomeration is $\Delta V(1) > 0$, or, equivalently, $\Delta V(0) < 0$. If all agents strictly prefer to reside in the core, then (by continuity of $\Delta V(h)$ at $h = 1$), after a small perturbation of the spatial distribution, they continue to prefer residing in the core. A sufficient condition for stability of dispersion is $\Delta V'(h^*) < 0$. After a small perturbation which increases (resp. decreases) h, the borderline agent strictly prefers to reside in region R (resp. L), thereby restoring the original distribution. This happens when the relative utility gain from the increase of local population, $\Delta U'(h^*)$, does not compensate the increase of the home-sweet-home effect, $\Delta t'(h^*)$.[7]

3 Stable long-run equilibria

The convexity properties of the overall stability differential determine the number of stable long-run equilibria. We obtain sharp characterizations about existence, stability and uniqueness of long-run equilibria if ΔV is either strictly quasi-convex or strictly quasi-concave. Recall that existence and stability of long-run equilibria only depends on the characteristics of ΔV:

[7] It is more complicated to assess stability of long-run equilibria that are *irregular*: agglomeration with $\Delta V(1) = 0$; and dispersion with $\Delta V'(h^*) = 0$. An agglomeration equilibrium with $\Delta V(1) = 0$ is stable if there exists $\epsilon > 0$ such that $\Delta V(1-\delta) > 0$ for all $\delta \in (0,\epsilon)$. A dispersion equilibrium with $\Delta V'(h^*) = 0$ is stable if there exists $\epsilon > 0$ such that $\Delta V(h^* - \delta) > 0$ and $\Delta V(h^* + \delta) < 0$ for all $\delta \in (0,\epsilon)$. Irregular equilibria do not exist generically, i.e., in a full measure subset of a suitably defined parameter space (which means that irregular equilibria cease to exist after a small perturbation achieved through a small change in parameters).

- *Agglomeration*, $h^* \in \{0,1\}$, is a long-run equilibrium if and only if $\Delta V(1) \geq 0$. It is stable if $\Delta V(1) > 0$, which is equivalent to $\Delta U(1) > \Delta t(1)$.

- *Dispersion*, $h^* \in (0,1)$, is a long-run equilibrium if and only if $\Delta V(h^*) = 0$, i.e., $\Delta U(h^*) = \Delta t(h^*)$. It is stable if $\Delta V'(h^*) < 0$, i.e., $\Delta U'(h^*) < \Delta t'(h^*)$.

Proposition 1. *If ΔV is strictly quasi-convex for $h \in [\frac{1}{2}, 1]$, there are stable long-run equilibria: agglomeration, symmetric dispersion, or both.*

Proof. There are three possible and mutually exclusive cases:

- If $\Delta V'(\frac{1}{2}) \geq 0$, then $\Delta V(h) > 0$ for all $h \in (\frac{1}{2}, 1]$. Agglomeration is the unique stable long-run equilibrium. Symmetric dispersion is not stable.
- If $\Delta V'(\frac{1}{2}) < 0$ and $\Delta V(1) < 0$, then $\Delta V(h) < 0$ for all $h \in (\frac{1}{2}, 1]$. Symmetric dispersion is the unique long-run equilibrium, and it is stable.
- If $\Delta V'(\frac{1}{2}) < 0$ and $V(1) > 0$, then $\exists!\ h^* \in (\frac{1}{2}, 1)$ s.t. $V(h^*) = 0$. Agglomeration and symmetric dispersion are stable long-run equilibria. Asymmetric dispersion with fraction h^* of agents in the core is a long-run equilibrium, but is not stable. □

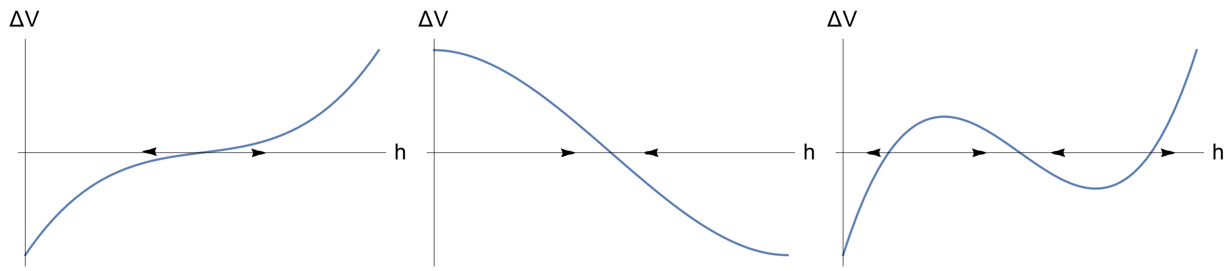

Figure 1 – Stable equilibria when ΔV is quasi-convex in $[\frac{1}{2}, 1]$: agglomeration (left); symmetric dispersion (middle); both (right).

Proposition 2. *If ΔV is strictly quasi-concave for $h \in [\frac{1}{2}, 1]$, there is a unique stable long-run equilibrium with $h^* \in [\frac{1}{2}, 1]$: agglomeration, symmetric dispersion, or asymmetric dispersion.*

Proof. There are three possible and mutually exclusive cases:

- If $\Delta V'(\frac{1}{2}) \leq 0$, then $\Delta V(h) < 0$ for all $h \in (\frac{1}{2}, 1]$. Symmetric dispersion is the unique long-run equilibrium, and it is stable.
- If $\Delta V'(\frac{1}{2}) > 0$ and $\Delta V(1) > 0$, then $\Delta V(h) > 0$ for all $h \in (\frac{1}{2}, 1]$. Agglomeration is the unique stable long-run equilibrium. Symmetric dispersion is not stable.
- If $\Delta V'(\frac{1}{2}) > 0$ and $\Delta V(1) < 0$, then $\exists!\ h^* \in (\frac{1}{2}, 1]$ s.t. $V(h^*) = 0$. Asymmetric dispersion with fraction h^* of agents in the core is the unique stable long-run equilibrium. Symmetric dispersion and agglomeration are not stable. □

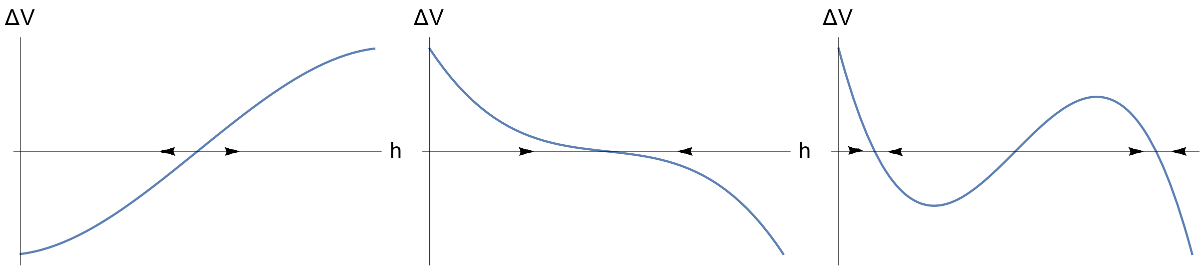

Figure 2 – Stable equilibria when ΔV is quasi-concave in $h \in [\frac{1}{2}, 1]$: agglomeration (left); symmetric dispersion (middle); asymmetric dispersion (right).

In Sections 3 and 4, we use well-known core-periphery models and different forms of heterogeneity in preferences for location to derive properties of ΔV, and apply Propositions 1 and 2 to characterize existence, stability and uniqueness of long-run equilibria.

4 Spatial patterns for different NEG models

We now consider three well-known analytically solvable core-periphery models which lead to different convexity properties of the utility differential, ΔU: the models of Pflüger (2004), Ottaviano (2001), and Ottaviano et al. (2002).

4.1 Common ground

The economy comprises: two regions, L and R; two sectors, manufactures and agriculture; and two types of agents, mobile and immobile.

The agricultural sector uses immobile labour to produce a perfectly tradable good under perfect competition and constant returns to scale (each agent produces one unit of the agricultural good). Choosing the agricultural good as numéraire, we set its price and the wage of immobile agents at unity in both regions.

In the manufacturing sector, many varieties of imperfectly tradable manufactured goods are produced under monopolistic competition and increasing returns to scale. Each firm produces a single variety using one unit of mobile labour (fixed cost) and, in addition, one unit of immobile labour per unit of output (variable cost). There is free entry in the manufacturing sector, thus firm profits are driven to zero (the nominal wage of mobile labour, w_i, totally absorbs operating profits).

There is a unit mass of mobile agents (who can migrate freely) and a mass $l/2$ of immobile agents in each region, who choose their consumption with the objective of maximizing a common utility function. Agents in region i maximize utility $u(C_i, A_i)$, where C_i is the consumption level of a composite good of manufactures and A_i is the consumption level of the agricultural good, subject to the budget constraint $P_iC_i + A_i = y_i$, where P_i is the regional price index of the manufacturing goods composite, and y_i is the nominal wage ($y_i = w_i$ for mobile agents and $y_i = 1$ for immobile agents).

Product market and labour market equilibrium yields unique short-run equilibrium wage, price and consumption levels as a function of the spatial distribution of agents, $h \in [0, 1]$. For each of the following models we use a different superscript for $\Delta U(h) \equiv U(h) - U(1-h)$.

4.2 The PF model

The PF model (Pflüger, 2004) assumes quasi-linear logarithmic utility from consumption:

$$u_i^{PF} = \alpha \ln C_i + A_i, \tag{3}$$

where $\alpha > 0$; and a CES composite of manufactures:

$$C_i = \left[\int_{s \in S} c_i(s)^{\frac{\sigma-1}{\sigma}} ds \right]^{\frac{\sigma}{\sigma-1}},$$

where $c_i(s)$ is consumption in region i of variety s of manufactures, and $\sigma > 1$ is the elasticity of substitution between varieties.

Trade across regions of manufactured varieties is subject to an iceberg cost: for each unit to arrive, it is necessary to ship $\tau \in (1, +\infty)$ units.

The short-run equilibrium utility differential is given by (Pflüger, 2004, p. 569):

$$\Delta U^{PF}(h) = \frac{\alpha}{\sigma - 1}\left(\left\{\frac{(2h-1)(\sigma-1)(1-\phi)\left[(l+2)\phi - l\right]}{2\sigma\left[1-h(1-\phi)\right]\left[(1-h)\phi + h\right]}\right\} + \ln\left[\frac{h(1-\phi)+\phi}{1-h(1-\phi)}\right]\right), \quad (4)$$

where $\phi = \tau^{1-\sigma}$ is the "freeness of trade" parameter.

Assumption 1. *The mass of immobile agents is relatively large:*

$$l > \frac{7\sigma\phi + \sigma - 6\phi}{3(\sigma-1)(1-\phi)}. \quad (5)$$

In Gaspar et al. (2018), who extend the PF model to an arbitrary number of regions, a condition for the stability of agglomeration in one region is provided in terms of (l, ϕ, σ) and the number of regions, n.[8] It can be shown that the condition always holds if $l < \frac{n\phi}{1-\phi}$. For $n = 2$, this is problematic because it means that no other stable equilibrium exists, which is implied by the fact that the PF model with two regions undergoes a supercritical pitchfork bifurcation at dispersion (Pflüger, 2004). Therefore, it is reasonable to assume that $l > \frac{2\phi}{1-\phi}$. This condition is implied by Assumption 1, which also yields strict concavity of ΔU^{PF}.

Lemma 1. *Under Assumption 1, $\Delta U^{PF}(h)$ is strictly concave in $h \in [\frac{1}{2}, 1]$.*

Proof. See Appendix A. □

The following result is a direct consequence of Proposition 2 and Lemma 1:

Corollary 1. *If the home-sweet-home effect, Δt, is convex in $h^* \in [\frac{1}{2}, 1]$ then the PF model under Assumption 1 has a unique stable long-run equilibrium with $h^* \in [\frac{1}{2}, 1]$: agglomeration, symmetric dispersion, or asymmetric dispersion.*

If the home-sweet-home effect, Δt, is concave, the overall utility differential, ΔV, is not necessarily either convex or concave, and thus Propositions 1 and 2 may not apply.

[8]Inequality (18) in Gaspar et al. (2018, pp. 871).

4.3 The FE model

The only difference of the FE model (Ottaviano, 2001) with respect to the PF model is that utility from consumption is:

$$u_i^{FE} = \mu \ln(C_i) + (1-\mu)\ln(A_i), \tag{6}$$

where $\mu \in (0,1)$ is the share of income spent on manufactures.

The short-run equilibrium utility differential is given by (Ottaviano, 2001, p. 57):

$$\Delta U^{FE}(h) = \ln\left[\frac{h\phi + (1-h)\psi}{(1-h)\phi + h\psi}\right] + \frac{\mu}{\sigma - 1}\ln\left[\frac{(1-h)\phi + h}{h\phi + (1-h)}\right], \tag{7}$$

where $\psi = \frac{1}{2}\left[1 + \phi^2 - \frac{\mu}{\sigma}(1-\phi^2)\right] = \frac{1}{2\sigma}[\sigma(1+\phi^2) - \mu(1-\phi^2)]$.

Assumption 2. *The following condition holds:*

$$\mu > g(\phi) \equiv \frac{(\sigma-1)\left[(1-\phi)\sigma^3 - \mu(\phi+1)\right]}{\left[\sigma(\phi+1) - \mu(1-\phi)\right]^3}. \tag{8}$$

The utility differential $\Delta U(h)$ may be concave or convex in $h \in [\frac{1}{2}, 1]$.[9] It is possible to show that the condition in Assumption 2 is equivalent to requiring the freeness of trade to be sufficiently high, $\phi > \phi_c \in (0,1)$, where ϕ_c solves $\mu = g(\phi)$ in (8) (cf. Appendix A).

Lemma 2. *Under Assumption 2, the utility differential $\Delta U^{FE}(h)$ is convex in $h \in [\frac{1}{2}, 1]$.*

Proof. See Appendix A. □

This property of ΔU^{FE} allows us to apply Proposition 1 to obtain

Corollary 2. *If the home-sweet-home effect, Δt, is concave then the FE model under Assumption 2 has stable long-run equilibria: agglomeration, symmetric dispersion, or both.*

[9] Ottaviano (2001, p. 59 and Appendix A) investigates possible shapes of the utility differential but only studies its properties up to the first derivative.

4.4 The OTT model

The OTT model (Ottaviano et al., 2002) has two important differences with respect to the PF and FE models. One is that preferences for manufactures are described by a quadratic aggregator instead of a CES aggregator. The other is that the cost of trading manufactured varieties, instead of being iceberg, is τ units of the numéraire good per unit shipped.

Utility from consumption is linear:

$$u_i^{OTT} = C_i + A_i, \tag{9}$$

with quadratic sub-utility:

$$C_i = \alpha \int_{s \in S} c_i(s) ds - \frac{\beta - \gamma}{2} \int_{s \in S} c_i(s)^2 ds - \frac{\gamma}{2} \left[\int_{s \in S} c_i(s) ds \right]^2 .$$

The short-run equilibrium differential in utility from consumption is:

$$\Delta U^{OTT} = C\tau \left(\tau^* - \tau\right) \left(h - \frac{1}{2}\right), \tag{10}$$

where $C > 0$ and $\tau^M > 0$ are bundling parameters that depend neither on h nor on τ.[10]

Lemma 3. *The utility differential $\Delta U^{OTT}(h)$ is linear in $h \in \left[\frac{1}{2}, 1\right]$.*

Since ΔU^{OTT} is linear, ΔV inherits the convexity properties of the home-sweet-home-effect.

Corollary 3. *If the home-sweet-home effect, Δt, is strictly quasi-convex then the OTT model has a unique stable long-run equilibrium with $h^* \in [\frac{1}{2}, 1]$: agglomeration, symmetric dispersion, or asymmetric dispersion. If Δt is strictly quasi-concave then the OTT model has stable long-run equilibria: agglomeration, symmetric dispersion, or both.*

[10] See Ottaviano et al. (2002, pp. 420) for more details on the two expressions.

5 Linear home-sweet-home effect

We now address the overall utility differential, $\Delta V(h) = \Delta U(h) - \Delta t(h)$ in the case of a linear home-sweet-home effect: $t(x) = \theta x$, where $\theta \geq 0$ (Hotelling, 1929; Mansoorian and Myers, 1993). This means that the increase in the utility differential ΔU necessary to attract an additional agent from the periphery to the core is independent of the level of agglomeration.[11] Note that with a linear home-sweet-home effect, the convexity properties of ΔU are inherited by ΔV.

5.1 The PF model

In the PF model, if the mass of immobile agents is sufficiently large, ΔU is strictly concave in $h \in (\frac{1}{2}, 1]$. Therefore, with a linear home-sweet-home effect, ΔV is also strictly concave. Hence, Proposition 2 holds and one equilibrium, out of three possible types, is stable. The next result provides more detail.

Proposition 3. *In the PF model under Assumption 1 with a linear home-sweet-home effect, there are threshold values θ_b (break point) and θ_s (sustain point) such that:*

- *Agglomeration is the unique stable equilibrium if $\theta < \theta_b$.*
- *Asymmetric dispersion is the unique stable equilibrium if $\theta \in (\theta_b, \theta_s)$.*
- *Symmetric dispersion is the unique stable equilibrium if $\theta > \theta_s$.*

Proof. See Appendix A. □

The PF model with a linear home-sweet-home effect thus predicts a smooth transition from symmetric dispersion to increasingly asymmetric dispersion and, finally, agglomeration as the home-sweet-home effect becomes weaker.

[11]It is constant and equal to 2θ in magnitude, because $\Delta t(h) = 2\theta h - \theta$.

5.2 The FE model

In the FE model, if the manufacturing sector is sufficiently large, ΔU is strictly convex in $h \in (\frac{1}{2}, 1]$. As a result, with a linear home-sweet-home effect, ΔV is also strictly convex. By Proposition 1 one or two equilibria may be stable. A more precise description is obtained in the next result.

Proposition 4. *In the FE model under Assumption 2 with a linear home-sweet-home effect, there are threshold values θ_b (break point) and θ_s (sustain point) such that:*

- *Agglomeration is the unique stable equilibrium if $\theta < \theta_b$.*
- *Agglomeration and symmetric dispersion are the only stable equilibria if $\theta \in (\theta_b, \theta_s)$.*
- *Symmetric dispersion is the unique stable equilibrium if $\theta > \theta_s$.*

Proof. See Appendix A. □

The FE model with a linear home-sweet-home effect thus predicts a catastrophic transition from symmetric dispersion to agglomeration as the home-sweet-home effect becomes weaker.

5.3 The OTT model

In the OTT model, ΔU is linear. This implies that, with a linear home-sweet-home effect, ΔV is also linear.

Proposition 5. *In the OTT model under Assumption 2 with a linear home-sweet-home effect, there is a threshold value θ_{bs} (break and sustain point) such that:*

- *Agglomeration is the unique stable equilibrium if $\theta < \theta_{bs}$.*
- *There are no stable equilibria if $\theta = \theta_{bs}$.*
- *Symmetric dispersion is the unique stable equilibrium if $\theta > \theta_{bs}$.*

Proof. See Appendix A. □

The OTT model with a linear home-sweet-home effect thus predicts a catastrophic transition from symmetric dispersion to agglomeration as the home-sweet-home effect becomes weaker. This conclusion is analogous to that concerning the FE model.

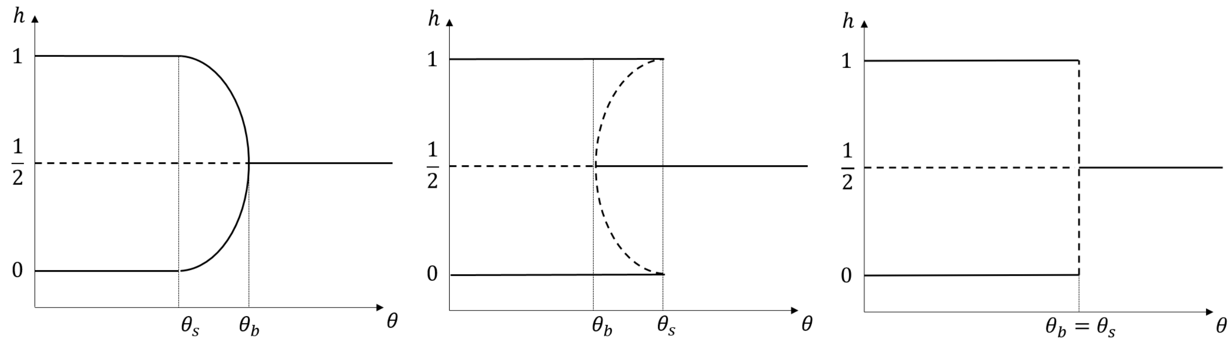

Figure 3 – Bifurcation diagrams: PF model (left); FE model (middle); OTT model (right).

6 Logit home-sweet-home effect

One of the most widely used discrete choice models is the logit. It was used to describe heterogeneity in preferences for location by Tabuchi and Thisse (2002) and Murata (2003). According to the logit model, the fraction of agents who choose to reside in region L is:

$$h = \frac{1}{1 + e^{-\frac{\Delta U}{\theta}}}, \tag{11}$$

where $\theta > 0$ is a scale parameter which measures the strength of heterogeneity. If $\theta \to 0$, agents do not care about location.

From $\Delta U = U(h) - U(1-h)$ and manipulating (11) yields:

$$U(h) - \theta \ln(h) = U(1-h) - \theta \ln(1-h). \tag{12}$$

which is our long-run equilibrium condition with $t(x) = \theta \ln(x)$.

The corresponding home-sweet-home effect, $\Delta t(h) = \theta \ln(h) - \theta \ln(1-h)$, is strictly convex for $h \in [\frac{1}{2}, 1]$. This leads to the following results.

Proposition 6. *In the PF model under Assumption 1 and in the OTT model, with a logit home-sweet-home effect, there is a threshold value θ_s (sustain point) such that:*

- *Asymmetric dispersion is the unique stable equilibrium if* $\theta < \theta_s$.

- *Symmetric dispersion is the unique stable equilibrium if* $\theta > \theta_s$

Proof. See Appendix A. □

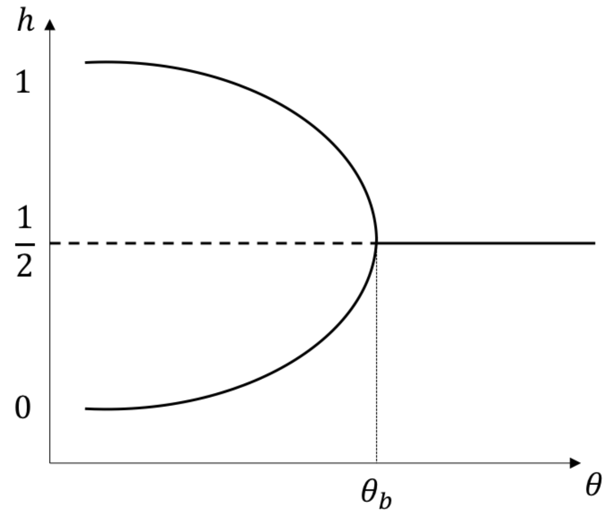

Figure 4 – Bifurcation diagram for the PF and OTT models as in Proposition 6.

With a logit home-sweet-home effect, both the PF model and the OTT model predict smooth transition from symmetric dispersion to increasingly asymmetric dispersion, tending to full agglomeration in the limit as the home-sweet-home effect becomes weaker. See Figure 4.

This is not the case with the FE model. Since ΔU^{FE} and Δt are both convex, ΔV^{FE} is not necessarily either convex or concave. Equilibrium configurations can arise that are different from the ones encountered so far. In Figure 5, we present an example where symmetric dispersion and asymmetric dispersion are both stable. The corresponding bifurcation diagram is presented in Figure 6. The configuration in Figure 5 occurs for the parameter range (θ_b, θ_f) in Figure 6. A detailed analysis of such a model is presented in Appendix A.1.

7 Concluding remarks

Individual idiosyncrasies governing preferences over specific locations with different cultural or historical amenities constitute an effective deterrent of inter-regional migration. This

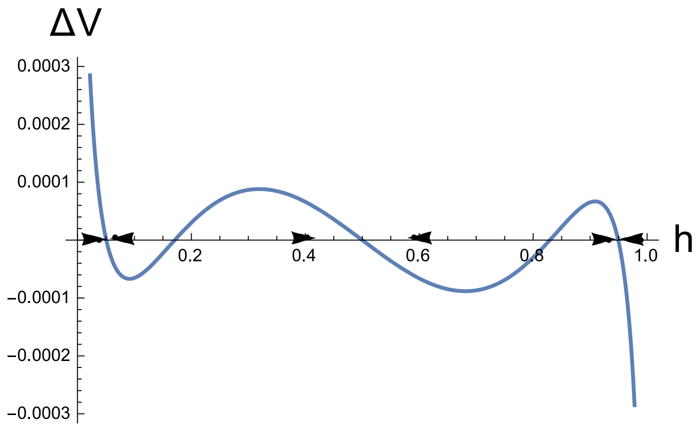

Figure 5 – A hypothetical form for the overall utility differential compatible with the FE model under a logit *home-sweet-home* effect. There are five interior equilibria, three of which are stable.

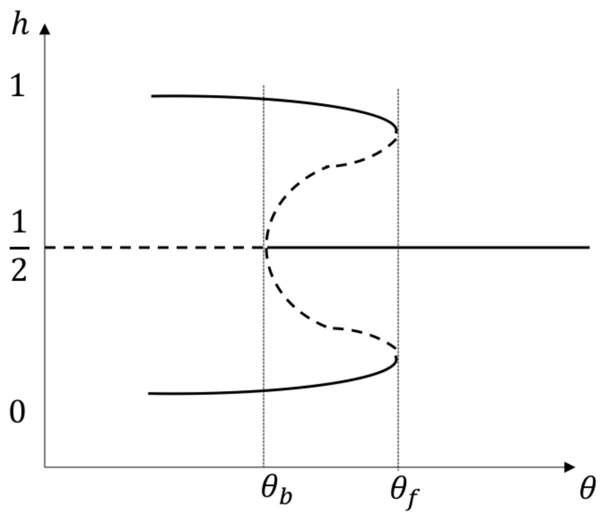

Figure 6 – Bifurcation diagram compatible with the FE model under a logit *home-sweet-home* effect. The phase diagram illustrating equilibrium configurations for the parameter range (θ_b, θ_f) is presented in Figure 5.

helps explaining why some people refuse to move to regions where they could otherwise improve their standard of living (as measured exclusively by pecuniary factors). Therefore, heterogeneity concerning preferences for residential location can be seen as a contributing factor for the reduced inter-regional mobility observed in some spatial contexts.

We have built a core-periphery model that allows us to arbitrarily specify how the utility from residing in a region changes across agents. Modelling the individual utility penalty of migrating to a given location is important because it impacts regional utility differentials with consequent implications on the spatial distribution of economic activities. Agent heterogeneity toward residential location is usually modelled through probabilistic migration according to the discrete choice logit model. This imposes an assumption on the distribution of agent preferences which implies that there are always agents who are unwilling to migrate. No matter how large the gains from agglomeration due to increasing returns and transportation costs, some people will always choose to live in a relatively poor region. In some geographical contexts, some people are in fact too attached to a given location, which would help sustain the claim that full agglomeration in one single region is unlikely. This is even more so when regions have their very own and distinct sets of cultural and historical amenities. However, the importance of these amenities is likely to vary both quantitatively and qualitatively according to the geographical scale. For instance, cultural and historical differences are generally more important at a transnational scale than at the national scale. This would make individuals more reluctant to move to another country than to move to another region within their country.[12]

We further illustrate our point of view using a very simple framework, where the utility penalty is linear. The more personally attached agents face a relatively lower utility penalty if they migrate to a less preferred region, which means they are more responsive to regional income differences. This provides a relationship between agents' idiosyncrasies and income inequalities that is potentially empirically relevant. Specifically, when regional asymmetries are small, income differences are not high enough to trigger migration of even the agents who have a just marginally higher preference for the smaller region. However, if regional asymmetries are very high, the pecuniary gains are large enough that they offset the personal attachment of any agent toward the less populated region. In this case, the initial spatial distribution will determine if there is a tendency towards spatial convergence or divergence. In other words, history matters.

The variety of possible spatial outcomes conveyed by just two different specifications for agent preferences, while overlooking other well-known potential determinants of spatial inequality, highlights the importance of the qualitative distribution of individual tastes.

[12]While changes in the logit model account for different heterogeneity scales (Scarpa et al., 2008; Train, 2009; Hess and Rose, 2012), they do not capture the fact that agent preferences may vary qualitatively. For instance, with the logit, agents with the highest personal attachment towards a region are always heavily penalized if they migrate to another region.

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Appendix A - Mathematical proofs

Proof of Lemma 1:

Differentiating $\Delta U^{PF}(h)$ in (4) twice with respect to h yields:

$$\Delta U^{PF''}(h) = \frac{(2h-1)\alpha(1-\phi)^3}{(\sigma-1)\sigma\left[1-h(1-\phi)\right]^3\left[h(1-\phi)+\phi\right]^3}\Phi,$$

where:

$$\begin{aligned}\Phi =& l(\sigma-1)(\phi-1)\left[h^2(1-\phi)^2 - h(1-\phi)^2 + \phi^2 + \phi + 1\right] + \\ & h^2(1-\phi)^2\left[\sigma(\phi-1)-2\phi\right] - h(1-\phi)^2\left[\sigma(\phi-1)-2\phi\right] + \\ & \phi\left\{\sigma\left[\phi(2\phi+3)+3\right] - 2\left(\phi^2+\phi+1\right)\right\}.\end{aligned}$$

It is readily observable that the first term in the product of $\Delta U^{PF''}(h)$ is positive if and only if $h > \frac{1}{2}$. Therefore, $\Delta U^{PF''}(h) < 0$ if and only if $\Phi < 0$. In terms of l, this becomes:

$$l > \frac{h\,(1-h)\,(1-\phi)^2\left[\sigma(1-\phi)+2\phi\right] - \phi\left\{2\left(\phi^2+\phi+1\right) - \sigma\left[\phi(2\phi+3)+3\right]\right\}}{(\sigma-1)(1-\phi)\left[h^2(1-\phi)^2 - h(1-\phi)^2 + \phi^2 + \phi + 1\right]}. \qquad (13)$$

Notice that the RHS of (13) is decreasing in h, which means that setting $h = \frac{1}{2}$ provides us the following sufficient condition, that is:

$$l > \frac{7\sigma\phi + \sigma - 6\phi}{3(\sigma-1)(1-\phi)}. \qquad (14)$$

The condition is satisfied under Assumption 1. Thus, we always have $\Phi < 0$. This implies that $\Delta U^{PF''}(h) < 0$ for $h > \frac{1}{2}$ and hence $\Delta U^{PF}(h)$ is concave. □

Proof of Lemma 2:

Differentiating $\Delta U^{FE}(h)$ in (7) with respect to h, we get:

$$\Delta U^{FE''}(h) = (2h-1)\left\{\frac{\mu(1-\phi)^3(\phi+1)}{(\sigma-1)\left[1-h(1-\phi)\right]^2\left[h(1-\phi)+\phi^2\right]} + \frac{(\phi-\psi)^3(\phi+\psi)}{\left[h(\phi-\psi)+\psi^2\right]\left[h(\psi-\phi)+\phi^2\right]}\right\} \qquad (15)$$

The first term inside the curved brackets is positive whereas the second one can be shown to be negative by noting that $\phi < \psi$ and $h \leq 1$. Using (15), we get that $\Delta U^{FE''}(h) > 0$ if:

$$\mu > \bar{\mu}(h) = \frac{(\sigma-1)\left[h(\phi-1)+1\right]^2\left[h(1-\phi)+\phi\right]^2(\psi-\phi)^3(\phi+\psi)}{(1-\phi)^3(\phi+1)\left[h(\phi-\psi)+\psi\right]^2\left[h(\psi-\phi)+\phi)^2\right]}.$$

Moreover, $\bar{\mu}(h)$ is decreasing in h if $h > \frac{1}{2}$ which means that a sufficient condition for $\Delta U^{FE''}(h)$ to be positive is that $\mu > \bar{\mu}\left(\frac{1}{2}\right)$. As a result, $\Delta U^{FE}(h)$ is convex for $h > \frac{1}{2}$ if:

$$\mu > \frac{(\phi+1)^3(\sigma-1)(\phi-\psi)^3}{(\phi-1)^3(\phi+\psi)^3}.$$

Substituting for the bundling parameter ψ we get that ΔU^{FE} is convex for $h > \frac{1}{2}$ if Assumption 2 is satisfied, that is, if inequality (8) holds:

$$\mu > g(\phi) \equiv \frac{(\sigma-1)\left[(1-\phi)\sigma^3-\mu(\phi+1)\right]}{\left[\sigma(\phi+1)-\mu(1-\phi)\right]^3}.$$

Since $g(\phi)$ is decreasing and $g(0) > 0$ and $g(1) < 0$, the inequality (8) in Assumption 2 provides a sufficient condition for convexity when $h > \frac{1}{2}$.

In order to show that inequality (8) is satisfied if the freeness of trade is high enough define $F(\phi) = \mu - g(\phi)$. Inequality (8) holds if and only if $F(\phi) > 0$. The properties of g are sufficient to ensure that F is increasing and that $F(1) > 0$. If $F(0) > 0$ the convexity of ΔU^{FE} is guaranteed. If $F(0) < 0$, since $F(\phi)$ is increasing and $F(1) > 0$, there is a unique value $\phi_c \equiv \phi \in (0,1)$ such that $F(\phi) > 0$ if $\phi > \phi_c$. Then ΔU^{FE} is convex for $h > \frac{1}{2}$ if $\phi \in (\phi_c, 1)$, where ϕ_c is implicitly defined by $\mu = g(\phi)$. □

Proof of Proposition 3:

Consider the utility differential in (4) and a linear *home-sweet-home*-effect $\Delta t(h) = (1-2h)\theta$. Then the overall utility differential is given by:

$$\Delta V^{PF} = \frac{\alpha}{\sigma-1}\left(\left\{\frac{(2h-1)(\sigma-1)(1-\phi)\left[(l+2)\phi-l\right]}{2\sigma\left[1-h(1-\phi)\right]\left[(1-h)\phi+h\right]}\right\}+\ln\left[\frac{h(1-\phi)+\phi}{1-h(1-\phi)}\right]\right)+(2h-1)\theta.$$

Accordingly, agglomeration $h^* = 1$ is stable if:

$$\theta < \theta_s \equiv \frac{\alpha(1-\phi)\left[l(\phi-1)+2\phi\right]}{2\sigma\phi} - \frac{\mu\ln\phi}{\sigma-1}.$$

Notice that $\lim_{\phi\to 0}\theta_s \to -\infty$, $\theta_s(\phi = 1) = 0$, and that θ_s is concave in ϕ. Therefore, θ_s has a zero for $\phi \in (0, 1)$. We thus require ϕ to be high enough so that $\theta_s > 0$.

Symmetric dispersion $h^* = \frac{1}{2}$ is stable if $\frac{\partial \Delta V^{PF}}{\partial h}\left(\frac{1}{2}\right) < 0$, that is, if:

$$\theta > \theta_b \equiv \frac{2\alpha(1-\phi)\left[l(\sigma-1)(\phi-1)+3\sigma\phi+\sigma-2\phi\right]}{3(\sigma-1)\sigma(\phi+1)^2}.$$

In order to ensure that $\theta_b > 0$, we assume that the following condition holds:

$$\phi > \frac{l(1-\sigma)+\sigma}{-(l+3)\sigma+l+2}.$$

In other words, we require the freeness of trade to be sufficiently high.[13] However, it cannot be too high so as to violate strict concavity of the PF model. Thus, using (5) from Assumption 1 and solving for ϕ, we assume that:

$$\frac{l(1-\sigma)+\sigma}{-(l+3)\sigma+l+2} < \phi < \frac{3l+\sigma-3l\sigma}{3l+6-3l\sigma-7\sigma}.$$

Next, we compute the following derivatives:

$$1.\ \frac{\partial \Delta V^{PF}}{\partial h}\left(\tfrac{1}{2};\theta_b\right) = 0;\ \ 2.\ \frac{\partial \Delta V^{PF}}{\partial \theta}\left(\tfrac{1}{2};\theta_b\right) = 0;\ \ 3.\ \frac{\partial^2 \Delta V^{PF}}{\partial h^2}\left(\tfrac{1}{2};\theta_b\right) = 0.$$

Next, we have that:

$$4.\ \frac{\partial^2 \Delta V^{PF}}{\partial h \partial \theta}\left(\tfrac{1}{2};\theta_b\right) = -2,$$

and finally:

$$5.\ \frac{\partial^3 \Delta V^{PF}}{\partial h^3}\left(\tfrac{1}{2};\theta_b\right) = \frac{32\alpha(1-\phi)^3\left[3l(\sigma-1)(\phi-1)+7\sigma\phi+\sigma-6\phi\right]}{(\sigma-1)\sigma(\phi+1)^4},$$

which is negative, since under Assumption 1 we have:

$$l > \frac{7\sigma\phi+\sigma-6\phi}{3(\sigma-1)(1-\phi)}.$$

According to Guckenheimer and Holmes (2002, pp. 150), the derivatives *(1.)* through *(5.)* ensure that symmetric dispersion undergoes a pitchfork bifurcation at the break point θ_b. Moreover, the negative sign at *(5.)* ensures that the pitchfork is supercritical. This implies

[13] It has to be higher than this threshold since evaluating θ_s at this threshold yields $\theta_s < 0$.

that $\theta_s < \theta_b$ and a branch of stable asymmetric equilibria exists for $\theta \in (\theta_s, \theta_b)$, which concludes the proof. □

Proof of Proposition 4:

Consider the utility differential in (7) and a linear *home-sweet-home*-effect. Then the overall utility differential is given by:

$$\Delta V^{FE} = \ln\left[\frac{h\phi + (1-h)\psi}{(1-h)\phi + h\psi}\right] + \frac{\mu}{\sigma - 1}\ln\left[\frac{(1-h)\phi + h}{h\phi + (1-h)}\right] + (2h-1)\theta.$$

Agglomeration $h^* = 1$ is stable if:

$$\theta < \theta_s \equiv \ln\left[\frac{2\sigma\phi}{\phi^2(\mu+\sigma) - \mu + \sigma}\right] - \frac{\mu \ln \phi}{\sigma - 1}.$$

Symmetric dispersion $h^* = \frac{1}{2}$ is stable if $\frac{\partial \Delta V^{FE}}{\partial h}\left(\frac{1}{2}\right) < 0$, i.e., if:

$$\theta > \theta_b \equiv \frac{2(1-\phi)\left[\mu^2(1-\phi) - \mu(2\sigma-1)(\phi+1) + (\sigma-1)\sigma(1-\phi)\right]}{(\sigma-1)(\phi+1)\left[\phi(\mu+\sigma) - \mu + \sigma\right]}, \tag{16}$$

where we require:

$$\phi > \frac{(\sigma-\mu)(\sigma-\mu-1)}{(\mu+\sigma-1)(\mu+\sigma)} \tag{17}$$

to ensure that $\theta_b > 0$, which, in turn, requires that $\sigma > 1 + \mu$ so that $\phi > 0$.

Next, we compute the following derivatives:

$$1.\ \frac{\partial \Delta V^{FE}}{\partial h}\left(\tfrac{1}{2};\theta_b\right) = 0;\ \ 2.\ \frac{\partial \Delta V^{FE}}{\partial \theta}\left(\tfrac{1}{2};\theta_b\right) = 0;\ \ 3.\ \frac{\partial^2 \Delta V^{FE}}{\partial h^2}\left(\tfrac{1}{2};\theta_b\right) = 0.$$

Further, we have:

$$4.\ \frac{\partial^2 \Delta V^{FE}}{\partial h \partial \theta}\left(\tfrac{1}{2};\theta_b\right) = 2,$$

and finally:

$$\frac{\partial^3 \Delta V^{FE}}{\partial h^3}\left(\tfrac{1}{2};\theta_b\right) = \frac{32(1-\phi)^3\left[\phi^3(\mu+\sigma-1)(\mu+\sigma)^3 - 3\phi^2(\mu-\sigma+1)(\mu+\sigma)^2(\mu-\sigma) + \eta\right]}{(\sigma-1)(\phi+1)^3\left[\phi(\mu+\sigma) - \mu + \sigma^3\right]},$$

where

$$\eta = 3\phi(\mu+\sigma-1)(\mu+\sigma)(\mu-\sigma)^2 - (\mu-\sigma)^3(\mu-\sigma+1)$$

This last derivative can be shown to be positive if the condition for convexity of ΔU^{FE} is satisfied (see Proof of Lemma 2):

$$\mu > \frac{(\sigma - 1)\left[(1-\phi)\sigma^3 - \mu(\phi+1)\right]}{\left[\sigma(\phi+1) - \mu(1-\phi)\right]^3}.$$

According to Guckenheimer and Holmes (2002, pp. 150), the derivatives *(1.)* through *(5.)* ensure that symmetric dispersion undergoes a pitchfork bifurcation at the break point θ_b. Moreover, the positive sign at *(5.)* ensures that the pitchfork is subcritical. This implies that $\theta_s > \theta_b$ and a branch of unstable asymmetric equilibria exists for $\theta \in (\theta_b, \theta_s)$. Moreover, in this interval, both agglomeration and symmetric dispersion are simultaneously stable. This concludes the proof. □

Proof of Proposition 5:

Consider the utility differential in (10) and a linear *home-sweet-home*-effect. Then the overall utility differential is given by:

$$\Delta V^{OTT} = \left[C\tau\left(\tau^* - \tau\right) + 2\theta\right]\left(h - \frac{1}{2}\right).$$

Agglomeration $h^* = 1$ is stable if:

$$\theta < \theta_{bs} \equiv \frac{1}{2} C\tau\left(\tau^* - \tau\right),$$

while symmetric dispersion $h^* = \frac{1}{2}$ is stable if

$$\theta > \theta_{bs}.$$

If $\theta = \theta_{bs}$, there are no stable equilibria, which concludes the proof. □

A.1. The FE model with logit *home-sweet-home* effect

We can use some of the calculations in the proof of Proposition 4 by noting that

$$\Delta V^{FE}_{logit} = \Delta U^{FE} - \theta \ln\frac{h}{1-h} \quad \text{whereas} \quad \Delta V^{FE}_{linear} = \Delta U^{FE} - \theta(2h-1).$$

Existence and multiplicity of equilibria

Notice that under the logit *home-sweet-home* effect, we have:

$$\frac{d\Delta V^{FE}}{dh}(h) = \frac{ah^4 + bh^3 + ch^2 + dh + e}{D},$$

where the numerator is a 4th degree polynomial whose coefficients depend on parameters μ, ϕ, and σ, and

$$\begin{aligned} D = & -(1-h)h(\sigma-1)\left[h(\phi-1)+1\right]\left[h(1-\phi)-\phi\right] \times \\ & \times \left\{(1-h)\mu\left(1-\phi^2\right)+\sigma\left[h(\phi-1)^2-\phi^2-1\right]\right\} \times \\ & \times \left\{h(\phi-1)\left[\mu(\phi+1)+\sigma(\phi-1)\right]+2\sigma\phi\right\} = \\ = & \ (1-h)h(\sigma-1)\left[h(\phi-1)+1\right]\left[h(1-\phi)-\phi\right] \times \\ & \times \left\{(1-h)(\mu+\sigma)\phi^2+(1-h)(\sigma-\mu)+2\phi\sigma h\right\} \times \\ & \times \left\{(\sigma+\mu)h\phi^2+(\sigma-\mu)h+2\sigma\phi(1-h)\right\} > 0. \end{aligned}$$

Hence, the sign of the derivative depends only on the sign of the numerator which has at most four real zeros. This means that ΔV^{FE} has at most four turning points and thus, at most four equilibria besides $h = \frac{1}{2}$. Due to symmetry, there are at most two asymmetric equilibria for $h \in \left(\frac{1}{2}, 1\right)$.

Bifurcation at symmetric dispersion

Symmetric dispersion, $h^* = \frac{1}{2}$, is stable if:

$$\theta > \theta_b \equiv \frac{(1-\phi)\left[\mu^2(\phi-1)+\mu(2\sigma-1)(\phi+1)+(\sigma-1)\sigma(\phi-1)\right]}{(\sigma-1)(\phi+1)\left[\phi(\mu+\sigma)-\mu+\sigma\right]}, \qquad (18)$$

where θ_b is the break-point, or the degree of consumer heterogeneity above which symmetric dispersion is stable. To ensure that the break-point is positive, we assume that the "no black-hole" condition (17) is satisfied. This, in turn, requires that the no black-hole condition from Fujita et al. (1999), $\sigma > 1 + \mu$, holds. Otherwise, the condition $\theta > \theta_b$ is trivially satisfied and symmetric dispersion is always stable.

Note that θ_b in (18) is not the same as in (16). We choose not to distinguish them since it is clear from the context and to avoid cumbersome notation.

At the break-point $\theta = \theta_b$, we have

$$\frac{\partial^2 \Delta V^{FE}}{\partial h^2}\left(\frac{1}{2};\theta_b\right) = 0, \; \frac{\partial \Delta V^{FE}}{\partial \theta}\left(\frac{1}{2};\theta_b\right) = 0, \; \frac{\partial^2 V^{FE}}{\partial h \partial \theta}\left(\frac{1}{2};\theta_b\right) = -4 < 0,$$

and:

$$\frac{\partial^3 \Delta V^{FE}}{\partial h^3}\left(\frac{1}{2};\theta_b\right) = -\frac{128(1-\phi)\phi\Psi}{(\sigma-1)(\phi+1)^3\left[\phi(\mu+\sigma)-\mu+\sigma)^3\right]},$$

where:

$$\begin{aligned}\Psi(\phi) =& \phi^3(\mu+\sigma)^2\left[\mu^2+\mu\sigma+2(\sigma-1)\sigma\right] + \phi^2(\sigma-\mu)(\mu+\sigma)\left[3\mu^2+3\mu\sigma-2(\sigma-1)\sigma\right] + \\ &+ \phi(\mu-\sigma)(\mu+\sigma)\left[3\mu^2-3\mu\sigma-2(\sigma-1)\sigma\right] - (\mu-\sigma)^2\left[\mu^2-\mu\sigma+2(\sigma-1)\sigma\right].\end{aligned}$$

The sign of the third derivative of ΔV^{FE} is opposite to the sign of Ψ. Notice that $\Psi(0) < 0$ and $\Psi(1) > 0$ and $\Psi'(\phi) > 0$. This means that Ψ has exactly one zero $\phi_c \equiv \phi \in (0,1)$ and that $\Psi(\phi) > 0$ if $\phi > \phi_c$. Therefore, we have $\frac{\partial^3 \Delta V^{FE}}{\partial h^3}\left(\frac{1}{2};\theta_b\right) > 0$ if $\phi < \phi_c$. According to Guckenheimer and Holmes (2002, pp. 150), the FE model with logit *home-sweet-home*-effect undergoes a pitchfork bifurcation at symmetric dispersion if $\phi \neq \phi_c$. If $\phi < \phi_c$ then the bifurcating branch is unstable and exists for $\theta > \theta_b$, as in Figure 6. The asymmetric equilibria arising through this bifurcation are unstable. If $\phi > \phi_c$ then a branch of stable asymmetric equilibria arises for $\theta < \theta_b$ – we do not pursue this case any further. When $\phi < \phi_c$ the primary branch may undergo another (secondary) bifurcation which we study next.

Bifurcation for asymmetric equilibria

Let $\phi < \phi_c$ and consider the half-branch of unstable asymmetric equilibria that exists for $h^* \in \left(\frac{1}{2}, 1\right)$. Then h^* interchanges stability at some value $\theta_f \equiv \theta > 0$ such that $\frac{d\Delta V^{FE}}{dh}(h^*;\theta_f) = 0$. This threshold value θ_f is given by:

$$\begin{aligned}\theta_f =& (1-h^*)h^*\left(\frac{(1-\phi)(\phi+1)\left[\phi^2(\mu+\sigma)^2-(\sigma-\mu)^2\right]}{\left[(1-h^*)\phi^2(\mu+\sigma)+(h^*-1)(\mu-\sigma)-2h^*\sigma\phi\right]\left\{h^*(\phi-1)\left[\phi(\mu+\sigma)+\mu-\sigma\right]+2\sigma\phi\right\}}\right. \\ &\left. + \frac{\mu-\mu\phi^2}{(\sigma-1)\left[h^*(\phi-1)+1\right]\left[h^*(1-\phi)+\phi\right]}\right) > 0.\end{aligned}$$

It can be shown that the sign of $\frac{\partial^2 \Delta V}{\partial h^2}^{FE}(h)$ depends on the sign of its numerator, which is a fifth degree polynomial, $P(h)$, and is zero at $h = \frac{1}{2}$. This means that the derivative has at most two positive roots for $h \in \left(\frac{1}{2}, 1\right)$. Cumbersome yet standard calculations permit to show that, for $(\phi, \sigma, \mu) = (0.5, 4, 0.3)$, we have:

$$P(h) \approx 48.6283h^5 - 121.571h^4 - 92.4189h^3 + 260.199h^2 + 17.9179h - 56.3778 > 0,\ \forall h \in \left(\frac{1}{2}, 1\right).$$

Therefore, there exists an open subset of parameter values (ϕ, σ, μ) such that, for any equilibrium $h^* \in \left(\frac{1}{2}, 1\right)$, we have:

$$\frac{\partial^2 \Delta V^{FE}}{\partial h^2}(h^*; \theta_f) = 0.$$

Finally, notice that:

$$\frac{\partial \Delta V^{FE}}{\partial \theta}(h^*; \theta_f) = \ln\left(\frac{1-h}{h}\right),$$

which is negative for any $h^* \in \left(\frac{1}{2}, 1\right)$. Thus, according to Guckenheimer and Holmes (2002, pp. 148), there exists a set of values in (ϕ, σ, μ) space such that an asymmetric equilibrium $h^* \in \left(\frac{1}{2}, 1\right)$ undergoes a saddle-node bifurcation at the limit point θ_f. Since the primary branch from symmetric dispersion is unstable, we can conclude that a curve of asymmetric equilibria exists tangent to the line $\theta = \theta_f$, lying to its left, such that the more asymmetric equilibria (higher h) are stable. See Figure 6. □