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State Space Stability Analysis of Unconstrained Decentralized Model Predictive Control Systems

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Abstract— **This paper analyzes stability properties of a decentralized MPC strategy. Assuming that control agents can exchange information with each other only once for every sampling interval, stability conditions on model and controller matrices are derived for the unconstrained case. Stability properties are analyzed either from a local or a global point of view and a comparison between the decentralized and the centralized control is introduced.**

I. INTRODUCTION

In process control it is usual to deal with different interconnected plants characterized by significant interactions. For these processes different control solutions can be developed: a centralized control solution, where all the interactions are considered, a decentralized control solution where each independent control agent is able to acquire estimations or measurements of the interactions [1], or a coordinator manages the information exchange among the agents and coordinates them in a hierarchical architecture [2].

Recently, the functionalities of a Decentralized Model Predictive Control (DMPC) solution has been tested in real plants. Preliminary results obtained by the proposed strategy on a gasifier are satisfactory if compared with those obtained by the classical centralized Model Predictive Control (MPC) [3]. Adopting a proper decentralized solution, the global computational effort can be reduced without significant deteriorations of the control performances and the fault tolerant issues can be improved [4].

In this paper a stability analysis of a model predictive control strategy is developed for the control agents operating in a decentralized control architecture. In general, the optimum policy for each agent does not guarantee the global optimum. Global objectives, such as closed-loop stability or some performance requirements for the global process require coordination among the control agents. The proposed solution is based on independent agents and on a local network used for exchanging a reduced set of information [5], [6], [7], [8].

This paper contain a set of preliminary results on the developed stability analysis and represent a contribution to decentralized implementation of model predictive control strategy. In order to define a behavior reference, a classical centralized control strategy is recalled in section II, then the decentralized case is analyzed in section III, either from a local or a global point of view. Concluding remarks end the paper.

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II. CENTRALIZED MPC

Model predictive control is a control technique based on the receding horizon principle that is also known as Receding Horizon Control (RHC) [9]. At each sampling time, using a predictive model of the system dynamics, the response of the process to changes in manipulated variables over a fixed horizon is predicted. Based on a proper objective function, a finite-horizon optimal control problem is solved to obtain current and future moves of the manipulated variables. Only the first computed move is actually applied to the real system whereas all other control actions in the optimal control sequence are discarded. The same procedure is repeated at the next control step based on the new measurement.

During the optimization procedure also constraints can be explicitly considered but, in this case, an explicit solution is not easy to find, even with linear processes [10]. Due to the finite horizon, MPC does not guarantee stability in general. If a terminal constraint and a terminal weight are added on the state, then the stability may be guaranteed if certain conditions are verified for the terminal weight [11]. However, as shown in [12], stability is even possible if no terminal weight is added to the formulation. In order to develop a general approach to the study of decentralized MPC able to provide an explicit solution to the problem, the simplifying assumption of unconstrained process variables is considered in this paper.

A. MPC problem

Let consider a plant $\mathscr{P}$ with fully accessible state and denote with $x \in \mathbb{R}^{n_x}$, $u \in \mathbb{R}^{n_u}$, $y(k) \in \mathbb{R}^{n_y}$ and $y^d \in \mathbb{R}^{n_y}$, its state, control input, output and desired output, respectively. Define the l-step ahead state prediction at time k as $\hat{x}(k+l|k) \in \mathbb{R}^{n_x}$, and the control input move at time k as $\Delta u(k) \triangleq u(k) - u(k-1)$. Finally, let assume that Q and R are two symmetric and positive definite matrices, and $p > 0$ and $m > 0$ are two integer values such that $m \leq p$.

Definition 1 (MPC): Given a linear plant $\mathscr{P}$:

$$x(k+1) = Ax(k) + Bu(k) \tag{1a}$$

$$y(k) = Cx(k), \tag{1b}$$

the centralized MPC problem with prediction horizon p and control horizon m consists of finding, at time k, a control