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Modeling and Simulation of Dewatering of Particulate Suspensions by Batch Pressure Filtration

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Abstract

A Darcy law based approach for simulation of batch constant pressure filtration of particulate suspensions when the feed solids concentration changes, is developed. Using filtration data obtained using a feed suspension with a not-very-low solids concentration, we propose procedures to predict changes in (i) kinetics of dewatering in the cake formation stage, and (ii) transition point between the cake formation and consolidation stages, as the initial solid concentration changes. The evolution of dewatering can be predicted using these two model parameters. The approach is simpler to implement in comparison to available pressure filtration models which require several characterization experiments.

Keywords: *Simulation, control and modeling.*

INTRODUCTION

Solid-liquid separation is an integral requirement of several processes in the chemical, mineral and water treatment industries. In most applications, the key objective is to increase the solid content of a suspension or to increase recovery of the liquid. Pressure filtration is routinely employed for dewatering of fine and colloidal suspensions, which can give rise to compressible cakes. This process, and constant pressure filtration in particular, has been extensively studied in the past 6 decades.

The modeling of dewatering by batch constant pressure filtration of materials which form compressible cakes is considered in this work. It is assumed that dewatering occurs in two stages. This feature can be clearly seen in several sets of published experimental data (Wakeman et al., 1991; Sis and Chander, 2000; de Kretser et al., 2001; Kapur et al., 2002; Brown and Zukoski, 2003; etc.). The progress of dewatering is commonly depicted using the temporal evolution of the cumulative volume of filtrate, V, or the instantaneous solid volume fraction, ϕ, etc.. Typical two stage filtration curves in $t-\phi$ coordinates are shown in Fig. 1. The first stage, stage 1, is characterized by the cake formation and growth, while the second stage, stage 2, consists of the cake consolidation stage. The initiation of stage 2 is typically characterized by an exponential increase in dewatering time (Fig. 1).

The overall dewatering process is governed by the following two quantities: compressibility of the material and the permeability. While the former governs the extent of dewatering, the latter governs the rate at which dewatering proceeds. Different mathematical theories of dewatering employ constitutive relationships – determined by experiments – for these two quantities, which are of the general form:

$$\phi_\infty = F_1(\Delta P) \tag{A}$$

$$k = F_2(\phi_\infty) \tag{B}$$

Here, ϕ_∞ is the final solid content at the specified constant pressure, ΔP, and k is the permeability of the cake. Equation (A) is only an engineering approximation since the experimentally observed dependence of compressive yield stress on the initial solid content has been suppressed (Green and Boger, 1997). Two different laboratory systems which are commonly used for determination of these constitutive relationships are the compressibility-permeability (CP) cell approach (Tien et al., 2001) and the step pressure filtration approach (de Kretser et al., 2001). The constitutive relationship thus determined, in conjunction with an appropriate filtration model (Stamatakis and Tien, 1991; Landman and White, 1997; Burger et al., 2001, etc.) can be used to simulate the filtration process under different operating conditions. *Even when the operating pressure is constant and the objective is only to investigate the effect of variation in initial solid concentration on the dynamics of dewatering, it is necessary to conduct experiments at different pressures, before the filtration process can be simulated. This information is needed to determine the dependence of the (i) kinetics of the cake formation stage and (ii) transition point between the cake formation and consolidation stages, on the solids concentration in the feed suspension.*

DESCRIPTION OF MODEL

Detailed derivation of the model has been presented earlier and hence is only briefly outlined here (Kapur et al., 2002). The model is formulated by assuming that dewatering is completed in two distinct stages (Fig 1, and also Fig. 1 of Kapur et al., 2002). Initial dewatering occurs by filtration and is characterized by the formation and growth of a filter cake. The first, i.e. the filtration stage (stage 1) ends when the filter cake which is moving upwards meets the piston which is moving down. At this time instant, t_c, the average solid volume fraction in the filter cake is denoted by ϕ_c. Subsequent dewatering, i.e. in stage 2 is characterized by compression or consolidation of the filter cake. In theory, this stage lasts from time t_c to infinite time, when the solid concentration ϕ_∞ becomes uniform throughout the cake and dewatering ceases. As demonstrated by Kapur et al. (2002), this model permits a unified treatment of both stages of the dewatering process.

Model for Cake Formation and Growth Stage (Stage 1)

The simplest of the models applicable to the cake formation and growth zone is that due to Ruth (Ruth, 1946). In terms of the solids concentration in the filtration cell, Ruth's equation can be written as

$$t = \frac{h_0^2}{\beta^2}\left[1 - \frac{\phi_0}{\phi}\right]^2, \text{ for } \phi \le \phi_c \tag{1}$$

The differential form of Eqn. (1) is given by

$$\frac{d\phi}{dt} = \frac{\beta^2 \phi^3}{2h_0^2 \phi_0 (\phi - \phi_0)}, \text{ for } \phi \le \phi_c \tag{2}$$

Here, β^2 is the reciprocal of the slope of the plot of t/V versus V, ϕ_0 and h_0 are the initial solid volume fraction and initial height, respectively. In the development of the model Eqns. (1) and (2), it is assumed that the membrane resistance and effects of sedimentation can be neglected.

Model for Cake Consolidation Stage (Stage 2)

In conjunction with Darcy law, a simple relationship between solid stress and solid concentration has been used to derive the following model for dewatering in the cake consolidation stage

$$\frac{d\phi}{dt} = k_c k(\phi)\phi^3(\phi_\infty - \phi), \text{ for } \phi > \phi_c \tag{3}$$

where the first two terms represent a suitable permeability correlation (eg. Kozeny-Carman, Brenner correlations, among others), with $k(\phi)$ denoting the ϕ-dependent part and k_c denoting the size dependent part (considered to be a constant) of the permeability correlation, and ϕ_∞ is the final solid content of the cake, i.e. the solid volume fraction as time, $t \to \infty$. A suitable form of the ϕ-dependent permeability term, $k(\phi)$, can be selected if experimental dewatering data at a single pressure is available. For modeling purposes, it is assumed that the form of $k(\phi)$ is only dependent on the chemistry of the suspension.

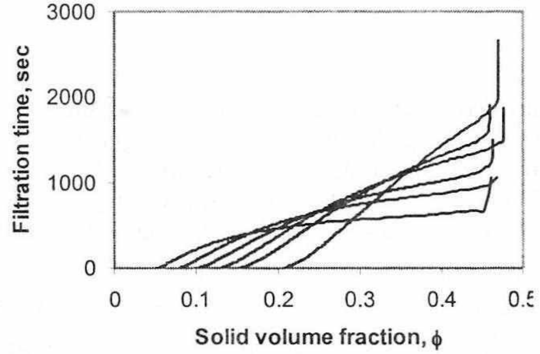

Fig. 1: Dewatering of Alumina A16-SG Suspensions. Experimentally Observed Evolution of Dewatering for Six Values of Initial Solids Concentration, Which Increase From 0.05 to 0.20 in Steps of 0.025. Each Curve Originates from its ϕ_0

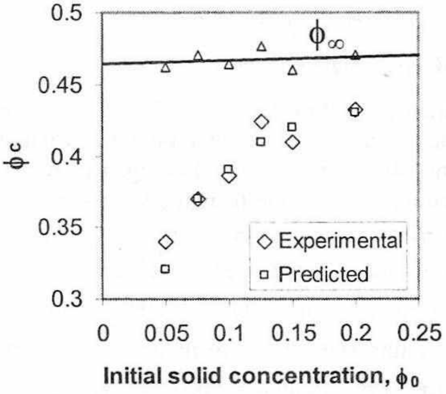

Fig. 2: Dependence of Parameter ϕ_c on Initial Solids Concentration, ϕ_0. For this Alumina A16-SG System, the Final Solids Volume Concentration, ϕ_∞, is Independent of ϕ_0

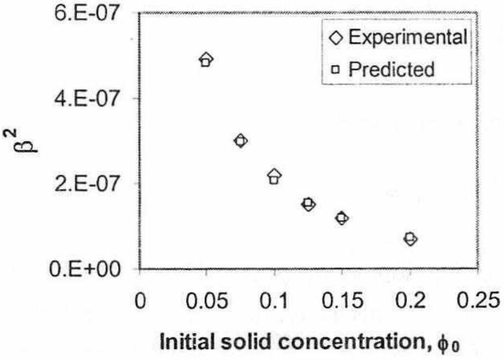

Fig. 3: Dependence of Parameter β^2 on Initial Solids Concentration, ϕ_0

It is well known that ϕ_c and β^2 depend on the initial solids concentration, ϕ_0, even when the operating pressure is held constant. ϕ_c is traditionally evaluated from experimental constant pressure filtration data using the procedure suggested by Shirato (Shirato et al., 1970). Evolution of dewatering and variation of the parameters ϕ_c and β^2 with change in ϕ_0 for alumina A16-SG suspensions (at a constant pressure of 100 kPa) is shown in Figs. 1-3.

OBJECTIVE

The objective of the present work is to develop Darcy law based methods for predicting the observed dependence of ϕ_c and β^2 on ϕ_0 (at some known pressure). It will be assumed that *one* set of experimental pressure filtration data at some not-very-low value of ϕ_0 is available. To the best of our knowledge, all available methods for predicting the dependences of ϕ_c and β^2 on ϕ_0 require a complete knowledge of the rheological behavior, i.e. the compressive yield stress and permeability characteristics of the system (Landman and White, 1997; Landman et al., 1999). As will be briefly demonstrated later in this paper, the ability to predict variation in ϕ_c and β^2 with change in ϕ_0, can be a valuable tool in practical situations.

RESULTS AND DISCUSSION

Experimental Procedure

Particulate suspensions for pressure filtration tests were prepared from A16 SG alumina of mean particle size 0.4 μm (supplied by Alcoa-ACC Industrial Chemical Ltd, India). In all experiments reported in this paper, the pH of the suspension was controlled close to 9. The suspensions were prepared by dispersing the powder with a magnetic stirrer for 2 min, followed by ultra-sonication with a Branson 450 sonicator for 2 min at 40 Watt power input, and conditioned for 24 hours using a magnetic stirrer. After conditioning, the suspension was again ultra-sonicated for 2 minutes using the sonicator. Whatman filter paper No. 42 was used as the filter medium. Pressure filtration experiments were carried out in a highly instrumented, programmable computer-driven laboratory scale test rig (de Kretser et al., 2001).

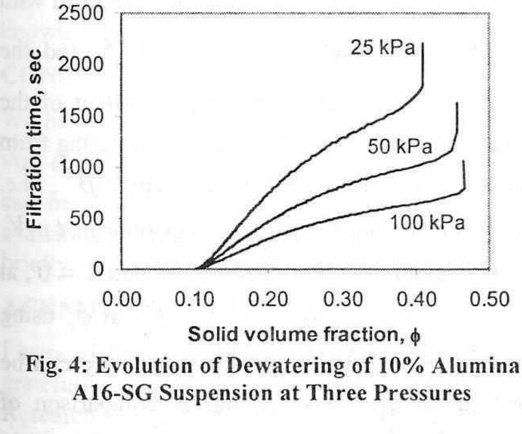

Fig. 4: Evolution of Dewatering of 10% Alumina A16-SG Suspension at Three Pressures

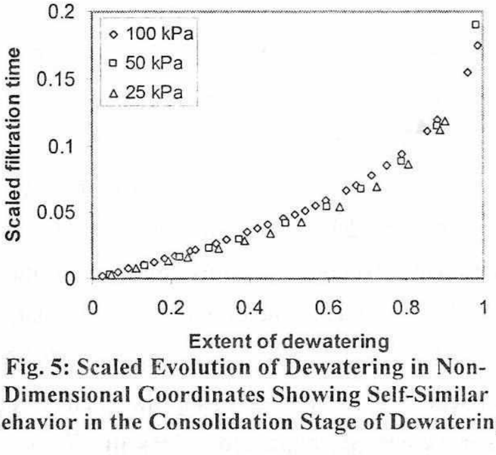

Fig. 5: Scaled Evolution of Dewatering in Non-Dimensional Coordinates Showing Self-Similar Behavior in the Consolidation Stage of Dewatering

Estimation of ϕ_c

The basis of the estimation process is the observation that when all input parameters other than the operating pressure are held constant, filtration curves in the cake consolidation stage (stage 2) show self-similar behavior when the filtration time is scaled with the rate of dewatering at the end of the cake formation stage, and the evolution of dewatering is represented in terms of a normalized solids concentration. The evolution of dewatering at three different pressures and the corresponding evolution in non-dimensional coordinates showing the self-similar nature of evolution are shown in Figs. 4 and 5. More interestingly, the self-similar behavior is preserved when the effect of changing the initial material load – either as a change in the initial solids concentration, or the volume of material to be dewatered – is accounted for.

Therefore, when it is assumed that (i) ϕ_∞ is independent of ϕ_0, and (ii) filtration data at some not-very-low value of ϕ_0 is available, the above discussion implies that the occurrence of self-similar behavior can be used to determine the solids concentration at the beginning of the cake consolidation stage, ϕ_c, as a function of ϕ_0. Determination of the parameter ϕ_c is essentially a single variable optimization problem involving minimizing the error between two curves, and this can also be easily carried out using a spread-sheet. The consolidation stage dewatering model needed for the iterative parameter estimation procedure is given by Eqn. (3), where the ϕ-dependent permeability term, i.e. $k(\phi)$, in Eqn. (3), has to be determined using the single set of experimental filtration data (in (ii) above). A comparison of experimental and predicted values of ϕ_c shown in Fig. 2 indicates that the method yields fairly accurate estimates of ϕ_c. We briefly digress to remark that even when the operating pressure is varied, the occurrence of self-similarity behavior can be used to predict ϕ_∞ if ϕ_c is known.

Estimation of β^2

The numerical scheme for estimating the kinetics of dewatering in the cake formation stage as ϕ_0 changes, is illustrated using a numerical example. Consider that a filtration test has been carried out for a $\phi_0^{Old} = 0.10$ suspension, and it is desired to estimate β^2 at a value of $\phi_0^{New} = 0.15$. The new value of β^2, i.e. for $\phi_0^{New} = 0.15$ can be estimated by (i) rescaling the filtration data obtained with $\phi_0^{Old} = 0.10$ with respect to the experimental time needed to reach $\phi = \phi_0^{New} = 0.15$ and the cumulative volume of filtrate collected at $\phi = \phi_0^{New} = 0.15$, (ii) accounting for the effect of the resistance resulting due to the presence of the cake, which has been deposited due to dewatering from $\phi_0^{Old} = 0.10$ to $\phi = \phi_0^{New} = 0.15$, and (iii) using Ruth's equation to estimate β^2, i.e. $\beta^2 = 2V(dV/dt)$. The method can be best visualized by considering the filtration behavior in $t - V$ coordinates. (i) above is equivalent to resetting the experimental conditions, i.e. $V = 0$ and $t = 0$, at $\phi = \phi_0^{New} = 0.15$. The effect of cake resistance can be accounted for by evaluating β^2 at ϕ_c using a suitable numerical approximation of dV/dt. It may be recalled that ϕ_c for $\phi_0^{New} = 0.15$ can be independently estimated using the method described in the previous section. A comparison of experimental and predicted values of β^2 shown in Fig. 3 indicates that the method yields fairly accurate estimates of β^2.

Determination of Batch Time

In order to demonstrate the utility of the methods described in the previous sections, we consider the situation when it is desired to produce a concentrated suspension of desired solids concentration, ϕ_d, using a fixed volume of dilute feed suspension whose concentration fluctuates from batch to batch. The batch time depends on the feed concentration, and an estimation of the required time is possible if ϕ_c and β^2 can be evaluated for a particular ϕ_0. Figure 6 shows the dependence of the time to reach four sets of desired ϕ_d, as ϕ_0 changes. It is clear that the batch time is significantly affected by changes in ϕ_0, especially at higher values of ϕ_d.

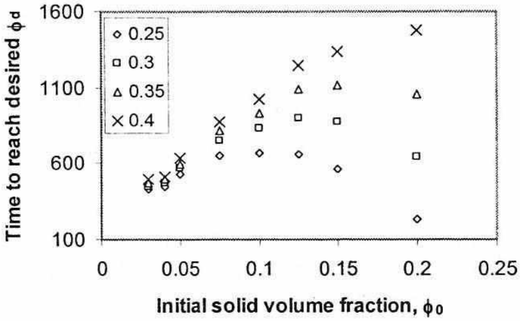

Fig. 6: Dependence of Dewatering Time to Achieve a Desired Product Concentration on the Initial Solids Concentration, ϕ_0

The methods described earlier in this work can be used to determine parameters, ϕ_c and β^2 for a given ϕ_0. These values can then be used in Eqns. (1) and (3) to estimate the batch time needed to maintain the desired product quality. For example, when the feed concentration drops from 0.125 to 0.1, the filtration time needed to produce a concentrated suspension containing 40% solids, i.e. $\phi_d = 0.40$, decreases from 1250 seconds to 1050 seconds. For this rather simple example, apriori knowledge of the batch time can be used to save about 20% of operating time.

CONCLUSIONS

We have presented simple methods for estimating two of the important filtration process parameters, viz. ϕ_c and β^2, when the feed concentration changes. It has been shown that these two parameters can be estimated if experimental information obtained from only a single filtration experiment is available. Results analyzing the implications for optimal operation of batch pressure filtration were briefly discussed.

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