A Simplest Fuzzy PID Controller: Analytical Structure and Stability Analysis

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***Abstract*— This paper reveals analytical structure for the simplest fuzzy PID controller which employs two fuzzy sets for each of the three input variables and four fuzzy sets for the output variable. Analytical structures are derived via left and right trapezoidal membership functions for each input, triangular membership functions for output, algebraic product triangular norm, bounded sum triangular co-norm, Mamdani minimum inference method, and center of area(COA) defuzzification method. Bounded-input bounded-output(BIBO) stability analysis is presented. Finally, a numerical example along with its simulation results is included to demonstrate the effectiveness of the simplest fuzzy PID controller.**

I. INTRODUCTION

Conventional(linear) PID controllers have been extensively used in industry due to their simplicity, low cost and effectiveness for linear systems. Generally, conventional PID controllers are not suitable for higher order and time-delay systems, nonlinear systems, complex and vague systems without precise mathematical models, and systems with uncertainties. It has been found that fuzzy logic based PID controllers have better capabilities of handling above mentioned systems.

A fuzzy self tuning PID control scheme has been presented(He et.al., 1993) for regulating industrial processes by first parameterizing a Ziegler-Nichols like tuning formula, and then using an on-line fuzzy inference mechanism. Attempts have been made (Carvajal et.al., 2000) to develop analytical structures for fuzzy PID controller by dividing the three dimensional(3D) input space into eight sectors, and deriving the structures for each sector. The main difficulty with this approach is the visualization of the state point in the 3D input space. Also, the suitability of these analytical structures from the control view point has not been studied. Very recently it has been shown that fuzzy PID control is not possible as long as intersection triangular norm is used(Mohan and Sinha) because the control surface has discontinuities at some points in the 3D input space.

As the fuzzy PID controllers obtained via Zadeh's minimum triangular norm exhibited undesirable properties, in this paper attempts are made to derive analytical structure for a fuzzy PID controller by employing algebraic product triangular norm, bounded sum triangular co-norm, left(Γ-type) and right(L-type) trapezoidal membership functions(Driankov et.al., 1993) for inputs, triangular membership functions for output, nonlinear control rules, Mamdani minimum inference method, and COA method of defuzzification. Conditions for BIBO stability of fuzzy PID control system are obtained. Finally, to demonstrate the superiority of fuzzy PID controller over the conventional PID controller, simulation results of an example are included.

The paper is organized as follows. The next section deals with the fundamental components of a typical fuzzy PID controller. Section 3 describes the fuzzy PID analytical structure. Section 4 is about BIBO stability analysis of fuzzy PID control systems. Section 5 includes simulation results while Section 6 consists of concluding remarks.

II. COMPONENTS OF FUZZY PID CONTROLLERS

The incremental control signal generated by a discrete-time PID controller is given by

$$\begin{aligned}\Delta u(kT) &= u(kT) - u[(k-1)T] \\ &= K_P^d v(kT) + K_I^d d(kT) + K_D^d a(kT) \qquad (1)\end{aligned}$$

where K_P^d, K_I^d, and K_D^d are respectively the proportional, integral and derivative constants of digital PID controller, the velocity v(kT), displacement d(kT) and acceleration a(kT) are given by Eqs. (2)-(4),

$$v(kT) = \{d(kT) - d[(k-1)T]\}/T \qquad (2)$$

$$d(kT) = e(kT) \qquad (3)$$

$$a(kT) = \{v(kT) - v[(k-1)T]\}/T \qquad (4)$$

$e(kT)$ is the error signal, and T is the sampling period. Eq. (1) is known as 'velocity algorithm' and it is a widely used form of digital PID control. The principal structure of a fuzzy PID controller(see Figure 1) consists of the following components.

A. Scaling Factors

Normalization is the process of mapping physical values of actual inputs and outputs of the controller into a normalized domain. N_d, N_v, N_a and $N_{\Delta u}$ are the normalization factors for d, v, a and Δu respectively. Denormalization maps the normalized output value into its physical output domain. $N_{\Delta u}^{-1}$ is the reciprocal of $N_{\Delta u}$, called denormalization factor. These scaling factors play a role similar to that of the gain coefficients K_P^d, K_I^d and K_D^d in a conventional PID controller.

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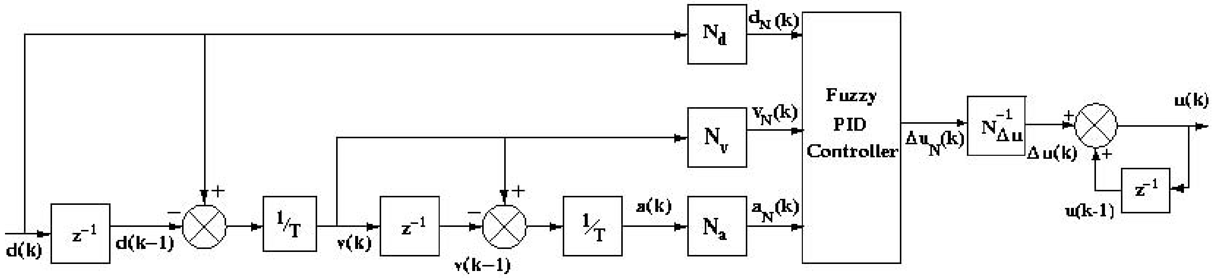

Fig. 1. The fuzzy PID control system

B. Fuzzification

Fuzzification converts crisp values of controller inputs into fuzzy sets that can be used by the inference engine(refer Section 2.4)to activate and apply the control rules. The fuzzy PID controller employs three inputs: the error signal $e(kT)$(displacement $d(kT)$), the first-order time derivative of $e(kT)$(velocity $v(kT)$), and the second-order time derivative of $e(kT)$(acceleration $a(kT)$). These inputs are fuzzified by a combination of L-type and Γ-type membership functions as illustrated in Figure 2 where d_N, v_N and a_N are the normalized inputs. The mathematical description of L-type and

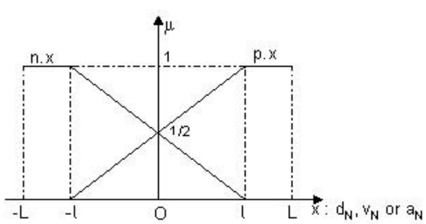

Fig. 2. The input membership functions

Γ-type membership functions is respectively given by

$$\mu_{n.x} = \begin{cases} 1 & -L \le x \le -l \\ \frac{(-x+l)}{2l} & -l \le x \le l \\ 0 & l \le x \le L \end{cases} \quad (5)$$

$$\mu_{p.x} = \begin{cases} 0 & -L \le x \le -l \\ \frac{(x+l)}{2l} & -l \le x \le l \\ 1 & l \le x \le L \end{cases} \quad (6)$$

Note that

$$\mu_{n.x} + \mu_{p.x} = 1 \quad (7)$$

The fuzzy controller has a single output, called incremental control output $\Delta u(kT)$. The membership functions for the normalized output Δu_N are shown in Figure 3. The constants l, L and M are chosen by the designer.

C. Control Rule Base

The following control rules are considered (Carvajal et.al., 2000 and Margaliot and Langholz, 2000) in terms of the abovementioned input and output fuzzy sets.

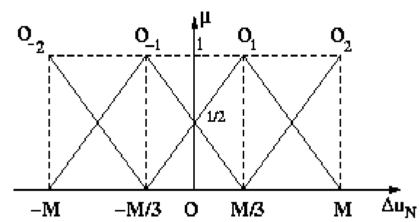

Fig. 3. The output membership functions

R1: If d_N=$n.d$ & v_N=$n.v$ & a_N=$n.a$then Δu_N=O_{-2}
R2: If d_N=$p.d$ & v_N=$n.v$ & a_N=$n.a$ then Δu_N=O_{-1}
R3: If d_N=$p.d$ & v_N=$n.v$ & a_N=$p.a$ then Δu_N=O_{+1}
R4: If d_N=$n.d$ & v_N=$n.v$ & a_N=$p.a$ then Δu_N=O_{-1}
R5: If d_N=$n.d$ & v_N=$p.v$ & a_N=$p.a$ then Δu_N=O_{+1}
R6: If d_N=$n.d$ & v_N=$p.v$ & a_N=$n.a$ then Δu_N=O_{-1}
R7: If d_N=$p.d$ & v_N=$p.v$ & a_N=$n.a$ then Δu_N=O_{+1}
R8: If d_N=$p.d$ & v_N=$p.v$ & a_N=$p.a$ then Δu_N=O_{+2}

where the & symbol in the antecedent part represents the fuzzy "AND" operation which is considered here as algebraic product triangular norm, and is defined as

$$\mu_{ant}(d_N, v_N, a_N) = \mu_i(d_N) \cdot \mu_j(v_N) \cdot \mu_k(a_N) \quad (8)$$

where i, j and k are the i^{th}, j^{th} and k^{th} fuzzy sets on d_N, v_N and a_N respectively. Notice that the control rules are nonlinear as the output fuzzy sets are not linearly related to the input fuzzy sets.

D. Inference Engine

Overall value of the incremental control output variable is computed by the inference engine by considering the individual contribution of each rule in the rule base. For this, corresponding to each rule, first the degree of match from the crisp input values is found by using the algebraic product triangular norm in Eq.(8). Then the degree of match is used to determine the inferred output fuzzy set using Mamdani minimum inference method which is defined as

$$\min(\hat{\mu}, \mu(\Delta u_N))$$

where $\hat{\mu}$ is the outcome of algebraic product triangular norm operator. The reference output fuzzy set(triangular), and the inferred output fuzzy set(shown with hatching) corresponding to this inference method are shown in Figure 4. The area of

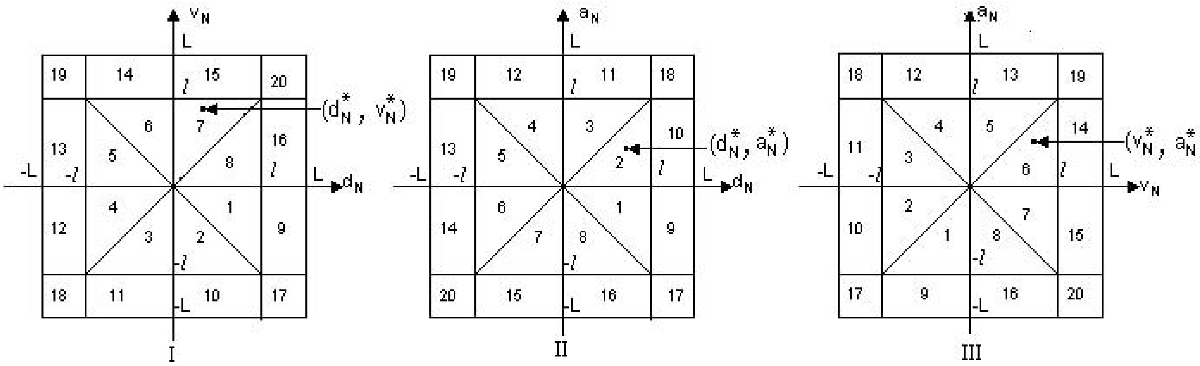

Fig. 5. Regions of the fuzzy PID controller input combinations

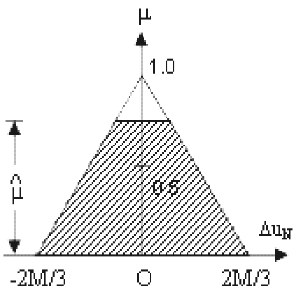

Fig. 4. Illustration of Mamdani minimum inference

inferred set is given by $(2M/3)\hat{\mu}(2-\hat{\mu})$.

As there are three inputs to the fuzzy PID controller, it is necessary to consider all possible combinations of these variables in a 3D space. A point, say(x_1,y_1,z_1), in a 3D space can always be distinctly shown by taking its projection on the *xy*-, *yz*-, and *zx*- planes. So, as shown in Figure 5, twenty input combinations are considered in each ($d_N v_N-$, $d_N a_N-$, and $v_N a_N-$) plane so that the state point (d_N^*, v_N^*, a_N^*) can be uniquely located in the 3D cell(subspace) represented by the triplet(n_I, n_{II}, n_{III}) where n_I, n_{II}, n_{III} = 1, 2, ..., 20. For example, the triplet (9,18,12) represents the 3D cell with 9 from I, 18 from II, and 12 from III of Figure 5.

The control rules (R1) to (R8) of the fuzzy PID controller are used to evaluate appropriate control law in each valid cell (n_I, n_{II}, n_{III}). By using the algebraic product triangular norm the outcome of premise part of each rule is found for all valid cells and is shown in Table I. There are altogether $20 \times 20 \times 20 = 8000$ cells in the 3D input space. Not all 8000 cells are valid cells; only a few of them are valid. A cell (n_I, n_{II}, n_{III}) is said to be valid if and only if the relations between d_N and v_N, and d_N and a_N produce the relation between v_N and a_N. For example, the cell (7, 2, 6) is a valid cell because the relations $d_N \leq v_N$ and $d_N \geq a_N$ produce the relation $v_N \geq d_N \geq a_N$ which is satisfied by the relation $v_N \geq a_N$.

It may be seen from the control rules that the output fuzzy sets O_{-1} and O_{+1} are fired three times each. In such a situation, a fuzzy triangular co-norm is used(Ying, 2000 and Patel and Mohan, 2002) to evaluate combined output fuzzy sets corresponding to the rule sets {(R2),(R4),(R6)} and {(R3), (R5), (R7)}. The bounded sum triangular conorm used here is defined as

$$\min\{1, \mu_A(x) + \mu_B(y)\}$$

Since the fuzzy controller has three inputs and algebraic product triangular norm is used, sum of all the outcomes corresponding to either rule set is less than unity. Therefore the combined memberships using bounded sum triangular co-norm are given by

$$\mu(R2) + \mu(R4) + \mu(R6) < 1$$
$$\text{and } \mu(R3) + \mu(R5) + \mu(R7) < 1$$

E. Defuzzification

Defuzzification module converts fuzzy information into crisp information. The most commonly used COA method is employed to defuzzify the incremental control output. This is expressed as

$$\Delta u_N(k) = \frac{\sum_{i=1}^{8} \{I_{MV}\} \times \{\text{output corresponding to } I_{MV}\}}{\sum_{i=1}^{8} \{I_{MV}\}} \tag{9}$$

where I_{MV} is input membership value.

III. ANALYTICAL STRUCTURE

In this section, we describe analytical structure of fuzzy PID controller derived using algebraic product triangular norm, bounded sum triangular co-norm, Mamdani minimum inference method, and triangular output fuzzy sets. In the following the sampling time 'kT' is shown as 'k' for simplicity.

Case(a) $-l \leq d_N(k),\, v_N(k),\, a_N(k) \leq +l$

$$\Delta u(k) = \left(\frac{2M}{3N_{\Delta u}}\right) \frac{N_1 v_N(k) + N_2 d_N(k) + N_3 a_N(k)}{D} \tag{10}$$

where

$$N_1 = 7l^5 - (a_N^2(k) + d_N^2(k))l^3 - a_N^2(k)d_N^2(k)l \qquad (11)$$
$$N_2 = 7l^5 - (v_N^2(k) + a_N^2(k))l^3 - v_N^2(k)a_N^2(k)l \qquad (12)$$
$$N_3 = 7l^5 - (d_N^2(k) + v_N^2(k))l^3 - d_N^2(k)v_N^2(k)l \qquad (13)$$
$$\text{and } D = 15l^6 - (d_N^2(k) + v_N^2(k) + a_N^2(k))l^4 - (d_N^2(k)v_N^2(k) + v_N^2(k)a_N^2(k) + a_N^2(k) d_N^2(k))l^2 - d_N^2(k)v_N^2(k)a_N^2(k) \qquad (14)$$

Case(b): The normalized inputs $d_N(k)$, $v_N(k)$ and $a_N(k)$ are not in the interval $[-l, l]$, see Figure 2. The $\Delta u(k)$ in different cells is as follows:

$$\Delta u(k) = \left(\frac{2lM}{3N_{\Delta u}}\right)\frac{x}{3l^2 - x^2} \qquad (15)$$
$$= \left(\frac{2M}{3N_{\Delta u}}\right)\frac{3l^2 + lx - x^2}{3l^2 - x^2} \qquad (16)$$
$$= \left(\frac{-2M}{3N_{\Delta u}}\right)\frac{3l^2 - lx - x^2}{3l^2 - x^2} \qquad (17)$$

with x as defined in Table 2.

$$\Delta u(k) = \left(\frac{-M}{3N_{\Delta u}}\right) \text{ for cells } (17,17,17), (18,19,18), (19,20,20) \qquad (18)$$
$$\Delta u(k) = \left(\frac{M}{3N_{\Delta u}}\right) \text{ for cells } (17,18,18), (19,19,19), (20,17,20) \qquad (19)$$
$$\Delta u(k) = \left(\frac{-M}{N_{\Delta u}}\right) \text{ for cell (18,20,17)} \qquad (20)$$
$$\Delta u(k) = \left(\frac{M}{N_{\Delta u}}\right) \text{ for cell (20,18,19)} \qquad (21)$$

A. Analysis of fuzzy Controller

a) : Eq.(10) can be rewritten as

$$\Delta u(kT) = \left(\frac{2M}{3N_{\Delta u}}\right)\left\{\frac{N_1 N_v}{D}v(kT) + \frac{N_2 N_d}{D}d(kT) + \frac{N_3 N_a}{D}a(kT)\right\} \qquad (22)$$

Now by comparing Eq.(22) with Eq.(1) one can easily recognize that fuzzy PID controllers are very much similar in structure to the linear PID controllers. Since N_1, N_2, N_3 and D are not just constants but are nonlinear functions of normalized inputs $d_N(kT)$, $v_N(kT)$ and $a_N(kT)$, the fuzzy PID controller is a nonlinear PID controller with the dynamic gains defined by

$$K_{Pd} = \frac{2MN_1N_v}{3N_{\Delta u}D}\ , \quad K_{Id} = \frac{2MN_2N_d}{3N_{\Delta u}D}$$
$$\text{and } K_{Dd} = \frac{2MN_3N_a}{3N_{\Delta u}D} \qquad (23)$$

where K_{Pd}, K_{Id} and K_{Dd} are respectively the dynamic proportional gain, dynamic integral gain and dynamic derivative gain of fuzzy controller. In the case of linear PID controller the gains K_P^d, K_I^d and K_D^d are constants, i.e. static. Hence, for comparison sake, we also define static gains of fuzzy PID controller by making $d_N(kT) = v_N(kT) = a_N(kT) = 0$ in Eq.(23) to get

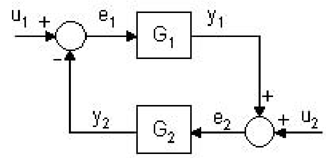

Fig. 6. Feedback control system

$$K_{Ps} = \beta N_v\ ,\quad K_{Is} = \beta N_d \text{ and } K_{Ds} = \beta N_a \qquad (24)$$

where K_{Ps}, K_{Is} and K_{Ds} are respectively the static proportional gain, static integral gain and static derivative gain, and

$$\beta = \left(\frac{14M}{45N_{\Delta u}l}\right) \qquad (25)$$

IV. BIBO STABILITY OF A FUZZY CONTROLLER

In this section BIBO stability analysis of the fuzzy PID control system, shown in Figure 1, is done using the Small Gain theorem(Vidyasagar, 1993).
Consider the system in Figure 6. The overall feedback system is described by the equations

$$e_1 = u_1 - y_2,\ \ e_2 = u_2 + y_1,\ \ y_1 = G_1e_1,\ \ y_2 = G_2e_2$$

Suppose that both subsystems G_1 and G_2 are causal and stable, and let $\gamma_1 = \gamma(G_1)$, the gain of G_1 and $\gamma_2 = \gamma(G_2)$, the gain of G_2. Also suppose that there are constants β_1, β_2, $\gamma_1 \geq 0$ and $\gamma_2 \geq 0$ so that

$$\|y_1\| = \|G_1e_1\| \leq \gamma_1\|e_1\| + \beta_1 \qquad (26)$$
$$\|y_2\| = \|G_2e_2\| \leq \gamma_2\|e_2\| + \beta_2 \qquad (27)$$

Under these conditions, the system is BIBO stable if $\gamma_1\gamma_2 < 1$, i.e. any bounded input pair (u_1, u_2) produces a bounded output pair (y_1, y_2).
We consider the general case where the process under control is nonlinear, denoted by $\mathcal{N}$. By defining $r(k) = u_1(k)$, $e(k) = d(k) = e_1(k)$, $\Delta u(k) = y_1(k)$, $u(k-1) = u_2(k)$, $u(k) = e_2(k)$ and $y(k) = y_2(k)$ in Figure 1, we obtain the equivalent closed loop system in Figure 6. Let

$$M_d = \sup_{k\geq 0}|d(k)|; M_v = \sup_{k\geq 0}|v(k)| \leq \frac{2}{T}M_d;$$
$$M_a = \sup_{k\geq 0}|a(k)| \leq \frac{2}{T}M_v \text{ or } \frac{4}{T^2}M_d$$
$$\tilde{M}_d = N_dM_d;\ \tilde{M}_v = N_vM_v;\ \tilde{M}_a = N_aM_a$$

When $d_N(k)$, $v_N(k)$ and $a_N(k)$ are in the interval $[-l, l]$ we have from Eqs. (10)-(14)

$$\|\Delta u(k)\| = \|y_1(k)\| = \|G_1e_1(k)\| \leq \left(\frac{2M}{3N_{\Delta u}}\right)\left\{\left|\frac{N_1}{DT} + \frac{N_2}{D} + \frac{N_3}{DT^2}\right| N_d|d(k)| + \left(\left|\frac{N_1}{DT}\right| + 3\left|\frac{N_3}{DT^2}\right|\right)\tilde{M}_d\right\}$$

TABLE I

Outcomes of 'algebraicproduct' operation of premise part of control rules $(R1)-(R8)$ for valid $3D$ cells

Cells	$(R1)$	$(R2)$	$(R3)$	$(R4)$	$(R5)$	$(R6)$	$(R7)$	$(R8)$
$(1,1,1)$ to $(8,8,8)$	μ_1	μ_2	μ_3	μ_4	μ_5	μ_6	μ_7	μ_8
$(9,17,9)$ $(16,17,16)$	0	$\mu_{n.v}$	0	0	0	0	$\mu_{p.v}$	0
$(9,18,12)$ $(16,18,13)$	0	0	$\mu_{n.v}$	0	0	0	0	$\mu_{p.v}$
$(10,11,18)$ $(11,12,18)$	0	0	$\mu_{p.d}$	$\mu_{n.d}$	0	0	0	0
$(10,16,17)$ $(11,15,17)$	$\mu_{n.d}$	$\mu_{p.d}$	0	0	0	0	0	0
$(12,19,12)$ $(13,19,13)$	0	0	0	$\mu_{n.v}$	$\mu_{p.v}$	0	0	0
$(12,20,9)$ $(13,20,16)$	$\mu_{n.v}$	0	0	0	0	$\mu_{p.v}$	0	0
$(14,12,19)$ $(15,11,19)$	0	0	0	0	$\mu_{n.d}$	0	0	$\mu_{p.d}$
$(14,15,20)$ $(15,16,20)$	0	0	0	0	0	$\mu_{n.d}$	$\mu_{p.d}$	0
$(17,9,10)$ $(17,10,11)$	0	$\mu_{n.a}$	$\mu_{p.a}$	0	0	0	0	0
$(17,17,17)$	0	1	0	0	0	0	0	0
$(17,18,18)$	0	0	1	0	0	0	0	0
$(18,13,11)$ $(18,14,10)$	$\mu_{n.a}$	0	0	$\mu_{p.a}$	0	0	0	
$(18,19,18)$	0	0	0	1	0	0	0	0
$(18,20,17)$	1	0	0	0	0	0	0	0
$(19,13,14)$ $(19,14,15)$	0	0	0	0	$\mu_{p.a}$	$\mu_{n.a}$	0	0
$(19,19,19)$	0	0	0	0	1	0	0	0
$(19,20,20)$	0	0	0	0	0	1	0	0
$(20,9,15)$ $(20,10,14)$	0	0	0	0	0	0	$\mu_{n.a}$	$\mu_{p.a}$
$(20,17,20)$	0	0	0	0	0	0	1	0
$(20,18,19)$	0	0	0	0	0	0	0	1

where $\mu_1=\mu_{n.d}\mu_{n.v}\mu_{n.a}$, $\mu_2=\mu_{p.d}\mu_{n.v}\mu_{n.a}$, $\mu_3=\mu_{p.d}\mu_{n.v}\mu_{p.a}$, $\mu_4=\mu_{n.d}\mu_{n.v}\mu_{p.a}$, $\mu_5=\mu_{n.d}\mu_{p.v}\mu_{p.a}$, $\mu_6=\mu_{n.d}\mu_{p.v}\mu_{n.a}$, $\mu_7=\mu_{p.d}\mu_{p.v}\mu_{n.a}$, *and* $\mu_8=\mu_{p.d}\mu_{p.v}\mu_{p.a}$

TABLE II

Attributes of x

x	*Equation*(15) *with cells*	*Equation*(16) *with cells*	*Equation*(17) *with cells*
$d_N(k)$	$(10,11,18),(11,12,18)$, (14,15,20),(15,16,20)	$(14,12,19),(15,11,19)$	$(10,16,17),(11,15,17)$
$v_N(k)$	$(9,17,9),(16,17,16)$, (12,19,12),(13,19,13)	$(9,18,12),(16,18,13)$	$(12,20,9),(13,20,16)$
$a_N(k)$	$(17,9,10),(17,10,11)$, $(19,13,14),(19,14,15)$	$(20,9,15),(20,10,14)$	$(18,13,11),(18,14,10)$

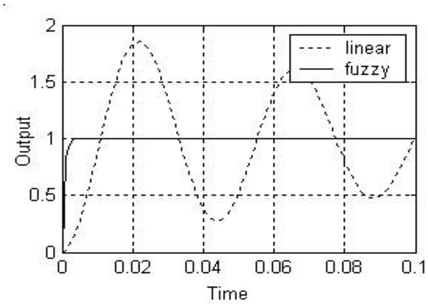

Fig. 7. Unit step response of closed loop system

which is in the form of Eq.(26) with

$$\gamma_1 = \left(\frac{2MN_d}{3N_{\Delta u}T^2}\right)\frac{A}{B} \tag{28}$$

where $A = \{7l^5 - l[\tilde{M}_a^2\tilde{M}_d^2 + l^2(\tilde{M}_a^2 + \tilde{M}_d^2)]\}T + \{7l^5 - l[\tilde{M}_v^2\tilde{M}_a^2 + l^2(\tilde{M}_v^2 + \tilde{M}_a^2)]\}T^2 + \{7l^5 - l[\tilde{M}_d^2\tilde{M}_v^2 + l^2(\tilde{M}_d^2 + \tilde{M}_v^2)]\}$
and $B = 15l^6 - [\tilde{M}_d^2\tilde{M}_v^2\tilde{M}_a^2 + (\tilde{M}_d^2\tilde{M}_v^2 + \tilde{M}_v^2\tilde{M}_a^2 + \tilde{M}_a^2\tilde{M}_d^2)l^2 + (\tilde{M}_d^2 + \tilde{M}_v^2 + \tilde{M}_a^2)l^4]$
Next, we have

$$\|y_2(k)\| = \|G_2(k)e_2(k)\| \quad \text{or} \quad \|\mathcal{N}e_2(k)\| \leq \|\mathcal{N}\| \ |e_2(k)|$$

which is in the form of Eq.(27) with

$$\gamma_2 = \|\mathcal{N}\| < \infty \tag{29}$$

So the sufficient condition for the nonlinear fuzzy PID control system to be BIBO stable is the parameters of the fuzzy PID controller must satisfy the inequality $\gamma_1\gamma_2 < 1$ where γ_1 and γ_2 are defined in Eqs. (28) and (29) respectively.

V. ILLUSTRATIVE EXAMPLES

Comparison of the performances of linear PID controller and the simplest fuzzy PID controller is done here by considering the following example:
A linear third order nonminimum phase system (Carvajal et. al., 2000)

$$G_p(s) = \frac{s^2 - s - 2}{s^3 + 3s^2 - 10s - 24} \tag{30}$$

with unit setpoint. In Eq.(30), $G_p(s)$ represents the transfer function of the plant to be controlled. For the above process, the values of sampling period T=0.001sec, proportional gain $K_P^d = 10.5T$, integral gain $K_I^d = 20000T$, derivative gain $K_D^d = 0.0005T$, absolute maximum displacement(error) $|d_{max}| = 1$, absolute maximum velocity $|v_{max}| = 131.343$, and absolute maximum acceleration $|a_{max}| = 19960.067$.

For the fuzzy PID controller, the parameters $N_d = 1800$, $N_v = 2.2$, $N_a = 0.41 \times 10^{-4}$, $N_{\Delta u} = 0.71$, and $l = M = 1900$ gave rise to the response in Figure 7, in which peak overshoot $M_p = 0.238\%$, rise time $t_r = 0.0019$ sec, and settling time $t_s = 0.0025$ sec. Figure 7 also shows the response with conventional PID controller, in which peak overshoot $M_p = 85.2357\%$, rise time $t_r = 0.009$ sec, and settling time $t_s = 0.4020$ sec. Upon comparison, it is evident from the plots that the fuzzy PID controller performs better, demonstrating its superiority over the conventional PID controller.

VI. CONCLUSIONS

In this paper, analytical structure for a fuzzy PID controller has been derived using L-type and Γ-type input fuzzy sets, triangular output fuzzy sets, algebraic product triangular norm, bounded sum triangular conorm, Mamdani minimum inference method and COA defuzzification method. BIBO stability results for fuzzy PID control have been given. The superiority of fuzzy PID controller over the linear PID controller has been demonstrated through a simulation study on a linear third order nonminimum phase system.

Since analytical structure of a fuzzy controller, in general, depends on the type of triangular norm, and "intersection" triangular norm has already been proved to be inappropriate for PID control(Mohan and Sinha), the important task that remains to be done is finding analytical structures for fuzzy PID controllers via other types of triangular norms, if possible, and studying their properties. It is believed that research in this direction is desirable to have a clear picture of fuzzy PID control systems.

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