PROCEEDINGS : CAMME-1996
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Advances in computer-aided crack length measurement during fatigue crack growth testing

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ABSTRACT

The accurate measurement of crack length is one of the most important aspect of fatigue crack growth rate (FCGR) testing. Of the various methods available for crack length measurement, compliance technique is very popular due to the facilities it provides for easy automation. In the compliance technique, compliance crack length (CCL) relations are used for correlating the compliance, computed from measurements of displacements and loads during fatigue cycling, to the crack length contained in the specimen. CCL relations are specific not only to the specimen geometry, but also to the location on the specimen body at which displacements are measured. This specificness is not very conducive to the experimentalist as it introduces errors in the measured crack length if the location of displacement measurement is not accurately maintained. With variations in specimen geometry and size, the accurate positioning of displacement measurement transducers is not an easy task. In order to provide greater flexibility in the use of the compliance technique, a new scheme has been proposed in this paper. Modelling the deformation of a fracture mechanics specimen during fatigue cycling as rotation of two rigid hinge about a hringe point, the relationship between the location of the hinge-point with crack length has been established using finite element analysis for the single-edge notched three point bend specimen. Further an iterative method has been developed which can be implemented in the background software for on-line crack length measurement. It has been shown that the iterative method converges rapidly to give the crack length with high accuracy.

INTRODUCTION

Fatigue crack growth rate (FCGR) tests are conducted in order to establish the resistance of a material to the propagation of cracks in it under fatigue loading. Such tests are necessary for the judicions selection of material for service under predominantly cyclic stresses and for the evaluation of the integrity, based on concepts of fracture mechanics, of structures and components that have experienced such stresses.

On of the most important aspect of the FCGR test is the accurate measurement of crack length as the test progresses. The accuracy in crack length monitoring determines the reliability of the FCGR data generated. Various methods are available for the measurement of crack length during FCGR testing. Of these, the compliance technique has proven to be the most popular because it lends itself to easy automation using a clip-on crack opening displacement (COD) gauge interfaced to a computer. The configuration of hardware and software necessary for such automated crack length measurement has therefore become industry-standard with most reputed manufacturers of servohydraulic systems used for FCGR testing.

In the compliance technique, displacements, V, at any location of the specimen body-usually at the crack mouth, at external knife-edges fixed over the crack mouth, or at the load point–is monitored continuously along with the load, P. From the (V,P) data obtained during a particular load cycle, the compliance V/P of the specimen is computed. The length of the crack contained in the specimen at the instant of the load cycle can be correlated to the compliance through a relation of the form

$$\frac{a}{W} = F(u)$$

$$u = \frac{1}{\sqrt{\frac{E'BV}{P}} + 1} \qquad \ldots 1$$

where W and B are the width and thickness of the specimen respectively, E' is the effective modulus and f is a polynomial function usually of the fifth order. Such relations are known as compliance crack length (CCL) relations. The form of u as given in eq. (1) was popularized by Saxena and Hudak[1] and has been used in CCL relation for a number of fracture mechanics specimen geometries. CCL relations are available for the single-edge notched three-point bend SE(B) specimen[2], the four-point bend 4PB specimen[3], the compact tension C(T) specimen[4], the arc tension A(T) specimen[2], the disc compact tension

DC(T) specimen[2] and the chord-supported arc bend A(B)–C specimen[5].

CCL relations are not only specific to specimen geometries, but, for a given specimen, to the location of measurement of displacements or compliance. For example, for the 4PB specimen, CCL relations are available not only for displacements measured at the crack mouth, but also for compliance measured at knife-edges of various sizes; and for each location, the polynomial f is distictly different[3]. Similarly, various unique location specific CCL relations are available for the C(T) specimen[4] and SE(B) specimen[6] as well.

To the experimentalist, the location specificness of CCL relations poses not an insignificant problem. For generation of reliable FCGR data, the experimentalist has to provide compliance measurement facilities as per a limited set of CCL relations available. In order to alleviate this restriction and to provide greater flexibility during FCGR testing, a scheme has been proposed in this paper by which it should be possible to obtain crack length from compliance measured at any location on the crack plane or its extension.

APPROACH

The gross deformations occurring in a specimen during fatigue loading are essentially elastic in nature. As elastic deformation is a conservative process, the displacements measured at various locations in a given specimen geometry can be related to one another. Hence it should be possible to measure displacements at a particular location, and calculate from these the displacements occurring at a reference location. Using the CCL relation for the reference location, the crack length in the specimen can then be obtained. It remains however to establish relations between displacements occurring at various locations, and to develop the CCL relation for the reference location.

Regarding the reference location to which measured displacements are translated, the crack mouth position can be considered. This has the advantage that for all specimen geometries, crack mouth CCL relations are readily available.

The elastic deformation of a cracked specimen, as during fatigue loading, can be visualized to be taking place due to the rotation of two rigid halves about a hing point. The situation is schematically portrayed in Fig. 1(a) for the SE(B) specimen. The displacements, there-

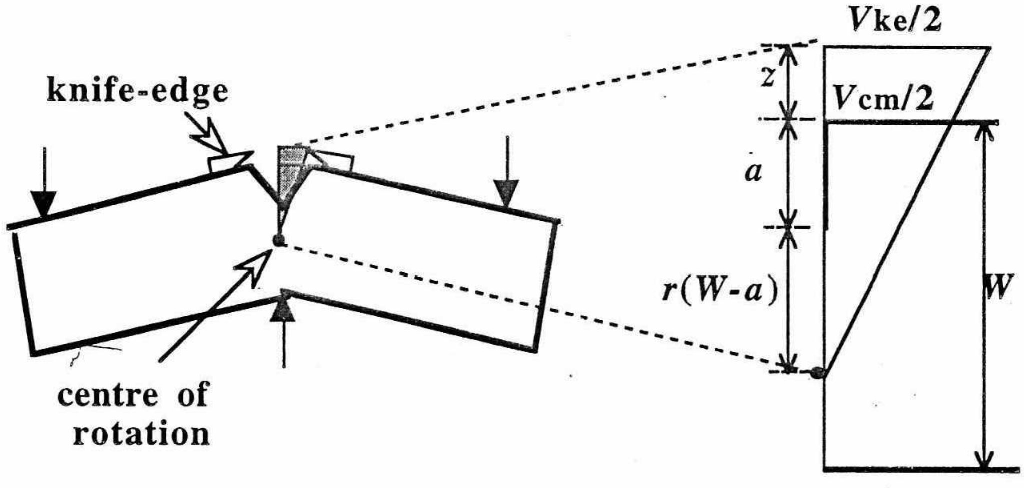

Fig. 1: Rigid body rotation of two specimen halves about a hinge-point and its geometric idealization.

fore, occurring at any location on the crack plane or its extension can be correlated using the method of similar triangles, as shown in Fig. 1(b). However, it is well known that the position of the virtual rotational hinge shifts as the crack grows[1]. Hence a direct solution for the crack length cannot be obtained.

In the present investigation, through finite element modelling of deformation of the SE(B) specimen, the variation in the position of the virtual rotational hinge has been established. Further, an iterative scheme has been proposed @, essentially based on the approach given above, by which it is possible to obtain the crack length in the SE(B) specimen geometry. It has been demonstrated that the iterative scheme converges to provide accurate measurement of the crack length. Details of the finite element modelling and analysis of and discussions on the results obtained are presented below.

FINITE ELEMENT MODELLING

The geometry of the SE(B) specimen considered for finite element analysis was as per the specification in ASTM standard[2]. Taking advantage of the symmetry of the geometry about the crack plane, a half model of the spacimen was meshed with iso-parametric triangular elements. The area surrounding the crack tip was meshed with quarter-point element (QPE). QPEs, introduced by Barsom[7] and Henshell and Shaw[8], are essentially six-noded triangular elements with thier midside nodes shifted to quarter-point positions. They are considered to be efficient in modelling the stress/strain singularity at the crack tip. The finite element mesh was optimized by progressively refining the mesh size until no significant change in the computed value of the stress intensity factor, K, was obtained (see Reference 5 for a discussion on obtaining K from FEM data). The K obtained from the optimized mesh was compared to that calculated from expression given in the ASTM standard[2] toconfirm the acceptibility of the mesh. In order to portray the situation in relatively thick fracture mechanics specimens, 2–D plane strain analyses were performed. Since displacements at locations remote from the crack tip are linearly proportional to the applied load during fatigue loading, assuming the absence of crack closure, it was adequate to perform clastic analyses. The commercial software ANSYS was used for implementation of the FEM. Fig. 2 shows at typical FEM mesh.

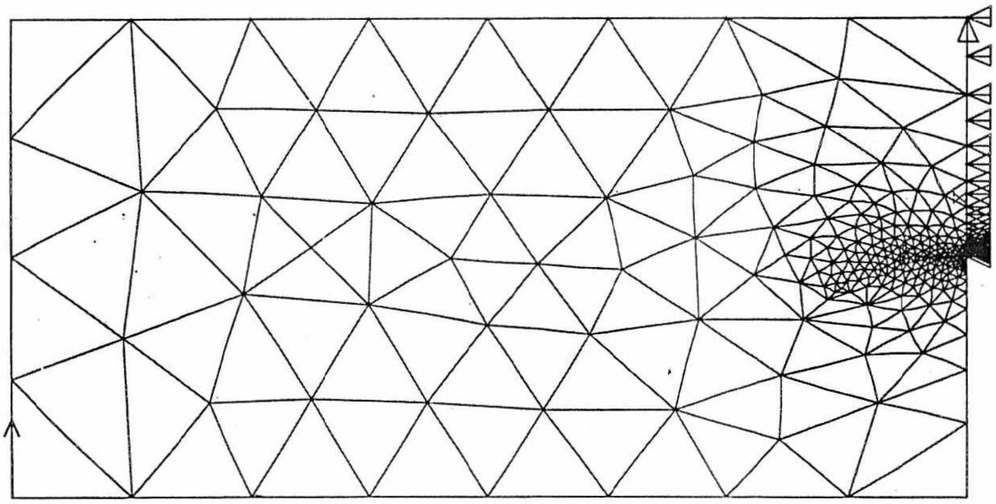

Fig. 2 : The FEM mesh for a half-model of the SE(B) specimen

For the SE(B) specimen geometry being considered, opening displacements were monitored at a number of locations for $0.2 \leq a/W \leq 0.8$, at intervals of 0.05. The displacement monitoring locations were characterized in terms of z/W, z being measured from the crack mouth away from the crack tip, along the crack plane, and being equal to the thickness of add-on knife-edges used. The z/W range considered included z/W = 0, 0.05, 0.075, 0.1, 0.125 and 0.15. It may be noted that z/W = 0 corresponds to the crack mouth location, while all the other values of z/W refer to external knife-edges. The add-on knife-edges were modelled in the finite element analysis by extra elements in the finite element meshes. For ease of obtaining the displacements, it was ensured that a node was available at each z/W position. As noted earlier, compliance, V/P, was calculated from displacements–load, (V,P), data obtained by FEM.

RESULTS AND DISCUSSIONS

The Routational-hinge Location

The position of the rotational-hinge (see Fig. 1) can be defined by the rotational factor, r, which is the fraction of the remaining ligament (W–a), ahead of the crack tip, at which it is located. If due to rigid rotation of specimen halves during fatigue loading, V_{ke} is the opening displacement at knife-edges of thickness z and V_{cm} is the corresponding displacement at the crack mouth, then from similar triangles, as shown in Fig. 1(b), it can be written

$$\frac{V_{ke}}{V_{cm}} = \frac{z + a + r\,(w-a)}{a + r\,(w-a)} \qquad \ldots\ 2$$

Eq. (2) can be rearranged and rationalized to give

$$\frac{\frac{z}{W}\frac{V_{cm}}{P} - \frac{a}{W}\left(\frac{V_{ke}}{P}\right) - \frac{V_{cm}}{P}}{\left(\frac{V_{ke}}{P} - \frac{V_{cm}}{P}\right)\left(1-\frac{a}{W}\right)} \qquad \ldots\ 3$$

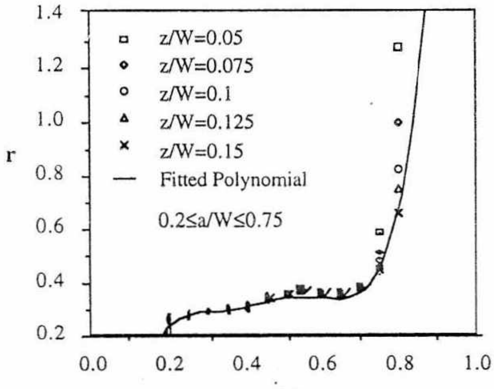

Fig. 3 : Variation of the rotational factor, r, with a/W, for various z/W

Using the compliance data, V_{cm}/P and V_{ke}/P, generated by FEM, r was calculated from eq. (3) for the entire range of a/W, for all the values of z/W being considered. It was observed, as expected, that r varied with a/W. Fig. 3 shows a plot of r against a/W. It can seen that similar variation is obtained for the various z/W cases. Hence, it is acceptable to represent this variation by an equation. A polynomial expression was found to be suitable for such representation, and accordingly by regression analysis the following relation was obtained :

$$r=-1.3141+20.227(a/W)-101.57(a/W)^2+249.30(a/w)^3$$
$$-295.0(a/W)^4+134.53(A/W)^5 \quad \ldots\ 4$$

It may be noted that in order to maintain a high fitting accuracy, only data points falling in the range $0.2 \leq a/W \leq 0.75$ were used to obtain eq. (4).

The Iterative Method for Crack Length Determination

As pointed out earlier, the dependence of r on a/W precluded a direct solution for crack length by translating compliance measured at a knife-edge location to a reference location like the crack mouth. In such a situation, it is common to use an iterative approach where initial values of mutually dependent variables are assumed, which are *refined* in successive iterations.

In the present case, the mutual dependence of the variables are given by:

(a) Eq.(2), which can be re-written as

$$V_{cm},P) = \frac{V_{ke}}{P}\left[\frac{\frac{a}{W}+r\left(1-\frac{a}{W}\right)}{\frac{z}{W}+\frac{a}{W}+r\left(1-\frac{a}{W}\right)}\right] \quad \ldots\ 5$$

(b) Eq. (1), which for the SE(B) specimen is given by the ASTM standard as[2]

$$(a/W) = 0.997-3.95u+2.982u^2-3.214u^3+51.52u^4-113.0u^5 \ \ldots\ 6$$

with u defined as $1/[(E'BV_{cm}/P)^{0.5}+1]$, as in eq. (1). and

(c) Eq. (4) which relates r to a/W.

The algorithm for the iterative scheme can be put down as follows :

i) Assume initial values of a/W and r. It has been found that taking a/W=0.4, the mid-point of the range of a/W considered, and r=0.32, corresponding to the a/W assumed, results in rapid convergence.

ii) Calculate the crack mouth compliance V_{cm}/P from the measured knife-edge compliance V_{ke}/P using eq. (5). It may be noted that z/W is known for any particular test. The current values of a/W and r are used.

iii) Calculated the crack length as a/W using eq.(6) and the computed crack mouth compliance V_{cm}/P

iv) Calculate the rotational factor r using eq.(4) with the current value of a/W.

v) Loop back to (ii) until satisfactory convengence, which can be specified by, say, less than $10^{-4}\%$ change in r

A flowsheet of the iterative scheme is given in Fig. 4.

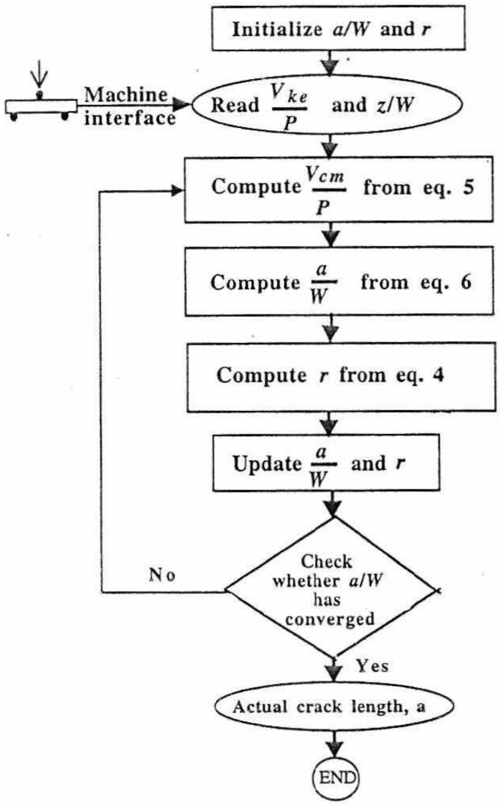

Fig. 4 ; Flowsheet of the iterative scheme

Using the knife-edge compliance data generated by FEM, the convengence characteristics of the iterative method was explored. It was observed that a/W converged to within 0.060 of the exact value in 10 iterations. It may be pointed out that in FCGR testing, measurement of crack length to within 1% is considered to be sufficiently accurate. Table-1 gives some sample results of the iterative scheme.

Table-1:Convergence characteristics of the iterative scheme for z/W=0.1

Iteration No.	a/W	r
1	0.40000	0.32000
2	0.21125	0.24547
3	0.20059	0.23462
4	0.19921	0.23303
5	0.19901	0.23280
6	0.19898	0.23276
7	0.19897	0.23276
8	0.19897	0.23276
9	0.19897	0.23276
10	0.19897	0.23276

Exact value of a/W = 0.2

Considering the simplicity of the computations involved and the few number of iterations that must be performed, the time required for the process is expected to be minimal. The iterative scheme was implemented in an existing FCGR testing software, running on a PC interfaced to a servohydraulic testing system, which used the compliance technique for crack length measurement. The performance of the modified software was found to be at per with the original software in which location specific CCL relations were used.

The iterative method for crack length measurement can be employed for other specimen geometries as well. It has been extended to the 4PB and the C(T) geometries also[9].

CONCLUSIONS

For the measurement of crack length during FCGR testing, and iterative method has been proposed which provides greater flexibility to the experimentalist. Using FEM, the deformation behaviour of the SE(B) specimen has been studied in order to develop a relation correlating the rotational factor r with the crack length. This relation along with the standard crack mouth CCL relation for the SE(B) specimen have been utilized in the iterative scheme to obtain accurate crack length in the specimen. The convergence characteristics of the iterative method has been studied and the iterative algorithm implemented in a FCGR testing software.

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