Prediction of dissolution time of gerromanganese in hot metal and steel bath

RASHMI KUMAR and SANJAY CHANDRA

The Tata Iron & Steel Company Limited, Jamshedpur - 831 001, India

ABSTRACT

Mathematical models enable us to study metallurgical processes in depth. Significant improvements in plant operation cannot, in general, be brought about without quantification of the impact of the operating parameters on the end product. It is for this reason that mathematical models have become popular in the steel industry, particularly with the advent of low-cost, high speed personal computers. Another reason why modelling is necessary is because increasingly alterations in operating parameters have to be made to bring about changes in heat or mass transfer. The gut-feelings of the plant operator are no longer able to deliver the goods because such feelings are developed by repeated exposure to quantifiable changes in the process brought about by alterations, deliberate or otherwise, in the operating parameters. Some of these basic parameters are not measured, for example heat transfer coefficient in case of a solid particle ·dissolving in a liquid bath, and therefore it would be futile to expect gut-feelings to offer solutions as to how dissolution time would be effected by a change in the type of ferroalloy or by change in its size. The determination of the dissolution time of ferroalloys is important as it indicates to the process control engineer the minimum time necessary before the bath can be subjected to the next step of processing. Changes in the bath super-heat, the size and the initial temperature of the ferroalloy and the bath hydrodynamics influence the dissolution time. Determining this dissolution time by experimentation is extremely difficult and would involve a series of experiments with different types of ferroalloys under varying conditions. On the other hand, the dissolution time can be obtained using a mathematical model of the dissolution kinetics of ferroalloys which after proper validation can be used to assess the dissolution behaviour of the ferroalloy under a variety of conditions. At TATA STEEL blast furnace hot metal is used to cast ingot molds. The com-

position of the blast furnace hot metal desired for making ingot moulds demands the addition of 20 kg each of ferromanganese and ferro-silicon in the 12 tonne transfer ladles into which the blast furnace metal is poured at the ingot mould foundry (IMF). Owing to the low temperature of hot metal at IMF, casting commences immediately and a complete dissolution of the ferroalloy is often not possible. A mathematical model for calculating dissolution time for a solid particle in a liquid metal has been developed at R&D Division of Tata Steel. The model has been developed with the feasibility of predicting dissolution times for cases where the ferroalloy has a melting point lower as well as higher than the bath temperature. The model has been validated against data published in literature and applied to predict dissolution times of ferroalloys in steel and blast furnace hot metal. This model was used to study dissolution of ferroalloys in foundry and to evolve suitable corrective measures. On the basis of this study additions are being optimized at the IMF in Tata Steel.

INTRODUCTION

Tata Steel has adopted a philosophy of steelmaking in which hot metal is desulphurised prior to its charging into the BOF vessel. Additionally, a low managanese hot metal is intentionally produced to minimise the detrimental impact of Mn on the phosphorus partition ratio during steelmaking. On the other hand, for adequate life of ingot moulds, which are cast in the captive foundry of the steel plant using blast furnace hot metal, high silicon and high manganese-to-sulphur ratio in the hot metal are necessary. To be able to meet these conflicting demands, one of the blast furnaces is burdened with manganese ore and a slightly higher ore to coke ratio so as to produce hot metal suited for casting of ingot mould. Nevertheless, in order to exactly meet the desired cast iron composition at the foundry, trimming additions of ferromanganese and ferrosilicon are necessary.

Addition of ferromanganese is also carried out in the BOF shop after the steel has been tapped from the LD vessel. The addition is done in the 130 tonne ladle into which the steel is transferred after heat making.

In both the cases discussed above, it becomes necessary to have a quantitative picture of the time taken for the ferroalloys to dissolve in the bath as a function of the superheat of the bath, the purging intensity employed and the size of the ferroalloy itself. Such a quantification is not possible without taking recourse to an elaborate mathemati-

cal model primarily on account of the fact that an external casing (or solidified shell) of the melt covers the ferroalloy as soon as it is immersed in the bath. Thereafter, the heat transfer to the steel (or cast iron) covered additive is no longer the large temperature difference between the molten bath and the cold ferroalloy. The driving force for heat transfer is considerably reduced to a small difference in temperature between the molten bath and the steel (or cast iron) casing around the additive.

Thus to quantify the effect of various process parameters on the dissolution period of ferroalloys, a mathematical model incorporating the initial formation of a solid shell, its growth and its subsequent melting has been developed at the R&D Division at Tata Steel. This paper details the development of the model, its validation and the benefits that are likely to accrue from the model's recommendations.

FERROALLOY DISSOLUTION MECHANISMS

It is well known that when a cold particle is immersed into a liquid metal at elevated temperatures a solid shell or crust is immediately formed around it. This shell continues to grow so long as the rate at which it transfers heat away from it is higher than the rate at which it receives heat from the bath. The growth stops and the melting of the solid crust begins when the rate of heat transfer from the melt by convection becomes greater than the rate of heat conduction through the crust.

There are various ways in which lumps of ferroalloys dissolve in a liquid melt. Two broad categories, depending on whether the melting range of the ferroalloy lies below or above the liquid metal bath temperature, have been identified by Guthrie and co-workers[1].

Class I Ferroalloy:

A typical class I ferroalloy has been defined as one whose melting point lies below the bath temperature of the melt. Absorption of such ferroalloys in the bath occurs via melting. This class includes ferromanganese, silicomanganese, ferrochrome and ferrosilicon addition to steel melts and tin, antimony, copper manganese, nickel and silicon additions to molten cast iron. During the assimilation of these ferroalloys into the bath, the most likely series of thermal events (four in number) are shown in an idealized form in Figure 1.
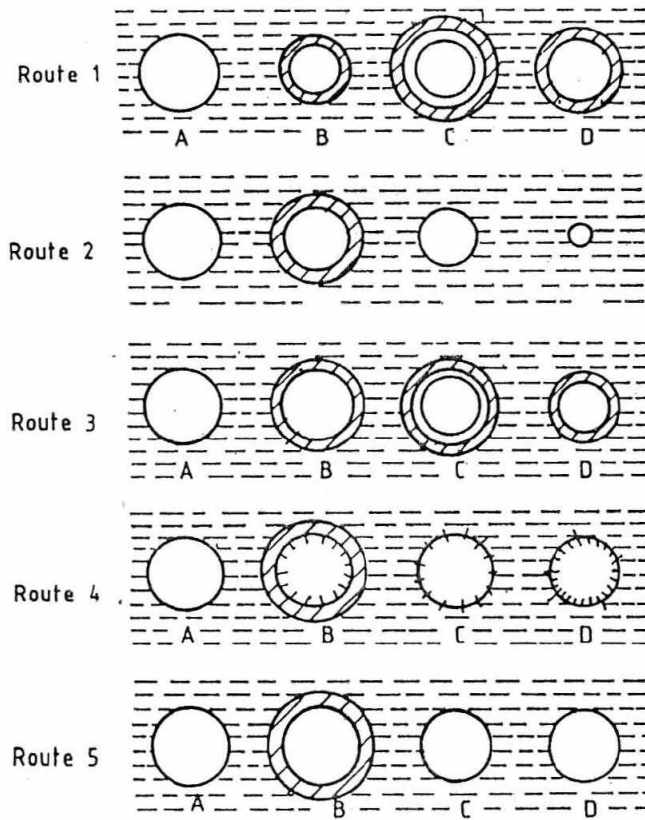
Fig. 1 : Five kinetic paths for alloy additions melting and/or dissolving in molten steel
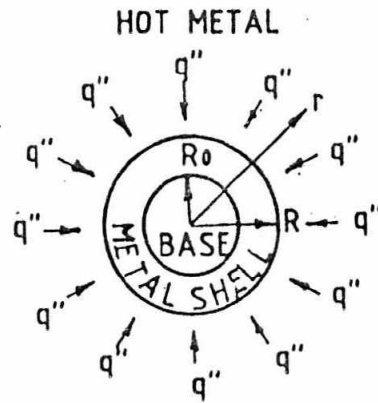

Fig. 2: Schematic representation of an addition with the solidified metal shell and hot metal (various interfaces and the coordinate system used in the mathematical model are also illustrated).

In route 1, as the cold addition (1A) is plunged into the metal bath, it gets covered by a shell of solidified metal (1B). The ferroalloy begins to melt within the steel shell, even as the steel shell continues to grow (1C). Very often, the solid added becomes totally molten before the shell melts back and releases the melted portion to the steel bath. The rate of shell melt back is governed by the convective heat transfer from the bath and thus, depends on the bath hydrodynamics (i.e., stirring).

Occasionally, alternative routes such as 2,3 or 4, may be followed depending on the specific thermal properties of the ferroalloy, size of the addition, and the composition and temperature of the bath. Conditions favouring route 2, in which no second metal shell is formed once the solid portion of the alloy gets exposed to the metal bath, include high superheat temperatures, larger lumps (~15 cm) and ferroalloys with low thermal conductivity (~4 W/m/K). If the ferroalloy is not fully molten by the time the first metal shell has melted back (3C), the exposed remainder of the lump generally follows route 3 and becomes covered once again with a second, smaller steel shell (3D). This type of phenomena is associated with large lumps of the additive and high superheat of the melts.

Route 4 depicts the case of ferroalloys having high exothermic heats of dissolution in liquid steel. The best known examples of this are the ferrosilicon alloys which, on being plunged into a steel bath (4A), exhibit an exothermic dissolution reaction triggered at the inner steel shell/solid ferrosilicon interface (4B). Route 4 is a possibility for both class I and II ferroalloys exhibiting exothermic dissolution behavior.

Class II Ferroalloy:

Class II ferroalloys are those whose melting points or melting ranges lie above the bath temperature of liquid melt. Typical examples include ferrovanadium, ferrotungsten and ferromolybdenum additions to a steel melt and chromium, molybdenum, vanadium and niobium additions to cast iron melts.

As is normal, a metal shell forms around such an addition following immersion (5B). In due course this melts back to expose the solid additive (5C). As the melting point of these ferroalloys lies above the bath temperature, a true dissolution process is thereafter involved. The diffusion of the solute material into the bulk of the melt by mass trans-
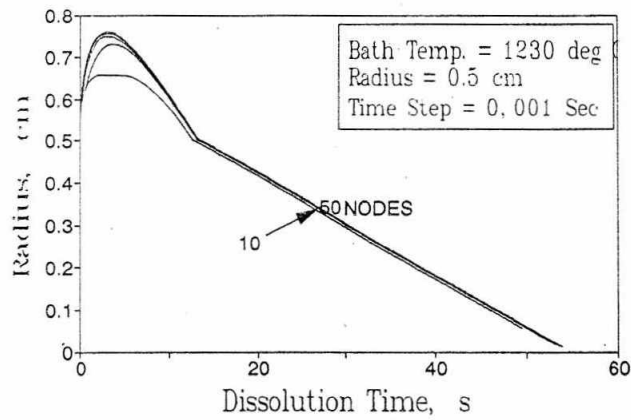
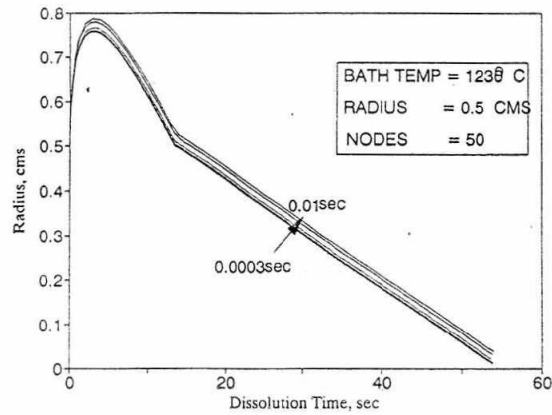*Fig. 3a : Node sensitivity analysis for dissolution of ferromanganese in hot metal*

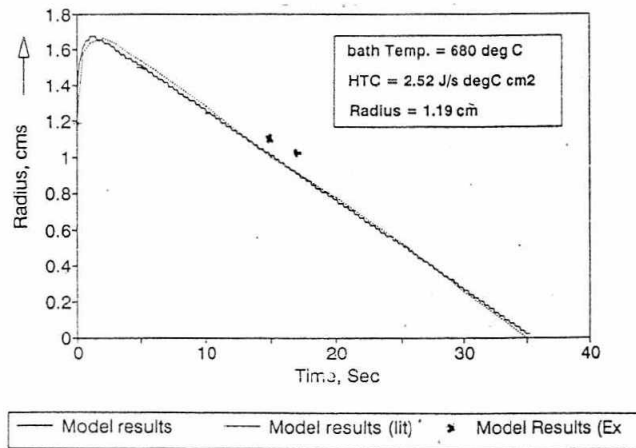*Fig. 3b : Time sensitivity analysis for dissolution of ferromanganese in hot metla*

Fig. 4 : change in size of aluminium spheres as a function of time

86

fer through the liquid phase boundary layer regulates the solution kinetics. Mass transfer kinetics being one order to magnitude slower than heat transfer kinetics for liquid metals systems, the time taken by the period (5C) is the rate controlling step. For class II ferroalloys therefore, one can consider a short "steel shell period", followed by a longer "free dissolution period".

MODEL FORMULATION

The heat transfer from a homogeneous liquid at a constant temperature, T_{bath}, to a solid sphere at an initial temperature, T_{sphere}, which is immersed in the liquid, can be expressed by Fourier's law. The temperature distribution inside the spherical shaped ferroalloy can be described by the heat condition equation expressed in spherical co-ordinates :

$$\frac{1}{r^2} \frac{\partial(k\, r^2\, \partial T/\partial r)}{\partial r} + \frac{1}{r^2 Sin\phi} \frac{\partial(K Sin\phi\, \partial T/\partial \phi)}{\partial \phi} + \frac{1}{r^2 Sin^2\theta} \frac{\partial(k \partial T/\partial \theta)}{\partial \theta} = \rho C_p \frac{\partial T}{\partial t}$$

$$\dots \quad 1$$

Since the heat flow is symmetrical with respect to the angular component (i.e., conditions independent of θ and ϕ) the above equation reduces to :

$$\frac{1}{r^2} \frac{\partial(k\, r^2\, \partial T/\partial r)}{\partial r} = \rho C_p \frac{\partial T}{\partial t} \qquad \dots \quad 2$$

Equation (2) was solved to determine the temperature distribution inside the solid additive assumed to be spherical and the solidified metal shell. Figure 2 is a schematic representation of the cross section through the center of the sphere. Equation (2) can be used to write appropriate expressions for transient heat conduction in the solid additive and the metal shell.

The expression to determine the temperature distribution inside the solid additive (region 1): $0<r<R_0$ and $0<t<t_{tot}$ is

$$\frac{1}{r^2} \frac{\partial(k_{base}\, r^2\, \partial T/\partial r)}{\partial r} = \rho_{base}\, C_{Pbase} \frac{\partial T}{\partial t} \qquad \dots \quad 3$$

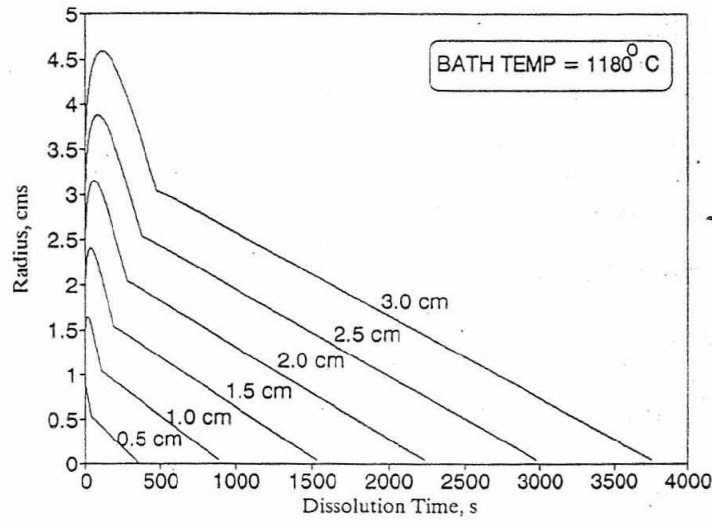The expression to determine the temperature distribution in the fro-

Fig. 5a Dissolution time of ferromanganese spheres of different sizes in hot metal at 1180°C.Graph shows solidification of shell and its subsequent melting
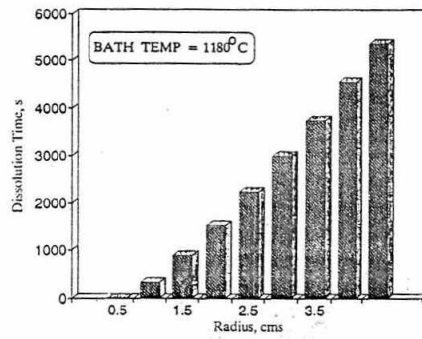

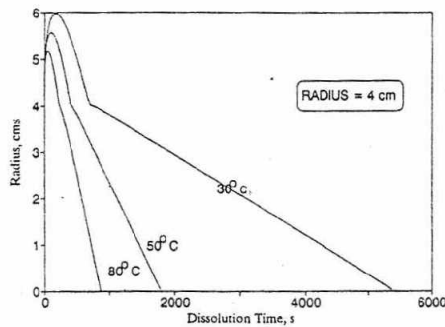*Fig. 5b Dissolution time of ferromanganese spheres of different sizes in hot metal at 1180°C.*

Fig. 6a : Effect of superheat on dissolution time of 2cm radius sphere of ferromanganese

88

zen crust or shell: $R_0 < r < R$ and $0 < t < t_b$ is

$$\frac{1}{r^2} \frac{\partial(k_{freez} r^2 \partial T/\partial r)}{\partial r} = \rho_{freez} C_{Pfreez} \frac{\partial T}{\partial t} \qquad \ldots 4$$

Initial conditions:

1. $t = 0$ and $0 < r < R_0$, $T = T_0$
2. $t = 0$ and $r > R$, $T = T_{bath}$

Boundary conditions :

1. $r = 0$ and $0 < t < t_{tot}$,

$$\frac{\partial T}{\partial r} = 0$$

2. $r = R_0$ and $0 < t < t_b$

$$k_{base} \frac{\partial T}{\partial r} \Big|_{base} = k_{freez} \frac{\partial T}{\partial r} \Big|_{freez}$$

3. $r = R$ and $0 < t < t_b$

$$T = T_{MPbath}$$

4. Boundary condition at the moving solidification front when,

$$0 < t < t_b$$

$$\left(k_{freez} \frac{\partial T}{\partial r} \right)_{freez,shell} = \rho_{freez} \lambda_{freez} \frac{\partial r}{\partial t} + h\,(T_{bath} - T_{M.P.shell})$$

$$t_b < t < t_{tot}$$

$$c(k_{base} \frac{\partial T}{\partial r})_{shell} = \rho_{freez} \lambda_{freez} \frac{\partial r}{\partial t} + h\,(T_{bath} - T_{M.P.shell})$$

The differential equations above were converted into finite difference equations using fully implicit scheme. The change in the outer shell thickness, ∂r, of the moving solidification front, can be calculated from the above equation. A positive value of ∂r indicates an increase in the thickness of the shell. When the shell thickness just exceeds the size of a node, a new node is added and the temperature of this new node is calculated by a linear interpolation of the adjoining
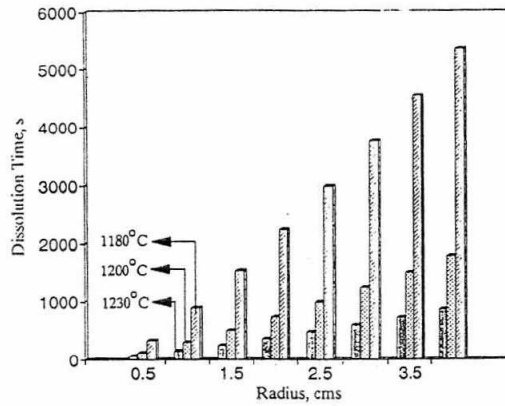
89

Fig.6b : Consolidated bar chart showing dissolution time of
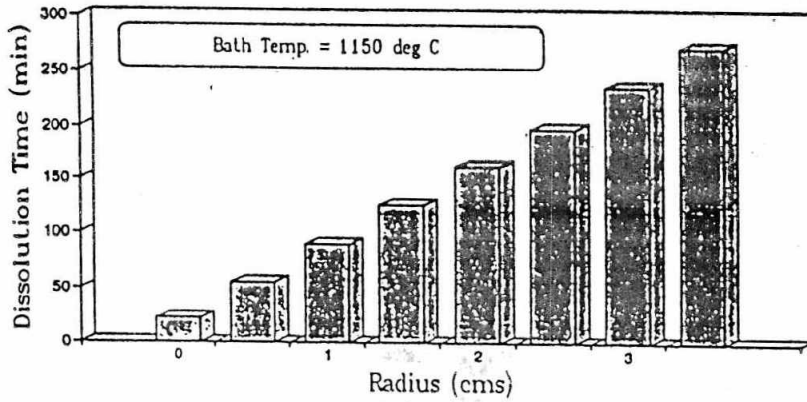different sizes of ferromanganese sphere at different bath temperature

Fig. 7a : Dissolution time of ferromanganese
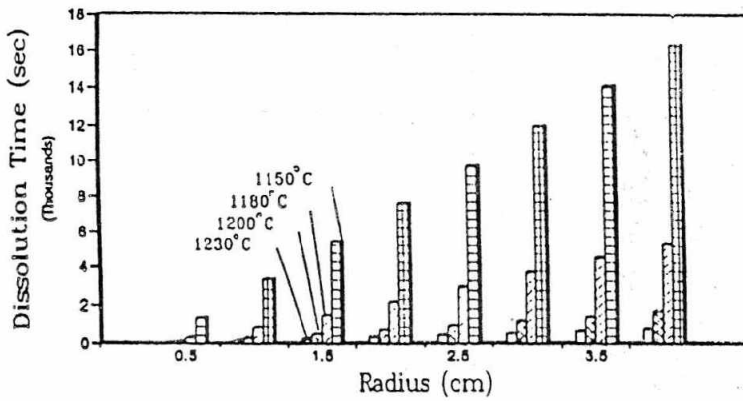spheres of different sizes in hot metal at 1150°C

Fig. 7b : Dissolution time of various sizes in of
ferromanganese spheres at different bath temperatures.

90

nodal temperatures. A negative value of ∂r indicates melting of the shell and a node is reduced when an appropriate amount of the shell melts. At any given moment the last node is a half node, but directly in contact with the bath. There is a region of the shell whose dimension is less than a node. This region is directly in contact with the bath and its temperature is equal to melting point of the freez (or base as the case may be). The temperature of the node immediately behind this shell changes as per equation (3) or (4).

In the event that the surface of the ferroalloy, enclosed within a steel casing reaches its melting point even as the solid shell around it has not fully melted, the temperature of the appropriate node is held constant at its melting point till the node has "accumulated" sufficient heat to melt.

VALIDATION OF THE MODEL

A computer program for the solution of the above equations was written in the FORTRAN language. Before the model could be used it was established that the formulation was mesh and time independent. For this purpose, a sensitivity analysis was carried out on the node size and the time step to ensure that the discretization in the space - time domain was appropriate and the results confirming this are shown in Figure 3(a) and 3(b).

The validity of the model was tested using the experimental data of Taniguchi et.al.[4] in which aluminum spheres of 2.38 cm diameter were immersed in a pure aluminum melt. The melt was not stirred and maintained at a fixed superheat during the test. Figure 4 shows the change in the size of the aluminium spheres as a function of time. The value of the heat transfer coefficient was determined by these researchers through an iterative process, to be 0.60 cal/cm²°C.

In order to check the validity of the model, it was tested under the experimental conditions adopted by Taniguchi *et.al.* An average value of heat transfer coefficient (0.6 cal/cm²°C) as obtained by the researchers was used in the present model to check if it predicts results close to experimental values obtained by Taniguchi *et.al.* A good match between the computed and experimental values as well as computation of the literature was obtained. Since our attempt is to match the predictions of the model developed independently by us with those obtained by the model used by Taniguchi; the heat transfer coefficient used in the present work was kept identical to that used by Taniguchi et.al.

It is intriguing to note that none of the many plots drawn by the investigators contain readings of the maximum thickness of the shell that solidifies on the addition and to that extent, the results from the present model could not be validated.

DISSOLUTION OF FERROMANGANESE IN CAST IRON

At the IMF in TATA STEEL, hot metal from blast furnaces is used for casting ingot moulds since the tonnage involved is large and a cupola of large size is not available. The compositions desired for casting ingot moulds demands the addition of 20 kg each of ferromanganese and ferrosilicon to every 12T transfer ladle of hot metal at the IMF. The low temperature of hot metal at the IMF forces casting to commence immediately and as a result, complete dissolution of the ferroalloys is not possible by the time the hot metal is poured into moulds. This leads to a lower silicon and a lower manganese to sulphur ratio in the ingot moulds which in turn, reduces the mould life. Sometimes, exogenous inclusions are also present in the ingot moulds owing to incomplete dissolution of the ferroalloys which also is detrimental to the quality of the moulds.

The hot metal is tapped at 1400-1450°C at the blast furnaces and the ladle is brought to the IMF. The track time is around 2 - 4 hours and it has been estimated that the temperature of the metal is around 1200°C by the time the ladle arrives at the IMF. Ferrosilicon and ferromanganese are kept in sacks at the bottom of the transfer ladle. Ferromanganese is added in the IMF in the form of chips. In carrying out the calculations, the shape of the ferroalloy was idealized to be spherical - an approach followed by other workers [2]. An equivalent surface area was taken for the spheres and its radius accurately computed. The properties of hot metal and ferromanganese are listed in Table 1.

Dissolution mechanisms of ferromanganese in hot metal is quite unique. Ferromanganese behaves differently at different bath temperatures i.e., if the hot metal bath temperature is above 1170°C (melting point of ferromanganese) then ferromanganese behaves as a class I ferroalloy. The ferroalloy melts in the hot metal bath and the process is heat transfer controlled. If the bath temperature is below 1170°C ferromanganese behaves as a class II ferroalloy and dissolves in the hot metal bath. The dissolution is then mass transfer controlled and is given by the following equation.

$N_A = \propto (C_A^* - C_A^B)$

$N_A =$ Dissolution molar flux of ferroalloy

$C_A^* =$ Interfacial concentration of solute

$C_A^B =$ bulk concentration of solute A

$\propto =$ mass transfer coefficient

As mass transfer kinetics is slower than heat transfer kinetics it takes a very long time for the ferroalloy to be assimilated in the hot metal bath at lower temperatures.

Figure 5(a & b) are plots for different initial sizes of ferromanganese sphere immersed in a stagnant bath of hot metal at 1180°C. It can be seen that the dissolution times increase with the size of the addition. It is to be emphasized that these dissolution times include the initial thermal period corresponding to the encasement of the alloy by a steel shell, together with the longer free dissolution period. Figure 6(a) shows the importance of bath superheat in minimizing the melting time. At zero superheat, the melting time would theoretically be infinite.

The combined effect of superheat of the bath and the size of the ferroalloy is indicated in Figure 6(b). Various inferences which can be drawn from this plot are

a) for a bath temperature of 1180°C, all sizes of additions show dissolution times in excess of 10 minutes i.e., a minimum of 10 minutes must be allowed for dissolution at this bath temperature.

b) as expected, the dissolution time is shorter at 1230°C than at 1180°C and the difference in dissolution time increases rapidly with increasing size of the ferroalloy.

In making these predictions, the heat transfer rates from the bath were estimated on the basis of natural convective heat transfer from a stagnant bath. The effect of change in the size of the sphere due to melting, on the heat transfer coefficient has been neglected. The correlation used to compute the heat transfer coefficients are given in Appendix I. In a stirred bath, it is expected that because of forced convection the average heat transfer rates would be higher.

When the bath temperature of hot metal is below 1170°C i.e. below the melting point of ferromanganese, the dissolution of the ferroalloy is controlled by mass transfer. Figure 7(a) shows the time taken for

ferromanganese to melt if the bath temperature is 1150°C. Here the "shell period" is very small compared with the "free dissolution period". Figure 7(b) is a consolidated bar chart showing the combined effects of the temperature of the bath and the size of the ferroalloy on the dissolution time. It can be clearly seen that the time taken for a ferroalloy to melt is considerably higher when the process is mass transfer controlled. Necessary correlation for the mass transfer coefficients used in the calculation are also given in Appendix I. In this case as well it is expected that in a stirred bath because of forced convection the average mass transfer rate would increase. However, as stirring the hot metal bath at the IMF is not a practical proposition (as it would exacerbate heat loss from the melt with its attendant problems) this option has not been modeled.

Tabel I : Physical, chemical and thermal properties of ferroalloys and hot metal

Properties	Hot Metal	Ferromanganese
Density (g/cm³ at 1200°C)	6.85	7.2
Heat Capacity (J/g°C)	0.67	0.7
Latent heat of fusion (J/g)	275.753	534.7
Freezing point (°C)	1150	1170
Thermal Conductivity (J/cm°C)	0.35	0.0753

Composition:

	Hot Metal	Ferromanganese
C	3.5%	6.4%
Mn	0.3%	79.5%
Si	0.8%	0.27%
S	0.06%	
P	0.3%	
Fe	94.5%	balance

DISSOLUTION OF FERROMANGANESE IN STEEL BATH

The modeling of the dissolution of ferromanganese in a steel bath is particularly demanding as the surface of the alloy begins to melt even when covered with a solid shell of steel. The mathematical model thus must allow for melting to take place not only from the surface of the additive (steel shell) but also from its interior (alloy). This is challenging task and the mathematical model was modified considerably to capture this "dual melting" phenomena[5].

The results of the computer simulation for ferromanganese in a steel bath at 1600°C is given in Figure 8 for three different sizes of the alloy. The plot appears different from the one depicting the dissolution of ferromanganese in cast iron bath (Figure 5a). This is on account of the fact that when the outer steel casing melts completely, it exposes a partially molten ferroalloy to the bath. The molten portion of the alloy joins the bath while the solid part melts in due course. The abrupt change in the slope of the graph depicts the sudden decrease in the size of the alloy concurrent with complete melting of the outer shell and the release of the molten portion of the alloy to the bath. At that instance the alloy, unlike the case of melting in cast iron, is not of the size at which it was originally added. Its radius is lower than the original size by an amount equal to the amount that was melted at the end of the steel shell melt-back period.
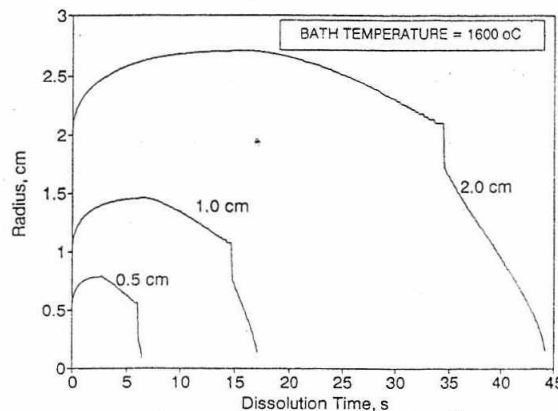

Fig.8 : Dissolution time of ferromanganese
spheres of different sizes in steel bath at 1600°C

RECOMMENDATIONS

Specific recommendations have been made for the addition of ferroalloys to cast iron bath. It is seen that if the addition can be carried out at a location enroute to the foundry shop when the bath temperature is above 1200°C, the time taken for dissolution can be considerably reduced. With improved manganese recovery ingot moulds are expected to have better life on account of a higher [Mn]/[S] ratio.

Thus it has been suggested to the operators that the addition of ferromanganese and ferrosilicon should be carried out at the blast furnace cast house itself. At a hot metal tempeerature of 1400-1450°C which is available at the furnace cast house, both ferroalloys would take 10-15 seconds only to dissolve.

CONCLUSION

A mathematical model for the dissolution of solid spheres of any ferroalloy in a liquid melt, incorporating heat transfer, mass transfer and dissolution kinetics has been developed. The model has been validated using the experimental data available in literature. The match has been found to be satisfactory. The model has been applied to the case of ferromanganese dissolution at the Ingot Mould Foundry and ferromanganese dissolution in Steel bath at TATA STEEL.

It is apparent from model predictions, that a 10mm sphere of ferromanganese takes 50 minutes to dissolve at the IMF which is in excess of the time available before casting. Thus for effective recovery, ferromanganese addition needs to be carried out at the blast furnace cast house itself as sufficient superheat and time for dissolution are both available.

NOMENCLATURE

r = variable radius (cm)

R = Maximum radius of the sphere after solidification (cm)

R_0 = initial radius of the solid sphere of the ferroalloy (cm)

K = thermal conductivity (J/cm/°C)

T = temperature (°C)

T_0 = initial temperature of the ferroalloy (°C)

t = time step (s)

C_p = specific heat (J/g/°V)

ρ = density (g/cm^3)

t_b = time for shell melt back (sec)

t_{tot} = time for complete dissolution of the sphere (s)

t_a = total time for freezing of shell (s)

A = area (cm^3)

V = volume (cm^3)

λ = latent heat (J/g)

T_{MPbath} = Melting point of bath °C

N.B : Subscript base and freeze to any symbol are for the additive and the shell respectively

96

REFERENCES

[1] S.A. Argyropoulos and R.I.L. Guthrie, "Dissolution kinetics of ferroalloys in steelmaking", *Steelmaking Proceedings,* Vol. 65, Pittsburgh, 1982.

[2] P.G. Sysmansis and S.A. Argyropoulos, "Mathematical modelling of exothermic dissolution", *Symposium Proceeding, Mathematical Modelling of Material Processing Operations,* Palm Springs, California, USA, 1987.

[3] Q. Jiao and N.J. Themelis, "Mathematical modelling of heat transfer during the melting of particle in a liquid slag or metal bath", *Canadian Metallurgical Quarterly,* Vol. 32, No. 1, 1993.

[4] S. Taniguchi, M. Ohmi and Shinji Ishiura, " A hot model study on the effect of gas injection upon the melting rate of solid sphere in a liquid bath", *Transaction ISIJ,* Vol. 23, 1983.

[5] Rashmi Kumar and S. Chandra, to be published.

[6] S.W. Churchill and H.H.S. Chu, "Correlating equations for laminar and turbulent free convection from a horizontal cylinder", *International Journal of Heat and Mass Transfer,* Vol. 18, 1975.

[7] G. Schutz, "Natural convection mass-transfer measurements on spheres and horizontal cylinders by an electrochemical method", *International Journal of Heat and Mass Transfer,* Vol. 6, 1963.

[8] R.L. Steinberger and R.E. Treybal, "Mass transfer from a solid soluble sphere to a flowing liquid stream", *Journal of A.I.Ch.E.,* Col. 6, No. 2, 1960.

APPENDIX I

The following heat and mass transfer correlation have been used in the model.

Calculation of the transfer coefficient

In order to estimate the amount of the heat transferred from the bath of molten steel to the interfaces of the ferroalloy spheres during the steel shell period, the relevant equation is

$$q = h\ (T_{bath} - T_{MPshell}) \qquad \ldots\ A1$$

where q = heat flux, T_{bath} = bath temperature, $T_{MPshell}$ = shell freezing temperature, h = convective heat transfer co-efficient.

For thermal natural convection to spheres, heat transfer coefficients were evaluated using the expression[5,6]:

97

$$N^{1/2} = 0.60 + 0.387 \left[\frac{Ra}{\{1+(0.559/Pr)^{9/16}\}^{16/9}} \right]^{1/6} \qquad \dots \text{A2}$$

$$N_{Fe} = \frac{hd}{K_{Fe}} \qquad \dots \text{A3}$$

$$Pr_{Fe} = \mu \frac{Cp}{K_{Fe}} \qquad \dots \text{A4}$$

$$\sqrt{Ra} = \frac{\rho^2 \beta g Cp \delta T d^3}{(\mu k)} \qquad \dots \text{A5}$$

Where N = Nusselt number, N_{fe} = Nusselt number for hot metal, Pr = Prandtl number, Ra = Rayleigh number, K_{fe} = thermal conductivity for hot metal, β = thermal diffusivity. Equation (A1) can be applied with equal precision to horozontal cylinders and spheres taking the characteristic length dimension as d, the diameter of the sphere or cylinder.

Calculation of the mass transfer coefficient

In order to estimate the amount of solute material diffusing into the bath of molten steel from the exposed interface of a dissolving Class II ferroalloy, the relevant equations is

$$N_A = \propto (C_A^* - C_A^B) \qquad \dots \text{A6}$$

where N_A = dissolution molar flux of ferroalloy, C_A^* = interfacial concentration of solute (i.e. solubility limit - liquidus concentration) and C_B^A = bulk concentration of solute A and α = mass transfer coefficient

For convective mass transfer of solute from the dissolving spheres to the bulk of the steel bath, the correlation which combines both natural and forced convective mass transfer effects, was adopted: It was assumed that any forced convection currents always augmented natural convection effects for the purpose of the present calculations[8].

$$Sh = 2 + 0.0254 (Gr_m \cdot Sc)^{0.33} Sc^{0.244} + 0.347 (Re\ Sc^{1/2})^{0.62} \qquad \dots \text{A7}$$

$$Sh = \frac{\propto d}{D_{A - Fe}}$$
$$Sc = \frac{\mu}{\rho D_{A - Fe}}$$
$$Re = \rho U_{relative} \frac{d}{\mu \cdot Gr_M}$$
$$Gr_M = \rho^2 \beta_M\ g\ \delta \times d^3/\mu^2$$