Propagation Characteristics of A_o Mode in a Metallic Beam Containing a Semi-infinite Crack

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ABSTRACT

When A_o Lamb wave propagating in a sub-beam encounters the edge of a semi-infinite horizontal crack in a metallic beam, reflection, and transmission into the main beam and sub-beam takes place. Lamb mode transmitted from one sub-beam to the other is termed as "Turning Lamb Mode" (TLM). The variations in reflection and transmission factors of A_o Lamb modes, defined based on Fourier Transform and Wavelet Transform, when a horizontal crack is situated at different positions across the thickness of a beam were studied through numerical simulations. In addition to this, the power reflection and transmission ratios were also estimated. It was observed that the transmission and reflection characteristics of A_o Lamb mode strongly depend on the thickness ratio. It was also observed that the nature of variations in transmission factor, and power transmission coefficient of TLM with thickness ratio is different.

Introduction

Lamb waves [1] can propagate long distance and interrogate the whole thickness of plate in which they propagate. Hence, Lamb waves have been explored for Non-destructive Evaluation (NDE) and Structural Health Monitoring (SHM) applications. One of the main difficulties associated with Lamb wave based NDE and SHM is interpretation and correlating the received signal with the defect. This is because of the multimodal and dispersive characteristics of Lamb waves.

Zhou and Yuan [2] carried out analytical studies on the reflection and transmission of flexural modes in an isotropic beam containing a semi-infinite axial crack. It was observed that the power of the reflection and transmission coefficients depend on both the frequency and the position of the crack across beam thickness. Finally the results were verified using conventional finite element method (FEM). Yuan et al. [3] carried out analytical studies on flexural wave reflection and transmission from main beam to sub-beams in uni-directional (UD) composite beams having a semi-infinite delamination of open and closed nature. It was found that the portion of reflected and transmitted power depends strongly on the frequency of incident wave and position of delamination across beam thickness. The results were verified using FEM. Wang and Rose [4] investigated wave propagation in isotropic beams containing closed semi-infinite delamination.

Ramadas *et al* [5] investigated new phenomenon of propagation of Lamb modes, named 'Turning Lamb Modes', in composite sub-laminates. The present work focuses on propagation characteristics of the fundamental anti-symmetric Lamb mode in metallic sub-beams. Variation of power reflection, transmission coefficients Lamb modes in sub-beams and main beam was computed and compared with reflection and transmission factors.

Numerically simulated transmission and reflection factors

Figure 1(a) shows specifications of the model used for simulations. Numerical modeling was carried on an aluminium beam having 200 mm length, 6 mm thickness, 100 mm semi-infinite crack and 14 mm width.

Table-1 lists the mechanical properties of aluminium material. A five cycle fundamental anti-symmetric mode (A_0) at 200 kHz central frequency was excited using appropriate displacement pattern across the thickness of sub-beam. Crack length was chosen in such a way that arrival time of reflection from beam and sub-beam edges were much higher than those from the crack edge. Since the crack extends from one of the edges to the center of beam, it is considered as 'semi-infinite crack'.

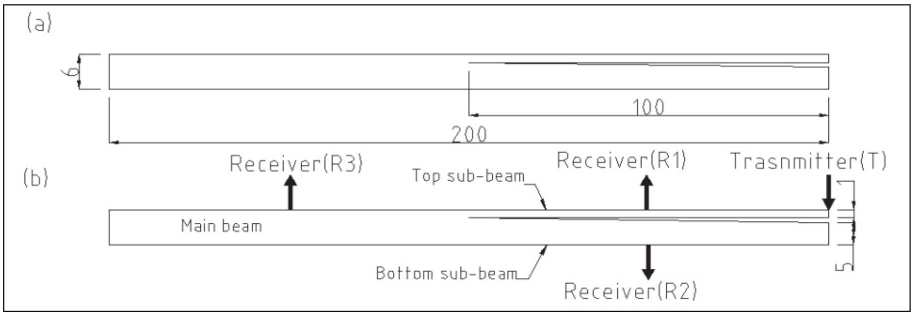

Fig. 1: (a) Model used for numerical simulations and (b) locations of the transmitter and receivers over the main and sub-beams.

At the location of crack, the beam is divided into two sub-beams, top sub-beam and bottom sub-beam. The thickness of each sub-beam depends on the location of crack across the thickness. Initially, the crack location was selected 1 mm below the top surface as shown in Figure 1(b). This resulted in thin (top) and thick (bottom) sub-beams having thicknesses 1 mm and 5 mm respectively. Thickness ratio is defined as the ratio of thickness of the top sub-beam on which the transmitter is mounted to the main beam thickness. The transmitter (T) was located over the thin beam and receivers R_1, R_2 and R_3 over the thin, thick sub-beams and main beam respectively, as shown in Fig 1(b).

Table 1: Material properties

Material	E in (GPa)	J	in kg/m3
Aluminium	70	0.30	2750

The phenomenon of propagation and interaction between the incident Ao Lamb mode and the edge of crack is shown in Figure 2(a). It was observed that when Ao mode is incident at the edge of delamination, reflection and transmission into the main beam takes place. Moreover, Lamb mode propagating in the top sub-beam takes 'U-turn' starts propagating in the bottom sub-laminate as shown in Figure 2. This mode of propagation is termed as 'Turning Lamb Mode' [5].

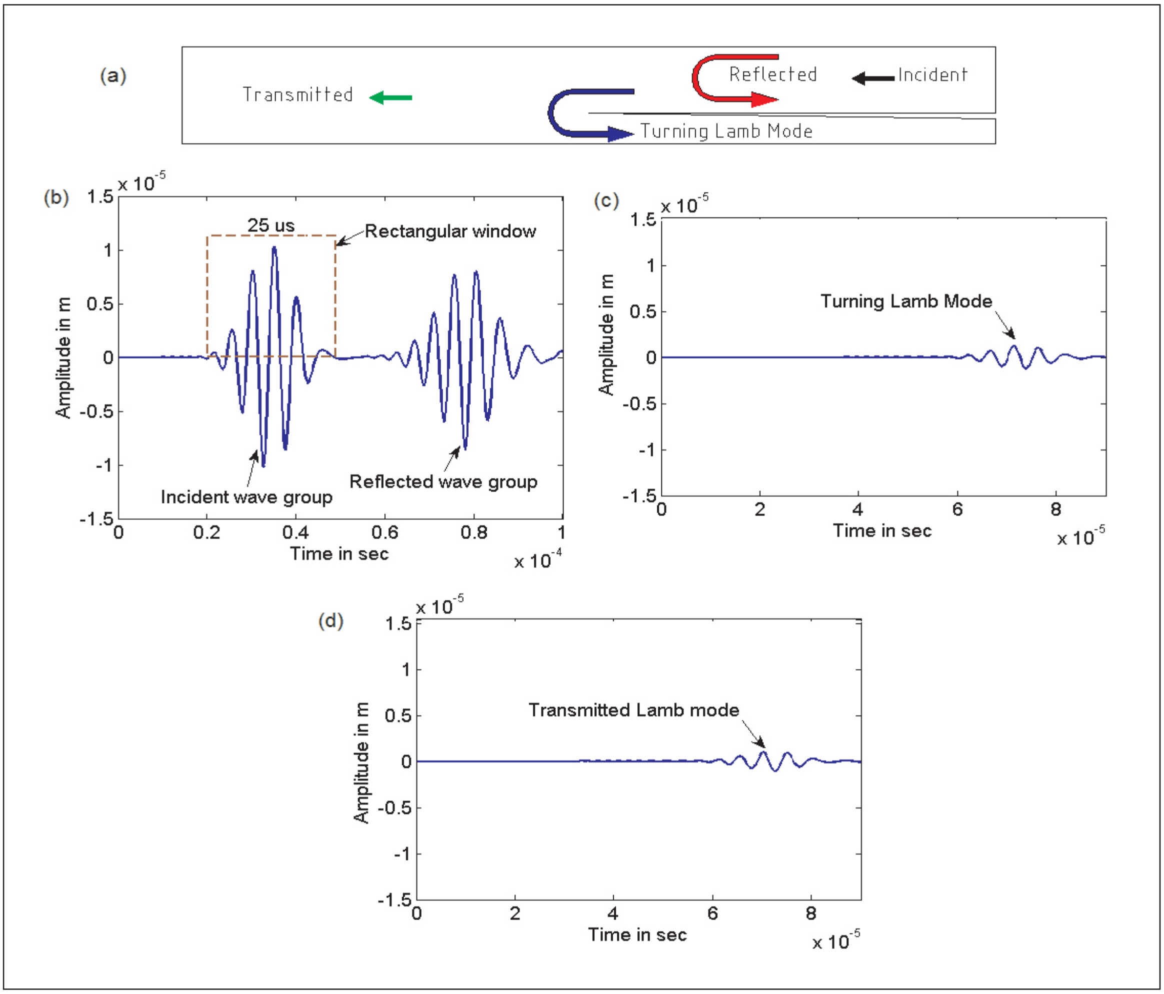

Fig. 2: (a) Interaction of Lamb mode with crack edge and propagation of 'Turning Lamb mode'. A-scans obtained at receivers (b) R$_1$, (c) R$_2$ and (d) R$_3$

Reflection in top sub-beams

Figure 2(b) shows A-scan captured at the receiver R1. In this A-scan, there are two A_0 wave groups. The first and second wave groups are the incident wave group at the crack edge and the reflection of the first wave group at the crack edge, respectively. Since the excitation pulse was for 25 µs of duration, Fourier Transform (FT) was carried out on a rectangular windowed signal of 25 µs duration as shown in Fig 2(b). 'Reflection factor' is estimated as ratio of amplitudes in frequency domain of the reflected wave group to the incident wave group.

Transmission from top sub-beam to bottom sub-beam

Receiver R_2, which was placed over the bottom sub-beam, captured the TLM. Fig 2(c) shows A-scan obtained at R_2. FT was carried out on rectangular windowed signal, which is shown in Fig 2(c). 'Transmission factor of TLM' is estimated as ratio of amplitudes in frequency domain of the transmitted TLM to the incident wave group.

Transmission from the top sub-beam to main beam

Figure 2(d) shows A-scan, captured by receiver R_3, of Lamb mode transmitted from top sub-beam to the main beam. Using FT technique, the signal in time domain was transformed to frequency domain. 'Transmission factor' of transmitted wave group into the main beam is defined as the ratio of amplitudes in frequency domain of the transmitted wave group into the main laminate to the incident wave group.

This completes one set of analyses for thickness ratio of 1/6. Now the crack location across the thickness is moved to 2 mm, measured from the top surface. This corresponds to thickness ratio of 2/6. Again similar numerical analysis, illustrated above, was carried out. This was continued for other thickness ratios 3/6, 4/6 and 5/6 as well. For each thickness ratio, reflection factor, transmission factor of TLM and transmission factor of wave group transmitted into the main beam were estimated.

Data analysis

Figure 3(a) shows the variation of the FT based reflection factor, transmission factor of TLM and transmission factor of wave group transmitted into the main beam with respect to the thickness ratio. With increase in thickness ratio, the reflection factor decreases and transmission factors increase as shown in Figure 3(a). In this work, wavelet coefficients of reflected and transmitted wave groups were computed using MATLABR Wavelet Tool Box [15]. Morlet was the mother wavelet selected for computing wavelet coefficients. WT was carried out on A-scans obtained at the receivers R_1, R_2 and R_3 for a given location of crack across the plate thickness. WT based reflection factor is estimated as the ratio of the wavelet coefficient of the reflected wave group to the incident wave group. Similarly, WT based transmission factors were also defined as the ratios of the coefficients of the respective transmitted wave groups to the incident wave group. Figure 3(b) shows the variation of WT based reflection and transmission factors. The trend in variation of factors based on FT and WT is almost same. In fact, the factors based on FT are almost equal to those counterparts based on WT for various thickness ratios.

Power reflection and transmission coefficients

Power reflection and transmission coefficients of the reflected and transmitted Lamb wave groups in the sub and main beams were estimated through numerical simulations. The following expression [8] gives the time averaged power flow, <P>, across any cross-section.

$$< P >= -\frac{1}{t_o}\int_{0}^{t_o}\int_{-\frac{h}{2}}^{+\frac{h}{2}}\left(\sigma_{xx}\,\dot{u}+\tau_{xz}\,\dot{w}\right)dzdt \qquad (1)$$

where, the stresses σ_{xx}, τ_{xz} and displacements u and w are functions of *x, y* and *t* (time). From equation (1) it is clear that the integration has to be performed across the beam thickness in the time interval, t_o. Since the duration of excitation was 25 µs, integration in equation (1) was carried out for this time interval. Power transmission coefficient is defined as ratio of power transmitted to the incident power. Similarly, power reflection coefficient is defined as ratio of power reflected to the incident power. Fig. 3(c) shows the variation of the reflection and transmission coefficients of Lamb modes in the sub-beams and main beam with respect to thickness ratio.

Results and discussion

Numerical simulations were carried out on aluminium beam containing semi-infinite crack revealed that reflection and transmission factors defined based on FT and WT were found to decrease and increase respectively with respect to thickness ratio as shown in Figs 3(a) and 3(b) respectively. It was observed that power reflection and transmission factors defined based on FT were almost equal to those defined based on WT for various thickness ratios.

Power associated with the incident, transmitted and reflected Lamb modes was calculated using equation (1). Fig 3(c) shows the variation of power reflection coefficient of Lamb mode reflected at the edge of delamination. Power reflection coefficient decreased with increase in thickness ratio. When thickness ratio was 1/6, the power reflection coefficient was around 0.724. With subsequent increase in thickness ratio from 1/6 to 5/6, power reflection coefficient decreased from 0.724 to 0.070 (approx). The reflected power, when thickness ratio was 5/6, was almost 1/10th of that power at 1/6 thickness ratio.

An interesting phenomenon was noticed in case of TLM. Power associated with the TLM was found to increase when the crack location across beam thickness was moved away from the top surface. When the crack was at 3 mm from the top surface, which corresponds to a thickness ratio of 0.5, the power transmission coefficient of TLM was high as shown in Fig 3(c). Power transmission coefficient was found to decrease when crack was moved further down. Initially, the power transmission coefficient of TLM increased from 0.096 to 0.304 when thickness ratio increased from 1/6 to 3/6, respectively. For thickness ratios 3/6 and 4/6, the power transmission coefficients were 0.304 and 0.299, respectively, which are nearly equal. When thickness ratio was further increased from 3/6 to 5/6, power transmission coefficients varied from 0.304 to 0.142, respectively as shown in Fig 3(c).

When A_o was incident at the edge of crack, it transmitted into the main laminate through the crack edge. Fig 3(c) shows power associated with the A_o Lamb mode transmitted into the main laminate. Power transmission coefficient of the A_o Lamb mode transmitted into the main laminate was found to increase with increase in thickness ratio. Power transmission coefficients vary from 0.07 to 0.766 for a variation of thickness ratio from 1/6 to 5/6 as shown in Fig 3(c).

Reflection factors, based on FT and WT, and power reflection coefficient based on power, of A_O Lamb modes exhibit a decreasing trend with increase in thickness ratio as shown in Figs 3(a) and 3(b), respectively. Transmission factor, based on FT and WT, and power transmission factor, based on power, of A_O TLM exhibit dissimilar behavior with increase in thickness ratio as shown in Figs 3(a) and 3(b), and 3(c) respectively. Transmission factor of Ao mode propagated into the main beam increases with increase in thickness ratio, whereas power transmission coefficient reaches a maximum value at thickness ratio 3/6, then starts decreasing with increase in thickness ratio as depicted in Fig 3(c).

At a given thickness ratio, the sum of power reflection coefficient and power transmission coefficients is not equal to unity because of the reason as follows. When A_o Lamb mode is incident at a crack edge, in addition to reflection and transmission, it also generates a new mode, S_o, which also propagates along with the reflected and transmitted A_o modes. Since there is some power associated with S_o mode as well, the total power carried by reflected and transmitted A_o modes is not equal to unity.

This cram has revealed the fact that albeit the trend in variation of reflection and transmission factors based on FT and WT is similar to power reflection and transmission coefficients, the variation of power transmission coefficients of A_o TLM is completely different from the variation of transmission factors based on FT and WT.

Conclusions

Detailed numerical simulations carried out on transmission and reflection characteristics of A_o Lamb modes in main and sub-beams revealed the fact that the variation of transmission coefficients of A_o TLM follow increase and decrease trend with increase in thickness ratio. Whereas the transmission factors of A_o TLM keep increasing with increase in thickness ratio.
In case of reflection coefficients and factors both keep decreasing with increase in thickness ratio. Transmission factors and coefficients of transmitted Lamb mode, A_o, into the main laminate also followed the increasing trend with increase in thickness ratio.

However, the transmission coefficients of A_o TLM exhibited a dissimilar trend when compared to the transmission factors of A_o TLM.

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