CORE
Metadata, citation and similar papers at core.ac.uk
Provided by eprints@NML

3rd International Conference on
Creep, Fatigue and Creep-Fatigue Interaction
September 24-26, 2008, Kalpakkam, India

A STOCHASTIC MODEL FOR EVOLUTION OF CREEP DAMAGE IN ENGINEERING MATERIAL

N. Roy[1], R.N. Ghosh[2] and S.C. Bose[2]
[1] Mathematical Modeling and Simulation Division
[2] Material Science and Technology Division
Burmamines, National Metallurgical Laboratory
Jamshedpur 831007, INDIA

Abstract

The scatter observed in creep deformation and failure data is of considerable technological importance because it greatly complicates the task of making accurate deformation and lifetime estimates for high temperature components. In this work a stochastic model for the creep damage evolution and associated scatter has been developed in terms of a discontinuous Markov process. The magnitude of damage has been described in the form of a probability distribution function whose evolution in time characterizes the nondeterministic nature of the damage accumulation process. The model is able to describe the state of damage along with the associated scatter at a given time at any stress level. The validity of the model has been established by comparing the predicted creep curves generated for a specific loading condition with those experimentally obtained for an austenitic stainless steel (Type 316, 18Cr 8 Ni 2Mo).

Keywords Creep; Damage; Stochastic Modeling; Markov Process;

1. Introduction

Creep tests conducted on engineering materials at a given stress and temperature exhibit considerable scatter in creep strain time data. This is often attributed to material variability. It is well known that modest variations in chemical composition, thermo-mechanical processing and heat treatment can lead to substantial variations in mechanical properties [1]. This variation or scatter is primarily responsible for the gap that exists between theoretical predictions of existing continuum damage model and experimental observations. Several studies employing the concept of probability have been developed to predict and characterize the variation in the evolution of creep damage [2]. Most of these are extensions of the existing deterministic models [3] with the assumption that the parameters are random variables. These are estimated from test data using statistical parameter estimation procedures.

The Weibull distribution function [4,5] frequently provides a reasonable fit to the probability distributions of life times (time to rupture) obtained from such tests. This explains to some extent the variability in life time. However the probability distribution of damage as it evolves during the test can not be reasonably represented by Weibull distribution in many cases. This is because it looks only at two limiting states, viz., the initial state when material is virgin and the final state when the material has failed (ruptured). This imposes severe limitations as it can not predict any intermediate states of damage accumulation.

Bogdanoff [6] and Ganeson [7] have developed an evolutionary probabilistic model for fatigue life time prediction. This gives the evolution of the probability distribution as a function of number of cycles.

This method has been suitably modified to develop a computationally simplified approach to represent creep damage evolution along with the associated scatter. The model developed has been applied to creep strain accumulation data on austenitic stainless steel (Type 316, 18Cr 8 Ni 2Mo) for different loading conditions.

2. Creep as a discrete Markov process:

Cavitation is one of the most common forms of creep damage. This has been correlated with creep stain accumulation by direct metallographic examination of power plant components exposed to creep. Figure1 shows a schematic representation of the nature and distribution of cavities during various stages of creep strain accumulation in a component. On the basis of this plot Neubauer and Wedel [8] proposed a method of monitoring creep damage in ageing power plant components by periodic metallographic examination. The process truly consists of both nucleation and growth, the former is probabilistic where as the latter is deterministic. A probabilistic mechanism is best represented as a discrete process whereas growth is continuous by nature.

With creep damage accumulation the distance between cavities keep decreasing and ultimately some of these may join together producing a micro-crack. This happens as and when a cavity nucleates in between two adjacent cavities. The process continues leading to the formation of macro-cracks. Ultimately the component fails as and when its load bearing cross section area is full of cavities/cracks. There are four distinct stages of cavitation. A virgin material passes through each of these before failure occurs. Let us represent these states by $\mathbf{S_1}$**,** $\mathbf{S_2}$**,** $\mathbf{S_3}$**,** $\mathbf{S_4}$**,** $\mathbf{S_5}$ and $\mathbf{S_6}$. Here $\mathbf{S_1}$ represents the virgin material and $\mathbf{S_6}$ denotes failure. Intermediate stages of damage are represented by $\mathbf{S_2}$ to $\mathbf{S_5}$. The time between the two consecutive states is the average inspection interval. Let us assume that damage accumulation starts from $\mathbf{S_1}$ and moves successively from one state to another. Here each move represents a step. In this case it denotes inspection interval. If the material is currently in state $\mathbf{S_i}$, it moves to state $\mathbf{S_j}$ with probability $\mathbf{p_{ij}}$ in the next step. This probability does not depend upon which state the material had been before its current state. This indeed is quite logical that it will be applicable for creep damage accumulation. The process of damage accumulation during creep depends only on its current state. Therefore it is possible to simulate this by a Markov chain process.

The transition probabilities ($\mathbf{p_{ij}}$) for moving from one state to another could be represented by the following transition matrix;

$$P = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{array} \begin{array}{c} \begin{array}{cccccc} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 \end{array} \\ \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} & p_{15} & p_{16} \\ p_{21} & p_{22} & p_{23} & p_{24} & p_{25} & p_{26} \\ p_{31} & p_{32} & p_{33} & p_{34} & p_{35} & p_{36} \\ p_{41} & p_{42} & p_{43} & p_{44} & p_{45} & p_{46} \\ p_{51} & p_{52} & p_{53} & p_{54} & p_{55} & p_{56} \\ p_{61} & p_{62} & p_{63} & p_{64} & p_{65} & p_{66} \end{bmatrix} \end{array} \qquad (1)$$

The elements of row 1 of the matrix represents probability of the material moving over to any of the states $\mathbf{S_1}$ to $\mathbf{S_6}$ if its current state is $\mathbf{S_1}$. $\mathbf{p_{ii}}$ is the probability of remaining in current state at time **t** and $\mathbf{p_{ii+1}}$ is the probability of shifting to the next state **i+1** at time **t+1**. These are the characteristic parameters determining the damage accumulation rate in this model. Let us assume for simplicity that the elements of the transition matrix does not change with time and incorporate some of the constraints imposed by mechanism of creep damage accumulation by nucleation of cavities. The process of cavitation is irreversible. It can not heal. This means if it is in state $\mathbf{S_i}$ it can not move to $\mathbf{S_j}$ where $\mathbf{j < i}$. This is represented mathematically as:

$$p_{ij} = 0 \quad if \quad j < i \tag{2}$$

Likewise even though the process of damage accumulation during creep is discrete it can not bypass a particular state. This means a virgin material ($\mathbf{S_1}$) before failure must pass through each of the successive steps from $\mathbf{S_1}$ to $\mathbf{S_6}$. This means if a material is in state $\mathbf{S_1}$ now it can either remain in its present state or move to state $\mathbf{S_2}$. Its probability of moving to state $\mathbf{S_3}$ and beyond is zero. This is mathematically expressed as:

$$p_{ij} = 0 \quad if \quad j > i+1 \tag{3}$$

If one imposes the conditions given in Eq.(2) or Eq.(3), many of the elements of **P** matrix will be zero. Therefore it is more convenient to represent this as in Eq.(4). Now onwards $\mathbf{p_{ii}}$ is referred as $\mathbf{p_i}$ and $\mathbf{p_{ii+1}}$ as $\mathbf{q_i}$.
With these constraints the transition matrix representing creep damage accumulation in a material becomes as follows:

$$P = \begin{array}{c} \\ S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{array} \begin{array}{c} \begin{array}{cccccc} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 \end{array} \\ \begin{bmatrix} p_1 & q_1 & 0 & 0 & 0 & 0 \\ 0 & p_2 & q_2 & 0 & 0 & 0 \\ 0 & 0 & p_3 & q_3 & 0 & 0 \\ 0 & 0 & 0 & p_4 & q_4 & 0 \\ 0 & 0 & 0 & 0 & p_5 & q_5 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \tag{4}$$

The Eq.(4) thus represents a transition matrix that could simulate accumulation of damage during creep. Here only four intermediate states of damage have been considered. Figure 2 gives a pictorial representation of the process. This can easily be extended by adding more number of intermediate states. This matrix can now be used to predict the state of the material after $\mathbf{n^{th}}$ step or during $\mathbf{n^{th}}$ inspection interval. Assuming that the elements in the matrix does not change with time, this is represented by:

$$\mathbf{P_n} = \mathbf{p_o P^n}, \qquad \mathbf{n} = 0,1,2,\ldots\ldots. \tag{5}$$

where $\mathbf{p_o}$ is a probability vector which represents the initial distribution of damage specified by the (**1 x b**) row vector

$$\mathbf{p_o} = \{ \pi_1 ,\ldots\ldots, \pi_b\} \text{ and } \sum_{j=1}^{b} \pi_j = 1 \tag{6}$$

For virgin material there is no damage. Therefore initial damage vector $\mathbf{p_o}$ should have components given by Eq. (7).

$$\mathbf{p_o} = \{ 1,0 ,\ldots\ldots, 0\} \tag{7}$$

In cases where elements of transition matrix changes with time this is given by

$$P_n = p_o \prod_{k}^{n} P_k \qquad (8)$$

Often in stead of representing the state of damage as features describing its characters it is possible to assign a numerical value to it. In damage mechanics it is customary to represent damage as the ratio of current creep strain to its rupture strength. Therefore $\mathbf{S_1 = 0}$ represents a virgin material where as $\mathbf{S_6 = 1}$ represents failure or fracture. Following the same analogy we could assign numerical values to each of the intermediate damaged states $\mathbf{S_2}$, $\mathbf{S_3}$, $\mathbf{S_4}$, and $\mathbf{S_5}$ as 0.2, 0.4, 0.6 and 0.8. Let the vector **V** represent a set of numeric values for the damage set **S**. Therefore at $\mathbf{n^{th}}$ time step it is possible to estimate the average magnitude of damage by the following expression.

$$Expected\ (Damage)^n = \sum_i P_i^n V_i \qquad (9)$$

The model developed here considers only six distinct stages of cavitation i.e., damage. To obtain a better accuracy in prediction, additional number of intermediate states can be incorporated. In that case, the damage is indexed by a finite set of states **1, 2, ..., *b***. State **1** means the initial state with no damage and state **b** means the final damage state, i.e. failure or replacement.

Estimation of the elements of the transition matrix is a major step in any attempt to simulate the evolution of creep damage at a given stress and temperature. Depending on the number of intermediate states to be considered and the magnitude of time to failure it can indeed be a computation intensive process. Bogdanoff [9] has suggested a simple procedure for estimating these elements to simulate fatigue damage evolution where as in this work it has been used to represent creep damage evolution.

To determine the probability transition matrix **P**, we introduce a random variable **W** in which $\mathbf{W_{1,b}}$ denotes the time to failure at state **b** given that the process starts in state **1**. The mean and the variance of $\mathbf{W_{1,b}}$ according to Bogdanoff [6] are

$$E\{W_{1,b}\} = (b-1) + \sum_{j=1}^{b-1} r_j \qquad (10)$$

$$Var\{W_{1,b}\} = \sum_{j=1}^{b-1} r_j (1 + r_j) \qquad (11)$$

where $r_j = \dfrac{p_j}{q_j}$

If r_j = **r**, a constant, the mean time to failure $\mathbf{W_m}$ and its standard deviation $\mathbf{S_m}$ can be expressed in terms of **r** and **b** as

$\mathbf{W_m}$ = (b-1)(1+r) (12)

$\mathbf{S^2_m}$ = (b-1)r(1+r) (13)

In the case of $\mathbf{r_j}$ is not equal to **r**, Eq.s(12,13) is not valid. Then dividing $\mathbf{W_m}$ into certain number of states, $\mathbf{W_1}$, $\mathbf{W_2}$,.. $\mathbf{W_m}$ with constant $\mathbf{r_j}$, mean time to reach a specific state and its standard deviation can be estimated by the following equations

$\mathbf{W_1}$ = $(b_1-1)(1+r_1)$

$\mathbf{S_1^2}$ = $(b_1-1)r_1(1+r_1)$ (14)

$\mathbf{W_2}$ = $(b_2-1)(1+r_2) + W_1$

$\mathbf{S_2^2}$ = $(b_2-1)r_2(1+r_2) + S_1^2$

Then from Eq.(14) ,

$$r_j = \frac{s_j^{\,2} - s_{j-1}^{\,2}}{W_j - W_{j-1}} \tag{15}$$

Using the definition of $r_j = \frac{p_j}{q_j}$ and Eq.(15), the probability transition matrix **P** is completely obtained.

3. Result & Discussion

The experimental results of Garofalo et al. [1] have been used to validate the proposed stochastic model. A number of replicated creep tests until rupture was performed on AISI type 316 stainless steel at several temperatures and stresses. These were constant load creep test data at 593 ^{o}C, 704 ^{o}C and 815 ^{o}C. Six repeat tests were conducted at each stress level at a given temperature. We have used these data to estimate the values of mean ($\mathbf{W_m}$) and standard deviation ($\mathbf{S_m}$) of time to reach a specified damage state. Using these, elements of the transition matrix were determined following the expression developed by Bogdanoff [6]. Table 1 gives estimates of mean time to reach a damage state and its standard deviation at 593 ^{o}C for two different loading conditions.

The transition probability matrix **P** for some of the test conditions (593 ^{o}C,218 MPa; 704 ^{o}C,66 MPa; 815 ^{o}C,49 MPa) are listed in Table 2. By substituting the values of **P** into the MC model, the probabilities of damage accumulation were determined at any instant of time for various test conditions. Figure 3 shows the probability distribution of damage accumulation at 593 ^{o}C, 218 MPa. This plot represent probability of the test sample being in a particular state of damage as a function of time. The states of damage (0, 0.2, 0.4, 0.6, 0.8, 1) are denoted by a set of lines having different symbols. For example, it is seen that at 200 hrs, the probabilities of the sample being in state 0, 0.2, 0.4, 0.8, 1 are 0, 0.416, 0.212, 0.044, 0.23 and 0.1 respectively. This implies that at **200** hrs, probability of remaining in state of damage **0.2** is maximum. Therefore this may be taken as the most likely state of damage in this test condition at **200** hrs. Similar kind of interpretation can be made from other test conditions.

The model has been used to simulate creep damage evolution as a function of time. Since the probability of reaching a specific state of damage at any instant of time is known, a random number generator can be used to estimate the time to reach a specific state. From this a set of normalized strain vs. time data is obtained. By repeating the method, several sets of predicted normalized strain vs. time data have been generated. The experimental and predicted sample data are shown in Fig.4(a-b). The predictions fall well within the scatter band of experimental data.

The number of intermediate stages of damage was arbitrarily selected. For example, in Table 1 we have used six levels of state of damage at which the model was made to fit mean and variance. It is possible to select any number of intermediate states. Higher number of intermediate states is likely to give smoother plots.

This model can also be used to predict creep damage evolution even under conditions not covered by test data. It is well known that stress rupture data are more readily available than creep strain time data. This procedure gives us a clue to generate the likely normalized creep strain time plots using these data. This will be of considerable help wherever creep strain based life prediction is required from more readily available stress rupture data.

In the present work the virgin material having no damage has been described in terms of a probability mass function represented by Eq.(7) where the first element is **1** and the rest of the elements are **0**. This may be valid for a virgin sample. It is also possible to have more than one non zero elements in this mass function. These elements can be so chosen to represent the initial state of damage of the test sample. Such a situation can arise if one is performing a creep test on samples drawn from service exposed high temperature components. The procedure suggested provides a method of assessing the state of damage in service exposed components by comparing the shape of its creep curve with those simulated by assigning a suitable initial probability mass function.

4. Conclusions

In the present study a stochastic model based on Markov process has been developed to describe creep damage evolution. The damage state at any time can be estimated if the initial state and the probability transition matrix are known. The model is able to describe the state of damage along with the associated scatter at a given time at any stress level. This can be used for creep strain based life prediction even if only stress rupture data are available.

References

[1] Garofalo F, Whitmore RW, Domis WF and Gemmingen FV. Creep and creep-rupture relationships in an austenitic stainless steel. Transaction of Metallurgical Society 1961; AIME (221): 310-319.

[2] Harlow DG and Delph TJ. A computational probabilistic model for creep damaging solids. Computers & Structures 1995; 54 (1): 161-166.

[3] Kachanov LM. Time to failure under creep conditions. Izv. ANSSSR 1958;OTN (8): 26-31 .

[4] Johnson RA. Probability and Statistics for Engineers. Pearson Education Asia, 2001.

[5] Weibull W. A statistical Distribution function of wide applicability. ASME J. Applied Mechanics 1951; 18 : 293-297.

[6] Bogdanoff JL. A new Cumulative Damage Model- Part 3. ASME J. Applied Mechanics 1978; 45 : 733-739.

[7] Ganesan R. A stochastic modeling and analysis methodology for quantification of fatigue damage. Computer methods in Applied Mechanics & Engineering 2000; 190 1005-1019.

[8] Neubauer B and Wedel U. Rest life estimation of creeping components by means of replicas. In: Woodford DA, Whitehead JR, editors. Advances in Life Prediction Methods. New York :American Society of Mechanical Engineers,1983, 307-314.

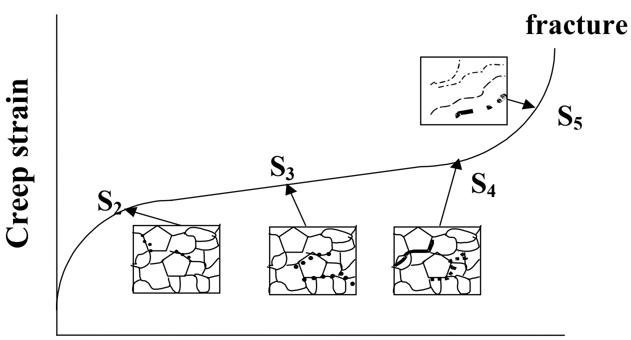
[9] Bogdanoff JL and Kozin F. A new Cumulative Damage Model- Part 4. ASME J. Applied Mechanics 1980; 47 : 40-44.

Table 1
Mean and Standard deviation of time to reach a specified state of damage at **593 °C.**

σ, MPa	State of damage	W_m	S_m
218	**0.0**	0	0
	0.2	5.440509	6.621964
	0.4	174.8096	191.2744
	0.6	303.9871	219.5607
	0.8	389.8558	222.9766
	1.0	439.1667	241.8871
287	**0.0**	0	0
	0.2	0.047873	0.033058
	0.4	0.486788	0.366294
	0.6	5.095581	5.696591
	0.8	11.1339	5.097553
	1	13.3	5.574226

Table 2
Transition Probability Matrix for various test conditions (listed only for $\mathbf{p_j,q_j}$)

Probability	**T : 593, °C σ : 218 MPa**		**T : 704, °C σ : 66 MPa**		**T : 815, °C σ : 49 MPa**	
p_1,q_1	0.889	0.111	0.99	0.01	0.368	0.632
p_2,q_2	0.995	0.005	0.982	0.018	0.619	0.381
p_3,q_3	0.989	0.011	0.992	0.008	0.653	0.347
p_4,q_4	0.946	0.054	0.994	0.006	0.682	0.318
p_5,q_5	0.994	0.006	0.994	0.006	0.704	0.296
p_6,q_6	0.889	0.111	0.99	0.01	0.368	0.632



Damage State	**Description**
S_1	Virgin
S_2	Isolated cavities
S_3	Aligned cavities
S_4	Linked cavities
S_5	Macro crack
S_6	Fracture

Fig. 1 A schematic representation of the nature of damage evolution in material subjected to creep loading and the phases used to represent the same.

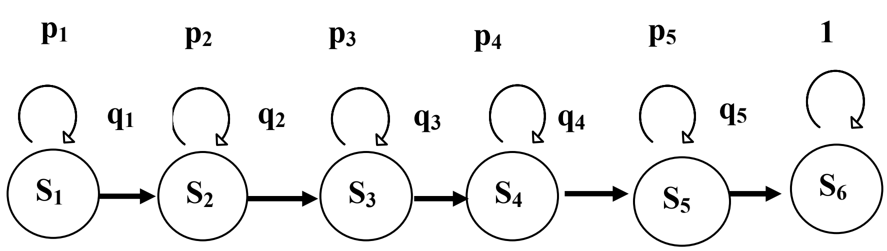


Fig. 2 A pictorial representation of Markov chain.

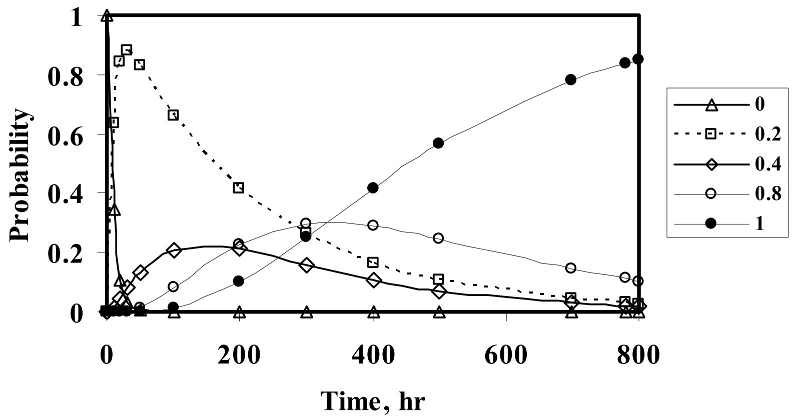


Fig. 3 Probability distribution of time to reach a specific damage state (0,0.2, 0.4, 0.6, 0.8, 1) at 593 ^{o}C, 218 Mpa.

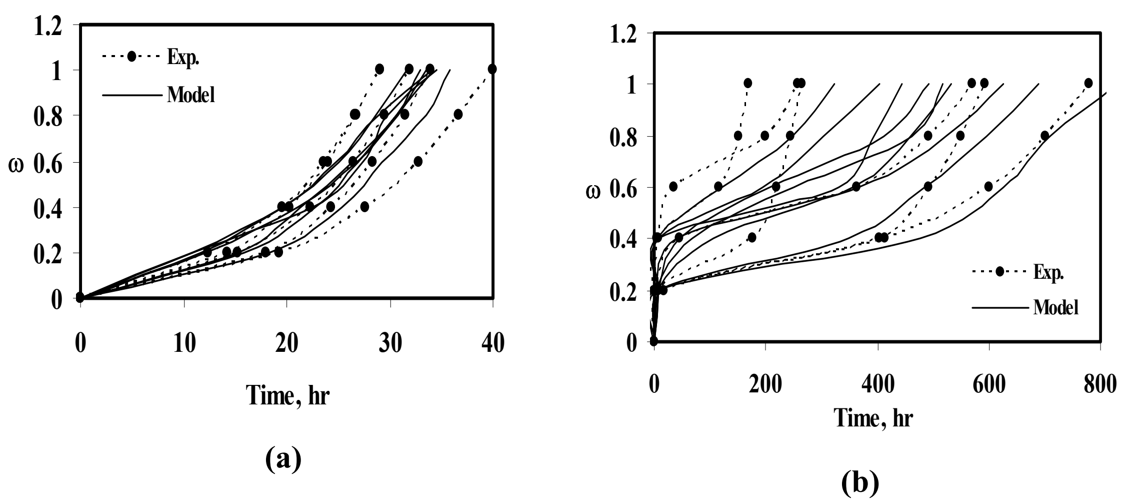


Fig. 4 Sample curves generated by model at **(a)** 704 ^{o}C ,91 Mpa and **(b)** 593 ^{o}C ,218 Mpa.